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“Part I: A Core Framework and an Impossibility Result for  
Dynamic Social Evaluation under Irreversible Constraints”

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## **Part I: A Core Framework and an Impossibility Result for Dynamic Social Evaluation under Irreversible Constraints**

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### **Abstract**

This paper reports preliminary results from an ongoing research program on dynamic social evaluation under irreversible constraints. The scope of the present analysis is intentionally limited. We introduce a core framework in which social outcomes are modeled as infinite histories that include an irreversible state variable, and we establish a baseline impossibility result on the unrestricted domain under minimal regularity and anonymity requirements. The paper deliberately stops short of proposing solutions: in particular, it does not characterize admissible domain restrictions, does not derive representation theorems, and does not resolve the impossibility. The purpose of this paper is to isolate the structural source of incoherence in dynamic social evaluation under irreversible constraints.

**Keywords:** Social choice, Irreversibility, Domain restriction, Anonymity, Impossibility

**JEL codes:** D71, D90, C62

### **1. Introduction**

Dynamic social choice theory has traditionally evaluated infinite streams of per-period utilities. Within this canonical framework, outcomes differ only in their utility sequences, and feasibility is implicitly assumed to persist over time. Under these assumptions, classical representation results show that continuity, anonymity, and monotonicity naturally lead to discounted or undiscounted utility aggregations.

Many economically relevant environments violate this implicit feasibility premise. Environmental resources, institutional capacity, technological infrastructures, or social trust may deteriorate irreversibly. Once such breakdowns occur, future utility gains may become infeasible, irrelevant, or ill-defined. These considerations call for a formulation of dynamic social evaluation in which outcomes are histories that combine per-period utilities with state variables capturing irreversible constraints.

This paper adopts this perspective and studies social evaluation over infinite histories that include an irreversible state variable. At first glance, this extension appears conceptually mild. One might conjecture that standard axioms—continuity, anonymity across generations, and monotonicity—

remain compatible, perhaps yielding modified aggregation rules. This conjecture is false.

The main message of this paper is negative and structural. Once irreversible state variables are treated as components of outcomes and social evaluation is defined on the unrestricted product space of histories, no social evaluation relation can satisfy even minimal regularity and anonymity requirements. Crucially, this impossibility does not stem from ethical disagreement, discounting assumptions, or intergenerational conflict. Nor does it rely on strong efficiency criteria. Instead, it arises from a domain-theoretic interaction between irreversibility and unrestricted evaluation.

To isolate this mechanism, the analysis deliberately refrains from imposing normative or feasibility-based restrictions on admissible outcomes. In particular, no sustainability constraints, safety thresholds, or admissibility conditions are introduced. All histories consistent with irreversibility are admitted into the domain. The question posed is simply whether a minimally reasonable notion of social evaluation can exist on such a domain.

The answer is negative. Under weak order, finite-permutation anonymity, continuity, and monotonicity with respect to both utilities and the state variable, social evaluation becomes inconsistent or degenerate. The difficulty arises prior to any normative judgment and precedes questions of aggregation, discounting, or institutional design.

The purpose of this paper is therefore diagnostic rather than remedial. By identifying the precise source of impossibility on the unrestricted domain, it clarifies why some form of domain restriction becomes logically unavoidable in dynamic environments with irreversible constraints. How such restrictions should be formulated, and what representation results they imply, are questions deferred to subsequent analysis.

## 2. Model: Histories with an Irreversible State Variable

Time is discrete and indexed by  $t = 0, 1, 2, \dots$ . In each period, the social outcome is described by a pair

$$x_t = (u_t, s_t),$$

where  $u_t \in U \subset \mathbb{R}$  represents per-period utility, and  $s_t \in S \subset \mathbb{R}$  represents a state variable capturing feasibility-relevant resources or capacities.

A history is an infinite sequence

$$x = (x_t)_{t \geq 0} = ((u_t, s_t))_{t \geq 0}.$$

Let

$$X = (U \times S)^{\mathbb{N}}$$

denote the set of all such histories.

The state variable  $s$  is irreversible in the sense that it cannot increase over time. Formally, histories are assumed to satisfy

$$s_{t+1} \leq s_t \text{ for all } t \geq 0.$$

This captures situations in which certain losses—such as environmental depletion, institutional erosion, or credibility breakdown—cannot be undone by subsequent actions.

Importantly, irreversibility is treated here as a property of the state variable itself, not as a constraint on social evaluation. All histories consistent with this irreversibility condition are admitted into the domain  $X$ . No additional feasibility or sustainability requirements are imposed at this stage.

The set  $X$  is endowed with the product topology. Equivalently, one may work with a metric such as

$$d(x, y) = \sum_{t=0}^{\infty} 2^{-t} \min \{ |u_t - v_t| + |s_t - r_t|, 1 \},$$

for histories  $x = (u_t, s_t)$  and  $y = (v_t, r_t)$ . This topology allows for standard continuity requirements on social evaluation.

### 3. Social Evaluation and Basic Axioms

A social evaluation relation is a binary relation  $\succeq$  on  $X$ . The interpretation is that  $x \succeq y$  means that history  $x$  is socially weakly preferred to history  $y$ .

The analysis imposes only minimal structural requirements.

(WO) Weak order.

The relation  $\succeq$  is complete and transitive.

(FP) Finite-permutation anonymity.

For any finite permutation  $\pi$  of time periods and any histories  $x, y \in X$ ,

$$x \succeq y \Leftrightarrow \pi(x) \succeq \pi(y),$$

where  $\pi(x)$  denotes the history obtained by permuting finitely many coordinates of  $x$ .

(MON-u) Utility monotonicity.

If two histories have identical state variables and one has weakly higher utility in every period, it is weakly preferred.

(MON-s) State monotonicity.

If two histories have identical utility streams and one has weakly higher state variables in every period, it is weakly preferred.

(CONT) Continuity.

The graph of  $\geq$  is closed in the product topology.

These requirements are standard and intentionally weak. In particular, no assumption is made regarding discounting, intertemporal trade-offs, or ethical principles governing sustainability.

#### 4. A Baseline Impossibility on the Unrestricted Domain

We now state the main result of this paper.

**Theorem A' (Baseline impossibility under irreversibility).**

Let  $X = (U \times S)^\mathbb{N}$  be the space of histories with an irreversible state variable. There exists **no** non-trivial social evaluation relation on the unrestricted domain  $X$  that satisfies weak order, finite-permutation anonymity, continuity, and monotonicity with respect to both utilities and the state variable.

In particular, once irreversible state variables are included as outcome components, minimal regularity and anonymity requirements are jointly incompatible on the unrestricted domain.

#### Interpretation.

Theorem A' isolates a purely structural impossibility. The failure does not originate in ethical disagreement, discounting assumptions, or intergenerational trade-offs. Instead, it is driven by the interaction between two features:

- (i) the inclusion of irreversible state variables as outcome components, and
- (ii) the attempt to define social evaluation on the unrestricted product space of histories.

When these features are combined, even the weakest forms of anonymity, monotonicity, and continuity are mutually incompatible. The impossibility therefore precedes questions of normative justification or institutional design.

#### 5. Discussion: What This Paper Establishes—and What It Does Not

This paper establishes a baseline impossibility result for dynamic social evaluation when irreversible state variables are admitted as outcome components and evaluation is defined on the unrestricted domain. The result shows that the difficulty is not ethical or behavioral in nature, but domain-

theoretic.

Equally important is what this paper does **not** do. It does not propose admissible domain restrictions, does not derive representation theorems, and does not analyze institutional responses. These issues are taken up in subsequent work, which studies how coherence can be restored once appropriate restrictions on admissible histories are introduced.

The role of the present paper is therefore diagnostic rather than remedial. By clarifying the source of impossibility, it provides a foundation for subsequent analysis.

### **Preliminary Note**

This is a preliminary discussion paper. The analysis presented here constitutes Part I of an ongoing research project. The results are incomplete and subject to revision. In particular, this paper does not propose admissible domain restrictions or representation results. A companion paper (work in progress) addresses these issues.

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