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"Shareholder Unanimity: A Survey from the Viewpoint of Incomplete Markets"

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Abstract Textbook treatments often identify the objective of the firm with profit maximization, presuming this would be in the interest of its shareholders. We study this hypothesis from the viewpoint of general equilibrium theory with incomplete markets. When the financial market is incomplete, profit maximization is not a well-defined concept, and conflicts of interest between shareholders are likely. We survey major contributions to the theory of shareholder unanimity, based on a categorization into four unanimity criteria. These criteria can be met in certain well-defined classes of economies, although not necessarily through the maximization of profit.

> "The typical form of business unit in the modern world is the corporation. Its most important characteristic is the combination of diffused ownership with concentrated control." (Knight, 1921, p. 291)

1 Introduction

Suppose a single manager makes production decisions in a firm on behalf of a large number of shareholders. A long-standing stance in economic thought is that managers should act in the interest of shareholders. But whose interest is the manager supposed to serve if shareholders change over time, or if the interests of different shareholders are not aligned? This question addresses a central aspect of financial economics. Changing share ownership is a natural feature of financial markets, and conflicts of interest are equally natural when these markets are incomplete. The answer of traditional textbooks is that the manager should choose the production plan

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that maximizes profit. Profit is the compensation that shareholders receive for their investment, and is defined as market value of future output, minus costs of present input. This leads to the next problem: How is the manager supposed to calculate the resulting profit of a production plan, if future output does not have an observable market price due to market incompleteness?

The purpose of this article is to systematically study when and how a firm can find production plans that are unanimously approved by its shareholders. In the course of this study, we survey fifty years of literature on the firm's decision making in general equilibrium with incomplete markets, and outline the current state of the art. Contrary to a large share of the traditional literature, we do not presuppose that profit maximization is a unanimously approved objective (or even social responsibility) of the firm, but treat this merely as a hypothesis. We are able to verify this hypothesis under certain restrictions on the parameters of the model, and for particular notions of shareholder unanimity. At the same time, we find that the hypothesis is not robust to relaxations of the restrictions identified: Several counterexamples illustrate cases in which shareholder reach no unanimous agreement, or agree on objectives other than profit maximization.

In our discussion of shareholder unanimity, we focus on the simplest possible setting with a single firm and two dates. At date 0, the firm chooses its production plan, and input is provided by the original shareholders, who may then trade shares for other assets in the financial market. We do not explain the nature and origin of these other financial assets, but treat them as exogenous objects whose presence allows us to study the implications of different market structures. As we assume that no consumer has any endowment of these assets (and their market-clearing condition is that the net demand is equal to zero), it is perhaps best to think of them as insurance contracts, futures, or options. At date 1, the final shareholders receive the output of the firm pro rata, all other assets pay off, the shareholders consume, and there is no further trade. The financial market is incomplete but free of other frictions. In particular, there are no short sale constraints or bid-ask spreads.

While this simple setting is rich enough to allow us to compare a variety of shareholder unanimity criteria and sufficient conditions proposed in the literature, some important problems are inevitably left out. First, we assume that the firm has not issued (corporate, defaultable) bonds. Then, there is no conflict of interest between shareholders and bondholders, simply because there is no bondholder. Second, there is no asymmetric information between shareholders and managers (or between any agents, for that matter). Then, there is no agency problem arising from the separation of ownership and control. In other words, we concentrate on the conflict of interest between shareholders about the production plan, and identify conditions under which this conflict can be resolved.

The remainder of this article is organized as follows. We introduce recurring notation in Section 2 and the setting in Section 3. In Section 4, we introduce four notions of unanimity, which differ in terms of when the decision is made and who is involved in it. In Section 5, we compare these notions of unanimity briefly with the most common welfare standards. We then present, in Section 6, sufficient conditions under which the decision is unanimously supported by shareholders. Section 7 concludes and gives a brief outlook on extensions to settings more general than ours.

2 Notation

For vectors x in S-dimensional Euclidean space $\mathbb{R}^S$, $x \geq 0$ means $x \in \mathbb{R}^S_+$, $x > 0$ means $x \in \mathbb{R}^S_+ \backslash \{0\}$, and $x \gg 0$ means $x \in \mathbb{R}^S_{++}$. For any two sets, $X \subset Y$ means that X is a proper subset of Y, whereas $X \subseteq Y$ also includes $X = Y$. For a closed, convex set Y, we denote by $N_Y[y] = \{p \in \mathbb{R}^S \mid p \cdot y \geq p \cdot x \text{ for every } x \in Y\}$ the normal cone to Y at the point of evaluation $y \in Y$. Moreover, $B_\varepsilon(y) = \{x \in \mathbb{R}^S \mid \|x - y\| < \varepsilon\}$ is the open ball with radius $\varepsilon > 0$ around y, and $\bar{B}_\varepsilon(y)$ is its closure. For any matrix M, we denote by $\langle M \rangle = \mathrm{Im}(M)$ the column span, and by $\mathrm{pr}_{\langle M \rangle}(x)$ the orthogonal projection of x onto the subspace $\langle M \rangle$. The identity matrix is denoted by $\boldsymbol{I}$, and $\mathbf{1}$ is the vector whose components are all one.

3 Setting

Consider a two-date finance economy with production. Uncertainty at date 1 is represented by a finite state space $\{1, 2, \ldots, S\}$. The economy is populated by a finite number $I \geq 2$ of consumers, and a single firm who owns the production technology. Production takes time: Input is paid at date 0, output is received at date 1. The financial market opens only at date 0.

3.1 Firm

The firm chooses a production plan y from a closed, convex production set $Y \subset \mathbb{R}_- \times \mathbb{R}^S_+$. The production plan is a vector $y = (y_0, y_\mathbf{1})$ that can be decomposed into an input quantity $y_0 \leq 0$ at date 0, and a vector of output quantities $y_\mathbf{1} = (y_1, \ldots, y_S) \geq 0$, one for each of the S states of nature that may realize at date 1.

3.2 Consumers

Consumers are index with superscripts i from an index set $\mathscr{I} = \{1, \ldots, I\}$. Every consumer is endowed with non-financial income (such as labor income)

$\omega^i = (\omega^i_0, \omega^i_{\mathbf{1}}) \in \mathbb{R}^{1+S}_{++}$ that may vary across states, and with an initial share $\delta^i \in [0, 1]$ in the firm. The autarky income of the consumer is thus $\omega^i + y\delta^i$, and it my be traded for a more preferred consumption plan $x^i = (x^i_0, x^i_{\mathbf{1}}) \in \mathbb{R}^{1+S}_{+}$ in the financial market. The consumption preferences of the consumer are represented by a utility function $U^i(x^i)$ of class C^2, that is strictly increasing, and strictly concave in x^i. This function associates with each plan a value on the real line.

Example 1 Additively separable expected utility functions of the von Neumann and Morgenstern (1944) type:

$$U^i(x^i) = u^i_0(x^i_0) + \rho \mathrm{E}(u^i_{\mathbf{1}}(x^i_{\mathbf{1}})) ,$$

in which $u^i_0 : \mathbb{R}_+ \to \mathbb{R}$ and $u^i_{\mathbf{1}} : \mathbb{R}_+ \to \mathbb{R}$ are strictly increasing, strictly concave C^2 functions, and $\rho \in (0, 1)$ is an impatience parameter.

Example 2 Quasilinear utility functions,

$$U^i(x^i) = x^i_0 + u^i_{\mathbf{1}}(x^i_{\mathbf{1}}) ,$$

in which $u^i_1 : \mathbb{R}^S_+ \to \mathbb{R}$ is a strictly increasing, strictly concave C^2 function.

Example 3 Mean-variance utility: There is a probability measure P on the state space $\{1, 2, \ldots, S\}$ such that

$$U^i(x^i) = U^i_{\mathbf{1}}(x^i_{\mathbf{1}}) = \mathrm{E}(x^i_{\mathbf{1}}) - \frac{1}{2\tau}\mathrm{Var}(x^i_{\mathbf{1}}), \tag{1}$$

in which $\tau > 0$ is the risk-tolerance parameter. Here, a distinction made between U^i and $U^i_{\mathbf{1}}$ in that the domain of the former is $\mathbb{R}_+ \times \mathbb{R}^S$ while that of the latter is is $\mathbb{R}^S$. The non-negativity constraint on the date-0 consumption in the domain of U^i is needed when considering the feasibility condition for inputs, and introduces an additional Lagrange multiplier in the first-order condition for a welfare maximization problem. Then,

$$DU^i_{\mathbf{1}}[x^i_{\mathbf{1}}] = [P]\left(\mathbf{1} - \frac{1}{\tau}(x^i_{\mathbf{1}} - \mathrm{E}^P[x^i_{\mathbf{1}}]\mathbf{1})\right), \tag{2}$$

where $[P]$ is the $S \times S$ diagonal matrix whose s-th diagonal element is equal to $P(s)$.[1] Note that $DU^i_{\mathbf{1}}[x^i_{\mathbf{1}}] \cdot \mathbf{1} = 1$ for every $x^i_{\mathbf{1}} \in \mathbb{R}^S$. This class of utility functions will be explored in Section 6.7.

[1] Since mean-variance utility is not strictly increasing, our coverage is limited to equilibria that satisfy the no-arbitrage condition $DU^i_{\mathbf{1}}[x^i] \gg 0$.

3.3 Markets

Trade in the financial market is limited to shares of the firm, and J additional assets whose state-dependent payoffs are collected in an $S \times J$ asset payoff matrix $A = (A^1, \ldots, A^J)$ that is assumed to satisfy $\mathrm{rank}(A) = J$ and is thus free of redundancy. The financial market is said to be *complete* if $\langle A, y_{\mathbf{1}} \rangle = \mathbb{R}^S$, and *incomplete* if $\langle A, y_{\mathbf{1}} \rangle \subset \mathbb{R}^S$. The number of shares outstanding is normalized to one, and the share price $p \geq 0$ thus represents the market value of the firm. Netting out production costs results in $p + y_0$, which is the *profit* or *net market value* of the firm. The remaining J assets are in zero net supply, and their prices are collected in a vector $q = (q_1, \ldots, q_J)$. Implicit in these observable prices, are stochastic discount factors $\pi_{\mathbf{1}} \in \mathbb{R}^S$ that solve the no-arbitrage pricing equation

$$(q, p) = \pi_{\mathbf{1}} \cdot (A, y_{\mathbf{1}}) \,. \tag{3}$$

By means of these stochastic discount factors, the price of an asset is expressed as the discounted present value of its future payoffs. The present value of a unit of consumption at date 0 is one. The combined discount factor vector $\pi = (1, \pi_{\mathbf{1}})$ can be used to rewrite profit in the equivalent form $\pi \cdot y = p + y_0$. Linear algebra implies that (3) has a unique solution when the market is complete, an a continuum of solutions when the market is incomplete.

Each consumer i chooses optimal shareholdings θ^i, and an optimal portfolio of the other assets $z^i = (z^i_1, \ldots, z^i_J)$ subject to the financial market budget constraint

$$\underbrace{x^i}_{\text{consumption}} = \underbrace{\omega^i + \begin{pmatrix} p + y_0 \\ 0 \end{pmatrix} \delta^i}_{\text{income}} + \underbrace{\begin{pmatrix} -q & -p \\ A & y_{\mathbf{1}} \end{pmatrix} \cdot \begin{pmatrix} z^i \\ \theta^i \end{pmatrix}}_{\text{market transaction}} \tag{4}$$

There are no short sale constraints and short sales are represented by negative holdings. Note that holders of initial shares δ^i pay the production input y_0, and holders of final shares θ^i receive the production output $y_{\mathbf{1}}$. Multiplying both sides of (4) with π from the left leads to

$$\pi \cdot x^i = \pi \cdot (\omega^i + y\delta^i) \,,$$

and thus the present value of consumption equals the present value of income. This simplified budget constraint serves as the basis for defining the budget set of the consumer, which contains all feasible and affordable consumption plans:

$$\mathscr{B}(\pi, y, \omega^i, \delta^i) = \left\{ (x^i_0, x^i_{\mathbf{1}}) \in \mathbb{R}^{1+S}_+ \;\middle|\; \begin{matrix} x^i_{\mathbf{1}} - \omega^i_{\mathbf{1}} \in \langle A, y_{\mathbf{1}} \rangle \\ \pi \cdot x^i = \pi \cdot (\omega^i + y\delta^i) \end{matrix} \right\} \tag{5}$$

Put differently: A consumption plan x^i that is not included in $\mathscr{B}(\pi, y, \omega^i, \delta^i)$ can either not be attained through trade in the financial market, or exceeds the budget of the consumer.

3.4 Economy

The economy is specified by the utility functions $U^1, \ldots, U^I$ of all consumers, the production set Y of the firm, and the set of exogenous parameters $\omega = (\omega^1, \ldots, \omega^I)$, $\delta = (\delta^1, \ldots, \delta^I)$, and A. For any fixed production plan y, the endogenous variables (π, x) are determined as a *no-arbitrage equilibrium* in the sense of Magill and Quinzii (1996), Section §10: Consumers take discount factors and payoffs as given, and demand a utility-maximizing consumption plan; discount factors are determined as market-clearing prices that equate demand with supply.

Definition 1 An *equilibrium* (π, x) for fixed production plan y is a tuple of prices and allocation such that

1. for each consumer $i \in \mathscr{I}$, x^i maximizes U^i on $\mathscr{B}(\pi, y, \omega^i, \delta^i)$
2. markets clear: $\sum_{i \in \mathscr{I}} (x^i - \omega^i) = y$.

Even though portfolios (z^i, θ^i) are left implicit in this formulation of equilibrium, these variables can be recovered, if necessary, by inverting Equation (4). Moreover, the sum of Equation (4) over all consumers, and the market clearing condition from Definition 1 jointly imply that $\sum_{i \in \mathscr{I}} z^i = 0$ and $\sum_{i \in \mathscr{I}} \theta^i = 1$. In words: When the economy is equilibrium, the financial market clears.

3.5 Valuation

Once a production plan $\bar{y}$ is chosen, and a corresponding equilibrium $(\pi, \bar{x})$ is computed, the profit of the firm is uniquely determined as $\pi \cdot \bar{y}$. In the same fashion, a value $\pi \cdot y$ can be assigned to any alternative production plan $y \in Y$, but whether this valuation is unique depends on the market structure. When financial markets are incomplete, the concept of no-arbitrage equilibrium is indeterminate: We can always define another price vector $\bar{\pi}_{\mathbf{1}} = \mathrm{pr}_{\langle A, \bar{y}_{\mathbf{1}} \rangle}(\pi_{\mathbf{1}})$, which results in the same budget set $\mathscr{B}(\bar{\pi}, \bar{y}, \omega^i, \delta^i) = \mathscr{B}(\pi, \bar{y}, \omega^i, \delta^i)$ for every consumer, and thus gives rise to another equilibrium $(\bar{\pi}, \bar{x})$ with identical allocation but different stochastic discount factors. In fact, every vector of stochastic discount factors from the subspace $R(\bar{\pi}_{\mathbf{1}}) = \mathrm{pr}^{-1}_{\langle A, \bar{y}_{\mathbf{1}} \rangle}(\bar{\pi}_{\mathbf{1}})$ gives rise to a different equilibrium. The dimension of this subspace is

$$\dim\left(R(\bar{\pi}_{\mathbf{1}})\right) = S - \dim\left(\langle A, \bar{y}_{\mathbf{1}} \rangle\right),$$

and only in the complete market case $\langle A, y_{\mathbf{1}} \rangle = \mathbb{R}^S$, this dimension is zero and the two price vectors from before must agree: $\bar{\pi}_{\mathbf{1}} = \pi_{\mathbf{1}}$. In case of incomplete markets, there is a multiplicity of stochastic discount factors $\bar{\pi}_{\mathbf{1}} \neq \pi_{\mathbf{1}}$ that agree on the valuation $\bar{\pi} \cdot \bar{y} = \pi \cdot \bar{y}$ of the status quo, as well as of any plan in the asset span,

$$\bar{\pi} \cdot y = \pi \cdot y \qquad \forall y \in \mathbb{R} \times \langle A, \bar{y}_{\mathbf{1}} \rangle, \tag{6}$$

but disagree on the valuation of plans outside the asset span $\mathbb{R} \times \langle A, \bar{y}_{\mathbf{1}} \rangle$. Such disagreement can be given a subjective interpretation by means of the marginal rates of substitution

$$\pi^i(x^i) = \frac{DU^i[x^i]}{D_{x_0}U^i[x^i]}.$$

By concavity of the utility function, the set $\{x^i \in \mathbb{R}_+^{1+S} \,|\, U^i(x^i) > U(\bar{x}^i)\}$ of consumption plans preferred to the status quo $\bar{x}^i$ is convex, and $\pi^i(\bar{x}^i)$ defines its supporting hyperplane. That is to say, if x^i is preferred to $\bar{x}^i$, its subjective valuation must be positive:

$$U^i(x^i) > U^i(\bar{x}^i) \quad \Longrightarrow \quad \pi^i(\bar{x}^i) \cdot (x^i - \bar{x}^i) > 0 \tag{7}$$

The first-order conditions of utility maximization imply that $\pi_{\mathbf{1}}^i(\bar{x}^i) \in R(\bar{\pi}_{\mathbf{1}})$ for every consumer (see Magill and Quinzii (1996), p. 85). Preferences are, so to speak, aligned along the asset span, and marginal rates $\pi_{\mathbf{1}}^i(\bar{x}^i)$ agree on the valuation of marketed assets. By contrast, valuation outside the asset span is subjective, and varies across consumers. A subjective valuation that is higher for an alternative production plan y than for the chosen plan $\bar{y}$ indicates a potential utility gain. This is illustrated in Figure 1: Consumer i is the only shareholder of the firm who owns the production set Y. If the firm produces $\bar{y} \in Y$, the shareholder receives its net market value $\bar{\pi} \cdot \bar{y}$ as a profit, and chooses the consumption plan $\bar{x}^i$ that attains the highest feasible utility level $U^i(\bar{x}^i)$. However, a deviation from $\bar{y}$ to the alternative production plan y is valued positively: $\pi^i(\bar{x}^i) \cdot (y - \bar{y}) > 0$. If this deviation is paid as a dividend to the shareholder, a consumption plan $x^i = \bar{x}^i + (y - \bar{y})\theta^i$ in the preferred set can be reached. As a consequence, the shareholder disapproves the current choice $\bar{y}$. In this simple example, $\theta^i = 1$ and thus a single shareholder receives the entire profit, but it should be clear that the argument remains valid for any positive share $\theta^i > 0$ in the firm.

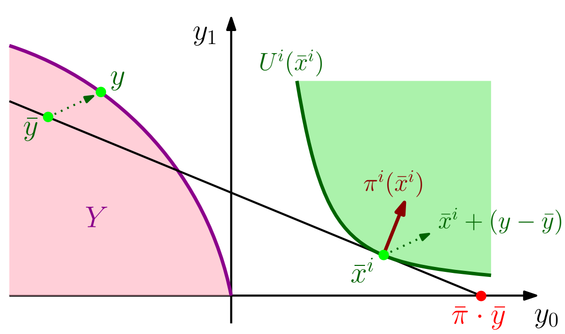

Fig. 1 Utility maximization at $\bar{x}^i$ implies that all preferred consumption plans lie on one side of the hyperplane supported by $\pi^i(\bar{x}^i)$. Subjective value is not maximized at $\bar{y}$ because Y is not entirely on the other side of the hyperplane.

4 Shareholder Unanimity

All unanimity criteria in the literature are based on a utility comparison between a status-quo production plan $\bar{y} \in Y$ and a feasible alternative $y \in Y$. There are, at least, two aspects that set different unanimity criteria apart. One is is the *timing* of this comparison: Ex-ante criteria study a deviation from $\bar{y}$ to y before the market opens. In response to such a deviation, prices will change and consumer will rebalance their portfolios. These adjustments have to be factored in when computing utility gains and losses. This mirrors imperfect competition since the market power of the firm is taken into account. Ex-post criteria study the same deviation after the market has closed. Changes in portfolios and prices can no longer occur, and are thus disregarded when computing utility gains and losses. This is much better aligned with perfect competition and price-taking behavior.

The other aspect that distinguishes different unanimity criteria is the *scope* of the utility comparison: Individual criteria compare the status quo $\bar{y}$ to a dictatorial solution. As a thought experiment, every shareholder is given the power to change the production plan unilaterally. If no single shareholder can gain utility through such an enforced deviation, the unanimity criterion is met. Group criteria compare the status quo to a cooperative solution. Shareholders can agree on a deviation from $\bar{y}$ to y as well as on mutual side payments that compensate the losers of the change. If no such agreement can realize a Pareto improvement among the group of shareholders, the unanimity criterion is met.

Group criteria permit us to address the hypothesis that shareholders want the firm to maximize its profit. As these criteria are based on a Pareto ranking, the first test is a local comparison: If the first-order conditions for profit maximization do not agree with the first-order conditions for Pareto optimality among shareholders, the hypothesis can be rejected immediately. If the first-order conditions agree, the second test is a global study of the maximization problem: Only when the typical assumptions of convex optimization are fulfilled, we can be sure that the first-order conditions are indeed sufficient for a solution, and not only necessary.

4.1 Ex-post individual unanimity

Suppose the status quo $\bar{y}$ has been decided, all input has been paid by the original shareholders, trade has taken place, and the market has closed. At this ex-post stage, feasible deviations are limited to production plans y that can be financed with the given input, and thus belong to the closed, convex set

$$\mathscr{Y}(\bar{y}) = \{y \in Y \mid y_0 = \bar{y}_0\}. \tag{8}$$

To what degree a consumer is affected by such a deviation depends on the share held, and this share depends on the market structure: If $\langle A \rangle \subset \langle A, y_{\mathbf{1}} \rangle$, shares are not redundant, and final shareholdings θ^i will generically differ from initial shareholdings δ^i. The change in consumption x^i is then $(y - \bar{y})\theta^i$ and thus proportional to final shareholdings. Suppose each member of the group of final shareholders $\mathscr{F} = \{i \in \mathscr{I} \mid \theta^i > 0\}$ had a veto right, and could enforce an alternative production plans y without the consent of the others. Unanimity is reached if all final shareholders forego their right.

Definition 2 A plan $(\bar{x}, \bar{y})$ satisfies *ex-post individual unanimity* if there is no alternative $y \in \mathscr{Y}(\bar{y})$ such that

$$U^i(\bar{x}^i + (y - \bar{y})\theta^i) \; > \; U^i(\bar{x}^i)$$

for some final shareholder $i \in \mathscr{F}$.

To understand the geometry of ex-post individual unanimity, it is helpful to note that $\bar{y}$ from Definition 2 must be a solution to

$$\max_y U^i(\bar{x}^i + (y - \bar{y})\theta^i) \quad \text{subject to} \quad y \in \mathscr{Y}(\bar{y})$$

for every shareholder $i \in \mathscr{F}$. This is a typical convex optimization problem, and the first-order condition

$$\theta^i DU^i[\bar{x}^i] \; \in \; N_{\mathscr{Y}(\bar{y})}[\bar{y}] \,. \tag{9}$$

characterizes its solution $\bar{y}$ (see Magill and Quinzii (1996), p. 410, Theorem A6.2). Since $\theta^i > 0$, the left-hand side can be divided by $\theta^i D_{x_0} U^i[\bar{x}^i]$, as a normalization. This leads to a simplified equivalent condition:

$$\pi^i(\bar{x}^i) \; \in \; N_{\mathscr{Y}(\bar{y})}[\bar{y}] \tag{10}$$

If $\langle A \rangle = \langle A, y_{\mathbf{1}} \rangle$, the share is redundant, and the budget set (5) would not change if trade in shares were restricted. This case permits us to concentrate our analysis on an equilibrium at which shares are not traded. The change in consumption x^i is then $(y - \bar{y})\delta^i$ and thus proportional to initial shareholdings. However, once we apply the same normalization, we again arrive at (10), which is necessary and sufficient for ex-post individual unanimity.

This condition is illustrated in Figure 2: Pick an arbitrary status quo $\bar{y}$ that satisfies condition (10) for each consumer $i \in \mathscr{F}$. Since $\pi^i(\bar{x}^i) \gg 0$ by assumption, $\bar{y}$ must be a boundary point of Y. If the input is fixed at $\bar{y}_0$, the set of feasible alternative production plans y reduces to a slice $\mathscr{Y}(\bar{y})$ of the production set Y. Provided the distance $\|y - \bar{y}\|$ is sufficiently small, every deviation along the boundary can be viewed as moving up or down the dashed tangent. The unanimity criterion is fulfilled if and only if the marginal rates of substitution $\pi^i(\bar{x}^i)$ of all final shareholders are

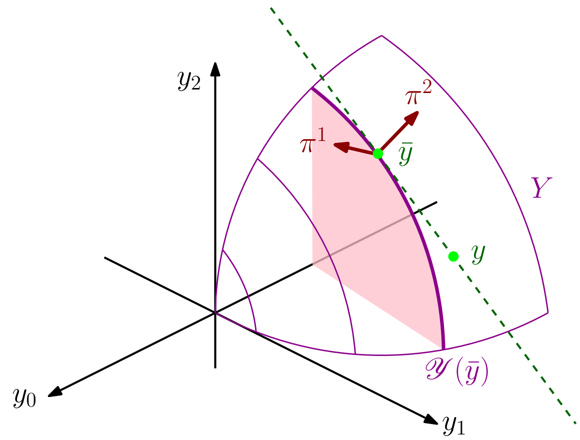

Fig. 2 Ex-post individual unanimity: Marginal changes in output (y_1, y_2) are tangential to $\mathscr{Y}(\bar{y})$ and thus orthogonal to the marginal rates vectors $\pi^i(\bar{x}^i)$ of all shareholders.

perpendicular to the tangent. This is illustrated for two shareholders in the figure: These two consumers disagree about the production scale. Consumer 1 would have preferred less input, while Consumer 2 would have preferred more input. However, given the fixed input $\bar{y}_0$, both agree that $\bar{y}_{\mathbf{1}}$ is the right output combination. The subjective valuation of every feasible deviation is nonpositive: $\pi^i(\bar{x}^i) \cdot (y - \bar{y}) \leq 0$ for both consumers.

Speaking of shareholder unanimity seems like an overstatement at this stage because the two consumers agree only on the output ray $\langle \bar{y}_{\mathbf{1}} \rangle$, but disagree about the production scale $\bar{y}_0$. Indeed, we have not yet factored in that $\bar{y}$ is not picked arbitrarily, but under the market clearing condition from Definition 1. Market clearing results in a price vector $\bar{\pi}$ that leads to congruent budget sets $\mathscr{B}(\bar{\pi}, \bar{y}, \omega^i, \delta^i)$, which differ across consumers only by a parallel shift due to differences in endowments (ω^i, δ^i). Figure 3 depicts one such budget set, for simplicity of a consumer who initially owns the entire firm $\delta^i = 1$ but has zero non-financial income $\omega^i = 0$. If the output ray $\langle \bar{y}_{\mathbf{1}} \rangle$ is fixed, the set of feasible alternative production plans y reduces to a change in production scale. Provided the distance $\|y - \bar{y}\|$ is sufficiently small, every deviation along the boundary consists of moving up or down the budget set. But since the marginal rates of substitution $\pi^i(\bar{x}^i)$ of all consumers are perpendicular to the budget set at an equilibrium, it is clear that the subjective valuation of every feasible deviation $\pi^i(\bar{x}^i) \cdot (y - \bar{y}) \leq 0$ is nonpositive.

As the production set Y is convex by assumption, this is not only true for local production changes $y \in B_\varepsilon(\bar{y})$, but extends to a global property. The first-order condition (10) is both necessary and sufficient for ex-post individual unanimity. Figure 3 also helps explain the frequent assertion in the unanimity literature, that

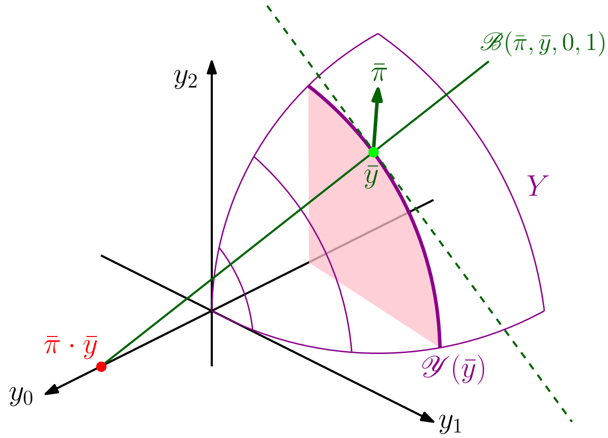

Fig. 3 Shareholders agree on the optimal production scale: Small deviations move along the budget set, and are thus orthogonal to all utility gradients.

shareholders want the firm to maximize its profit $\bar{\pi} \cdot \bar{y}$. If both the output ray $\langle \bar{y}_{\mathbf{1}} \rangle$ and the price vector $\bar{\pi}$ were fixed, every feasible alternative y would indeed lead to a lower profit $\bar{\pi} \cdot y \leq \bar{\pi} \cdot \bar{y}$, which shifts the budget set downward, and thus reduces the purchasing power of the consumer. What this does not explain though, just like the literature in question does not explain it, is why consumers would still care about their nominal purchasing power after the market has already closed. We shall defer a thorough critique of this point for the time being, and rather complement the ex-post unanimity criterion with a variant that permits side payments.

4.2 Ex-post group unanimity

Suppose the group of final shareholders $\mathscr{F}$ controls the firm, and may implement a scheme of side payments $\upsilon = (\upsilon^1, \ldots, \upsilon^I) \in \mathbb{R}^I$ to make all members agree on a deviation from $\bar{y}$ to y. All side payments take place at date 0, such that no commitment problems arise. Unanimity is reached if there is no alternative y and no such scheme that makes all members of $\mathscr{F}$ better off.

Definition 3 A plan $(\bar{x}, \bar{y})$ satisfies *ex-post group unanimity* if there are no alternative $y \in \mathscr{Y}(\bar{y})$ and no side payments $\upsilon \in \mathbb{R}^I$ such that $\sum_{i \in \mathscr{F}} \upsilon^i = 0$ and

$$U^i(\bar{x}^i_0 + \upsilon^i, \bar{x}^i_{\mathbf{1}} + (y_{\mathbf{1}} - \bar{y}_{\mathbf{1}})\theta^i) \;>\; U^i(\bar{x}^i)$$

for every final shareholder $i \in \mathscr{F}$.

It follows from Definition 3, that $\bar{y}$ must be a solution to the vector maximization problem

$$\operatorname{vec}\max_{y,\upsilon} \left\{ U^i \left(\bar{x}^i + \begin{pmatrix} \upsilon^i \\ \mathbf{0} \end{pmatrix} + (y - \bar{y})\theta^i \right) \right\}_{i \in \mathscr{F}} \quad \text{subject to} \quad y \in \mathscr{Y}(\bar{y}) \quad \text{and} \quad \sum_{i \in \mathscr{F}} \upsilon^i = 0\,.$$

The system of first-order conditions is of the form

$$\begin{aligned} \sum_{i \in \mathscr{F}} \alpha^i \theta^i D U^i[\bar{x}^i] \;&\in\; N_{\mathscr{Y}(\bar{y})}[\bar{y}] \\ \alpha^i D_{x_0} U^i[\bar{x}^i] \;&=\; \lambda \qquad \forall i \in \mathscr{F} \end{aligned}$$

in which $(\alpha, \lambda) > 0$ is a vector of multipliers (see Magill and Quinzii (1996), p. 412, Theorem A6.5). Since the system of first-order conditions is invariant to rescaling the multiplier vector (α, λ), one can set $\lambda = 1$ and determine the remaining components as $\alpha^i = (D_{x_0} U^i[\bar{x}^i])^{-1}$. This leads to the equivalent condition

$$\sum_{i \in \mathscr{F}} \theta^i \pi^i(\bar{x}^i) \;\in\; N_{\mathscr{Y}(\bar{y})}[\bar{y}]\,. \tag{11}$$

Since $\mathscr{Y}(\bar{y})$ is convex and utility functions U^i are strictly concave, (11) is necessary and sufficient for ex-post group unanimity. Moreover, the first-order conditions for ex-post individual unanimity (10) imply the above condition (11) by means of weighted summation, which shows that ex-post group unanimity is the weaker criterion. If $\mathscr{F}$ is a singleton because one consumer buys all shares, this criterion reduces to ex-post individual unanimity. The geometry of ex-post group unanimity is illustrated in Figure 4: Neither of the two consumers individually agrees with the status quo $\bar{y}$. Consumer 1 would rather have more output in the second state, while Consumer 2 would rather have more output in the first state. However, they cannot agree on side payments that would put the partner in favor of such a change: The share-weighted sum $\bar{\pi} \in N_{\mathscr{Y}(\bar{y})}[\bar{y}]$ exactly satisfies the first-order condition for ex-post group unanimity.

This has implications for the objective of the firm. Since $N_Y[y] \subseteq N_{\mathscr{Y}(\bar{y})}[y]$ holds by construction, (11) is met if the firm applies a value maximization criterion proposed by Drèze (1974):

$$\max_y \sum_{i \in \mathscr{F}} \theta^i \pi^i(\bar{x}^i) \cdot y \quad \text{subject to} \quad y \in Y \tag{12}$$

The stochastic discount factors used for computing the value of the firm, are a share-weighted sum of marginal rates of substitution. Final shares are used as weights. As the objective function is linear in y, and Y is a convex set, the first-order condition for ex-post group unanimity (11) is both necessary and sufficient for a solution to the above maximization problem. As a consequence, the standard of ex-post group unanimity can be met in *any* economy, provided the firm chooses its production plan according to the Drèze criterion (12). Moreover, such *Drèze equi-*

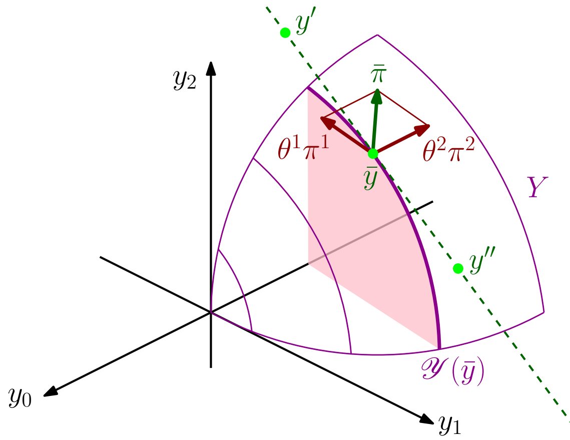

Fig. 4 Ex-post group unanimity: Consumer 1 prefers a change to y' because $\boldsymbol{\pi}^1(\bar{x}^1)\cdot(y'-\bar{y})>0$; conversely, Consumer 2 prefers a change to y''. The share-weighted sum $\bar{\boldsymbol{\pi}}$ is exactly orthogonal to the tangent: $\bar{\boldsymbol{\pi}}\cdot(y'-\bar{y})=0$

libria are the only candidates for market outcomes that satisfy ex-post individual unanimity, simply because that is a stronger criterion. It should be noted that set of such candidates may be empty: Unless $Y\subset\mathbb{R}\times\langle A\rangle$, a condition that we will study in detail in Section 4, the dimension of the asset span $\langle A, y_{\mathbf{1}}\rangle$ is endogenous. As Momi (2001) points out, Drèze equilibria need not exist in that case.

The hypothesis that shareholders want the firm to maximize profit is only supported in a weak sense: The firm in (12) maximizes a weighted sum of purely subjective valuations of its production plan. In equilibrium, $\bar{y}_{\mathbf{1}}\in\langle A,\bar{y}_{\mathbf{1}}\rangle$ and these subjective valuations indeed agree with the profit of the firm. However, outside the asset span, the market provides no price signals, and subjective valuations disagree. Ex-post group unanimity can only be reached because the Drèze criterion provides a blueprint for reaching consensus. Examples of economies in which this criterion selects production plans that satisfy ex-post group unanimity can be found in the original contribution of Drèze (1974), and in Section §31 of the textbook of Magill and Quinzii (1996). The latter authors were the first to prove that ex-post group unanimity implies the Drèze criterion.

4.3 Ex-ante individual unanimity

Suppose the market has not yet opened, and consider the following thought experiment: The manager of the firm is proposing a production plan $\bar{y}\neq 0$ to the group of original shareholders $\mathscr{O}=\{i\in\mathscr{I}\mid\delta^i>0\}$. These shareholders take the market power of the firm into account, and understand how the choice of $\bar{y}$ affects market clearing prices. It is therefore easy for them to make consumption plans $\bar{x} = (\bar{x}^1, \ldots, \bar{x}^I)$. Unanimity is reached if no member of the original shareholder benefits from exercising a veto right.

Definition 4 A plan $(\bar{x}, \bar{y})$ satisfies *ex-ante individual unanimity* if there is no alternative $y \in Y \backslash \{0\}$ with a resulting equilibrium (π, x) such that

$$U^i(x^i) \ > \ U^i(\bar{x}^i)$$

for some original shareholder $i \in \mathscr{O}$.

This criterion is particularly demanding if there are multiple equilibria. In that case, Definition 4 requires that even the most favorable market outcome does not lead to a utility improvement. Moreover, it should be noted that the definition does not permit shareholders to shut down production completely. Such a restriction was not necessary at the ex-post stage: Consumers who are better off if the firm does not produce would not buy shares in the first place. By contrast, at the ex-ante stage, original shareholders make the decisions. But the assignment of initial shares is exogenous and need not fulfill any rationality criterion. It is possible that initial shares are held by a consumer who prefers an inactive firm and thus vetoes against any nonzero production plan. Since aggregate demand exhibits a discontinuity at zero, a problem discussed by Zierhut (2019), we could not even describe this unanimity criterion locally, be means of first-order conditions. However, when $\langle A \rangle \subset \langle A, \bar{y}_{\mathbf{1}} \rangle$, we can usually apply the implicit function theorem to solutions of the equilibrium equations. That way, we can find for every equilibrium $(\bar{\pi}, \bar{x})$ a family of C^1 functions $(x^{1*}, \ldots, x^{I*})$ that solve the equilibrium equation $\sum_{i \in \mathscr{I}} (x^{i*}(y) - \omega^i) = y$ in a neighborhood $B_\varepsilon(\bar{y})$. In particular, $x^{i*}(\bar{y}) = \bar{x}^i$ and ex-ante individual unanimity can be locally expressed as a solution to

$$\max_y U^i(x^{i*}(y), y) \quad \text{subject to} \quad y \in Y \cap \bar{B}_\varepsilon(\bar{y})$$

for every shareholder $i \in \mathscr{O}$. This leads to the first-order condition

$$\pi^i(\bar{x}^i) \cdot Dx^{i*}[\bar{y}] \ \in \ N_Y[\bar{y}] \, . \tag{13}$$

It should be emphasized that we have no reason to believe that $x^{i*}(y)$ is quasi-concave, and thus the first-order condition (13) is only necessary but by no means sufficient for ex-ante individual unanimity. Nevertheless, it is sufficient to reject the hypothesis that powerful shareholders favor net market value maximization. If Equation (13) suggests any value maximizing behavior, then at best that consumer want to maximize the subjective valuation of their consumption plan, not the market valuation of the production plan.

4.4 Ex-ante group unanimity

Suppose the group of original shareholders $\mathscr{O}$ controls the firm, and may use a scheme of side payments. Unanimity is reached if there is no scheme under which a deviation from $\bar{y}$ makes all members of $\mathscr{O}$ better off. The market power of the firm is taken into account.

Definition 5 A plan $(\bar{x}, \bar{y})$ satisfies *ex-ante group unanimity* if there are no alternative $y \in Y\backslash\{0\}$ with a resulting equilibrium (π, x), and no side payments $\upsilon \in \mathbb{R}^I$ such that $\sum_{i\in\mathscr{O}} \upsilon^i = 0$ and

$$U^i(x_0^i + \upsilon^i,\, x_{\mathbf{1}}^i) \;>\; U^i(\bar{x}^i)$$

for every original shareholder $i \in \mathscr{O}$.

If a plan satisfies this criterion, then $\bar{y}$ must be a solution to the local vector maximization problem

$$\operatorname*{vec\,max}_{y,\upsilon} \left\{U^i\left(x^{i*}(y) + \begin{pmatrix}\upsilon^i \\ \mathbf{0}\end{pmatrix}\right)\right\}_{i\in\mathscr{O}} \qquad \text{subject to} \qquad y \in Y \cap \bar{B}_\varepsilon(\bar{y})$$
$$\text{and} \qquad \sum_{i\in\mathscr{O}} \upsilon^i = 0\,.$$

Analogously to the ex-post criterion, the system of first-order conditions can be consolidated to the a single inclusion

$$\sum_{i\in\mathscr{O}} \pi^i(\bar{x}^i)\cdot Dx^{i*}[\bar{y}] \;\in\; N_Y[\bar{y}]\,. \tag{14}$$

If $\mathscr{O}$ is a singleton because one consumer initially owns the entire firm, this criterion reduces to ex-ante individual unanimity. The criterion of ex-ante group unanimity is studied by Bejan (2020), who defines it under the name C-efficiency for arbitrary coalitions of controlling shareholders, in the present case $C = \mathscr{O}$. One of her key findings is that the firm must maximize shareholders' surplus from trading in the financial market, and not just the market value of the production plan. The objective of the firm then assumes the following form:

$$\max_{y\in Y} \sum_{i\in\mathscr{O}} \pi^i(\bar{x}^i)\cdot\left(x^{i*}(y) - \omega^i\right) \tag{15}$$

In equilibrium, the objective is indeed a market value because $x_{\mathbf{1}}^{i*}(\bar{y}) - \omega_{\mathbf{1}}^i \in \langle A, \bar{y}_{\mathbf{1}}\rangle$ by definition of the budget set (5), and subjective valuations agree on the asset span. It is easy to see that the first-order condition of this maximization problem coincides with the first-order condition (14) for ex-ante group unanimity. The difficulty with the objective (15) is that it only lends itself to a concept of equilibrium if $x^{i*}(y)$ is not only locally, but also globally a well-behaved function. It can only be globally C^1 if every choice of production plan y leads to a unique equilibrium.

Equilibrium uniqueness is guaranteed under a few particular forms of utility function, including quasilinear utility, quadratic expected utility, and expected utility with relative risk aversion less than one (see Hens and Pilgrim (2002), Chapter 6.4). Surprisingly, there are simple sufficient conditions on the parameters (Y, ω, δ, A) that turn (15) into a classical profit maximization objective. We discuss these conditions in Section 6.

5 Unanimity versus welfare

Shareholder unanimity is not a welfare standard. Unanimity criteria can always be thought of addressing a question of the following kind: Suppose production decisions are made by a manager. Can this manager choose a production plan, possibly in combination with side payments, that is in the interest of the firm's shareholders? By contrast, the question behind welfare standards is along the following lines: Suppose production decisions are made by a social planner. Can this planner choose a production plan, possibly in combination with side payments, that is in the interest of society? Unless every member of society happens to be a shareholder, these two questions are clearly distinct. In particular, the social planner has a much larger choice set. Contrary to the manager of a firm, the planner can enforce transfers from and to outsiders, who do not hold shares.

In the context of incomplete markets, the usual welfare standards is constrained efficiency. This standard constrains the planner to transfers that are also feasible in the financial market: The constrained planner can reallocate initial shares, final shares, and consumption at date 0 freely, but cannot make any transfers outside the asset span at date 1. A plan $(\bar{x}, \bar{y})$ is called *constrained efficient* if such a planner cannot implement an alternative plan that makes every consumer better off. The first-order conditions of the constrained planner are of the following form (see Magill and Quinzii (1996), p. 370):

$$\pi_{\mathbf{1}}^{i}(\bar{x}^{i}) \cdot A = q \qquad \forall i \in \mathscr{I} \tag{16}$$

$$\sum_{i \in \mathscr{I}} \theta^{i} \pi^{i}(\bar{x}^{i}) \in N_{Y}[\bar{y}] \tag{17}$$

In spite of the strong similarity between (17) and the first-order condition of ex-post group unanimity (11), the relevant consumer base is different: Whereas constrained efficiency requires that preferences of short sellers are taken into account, these are disregarded in Definition 3. Only in the particular case of $\mathscr{F} = \mathscr{I}$, the first-order conditions for constrained efficiency and ex-post group unanimity agree. However, this does not render the concepts equivalent. While the first-order conditions are sufficient for the unanimity criterion, they are not sufficient for constrained efficiency, owing to a nonconvexity in the choice set of the constrained planner (see

Magill and Quinzii (1996), Section §31 for a discussion). As a consequence, every constrained efficient plan satisfies ex-post group unanimity, but the converse does not hold. This is underlined by an example of Dierker, Dierker, and Grodal (2002), whose unique Drèze equilibrium indeed satisfies $\mathscr{F} = \mathscr{I}$ as well as ex-post group unanimity, but fails to be constrained efficient.

The same example is illustrative of another special case, namely when every consumer initially holds some share of the firm, and thus $\mathscr{O} = \mathscr{I}$. The example of Dierker, Dierker, and Grodal (2002) also fits here because no profit is made at the Drèze equilibrium, and thus initial shares drop out of the budget constraint. Therefore, the distribution of initial shares is irrelevant, every consumer can be viewed as an original shareholder, and in this case constrained efficiency coincides with ex-ante group unanimity. This leads to the surprising outcome that ex-post group unanimity can be met, but ex-ante group unanimity cannot, even though the first-order conditions of both criteria are satisfied. This shows that ex-ante group unanimity is a stronger criterion, even when the groups of original and final shareholders coincide.

Matters are different when the higher welfare standard of Pareto efficiency is met. The social planner behind Pareto efficiency is more powerful than its constrained counterpart, and can also redistribute consumption at date 1 freely. A plan $(\bar{x}, \bar{y})$ is called *Pareto efficient* if this powerful planner cannot implement an alternative plan that makes every consumer better off. The first-order conditions for Pareto efficiency are:

$$\pi^1(\bar{x}^1) \; = \; \cdots \; = \; \pi^I(\bar{x}^I) \tag{18}$$

$$\pi^i(\bar{x}^i) \in N_Y[\bar{y}] \qquad \forall i \in \mathscr{I} \tag{19}$$

These conditions are highly unlikely to be met when the financial market is incomplete: For any combination of strictly increasing, strictly concave utility functions that satisfy a boundary condition, and for any fixed production plan $\bar{y}$, condition (18) is generically violated (see Magill and Quinzii (1996), p. 102, Theorem 11.6). Here *generic* means that the condition holds for some combinations of endowments $\omega = (\omega^1, \ldots, \omega^I)$, but these combinations have Lebesgue measure zero in the space of endowments, and thus correspond to an exceptional constellation of model parameters. However, as will be discussed in Section 6.7, there are certain joint restrictions on utility functions and endowments that guarantee (18) even when markets are incomplete, and this leads to stronger forms of shareholder unanimity.

In absence of such restrictions on utility functions and endowments, stronger unanimity criteria are at odds with welfare standards. Zierhut (2017) shows that ex-post individual unanimity is generically incompatible with constrained efficiency. This result is obtained in an economy with no other assets A. The presence or absence of other assets matters: As will be clarified in the following section, there are certain spanning conditions for A and Y that help attain stronger unanimity criteria.

6 Sufficient conditions

This section identifies, on the one hand, families of production sets Y and market structures A that guarantee some form of shareholder unanimity for arbitrary consumer characteristics, and on the other hand, families of utility functions U^i that guarantee the same for arbitrary production technologies. All of these conditions are empirically verifiable in nature.

6.1 Ex-post spanning

The ex-post spanning condition of Ekern and Wilson (1974) requires the production set to be fully covered by the asset span. Since the asset span is endogenous, this condition can only be verified after the production plan $\bar{y}$ has been chosen.

Definition 6 A production set Y satisfies the *ex-post spanning* condition if

$$Y \subset \mathbb{R} \times \langle A, \bar{y}_\mathbf{1} \rangle$$

for the status-quo production plan $\bar{y} \in Y$.

Under ex-post spanning, every feasible output combination $y_\mathbf{1}$ is contained in the asset span and thus has a market value $\bar{\pi}_\mathbf{1} \cdot y_\mathbf{1}$. Therefore, the objective of profit maximization is well-defined and leads to ex-post individual unanimity:

Proposition 1 *Let $(\bar{\pi}, \bar{x}) \gg 0$ be an equilibrium for fixed production plan $\bar{y} \in Y$. If Y satisfies the ex-post spanning condition and*

$$\bar{y} = \arg\max_{y \in Y} \bar{\pi} \cdot y \,,$$

then $(\bar{x}, \bar{y})$ satisfies ex-post individual unanimity.

Proof. Combining Definition 6 with Equation (6) leads to $\bar{\pi} \cdot y = \pi^i(\bar{x}^i) \cdot y$ for every production plan $y \in Y$ and every consumer $i \in \mathscr{F}$. Thus, $\pi^i(\bar{x}^i)$ can be substituted for $\bar{\pi}$ in the above maximization problem, whose first-order condition then implies that $\pi^i(\bar{x}^i) \in N_Y[\bar{y}]$. Since $N_Y[\bar{y}] \subset N_{\mathscr{Y}(\bar{y})}[\bar{y}]$ by definition (8), the first-order condition for ex-post individual unanimity (10) is satisfied. This first-order condition is necessary and sufficient because $\mathscr{Y}(\bar{y})$ is closed and convex. □

The logic of Proposition 1 is illustrated in Figure 5: Profit maximization leads to a status quo $\bar{y}$ at the boundary of the production set Y. The share payoffs $\bar{y}_\mathbf{1}$ and one additional asset with linearly independent payoffs A^1 are sufficient to span the two-dimensional production set. The budget plane separates higher utility levels from feasible alternatives $y \in Y$. Since subjective valuations agree on that span, every consumer assigns a nonpositive value $\pi^i(\bar{x}^i) \cdot (y - \bar{y}) \leq 0$ to feasible deviations, which precludes a utility improvement. Since ex-post group unanimity is a weaker concept, this proposition has the following immediate corollary.

Corollary 1 *Under the same assumptions as Proposition 1, the plan* $(\bar{x}, \bar{y})$ *satisfies ex-post group unanimity.*

Indeed, the ex-post spanning condition guarantees that market valuations $\bar{\pi} \cdot y$ and subjective valuations $\pi^i(\bar{x}^i) \cdot y$ agree for every production plan $y \in Y$. In this case, the objective from Proposition 1 yields exactly the same production plan as the Drèze criterion (12).

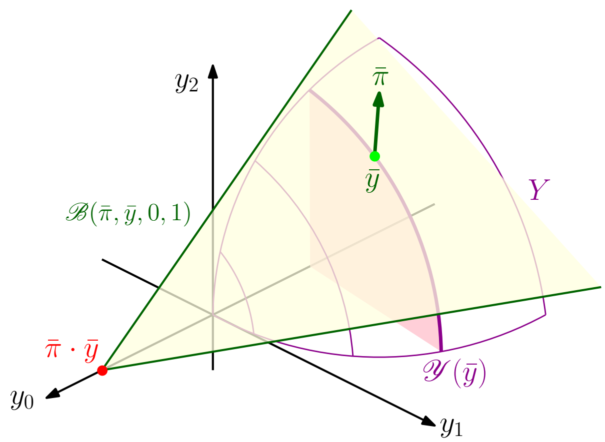

Fig. 5 The spanning condition: There is no disagreement in subjective valuation when the asset span covers the entire budget set.

6.2 Ex-ante spanning

The ex-ante spanning condition requires that the production set is covered by the span of all other assets. This condition is fully stated in terms of exogenous objects, and is therefore a property of the economy, not of a particular equilibrium. The formal definition of ex-ante spanning is due to Zierhut (2019), but the concept itself was verbally described before by DeAngelo (1981).

Definition 7 A production set Y satisfies the *ex-ante spanning* condition if

$$Y \subset \mathbb{R} \times \langle A \rangle.$$

This condition is already verifiable at the ex-ante stage, and leads to a stronger unanimity criterion provided *every* consumer is endowed with initial shares of the firm, and thus a member of $\mathscr{O}$.

Proposition 2 *Let $(\bar{\pi}, \bar{x}) \gg 0$ be an equilibrium for fixed production plan $\bar{y} \in Y$. If $\mathscr{O} = \mathscr{I}$, and Y satisfies the ex-ante spanning condition, and*

$$\bar{y} = \arg\max_{y \in Y} \bar{\pi} \cdot y \,,$$

then $(\bar{x}, \bar{y})$ satisfies ex-ante group unanimity.

Proof. Suppose there were an alternative $y \in Y$, a resulting equilibrium (π, x), and side payments $\upsilon \in \mathbb{R}^I$ that sum up to zero such that

$$U^i\left(x^i + \begin{pmatrix} \upsilon^i \\ \mathbf{0} \end{pmatrix}\right) \;>\; U^i(\bar{x}^i) \qquad \forall i \in \mathscr{I}\,.$$

By Equation (7), this implies

$$\pi^i(\bar{x}^i) \cdot \left(x^i + \begin{pmatrix} \upsilon^i \\ \mathbf{0} \end{pmatrix} - \bar{x}^i\right) > 0 \qquad \forall i \in \mathscr{I}\,. \tag{20}$$

By definition of budget set (5), $x^i_{\mathbf{1}} - \omega^i_{\mathbf{1}} \in \langle A, y_{\mathbf{1}} \rangle$ for every $y \in Y$. In conjunction with Definition 7, this reduces to

$$\begin{aligned} x^i_{\mathbf{1}} - \omega^i_{\mathbf{1}} &\in \langle A \rangle \\ \bar{x}^i_{\mathbf{1}} - \omega^i_{\mathbf{1}} &\in \langle A \rangle \,. \end{aligned}$$

Subtracting the second line from the first leads to $x^i_{\mathbf{1}} - \bar{x}^i_{\mathbf{1}} \in \langle A \rangle$. It follows from Equation (6) that $\bar{\pi}$ can be substituted for $\pi^i(\bar{x}^i)$ in (20). Summing over all consumers results in

$$\bar{\pi} \cdot \sum_{i \in \mathscr{I}} (x^i - \bar{x}^i) + \underbrace{\sum_{i \in \mathscr{I}} \upsilon^i}_{=0} > 0\,.$$

By the market clearing condition from Definition 1, this is equivalent to $\bar{\pi} \cdot (y - \bar{y}) > 0$, but if that were the case, $\bar{y}$ could not have been a solution to the maximization problem – a contradiction. □

The intuition behind Proposition 2 is easy to see in the construction of the objective function. Ex-ante group unanimity requires that the objective of the firm is of the form (15), and under ex-ante spanning this becomes

$$\max_{y \in Y} \bar{\pi} \cdot \sum_{i \in \mathscr{O}} (x^{i*}(y) - \omega^i)$$

Deviations at the ex-ante stage cause price changes, which affect not only shareholders but also all other consumers who trade in some of the remaining J assets. However, if $\mathscr{O} = \mathscr{I}$, all utility effects of price changes are internalized within the control group of the firm. In this case, the above objective can be combined with the market clearing condition from Definition 1, which results in

$$\max_{y \in Y} \bar{\pi} \cdot \sum_{i \in \mathscr{I}} (x^{i*}(y) - \omega^i) = \max_{y \in Y} \bar{\pi} \cdot y.$$

As a consequence, even a maximization problem as in Proposition 2, in which the firm acts as a price taker and ignores changes in demand, can lead to a production plan that satisfies every shareholder. However, if the control group were smaller, the internalization argument would break down: As Dierker and Dierker (2012) explain, original shareholders would then benefit from manipulating prices in such a way that outsiders are exploited. This is in stark contrast with the ex-post stage, at which prices can no longer be manipulated because the market has already closed, and unanimity can be obtained without restrictions on the composition of shareholders. This is summarized in the next corollary, which only utilizes that ex-ante spanning implies ex-post spanning, and can be stated without proof.

Corollary 2 *Under the same assumptions as Proposition 2, and even if $\mathscr{O} \subset \mathscr{I}$, the plan $(\bar{x}, \bar{y})$ satisfies ex-post individiual unanimity and ex-post group unanimity.*

That is to say, as long as we maintain the assumption of ex-ante spanning, we will fulfill unanimity criteria even if we relax the assumption that $\mathscr{O} = \mathscr{I}$, but these criteria are weaker. This leads to natural follow-up question: Suppose, to the contrary, that every consumer is an original shareholder, but we relax the assumption of ex-ante spanning. Can we also expect some form of shareholder unanimity in that case? This question is answered in the affirmative for a particular family of production technologies.

6.3 Diamond ray technologies

Diamond (1967) considers a specific class of production technologies, whose feasible output vectors $y_{\mathbf{1}}$ are all contained in a single ray. Suppose the output of the firm is nonzero; then, the asset span $\langle A, y_{\mathbf{1}} \rangle$ is invariant to changes in production scale. Moreover, since $Y \subset \mathbb{R} \times \langle A, y_{\mathbf{1}} \rangle$ in this case, ex-post spanning is trivially implied.

Definition 8 A production set Y represents a *Diamond ray technology* if

$$Y \subset \mathbb{R} \times \langle y_{\mathbf{1}} \rangle$$

for every nonzero production plan $y \in Y$.

This class of technologies includes production sets of the following kind.

Example 4 Production sets that are literally a ray:

$$Y = \{\lambda(y_0, y_{\mathbf{1}}) \mid \lambda \geq 0\}$$

for some $y_0 < 0$ and $y_{\mathbf{1}} > 0$.

Example 5 Multiplicative production functions as in Diamond (1967), that specify production scale $g(c)$ as a function of the capital stock c:

$$Y = \{(-c, y_{\mathbf{1}} g(c)) \mid c \geq 0\}$$

for some $y_{\mathbf{1}} > 0$. The function $g : \mathbb{R}_+ \to \mathbb{R}_+$ is strictly increasing, concave and satisfies $g(0) = 0$.

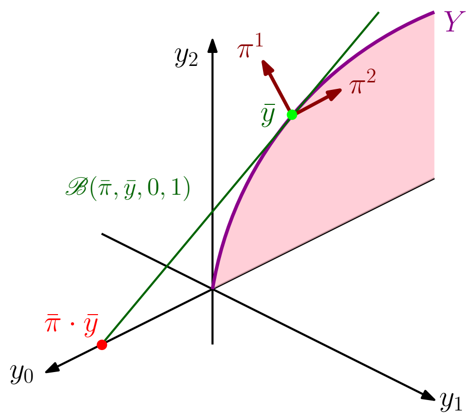

Fig. 6 Diamond ray technology: Shareholders agree on the production scale $\bar{y}_0 > 0$, and each scale permits only one efficient output combination $\bar{y}_{\mathbf{1}}$. Unanimity follows from a lack of feasible alternatives.

These production sets are so thin that any disagreement between shareholders is confined to a single dimension. This is illustrated in Figure 6: In comparison with Consumer 1, Consumer 2 prefers more output in the first state. Nevertheless, this is not a source of disagreement because an output adjustment in the first state is technologically infeasible. Since the proportion of output in the two states is fixed, disagreement could only arise about the scale of production. However, since all feasible deviations from the status quo $\bar{y}$ are contained in $\mathbb{R} \times \langle y_{\mathbf{1}} \rangle$, such deviations do not alter the budget set (5), and preferences over production scales are aligned through the market mechanism.

Proposition 3 *Let $(\bar{\pi}, \bar{x}) \gg 0$ be an equilibrium for fixed production plan $\bar{y} \in Y \backslash \{0\}$. If $\mathscr{O} = \mathscr{I}$, and Y represents a Diamond ray technology, and*

$$\bar{y} = \arg\max_{y \in Y} \bar{\pi} \cdot y\,,$$

then $(\bar{x}, \bar{y})$ satisfies ex-ante group unanimity.

Proof. Suppose there were an alternative $y \in Y$, a resulting equilibrium (π, x), and side payments $\upsilon \in \mathbb{R}^I$ that sum up to zero such that

$$U^i\left(x^i + \begin{pmatrix} \upsilon^i \\ \mathbf{0} \end{pmatrix}\right) > U^i(\bar{x}^i) \qquad \forall i \in \mathscr{I}.$$

By Equation (7), this implies

$$\pi^i(\bar{x}^i) \cdot \left(x^i + \begin{pmatrix} \upsilon^i \\ \mathbf{0} \end{pmatrix} - \bar{x}^i\right) > 0 \qquad \forall i \in \mathscr{I}. \tag{21}$$

By the definition of budget set (5), $x^i_{\mathbf{1}} - \omega^i_{\mathbf{1}} \in \langle A, y_{\mathbf{1}} \rangle$ and $\bar{x}^i_{\mathbf{1}} - \omega^i_{\mathbf{1}} \in \langle A, \bar{y}_{\mathbf{1}} \rangle$. Definition 8 implies that $\langle A, y_{\mathbf{1}} \rangle = \langle A, \bar{y}_{\mathbf{1}} \rangle$, and the two inclusion can be joined into $x^i_{\mathbf{1}} - \bar{x}^i_{\mathbf{1}} \in \langle A, \bar{y}_{\mathbf{1}} \rangle$. According to Equation (6), $\bar{\pi}$ can be substituted for $\pi^i(\bar{x}^i)$ in (21). Summing over all consumers results in

$$\bar{\pi} \cdot \sum_{i \in \mathscr{I}} (x^i - \bar{x}^i) + \underbrace{\sum_{i \in \mathscr{I}} \upsilon^i}_{=0} > 0\,.$$

By the market clearing condition from Definition 1, this is equivalent to $\bar{\pi} \cdot (y - \bar{y}) > 0$, but if that were the case, $\bar{y}$ could not have been a solution to the maximization problem – a contradiction. □

Note that Proposition 3 only holds for a nonzero status quo, for if no production takes place and thus $\bar{y} = 0$, production set Y and budget set $\mathscr{B}(\bar{\pi}, \bar{y}, \omega^i, \delta^i)$ may extend into different subspaces. However, when $\bar{y} \neq 0$, ex-post spanning is ensured, and the following corollary is a straightforward consequence.

Corollary 3 *Under the same assumptions as Proposition 3, and even if $\mathscr{O} \subset \mathscr{I}$, the plan $(\bar{x}, \bar{y})$ satisfies ex-post individual unanimity and ex-post group unanimity.*

That Diamond's model satisfies the ex-post spanning condition and thus guarantees ex-post shareholder unanimity is already noted by Ekern and Wilson (1974), and formally proven by Forsythe (1979).

6.4 Constant returns to scale

In a comment on the contribution of Ekern and Wilson (1974), Radner (1974) argues that their ex-post individual unanimity result can be strengthened to ex-ante individual unanimity under the same spanning condition. His line of reasoning is based on a casual analogy with the complete-market Arrow-Debreu model, rather than on formal rigor. This reflects the mindset of the early unanimity literature from the 1970s, which viewed disagreement between shareholders as a new and exclusive feature of incomplete markets. That shareholders in complete markets would unanimously approve net value maximization used to be a folk theorem of that time.[2] This folklore seems to be based on a misguided understanding of consumers' price-taking behavior. To set it straight: Price taking means that consumers take prices π, just like all other arguments of their budget set, as given when they choose their optimal consumption plan. What it does not mean, is that consumers believe that prices π are independent of aggregate production y. The economic relation between π and y is just of no concern to the utility maximization problem because neither is a choice variable of the consumer.

Any argument that consumers must favor a higher net market value $\pi \cdot y$ since it relaxes their budget constraint as a right-hand side term in (5), is made in blatant ignorance of the fact that π also shows up on the left-hand side. At the ex-post stage, we could disregard prices in the analysis of shareholder unanimity, simply because the market is no longer open. However, at the ex-ante stage, the utility effect of a deviation from a status quo production plan $\bar{y}$ depends on both sides of the budget constraint, and can only be understood if the resulting price response is taken into account. Grossman and Stiglitz (1977) account for this price response, and point out that Radner's reasoning is based on an implicit assumption of *competitivity*: Even if production plans change from $\bar{y}$ to y, the resulting market prices $\bar{\pi}$ remain the same. This can be stated formally as follows.

Definition 9 The economy is *competitive* if there are prices $\bar{\pi}$ with the following property: If (π, x) is an equilibrium for fixed production plan $y \in Y$ with $y \neq 0$, then $(\bar{\pi}, x)$ is an equilibrium for the same production plan.

Competitivity in the sense of Grossman and Stiglitz (1977) must be carefully distinguished from a concept that has been developed at the same time: competitive price perceptions in the sense of Leland (1977) and Grossman and Hart (1979), which will be discussed in detail Section 6.6. In short, competitivity is a supplementary condition to ex-ante spanning which ensures that the observed discount factors are invariant to production changes. By contrast, competitive price perceptions substitute subjective valuations for unobservable discount factors in unspanned regions of the consumption space. The former concept is objective and based on price taking, whereas the latter concept is subjective and based on beliefs.[3] The combination of ex-ante spanning and competitivity leads to the following strong result.

Proposition 4 *Let* $(\bar{\pi}, \bar{x}) \gg 0$ *be an equilibrium for fixed production plan* $\bar{y} \in Y$*. If* Y *satisfies ex-ante spanning, and the economy is competitive, then* $(\bar{x}, \bar{y})$ *satisfies ex-ante individual unanimity.*

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[2] In spite of first contrary result during that decade, Baron (1979) still contends in his survey article that "it is evident from the budget constraint that all ex ante shareholders [...] prefer that the firm maximize its perceived net market value, so shareholder unanimity is attained" (p. 108).

[3] Remarkably, in an extended reprint of their original article, Grossman and Stiglitz (1980) lean their word choice toward perceptions and describe competitivity as "the assumption that each firm perceives the market price of the basis (i.e., composite) commodities to be unaffected by its production decisions" (p. 544).

Proof. Suppose there were an alternative $y \in Y$ and a resulting equilibrium (π, x) such that $U^i(x^i) > U^i(\bar{x}^i)$. This implies that $\pi^i(\bar{x}^i) \cdot (x^i - \bar{x}^i) > 0$. Since $\pi^i_0(\bar{x}^i) = \bar{\pi}_0 = 1$ by definition and $x^i_{\mathbf{1}} - \bar{x}^i_{\mathbf{1}} \in \langle A \rangle$ by ex-ante spanning, the previous inequality can be rewritten as $\bar{\pi} \cdot x^i - \bar{\pi} \cdot \bar{x}^i > 0$. By competitivity, we can equalize $\pi = \bar{\pi}$, and the inequality is equivalent to $\pi \cdot x^i - \bar{\pi} \cdot \bar{x}^i > 0$. However, the budget constraints from (5) at the two equilibria are

$$\pi \cdot x^i = \pi \cdot (\omega^i + y\delta^i)$$
$$\bar{\pi} \cdot \bar{x}^i = \bar{\pi} \cdot (\omega^i + y\delta^i)$$

and since the right-hand sides are identical under competitivity, this implies that $\pi \cdot x^i - \bar{\pi} \cdot \bar{x}^i = 0$ – a contradiction. □

Makowski (1980, 1983a) argues passionately that Proposition 4 should also hold without the assumption of ex-ante spanning, as long as shares are redundant for every shareholder. However, at least in the present setting with strictly concave utility functions U^i, shares are only redundant if $y_{\mathbf{1}} \in \langle A \rangle$ for every $y \in Y$, which is again the ex-ante spanning condition. Contrary to the earlier literature, Makowski (1983a,b) makes an effort to outline conditions on exogenous objects that result in a competitive economy. From his examples it becomes clear that he has utility functions with flat segments in mind, although this does not seem to be a robust condition. A more promising starting point is the following class of constant returns to scale production technologies:

Definition 10 A production set Y represents a *linear activity technology* if

$$Y = \left\{ y \in \mathbb{R}_- \times \mathbb{R}^S_+ \mid y \in \langle L \rangle \right\}$$

for some matrix $L = (\ell^1, \ldots, \ell^N)$ of activity vectors $\ell^n \in \mathbb{R}_- \times \mathbb{R}^S_+$ with $\operatorname{rank}(L_{\mathbf{1}}) = N \leq S$.

One trivial case of a linear activity technology with $N = 1$ is the ray technology from Example 4. Such a technology is illustrated as production set Y in Figure 7: The only activity is production of output in the second state. The production plan $\bar{y}$ maximizes net market value, which is a well-defined because the self-spanning property guarantees that the production set is fully contained in the budget set of the consumer. Due to constant returns to scale, the resulting market value $\bar{\pi} \cdot \bar{y}$ is always zero, and initial shareholdings are irrelevant because they do not affect wealth. The first-order conditions of utility maximization guarantee that the marginal rates vectors of all consumers are orthogonal to Y at the status quo $\bar{y}$. Now suppose the firm tried to deviate to an alternative plan y that uses only half of the input, and thus only generates half of the output. Since the budget set (5) would not change, the marginal rates vectors would still be orthogonal to Y at the new production plan. However, to maintain the same consumption plan $\bar{x}$, each consumer would now have to buy *twice* the amount of final shares. But then the financial market would no longer clear: $\sum_{i \in \mathscr{I}} (\bar{x}^i - \omega^i) \neq y$. Moreover, there is no price-adjustment π that could restore market clearing. If $\pi_2 > \bar{\pi}_2$, the profit-maximizing production scale would become infinite, which precludes existence of equilibrium. But if $\pi_2 < \bar{\pi}_2$, the profit-maximizing production scale would drop to zero, aggregate demand would exhibit a discontinuity, and equilibrium existence would fail again. As this nonexistence problem occurs for any alternative production plan $y \neq \bar{y}$, no production plan other than $\bar{y}$ gives rise to an equilibrium $(\bar{\pi}, \bar{x})$. Therefore, the price vector $\bar{\pi}$ satisfies Definition 9, and the economy is competitive.

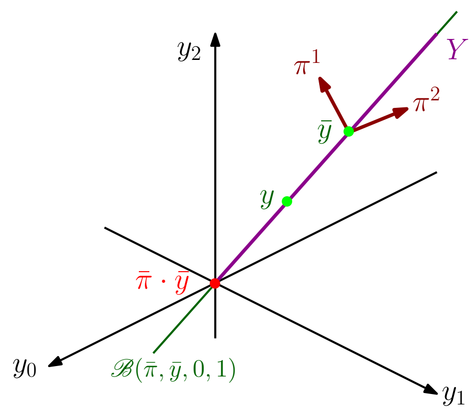

Fig. 7 Linear activity model: Every orthogonal vector $\bar{\pi}$ at $\bar{y}$ is still orthogonal at any other efficient plan y. Equilibrium prices are thus invariant to production changes.

If there are multiple activities, there is no self-spanning. To obtain competitivity in the case of $N > 1$, all activities must be made redundant by means of other assets. The following proposition shows that if profit maximization is taken as an institutional convention, spanned linear activity technologies indeed give rise to competitivity. It is worth emphasizing that the proposition itself does not provide any justification for profit maximization. If side payments are not possible, some shareholders might be better off at an equilibrium, at which profits are not maximized.

Proposition 5 *Let Y represent a linear activity technology with $\langle L_{\mathbf{1}} \rangle = \langle A \rangle$, and let $\omega^i_{\mathbf{1}} \in \langle A \rangle$ for every consumer $i \in \mathscr{I}$. If every equilibrium $(\bar{\pi}, \bar{x})$ for every fixed production plan $\bar{y} \neq 0$ satisfies*

$$\bar{y} = \arg\max_{y \in Y} \bar{\pi} \cdot y ,$$

then the economy is competitive.

Proof. In case of a linear activity technology, the first-order condition $\bar{\pi} \in N_Y[\bar{y}]$ becomes $\bar{\pi} \cdot L = 0$ for any $\bar{y} \neq 0$, and is therefore independent of the point of evaluation $\bar{y}$. Accordingly, for any production plan y and resulting equilibrium (π, x), the condition $\pi \cdot L = 0$ must hold. Since $\omega^i_{\mathbf{1}} \in \langle A \rangle = \langle L_{\mathbf{1}} \rangle$, the budget set from (5) reduces to $\mathscr{B}(\pi, y, \omega^i, \delta^i) = \{x^i \in \mathbb{R} \times \langle L_{\mathbf{1}} \rangle \mid \pi \cdot x^i = \pi \cdot \omega^i\}$, which is independent of the production plan y. Now $\pi \cdot L = \bar{\pi} \cdot L = 0$ leads to an equivalence of budget sets:

$$\mathscr{B}(\pi, y, \omega^i, \delta^i) = \mathscr{B}(\bar{\pi}, y, \omega^i, \delta^i)$$

Identical budget sets lead to identical utility maxima. Therefore, for any equilibrium (π, x), the tuple $(\bar{\pi}, x)$ is an equilibrium with identical consumption. □

The additional requirement of spanned endowments $\omega^i_{\mathbf{1}} \in \langle A \rangle$ goes unnoticed in the analysis of Grossman and Stiglitz (1977, 1980), who simply assume that endowments at the terminal date are zero. The observation that certain constant returns to scale technologies and ex-ante spanning jointly imply ex-ante individual unanimity seems to be due to DeAngelo (1981), who generously attributes the intellectual contribution to Fama and Laffer (1972). Since the latter authors address topics other than unanimity, a more natural predecessor is Rubinstein (1978), who points out the connection between constant returns to scale and ex-ante individual unanimity in a complete-market economy. Carceles-Poveda and Coen-Pirani (2009) show that the logic of Propositions 4 and 5 extends to settings with two input commodities, capital and labor. The main ingredients of their shareholder unanimity result are essentially the same: Nonzero production, a ray technology, and constant returns to scale. St-Pierre (2018) attempts a further extension to an arbitrary number of input commodities, and presents a shareholder unanimity theorem that supposedly does not require constant returns to scale. Both of these results depend on competitivity, but the approaches could not be more different: While Carceles-Poveda and Coen-Pirani (2009) design a setting whose explicit assumptions result in competitivity, St-Pierre (2018) entertains competitivity as an implicit assumption, unlikely to result from the design of his setting.

6.5 Large economies

A different approach to endogenizing competitivity is pursued by Hart (1979). He considers an economy with an infinite number of firms that have different fixed costs of production. The baseline economy is populated by a finite number of consumers, who are then replicated in order to enlarge the economy. The fixed costs ensure that only a finite number of firms are active, but this number of active firms grows in parallel to the population of consumers. As the contribution of every single firm to aggregate output becomes negligible, also the price impact of a change in its production plan becomes very small. Such a replication economy can become competitive in the limit.

Hart studies the foundations of net market value maximization from an ex-ante individual perspective. His main finding is that in sufficiently large economies, all original shareholders of a firm are better off at an equilibrium where its net market value is higher, than at an equilibrium where it is lower. However, the scope of this result is not quite easy to grasp. It does not conform with any of the usual unanimity criteria because production plans with identical net market values are not even compared. Moreover, the result is derived under a sizable number of ad-hoc assumptions on endogenous objects, and it is not clear how the exogenous objects of the model would have to look if these ad-hoc assumptions were to be satisfied. One absolutely indispensable ad-hoc assumption is that every fixed production plan results in a unique equilibrium. In absence of such uniqueness, Roberts (1980) shows that competitivity need not arise in the limit because price effects are always large near critical equilibria, and these do not vanish in the course of replication. Hart describes the scope of his result as follows:

> "What we have shown is that all shareholders will agree that firms should maximize net market value. However, this does not mean that there will be agreement as to how this goal is to be achieved. It is quite possible that some shareholder will believe that plan A yields a higher net market value than plan B, while other shareholders believe the opposite. Interestingly, this is less likely to happen in economies where the spanning condition holds (or in economies with complete markets). For in this case the implicit prices which are required to evaluate changes in production plans are quoted in the market. In contrast, when the spanning condition does not hold, some of these implicit prices will have to be guessed and the possibilities for disagreement are much greater." (Hart, 1979, p. 1076)

Our reading of his result is as follows: By the convexifying effect of large numbers, the growing number of active firms lets the replication economy approach a constant returns to scale economy, just like illustrated by Novshek and Sonnenschein (1987). The comparison of profit-maximizing and non-maximizing behavior then boils down to a comparison of production plans at the boundary and production plans in the relative interior of the aggregate production set Y. Since interior plans involve productive inefficiency, all original shareholders can be made better off if the firm deviates toward the boundary, in particular if the resulting price effects are negligible. Nevertheless, different shareholders will prefer different boundary points, unless Y is covered by the asset span.

Hart's analysis does not clarify what objective the firm should pursue when there is no ex-ante spanning; that is to say, when relevant discount factors are not quoted in the market. Makowski (1983a,b) addresses this question and presents an equilibrium concept that ensures ex-post individual unanimity in any economy, and ex-ante individual unanimity at least in competitive economies. In Makowski equilibrium, the firm maximizes the subjective valuation of the consumer who assigns the highest value to the production plan. This objective is well-defined regardless of whether a spanning condition is satisfied or not. Unfortunately, the resulting equilibrium concept is very fragile: For finite economies of any size, Zierhut (2017) shows that in absence of spanning, Makowski equilibria generically fail to exist. Even though the properties of Makowski equilibrium suggest that a sequence of such equilibria in a replication economy must attain ex-ante individual unanimity in the limit, this cannot be ascertained because the very existence of such a sequence is highly unlikely.

To conclude: Even in large economies there can be no ex-ante unanimity without ex-ante spanning. A large number of firms can make some economies competitive, but these economies have rather particular parameterizations. Especially the requirement of equilibrium uniqueness limits utility functions to a few selected functional forms, including quasilinear utility, quadratic expected utility, and expected utility with relative risk aversion less than one (see Hens and Pilgrim (2002), Chapter 6.4).

6.6 Competitive price perceptions

Prices of shares and other assets are determined in the financial market by the forces of supply and demand. Any variation in the production plan of the firm changes the supply side, and asset prices will have to adjust in order to meet the market clearing condition. Since production plans and prices are exogenous to the decision problem of the consumer, there is usually no need to model how consumers understand the interrelation of the two. In an important contribution, Gevers (1974) points out that this logic changes when shareholder voting is integrated into the model. If shareholders vote on production plans, they no longer take y as an exogenous variable, but factor in how the choice of y affects their budget set.

This is easy in a competitive economy with a complete financial market, in which a unique vector $\bar{\pi}$ of discount factors can be observed. Competitivity guarantees that these discount factors are invariant to changes in supply, and only the direct effect of a change from $\bar{y}$ to y has to be taken into account. However, in case of an incomplete financial market, observable prices only determine $\bar{\pi}$ on the asset span, but give no guidance how the market, be it competitive or not, would value production plans outside the asset span. To keep the shareholder voting problem well-specified, Gevers (1974) introduced a concept of *competitive price perceptions*: Every consumers i believes that the market determines asset prices by the formulas

$$q^{i*}(y) = \pi^i_{\mathbf{1}}(\bar{x}^i) \cdot A\,, \qquad p^{i*}(y) = \pi^i_{\mathbf{1}}(\bar{x}^i) \cdot y_{\mathbf{1}}\,. \tag{22}$$

That is to say, consumers substitute their subjective valuations for unobservable discount factors when pricing a production plan outside the asset span. On the asset span, these subjective valuations indeed agree with how a competitive market would price these assets. Since different consumers have different subjective valuations, competitive price perceptions usually diverge, and are thus biased. However, in equilibrium with shareholder voting, biased perceptions do not translate into biased behavior: At the status quo $(\bar{x}, \bar{y})$, all perceptions agree with the actual market clearing prices, and subjective divergence only manifests in out-of-equilibrium behavior.

Several authors have adopted the concept of competitive price perceptions, to derive an objective function of the firm: Leland (1974, 1977) and Makowski and Pepall (1985) on the basis of ex-ante individual unanimity, and Grossman and Hart (1979) on the basis of ex-ante group unanimity. We will use the Grossman-Hart setting for discussing the role of price perceptions. Ex-ante group unanimity requires the firm to adopt the objective function in (15). Price perceptions simplify this function: Substituting the formulas (22) for q and p in Equation (4), and substituting its right-hand side for $x^{i*}(y)$ in the objective function of the firm leads to:

$$\sum_{i \in \mathcal{O}} \pi^i(\bar{x}^i) \cdot (x^{i*}(y) - \omega^i) = \sum_{i \in \mathcal{O}} \delta^i \pi^i(\bar{x}^i) \cdot y$$

Competitive price perceptions thus suggest that subjective valuations should be weighted by initial shares in order to be consistent with ex-ante group unanimity. This leads to a value maximization criterion proposed by Grossman and Hart (1979):

$$\max_y \sum_{i \in \mathcal{O}} \delta^i \pi^i(\bar{x}^i) \cdot y \quad \text{subject to} \quad y \in Y \tag{23}$$

While the appeal of this criterion is its simplicity, it raises several conceptual issues. First and foremost, if perceptions are part of the objective function, they manifest in equilibrium behavior, and not just out of equilibrium like in the model of Gevers (1974). The question is then why subjective biases should be hardwired into the decision criterion of the firm. Grossman and Hart (1979) argue that this reflects the behavior of a manager who acts in the interest of the original shareholders. However, this view is challenged by Dierker and Dierker (2012), who construct an example in which all original shareholders are better off after a deviation from the Grossman-Hart criterion, and suitable side payments. Contrary to subjective beliefs, ex-ante group unanimity is not attained in Grossman-Hart equilibrium. In other words, the manager does not act in the interest of the original shareholders but is mislead by their subjective biases.

In light of this finding, it is worth studying economies that are free of subjective biases. Such an economy is constructed in an example by Bettzüge, Hens, and Zierhut (2022). The example features two consumers, two firms, and competitive price perceptions that are correct for every production plan. In absence of subjective bias, a different phenomenon surfaces: The dimension of the asset span drops whenever production plans are chosen as in (23), demand exhibits a discontinuity, and no Grossman-Hart equilibrium exists. The source of this nonexistence problem is lack of ex-ante spanning. Similar reservations arise about the approach of Makowski and Pepall (1985), which leads to the concept of Makowski equilibrium, and thus again to a nonexistence problem in absence of ex-ante spanning. The obvious solution to both nonexistence problems would be adding sufficiently many assets to meet the ex-ante spanning condition, but that turns competitive price perceptions into a void concept as all relevant discount factors can then be observed in the market.

6.7 Mean-variance utility

It has been noted by Stiglitz (1972) and Ekern and Wilson (1974) that consumers with mean-variance utility reach some forms of unanimity about production plans. We would like to emphasize here that this is because mean-variance economies are one particular case of what Krouse (1985) calls *equivalently complete markets*, or what LeRoy and Werner (2014) in Chapter 16 of their textbook call *effectively complete markets*: Provided that $\mathbf{1} \in \langle A \rangle$, such that riskless borrowing and saving is possible, and that $\omega^i_{\mathbf{1}} \in \langle A \rangle$ for each consumer i, such that endowments can be replicated by portfolios of traded assets, there is a unique equilibrium $(\bar{\pi}, \bar{x})$ for any production plan $\bar{y} \in Y$, and its allocation satisfies the necessary condition (18) for Pareto efficiency (see Magill and Quinzii (1996), p. 181, Theorem 17.3).

While the following proposition is valid not only for mean-variance utility functions but for general utility functions, this is a good place to state it as the subsequent proposition relies on it.

Proposition 6 *Let $(\bar{x}, \bar{y})$ be a Pareto efficient plan.*

1. *$(\bar{x}, \bar{y})$ satisfies ex-post individual unanimity.*
2. *$(\bar{x}, \bar{y})$ satisfies ex-post group unanimity.*
3. *If $\mathscr{O} = \mathscr{I}$, then $(\bar{x}, \bar{y})$ satisfies ex-ante group unanimity.*

Proof. The first-order conditions of Pareto efficiency (18) and (19) imply the first-order condition of ex-post individual unanimity (10) because $N_Y[\bar{y}] \subset N_{\mathscr{Y}(\bar{y})}[\bar{y}]$. Since the latter condition is sufficient for ex-post individual unanimity, which in turn implies ex-post group unanimity, the first two properties are proven. Moreover, Pareto efficiency means there is no other feasible plan (x, y) such that $U^i(x^i) > U^i(\bar{x}^i)$ for every consumer $i \in \mathscr{I}$. If $\mathscr{O} = \mathscr{I}$, this implies Definition 5, and the third property is proven. □

As a consequence, the unanimity criteria from Proposition 6 are all met in the mean-variance economy outlined above, provided the firm maximizes profit such that (19) is satisfied. The next proposition shows that there is, indeed, a no-arbitrage equilibrium where (19) is satisfied.

Proposition 7 *Suppose that for each consumer i, U^i is represented by* (1) *with a risk-tolerance parameter τ^i. Write $\tau = \sum_i \tau^i$ and define U by* (1). *Let $\bar{y}$ be a solution to the problem of maximizing $U(\omega + \delta y)$ subject to $y \in Y$. Assume that $\pi_0 > 0$ for every $\pi = (\pi_0, \pi_{\mathbf{1}}) \in N_Y(\bar{y})$ and that $\mathbf{1} \in \langle A, \bar{y}_{\mathbf{1}} \rangle$ and $\omega^i_{\mathbf{1}} \in \langle A, \bar{y}_{\mathbf{1}} \rangle$ for every i. Then, there is an $r > 0$ and a consumption allocation $\bar{x}$ such that for $\bar{\pi} = (1, r^{-1} DU_{\mathbf{1}}[\bar{\omega}_{\mathbf{1}} + \bar{y}_{\mathbf{1}}])$, we have $\bar{\pi} \in N_Y(\bar{y})$ and:*

1. *$(\bar{\pi}, \bar{x})$ is a no-arbitrage equilibrium under $\bar{y}$.*
2. *$(\bar{x}, \bar{y})$ satisfies ex-post individual unanimity.*
3. *$(\bar{x}, \bar{y})$ satisfies ex-post group unanimity.*
4. *If $\delta^i > 0$ for every i, then $(\bar{x}, \bar{y})$ satisfies ex-ante group unanimity.*

It is well known that U is a utility function of the representative consumer; that is, once the production plan $\bar{y}$ is chosen, the date-1 state prices at equilibrium are equal to those of the representative-consumer economy with U. The assumption on $\langle A, \bar{y}_{\mathbf{1}} \rangle$ means that every consumer's date-1 endowment, as well as the risklss bond, can be traded or replicated. This is weaker than the complete-market assumption, $\langle A, \bar{y}_{\mathbf{1}} \rangle = \mathbb{R}^S$, but sufficiently strong to guarantee that the equilibrium consumption allocation is Pareto-efficient given any production plan, thanks to the mean-variance utility functions. The financial market is equivalently complete in the sense of Krouse (1985) and effectively complete in the sense of LeRoy and Werner (2014). Markets in oft-used models in economics and finance have this property.[4] This efficiency property justifies our use of the representative consumer in deriving the equilibrium price vector $\bar{\pi}$.

The assumption that $\pi_0 > 0$ for every $\pi = (\pi_0, \pi_{\mathbf{1}}) \in N_Y(\bar{y})$ means, roughly, that the marginal productivity of the date-0 zero input is strictly positive. We will see in the proof that r is the equilibrium riskless rate.

Proof. 1. By the assumption on $N_Y(\bar{y})$, $\bar{\omega}_0 + \bar{y}_0 = 0$. By the first-order condition for the representative consumer's utility maximization problem, $DU[\bar{\omega} + \bar{y}] = (0, DU_{\mathbf{1}}[\bar{\omega}_{\mathbf{1}} + \bar{y}_{\mathbf{1}}])$ belongs to the sum $N_Y(\bar{y}) + (\mathbb{R}_- \times \{0\})$ of the normal cone $N_Y(\bar{y})$ and the cone $\mathbb{R}_- \times \{0\} \subset \mathbb{R} \times \mathbb{R}^S$.[5] Thus, there is a $\pi \in N_Y(\bar{y})$ and $r \geq 0$ such that $DU[\bar{\omega} + \bar{y}] = \pi - (r, 0)$; that is, $(r, DU_{\mathbf{1}}[\bar{\omega}_{\mathbf{1}} + y_{\mathbf{1}}]) = \pi$. By the assumption on $N_Y(\bar{y})$, $r > 0$. Define $\bar{\pi} = r^{-1}\pi$, then $\bar{\pi} \in N_Y(\bar{y})$, $\bar{\pi}_0 = 1$, and $\bar{\pi}_{\mathbf{1}} = r^{-1} DU_{\mathbf{1}}[\bar{\omega}_{\mathbf{1}} + \bar{y}_{\mathbf{1}}]$.[6] By (2),

$$DU_{\mathbf{1}}\left[\bar{y}_{\mathbf{1}} + \bar{\omega}_{\mathbf{1}}\right] = [P]\left(\mathbf{1} - \frac{1}{\tau}\left((\bar{y}_{\mathbf{1}} + \bar{\omega}_{\mathbf{1}}) - \mathrm{E}^P\left[\bar{y}_{\mathbf{1}} + \bar{\omega}_{\mathbf{1}}\right]\mathbf{1}\right)\right). \tag{24}$$

For each i, define $\bar{x}^i = \left(\bar{x}_0^i, \bar{x}_{\mathbf{1}}^i\right)$ so that $\bar{x}_0^i = 0$, $\bar{\pi}_{\mathbf{1}} \cdot \bar{x}_{\mathbf{1}}^i = \bar{\pi} \cdot (\omega^i + \delta^i \bar{y})$, and

$$\bar{x}_{\mathbf{1}}^i - \mathrm{E}^P[\bar{x}_{\mathbf{1}}^i]\mathbf{1} = \frac{\tau^i}{\tau}\left((\bar{y}_{\mathbf{1}} + \bar{\omega}_{\mathbf{1}}) - \mathrm{E}^P\left[\bar{y}_{\mathbf{1}} + \bar{\omega}_{\mathbf{1}}\right]\mathbf{1}\right) \tag{25}$$

for each i. Such an $\bar{x}_{\mathbf{1}}^i$ indeed exists. The equality (25) is equivalent to saying that there is a $b^i \in \mathbb{R}$ such that

$$\bar{x}_{\mathbf{1}}^i = \frac{\tau^i}{\tau}\left((\bar{y}_{\mathbf{1}} + \bar{\omega}_{\mathbf{1}}) - \mathrm{E}^P\left[\bar{y}_{\mathbf{1}} + \bar{\omega}_{\mathbf{1}}\right]\mathbf{1}\right) + b^i\mathbf{1},$$

---

[4] Examples include the cases where all consumers exhibit constant absolute risk aversion and where all consumers exhibit constant relative risk aversion with a common coefficient of relative risk aversion.

[5] The addition of the cone $\mathbb{R}_- \times \{0\}$ is needed because the non-negativity constraint $\bar{\omega}_0 + y_0 \geq 0$ binds at $\bar{y}$.

[6] Note that $\pi \cdot (1, 0) = r$ and, by (2), $\pi \cdot (0, \mathbf{1}) = 1$. Hence, r is equal to one plus the riskless rate.

and we can determine the value of b^i to satisfy

$$\bar{\pi}_{\mathbf{1}} \cdot \bar{x}^i_{\mathbf{1}} = \frac{1}{r}\left(\frac{\tau^i}{\tau^2}\mathrm{Var}[\bar{y}_{\mathbf{1}} + \bar{\omega}_{\mathbf{1}}] + b^i\right) = \bar{\pi} \cdot (\omega^i + \delta^i \bar{y}).$$

Also, since $\mathbf{1} \in \langle A, \bar{y}_{\mathbf{1}} \rangle$ and $\omega^i_{\mathbf{1}} \in \langle A, \bar{y}_{\mathbf{1}} \rangle$, we have $\bar{x}^i_{\mathbf{1}} - \omega^i_{\mathbf{1}} \in \langle A, \bar{y}_{\mathbf{1}} \rangle$.

Thus, the two-fund theorem holds for this economy: each consumer consumes a linear combination of the mean-zero part of the aggregate consumption $(\bar{y}_{\mathbf{1}} + \bar{\omega}_{\mathbf{1}}) - \mathrm{E}^P[\bar{y}_{\mathbf{1}} + \bar{\omega}_{\mathbf{1}}]\mathbf{1}$ and the riskless payoff $\mathbf{1}$. Since $\sum_i \tau^i/\tau = 1$, the market for the mean-zero part clears. Hence, by Walras Law and $\sum_i \bar{\pi} \cdot (\omega^i + \delta^i \bar{y}) = \bar{\pi} \cdot (\bar{\omega} + \bar{y})$, the market for the riskless bond clears. Since $\bar{x}^i_0 = 0$ for all i and $\bar{\omega}_0 + \bar{y}_0 = 0$, the market for the date-0 consumption also clears.

By (2) and (25),

$$DU^i_{\mathbf{1}}[\bar{x}^i_{\mathbf{1}}] = [P]\left(\mathbf{1} - \frac{1}{\tau}\left((\bar{y}_{\mathbf{1}} + \bar{\omega}_{\mathbf{1}}) - \mathrm{E}^P[\bar{y}_{\mathbf{1}} + \bar{\omega}_{\mathbf{1}}]\mathbf{1}\right)\right), \tag{26}$$

for every i. Hence, $\bar{\pi} = (1, R^{-1}DU^i_{\mathbf{1}}[\bar{x}^i_{\mathbf{1}}])$ for every i. This implies two things. First, since $\bar{x}^i_{\mathbf{1}}$ satisfies the budget constraint, it also satisfies the utility maximization condition. This proves part 1. Second, the plan $(\bar{x}, \bar{y})$ is Pareto-efficient. By Proposition 6, this proves part 2, part 3 (because $\tau^i/\tau > 0$), and part 4. □

Unlike the three unanimity conditions in Proposition 7, ex-ante individual unanimity is, in general, not satisfied. We now give a worked-out example of this fact.

Example 6 Let $v \in \mathbb{R}_{++}$ satisfy $\mathrm{E}^P[v] = \sum_s P(s)v_s = 1$. Define $F : \mathbb{R}^S \to \mathbb{R}$ by $F(y_{\mathbf{1}}) = \sum_s P(s)v_s y_s$. Define $Y = \{y = (y_0, y_{\mathbf{1}}) \in \mathbb{R}_- \times \mathbb{R}^S \mid y_0 + F(y_{\mathbf{1}}) \leq 0\}$. Then Y exhibits constant returns to scale and the marginal rates of transformation are constant, with a normal vector $(1, [P]v)$. Hence, the assumption on $N_Y(y)$ in Proposition 7 is met for every $y \in Y$. Moreover, $[P]v$ is the equilibrium state price vector for the date-1 consumption and the equilibrium riskless rate is zero.

Proposition 8 *Suppose that for each consumer i, U^i is represented by* (1) *with the risk-tolerance parameter τ^i. Write $\tau = \sum_i \tau^i$ and define U by* (1)*. Suppose that the production set Y satisfies Example 6 and that $\mathbf{1} \in \langle A \rangle$ and $\omega^i_{\mathbf{1}} = 0$ for every i.*

1. *The solution $\bar{y}$ to the problem of maximizing $U(\omega + \delta y)$ subject to $y \in Y$ is given by*

$$\bar{y} = (\bar{y}_0, \bar{y}_{\mathbf{1}}) = \left(-\bar{\omega}_0, \tau(\mathbf{1} - v) + \left(\bar{\omega}_0 + \tau\mathrm{Var}^P[v]\right)\mathbf{1}\right).$$

2. *The no-arbitrage equilibrium $(\bar{\pi}, \bar{x})$ under $\bar{y}$ is given by*

$$\bar{\pi} = (\bar{\pi}_0, \bar{\pi}_{\mathbf{1}}) = (1, [P]v),$$
$$\bar{x}^i = (\bar{x}^i_0, \bar{x}^i_{\mathbf{1}}) = \left(0, \tau^i(\mathbf{1} - v) + \left(\tau^i\mathrm{Var}^P[v] + \omega^i_0\right)\mathbf{1}\right) \textit{ for every } i.$$

3. *$(\bar{x}, \bar{y})$ satisfies ex-ante individual unanimity if and only if either $\tau^i/\tau = \delta^i$ for every i or $\nu = \mathbf{1}$.*

Since the production set Y of Example 6 satisfies the assumptions of Proposition 7, the plan $(\bar{x}, \bar{y})$ satisfies ex-post group unanimity, ex-ante group unanimity if $\delta^i > 0$, and ex-post individual unanimity. Proposition 8 shows, however, that ex-ante individual rationality is violated except for the rare cases of $\tau^i/\tau = \delta^i$ for every i and of $\nu = \mathbf{1}$. In the first case, there is no share trade and, thus, no bond trade either, at equilibrium. In the second case, the output $\bar{y}_{\mathbf{1}}$ at date 1 is deterministic. The assumption that $\omega^i_{\mathbf{1}} = 0$ for every i means that the date-1 consumption is not endowed but produced. It is imposed to make the rare cases as simple as possible and can be dispensed with as long as we assume that $\omega^i_{\mathbf{1}} \in \langle A \rangle$ for every i.

Proof. Parts 1 and 2 can be proved by direct calculation. Part 2 can also be confirmed by the proof for Part 3.

To prove part 3, note that for every $y \in Y$, (π, x) is a no-arbitrage equilibrium under y if and only if

$$[P]\left(\mathbf{1} - \frac{1}{\tau}\left(y_{\mathbf{1}} - \mathrm{E}^P[y_{\mathbf{1}}]\mathbf{1}\right)\right) \in R(\pi_{\mathbf{1}}),$$

where $R(\pi_{\mathbf{1}}) = \mathrm{pr}^{-1}_{\langle A, y_1 \rangle}(\pi_{\mathbf{1}})$; and $x^i_0 = 0$ and

$$x^i_{\mathbf{1}} = \frac{\tau^i}{\tau} y_{\mathbf{1}} + \left(\omega^i_0 + \delta^i y_0 + \left(\delta^i - \frac{\tau^i}{\tau}\right)\left(\mathrm{E}[y_{\mathbf{1}}] - \frac{1}{\tau}\mathrm{Var}[y_{\mathbf{1}}]\right)\right)\mathbf{1}$$

for every i.[7] Hence,

$$\begin{aligned} U^i(x^i) = U^i_{\mathbf{1}}(x^i_{\mathbf{1}}) &= \frac{\tau^i}{\tau}\mathrm{E}^P[y_{\mathbf{1}}] + \left(\omega^i_0 - \delta^i F(y_{\mathbf{1}}) + \delta^i - \frac{\tau^i}{\tau}\right)\left(\mathrm{E}[y_{\mathbf{1}}] - \mathrm{Var}^P[y_{\mathbf{1}}]\right) \\ &\quad - \frac{1}{2\tau^i}\left(\frac{\tau^i}{\tau}\right)^2 \mathrm{Var}^P[y_{\mathbf{1}}] \\ &= \omega^i_0 + \delta^i\left(\mathrm{E}^P[y_{\mathbf{1}}] - \frac{1}{\tau}\mathrm{Var}^P[y_{\mathbf{1}}] - F(y_{\mathbf{1}})\right) + \frac{\tau^i}{2\tau^2}\mathrm{Var}^P[y_{\mathbf{1}}]. \end{aligned}$$

Thus, the first-order condition for a solution to the problem of maximizing this utility level with respect to $y_{\mathbf{1}}$ is that

$$\delta^i\left(\mathbf{1} - \frac{2}{\tau}(y_{\mathbf{1}} - \mathrm{E}^P[y_{\mathbf{1}}]\mathbf{1}) - \nu\right) + \frac{2\tau^i}{2\tau^2}(y_{\mathbf{1}} - \mathrm{E}^P[y_{\mathbf{1}}]\mathbf{1}) = 0,$$

which can be rewritten as

[7] This confirms the equality for $\bar{x}^i_{\mathbf{1}}$ in part 2.

$$\mathbf{1} - \left(2 - \frac{\tau^i}{\tau}\frac{1}{\delta^i}\right)\frac{1}{\tau}(y_{\mathbf{1}} - \mathrm{E}^P[y_{\mathbf{1}}]\mathbf{1}) = \nu. \tag{27}$$

If $y = \bar{y}$, then this can be rewritten as

$$\mathbf{1} - \left(2 - \frac{\tau^i}{\tau}\frac{1}{\delta^i}\right)\frac{\tau}{\tau}(\mathbf{1} - \nu) = \nu,$$

which can further be rewritten as

$$\left(1 - \frac{\tau^i}{\tau}\frac{1}{\delta^i}\right)(\mathbf{1} - \nu) = 0.$$

This last equality holds for every i if and only if $\tau^i/\tau = \delta^i$ for every i or if $\nu = \mathbf{1}$. □

While we do not develop a formal argument here, we would like to point out that the same result, that all the unanimity conditions except for ex-ante individual unanimity are satisfied at the equilibrium where the representative consumer's utility is maximized, is valid for other types of economies. Examples were mentioned in in Footnote 4: all consumers exhibit constant absolute risk aversion, or all consumers exhibit constant relative risk aversion with a common coefficient of relative risk aversion. This is because the financial market is, then, equivalently (or effectively) complete and the equilibrium consumption allocation is Pareto-efficient given any production plan, and, yet, the change in production plans may well affect its share price.

7 Concluding remarks

Strong forms of shareholder unanimity are attainable in a complete and competitive financial market. In face of market imperfections, adding parameter restrictions, or weakening the unanimity criterion becomes necessary for affirmative results. The more imperfect the market, the weaker the attainable criterion. The connection between shareholder unanimity and profit maximization is limited to cases in which profit is a determinate concept because all relevant discount factors can be inferred from observable prices.

An aspect that has received only limited attention in the past fifty years of unanimity literature, as well as in the present study, is the lifetime of consumers and firms. The research gap is not so much on consumers and firms with *longer* lifetimes. In fact, economies in which the market opens more than once have been studied by Grossman and Stiglitz (1977, 1980), Grossman and Hart (1979), Ohlson (1985), and Dierker (2015). The most noteworthy new features is a new group of shareholders, the current shareholders of the firm, who need not agree with their original or final counterparts. But otherwise, many results from the two-date setting go through with minor modifications.

The more promising avenue for future research are consumers and firms with *different* lifetimes. A particular economy where consumers live shorter than firms has been suggested by Grossman and Stiglitz (1977), who conjecture that the previous unanimity results extend into an overlapping-generations setting in which every consumer lives for two dates. At every point in time, the old generation would sell all of their shares, while the young generation would be the natural buyers. Shareholder unanimity would then reduce to agreement within the same generation, whose planning horizon ends after one period. Conversely, result in favor of profit maximization are more likely in economies where consumers live longer than firms: If firms are repeatedly founded and every firms lives for two dates, the existing spanning conditions can be extended quite easily to economies with longer horizons. Such spanning also eliminates equilibrium nonexistence problems that would otherwise occur in incomplete market economies with multiple future dates.

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