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"Why Aggregate Indicators Fail in Fiscal Sustainability Evaluation: Tax Base Heterogeneity, Reweighting, and the Limits of GDP Elasticity"

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Why Aggregate Indicators Fail in Fiscal Sustainability Evaluation: Tax Base Heterogeneity, Reweighting, and the Limits of GDP Elasticity

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Abstract

Fiscal sustainability is commonly evaluated using aggregate indicators such as GDP growth and tax revenue elasticity, yet this paper shows that such indicators can be fundamentally insufficient once fiscal capacity depends on the evolving structure of underlying tax bases. When tax revenue is generated by heterogeneous tax bases whose growth rates and revenue weights evolve over time, aggregation induces an intrinsic informational loss that cannot be resolved by refining elasticity estimates. To establish this result, tax revenue is modeled as the aggregation of industry-specific tax bases subject to heterogeneous growth dynamics and institutional features of corporate taxation. Within this framework, we show that tax revenue growth cannot, in general, be characterized by a single, time-invariant elasticity with respect to aggregate output. The analysis clarifies the structural conditions under which conventional benchmarks, including the Domar condition, remain valid. When tax bases evolve proportionally with aggregate output and industrial composition is stable, the Domar condition emerges as a special case of a more general stability condition. Once these restrictive assumptions are relaxed, changes in tax base composition and sectoral profit dynamics can generate systematic divergences between output growth and tax revenue growth. The paper derives a structural fiscal stability condition in which debt sustainability depends on the weighted growth dynamics of underlying tax bases rather than on aggregate output alone. Stylized facts from Japan and the United States illustrate how differences in industrial structure and tax institutions shape revenue dynamics in practice. More fundamentally, the analysis highlights the structural limitations of aggregate-indicator–based fiscal evaluation: fiscal sustainability is a property of how fiscal capacity is generated through the composition and dynamics of underlying tax bases.

Keywords: Fiscal sustainability; Tax revenue elasticity; Aggregation-induced informational loss; Reweighting; Domar condition; Tax base heterogeneity

JEL Codes: H20; H63; E62; O40

1. Introduction

Fiscal sustainability is commonly assessed through evaluation rules that relate the growth rate of government revenue to that of public debt. Among these, the Domar condition has long served as a

benchmark, asserting that public debt remains stable provided that the growth rate of tax revenue exceeds the effective interest rate on government debt (Domar, 1944). In practice, the Domar condition is implemented through aggregate-indicator–based criteria, most notably by assuming that tax revenue grows proportionally with aggregate economic activity, typically summarized by a constant tax revenue elasticity with respect to GDP.

This elasticity-based representation underlies a wide range of fiscal sustainability analyses, both in academic research and in policy applications (Barro, 1979; Bohn, 1998, 2007). Within these frameworks, aggregate output growth is treated as a sufficient statistic for revenue dynamics, and tax revenue is modeled as a reduced-form outcome of GDP growth, abstracting from the underlying structure and composition of tax bases.

However, recent fiscal developments cast doubt on the adequacy of this approach. Empirically, tax revenue has exhibited substantial and persistent deviations from GDP-based projections, even in periods without major tax reforms. These deviations point to a more fundamental issue: aggregate indicators fail to capture changes in the underlying structure of tax bases. In particular, systematic shifts in the composition of corporate profits across industries, differences in loss-offset mechanisms, and changes in the institutional structure of corporate taxation (Shimada, 2022; OECD, 2022) alter the mapping from output growth to revenue growth. Such evidence suggests that fiscal sustainability cannot be coherently evaluated through a single aggregate elasticity parameter alone.

This paper revisits the Domar condition by making explicit the structural assumptions under which it provides a reliable benchmark for fiscal sustainability. The standard formulation implicitly assumes that tax bases evolve proportionally with aggregate output and that industrial composition is stable over time. Under these assumptions, tax revenue can be represented as a simple reduced-form function of GDP, and fiscal sustainability reduces to a comparison between macroeconomic growth and interest rates, as in standard debt sustainability analysis frameworks. Once these assumptions are relaxed, however, the logic underlying the Domar condition no longer applies.

We develop a structural characterization of tax revenue stability that nests the elasticity-based approach as a special case and clarifies the conditions under which it remains informative. Instead of modeling tax revenue as a reduced-form function of GDP, we explicitly decompose it into industry-specific tax bases and institutional parameters. In this framework, tax revenue is determined by the interaction between sectoral profit growth, changes in industrial composition, and the tax system that maps profits into revenue. Fiscal sustainability therefore depends on the stability of these structural

components rather than on aggregate growth alone.

The contribution of this paper is threefold. First, we formally demonstrate that the Domar condition represents a special and restrictive case of a more general fiscal stability condition. Second, we show that the commonly used concept of tax revenue elasticity is not a structural parameter but an endogenous and time-varying statistic reflecting underlying changes in economic structure and tax institutions. Third, we derive an alternative sustainability condition that replaces aggregate output growth with a weighted measure of tax base dynamics.

The analysis is primarily theoretical. Rather than pursuing extensive cross-country estimation, we use stylized facts from Japan and the United States to illustrate how differences in industrial composition and tax institutions affect the stability of tax revenue. These examples serve to clarify the mechanisms emphasized by the theory, not to provide exhaustive empirical validation. More broadly, the analysis reframes fiscal sustainability as a structural property of tax revenue generation, rather than a reduced-form relationship between aggregate growth and public debt.

In this sense, this paper contributes to a line of research that examines the logical limits of evaluation based on aggregate indicators in dynamic economic environments where feasibility and sustainability depend on the evolution of underlying structural constraints.

2. A Diagnostic Framework: Why Aggregate Indicators Lose Information

This section provides a diagnostic perspective on why aggregate indicators—such as GDP growth and tax revenue elasticity—can fail as sufficient statistics for long-run fiscal capacity. The key mechanism is aggregation-induced informational loss: when total revenue is an aggregation of heterogeneous tax bases whose growth rates and revenue weights evolve, the mapping from aggregate output growth to revenue growth is not stable in general.

2.1 Aggregation-induced informational loss

Aggregate indicators summarize complex underlying economic processes into a small number of observable statistics. By construction, this aggregation involves a many-to-one mapping: distinct underlying structures can generate identical aggregate outcomes at a given point in time. When fiscal capacity depends on heterogeneous tax bases whose growth rates and relative importance evolve, this aggregation inevitably entails informational loss.

In particular, tax revenue elasticity summarizes the co-movement between aggregate output and tax revenue at a given date. It does not encode how revenue is generated across underlying tax bases,

nor how changes in their composition affect future revenue dynamics. As a result, identical values of aggregate output growth and observed tax revenue elasticity may correspond to fundamentally different underlying fiscal structures.

The key mechanism driving this informational loss is reweighting. Even if sectoral tax bases grow at stable rates, changes in their revenue shares mechanically alter aggregate revenue growth. Because aggregate indicators abstract from these compositional dynamics, they cannot provide sufficient statistics for long-run fiscal capacity.

This aggregation-induced informational loss implies a diagnostic limitation of aggregate-indicator–based fiscal evaluation. Prior to any stability assessment, aggregate indicators alone are insufficient to distinguish economies with structurally different fiscal capacity.

Figure 1 provides a graphical illustration of this aggregation-induced informational loss, showing how identical aggregate indicators can arise from distinct underlying tax base dynamics and hence imply divergent fiscal capacity.

2.2 Two Economies with Identical (g, ε) but Different Fiscal Capacity

To fix ideas, consider two economies that share the same aggregate GDP growth rate g and the same observed tax revenue elasticity ε at a given date t^*. From the perspective of aggregate-indicator–based evaluation, these economies appear fiscally indistinguishable.

Suppose, however, that the underlying structure of tax revenue differs across the two economies. In Economy A, taxable profits are concentrated in a small number of sectors, implying that revenue weights adjust substantially over time. In Economy B, taxable profits are more evenly distributed across sectors, with relatively stable revenue shares. Even if aggregate output growth and observed elasticity coincide at date t^*, the evolution of tax revenue will differ across the two economies as revenue weights evolve.

Because aggregate indicators summarize only contemporaneous co-movements between output and revenue, they do not encode these structural differences. As a result, identical values of (g, ε) can correspond to divergent paths of tax revenue growth and, hence, different degrees of fiscal capacity and debt sustainability. This example illustrates the diagnostic limitation identified in Section 2.1. Figure 1 provides a graphical illustration of this point, showing how identical aggregate indicators can correspond to distinct underlying tax base dynamics.

3. A Structural Model of Tax Revenue

Tax Revenue Decomposition

We model government tax revenue as the aggregation of industry-specific tax bases. Let T_t denote total tax revenue at time t, given by

$$T_t = \sum_{i=1}^{N} \tau_i \, \Pi_{i,t},$$

where $\Pi_{i,t}$ denotes the taxable profit (tax base) of industry i, and τ_i represents the effective institutional parameter governing the mapping from profits to tax revenue, including statutory tax rates, deductions, and loss-offset provisions (OECD, 2022).

Dynamics of Tax Bases

Each industry's tax base evolves according to

$$\Pi_{i,t+1} = \Pi_{i,t}(1 + g_i + \eta_t),$$

where g_i denotes the industry-specific growth component and η_t captures common macroeconomic shocks affecting all industries.

4. Structural Fiscal Stability Condition

This section shifts from the diagnostic perspective developed in Section 2 to a structural characterization of fiscal sustainability. The preceding analysis showed that aggregate indicators fail to provide sufficient information for long-run fiscal evaluation when tax revenue is generated by heterogeneous tax bases whose growth rates and revenue weights evolve over time. The purpose of this section is to identify a minimal structural condition under which fiscal sustainability can be assessed once this heterogeneity is taken seriously.

Rather than proposing a new fiscal rule or empirical benchmark, the analysis derives a general stability condition that explicitly incorporates the growth dynamics and reweighting of underlying tax bases. This condition nests conventional aggregate benchmarks as special cases and clarifies the precise sense in which aggregate growth-based criteria are informative only under restrictive structural assumptions.

Theorem 1 (A Structural Fiscal Stability Condition).

Consider an economy in which tax revenue is generated by multiple tax bases whose growth rates and revenue weights evolve over time. Fiscal sustainability—defined as the non-divergence of the debt-to-GDP ratio—requires that the weighted average growth rate of tax bases satisfies

$$\sum_{i=1}^{N} \omega_{i,t} \left(g_i + \eta_t\right) \geq r - g,$$

where $\omega_{i,t}$ denotes the revenue share of tax base i, r is the effective interest rate on government debt, and g is the growth rate of aggregate output.

The proof is provided in Appendix A, where the government budget constraint is expressed in debt-to-GDP form and fiscal sustainability is formally defined as the non-divergence of the debt ratio.

Figure 1 provides a graphical representation of the structural fiscal stability condition stated in Theorem 1, illustrating how debt sustainability depends on the weighted growth dynamics of underlying tax bases relative to the interest–growth differential. In the next section, this framework is used to clarify the precise sense in which conventional aggregate benchmarks—most notably the Domar condition—can be interpreted as special cases arising under restrictive structural assumptions.

5. Conventional Fiscal Benchmarks and Their Domain of Validity

When Conventional Benchmarks Work

The substantive content of the Domar condition enters at the next step, when the primary balance s_t is linked to aggregate economic growth. In conventional formulations, tax revenue is assumed to grow proportionally with aggregate output, often summarized by a constant tax revenue elasticity with respect to GDP. Under this assumption, primary balance dynamics can be expressed as a simple function of g, and fiscal sustainability reduces to a comparison between the interest rate and the growth rate of the economy, yielding the well-known Domar condition (Domar, 1944).

This step, however, relies on a strong and largely implicit structural assumption: that the tax base evolves proportionally with aggregate output and that changes in industrial composition can be ignored. Once this assumption is made, tax revenue behaves as if it were generated by a single aggregate tax base, and the mapping from aggregate growth to revenue growth becomes stable by construction.

The present paper replaces this reduced-form link with a structural decomposition of tax revenue. Instead of treating tax revenue as a direct function of GDP, we model it as the aggregation of heterogeneous, industry-specific tax bases,

$$T_t = \sum_{i=1}^{N} \tau_i \, \Pi_{i,t},$$

whose growth rates and relative importance evolve over time. Under this formulation, the primary balance s_t is no longer pinned down by aggregate output growth alone. Rather, revenue growth depends on the weighted growth dynamics of underlying tax bases.

This distinction is formalized in Theorem 1. Fiscal sustainability requires that the weighted average growth rate of taxable profits,

$$\sum_{i=1}^{N} \omega_{i,t} \, (g_i + \eta_t),$$

exceeds the effective interest–growth differential $r - g$, where the weights $\omega_{i,t}$ reflect each industry's contribution to total tax revenue. Aggregate output growth plays no privileged role in this condition beyond its effect on the evolution of tax bases.

The Domar condition emerges as a special and degenerate case of this more general result. If taxable profits are proportional to aggregate output for all industries, if industry-specific growth rates are identical, and if institutional parameters are uniform, then the weighted growth rate of tax bases collapses to aggregate output growth. Only under these restrictive assumptions does the conventional Domar condition characterize fiscal sustainability.

The implication is immediate and sharp. The Domar condition is not a general law of fiscal sustainability, but a special-case result that holds only when tax revenue can be treated as if it were generated by a single, aggregate tax base growing proportionally with GDP. Once tax bases are heterogeneous and their composition evolves over time—as is empirically the case in modern economies—elasticity-based benchmarks lose their structural meaning, and fiscal sustainability must be assessed through the underlying dynamics of tax bases rather than aggregate output growth alone. Importantly, the failure of the Domar condition in this framework is not due to behavioral responses, nonlinearities, or unconventional debt dynamics, but arises mechanically from the heterogeneity and reweighting of tax bases.

6. Aggregation Bias and the Instability of Tax Revenue Elasticity

Why Elasticity Is Not Structural

Theorem 2 (Instability of Aggregate Tax Revenue Elasticity).

The concept of tax revenue elasticity presumes that aggregate output growth provides a stable and

sufficient summary of the forces driving revenue dynamics. When tax revenue is generated by heterogeneous tax bases whose relative importance evolves over time, this presumption fails. Formally, the aggregate tax revenue elasticity

$$\varepsilon_t = \frac{d\ln\ T_t}{d\ln\ Y_t}$$

is not a structural parameter but an endogenous statistic given by

$$\varepsilon_t = \sum_i \omega_{i,t} \frac{d\ln\ \Pi_{i,t}}{d\ln\ Y_t},$$

which varies mechanically with changes in industry composition and tax base dynamics.

7. Intuition and Illustration

This section provides an intuitive illustration of the diagnostic problem identified in the preceding analysis. While Sections 2–5 established that aggregate indicators such as GDP growth and tax revenue elasticity are not structurally sufficient for assessing fiscal sustainability, the present section illustrates how this informational limitation arises in a simple and transparent way.

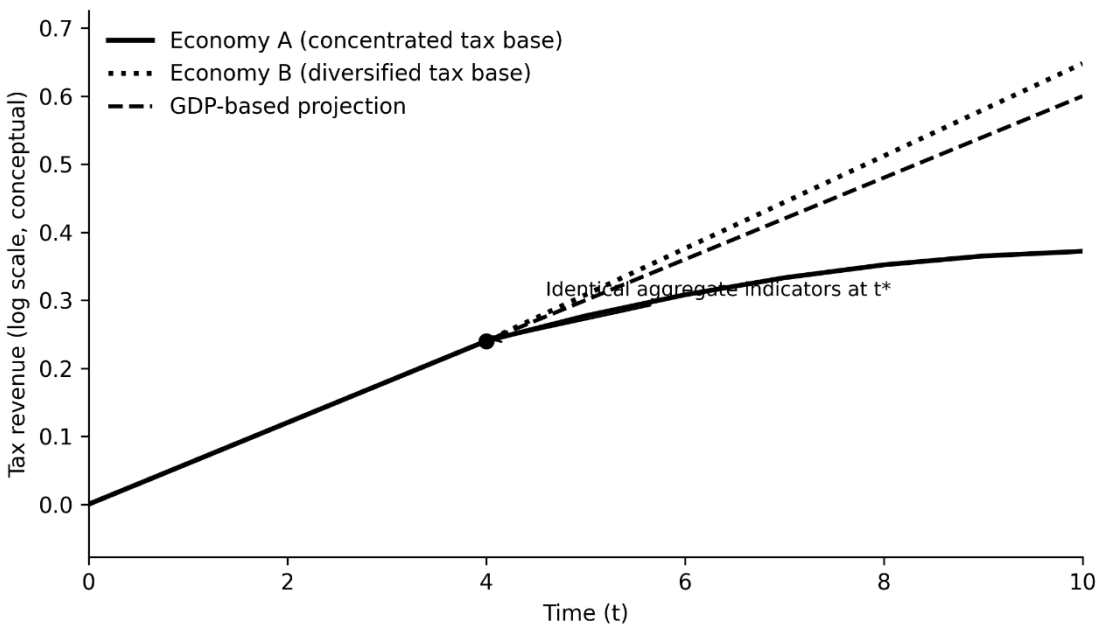

Figure 1. Aggregation-Induced Informational Loss in Fiscal Sustainability Evaluation.

This figure illustrates how economies with identical aggregate indicators at a given date can exhibit divergent revenue dynamics when underlying tax bases differ in concentration and reweighting over

time. At time t^*, Economy A and Economy B share the same aggregate GDP growth rate and observed tax revenue elasticity, yet their subsequent tax revenue paths diverge as revenue weights evolve. Aggregate indicators fail to capture these structural differences, motivating the diagnostic framework developed in this paper.

8. Stylized Facts: Japan and the United States

This section provides stylized illustrations intended to demonstrate how aggregation-induced informational loss can arise in practice, even in the absence of policy changes or unusual macroeconomic shocks. Rather than offering systematic empirical evidence, the purpose is to show how economies with different tax base structures can exhibit markedly different revenue dynamics under similar aggregate growth conditions.

In Japan, corporate tax revenue exhibits pronounced sensitivity to sectoral profit fluctuations, particularly in manufacturing and export-oriented industries, amplified by loss carryforward provisions and cyclical volatility (Shimada, 2022). These features correspond to substantial variation in the revenue weights $\omega_{i,t}$.

By contrast, the United States displays a more diversified industrial structure and different institutional features of corporate taxation, resulting in distinct revenue dynamics even under similar aggregate growth conditions (OECD, 2022). These differences are difficult to reconcile with a single elasticity parameter but are naturally interpreted within the structural tax base framework.

9. Relation to Existing Empirical Work

This section situates the present analysis within the existing empirical literature on fiscal sustainability and tax revenue dynamics. In particular, it clarifies how the structural framework developed here complements and rationalizes earlier empirical findings that document instability in aggregate tax revenue elasticities. This paper builds on Shimada (2022), which documented persistent deviations between GDP-based tax revenue projections and realized revenue using industry-level data. While the earlier study established the empirical relevance of unstable tax revenue elasticity, the present paper develops a general theoretical framework explaining why such instability is mechanically implied in economies with heterogeneous tax bases and institutions. The two papers are complementary: the earlier work motivates the problem empirically, while the present analysis provides a structural resolution.

The present framework also provides a structural interpretation of why empirical estimates of tax

revenue elasticity and fiscal reaction functions often appear unstable across samples and time periods. In reduced-form regressions, such instability is frequently attributed to measurement error or regime shifts. By contrast, the theory developed here shows that time variation in estimated elasticities can arise mechanically from changes in tax base composition and revenue weights, even in the absence of policy changes or behavioral responses. This perspective helps reconcile mixed empirical findings in the literature without requiring ad hoc assumptions.

10. Conclusion: Implications for Measurement and Monitoring

Sections 7–9 complemented the formal analysis by providing intuitive, illustrative, and contextual perspectives on the structural limitations of aggregate-indicator–based fiscal evaluation, clarifying how aggregation-induced informational loss arises in practice and how the present framework relates to existing empirical findings. More generally, aggregate indicators fail as a basis for coherent long-run evaluation when the objects being evaluated depend on underlying structural constraints whose evolution is heterogeneous and potentially irreversible. This paper has reconsidered fiscal sustainability from a structural perspective on tax revenue generation. By decomposing tax revenue into industry-specific tax bases and institutional parameters, the analysis clarifies the conditions under which standard elasticity-based benchmarks, including the Domar condition, provide meaningful guidance.

The main theoretical result demonstrates that fiscal stability depends not on aggregate output growth alone, but on the weighted growth dynamics of underlying tax bases. Aggregate GDP growth enters the analysis only indirectly, through its impact on sectoral profits and their composition. Once tax bases are heterogeneous and their relative importance evolves over time, there is no reason to expect a stable mapping from GDP growth to tax revenue growth.

From an operational perspective, the framework suggests that fiscal sustainability assessments should move beyond aggregate output-based indicators and explicitly monitor the composition and growth dynamics of underlying tax bases. In practice, this implies focusing on the sectoral sources of taxable income and their evolving revenue weights $\omega_{i,t}$, rather than relying on a single aggregate elasticity with respect to GDP. Even without precise estimation, tracking changes in tax base concentration and sectoral volatility can provide early warnings of fiscal fragility that are invisible in GDP-centered analyses.

This finding highlights a fundamental limitation of GDP-centered approaches to fiscal sustainability. By collapsing tax revenue dynamics into a single elasticity with respect to output, such approaches

implicitly assume away the structural features that determine the actual evolution of the tax base. In economies undergoing structural change—through shifts in industrial composition, profit concentration, or tax institutions—these assumptions are unlikely to hold.

The structural framework developed in this paper reframes fiscal sustainability as a problem of tax base composition and dynamics, rather than a reduced-form comparison between aggregate growth and interest rates. More fundamentally, fiscal sustainability is a structural property of how fiscal capacity is generated within the economy. GDP-centered benchmarks obscure this structure by construction. Once tax bases are heterogeneous and evolving, sustainability becomes inseparable from the composition and dynamics of taxable income itself. This perspective does not reject traditional benchmarks such as the Domar condition, but precisely delineates the narrow circumstances under which they apply—and the far more general environments in which they do not.

Beyond the specific case of tax revenue, the analysis highlights a general measurement lesson: aggregate indicators can obscure structural constraints when underlying components are heterogeneous and their weights evolve. For fiscal monitoring, this suggests complementing GDP-centered benchmarks with systematic tracking of tax-base concentration and revenue-weight reallocation over time.

Appendix

Appendix A. Proof of Theorem 1

This appendix provides a proof of Theorem 1. We consider a standard government budget constraint and analyze the conditions under which the debt-to-GDP ratio remains stable over time, following the intertemporal approach to fiscal sustainability (Barro, 1979; Bohn, 1998). Fiscal sustainability is formally defined as the non-divergence of the debt-to-GDP ratio.

Consider the government budget constraint expressed in nominal terms:

$$B_{t+1} = (1 + r)B_t - T_t,$$

where B_t denotes the stock of government debt at time t, r is the effective interest rate, and T_t is total tax revenue.

Let Y_t denote aggregate output growing at rate g, so that

$$Y_{t+1} = (1 + g)Y_t.$$

Define the debt-to-GDP ratio as $b_t = B_t/Y_t$. Dividing the budget constraint by Y_{t+1}, we obtain

$$b_{t+1} = \frac{(1+r)}{(1+g)} b_t - \frac{T_t}{(1+g)Y_t}.$$

Debt sustainability requires that the debt-to-GDP ratio does not diverge, i.e., $b_{t+1} \leq b_t$ in the long run. This condition can be written as

$$\frac{T_t}{Y_t} \geq (r-g)b_t.$$

Tax revenue is given by the structural decomposition

$$T_t = \sum_{i=1}^{N} \tau_i \, \Pi_{i,t},$$

where $\Pi_{i,t}$ denotes the tax base of industry i and τ_i is the corresponding institutional parameter.

The growth rate of total tax revenue is therefore

$$\frac{\dot{T}_t}{T_t} = \sum_{i=1}^{N} \omega_{i,t} \frac{\dot{\Pi}_{i,t}}{\Pi_{i,t}}, \text{where } \omega_{i,t} = \frac{\tau_i \Pi_{i,t}}{T_t}.$$

Using the assumed tax base dynamics

$$\Pi_{i,t+1} = \Pi_{i,t}(1 + g_i + \eta_t),$$

the growth rate of each tax base is $g_i + \eta_t$. Hence, the growth rate of total tax revenue is

$$\sum_{i=1}^{N} \omega_{i,t} \, (g_i + \eta_t).$$

Fiscal stability therefore requires that the weighted growth rate of tax bases is at least as large as the effective interest-growth differential:

$$\sum_{i=1}^{N} \omega_{i,t} \, (g_i + \eta_t) \geq r - g.$$

This establishes Theorem 1. □

Appendix B. Aggregation Bias in Tax Revenue Elasticity

This appendix formalizes the aggregation bias inherent in the concept of tax revenue elasticity.

Tax revenue elasticity with respect to aggregate output is commonly defined as

$$\varepsilon_t = \frac{d\ln\ T_t}{d\ln\ Y_t}.$$

Using the structural decomposition $T_t = \sum_i \tau_i\, \Pi_{i,t}$, total differentiation yields

$$\frac{dT_t}{T_t} = \sum_{i=1}^{N} \omega_{i,t} \frac{d\Pi_{i,t}}{\Pi_{i,t}}, \omega_{i,t} = \frac{\tau_i \Pi_{i,t}}{T_t}.$$

Dividing both sides by $d\ln\ Y_t$, we obtain

$$\varepsilon_t = \sum_{i=1}^{N} \omega_{i,t} \frac{d\ln\ \Pi_{i,t}}{d\ln\ Y_t}.$$

This expression shows that aggregate tax revenue elasticity is a weighted average of industry-specific elasticities. However, the weights $\omega_{i,t}$depend on the relative size of each tax base in total revenue and therefore vary over time as industrial composition and profitability change.

As a result, even if each industry-specific elasticity were constant, the aggregate elasticity $\varepsilon_t$would generally be time-varying due to changes in the weights. Conversely, observed stability of aggregate elasticity would require restrictive conditions on both tax base growth and industrial composition.

This aggregation bias implies that tax revenue elasticity is not a structural parameter but an endogenous statistic reflecting underlying economic and institutional structure.

Appendix C. Sufficient Conditions for the Domar Case

This appendix provides a detailed algebraic derivation of the standard Domar condition and clarifies the structural assumptions embedded in it.

We begin with the government budget constraint in nominal terms:

$$B_{t+1} = (1 + r)B_t - (T_t - G_t),$$

where $B_t$denotes the stock of government debt, $r$is the effective interest rate, $T_t$tax revenue, and $G_t$government expenditure.

Let aggregate output Y_t grow at rate g, so that

$$Y_{t+1} = (1 + g)Y_t.$$

Define the debt-to-GDP ratio as $b_t = B_t/Y_t$. Dividing the budget constraint by Y_{t+1}, we obtain

$$b_{t+1} = \frac{(1+r)B_t}{(1+g)Y_t} - \frac{T_t - G_t}{(1+g)Y_t} = \frac{1+r}{1+g} b_t - \frac{T_t - G_t}{(1+g)Y_t}.$$

Using a first-order approximation for small growth rates, the change in the debt ratio can be written as

$$\Delta b_t \equiv b_{t+1} - b_t \approx (r-g)b_t - s_t,$$

where

$$s_t \equiv \frac{T_t - G_t}{Y_t}$$

denotes the primary balance as a share of GDP.

Fiscal sustainability, defined as the non-divergence of the debt ratio, requires

$$\Delta b_t \leq 0 \Leftrightarrow s_t \geq (r-g)b_t.$$

Up to this point, the derivation relies only on accounting identities and definitions. The critical step arises when the primary balance $s_t$is linked to aggregate economic growth.

Using the definition of the primary balance,

$$s_t = \frac{T_t}{Y_t} - \frac{G_t}{Y_t},$$

the standard approach implicitly assumes that tax revenue grows proportionally with aggregate output, so that

$$\frac{T_{t+1}}{T_t} = \frac{Y_{t+1}}{Y_t},$$

or equivalently,

$$\frac{d\ln\ T_t}{d\ln\ Y_t} = \varepsilon, \varepsilon = 1.$$

Under this assumption, tax revenue behaves as if it were generated by a single aggregate tax base proportional to GDP. The primary balance-to-GDP ratio can then be treated as a stable function of aggregate growth, and the fiscal sustainability condition reduces to

$$g \geq r - \frac{S_t}{B_t},$$

which can be rearranged as

$$s_t \geq \frac{B_t}{Y_t}(r - g).$$

This is the standard Domar condition.

The key observation is that this conclusion hinges on the assumption that tax revenue is proportional to GDP. Once tax revenue is instead decomposed into heterogeneous tax bases,

$$T_t = \sum_i \tau_i \, \Pi_{i,t},$$

the proportionality between T_t and Y_t no longer holds in general. In that case, the primary balance cannot be expressed as a simple function of aggregate growth, and the Domar condition fails to characterize fiscal sustainability.

Thus, the Domar condition is valid only under highly restrictive structural assumptions: proportional tax bases, homogeneous growth rates, and stable industrial composition. Outside this special case, fiscal sustainability must be assessed through the underlying dynamics of tax bases rather than aggregate output growth.

Appendix Note. While Figure 1 in this discussion paper highlights aggregation-induced informational loss in fiscal sustainability evaluation, the companion paper provides a formal structural stability condition and a corresponding mapping between tax base dynamics and debt sustainability.

References

Barro, R. J. (1979).On the determination of the public debt.*Journal of Political Economy*, **87**(5), 940–971.

Bohn, H. (1998).The behavior of U.S. public debt and deficits.*Quarterly Journal of Economics*, **113**(3), 949–963.

Bohn, H. (2007). Are stationarity and cointegration restrictions really necessary for the intertemporal budget constraint? *Journal of Monetary Economics*, **54**(7), 1837–1847.

Domar, E. D. (1944).The “burden of the debt” and the national income.*American Economic Review*, **34**(4), 798–827.

Koopmans, T. C. (1960).Stationary ordinal utility and impatience. *Econometrica*, **28**(2), 287–309.

OECD (2022).*Corporate Tax Statistics*.OECD Publishing, Paris.

Shimada, E. (2022).Industry-specific analysis of the impact of changes in the macroeconomic environment on corporate profits and estimation of corporate tax revenue. *International Journal of Economic Policy Studies*, **16**(2), 189–213.