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“Automation and Growth Patterns in an Open Economy”

Shohei Momoda, Takayuki Ogawa, and Ryosuke Shimizu

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Automation and Growth Patterns in an Open Economy

Shohei Momoda[*], Takayuki Ogawa[†], and Ryosuke Shimizu[‡]

Abstract

Recent data suggest that countries with a higher accumulation of robots achieve higher economic growth . This study analyzes the international growth patterns in a two-country economy with task-based automation technology. We show that whenever one country can achieve perpetual growth by fully automating all tasks, another country cannot. Thus, automation widens the international disparities in output growth. Using panel data covering 62 countries from 1994 to 2019, we empirically find that countries with more industrial robots are associated with higher economic growth through the increased accumulation of robots.

Keywords : Automation; Growth patterns; International trade.
JEL classification: F43, F62, O33.

[*]Corresponding author. Graduate School of Humanities and Social Sciences, Hiroshima University, 1-3-2 Kagamiyama, Higashi-Hiroshima City, Hiroshima 739-8511, JAPAN. E-mail: shohei.momoda.econ@gmail.com.

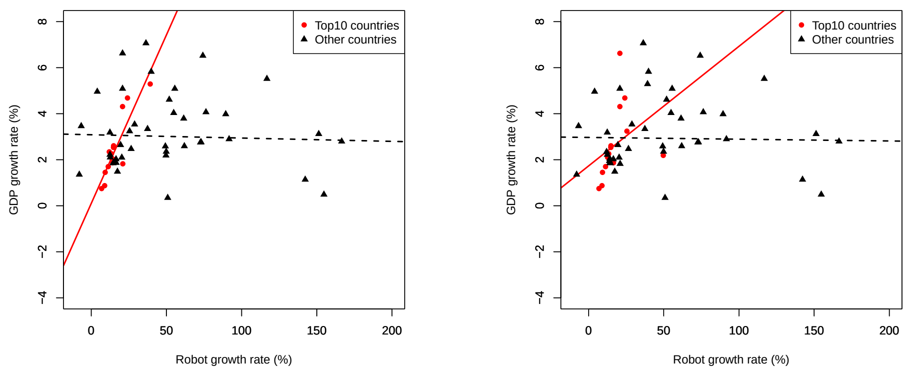
[†]Faculty of Economics, Osaka University of Economics, 2-2-8 Osumi, Higashiyodogawa-ku, Osaka City, Osaka 533-8533, JAPAN. E-mail: tkogawa@osaka-ue.ac.jp.

[‡]Department of Humanities and Social Science, Faculty of Law and Letters, Ehime University, 3 Bunkyo-cho, Matuyama City, Ehime 790-8577, JAPAN. E-mail: R.shimizuecon@gmail.com.

1 Introduction

Advances in production automation, including the use of industrial robots, which have recently been installed in various production processes, are expected to result in unprecedented economic growth unconstrained by labor input (Aghion et al., 2019). However, although automation technologies are gradually spreading worldwide, the density of robots, measured as the number of robots per capita, varies significantly across countries (De Backer et al., 2018). As Figure 1a shows, countries with higher robot density achieve higher economic growth by accumulating more robots. This tendency also holds true in Figure 1b, where countries are grouped according to their amounts of robot stock. These observations suggestthat the use of robots accelerates economic growth and widens economic disparities across countries. In this study , we analyze the international growth patterns using a two-country model in which production tasks are automatable by robots. We also empirically test our theoretical findings based on panel data covering 62 countries from 1994 to 2019.

Industrial robots are distinguished from traditional capital such as plants owing to their substitutability for labor: robots can perfectly substitute for labor when conducting production tasks. An expanding body of the literature now investigates the impact of automation technology on wages, the share of labor income, and unemployment (e.g., Autor et al., 2003; Graetz and Michaels, 2018; Acemoglu and Restrepo, 2020). In this study, we employ the workhorse model developed by Acemoglu and Autor (2011) and Acemoglu and Restrepo (2018b) in which production is performed by combining various tasks, with some tasks fully automated using robots. A remarkable feature of their task-based model is that robots can outperform labor in productivity, thereby leading to persistent economic growth through the accumulation of reproducible robot capital (Nakamura and Nakamura, 2008). Moreover, although it has not received much attention, the task-



(a) Grouping by robot density (b) Grouping by robot stock

Figure 1 : Relationship between robot growth and GDP growth, 1994–2019.

Note: This figure plots the robot and GDP growth rates, averaged from 1994 to 2019, for 62 countries. The robot growth rate is the average annual growth rate of the "operational stock" of industrial robots, sourced from the International Federation of Robotics . The average GDP growth rate comes from the 2023 World Economic Outlook published by the International Monetary Fund (IMF). The sample is divided into two groups. The top 10 group, red circles in Panel (a), includes countries ranked in the top 10 for robot density at least once between 2000 and 2019, where robot density is the number of operational industrial robots per 10,000 employees in the manufacturing industry according to the International Federation of Robotics. The top 10 group in Panel (b) includes countries ranked based on the "operational stock" instead of robot density. The red (black) line represents the ordinary least squares regression line for the countries in the top 10 group (other countries).

based model also allows for production to be carried out only by labor.

This study aims to clarify the patterns of automation and economic growth in a financially integrated world. We theoretically find that at the stage of low robot productivity, production relies solely on labor (and traditional capital) and the economy remains at a low level of output. As robots become more productive, the economy shifts to a stage in which robots replace labor in certain tasks. Eventually, with a substantial increase in robot productivity, (almost) all tasks are automated by robots, leading to persistent economic growth. However, as long as one country can achieve persistent growth by fully automating tasks, the other country cannot. This is because in a financially integrated world, robots move to the most productive country, namely, the country with the highest comparative advantage of robots over labor. Therefore, automation technology causes disparities in output growth internationally.

We empirically test our theoretical findings suggesting that countries with higher robot stock achieve more rapid output growth through the accumulation of robots. This contrasts with the implications of neoclassical models with capital accumulation as an engine of economic growth in which the growth rate of per capita output converges to zero in the absence of exogenous technological progress because of the diminishing marginal product of capital (Barro, 1991; Barro and Sala-i-Martin, 1992; Mankiw et al., 1992; Johnson and Papageorgiou, 2020).[1] Our regression results using the model with country and time fixed effects show that the association between the growth rates of robot stock and GDP is statistically significant and positive for countries with high robot density, whereas it is insignificant for countries with low robot density. In the estimation, we control for the growth effects of increases in traditional capital and labor employment. Notably, we find that the accumulation of traditional capital has no statistically significant impact on output growth when country and time fixed effects are controlled for. Unlike traditional capital, the use of robots accelerates economic growth in countries with high robot density. This is consistent with the quantitative findings of Autor and Salomons (2018), Berg et al. (2018), and Acemoglu (2024). The result remains robust to changing the method of grouping the

[1] The endogenous growth literature theoretically and empirically explores the factors leading to technological progress. Examples include human capital accumulation (Lucas, 1988; Barro, 1991; Griliches, 1998; Bils and Klenow, 2000), R&D activities (Romer, 1990; Bloom et al., 2013), and public investment and capital (Grier and Tullock, 1989; Barro, 1990; Devarajan et al., 1996). See Barro and Sala-i-Martin (2004, chapter 8) and Aghion and Howitt (2008, chapter 7) for the convergence and divergence in cross-country income due to international technological transfers.

sample and altering the source of the employment data.

This study contributes to the literature on economic growth driven by automated production tasks.[2] The pioneering work by Zeira (1998) shows that with technological innovations, labor-saving technologies are chosen over capital-saving technologies. Acemoglu and Autor (2011) develop a task-based model to investigate how automation progress affects wages depending on workers' skill types. Nakamura and Nakamura (2008) suggest that automated tasks may continue to increase and the economy may grow perpetually. The key mechanism is that the marginal product of robots does not diminish sufficiently and approaches a constant value asymptotically. Nakamura (2010) finds that multiple steady states arise in the presence of an externality of mechanization on robot productivity: the economy either falls into a poverty trap with no growth or achieves persistent growth depending on the initial level of robot stock. Ikeshita et al. (2023) point out that task automation leads to perpetual growth within the Solow growth framework. Ray and Mookherjee (2022) analyze the impact of automation on the labor income share using a model with robot capital, traditional capital, and human capital. They establish a condition under which endogenous economic growth occurs; however, the labor income share does not converge to zero when robots are reproducible. These studies have primarily focused on a closed economy, whereas we examine international growth patterns and show that perpetual economic growth can occur only in the country with the highest comparative advantage of robots over labor, while other countries invest in this leading country and cease to grow.[3]

[2]There is an alternative analytical framework in which robots serve the same role as labor as inputs for production (e.g., Steigum, 2011; Prettner and Strulik, 2020; Sasaki, 2023).

[3]Various sources of economic growth, other than capital accumulation, are investigated. For instance, Acemoglu and Restrepo (2018b) and Hémous and Olsen (2022) consider new task creation to be a driver of economic growth. Nakamura and Zeira (2024) clarify the mechanism by which new task creation covers unemployment caused by automation advancements. Aghion et al. (2019) provide the condition for *technological singularity* to emerge through an expansion of new ideas.

This study is also closely related to the literature analyzing automation in open economy settings. Acemoglu and Restrepo (2022a) show that population aging induces automation advancements, which spill over to foreign countries through international trade. Using a quantitative model with global value chains, Furusawa et al. (2021) calibrate the labor market impact of automation. Mandelman and Zlate (2022) demonstrate that the employment polarization in the United States with low-skilled immigrants in the workforce is caused by lower production costs due to automation and offshoring. Artuc et al. (2023) show numerically that automation enhances welfare more in more automated countries than in less automated countries. Auray and Eyquem (2019) consider two countries with heterogeneous firms and workers to explain why rapid progress in automation is not necessarily associated with substantial improvement in total factor productivity. Momoda et al. (forthcoming) reveal that technological innovations have asymmetric spillover effects on the degree of automation and welfare in robot-producing and non-robot-producing countries. We complement this body of research by showing the determination of international growth patterns, including the stages in which production is performed only by either labor or robots. Moreover, we provide empirical evidence that the accumulation of robots accelerates economic growth. Our findings thus offer new insights into global disparities in economic growth driven by advancements in automation owing to the use of industrial robots.

The remainder of this paper is organized as follows. Section 2 presents the basic structure of the theoretical model. Section 3 derives the equilibrium condition. Section 4 analyzes the dynamics to clarify international growth patterns. Section 5 presents empirical evidence that the accumulation of industrial robots accelerates economic growth in some countries with higher robot density, as implied in our theoretical model. Section 6 concludes.

2 The Model

Consider an economy with two countries h and f. Their labor supply is constant and denoted by $L^h(>0)$ and $L^f(>0)$, respectively. A homogeneous tradable good is produced in both countries using task-based technologies à la Zeira (1998), Acemoglu and Autor (2011), and Acemoglu and Restrepo (2018b). Production is required to combine various types of tasks, each of which is performed by either industrial robots or labor. Robots are freely mobile across the two countries, whereas labor is not. As per factor prices and factor productivities, there are three possible production patterns within a country: it uses (i) only labor, (ii) both labor and robots, and (iii) only robots. Within this framework, we investigate international growth patterns.

2.1 Household

A representative household in country $i(=h,f)$ lives infinitely and has the following lifetime utility:

$$U^i = \int_0^\infty e^{-\rho t} \ln c^i \mathrm{d}t,$$

where $\rho(>0)$ and c^i denote the rate of time preference and individual consumption in country i, respectively. Each household inelastically supplies one unit of labor and is subject to the following budget constraint:

$$\dot{a}^i = ra^i + w^i - c^i, \tag{1}$$

where a dot represents a time derivative, w^i is the wage rate in country i, and a^i is the individual financial wealth in country i, which yields interest rate r and consists of equities and foreign assets/liabilities. Owing to arbitrage in the financial market, the interest rate is equalized across the two countries.

The optimality conditions for utility maximization are

$$\frac{\dot{c}^i}{c^i} = r - \rho, \qquad \lim_{s\to\infty} a_s^i e^{-\int_t^s r_v \mathrm{d}v} = 0, \tag{2}$$

where the subscript represents the time index. Because the first condition holds for both countries and labor supply is constant, global total consumption, C, evolves according to

$$\frac{\dot{C}}{C} = r - \rho, \qquad \text{where} \quad C \equiv L^h c^h + L^f c^f. \tag{3}$$

2.2 Production

A representative firm in country i assembles final output Y^i by combining a mass one of tasks Acemoglu and Autor (2011):

$$Y^i = \exp\left[\int_0^1 \ln y^i(z)\mathrm{d}z\right],$$

where $y^i(z)$ represents the input of task z in country i. Each task is performed by employing industrial robot $m^i(z)$ and labor $l^i(z)$ within a country:

$$y^i(z) = \theta_m^i m^i(z) + \theta_l^i \gamma(z) l^i(z),$$

where $\theta_m^i(> 0)$ and $\theta_l^i\gamma(z)(> 0)$ measure the productivity levels of robots and labor in country i, respectively.

Without loss of generality, we assume that (i) robot productivity is higher in country h than in country f and (ii) labor productivity is task-dependent, ordered from lowest to highest:

Assumption 1. $\theta_m^h > \theta_m^f$.

Assumption 2. $\gamma'(z) > 0, \quad \gamma(0) > 0, \quad \gamma(1) < \infty.$

Assumption 2 implies that robots have a comparative advantage over labor in lower-indexed tasks. Because robots and labor are perfectly substitutable in task production, each task requires one of the two production factors. Let I^i be the threshold value such that for $z \leq (\geq) I^i$, task z is conducted by employing robots (labor) in country i. An increase in I^i implies that more tasks are automated in that country.

The firm's profit maximization problem involves choosing the degree of automation I^i and the levels of factor inputs in task z, $m^i(z)$ and $l^i(z)$, respectively:

$$\max_{I^i, m^i(z), l^i(z)} \exp\left[\int_0^{I^i} \ln \theta_m^i m^i(z) \mathrm{d}z + \int_{I^i}^1 \ln \theta_l^i \gamma(z) l^i(z) \mathrm{d}z\right] - q \int_0^{I^i} m^i(z) \mathrm{d}z - w^i \int_{I^i}^1 l^i(z) \mathrm{d}z,$$

where q is the rental price of robots, which is the same across countries because of international robot mobility. The optimality conditions are

$$\frac{q}{\theta_m^i} - \frac{w^i}{\theta_l^i \gamma(I^i)} \begin{cases} \geq 0 & \text{if } I^i = 0, \\ = 0 & \text{if } 0 < I^i < 1, \\ \leq 0 & \text{if } I^i = 1, \end{cases} \tag{4}$$

$$m^i(z) = \frac{Y^i}{q} \qquad \text{for } 0 < I^i \leq 1, \tag{5}$$

$$l^i(z) = \frac{Y^i}{w^i} \qquad \text{for } 0 \leq I^i < 1. \tag{6}$$

The first condition indicates that the firm uses only labor (robots) for all tasks if the marginal cost of robots is higher (lower) than that of labor in an efficiency unit; otherwise, both labor and robots are employed within a country, meaning that the threshold I^i lies between 0 and 1. The second and third conditions provide the demand for each input.

Regarding the robot supplier, we consider a single firm that invests in and rents robots to both countries at the same rental price q.[4] The firm accumulates robots to maximize its value:

$$V = \int_t^\infty (qM - X) e^{-\int_t^s r \mathrm{d}v} \mathrm{d}s, \qquad \text{subject to} \quad \dot{M} = X - \delta M, \quad M \geq 0,$$

[4]Alternatively, we could model that robots are produced within each country and are immobile internationally. In this case, the rental prices in both countries may differ (e.g., Ono and Shibata, 2006, 2010). However, this consideration does not alter our main implications. We believe that our assumption of a single robot producer is not unrealistic because industrial robots are more mobile than traditional capital such as plants.

where M, X, and $\delta(\geq 0)$ denote robot stock, investment in robots, and the depreciation rate of robots, respectively.[5] Households can freely sell or buy equity of this firm in an international financial market.

Optimality requires the transversality condition and

$$r \geq q - \delta \qquad \text{with equality whenever } M > 0. \tag{7}$$

If the marginal cost of capital procurement (r) exceeds the marginal revenue of the accumulation of robots, which is equal to the rental price (q) net of the depreciation rate (δ), then the firm shuts down and is liquidated ($M = 0$). As long as $M > 0$, it holds that $r = q - \delta$.

3 Equilibrium

In this section, we derive the equilibrium conditions. Because the factor demand levels are symmetric across tasks within a country from (5) and (6), we obtain the following relationship:

$$m^i(z) = \frac{M^i}{I^i} \quad \text{for } 0 < I^i \leq 1, \qquad l^i(z) = \frac{L^i}{1 - I^i} \quad \text{for } 0 \leq I^i < 1, \tag{8}$$

where M^i is country i's total demand for robots and satisfies

$$M^h + M^f = M.$$

Using these factor demand, we rewrite the production function of the final good as

$$Y^i = A^i\left(I^i\right)\left(M^i\right)^{I^i}\left(L^i\right)^{1-I^i}, \tag{9}$$

$$\text{where} \quad A^i\left(I^i\right) \equiv \left(\frac{\theta^i_m}{I^i}\right)^{I^i}\left(\frac{\theta^i_l}{1 - I^i}\right)^{1-I^i} \exp\left[\int_{I^i}^1 \ln\gamma(z)\mathrm{d}z\right]. \tag{10}$$

[5] By describing the dynamic motion of the accumulation of robots as $\dot{M} = \zeta X - \delta M$, we can distinguish investment productivity ζ from robot productivity in final good production, θ^i_m. In our setting, an increase in ζ simultaneously increases θ^h_m and θ^f_m. Hence, for simplicity, we omit the investment productivity parameter.

If I^i lies between 0 and 1, this aggregate production technology exhibits decreasing returns to scale with respect to robot and labor inputs. Otherwise, it has constant returns to scale, being only one of the two factors employed:

$$
\begin{aligned}
&Y^i = A^i(0)L^i, && A^i(0) = \theta_l^i \exp\left[\int_0^1 \ln\gamma(z)\mathrm{d}z\right], && \text{if } I^i = 0;\\
&Y^i = A^i(1)M^i, && A^i(1) = \theta_m^i, && \text{if } I^i = 1.
\end{aligned}
$$

We call the former *no automation* and the latter *full automation*. With full automation, the country continues to grow by perpetually accumulating reproducible robots. However, we later show that full automation is only asymptotically achieved in equilibrium and does not occur simultaneously in both countries as long as there is even a slight international difference in robot productivity ($\theta_m^h \neq \theta_m^f$). In other words, whenever one country reaches full automation asymptotically, the other country ceases to expand, employing non-reproducible labor.

Applying (8) and (9) to the factor demand functions (5) and (6) yields

$$
q = I^i A^i(I^i)\left(\frac{M^i}{L^i}\right)^{I^i-1} \qquad \text{for } 0 < I^i \leq 1, \tag{11}
$$

$$
w^i = \left(1 - I^i\right) A^i(I^i)\left(\frac{M^i}{L^i}\right)^{I^i} \qquad \text{for } 0 \leq I^i < 1, \tag{12}
$$

which means that the factor prices equal the marginal product of each input. These relationships rewrite (4), where $0 < I^i < 1$, as

$$
\frac{I^i\theta_l^i\gamma(I^i)}{(1-I^i)\theta_m^i} = \frac{M^i}{L^i} \qquad \Rightarrow \qquad I^i = I^i\left(\frac{M^i}{L^i}\right). \tag{13}
$$

Under Assumption 2, the function $I^i\left(M^i/L^i\right)$ has the following properties:

$$
\begin{aligned}
&\frac{\partial I^i}{\partial\left(M^i/L^i\right)} = \frac{(1-I^i)^2\theta_m^i}{\left[\gamma(I^i) + I^i(1-I^i)\gamma'(I^i)\right]\theta_l^i} > 0;\\
&I^i \to 0 \text{ as } \frac{M^i}{L^i} \to 0;\\
&I^i \to 1 \text{ as } \frac{M^i}{L^i} \to \infty.
\end{aligned} \tag{14}
$$

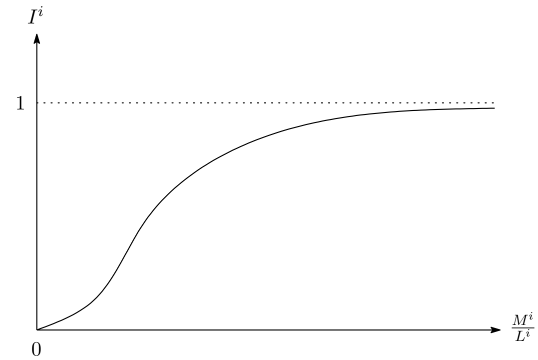


Figure 2 : The relationship between $\frac{M^i}{L^i}$ and I^i.

The sign of the second derivative of the function $I^i\left(M^i/L^i\right)$ is undetermined partly because the sign of $\gamma''(I^i)$ is ambiguous.

Figure 2 illustrates the shape of the function $I^i\left(M^i/L^i\right)$. The last property in (14) shows that the economy never reaches full automation ($I^i = 1$) for a finite M^i and a positive constant L^i. Acemoglu and Restrepo (2018b) derive the condition for full automation to emerge by introducing leisure utility under which households choose zero labor supply as I^i reaches 1. With $L^i = 0$, full automation ($I^i = 1$) is consistent with condition (13), even if M^i is finite. Although our model assumes an inelastic positive labor supply and cannot analyze full automation, we can examine the situation in which M^i continues to increase and I^i asymptotically approaches 1. We call this situation *asymptotic full automation*.

Next, we examine the international allocation of robots depending on their rental price. Substituting $A^i(I^i)$ in (10) and M^i/L^i in (13) into (11) and applying the function $I^i(M^i/L^i)$ in (13) to the result, we obtain the following relationship between robot stock and the rental price:

$$q = \theta_m^i \exp\left[\int_{I^i(M^i/L^i)}^{1} \ln \frac{\gamma(z)}{\gamma\left(I^i(M^i/L^i)\right)} \mathrm{d}z\right] \qquad \text{for } 0 < I^i \leq 1. \tag{15}$$

Assumption 2 and the property in (14) guarantee that

$$\begin{aligned} &\frac{\partial q}{\partial(M^i/L^i)} = -\frac{(1-I^i)\gamma'(I^i)q}{\gamma(I^i)} \frac{\partial I^i}{\partial(M^i/L^i)} < 0; \\ &q \to \overline{q}^i \text{ as } \frac{M^i}{L^i} \to 0, \text{ or } I^i \to 0; \\ &q \to \underline{q}^i \text{ as } \frac{M^i}{L^i} \to \infty, \text{ or } I^i \to 1, \end{aligned} \tag{16}$$

where $\overline{q}^i$ and $\underline{q}^i$ are the upper and lower bounds of the robot rental price, respectively, given by

$$\overline{q}^i \equiv \theta_m^i \exp\left[\int_0^1 \ln \frac{\gamma(z)}{\gamma(0)} \mathrm{d}z\right] \ (> 0), \qquad \underline{q}^i \equiv \theta_m^i \ (\in (0, \overline{q}^i)).$$

Assumption 1 implies that

$$\overline{q}^h > \overline{q}^f, \qquad \underline{q}^h > \underline{q}^f.$$

In Figure 3, Case 1 (Case 2) describes the relationship equation (15) for $i = h, f$ when $\underline{q}^h \geq (<) \overline{q}^f$ due to relatively low (high) θ_m^f. In Cases 1 and 2, as long as robots exist somewhere in the world (i.e., $M > 0$), the equilibrium rental price must lie within a bounded interval between $\overline{q}^h$ and $\underline{q}^h$ because it is profitable for the robot supplier to rent robots to the more productive country h. In contrast to the neoclassical production function, task-based technology does not fulfill the Inada condition; that is, the marginal product of robots does not fall to zero (rise to infinity) as robot stock approaches infinity (zero). This is because the complementarity between the accumulation of robots and automation makes the marginal product of robots decrease less as automation progresses Nakamura and Nakamura (2008). Consequently, there is a possibility that all robots are concentrated in one country. In Case

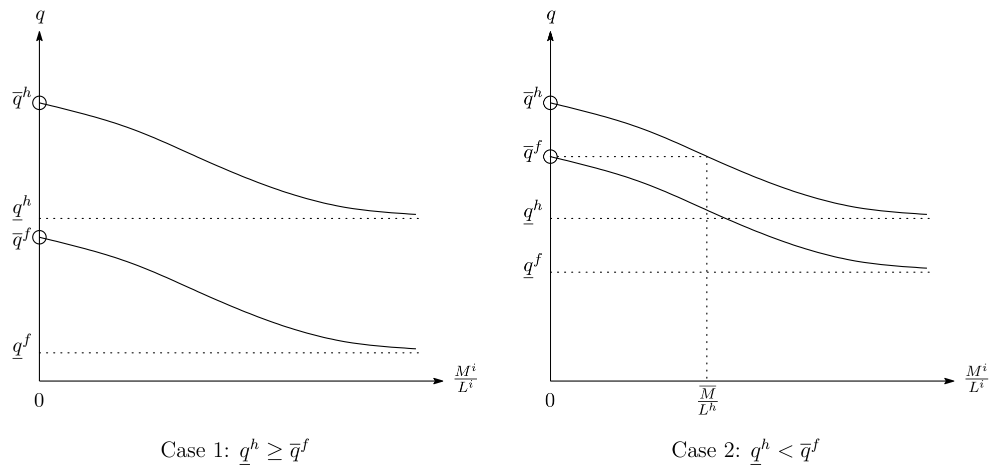


Figure 3 : The relationship between $\frac{M^i}{L^i}$ and q.

1, country h uses all existing robots for all $q \in (\underline{q}^h, \overline{q}^h)$. In Case 2, the concentration of all robots in country h occurs for relatively high $q \in [\overline{q}^f, \overline{q}^h)$; whereas robots are employed in both countries if the rental price is sufficiently low to satisfy $q \in (\underline{q}^h, \overline{q}^f)$.

We now determine the equilibrium rental price. If country h employs all existing robots, then (15) in which $i = h$ and $M^h = M$ determines the equilibrium rental price as

$$q = \widehat{q}\,(M) \equiv \theta_m^h \exp\left[\int_{I^h(M/L^h)}^1 \ln \frac{\gamma(z)}{\gamma\left(I^h(M/L^h)\right)} \mathrm{d}z\right] \qquad \text{for } 0 < I^h \leq 1. \tag{17}$$

This holds for all $M \in (0, \infty)$ in Case 1 in Figure 3.

Consider Case 2 in Figure 3. From (16), the rental price decreases as robots accumulate. As the rental price falls sufficiently to meet $\widehat{q}(M) \in (\underline{q}^h, \overline{q}^f)$, country f, which has low robot productivity, begins to use robots. In other words, robots are also allocated to country f if M rises above the threshold value $\overline{M}$ that fulfills

$$\theta_m^h \exp\left[\int_{I^h\left(\overline{M}/L^h\right)}^1 \ln\frac{\gamma(z)}{\gamma\left(I^h\left(\overline{M}/L^h\right)\right)}\mathrm{d}z\right] = \overline{q}^f. \tag{18}$$

A positive finite $\overline{M}$ exists if and only if $\underline{q}^h < \overline{q}^f$, as shown in Case 2 in Figure 3.

As M exceeds $\overline{M}$, (15) must hold for both $i = h$ and f. Thus, M^f can be expressed as a function of M^h. Furthermore, considering the equilibrium condition in the robot market ($M = M^h + M^f$), we can express M^h and M^f as a function of M, respectively:

$$M^h = \widetilde{M}^h(M), \qquad M^f = \widetilde{M}^f(M) \qquad \text{if } \overline{M} < M. \tag{19}$$

These functions indicate that the robot stock of each country increases with global robot stock: for $i = h$ (or f) and $j = f$ (or h),

$$\frac{\partial M^i}{\partial M} = \left\{1 + \frac{I^i(1-I^i)\gamma'(I^i)\left[\gamma(I^j) + I^j(1-I^j)\gamma'(I^j)\right]M^j}{I^j(1-I^j)\gamma'(I^j)\left[\gamma(I^i) + I^i(1-I^i)\gamma'(I^i)\right]M^i}\right\}^{-1} > 0 \quad \text{if } \overline{M} < M. \tag{20}$$

Thus, we can represent the rental price as a function of M from (15):

$$q = \widetilde{q}(M) \equiv \theta_m^h \exp\left[\int_{I^h\left(\widetilde{M}^h(M)/L^h\right)}^1 \ln\frac{\gamma(z)}{\gamma\left(I^h\left(\widetilde{M}^h(M)/L^h\right)\right)}\mathrm{d}z\right]$$

for $0 < I^h \leq 1$ and $\overline{M} < M$.

Global robot stock M rises according to the equilibrium condition in the final good market:

$$\dot{M} = Y(M) - C - \delta M, \tag{21}$$

where $Y(M) \equiv Y^h + Y^f$ satisfies (9) into which the functions in (13) and (19) are substituted:

$$Y(M)\begin{cases} = A^h\left(I^h\left(\frac{M}{L^h}\right)\right)(M)^{I^h\left(M/L^h\right)}\left(L^h\right)^{1-I^h\left(M/L^h\right)} + A^f(0)L^f & \text{if } M \leq \overline{M}, \\ = A^h\left(I^h\left(\frac{\widetilde{M}^h(M)}{L^h}\right)\right)\left[\widetilde{M}^h(M)\right]^{I^h\left(\widetilde{M}^h(M)/L^h\right)}\left(L^h\right)^{1-I^h\left(\widetilde{M}^h(M)/L^h\right)} & \\ \quad +A^f\left(I^f\left(\frac{\widetilde{M}^f(M)}{L^f}\right)\right)\left[\widetilde{M}^f(M)\right]^{I^f\left(\widetilde{M}^f(M)/L^f\right)}\left(L^f\right)^{1-I^f\left(\widetilde{M}^f(M)/L^f\right)} & \text{if } \overline{M} < M. \end{cases}$$

As shown in Appendix A, global total production increases with global robot stock, independent of the relative levels of M and $\overline{M}$:

$$Y'(M) = q > 0. \tag{22}$$

As M approaches infinity (zero), the slope of $Y(M)$ converges to the minimum (maximum) level of rental price $\underline{q}^h$ ($\overline{q}^h$). As illustrated in Figure 4, the $\dot{M}$ locus strictly increases if $\underline{q}^h \geq \delta$; it has a single peak if $\underline{q}^h < \delta \leq \overline{q}^h$; and it decreases strictly if $\overline{q}^h < \delta$. For analytical simplicity, we exclude the last case in which the supplier has no incentive to accumulate robots from (7) because any equilibrium rental price is below the robot depreciation rate. That is, we assume that

Assumption 3. $\delta \leq \overline{q}^h$.

It is noted that, deviating from the neoclassical framework, production is possible even if robots are not used at all in either country. This implies that the intercept of the $\dot{M}$ locus in Figure 4 has a positive value:

$$Y(0) = A^h(0)L^h + A^f(0)L^f > 0. \tag{23}$$

As (7) holds with equality if $M > 0$, the dynamic equation of aggregate consumption, (3), reduces to

$$\frac{\dot{C}}{C} = q - \rho - \delta, \qquad \text{where} \quad q = \begin{cases} \widehat{q}(M) & \text{if } 0 < M \leq \overline{M}, \\ \widetilde{q}(M) & \text{if } \overline{M} < M. \end{cases} \tag{24}$$

Equations (21) and (24) constitute autonomous dynamics with respect to M and C. Because the rental price has the lower bound $\underline{q}^h$ and the upper bound $\overline{q}^h$, the economy does not necessarily converge to the steady state in which $q = \rho + \delta$. If $\overline{q}^h \leq \rho + \delta$ ($\rho + \delta \leq \underline{q}^h$), aggregate consumption continues to decrease (increase) regardless of robot stock level M.

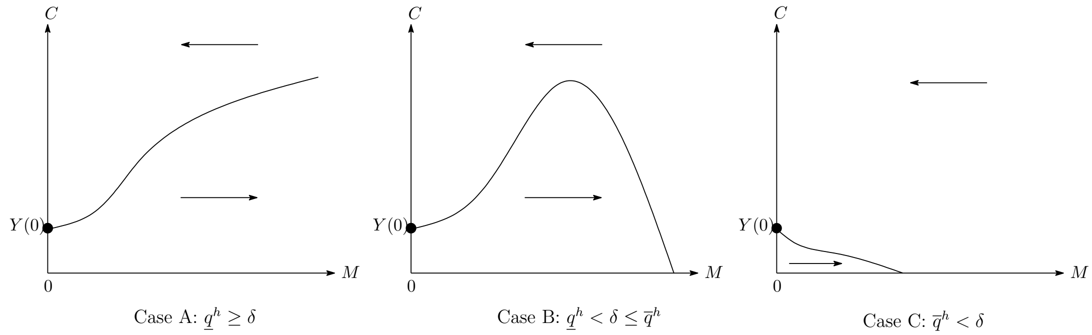


Figure 4 : The $\dot{M} = 0$ locus.

4 International Growth Patterns

This section shows that the following three types of international growth patterns arise:

1. **No automation**: If $\overline{q}^h \leq \rho + \delta$, the economy converges to the steady state in which neither country uses robots, instead employing only labor ($I^h = I^f = 0$).

2. **Partial automation**: If $\underline{q}^h < \rho + \delta < \overline{q}^h$, the economy converges to the steady state in which at least country h uses robots as well as labor ($0 < I^h < 1$ and $0 \leq I^f < I^h$).

3. **Asymptotic full automation**: If $\rho + \delta \leq \underline{q}^h$, the economy converges to a balanced growth path along which country h asymptotically achieves full automation and perpetual production growth but country f does not ($I^h \to 1$ and $0 \leq I^f < 1$).

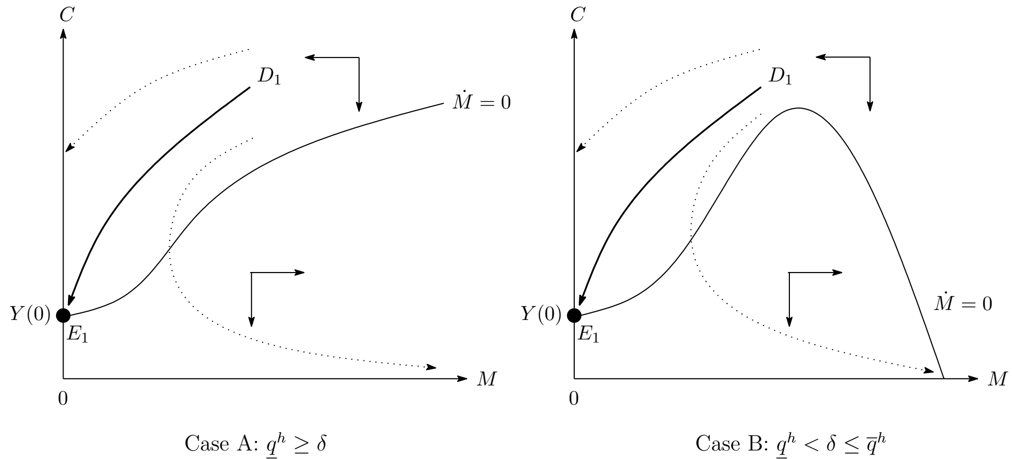


Figure 5 : The phase diagram of the dynamics when $\overline{q}^h \leq \rho + \delta$.

4.1 No automation

We begin with the case in which the robot productivity of country h, θ_m^h, is sufficiently low to satisfy $\overline{q}^h \leq \rho + \delta$, where $\overline{q}^h \geq \delta$ from Assumption 3. Because the equilibrium rental price q is always under $\rho + \delta$, global total consumption continues to fall from (24) as long as $M > 0$. The phase diagram of the dynamics is illustrated in Figure 5. Along the saddle-point stable path D_1E_1, the economy reaches the steady state at which both countries produce output by employing only labor ($M = M^h = M^f = 0$ and $I^h = I^f = 0$) and total consumption is given by $Y(0)$ in (23).[6] In Appendix C, we prove that the trajectories except for D_1E_1 are incompatible with the household's optimality conditions in (2).

[6]In Appendix B, we analyze the local stability of the dynamics around the steady state E_1 and prove that the dynamic system is saddle-point stable at least under Assumption 3. At E_1 in which $M = 0$, we have $C = Y(0)$ from (21) and r can deviate from $\overline{q}^h - \delta$ from (7). If $r > (<)\rho$, over-savings (under-savings) occurs so that future consumption increases (decreases) from (3), and then r immediately falls (rises) to clear the market. As a result, r equals ρ and C remains constant at $Y(0)$ in the steady state E_1.

In transition, country h employs robots and labor until reaching the steady state E_1, whereas country f may or may not use robots. If $\underline{q}^h \geq \overline{q}^f$ (see Case 1 in Figure 3), country f cannot employ robots even in transition because of its low robot productivity. By contrast, if $\underline{q}^h < \overline{q}^f$ (see Case 2 in Figure 3), country f is sufficiently productive to employ robots for a large $M > \overline{M}$ but not for a small $M \leq \overline{M}$, where $\overline{M}$ is defined in (18).

The result is summarized as follows.

Proposition 1. *If the robot productivity of country h is sufficiently low to satisfy* $\overline{q}^h \in [\delta, \rho + \delta]$, *there exists a unique equilibrium path along which neither country eventually uses robots* ($I^h = I^f = 0$)*; that is, no automation prevails in the steady state.*

This type of equilibrium never emerges under neoclassical production functions in which the marginal product of capital approaches infinity as capital stock approaches zero. Our result shows that production automation continues to decline as long as robot technology is scarce.

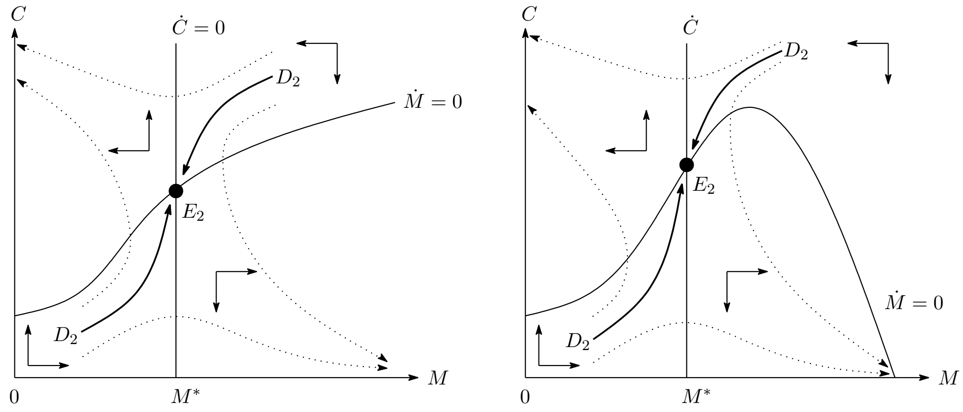
4.2 Partial automation

Consider an advancement of robot technology in country h that leads to $\underline{q}^h < \rho + \delta < \overline{q}^h$. In this case, there is an equilibrium rental price such that $q = \rho + \delta$. The phase diagram of the dynamics is illustrated in Figure 6.

As in the Ramsey–Cass–Koopmans model (see Blanchard, 1989, chapter 2), there exists a unique stable path D_2E_2 that reaches the steady state with a positive and finite level of robot stock M^*.[7]

As robots accumulate along path D_2E_2, the rental price decreases and the number of automated tasks increases in country h. By contrast, this diffusion mechanism may or may not work in country f. If the robot productivity of country f is so low that $\overline{q}^f \leq \underline{q}^h$ (Case 1 in Figure 3), robots

[7] We can rule out the divergent paths starting above or below the path D_2E_2. See Appendix C.



Case A: $\underline{q}^h \geq \delta$ Case B: $\underline{q}^h < \delta \leq \overline{q}^h$

Figure 6 : The phase diagram of the dynamics when $\underline{q}^h < \rho + \delta < \overline{q}^h$.

are not used in country f throughout the entire path ($0 < I^h < 1$ and $I^f = 0$). Even if $\overline{q}^f > \underline{q}^h$ (Case 2 in Figure 3), the same phenomenon with no automation occurs, at least around the steady state E_2 as long as $\overline{q}^f \leq \rho + \delta$ (i.e., $M^* \leq \overline{M}$). However, if country h is sufficiently productive to be $\overline{q}^f > \rho + \delta$, we have $M^* > \overline{M}$, which leads to partial automation in both countries around the steady state ($0 < I^h < 1$ and $0 < I^f < 1$).[8]

We establish the following proposition.

Proposition 2. *If the robot productivity of country h satisfies $\underline{q}^h < \rho + \delta < \overline{q}^h$, there exists a unique equilibrium path along which country h uses both robots and labor ($0 < I^h < 1$); that is, partial automation prevails in country*

[8]Suppose that the economy is initially in the steady state E_1 in Figure 5, where robot stock M is zero, and that an improvement in θ_m^h unanticipatedly takes place. The equilibrium dynamics shift from those in Figure 5 to those in Figure 6. In our setting, this shift is possible because production is feasible using only labor and thus output $Y(0)$ is positive even with no robot stock ($M = 0$). Such a shift is impossible according to the neoclassical production function in which capital and labor are complements and output is zero with no capital stock.

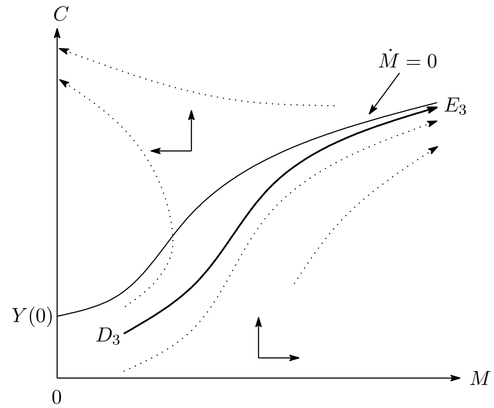


Figure 7 : The phase diagram of the dynamics when $\rho + \delta \leq \underline{q}^h$.

h. In the steady state, country f does not use robots ($I^f = 0$) if the robot productivity of country f is sufficiently low to satisfy $\overline{q}^f \leq \rho + \delta$, but uses both robots and labor ($0 < I^f < 1$) if $\overline{q}^f > \rho + \delta$.

4.3 Asymptotic full automation

The last case is where country h is highly productive to the extent that $\rho + \delta \leq \underline{q}^h$. Because the equilibrium rental price exceeds $\rho + \delta$ for any $M > 0$, global total consumption continues to rise from (24). The $\dot{M} = 0$ locus is strictly increasing, as illustrated in Case A in Figure 4, because $\delta < \underline{q}^h$ is ensured. Thus, we obtain the phase diagram of the dynamics in Figure 7. In Appendix C, we show that the path D_3E_3 is a unique equilibrium path that fulfills the household's optimality conditions (2). Along the path D_3E_3, consumption and robot stock continue to increase and consequently the degree of automation in country h approaches full automation asymptotically ($I^h \to 1$).

Perpetual economic growth is sustained by asymptotic full automation in country h, while country f ceases to grow. Noticing that the rental price

q declines toward $\underline{q}^h$ along the equilibrium path, no automation prevails in country f (i.e., $I^f = 0$) if $\overline{q}^f \leq \underline{q}^h$. Even if country f is sufficiently productive to satisfy $\overline{q}^f > \underline{q}^h$, automation advancements in country f eventually stop around $I^{f*} \in (0, 1)$ that fulfills

$$\theta_m^f \exp\left[\int_{I^{f*}}^{1} \ln \frac{\gamma(z)}{\gamma\left(I^{f*}\right)} \mathrm{d}z\right] = \underline{q}^h, \tag{25}$$

where the left-hand side follows from (15). This is because renting robots over I^{f*} to the less productive country f is unprofitable.

Proposition 3. *If the robot productivity of country h is sufficiently high to satisfy $\rho+\delta \leq \underline{q}^h$, there exists a unique equilibrium path along which country h uses only robots asymptotically ($I^h \to 1$) but country f does not; that is, asymptotic full automation prevails only in country h. Along the equilibrium path, country f does not use robots ($I^f = 0$) if the robot productivity of country f is sufficiently low to satisfy $\overline{q}^f \leq \underline{q}^h$, but uses both robots and labor ($0 < I^f < 1$) if $\overline{q}^f > \underline{q}^h$.*

A key feature of our task-based technology is that the marginal product of robots is bounded from above and below, resulting in uneven economic growth in the financially integrated world. Some other production functions share the similar property, as pointed out by Jones and Manuelli (1990) and Barro and Sala-i-Martin (2004, pp. 66–71, 226–232), but yield different outcomes. For example, if we assume a combination of AK and Cobb–Douglas technologies instead of (9):

$$Y^i = B^i M^i + \left(M^i\right)^\alpha \left(L^i\right)^{1-\alpha}, \qquad \text{where} \quad \frac{\partial Y^i}{\partial M^i} = B^i + \alpha \left(\frac{M^i}{L^i}\right)^{\alpha-1},$$

$B^h > B^f > 0$ and $0 < \alpha < 1$, then the marginal product of robots has a positive lower bound but no finite upper bound:

$$\overline{q}^i = \lim_{\frac{M^i}{L^i} \to 0} \frac{\partial Y^i}{\partial M^i} = \infty, \qquad \underline{q}^i = \lim_{\frac{M^i}{L^i} \to \infty} \frac{\partial Y^i}{\partial M^i} = B^i > 0.$$

In other words, the Inada condition is violated as M^i/L^i approaches infinity. When $\rho+\delta \leq \underline{q}^h$, country h perpetually grows and the rental price converges to the lower bound $\underline{q}^h$. However, along this path, robots are also used in country f; that is, there is no case in which country f employs only labor, unlike in Proposition 3. Therefore, we conclude that our automation technology is more likely to generate uneven international growth.

The same result is obtained if we assume a production function with a constant elasticity of substitution between robots and labor:

$$Y^i = B^i \left[\alpha \left(M^i\right)^{\psi} + (1-\alpha)\left(L^i\right)^{\psi}\right]^{\frac{1}{\psi}},$$

$$\text{where} \quad \frac{\partial Y^i}{\partial M^i} = B^i \alpha \left[\alpha + (1-\alpha)\left(\frac{M^i}{L^i}\right)^{-\psi}\right]^{\frac{1-\psi}{\psi}}.$$

With a high degree of substitution such that $1 < 1/(1-\psi) < \infty$, the marginal product of robots is strictly positive but unbounded from above:

$$\overline{q}^i = \lim_{\frac{M^i}{L^i}\to 0} \frac{\partial Y^i}{\partial M^i} = \infty, \qquad \underline{q}^i = \lim_{\frac{M^i}{L^i}\to \infty} \frac{\partial Y^i}{\partial M^i} = B^i \alpha^{\frac{1}{\psi}} > 0.$$

This production function has a different property when the degree of substitution is low $(1/(1-\psi) < 1)$: the marginal product of robots is bounded from above but converges to zero as M^i/L^i approaches infinity:

$$\overline{q}^i = \lim_{\frac{M^i}{L^i}\to 0} \frac{\partial Y^i}{\partial M^i} = B^i \alpha^{\frac{1}{\psi}} > 0, \qquad \underline{q}^i = \lim_{\frac{M^i}{L^i}\to \infty} \frac{\partial Y^i}{\partial M^i} = 0.$$

This allows for no automation but not for perpetual growth simultaneously. Therefore, we can maintain the previous implication: automation leads to uneven international growth relative to other production technologies.[9]

[9] Nakamura and Nakamura (2008) and Nakamura (2010) point out that the elasticity of substitution between robots and labor exceeds unity through changes in the degree of automation, even if the production function of a final good takes a Cobb–Douglas form, as in (9). To see this, we divide (11) by (12) to obtain q/w^i and totally differentiate the

Table 1 : International growth patterns in the steady state under Assumptions 1–3.

Condition		Country h	Country f
$\delta \leq \overline{q}^h \leq \rho + \delta$		no automation ($I^h = 0$)	no automation ($I^f = 0$)
$\underline{q}^h < \rho + \delta < \overline{q}^h$	$\overline{q}^f \leq \rho + \delta$	partial automation ($0 < I^h < 1$)	no automation ($I^f = 0$)
	$\overline{q}^f > \rho + \delta (> \underline{q}^f)$		partial automation ($0 < I^f < I^h < 1$)
$\rho + \delta \leq \underline{q}^h$	$\overline{q}^f \leq \underline{q}^h$	asymptotic full automation ($I^h \to 1$)	no automation ($I^f = 0$)
	$\overline{q}^f > \underline{q}^h$		partial automation ($0 < I^f < 1$)

4.4 Summary

Table 1 summarizes the results of Propositions 1–3. The improvement in robot productivity in the more productive country h shifts the equilibrium from no automation to partial automation and to asymptotic full automation. In other words, production automation serves as a pathway for sustaining perpetual economic growth. However, the same phenomenon does not occur in the less productive country f. Country f's production ceases to grow, remaining in either no automation or partial automation. If the robot productivity of country f improves to surpass that of country h and Assumption 1 is no longer satisfied, the production growth of country h is halted. In any case, we conclude that automated production technologies cause international dispersion in output growth, unlike models with a neoclassical production function.

Our contributions complement those of Nakamura (2010), who finds

result by considering the relationship between I^i and M^i/L^i in (14). This yields

$$-\frac{\mathrm{d}\frac{M^i}{L^i}/\frac{M^i}{L^i}}{\mathrm{d}\frac{q}{w^i}/\frac{q}{w^i}} = 1 + \frac{\gamma(I^i)}{I^i(1-I^i)\gamma'(I^i)}(> 1).$$

Accordingly, the elasticity of substitution between robots and labor is greater than unity and varies with the degree of automation I^i.

that automation sustains perpetual growth in a closed economy setting. However, while Nakamura focuses solely on asymptotic full automation, we establish the condition under which growth patterns with no automation, partial automation, and asymptotic full automation arise. Notably, we show that asymptotic full automation is achievable by one of the two countries when international technological differences exist (Assumption 1), implying inevitable uneven output growth in an open economy with automation technologies.

One may consider that country f is deprived of opportunities for growth through automation technology. This is rather desirable in terms of welfare. For example, consider the case in which country f is highly productive, but less productive than country h: $\rho + \delta \leq \underline{q}^f$ and $\overline{q}^f \leq \underline{q}^h$. Proposition 3 indicates that country f's production remains constant using only labor, which is referred to as no automation ($I^f = 0$). In this case, if country f prohibits international asset trade, it achieves asymptotic full automation ($I^f \to 0$) because $\rho + \delta \leq \underline{q}^f$, thereby promoting economic growth perpetually. Nevertheless, the closed economy is inferior in terms of country f's welfare. This is because consumption grows at the rate $\underline{q}^f - \rho - \delta$ in the closed economy, which is lower than the rate $\underline{q}^h - \rho - \delta$ in the open economy in which country f can access more profitable investment opportunities. Thus, international financial integration impedes country f's production growth but enhances its welfare. Similar phenomena occur in all the cases in Table 1. We state this result in the following proposition.

Proposition 4. *The integration of financial markets may hamper asymptotic full automation in the less productive country f but does improve its welfare.*

5 Empirical Evidence

The theoretical results in Propositions 1–3 imply that (i) under low robot productivity, economic growth gradually slows and eventually stops unless exogenous technological improvement continues, and (ii) with sufficiently high robot productivity, most tasks become automated and the economy continues to grow perpetually through the accumulation of robots. Thus, as illustrated in Figure 1, the growth impact of increases in robot capital may differ significantly among countries depending on their level of robot productivity. In this section, we present the results of empirically investigating whether the accumulation of industrial robot capital accelerates real GDP growth using panel data on 62 countries from 1994 to 2019.

5.1 Regression model

For simplicity, the theoretical model in the previous sections omitted traditional capital such as plants. To clarify the difference between the empirical implications of traditional and robot capital, we consider the following production function instead of (9):

$$Y_t^i = \left(K_t^i\right)^{\alpha} \left[A_t^i (M_t^i)^{I_t^i} (L_t^i)^{1-I_t^i}\right]^{1-\alpha}, \qquad \text{with} \quad 0 < \alpha < 1,$$

where the subscript t denotes the period, K_t^i represents traditional capital, and

$$A_t^i = A^i(I_t^i), \qquad \frac{\mathrm{d}A_t^i}{\mathrm{d}I_t^i} = -A_t^i \ln \frac{M_t^i}{L_t^i} \gtreqless 0,$$

$$I_t^i = I^i\left(\frac{M_t^i}{L_t^i}\right), \qquad \frac{\mathrm{d}I_t^i}{\mathrm{d}\left(\frac{M_t^i}{L_t^i}\right)} = \frac{(1-I_t^i)^2 \theta_m^i}{\left[\gamma(I_t^i) + I_t^i(1-I_t^i)\gamma'(I_t^i)\right]\theta_l^i} > 0,$$

from (10) and (13), respectively. Robots and labor are perfectly substitutable at each level of task production. The share of income for robots is given by $(1-\alpha)I_t^i$ and changes according to the degree of automation, I_t^i. On the contrary, traditional capital complements both labor and robots. The income share of traditional capital remains constant at α.

Differentiating this production function, we obtain

$$\frac{\dot{Y}_t^i}{Y_t^i} = \alpha\frac{\dot{K}_t^i}{K_t^i} + (1-\alpha)I_t^i\frac{\dot{M}_t^i}{M_t^i} + (1-\alpha)(1-I_t^i)\frac{\dot{L}_t^i}{L_t^i}, \tag{26}$$

where the effects of a change in I_t^i cancel out. The coefficient of the second term on the right-hand side, $(1-\alpha)I_t^i$, shows that the accumulation of robot stock has a stronger impact on the output growth rate in a country with a higher degree of automation, with other factors held constant.

To test this implication, we consider the following regression model with the country fixed effect μ_i and time fixed effect λ_t:

$$g_{it}^Y = \beta_1 g_{it}^K + (\beta_2 + \beta_3 d_i)g_{it}^M + (\beta_4 + \beta_5 d_i)g_{it}^L + \mu_i + \lambda_t + u_{it}, \tag{27}$$

where $\beta_1, \dots, \beta_5$ are the unknown coefficients to be estimated, u_{it} is the error term, and

$$g_{it}^Y \equiv \frac{\dot{Y}_t^i}{Y_t^i}, \quad g_{it}^K \equiv \frac{\dot{K}_t^i}{K_t^i}, \quad g_{it}^M \equiv \frac{\dot{M}_t^i}{M_t^i}, \quad g_{it}^L \equiv \frac{\dot{L}_t^i}{L_t^i}.$$

We classify the sample into two groups based on the data related to their amounts of robot capital. The dummy variable d_i takes 1 if a country is classified as a country group with high robot capital and 0 otherwise. From a theoretical perspective, we hypothesize the following.[10]

Hypothesis 1. *A country group with high robot capital has a greater impact of g_{it}^M on g_{it}^Y than a country group with low robot capital. That is, $\beta_3 > 0$.*

According to our theory, the economy with low robot productivity converges to the steady state with no economic growth. Hence, the effect of g_{it}^M on g_{it}^Y may be negligible in countries with low robot capital, implying that β_2 may be insignificant or a small value. The theory predicts $\beta_1 > 0$, $\beta_4 > 0$, and $\beta_5 < 0$.

[10] The growth-accelerating effect of robot capital is valid even if asymptotic full automation is a distant prospect. Hence, our empirical analysis does not contradict the conclusion of Nordhaus (2021), who shows that singularity, interpreted as asymptotic full automation in our context, is not imminent by checking the growth trajectories of capital-related indicators, particularly the share of capital in total income and share of informational capital in total capital.

5.2 Data

The dataset is an unbalanced panel covering 62 countries from 1994 to 2019, with missing data for some years and countries. Y_t^i represents real Gross Domestic Product (GDP), calculated by dividing nominal GDP (in current U.S. dollars) by the GDP deflator. The data are sourced from the World Economic Outlook (April 2023) published by the International Monetary Fund (IMF). The amount of traditional capital, K_t^i, is computed as the sum of general government capital stock (kgov_n), private capital stock (kpriv_n), and public–private partnerships capital stock (kppp_n), all measured in billions of nominal national currencies. The data are based on the Investment and Capital Stock Dataset 1960–2019 compiled by the IMF. The number of robots, M_t^i, is provided by "operational stock," which measures the number of robots currently deployed, reported by the International Federation of Robotics 2021.[11]

For the data on labor L_t^i, we employ four alternative measures. As a benchmark, we use the employment data for all sexes and age groups (in thousands of people) sourced from the International Labour Organization (ILO). To assess the robustness of our regression results, we also use three alternative measures. First, we replace the employment data with the working-age population data (in millions of people) from the World Economic Outlook (April 2023) compiled by the IMF. Second, we use different employment data (in millions of people), taken from the World Economic Outlook (April 2023). The coverage of countries in the IMF's employment

[11] In a broader sense, automation-related capital contains Information and Communication Technology (ICT) assets. The Conference Board's Total Economy Database provides ICT data only on a flow service basis. EU KLEMS, compiled by the European Commission, includes the ICT stock data but covers only 26 countries, with some missing values. Eden and Gaggl (2018, 2020) employ these ICT data to examine its growth effects. Simulating the United States, Eden and Gaggl (2018) reveal the recent increasing contribution of ICT to per capita income, which improves the welfare of the representative household. However, Eden and Gaggl (2020) find no systematic relationship between ICT and per capita income when considering cross-country differences in skilled and unskilled labor endowments.

data is more limited than that of the ILO's. While the IMF relies on employment data submitted by individual countries, the ILO's data are based on statistics compiled by national authorities in accordance with ILO standards.[12] Therefore, in countries in which both IMF and ILO employment data are available, discrepancies exist between the values of each dataset. Third, we calculate the total hours worked by multiplying the employment data by the mean weekly hours actually worked per employed person, covering all sexes, age groups, and economic activities, as reported by the ILO.

The dummy variable d_i takes 1 if a country is classified as having high robot capital and 0 otherwise. We consider two types of classifications using the industrial robot data from the International Federation of Robotics 2021. In the baseline regression, we classify countries based on robot density, defined as the number of operational industrial robots per 10,000 employees in the manufacturing industry. As a robustness check of our regression results, we alternatively classify countries using the value of the "operational stock" of industrial robots instead of robot density. See Table 2 for a classification and list of countries with high robot capital.

Table 3 reports the summary statistics of the sample used in the baseline regression analysis. The growth rate of robot capital, g_{it}^M, is extremely high and considerably volatile, with an average rate of 42.15% and standard deviation of 6.85%. This reflects the fact that the number of operational robots is low in many countries; hence, even a slight change in robot stock can vary the growth rate markedly. The correlation matrix in Table 4 shows that there is no perfect multicollinearity between the explanatory variables.

[Tables 2–4]

[12]Further details can be found in the International Labour Standards, available at https://www.ilo.org/international-labour-standards.

5.3 Baseline regression results

Table 5 presents the baseline results of the regression model (27) using the Ordinary Least Squares (OLS) estimation. As a benchmark, we use the employment data from the ILO as a proxy for L_t^i and categorize countries based on their robot density. The dummy variable d_i equals one if a country is ranked among the top 10 in robot density at least once between 2000 and 2019. Table 2 lists the top 10 countries by robot density.

[Table 5]

Columns (1)–(4) report the results under the different fixed effects specifications: without fixed effects, with country fixed effects only, with time fixed effects only, and with both country and time fixed effects, respectively. The coefficient of the interaction term $d_i \times g_{it}^M$ is positive and statistically significant at the 1% level across all the regressions. By contrast, g_{it}^M is not statistically significant in all the regressions except for that without fixed effects, indicating its limited explanatory power. These results support Hypothesis 1, suggesting that the accumulation of industrial robots drives real GDP growth in high robot-density countries, but not in countries with low robot density. Hence, international disparities in GDP growth would widen owing to the accelerated economic growth in countries with high robot density. As shown in column (4), which presents the OLS regression result with country and time fixed effects, the estimated coefficient of $d_i \times g_{it}^M$ is 0.010. This implies that a 1% increase in the growth rate of robot stock leads to a 0.01% increase in the GDP growth rate in countries with high robot density.

Our findings are consistent with those of previous studies. Berg et al. (2018) simulate how improved robot productivity affects GDP by considering various cases of the substitutability between robots and labor. They reveal that when robots compete with labor in all tasks and traditional capital remains unchanged in the initial state, GDP increases by 12–21% in the long run, accompanied by the accumulation of robot capital. Acemoglu (2024) evaluates the potential impact of artificial intelligence on future GDP using the task-based model of Acemoglu and Restrepo (2022b). Under a moderate increase in capital stock driven by artificial intelligence, real GDP growth is projected to fall by 0.93–1.16% over the next decade in total. In an alternative scenario with a substantial investment boom, these estimates rise to 1.40–1.56%. This calculation is consistent with our estimations from data on the accumulation of robots. The cumulative growth rate of robot stock in countries ranked among the top 10 in robot density is 124% on average over the 10 years from 2010 to 2019. Hence, according to our estimated coefficient of $d_i \times g_{it}^M$, these countries experience a 1.24% increase in real GDP growth over those 10 years.

No definitive results are obtained for the other variables. The positive relationship between g_{it}^K and g_{it}^Y is significant in columns (1)–(3), but not significant in column (4), which shows that results based on the regression with country and time fixed effects. Further, g_{it}^L has no explanatory power in any of the regressions except in the specification without fixed effects. The coefficient of $d_i \times g_{it}^L$ is significant but exhibits the wrong sign.

5.4 Robustness check

We assess the robustness of the regression results in Table 5 using four approaches: (i) using alternative data for labor L_t^i, (ii) altering the classification criteria for countries with high robot capital, (iii) trimming the top and bottom 5% of the data on the growth rate of robot stock, and (iv) re-estimating the model using per capita variables.

Alternative labor data

In the baseline regression, we use the employment data from the ILO as a proxy for labor L_t^i to calculate the growth rate g_{it}^L. Alternative proxies are available. Table 6 presents the regression results using the working-age population data from the IMF instead of the employment data. Table 7 presents the results using the alternative employment data from the IMF. Table 8 considers the hours worked by multiplying the employment data in Table 5 by the mean hours worked per employed person.

[Tables 6–8]

In all cases, the core finding is robust: the coefficient of $d_i \times g_{it}^M$ remains significantly positive, whereas the explanatory power of g_{it}^M is limited. Importantly, the alternative labor data alter the significance of the coefficient of g_{it}^L, aligning with the sign predicted by the theoretical model (see Tables 7 and 8); g_{it}^L is positively associated with g_{it}^Y and $d_i \times g_{it}^L$ has a negative coefficient. In summary, while these alternative labor data are desirable, they have the disadvantage of a substantial decrease in the number of observations.

Classification criteria for high robot-capital countries

The baseline regression designates countries as installing high robot capital if their robot density ranks within the top 10 at least once between 2000 and 2019. In this robustness check, we explore alternative classification criteria. In Tables 9 and 10, the criteria for robot density are changed from the top 10 to the top 5 and to top 15, respectively. In Table 11, countries are instead ranked by their operational stock of industrial robots and countries whose robot stocks are ranked in the top 10 at least once between 2000 and 2019 are classified as high robot-capital countries. Table 2 lists the countries with high robot capital under each classification criterion. To allow for a comparison with Table 5, we use the employment data from the ILO as a proxy for labor L_t^i in Tables 9–11.

[Tables 9–11]

The regression results in Tables 9 and 11 are essentially unchanged from those in Table 5. However, in Table 9, which uses a classification criterion based on a robot density ranking in the top 5, all the coefficients of $d_i \times g_{it}^M$ are approximately 0.002 higher than those in Table 5. By contrast, as shown in Table 10, when countries with less high robot density are included in the high robot-capital country group, the robot growth rate $d_i \times g_{it}^M$ loses explanatory power and the labor growth rate g_{it}^L becomes a more dominant explanatory factor.

Trimmed data

As production using industrial robots has advanced rapidly, the sample includes extremely high growth rates of robot capital in some countries over certain years. Although such data could be informative, we test the model by trimming the data on the growth rates of robot capital to confirm the robustness of the regression results in Table 5. We use the employment data from the ILO as a proxy for L_t^i and classify countries using a criterion based on a robot density ranking in the top 10.

Table 12 reports the results obtained by trimming the top and bottom 5% of the data on the growth rate of robot capital. The coefficient of g_{it}^M becomes significantly positive as a result of removing the bottom 5% of the data. By contrast, while the coefficient of $d_i \times g_{it}^M$ remains significant, its estimates decrease because of the exclusion of the top 5% of the data. In Table 13 where we trim the top 10% of the data on the growth rates of robot capital, this tendency is more evident: the coefficient of $d_i \times g_{it}^M$ is no longer significant because of the exclusion of a greater number of high robot-density countries.

[Tables 12–13]

Per capita variables

By subtracting $\dot{L}_t^i/L_t^i$ from both sides and rearranging the result, we can express (26) in terms of per capita variables as follows:

$$\frac{\dot{y}_t^i}{y_t^i} = \alpha\frac{\dot{k}_t^i}{k_t^i} + (1-\alpha)I_t^i\frac{\dot{m}_t^i}{m_t^i},$$

where $y_t^i \equiv Y_t^i/L_t^i$, $k_t^i \equiv K_t^i/L_t^i$, and $m_t^i \equiv M_t^i/L_t^i$. Accordingly, in this robustness check, we test the following regression model instead of (27):

$$g_{it}^y = \delta_1 g_{it}^k + (\delta_2 + \delta_3 d_i)g_{it}^m + \mu_i + \lambda_t + e_{it}, \tag{28}$$

where $g_{it}^y \equiv \dot{y}_t^i/y_t^i$, $g_{it}^k \equiv \dot{k}_t^i/k_t^i$, $g_{it}^m \equiv \dot{m}_t^i/m_t^i$; $\delta_1, \ldots, \delta_3$ are the unknown coefficients to be estimated; and e_{it} is the error term. We predict $\delta_1 > 0$, $\delta_2 > 0$, and $\delta_3 > 0$.

As shown in Table 5, we classify high robot-capital countries using a criterion based on a robot density ranking in the top 10. To calculate a per capita value of each variable, we use four alternative measures for labor L_t^i: the IMF's employment data in Table 14, the population data in Table 15, the ILO's employment data in Table 16, and the hours worked data in Table 17. Importantly, in Tables 14–17, the interaction term $d_i \times g_{it}^m$ is significantly and positively associated with g_{it}^y, implying the growth-accelerating effect of robot capital (Hypothesis 1). The coefficient of g_{it}^m is not significant in Tables 14–17. Compared with the results in Tables 5–8 derived from the regression model (27), the notable distinction is that g_{it}^k has a significant positive coefficient (except for in column (4) in Table 15), thus showing the GDP growth effect of traditional capital.

[Tables 14–17]

6 Conclusion

This study examines the international growth patterns in an open economy with task-based automation technologies. In our framework, production can be performed by using only labor or, asymptotically, only robots. We show that while the more productive country can achieve asymptotic full automation, the less productive country cannot. This implies that automation technology becomes a factor that can cause international dispersion in production growth, which occurs as long as the international difference in robot productivity is even slight. However, while restricting international asset trade may open the door for the low productivity country to achieve asymptotic full automation, this is undesirable in terms of welfare.

We also present empirical results that support the theoretical implications of the relationship between automation and production growth. According to our panel data analysis including 62 countries from 1994 to 2019, countries with more industrial robot installations exhibit higher production growth as the accumulation of robots increases. In other words, the accumulation of robots accelerates economic growth and widens the international disparities in economic growth. The robustness of our regression results is ensured even if we (i) use alternative labor proxies, (ii) redefine the criteria of a country group with high robot capital, (iii) trim the top and bottom 5% of the data on the growth rate of robot stock, and (iv) rewrite and regress the model in terms of per capita variables.

Our model has potential for future research. We assume that the task-dependent component of labor productivity, $\gamma(z)$, has a common curvature across countries. Relaxing this assumption generates richer implications; for example, the comparative advantages between countries change over time, thereby providing more complicated dynamics of automation. Second, we exclude the convergence and divergence in cross-country income that stem from the international diffusion of robot and labor productivities, facilitated by technological transfers through, for example, R&D activities, imitation, and foreign direct investment (Barro and Sala-i-Martin, 2004, chapter 8; Aghion and Howitt, 2008, chapter 7). Integrating these elements with automation technology would yield a more comprehensive understanding of the global income distribution, whereas our model isolates a distinct mechanism driven by production automation. Third, new task creation is vital as an engine of economic growth (Acemoglu and Restrepo, 2018b; Nakamura and Zeira, 2024), whereas our model relies on automation through the accumulation of robots. Considering both engines would have additional implications for labor allocation. Fourth, a more accurate evaluation of welfare in the presence of automated technologies requires considering the possibility of unemployment (Cords and Prettner, 2022; Ogawa and Shimizu, 2022), changes in the terms of trade (Momoda et al., forthcoming), and the effects of task offshoring (Mandelman and Zlate, 2022). Finally, the introduction of low- and high-skilled workers into the model, as in Acemoglu and Restrepo (2018a), would provide insights into the impact of automation on the income distribution.

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Declaration of Interests

None

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Appendix A. Slope of $Y(M)$

In this appendix, we derive the slope of the $Y(M)$ function, represented in (22). Using (10) and (13) to substitute for $A^i(I^i)$ and M^i/L^i from (9), we have

$$Y^i = \frac{\theta_l^i \left[\gamma(I^i)\right]^{I^i} L^i}{1 - I^i} \exp\left[\int_{I^i}^1 \ln\gamma(z)\mathrm{d}z\right].$$

The partial differentiation of this equation with respect to I^i yields

$$\frac{\partial Y^i}{\partial I^i} = \frac{\left[\gamma(I^i) + I^i(1 - I^i)\gamma'(I^i)\right] Y^i}{(1 - I^i)\gamma(I^i)} > 0.$$

Multiplying this result by $\partial I^i/\partial(M^i/L^i)$ in (14), $\partial(M^i/L^i)/\partial M^i = 1/L^i$ and $\partial M^i/\partial M$, we obtain

$$\frac{\partial Y^i}{\partial M} = \frac{I^i Y^i}{M^i}\frac{\partial M^i}{\partial M} = q\frac{\partial M^i}{\partial M},$$

where the second equality is derived from (9) and (11). Because $M^h = M$ when country h uses all existing robots (more precisely, when either $\underline{q}^h \geq \overline{q}^h$ or $\underline{q}^h < \overline{q}^h$ and $M \leq \overline{M}$), it holds that $\partial M^h/\partial M = 1$ and $\partial M^f/\partial M = 0$. On the contrary , when both countries use robots (i.e., when $\underline{q}^h < \overline{q}^h$ and $\overline{M} < M$), $\partial M^i/\partial M$ is given by (20) and we have $(\partial M^h/\partial M) + (\partial M^f/\partial M) = 1$. Accordingly, irrespective of the relative magnitude between $\underline{q}^h$ and $\overline{q}^h$ and the level of M, the slope of $Y(M)$ is represented by (22):

$$Y'(M) = \frac{\partial Y^h}{\partial M} + \frac{\partial Y^f}{\partial M} = q.$$

Appendix B. Local Stability of the Dynamics in Figure 5

This appendix analyzes the local stability of the dynamics around the no-automation steady state E_1 in Figure 5. Linearizing the dynamic system (21) and (24) around $M = 0$ and $C = Y(0)$, we obtain

$$\begin{pmatrix} \dot{M} \\ \dot{C} \end{pmatrix} = \begin{bmatrix} \overline{q}^h - \delta & -1 \\ \widehat{q}'(0)Y(0) & \overline{q}^h - \rho - \delta \end{bmatrix} \begin{pmatrix} M - 0 \\ C - Y(0) \end{pmatrix}.$$

Recall that (i) the rental price approaches $\overline{q}^h$ toward $M = 0$ and (ii) the relationship between q and M is represented by the function $\widehat{q}(M)$ in (17) because all existing robots are employed in country h in the neighborhood of sufficiently small M, irrespective of either $\underline{q}^h \geq \overline{q}^h$ or $\underline{q}^h < \overline{q}^h$. The determinant of the coefficient matrix is as follows:

$$\text{Det} = \left(\overline{q}^h - \delta\right)\left(\overline{q}^h - \rho - \delta\right) + \widehat{q}'(0)Y(0).$$

We have (i) $\overline{q}^h - \delta \geq 0$ from Assumption 3, (ii) $\overline{q}^h - \rho - \delta \leq 0$ in the present case, and (iii) $Y(0) > 0$ from (23). Substitute the first equation in (14) into the first equation in (16) and evaluate the result by $i = h$, $M^h = M$, $I^h = 0$, and $q = \overline{q}^h$. This yields

$$\widehat{q}'(0) = -\frac{\gamma'(0)\overline{q}^h\theta_m^h}{[\gamma(0)]^2\,\theta_l^h L^h} < 0.$$

Therefore, the determinant is negative and the dynamic system has one positive and one negative characteristic root. Because C is jumpable and M is not, the dynamic path is saddle-point stable at least under Assumption 3.

Appendix C. Uniqueness of the Equilibrium Path

In this appendix, we show that path D_jE_j ($j = 1, 2, 3$) described in Figures 5–7 is a unique equilibrium path. Starting above path D_jE_j, the trajectory hits the vertical axis in finite time and then aggregate consumption must fall to $Y(0)$ discontinuously. This discrete jump is incompatible with the consumption-smoothing condition (3). Blanchard and Fischer (1989, appendix A in p. 75) provide the mathematical proof to rule out such paths. By contrast, the trajectory starting below the path D_jE_j violates the transversality condition in (2), as we prove below.

Before proceeding with the proof, we show that the wage rate w^i ($i = h, f$) is bounded from above and below. Substituting $A^i(I^i)$ in (10) and

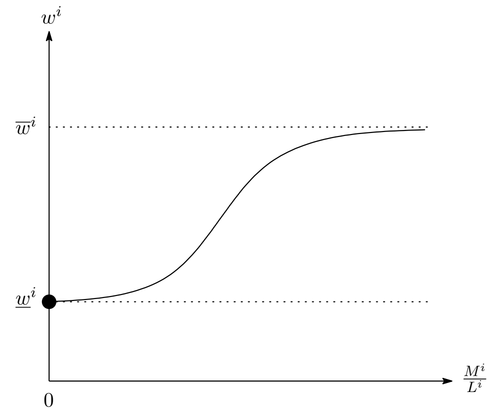

Figure C1 : The relationship between $\frac{M^i}{L^i}$ and w^i.

M^i/L^i in (13) into w^i in (12) and applying the function $I^i(M^i/L^i)$ in (13) to the result, we can relate the wage rate to the capital-to-labor ratio as follows:

$$w^i = \theta_l^i \gamma\left(I^i\left(\frac{M^i}{L^i}\right)\right)\exp\left[\int_{I^i(M^i/L^i)}^1 \ln\frac{\gamma(z)}{\gamma\left(I^i(M^i/L^i)\right)}\mathrm{d}z\right] \qquad \text{for } 0 \leq I^i < 1.$$

As described in Figure C1, the wage rate is positive, bounded between $\underline{w}^i$ and $\overline{w}^i$, and strictly increasing with the robot-to-labor ratio:

$$\frac{\partial w^i}{\partial(M^i/L^i)} = -\frac{I^i\gamma'(I^i)w^i}{\gamma(I^i)}\frac{\partial I^i}{\partial(M^i/L^i)} > 0 \quad \text{from Assumption 2 and (14);}$$
$$w^i = \underline{w}^i \text{ as } \frac{M^i}{L^i} = 0, \text{ or } I^i = 0; \qquad w^i \to \overline{w}^i \text{ as } \frac{M^i}{L^i} \to \infty, \text{ or } I^i \to \infty,$$

where the lower and upper bounds of the wage rate, $\underline{w}^i$ and $\overline{w}^i$, are given by

$$\underline{w}^i \equiv \theta_l^i \exp\left[\int_0^1 \ln\gamma(z)\mathrm{d}z\right] \ (\in (0, \overline{w}^i)), \qquad \overline{w}^i \equiv \theta_l^i\gamma(1) \ (\in (0,\infty)). \ \text{(C1)}$$

Cases of no automation and partial automation in Figures 5 and 6

Let us return to the proof. First, we consider the dynamics of no automation and partial automation, illustrated in Figures 5 and 6. Integrate the household's budget equation (1) from time t to ∞ to obtain

$$\lim_{s\to\infty} a_s^i e^{-\int_t^s r_v \mathrm{d}v} - a_t^i = \int_t^\infty \left(w_s^i - c_s^i\right) e^{-\int_t^s r_v \mathrm{d}v} \mathrm{d}s.$$

Because the integration of the Euler equation (2) from time t to s generates

$$c_s^i = c_t^i e^{\int_t^s r_v \mathrm{d}v},$$

we have the following consumption function:

$$c_t^i = \rho \left(a_t^i + \int_t^\infty w_s^i e^{-\int_t^s r_v \mathrm{d}v} \mathrm{d}s - \lim_{s\to\infty} a_s^i e^{-\int_t^s r_v \mathrm{d}v} \right).$$

Summing up this equation yields the following aggregate consumption function:

$$C_t = \rho \left(M_t + H_t - \lim_{s\to\infty} M_s e^{-\int_t^s r_v \mathrm{d}v} \right), \tag{C2}$$

$$\text{where} \quad M_t = L^h a_t^h + L^f a_t^f,$$

$$H_t \equiv L^h \int_t^\infty w_s^h e^{-\int_t^s r_v \mathrm{d}v} \mathrm{d}s + L^f \int_t^\infty w_s^f e^{-\int_t^s r_v \mathrm{d}v} \mathrm{d}s.$$

The second equation represents the equilibrium condition in the financial market and H_t is aggregate human capital.

Along the trajectory starting below the path $D_j E_j$ ($j = 1, 2$), aggregate consumption decreases toward zero ($C_t \to 0$), keeping robot stock positive ($M_t > 0$). Moreover, as shown in Figure C1, the wage rates of both countries are bounded at positive values; hence, aggregate human capital is positive ($H_t > 0$). Accordingly, the consumption function (C2) implies that

$$\lim_{s\to\infty} M_s e^{-\int_t^s r_v \mathrm{d}v} > 0,$$

or, equally, that at least the transversality condition of either country is violated along this trajectory. In other words, the over-accumulation of robots occurs; thus, it is optimal to increase consumption. Consequently, the path D_jE_j $(j = 1, 2)$ is shown to be a unique equilibrium path.

Case of asymptotic full automation in Figure 7

Next, we consider the dynamics of asymptotic full automation in Figure 7. Over time, the rental price and country h's wage rate converge toward $\underline{q}^h = \theta_m^h$ and $\overline{w}^h = \theta_l^h \gamma(1)$, respectively (see Figures 3 and C1). Country f's wage rate is bounded and lies at a certain constant level, w^{f*}, between $\overline{w}^f$ and $\underline{w}^h$. Hence, aggregate human capital converges to the following positive constant value:

$$\lim_{t\to\infty} H_t = \frac{L^h \overline{w}^h + L^f w^{f*}}{\underline{q}^h - \delta} (> 0),$$

where we used $r_s = \underline{q}^h - \delta (> 0)$ for $M_t > 0$ from (7) and $\rho + \delta \leq \underline{q}^h$ in the case of asymptotic full automation (see Proposition 3).

Given this $\lim_{t\to\infty} H_t (> 0)$ and robot stock $M_t (> 0)$, the optimal level of consumption that satisfies the individual transversality conditions is uniquely determined by

$$C_t = \rho \left(M_t + \lim_{t\to\infty} H_t \right),$$

which is from (C2). This consumption level is on the path D_3E_3 in Figure 7. Lower consumption leads to the violation of optimality:

$$\lim_{s\to\infty} M_s e^{-\int_t^s r_v \mathrm{d}v} > 0.$$

This implies that robots accumulate too quickly and consumption grows too slowly. Thus, the path D_3E_3 is a unique equilibrium path.

Appendix D. International Labor Mobility and Full Automation

In the main text, we assume away international labor mobility. This appendix clarifies that labor mobility causes the population to be concentrated in either of the two countries, thereby generating the possibility of *(non-asymptotic) full automation* in the more productive country, $I^h = 1$. The driving force behind this population concentration is that wages do not rise to infinity (do not fall to zero) as labor decreases to zero (increases to infinity), unlike under the neoclassical production function. This property of wages is illustrated in Figure C1.

Because households can freely access the international financial markets in both countries, the incentive for labor mobility is governed solely by the international difference in wage rates—people move to the country with the higher wage rate. Here, we consider that L^h and L^f represent the population of each country satisfying

$$L^h + L^f = \overline{L},$$

where $\overline{L}$ is the constant world population.

Population concentration under no automation

First, we consider the case in which both countries initially remain at the steady state with no automation, E_1, in Figure 5 and suppose that restrictions on migration are unanticipatedly removed. In this case, wage rates are at each lower bound $\underline{w}^i$ and then all people migrate to the country with the higher wage rate or higher labor productivity θ_l^i.

Proposition D.1. *Suppose that the economy is initially in the steady state with no automation under the condition $\delta \leq \overline{q}^h \leq \rho+\delta$. Then, international labor mobility causes the population to be concentrated in country f (country h); that is, $L^f = \overline{L}$ ($L^h = \overline{L}$) if its labor productivity is sufficiently high to satisfy $\underline{w}^f > (<) \underline{w}^h$.*

Table D1 : Labor mobility and automation under $\rho + \delta \leq \underline{q}^h$.

<table>
<tr><th colspan="3">Conditions</th><th>Country h</th><th>Country f</th></tr>
<tr><td rowspan="2">$\overline{q}^f \leq \underline{q}^h$</td><td colspan="2">$\underline{w}^f < \overline{w}^h$</td><td>asymptotic full automation
($L^h = \overline{L}$, $I^h \to 1$)</td><td>no production
($L^f = 0$, $I^f = 0$)</td></tr>
<tr><td colspan="2">$\underline{w}^f > \overline{w}^h$</td><td>full automation
($L^h = 0$, $I^h = 1$)</td><td>no automation
($L^f = \overline{L}$, $I^f = 0$)</td></tr>
<tr><td rowspan="4">$\overline{q}^f > \underline{q}^h$</td><td colspan="2">$\underline{w}^f < \underline{w}^h$</td><td rowspan="2">asymptotic full automation
($L^h = \overline{L}$, $I^h \to 1$)</td><td rowspan="2">no production
($L^f = 0$, $I^f = 0$)</td></tr>
<tr><td rowspan="2">$\underline{w}^h < \underline{w}^f \leq \overline{w}^h$</td><td>$w^{f*} < \overline{w}^h$</td></tr>
<tr><td>$w^{f*} > \overline{w}^h$</td><td rowspan="2">full automation
($L^h = 0$, $I^h = 1$)</td><td rowspan="2">no automation
($L^f = \overline{L}$, $I^f = 0$)</td></tr>
<tr><td colspan="2">$\underline{w}^f > \overline{w}^h$</td></tr>
</table>

If the production function satisfies the Inada condition, such population concentration never occurs; that is, labor is allocated to both countries so that wage rates are equalized internationally.

Population concentration leads to full automation

Next, we focus on the situation in which asymptotic full automation initially prevails in country h, as Figure 7 illustrates. When labor mobility is restricted, the wage rate in country h approaches the upper bound $\overline{w}^h$, whereas that in country f takes a value between the upper bound $\overline{w}^f$ and lower bound $\underline{w}^f$ depending on its labor productivity.

In the first case, $\overline{q}^f \leq \underline{q}^h$, implying no automation in country f (see Proposition 3). Its wage rate is given by the lower bound $\underline{w}^f$. Therefore, if $\underline{w}^f > \overline{w}^h$ because of the high labor productivity of country f, all people move there and thereby full automation is realized in country h (i.e., $L^h = 0$, $L^f = \overline{L}$, $I^h = 1$, $I^f = 0$). Otherwise, full automation remains asymptotic even after labor mobility because all people prefer to work in country h (i.e., $L^h = \overline{L}$, $L^f = 0$, $I^h \to 1$, $I^f = 0$).

In the second case, partial automation prevails in country f because $\overline{q}^f > \underline{q}^h$ (see Proposition 3). From (13) and (25), the robot-to-labor ratio of country f is in the neighborhood of m^{f*} such that the marginal productivity

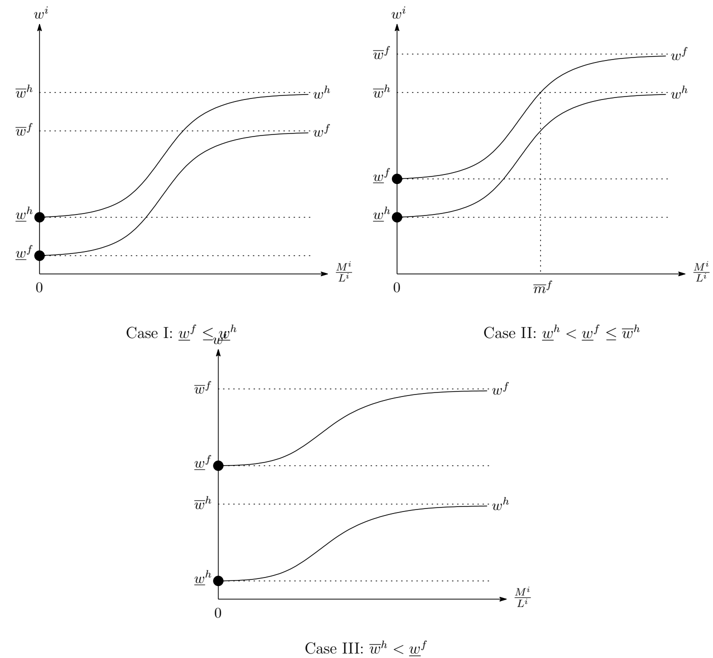

Figure D1 : The relationship between w^h and w^f.

of robots equals the rental price $\underline{q}^h$:

$$\theta_m^f \exp\left[\int_{I^f(m^{f*})}^{1} \ln \frac{\gamma(z)}{\gamma\left(I^f(m^{f*})\right)} \mathrm{d}z\right] = \underline{q}^h.$$

Given this m^{f*}, the functional relationship between w^i and M^i/L^i, given in Appendix C, determines the equilibrium wage rate of country f, denoted by w^{f*}, between $\underline{w}^f$ and $\overline{w}^f$:

$$w^{f*} = \theta_l^f \gamma\left(I^f\left(m^{f*}\right)\right) \exp\left[\int_{I^f(m^{f*})}^{1} \ln \frac{\gamma(z)}{\gamma\left(I^f(m^{f*})\right)} \mathrm{d}z\right] \in (\underline{w}^f, \overline{w}^f).$$

Figure D1 depicts three types of wage schedules. Case I fulfills $\underline{w}^f \leq \underline{w}^h$ because of the low labor productivity of country f. Any level of w^{f*} is below country h's wage rate, $\overline{w}^h$. Hence, country h attracts people and retains asymptotic full automation (i.e., $L^h = \overline{L}$, $L^f = 0$, $I^h \to 1$, $I^f = 0$). As country f's labor productivity θ_l^f increases, the wage schedule shifts to Case II, where $\underline{w}^h < \underline{w}^f \leq \overline{w}^h$ and the marginal labor productivity of country f equals country h's wage rate $\overline{w}^h$ at the threshold value $\bar{m}^f$:

$$\theta_l^f \gamma \left(I^f \left(\overline{m}^f \right) \right) \exp \left[\int_{I^f(\overline{m}^f)}^1 \ln \frac{\gamma(z)}{\gamma \left(I^f \left(\overline{m}^f \right) \right)} \mathrm{d}z \right] = \overline{w}^h.$$

If $w^{f*} > \overline{w}^h$, country h achieves full automation (i.e., $L^h = 0$, $L^f = \overline{L}$, $I^h = 1$, $I^f = 0$); otherwise, it retains asymptotic full automation (i.e., $L^h = \overline{L}$, $L^f = 0$, $I^h \to 1$, $I^f = 0$). Case III corresponds to high labor productivity in country f: $\overline{w}^h < \underline{w}^f$. Because any w^{f*} exceeds country h's wage rate $\overline{w}^h$, country h attracts all people and achieves full automation (i.e., $L^h = 0$, $L^f = \overline{L}$, $I^h = 1$, $I^f = 0$).

The conditions for full automation are summarized in Table D1 and in the following proposition.

Proposition D.2. *Suppose that the economy is initially in the steady state with asymptotic full automation under the condition $\rho + \delta \leq \underline{q}^h$. Then, international labor mobility leads to (non-asymptotic) full automation in country h, causing the population to be concentrated in country f if the labor productivity of country f is sufficiently high, more concretely, if one of the following three conditions is met: (i) $\overline{q}^f \leq \underline{q}^h$ and $\underline{w}^f > \overline{w}^h$; (ii) $\overline{q}^f > \underline{q}^h$ and $\underline{w}^h < \underline{w}^f \leq \overline{w}^h < w^{f*}$; and (iii) $\overline{q}^f > \underline{q}^h$ and $\underline{w}^f > \overline{w}^h$.*

Our theory predicts the emergence of a production-specializing country in which no workers live because production is fully automated. This geographical implication also applies to the population agglomeration among the regions within a country. Alternatively, our result can be interpreted as meaning that internal labor mobility between two firms with heterogeneous productivities can lead to full automation in the firm with high robot productivity. In any case, free labor mobility benefits all people through the specialization of production.

Table 2 : List of countries with high robot capital

Country	**Robot density ranking**			**Robot stock ranking Top 10**
	Top 10	**Top 5**	**Top 15**	
Australia			○	
Austria	○		○	
Belgium	○	○	○	
Canada	○		○	
China				○
Denmark	○	○	○	
Finland	○		○	
France	○		○	○
Germany	○	○	○	○
Italy	○	○	○	○
Japan	○	○	○	○
Korea	○	○	○	○
Mexico				○
Netherlands			○	
Russia				○
Singapore	○	○	○	
Slovak Republic			○	
Slovenia			○	
Spain	○		○	○
Sweden	○	○	○	○
Switzerland			○	
Taiwan	○		○	○
Thailand				○
U.K.			○	○
U.S.	○	○	○	○

Note: This table lists countries classified as having high robot capital based on data provided by the International Federation of Robotics 2021. Robot density is the number of operational industrial robots per 10,000 employees in the manufacturing industry. The first three columns labeled "Robot density ranking" and the last column labeled "Robot stock ranking" list high robot-capital countries classified according to robot density and the value of the operational stock of industrial robots, respectively. In each of the "Top 10," "Top 5," and "Top 15" lists, countries marked with a circle are those ranked in the top 10, top 5, and top 15 at least once between 2000 and 2019 based on each classification criterion.

Table 3 : Summary statistics

Variable	N	Mean	SD	Min	Max
All 62 countries					
g_{it}^{Y}	954	2.74	0.10	-14.84	14.52
g_{it}^{K}	954	6.10	0.19	-8.51	66.36
g_{it}^{M}	954	42.15	6.85	-100.00	4800.00
g_{it}^{L}	954	1.37	0.33	-74.96	294.30
Top 10 countries with high robot density					
g_{it}^{Y}	370	2.44	0.13	-8.07	14.52
g_{it}^{K}	370	3.85	0.16	-3.75	18.11
g_{it}^{M}	370	15.95	2.41	-82.33	408.33
g_{it}^{L}	370	0.96	0.09	-6.66	9.07
Non Top 10 countries with high robot density					
g_{it}^{Y}	584	2.93	0.14	-14.84	11.11
g_{it}^{K}	584	7.53	0.28	-8.51	66.36
g_{it}^{M}	584	58.76	11.03	-100.00	4800.00
g_{it}^{L}	584	1.64	0.54	-74.96	294.30

Note: This table presents the summary statistics of the sample used in the baseline empirical analysis. Values are rounded to two decimal places. The data cover 62 countries from 1994 to 2019. g_{it}^{Y}, g_{it}^{K}, g_{it}^{M}, and g_{it}^{L} denote the growth rates of real GDP, traditional capital, robot capital, and labor, respectively. To calculate g_{it}^{L}, we use the employment data from the ILO as a proxy for labor L_t^i. The top 10 countries with high robot density include those ranked among the top 10 in robot density at least once between 2000 and 2019, while the other countries are classified as non-top 10 countries with high robot density.

Table 4 : Correlation matrix

	g_{it}^{Y}	g_{it}^{K}	g_{it}^{M}	g_{it}^{L}
g_{it}^{Y}	1.00	0.27	0.10	0.10
g_{it}^{K}	0.27	1.00	0.03	0.07
g_{it}^{M}	0.10	0.03	1.00	-0.00
g_{it}^{L}	0.10	0.07	-0.00	1.00

Note: This table presents the correlation matrix of the variables used in the baseline empirical analysis. Values are rounded to two decimal places. g_{it}^{Y}, g_{it}^{K}, g_{it}^{M}, and g_{it}^{L} denote the growth rates of real GDP, traditional capital, robot capital, and labor, respectively. To calculate g_{it}^{L}, we use the employment data from the ILO as a proxy for labor L_t^i.

Table 5 : Baseline regression results

	(1)	(2)	(3)	(4)
Intercept	1.935***			
	(0.168)			
d_i	−0.972***		−0.903***	
	(0.220)		(0.292)	
g_{it}^K	0.121***	0.101**	0.121***	0.053
	(0.016)	(0.048)	(0.043)	(0.043)
g_{it}^M	0.001**	0.001	0.000	0.000
	(0.000)	(0.001)	(0.000)	(0.000)
$d_i \times g_{it}^M$	0.019***	0.017***	0.013***	0.010***
	(0.003)	(0.002)	(0.002)	(0.002)
g_{it}^L	0.017**	0.011	0.008	0.000
	(0.009)	(0.020)	(0.019)	(0.015)
$d_i \times g_{it}^L$	0.712***	0.665***	0.452***	0.374***
	(0.088)	(0.082)	(0.128)	(0.095)
Num.Obs.	954	954	954	954
Num.Country	62	62	62	62
F statistic	34.19	27.12	28.96	10.98
R squared	0.18	0.13	0.16	0.06
Adjusted R squared	0.17	0.07	0.13	-0.04
Country fixed effect	No	Yes	No	Yes
Time fixed effect	No	No	Yes	Yes

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Note: This table presents the OLS estimates using the regression model (27) with country and time fixed effects. The sample is an unbalanced panel dataset covering 62 countries from 1994 to 2019. Columns (1)–(4) report the regression results without country and time fixed effects, with country fixed effects only, with time fixed effects only, with both country and time fixed effects, respectively. g_{it}^Y, g_{it}^K, g_{it}^M, and g_{it}^L denote the growth rates of real GDP, traditional capital, robot capital, and labor, respectively. To calculate g_{it}^L, we use the employment data from the ILO as a proxy for labor L_t^i. The dummy variable d_i takes 1 if a country is ranked among the top 10 in robot density at least once between 2000 and 2019 and 0 otherwise. Robust standard errors are listed in parentheses: columns (1) and (3) use White standard errors, whereas columns (2) and (4) base on clustered standard errors.

Table 6 : Using population as a proxy for labor

	(1)	(2)	(3)	(4)
Intercept	1.598***			
	(0.185)			
d_i	-0.383		-0.615*	
	(0.254)		(0.344)	
g_{it}^K	0.120***	0.115**	0.111***	0.054
	(0.016)	(0.050)	(0.040)	(0.043)
g_{it}^M	0.001**	0.001	0.000	0.000
	(0.000)	(0.001)	(0.000)	(0.000)
$d_i \times g_{it}^M$	0.020***	0.018***	0.014***	0.011***
	(0.003)	(0.002)	(0.002)	(0.002)
g_{it}^L	0.591***	0.390	0.656***	0.745*
	(0.139)	(0.559)	(0.223)	(0.423)
$d_i \times g_{it}^L$	0.162	-0.243	0.171	-0.588
	(0.257)	(0.604)	(0.251)	(0.441)
Num.Obs.	945	945	945	945
Num.Country	60	60	60	60
F statistic	25	13.72	30.37	7.33
R squared	0.14	0.07	0.17	0.04
Adjusted R squared	0.13	0	0.14	-0.06
Country fixed effect	No	Yes	No	Yes
Time fixed effect	No	No	Yes	Yes

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Note: This table presents the OLS estimates using the regression model (27) with country and time fixed effects. The sample is an unbalanced panel dataset covering 60 countries from 1994 to 2019. Columns (1)–(4) report the regression results without country and time fixed effects, with country fixed effects only, with time fixed effects only, with both country and time fixed effects, respectively. g_{it}^Y, g_{it}^K, g_{it}^M, and g_{it}^L denote the growth rates of real GDP, traditional capital, robot capital, and labor, respectively. To calculate g_{it}^L, we use the working-age population data from the World Economic Outlook (April 2023), published by the IMF, as a proxy for labor L_t^i. The dummy variable d_i takes 1 if a country is ranked among the top 10 in robot density at least once between 2000 and 2019 and 0 otherwise. Robust standard errors are listed in parentheses: columns (1) and (3) use White standard errors, whereas columns (2) and (4) base on clustered standard errors.

Table 7 : Using alternative employment data as a proxy for labor

	(1)	(2)	(3)	(4)
Intercept	0.866***			
	(0.157)			
d_i	−0.017		0.023	
	(0.179)		(0.238)	
g_{it}^K	0.136***	0.059*	0.149***	0.029
	(0.024)	(0.034)	(0.030)	(0.031)
g_{it}^M	0.001**	0.000	0.000	0.000
	(0.000)	(0.000)	(0.000)	(0.000)
$d_i \times g_{it}^M$	0.016***	0.015***	0.010***	0.008***
	(0.002)	(0.002)	(0.002)	(0.002)
g_{it}^L	0.987***	1.078***	0.859***	0.950***
	(0.053)	(0.083)	(0.066)	(0.080)
$d_i \times g_{it}^L$	−0.200**	−0.301**	−0.215*	−0.371***
	(0.082)	(0.149)	(0.130)	(0.132)
Num.Obs.	683	683	683	683
Num.Country	35	35	35	35
F statistic	127.24	127.12	109.74	106.18
R squared	0.53	0.5	0.5	0.46
Adjusted R squared	0.53	0.47	0.48	0.41
Country fixed effect	No	Yes	No	Yes
Time fixed effect	No	No	Yes	Yes

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Note: This table presents the OLS estimates using the regression model (27) with country and time fixed effects. The sample is an unbalanced panel dataset covering 35 countries from 1994 to 2019. Columns (1)–(4) report the regression results without country and time fixed effects, with country fixed effects only, with time fixed effects only, with both country and time fixed effects, respectively. g_{it}^Y, g_{it}^K, g_{it}^M, and g_{it}^L denote the growth rates of real GDP, traditional capital, robot capital, and labor, respectively. To calculate g_{it}^L, we use employment data from the World Economic Outlook (April 2023), published by the IMF, as a proxy for labor L_t^i. The dummy variable d_i takes 1 if a country is ranked among the top 10 in robot density at least once between 2000 and 2019 and 0 otherwise. Robust standard errors are listed in parentheses: columns (1) and (3) use White standard errors, whereas columns (2) and (4) base on clustered standard errors.

Table 8 : Using hours worked as a proxy for labor

	(1)	(2)	(3)	(4)
Intercept	1.214***			
	(0.189)			
d_i	-0.282		-0.335	
	(0.230)		(0.271)	
g_{it}^K	0.123***	0.130**	0.146***	0.116*
	(0.020)	(0.057)	(0.054)	(0.064)
g_{it}^M	0.001	0.001	0.000	-0.001
	(0.001)	(0.001)	(0.001)	(0.001)
$d_i \times g_{it}^M$	0.014***	0.014***	0.006***	0.005***
	(0.005)	(0.004)	(0.002)	(0.002)
g_{it}^L	0.566***	0.578***	0.417***	0.411***
	(0.039)	(0.161)	(0.119)	(0.123)
$d_i \times g_{it}^L$	-0.158**	-0.178	-0.248**	-0.273**
	(0.078)	(0.167)	(0.116)	(0.124)
Num.Obs.	578	578	578	578
Num.Country	42	42	42	42
F statistic	62.81	67.05	52.66	39.66
R squared	0.4	0.39	0.37	0.28
Adjusted R squared	0.39	0.33	0.33	0.18
Country fixed effect	No	Yes	No	Yes
Time fixed effect	No	No	Yes	Yes

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Note: This table presents the OLS estimates using the regression model (27) with country and time fixed effects. The sample is an unbalanced panel dataset covering 42 countries from 1994 to 2019. Columns (1)–(4) report the regression results without country and time fixed effects, with country fixed effects only, with time fixed effects only, with both country and time fixed effects, respectively. g_{it}^Y, g_{it}^K, g_{it}^M, and g_{it}^L denote the growth rates of real GDP, traditional capital, robot capital, and labor, respectively. To calculate g_{it}^L, we use the total hours worked data from the ILO as a proxy for labor L_t^i. The total hours worked are computed by multiplying the employment data by the mean weekly hours actually worked per employed person. The dummy variable d_i takes 1 if a country is ranked among the top 10 in robot density at least once between 2000 and 2019 and 0 otherwise. Robust standard errors are listed in parentheses: columns (1) and (3) use White standard errors, whereas columns (2) and (4) base on clustered standard errors.

Table 9 : Classification criterion based on a robot density ranking in the top 5.

	(1)	(2)	(3)	(4)
Intercept	1.898***			
	(0.151)			
d_i	−1.100***		−1.014***	
	(0.251)		(0.282)	
g_{it}^K	0.126***	0.112**	0.127***	0.058
	(0.016)	(0.050)	(0.043)	(0.044)
g_{it}^M	0.001**	0.001	0.000	0.000
	(0.000)	(0.001)	(0.000)	(0.000)
$d_i \times g_{it}^M$	0.021***	0.019***	0.015***	0.012***
	(0.004)	(0.002)	(0.002)	(0.002)
g_{it}^L	0.020**	0.013	0.009	0.001
	(0.009)	(0.022)	(0.020)	(0.016)
$d_i \times g_{it}^L$	0.855***	0.733***	0.602***	0.429***
	(0.120)	(0.113)	(0.159)	(0.145)
Num.Obs.	954	954	954	954
Num.Country	62	62	62	62
F statistic	30.37	21.19	28.56	9.07
R squared	0.16	0.11	0.16	0.05
Adjusted R squared	0.16	0.04	0.13	-0.05
Country fixed effect	No	Yes	No	Yes
Time fixed effect	No	No	Yes	Yes

* p < 0.1, ** p < 0.05, *** p < 0.01

Note: This table presents the OLS estimates using the regression model (27) with country and time fixed effects. The sample is an unbalanced panel dataset covering 62 countries from 1994 to 2019. Columns (1)–(4) report the regression results without country and time fixed effects, with country fixed effects only, with time fixed effects only, with both country and time fixed effects, respectively. g_{it}^Y, g_{it}^K, g_{it}^M, and g_{it}^L denote the growth rates of real GDP, traditional capital, robot capital, and labor, respectively. To calculate g_{it}^L, we use the employment data from the ILO as a proxy for labor L_t^i. The dummy variable d_i takes 1 if a country is ranked among the top 5 in robot density at least once between 2000 and 2019 and 0 otherwise. Robust standard errors are listed in parentheses: columns (1) and (3) use White standard errors, whereas columns (2) and (4) base on clustered standard errors.

Table 10 : Classification criterion based on a robot density ranking in the top 15

	(1)	(2)	(3)	(4)
Intercept	1.541***			
	(0.198)			
d_i	0.459**		0.012	
	(0.206)		(0.377)	
g_{it}^K	0.115***	0.083*	0.111**	0.039
	(0.017)	(0.043)	(0.045)	(0.041)
g_{it}^M	0.001*	0.001	0.000	0.000
	(0.001)	(0.001)	(0.001)	(0.001)
$d_i \times g_{it}^M$	0.000	0.000	0.001	0.000
	(0.001)	(0.001)	(0.001)	(0.001)
g_{it}^L	0.336***	0.303**	0.272**	0.232**
	(0.034)	(0.136)	(0.111)	(0.106)
$d_i \times g_{it}^L$	−0.334***	−0.304**	−0.279**	−0.243**
	(0.035)	(0.136)	(0.111)	(0.106)
Num.Obs.	954	954	954	954
Num.Country	62	62	62	62
F statistic	31.12	23.26	33.92	17.26
R squared	0.16	0.12	0.18	0.09
Adjusted R squared	0.16	0.05	0.15	0
Country fixed effect	No	Yes	No	Yes
Time fixed effect	No	No	Yes	Yes

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Note: This table presents the OLS estimates using the regression model (27) with country and time fixed effects. The sample is an unbalanced panel dataset covering 62 countries from 1994 to 2019. Columns (1)–(4) report the regression results without country and time fixed effects, with country fixed effects only, with time fixed effects only, with both country and time fixed effects, respectively. g_{it}^Y, g_{it}^K, g_{it}^M, and g_{it}^L denote the growth rates of real GDP, traditional capital, robot capital, and labor, respectively. To calculate g_{it}^L, we use the employment data from the ILO as a proxy for labor L_t^i. The dummy variable d_i takes 1 if a country is ranked among the top 15 in robot density at least once between 2000 and 2019 and 0 otherwise. Robust standard errors are listed in parentheses: columns (1) and (3) use White standard errors, whereas columns (2) and (4) base on clustered standard errors.

Table 11 : Classification criterion based on a robot stock ranking in the top 10

	(1)	(2)	(3)	(4)
Intercept	1.794***			
	(0.165)			
d_i	−0.537**		−0.507	
	(0.216)		(0.310)	
g_{it}^K	0.125***	0.107**	0.126***	0.054
	(0.016)	(0.049)	(0.042)	(0.044)
g_{it}^M	0.001**	0.001	0.000	0.000
	(0.000)	(0.001)	(0.000)	(0.000)
$d_i \times g_{it}^M$	0.017***	0.014***	0.012***	0.008***
	(0.003)	(0.003)	(0.002)	(0.003)
g_{it}^L	0.018**	0.011	0.007	0.000
	(0.009)	(0.020)	(0.018)	(0.015)
$d_i \times g_{it}^L$	0.488***	0.432***	0.359***	0.271***
	(0.074)	(0.131)	(0.095)	(0.085)
Num.Obs.	954	954	954	954
Num.Country	62	62	62	62
F statistic	30.97	20.92	28.79	8.76
R squared	0.16	0.11	0.16	0.05
Adjusted R squared	0.16	0.04	0.13	-0.05
Country fixed effect	No	Yes	No	Yes
Time fixed effect	No	No	Yes	Yes

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Note: This table presents the OLS estimates using the regression model (27) with country and time fixed effects. The sample is an unbalanced panel dataset covering 62 countries from 1994 to 2019. Columns (1)–(4) report the regression results without country and time fixed effects, with country fixed effects only, with time fixed effects only, with both country and time fixed effects, respectively. g_{it}^Y, g_{it}^K, g_{it}^M, and g_{it}^L denote the growth rates of real GDP, traditional capital, robot capital, and labor, respectively. To calculate g_{it}^L, we use the employment data from the ILO as a proxy for labor L_t^i. The dummy variable d_i takes 1 if a country is ranked among the top 10 in the amount of operational stock of industrial robots at least once between 2000 and 2019 and 0 otherwise. Robust standard errors are listed in parentheses: columns (1) and (3) use White standard errors, whereas columns (2) and (4) base on clustered standard errors.

Table 12 : Top and bottom 5% trimmed data

	(1)	(2)	(3)	(4)
Intercept	1.587***			
	(0.169)			
d_i	−0.707***		−0.737***	
	(0.212)		(0.284)	
g_{it}^K	0.133***	0.100***	0.140***	0.058
	(0.016)	(0.037)	(0.049)	(0.036)
g_{it}^M	0.013***	0.012***	0.007***	0.006***
	(0.002)	(0.002)	(0.002)	(0.002)
$d_i \times g_{it}^M$	0.009**	0.008**	0.009**	0.007**
	(0.004)	(0.003)	(0.004)	(0.003)
g_{it}^L	0.009	0.002	0.002	−0.005
	(0.008)	(0.012)	(0.014)	(0.010)
$d_i \times g_{it}^L$	0.717***	0.682***	0.483***	0.428***
	(0.081)	(0.078)	(0.126)	(0.087)
Num.Obs.	859	859	859	859
Num.Country	61	61	61	61
F statistic	46.55	40.88	36.31	17.16
R squared	0.25	0.2	0.21	0.1
Adjusted R squared	0.24	0.14	0.18	0
Country fixed effect	No	Yes	No	Yes
Time fixed effect	No	No	Yes	Yes

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Note: This table presents the OLS estimates using the regression model (27) with country and time fixed effects. The sample is an unbalanced panel dataset from 1994 to 2019. We trim the top and bottom 5% of the data on the growth rate of robot stock. As a result, the number of countries is reduced from 62 to 61. Columns (1)–(4) report the regression results without country and time fixed effects, with country fixed effects only, with time fixed effects only, with both country and time fixed effects, respectively. g_{it}^Y, g_{it}^K, g_{it}^M, and g_{it}^L denote the growth rates of real GDP, traditional capital, robot capital, and labor, respectively. To calculate g_{it}^L, we use the employment data from the ILO as a proxy for labor L_t^i. The dummy variable d_i takes 1 if a country is ranked among the top 10 in robot density at least once between 2000 and 2019 and 0 otherwise. Robust standard errors are listed in parentheses: columns (1) and (3) use White standard errors, whereas columns (2) and (4) base on clustered standard errors.

Table 13 : Top and bottom 10% trimmed data

	(1)	(2)	(3)	(4)
Intercept	1.873***			
	(0.168)			
d_i	−0.889***		−0.859***	
	(0.216)		(0.293)	
g_{it}^K	0.117***	0.097**	0.118**	0.052
	(0.016)	(0.040)	(0.047)	(0.040)
g_{it}^M	0.013***	0.013***	0.007**	0.006***
	(0.002)	(0.003)	(0.003)	(0.002)
$d_i \times g_{it}^M$	0.006	0.007	0.005	0.005
	(0.005)	(0.005)	(0.004)	(0.003)
g_{it}^L	0.012	0.005	0.003	−0.005
	(0.009)	(0.015)	(0.016)	(0.012)
$d_i \times g_{it}^L$	0.711***	0.669***	0.447***	0.376***
	(0.086)	(0.079)	(0.118)	(0.088)
Num.Obs.	858	858	858	858
Num.Country	62	62	62	62
F statistic	33.21	30.84	25.28	11.33
R squared	0.19	0.16	0.16	0.07
Adjusted R squared	0.18	0.09	0.12	-0.04
Country fixed effect	No	Yes	No	Yes
Time fixed effect	No	No	Yes	Yes

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Note: This table presents the OLS estimates using the regression model (27) with country and time fixed effects. The sample is an unbalanced panel dataset from 1994 to 2019. We trim the top 10% of the data on the growth rate of robot stock. Columns (1)–(4) report the regression results without country and time fixed effects, with country fixed effects only, with time fixed effects only, with both country and time fixed effects, respectively. g_{it}^Y, g_{it}^K, g_{it}^M, and g_{it}^L denote the growth rates of real GDP, traditional capital, robot capital, and labor, respectively. To calculate g_{it}^L, we use the employment data from the ILO as a proxy for labor L_t^i. The dummy variable d_i takes 1 if a country is ranked among the top 10 in robot density at least once between 2000 and 2019 and 0 otherwise. Robust standard errors are listed in parentheses: columns (1) and (3) use White standard errors, whereas columns (2) and (4) base on clustered standard errors.

Table 14 : Using per capita variables based on the ILO's employment data

	(1)	(2)	(3)	(4)
Intercept	-3.440^{***}			
	(0.235)			
d_i	2.325^{***}		2.257^{***}	
	(0.360)		(0.687)	
g_{it}^k	0.789^{***}	0.869^{***}	0.800^{***}	0.883^{***}
	(0.015)	(0.131)	(0.160)	(0.118)
g_{it}^m	0.001^{*}	0.002^{*}	0.001	0.001
	(0.001)	(0.001)	(0.001)	(0.001)
$d_i \times g_{it}^m$	0.020^{***}	0.021^{***}	0.013^{***}	0.014^{***}
	(0.006)	(0.003)	(0.003)	(0.003)
Num.Obs.	954	954	954	954
Num.Country	62	62	62	62
F statistic	724.23	1334.21	795.08	1550.77
R squared	0.75	0.82	0.77	0.84
Adjusted R squared	0.75	0.81	0.77	0.83
Country fixed effect	No	Yes	No	Yes
Time fixed effect	No	No	Yes	Yes

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Note: This table presents the OLS estimates using the regression model (28) with country and time fixed effects. The sample is an unbalanced panel dataset covering 62 countries from 1994 to 2019. Columns (1)–(4) report the regression results without country and time fixed effects, with country fixed effects only, with time fixed effects only, with both country and time fixed effects, respectively. g_{it}^y, g_{it}^k, and g_{it}^m denote the growth rates of real GDP per capita, traditional capital per capita, and robot capital per capita, respectively. To calculate a per capita value of each variable, we use the employment data from the ILO as a proxy for labor L_t^i. The dummy variable d_i takes 1 if a country is ranked among the top 10 in robot density at least once between 2000 and 2019 and 0 otherwise. Robust standard errors are listed in parentheses: columns (1) and (3) use White standard errors, whereas columns (2) and (4) base on clustered standard errors.

Table 15 : Using per capita variables based on population

	(1)	(2)	(3)	(4)
Intercept	1.430***			
	(0.165)			
d_i	−0.282		−0.512*	
	(0.206)		(0.292)	
g_{it}^k	0.122***	0.114**	0.113***	0.054
	(0.016)	(0.048)	(0.040)	(0.042)
g_{it}^m	0.001**	0.001	0.000	0.000
	(0.000)	(0.001)	(0.000)	(0.000)
$d_i \times g_{it}^m$	0.020***	0.018***	0.014***	0.011***
	(0.003)	(0.002)	(0.002)	(0.002)
Num.Obs.	945	945	945	945
Num.Country	60	60	60	60
F statistic	27.08	22.48	28.66	9.38
R squared	0.1	0.07	0.11	0.03
Adjusted R squared	0.1	0.01	0.08	-0.07
Country fixed effect	No	Yes	No	Yes
Time fixed effect	No	No	Yes	Yes

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Note: This table presents the OLS estimates using the regression model (28) with country and time fixed effects. The sample is an unbalanced panel dataset covering 60 countries from 1994 to 2019. Columns (1)–(4) report the regression results without country and time fixed effects, with country fixed effects only, with time fixed effects only, with both country and time fixed effects, respectively. g_{it}^y, g_{it}^k, and g_{it}^m denote the growth rates of real GDP per capita, traditional capital per capita, and robot capital per capita, respectively. To calculate a per capita value of each variable, we use the working-age population data from the World Economic Outlook (April 2023), published by the IMF, as a proxy for labor L_t^i. The dummy variable d_i takes 1 if a country is ranked among the top 10 in robot density at least once between 2000 and 2019 and 0 otherwise. Robust standard errors are listed in parentheses: columns (1) and (3) use White standard errors, whereas columns (2) and (4) base on clustered standard errors.

Table 16 : Using per capita variables based on the IMF's employment data

	(1)	(2)	(3)	(4)
Intercept	0.995***			
	(0.147)			
d_i	−0.181		−0.161	
	(0.163)		(0.229)	
g_{it}^k	0.128***	0.063**	0.155***	0.066**
	(0.023)	(0.032)	(0.025)	(0.028)
g_{it}^m	0.001*	0.000	0.000	0.000
	(0.000)	(0.000)	(0.000)	(0.000)
$d_i \times g_{it}^m$	0.016***	0.015***	0.009***	0.007***
	(0.002)	(0.002)	(0.002)	(0.002)
Num.Obs.	683	683	683	683
Num.Country	35	35	35	35
F statistic	20.96	16.03	20.9	6.94
R squared	0.11	0.07	0.11	0.03
Adjusted R squared	0.1	0.02	0.07	-0.06
Country fixed effect	No	Yes	No	Yes
Time fixed effect	No	No	Yes	Yes

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Note: This table presents the OLS estimates using the regression model (28) with country and time fixed effects. The sample is an unbalanced panel dataset covering 35 countries from 1994 to 2019. Columns (1)–(4) report the regression results without country and time fixed effects, with country fixed effects only, with time fixed effects only, with both country and time fixed effects, respectively. g_{it}^y, g_{it}^k, and g_{it}^m denote the growth rates of real GDP per capita, traditional capital per capita, and robot capital per capita, respectively. To calculate a per capita value of each variable, we use the employment data from the World Economic Outlook (April 2023), published by the IMF, as a proxy for labor L_t^i. The dummy variable d_i takes 1 if a country is ranked among the top 10 in robot density at least once between 2000 and 2019 and 0 otherwise. Robust standard errors are listed in parentheses: columns (1) and (3) use White standard errors, whereas columns (2) and (4) base on clustered standard errors.

Table 17 : Using per capita variables based on hours worked

	(1)	(2)	(3)	(4)
Intercept	0.571***			
	(0.190)			
d_i	-0.132		-0.101	
	(0.240)		(0.337)	
g_{it}^k	0.187***	0.230***	0.227***	0.286***
	(0.020)	(0.075)	(0.076)	(0.090)
g_{it}^m	0.001	0.001	0.000	0.000
	(0.001)	(0.001)	(0.001)	(0.001)
$d_i \times g_{it}^m$	0.014***	0.014***	0.007***	0.006***
	(0.005)	(0.004)	(0.002)	(0.002)
Num.Obs.	578	578	578	578
Num.Country	42	42	42	42
F statistic	27.61	32.46	39.8	49.59
R squared	0.16	0.15	0.22	0.23
Adjusted R squared	0.16	0.08	0.19	0.12
Country fixed effect	No	Yes	No	Yes
Time fixed effect	No	No	Yes	Yes

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Note: This table presents the OLS estimates using the regression model (28) with country and time fixed effects. The sample is an unbalanced panel dataset covering 42 countries from 1994 to 2019. Columns (1)–(4) report the regression results without country and time fixed effects, with country fixed effects only, with time fixed effects only, with both country and time fixed effects, respectively. g_{it}^y, g_{it}^k, and g_{it}^m denote the growth rates of real GDP per capita, traditional capital per capita, and robot capital per capita, respectively. To calculate a per capita value of each variable, we use the total hours worked data from the ILO as a proxy for labor L_t^i. The total hours worked are computed by multiplying employment data by the mean weekly hours actually worked per employed person. The dummy variable d_i takes 1 if a country is ranked among the top 10 in robot density at least once between 2000 and 2019 and 0 otherwise. Robust standard errors are listed in parentheses: columns (1) and (3) use White standard errors, whereas columns (2) and (4) base on clustered standard errors.