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"When Does Optimization Become Incoherent?
Irreversibility, Non-Compensability, and Feasible Choice"

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When Does Optimization Become Incoherent?

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Abstract

Optimization is the central organizing principle of economic analysis. Individual choice, social evaluation, and policy design are routinely formulated as the maximization of an objective function over a feasible set. This paper identifies a class of environments in which optimization itself ceases to be a coherent principle of evaluation. We study decision problems in which the domain of admissible actions includes losses that are irreversible, non-substitutable, and non-compensable. We show that, under minimal regularity conditions, no refinement of the objective function—such as state dependence, option values, or intertemporal trade-offs—can generally sustain coherent maximization on an unrestricted feasible set. In such environments, optimization necessarily leads to inconsistency: actions generating non-compensable losses may be selected as optimal whenever short-run gains dominate, regardless of how the evaluation criterion is specified. The main result is an impossibility theorem establishing that coherence failure is structural and does not stem from informational limitations, computational constraints, or ethical disagreement. We then provide a necessity result showing that coherence can be restored if and only if the feasible set is restricted so as to exclude actions that generate non-compensable losses. On the resulting restricted domain, standard optimization methods apply without contradiction. The analysis reframes irreversibility as a problem of feasibility design rather than objective-function design. It clarifies the limits of optimization-based evaluation and characterizes the minimal conditions under which optimization remains a valid principle of choice. This paper also serves as a foundational contribution to a broader research agenda on the structural limits of evaluation. By isolating the conditions under which optimization-based evaluation becomes incoherent, the analysis provides a unifying framework for understanding feasibility-based constraints across diverse economic domains. The analysis is intended to serve as a conceptual reference point for further work on feasibility, irreversibility, and evaluation, rather than to exhaust their possible applications.

Keywords: Optimization, Non-compensability, Feasible sets, Irreversibility, Impossibility theorem

JEL codes: D01, D81, D90

1. Introduction

Optimization is the central organizing principle of economics—yet this paper highlights that there exist environments in which optimization itself ceases to be a coherent mode of evaluation.

This paper identifies a class of environments in which optimization itself ceases to be a coherent principle of evaluation. The issue does not arise from computational complexity, imperfect information, or ethical disagreement. Rather, it emerges when the domain of choice includes losses that are irreversible, non-substitutable, and non-compensable. In such environments, no refinement of the objective function—however sophisticated—can generally restore coherence to optimization-based evaluation unless the feasible set is restricted in a particular way.

A standard response in economics to irreversibility or complexity is to enrich the objective function. Option values, state-dependent utilities, and intertemporal trade-offs are introduced so that all relevant considerations are reflected in a single scalar criterion. This approach presumes that any loss can, at least in principle, be offset by sufficiently large gains along other dimensions. The present paper shows that this presumption fails once certain losses are non-compensable: that is, once they cannot be traded off against gains elsewhere without violating basic coherence requirements of evaluation.

The central result is an impossibility theorem. Under minimal regularity conditions, if the domain of admissible actions includes irreversible and non-compensable losses, then optimization over an unrestricted feasible set necessarily leads to inconsistency. In particular, there exist environments in which any evaluation functional that permits trade-offs will rank a violating action as optimal, even though the loss it entails cannot be meaningfully compensated. This result holds regardless of how the objective function is specified or refined.

Importantly, the paper's conclusion is not that optimization should be abandoned. Rather, it characterizes the conditions under which optimization remains viable. The analysis shows that coherence can be restored if and only if the feasible set is restricted so as to exclude actions that generate non-compensable losses. On the restricted domain, standard optimization techniques apply without contradiction. In this sense, the paper reframes the problem of irreversibility from one of objective-function design to one of feasibility design.

This shift has broad implications. Many economically relevant objects—environmental stocks, institutional integrity, and certain rights or commitments—are routinely treated as if they were commensurable with standard welfare measures. The present analysis demonstrates that, when such objects are non-compensable, treating them as trade-offable quantities undermines the logical foundations of evaluation itself. The need for inviolable feasibility constraints is therefore not a normative add-on, but a logical requirement for maintaining coherence.

The paper contributes to several literatures. First, it complements work on irreversibility and option value by showing that objective-function refinements are insufficient in the presence of non-compensable losses. Second, it provides a formal foundation for threshold-based approaches—such as safe-minimum standards and critical-capital doctrines—by deriving them as necessary conditions for coherent optimization. Third, it connects to social choice theory by identifying a structural role for constraints that operate prior to ranking, rather than as arguments of the ranking itself.

More broadly, the paper addresses a foundational question: When does optimization cease to be a coherent principle of choice? By answering this question, it clarifies the limits of a core methodological commitment of economics and identifies the minimal adjustments required to preserve its applicability.

The present paper serves as a foundational contribution to a broader research agenda on the structural limits of evaluation. While subsequent work examines specific manifestations of evaluation failure in dynamic social choice, fiscal sustainability, organizational decision-making, and statistical inference, the analysis here isolates the core logical mechanism through which optimization-based evaluation itself becomes incoherent. By identifying the structural conditions under which evaluation fails, the paper provides a conceptual anchor for feasibility-based approaches across diverse economic settings.

Contribution Paragraph

The contribution of this paper is not to reject optimization, but to characterize the minimal conditions under which optimization remains coherent. The impossibility result establishes that, in the presence of irreversible and non-compensable losses, no objective-function refinement can generally sustain coherent maximization on an unrestricted domain. The accompanying necessity result shows that coherence is restored once the feasible set is appropriately restricted. Together, these results reframe irreversibility as a problem of feasibility design rather than objective-function design, thereby providing a constructive foundation for optimization-based analysis in environments with non-compensable losses.

Examples (Scope Illustration; not motivations)

The following examples are not offered as motivations, but as instances illustrating the scope of the result.

Example 1: Environmental Irreversibility

Consider a policy choice affecting an environmental stock whose degradation is irreversible and non-substitutable. Standard approaches incorporate irreversibility by adding option values or state-contingent utilities to the objective function. The present result shows that, if environmental loss is non-compensable, such refinements are insufficient: for any evaluation that allows trade-offs, there exist circumstances in which short-run gains dominate and irreversible degradation is selected as optimal. Coherent optimization therefore requires excluding such actions from the feasible set, providing a formal foundation for threshold-based environmental constraints.

Example 2: Institutional Integrity

Many economic systems rely on institutional features—such as legal credibility or enforcement capacity—that, once undermined, cannot be readily restored. If institutional breakdown is treated as a compensable loss, optimization-based evaluation may rationalize actions that yield immediate benefits at the cost of long-run institutional collapse. The analysis implies that, when institutional integrity is non-compensable, maintaining coherent evaluation necessitates feasibility restrictions that rule out actions threatening irreversible institutional damage.

Example 3: Inviolable Constraints in Social Choice

In social evaluation, certain actions may be regarded as impermissible regardless of aggregate gains. Rather than interpreting such constraints as external ethical impositions, the present framework shows that they can arise endogenously from coherence requirements. When violations entail non-compensable losses, permitting them in the choice set leads to inconsistency in ranking. Excluding them as infeasible actions restores coherent optimization, aligning rights-like constraints with the logic of evaluation rather than with ad hoc normative assumptions.

2. Model

This section introduces a formal framework designed to make the tension identified above precise. The framework is not intended to represent the only possible approach to evaluation, but rather to provide a tractable setting in which the structural difficulty associated with irreversibility and non-compensability can be examined formally.

We consider a decision maker (or a social evaluator) choosing among a set of actions that generate outcomes over time. Outcomes are evaluated using a general criterion that allows for intertemporal trade-offs, state dependence, and other refinements standard in economic analysis. Crucially, the domain of actions includes some that induce losses which, once realized, cannot be undone, replaced, or meaningfully offset by gains along other dimensions. No assumptions are made about preferences beyond minimal regularity conditions required for ranking and comparison.

The key modeling choice concerns the domain of admissible actions. Rather than presupposing feasibility restrictions, the model allows all actions to be initially admissible. This permits an examination of whether optimization over an unrestricted domain remains coherent once non-compensable losses are present, and if not, which restrictions on the feasible set are necessary to restore coherence.

All subsequent results follow from this minimal structure. The analysis does not rely on informational imperfections, uncertainty, or ethical disagreement, but solely on the interaction between optimization and non-compensable losses within the domain of choice.

2.1 Action Set and Evaluation Functions

Let X denote the set of actions (policies, institutions, or social alternatives). Social evaluation is conducted using a class of evaluation functions $\mathcal{U}$. Each $U \in \mathcal{U}$ takes the form

$$U(x) = u(x) + \alpha v(x),$$

where $u(x)$ represents lifetime utility of the current decision-maker or generation, $v(x)$ represents evaluations of others, future generations, or social states, and $\alpha \geq 0$ captures the degree of altruism. Discounting and finite horizons are embedded in the definitions of u and v. While altruism is allowed, complete identification is not assumed.

This formulation encompasses a broad class of models, including discounted utility, intergenerational altruism, and social welfare functions.

2.2 Violations and Axioms

Consider a state S (such as a natural environment, a personal existence, or a life path). Let $B \subset X$ denote the subset of actions that constitute violations. Violations are characterized by the following three properties.

Axiom 1 (Irreversibility).

Losses to S caused by violating actions cannot be reversed ex post.

Axiom 2 (Non-Substitutability).

Such losses cannot be substituted by other goods or states.

Axiom 3 (Non-Compensability).

Such losses cannot be offset by monetary compensation or by increases in other components of utility.

These axioms describe structural properties of outcomes related to time, choice, and exchange. Non-compensability is treated here as a property of the domain—an assumption about admissible losses—and the analysis characterizes its implications for the coherence of optimization, without assessing its normative justification.

3. Main Result

Within the framework described above, the paper derives an impossibility result concerning optimization-based evaluation. The result establishes that, once irreversible and non-compensable losses are admissible, no finite-valued objective function can provide a coherent ranking of all feasible alternatives while satisfying minimal regularity conditions.

The analysis focuses on whether coherent maximization is possible when all such actions are treated as admissible. The result establishes a sharp boundary. Under minimal regularity conditions, if actions generating non-compensable losses are included in the feasible set, then optimization over that set necessarily becomes incoherent. This failure does not depend on the specification of the objective function or on informational or computational limitations.

The theorem therefore identifies a structural impossibility: coherence cannot be restored by refining the evaluation criterion alone. At the same time, the result points toward a constructive resolution. By characterizing the minimal restrictions on the feasible set required to exclude non-compensable losses, the analysis clarifies the domain on which standard optimization methods apply without contradiction.

The formal statement of the result follows.

Intuition

The intuition underlying the main result is straightforward. Even if a loss is irreversible and non-substitutable, as long as it enters the evaluation function as a finite negative term, discounting and finite horizons allow its weight to become arbitrarily small. At the same time, the contemporaneous benefits generated by a violating action can be made arbitrarily large. Hence, under some admissible evaluation function, violating actions are necessarily preferred. If violations are to be excluded under all evaluation functions, the issue cannot be resolved by modifying the objective function, but must instead be addressed at the level of admissible alternatives, that is, through the design of the feasible set.

Theorem 1 (Optimization Failure under Non-Substitutable and Irreversible Externalities)

Optimization-based evaluation becomes structurally incoherent when feasible sets include irreversible and non-compensable losses that cannot be consistently traded off within any finite-valued objective function.

The result does not rely on unbounded utilities; it arises from the admissibility of non-compensable losses in the domain.

Let X, $\mathcal{U}$, and $B \subset X$be defined as above. Then the following statements hold.

1. **Impossibility.**
   For any evaluation function $U \in \mathcal{U}$, there exists an environment in which a violating action is optimal.

2. **Necessity.**
   A necessary and sufficient condition for excluding all violating actions under every $U \in \mathcal{U}$is to restrict the feasible set to
   $$X^D = X \setminus B.$$

3. **Invariance.**
   Under this restriction, outcomes are independent of discount rates, degrees of altruism, and preference heterogeneity.

See Appendix A.5 for the proof.

4. Discussion and Implications

The impossibility result admits several interpretations. One natural reading emphasizes the role of non-compensable losses in undermining the standard logic of optimization. When such losses are admissible, evaluation cannot rely on finite trade-offs, regardless of how objective functions are specified or calibrated.

From this perspective, the difficulty lies not in the choice of an objective function, but in the structure of the feasible set over which evaluation is conducted. When feasibility itself is compromised by irreversible losses, the evaluative task may no longer be well defined within the standard optimization framework. In such environments, the problem is not how to improve the objective function, but whether objective-based evaluation remains a coherent mode of assessment.

This interpretation suggests that meaningful evaluation under irreversibility may require restrictions on admissible alternatives rather than refinements of evaluative criteria. Constraints, thresholds, or viability conditions may play a role in restoring coherence, not as normative prescriptions, but as structural features necessary for evaluation to be well defined.

4.1 Optimization versus Feasibility Design

The impossibility result does not imply that optimization should be abandoned. Rather, it identifies a fundamental limitation of optimization when applied to an unrestricted domain that includes non-compensable losses. In such environments, coherence failure is unavoidable: any evaluation that permits trade-offs may select actions generating losses that cannot be meaningfully offset.

The necessity result clarifies how optimization can be restored. Coherence is recovered if and only if the feasible set excludes actions that generate non-compensable losses. On the restricted domain, standard maximization techniques apply without contradiction. In this sense, the problem is not one of objective-function design, but of feasibility design.

This distinction is critical. Much of economic analysis implicitly assumes that all relevant considerations can be incorporated into a sufficiently rich objective function. The present analysis shows that this presumption fails once non-compensable losses are admitted. No refinement of the objective function can substitute for appropriate restrictions on the feasible set.

4.2 Relation to Irreversibility and Option Value

A large literature addresses irreversibility by enriching the objective function—for example, through option values or state-dependent utilities. These approaches preserve optimization by accounting for the value of waiting or flexibility.

The present results complement this literature by identifying a boundary. When losses are compensable, objective-function refinements may suffice. When losses are non-compensable, however, such refinements are generally insufficient. Even with option values included, optimization over an unrestricted domain may select actions that generate irreversible losses whenever short-run gains dominate.

Thus, the analysis does not contradict option-value reasoning. Instead, it shows that option values operate within a domain where losses remain compensable. Once this condition fails, feasibility restrictions become logically necessary.

4.3 Thresholds, Safe Minimum Standards, and Critical Domains

Threshold-based approaches—such as safe minimum standards or doctrines of critical capital—have often been justified on precautionary or normative grounds. The present framework provides a complementary foundation.

When certain losses are non-compensable, excluding actions that cross a threshold is not a matter of prudence, but a requirement for coherent evaluation. Thresholds emerge endogenously as boundaries of the admissible domain, beyond which optimization loses meaning.

This perspective shifts the interpretation of such standards. They are not external constraints imposed on an otherwise well-defined optimization problem; they are conditions under which optimization itself remains well-defined.

4.4 Inviolable Constraints in Social Evaluation

In social evaluation, certain decision problems involve instances where non-compensable losses are present, and actions generating such losses may be regarded as impermissible regardless of aggregate gains.

The present analysis offers an alternative interpretation. When violations entail non-compensable losses, permitting them in the choice set leads to incoherent rankings. Excluding such actions restores coherent optimization. In this sense, inviolable constraints can be understood as feasibility restrictions required for logical consistency, rather than as ad hoc normative additions.

4.5 Broader Implications

The results have implications for a wide range of economic environments, including irreversible environmental change, institutional breakdown, and social decision problems involving inviolable constraints. Across these settings, the common structure is the presence of non-compensable losses.

By reframing irreversibility as a problem of feasibility design, the analysis clarifies the limits of optimization-based evaluation and identifies the minimal adjustments required to preserve its coherence. Optimization does not disappear; it survives on a restricted but well-defined domain.

5. Conclusion

Optimization-based evaluation becomes structurally incoherent when feasible sets include irreversible and non-compensable losses that cannot be consistently traded off within any finite-valued objective function. The analysis presented in this paper points to a tension at the heart of optimization-based evaluation. In environments characterized by irreversibility and non-compensability, the familiar framework of objective maximization may fail to provide coherent rankings of feasible alternatives.

Rather than proposing a definitive replacement for optimization-based evaluation, the paper isolates a structural difficulty that warrants further examination. One possible implication is that evaluation in such environments may depend on restrictions to the feasible set rather than on increasingly sophisticated objective functions. The nature and scope of such restrictions, however, remain open questions.

More broadly, the analysis invites reconsideration of the relationship between evaluation and feasibility in dynamic settings. Similar tensions may arise in other contexts in which losses permanently alter the space of admissible outcomes. The extent to which such tensions can be addressed through alternative feasibility restrictions, evaluative criteria, or institutional designs remains an open question, and the framework developed here is intended to provide a starting point for future investigation rather than a definitive resolution.

Appendix A: Independence and Role of the Axioms

This appendix collects proofs and technical details clarifying that the impossibility result established in the main text is structural in nature and does not hinge on particular ethical assumptions, functional forms, or ad hoc restrictions.

Independence of the Axioms and Restoration of Optimization

This appendix clarifies the role played by each axiom introduced in Section 2.2 by examining what happens when individual axioms are relaxed. The purpose is to show that the main impossibility result is not driven by any single assumption, but by their joint presence.

A.1 Absence of Irreversibility

If losses are reversible, standard dynamic optimization applies. Future restoration possibilities allow losses to be evaluated as temporary costs, which can be traded off against future benefits. In such environments, optimization-based evaluation remains coherent, and violations need not be excluded from the feasible set.

A.2 Absence of Non-Substitutability

If losses are substitutable by other goods or states, shadow prices can be assigned to losses, and trade-offs can be evaluated through standard welfare comparisons. In this case, losses can be internalized within the objective function, and coherent maximization is restored through appropriate pricing.

A.3 Absence of Non-Compensability

If losses are compensable—whether through monetary transfers, future benefits, or other dimensions of utility—then refinements of the objective function, such as option values or Pigouvian corrections, are sufficient to sustain coherent optimization. The infeasibility of trade-offs is precisely what distinguishes the present analysis from standard irreversibility models.

A.4 Joint Necessity

The failure of optimization identified in the paper arises only when irreversibility, non-substitutability, and non-compensability are jointly present. Dropping any one of these properties restores a domain on which optimization-based evaluation remains coherent. This confirms that the impossibility result is structural rather than definitional.

A.5 Proof of Invariance (Theorem 1, Part 3)

This subsection establishes that, once violations are excluded from the feasible set, the resulting optimization outcome is invariant to discounting, altruism, and preference heterogeneity.

Proof.

Under the admissible domain restriction, all feasible actions preserve the irreversible state above the critical threshold. Consequently, the comparison among feasible actions depends solely on whether an action violates the irreversibility constraint, not on how utility gains are aggregated once feasibility is ensured. Discount rates, degrees of altruism, and preference heterogeneity affect only the aggregation of utility streams conditional on feasibility, and therefore do not alter the set of admissible actions or the resulting optimal choice. This establishes invariance with respect to these parameters. ■

Appendix B Structural and Normative Interpretations of Non-Compensability

This appendix clarifies the logical role of non-compensability in the analysis. The purpose is not to deny that non-compensability may reflect normative commitments in some contexts, but to show that the main results of the paper do not depend on whether it is interpreted normatively or structurally.

B.1 Two Interpretations of Non-Compensability

Non-compensability can arise under two analytically distinct interpretations.

First, non-compensability may reflect structural properties of the environment. In some decision problems, losses are non-compensable because no mechanism exists by which they can be reversed, substituted, or reconstructed. Physical irreversibility, institutional fragility, and the absence of transferable or reproducible substitutes are examples of such structural features. Under this interpretation, non-compensability is a descriptive property of the domain of actions.

Second, non-compensability may be adopted as a normative commitment. Certain losses—such as violations of basic existence, dignity, or fundamental institutional integrity—may be regarded as unacceptable regardless of the magnitude of compensating gains. In this case, non-compensability enters as a value axiom restricting admissible trade-offs.

These interpretations are conceptually distinct. One concerns the structure of feasible outcomes; the other concerns the ethical admissibility of certain comparisons.

B.2 Independence of the Main Result from Normative Commitment

The main results of the paper do not hinge on which interpretation is adopted.

If non-compensability is treated as a structural property of the domain, the impossibility result follows directly. As long as losses enter admissible evaluation criteria as finite negative terms, while contemporaneous gains can dominate under discounting or finite horizons, optimization over an unrestricted domain necessarily leads to incoherence.

If non-compensability is instead adopted as a normative restriction, the conclusion remains unchanged. Unless violations are assigned lexicographic priority or infinite penalties—which lie outside standard optimization frameworks—there exist environments in which trade-off–based evaluation selects actions generating non-compensable losses. Excluding such actions under all admissible evaluations therefore still requires restricting the feasible set.

Thus, whether non-compensability is interpreted structurally or normatively, refining the objective function alone cannot generally restore coherent optimization.

B.3 Role of Feasibility Restrictions

This observation clarifies the role of feasibility constraints in the analysis. The introduction of inviolable constraints is not a way of embedding moral conclusions into the objective function. Rather, constraints emerge as the only mechanism capable of ensuring coherence across a broad class of admissible evaluations, independently of their ethical interpretation.

Accordingly, the paper advances a conditional claim rather than a moral assertion: if actions generating non-compensable losses are to be excluded under all admissible evaluations within standard optimization frameworks, then such exclusions must be implemented at the level of the feasible set.

This conditional structure allows the analysis to remain agnostic about ultimate moral commitments while identifying a precise structural limitation of optimization-based evaluation.

Appendix C: Lemmas Supporting Theorem 1

Supporting Lemmas

This appendix collects formal lemmas that support the main impossibility result.

Lemma C.1 (Existence of Dominating Violations)

For any admissible evaluation function $V \in \mathcal{V}$, there exists an environment and a violating action $a \in B$ such that a is ranked strictly above all non-violating actions.

Proof sketch.

The argument relies on standard dominance reasoning and does not require any nonstandard continuity or compactness assumptions; it is therefore only sketched here for brevity.
Because losses enter V as finite negative terms, while contemporaneous gains can dominate under discounting or finite horizons, it is always possible to construct an environment in which the net evaluation of a violating action exceeds that of any non-violating alternative. ■

Lemma C.2 (Independence from Boundedness)

The construction in Lemma C.1 does not rely on unbounded utility levels. The result holds even if utilities are uniformly bounded above.

Proof sketch.

The argument relies on standard dominance reasoning and does not require any nonstandard continuity or compactness assumptions; it is therefore only sketched here for brevity.
The argument depends on relative dominance rather than absolute magnitude. Finite penalties remain dominated by sufficiently favorable configurations of admissible gains within the bounded domain. ■

Lemma C.3 (Finite Penalties and Discounting)

For any discount factor and any finite penalty assigned to violations, there exists an admissible evaluation under which a violating action is optimal.

Proof sketch.

The argument relies on standard dominance reasoning and does not require any nonstandard continuity or compactness assumptions; it is therefore only sketched here for brevity.
Discounting allows the effective weight of future losses to be made arbitrarily small relative to contemporaneous gains, regardless of the discount rate, as long as penalties remain finite. ■

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