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Yasuhiro Nakamoto, Kazuo Mino, Yunfang Hu

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Preference Structures, Wealth Distribution, and Patterns of Trade in a Global Economy*

Yasuhiro Nakamoto†, Kazuo Mino‡ and Yunfang Hu§

Abstract

This study integrates the dynamic $2\times2\times2$ Hechscher-Ohlin model with the neoclassical growth model, considering heterogeneous households, to explore the relationship between preference structures, wealth distribution, and international trade in a unified setting. We demonstrate that if households have homothetic utility functions, the long-run trade pattern depends solely on the initial distribution of capital between two countries. Conversely, if preferences are non-homothetic, the initial distribution of wealth among households also influences long-run trade patterns. Numerical examples further examine the wealth distribution in each country, showing that the initially poor can catch up with the initially rich.

Keywords: Dynamic Hecksher-Ohlin model, Non-homothetic preference, Heterogeneous households, Wealth distribution.

JEL Classification Code: D31, E20, F11, F43, O41.

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†Corresponding Author. Faculty of Informatics, Kansai University, 2-1-1 Reizanji-cho,Takatsuki-shi, Osaka, 569-1095, Japan, e-mail: nakamoto@kansai-u.ac.jp

‡Institute of Economic Research, Kyoto University, Yoshida Honmachi, Sakyo-ku, Kyoto, 606-8501 Japan, e-mail: mino@kier.kyoto-u.ac.jp

§Graduate School of Economics, Kobe University, 2-1 Rokkodai, Nada-ku, Kobe 657-8501 Japan, email: yhu@econ.kobe-u.ac.jp

1 Introduction

In the standard 2×2×2 Hechscher-Ohlin (HO) model with capital accumulation, the global economy ultimately reaches a singular steady state. Here, the long-term distribution of capital between two countries depends on their initial capital and labor holdings. Consequently, the country initially endowed with relatively capital-intensive resources maintains its comparative advantage in capital-intensive goods over the long term, thus affirming the HO theorem of comparative advantage within a dynamic framework. However, this study seeks to revisit this conclusion within a broader context.

Adhering to the standard HO model, we posit that each country has two production sectors: one for pure investment goods and the other for pure consumption goods. Both sectors utilize capital and labor under technologies exhibiting constant returns to scale. Additionally, we assume symmetry in production technologies across both countries and identical preferences among households. Departing from the conventional framework, our study introduces heterogeneity among households within each country. Specifically, households possess varying levels of wealth at the outset, a departure from the standard assumption of homogeneity.

Existing literature demonstrates that if the utility function is homothetic, the wealth distribution among households does not influence macroeconomic variables (e.g., Caselli and Ventura, 2000). Consequently, the determination of long-run trade patterns relies solely on the initial distribution of capital and labor between two countries, thus confirming the dynamic HO theorem. Conversely, if households have non-homothetic preferences, both the initial distribution of capital and labor between countries and the initial wealth distribution among households determine long-run trade patterns in the global economy. This finding can be viewed as an extended HO theorem applicable in a dynamic context. Our study aims to elucidate the connections between preference structure, wealth distribution, and trade patterns within a unified framework.

We first show that the global economy has a unique steady state that exhibits saddle-point stability, regardless of preference structures and initial wealth distribution among households. Subsequently, we investigate the role of preference structure in the dynamic HO model. Assuming homothetic utility functions, we reaffirm that the initial distribution of capital and labor determines the steady-state capital distribution between two countries. For homothetic preferences, each household's optimal consumption is a linear function of its total wealth (i.e., asset holdings plus human wealth), with a common coefficient across all households. This allows us to derive an aggregate consumption function independent of wealth distribution, meaning the Euler equation of aggregate consumption is unaffected by wealth distribution. In contrast, if households' utility functions are non-homothetic, each household's optimal consumption becomes a nonlinear function of its total wealth. This implies that Gorman's aggregation theorem does not apply, and distribution influences the behavior of aggregate consumption and capital. This results in the extended HO theorem discussed earlier.

To illustrate the pivotal role of preference structure in the dynamic HO model, we engage in several thought experiments. Let us assume two countries initially operate in autarky, with identical levels of constant labor supply and capital holdings. Given symmetric technologies, the relative price between investment and consumption goods is the same in both countries, and their dynamic behaviors mirror each other. Consequently, even upon opening up their economies, international trade does not occur. Next, if households possess non-homothetic utility functions, wealth distribution influences aggregate capital behavior. Thus, if one country exhibits initially greater wealth inequality than the other, the dynamic trajectories of aggregate capital in each country diverge, leading to disparate autarkic prices. Consequently, commodity trade emerges, regardless of the countries' initial capital holdings. Specifically, we demonstrate that if wealth distribution is more unequal in a foreign country compared to a home country, foreign households may consume less, leading to a larger capital stock in the foreign country over time. Consequently, in the long-run equilibrium, the foreign country maintains its comparative advantage in capital-intensive goods. To complement our theoretical discussion, we provide numerical examples of the model with non-homothetic preferences.

Related Literature

Our study is closely aligned with two main strands of the literature. First, it relates to a category of studies focusing on dynamic HO models involving two countries. Early contributions include Oniki and Uzawa (1965) and Stiglitz (1970). Oniki and Uzawa (1965) provided a comprehensive analysis of global trade patterns, including complete specialization, assuming exogenously given household saving rates. Stiglitz (1970) explored similar issues using a model with optimizing households. Renewed interest in dynamic HO models emerged in the early 1990s. For instance, Chen (1992) investigated the establishment of the dynamic HO theorem of comparative advantage under the assumption of homothetic preferences and incomplete specialization. Subsequent studies introduced modifications to the dynamic HO framework. For example, Nishimura and Shimomura (2002) considered a model incorporating production externalities, while Chen et al. (2009) examined a model with an endogenous time discount rate. Other notable contributions include those by Hu et al. (2009), Bond et al. (2011), and Caliendo (2011). Among these, Bond et al. (2011, 2013) analyzed models with non-homothetic preferences.[1] However, in this line of research, the assumption of homogeneous households within each country has rendered the examination of individual wealth distribution unfeasible.

The second strand of literature relevant to our investigation examines neoclassical growth models with heterogeneous households. An early contribution in this field was by Stiglitz (1969), who studied wealth distribution within the framework of the Solow (1956) neoclassical growth model. Using a neoclassical growth model where agents have different time discount rates, Becker (1980) confirmed Ramsey's (1928) conjecture that the most patient household eventually accumulates the entire capital stock. Further studies on growth models with heterogeneous households include Bourguignon (1980), Chatterjee (1994), Foellmi (2011), Kraus and Serve (2000), and Sorger (2002). Additionally, Nakamoto and Mino (2012) and Mino and Nakamoto (2015) investigated wealth distribution and equilibrium dynamics in neoclassical growth models with consumption externalities. Other notable contributions include works by Garcia-Penerosa and Turnovsky (2006, 2009) and Caselli and Ventura (2000), which examined distributional dynamics in models with homothetic preferences.[2] All of the aforementioned studies focus exclusively on closed economies.[3] Furthermore, these studies typically consider one-sector rather than two-sector models.

Our study integrates these two strands of literature. This integration facilitates a unified

[1]See also Hunter (1991) for the role of non-homothetic preferences in the static HO model.

[2]Bertola et al. (2006) provide a comprehensive survey of income distribution in dynamic macroeconomic models, with Chapter 4 detailing wealth and income distribution in neoclassical growth models with heterogeneous households.

[3]An exception is Chen and Turnovsky (2011), who analyzed income distribution in a small open economy with homothetic preferences.

analysis of the interactions between wealth distribution, trade, and growth.

The remainder of this paper is organized as follows: Section 2 constructs the base model and characterizes the steady-state equilibrium of the global economy. Section 3 examines wealth distribution under different preference structures. We first assume homothetic preferences and provide an intuitive discussion of the dynamic HO theorem. We then explore the case of non-homothetic preferences, highlighting the relationship between the initial wealth distribution among households and the steady-state capital distribution between two countries. Section 4 presents a numerical analysis of the model with non-homothetic preferences. Finally, Section 5 concludes.

2 Analytical Framework

We consider a global economy with two countries: home and foreign. Both countries have the identical technologies. In each country, there is a continuum of households of unit size, where households are heterogeneous in terms of their initial wealth levels. We assume that households in both countries have identical time preferences and identical utility functions. Given the symmetric production technologies and identical utility functions across both countries, our analysis primarily focuses on the home country unless otherwise specified.

2.1 Production

Our economy consists of two production sectors: sector i in each country produces investment goods, while sector c manufactures consumption goods. Our model adopts a standard production structure, characterized by constant-return-to-scale neoclassical production technologies in both sectors. The production function in sector ($x = i, c$), denoted as $Y^x = F_x(K_x, L_x)$, where Y^x represents output, K_x is capital, and L_x is labor. In a competitive factor and product market setting, the real rent, r, and real wage rate, w, are determined by:

$$r = f_i'(k_i) = pf_c'(k_c), \quad w = f_i(k_i) - k_i f_i'(k_i) = p\left\{f_c(k_c) - k_c f_c'(k_c)\right\}, \tag{1}$$

where $k_x = K_x/L_x$, $f_x(k_x) = F_x(K_x, L_x)/L_x$ ($x = i, c$) and p denotes the price of consumption goods relative to investment goods. From these equations (1), optimal factor intensity is determined by the function of relative price:

$$k_i = k_i(p), \quad k_c = k_c(p). \tag{2}$$

By substituting (2) into (1), we derive $r = r(p)$ and $w = w(p)$.

The following discussion assumes that both countries always produce both goods; hence, we do not consider the case of specialization. For simplicity, we focus on a scenario where the investment goods sector employs more capital-intensive technology than the consumption goods sector.

Assumption 1. *The investment goods sector uses more capital-intensive technology than the consumption goods sector: $k_i(p) > k_c(p)$ for all feasible levels of p.*

From Assumption 1, it holds that $k_x'(p) > 0,\ x = i,\ c$, where

$$\frac{\partial k_i}{\partial p} = -\frac{1}{p(k_i - k_c)} \frac{f_i'(k_i) f_c(k_c)}{f_i''(k_i) f_c'(k_c)} (> 0), \quad \frac{\partial k_c}{\partial p} = -\frac{1}{p(k_i - k_c)} \frac{f_c'(k_c) f_i(k_i)}{f_c''(k_c) f_i'(k_i)} (> 0). \tag{3}$$

Intuitively, these results are consistent with the well-known Stolper-Samuelson theorem. Under Assumption 1, an increase in p raises the wage rate $w(p)$, but lowers the rate of return to capital $r(p)$. Hence, with relatively cheaper capital, both k_i and k_c will increase.

We assume that the production factors can freely shift between the sectors within a country but cannot move between countries. Denoting the level of aggregate capital in the home country as K, we establish the full employment conditions for capital and labor, respectively:

$$L_i + L_c = 1, \quad K_i + K_c = K. \tag{4}$$

Here, we assume that labor supply is constant and normalized to unity. Using (2) and (4), we derive the following:

$$K = (1 - L_c) k_i(p) + L_c k_c(p), \tag{5}$$

and furthermore, the equation (5) can be rewritten as

$$L_c = \frac{k_i(p) - K}{k_i(p) - k_c(p)}, \tag{6}$$

where we suppose $L_c \in (0, 1)$, so that the two countries produce both consumption and investment goods. In addition, taking account of $L_c \in (0, 1)$, Assumption 1 derives the following:

$$k_i(p) > K > k_c(p), \quad \text{for all } p. \tag{7}$$

Consequently, the labor allocation for the investment sector is given by:

$$L_c = L_c(K, p) \equiv L(K, p). \tag{8}$$

Taking account of Assumption 1, we obtain each differential as follows:

$$\frac{\partial L}{\partial K}(\equiv L_K) = \frac{-1}{k_i(p) - k_c(p)}(< 0), \quad \frac{\partial L}{\partial p}(\equiv L_p) = \frac{k_c'(p)(k_i(p) - K)}{(k_i(p) - k_c(p))^2} + \frac{k_i'(p)(K - k_c(p))}{(k_i(p) - k_c(p))^2}(> 0).$$

Utilizing (8), each output function of investment and consumption goods is given by:

$$Y^i(K, p) = (1 - L(K, p))f_i(k_i(p)), \quad Y^c(K, p) = L(K, p)f_c(k_c(p)). \tag{9}$$

Under Assumption 1, we obtain

$$\frac{\partial Y^i}{\partial K}(\equiv Y_K^i) = -L_K(K, p)f_i(p)(> 0), \quad \frac{\partial Y^c}{\partial K}(\equiv Y_K^c) = L_K(K, p)f_c(p)(< 0),$$

which is the usual Rybczynski result. In this case, when the supply of capital stock increases, while all other factors remain constant, the output of the investment goods that use capital intensively increases, and the output of consumption goods decreases.

For a given K, an increase in p raises the production of Y^c and decreases that of Y^i :

$$\frac{\partial Y^i}{\partial p}(\equiv Y_p^i) = \frac{k_c'(p)(K - k_i(p))f_i(k_i(p))}{(k_i(p) - k_c(p))^2} + \frac{k_i'(p)(k_c(p) - K)f_i(k_i(p))}{(k_i(p) - k_c(p))^2}\left(1 - \frac{f_i'(k_i(p))(k_i(p) - k_c(p))}{f_i(k_i(p))}\right)(< 0),$$

$$\frac{\partial Y^c}{\partial p}(\equiv Y_p^c) = \frac{k_i'(p)(K - k_c(p))f_c(k_c(p))}{(k_i(p) - k_c(p))^2} + \frac{f_c(k_c(p))(k_i(p) - K)k_c'(p)}{(k_i(p) - k_c(p))^2}\left(1 + \frac{f_c'(k_c(p))(k_i(p) - k_c(p))}{f_c(k_c(p))}\right)(> 0).$$

2.2 Households

There is a continuum of households in each country, each with unit measure. The objective function of household j in the home country is given by

$$U^j = \int_0^\infty u(C_j)e^{-\rho t}dt, \quad \rho > 0, \quad j \in [0, 1], \tag{10}$$

where ρ is the constant rate of time preference, common among the households, and C_j represents the consumption level of household j. The instantaneous utility function $u(C_j)$ is monotonically increasing and strictly concave in C_j and it satisfies the Inada conditions. The flow budget constraint is:

$$\dot{K}_j = (r(p) - \delta)K_j + w(p) - pC_j, \tag{11}$$

where δ denotes the rate of depreciation.

Denoting the (private) utility price of capital as q_j, maximizing (10) subject to (11) yields the necessary conditions for an optimum:

$$u'(C_j) = q_j p, \tag{12a}$$

$$-\frac{\dot{q}_j}{q_j} = r(p) - \delta - \rho, \tag{12b}$$

together with the transversality condition $\lim_{t\to\infty} e^{-\rho t} q_j K_j = 0$. From (12$b$), we observe $\frac{\dot{q}_j}{q_j} = \frac{\dot{q}_n}{q_n}$ where $j, n \in [0, 1]$. This indicates that the ratio of marginal utility between households j and n remains constant over time. Consequently, $\frac{u'(C_j)}{u'(C_n)}$ = constant, requiring our model to specify the trajectory starting from a specific set of the different levels of initial capital holdings.

Finally, when we aggregate (11) across all agents in the home country, we arrive at the dynamic equation for aggregate capital:

$$\dot{K} = (r(p) - \delta)K + w(p) - pC, \tag{13a}$$

where $K = \int_0^1 K_j dj$ and $C = \int_0^1 C_j dj$. Additionally, from (13a), the equation governing capital accumulation for the global economy is:

$$\dot{K}_w = (r(p) - \delta)K_w + 2w(p) - pC_w, \tag{13b}$$

where

$$K_w = K + K^*, \quad \text{and} \quad C_w = C + C^*. \tag{14}$$

2.3 Global Market Equilibrium and the Capital Accumulation Equation

Let us examine the equilibrium of the global market. Since both home and foreign countries produce both goods, all firms in the global economy face an identical relative price, p. Moreover, assuming symmetric production structures in these countries ensures an identical capital intensity, implying $k_x(p) = k_x^*(p)$ for $(x = i, c)$ at all times. Consequently, since the inequality (7) holds in both countries, we observe

$$2k_i(p) > K_w > 2k_c(p). \tag{15}$$

Now, assuming that investment and consumption goods freely cross borders under free trade, the global-market equilibrium conditions for both goods are given by:

$$Y^c(K,p) + Y^c(K^*,p) = C + C^*, \tag{16a}$$

$$Y^i(K,p) + Y^i(K^*,p) = \dot{K} + \dot{K}^* + \delta(K + K^*), \tag{16b}$$

where the levels of aggregate consumption in each country are given by $C = \int_0^1 C_j dj$ and $C^* = \int_0^1 C_j^* dj$. Defining the levels of global consumption and capital as (14) and utilizing $k_x(p) = k_x^*(p)$ $(x = i, c)$ and (16a), the global market condition for consumption goods can be rewritten as:

$$C_w = \underbrace{\frac{2k_i(p) - K_w}{k_i(p) - k_c(p)}}_{L+L^*} f_c(k_c(p)), \tag{17}$$

where $L^* = \frac{k_i(p) - K^*}{k_i(p) - k_c(p)}$.

Using (17) yields the relative price which depends on the levels of global consumption and capital as follows:

$$p = p(K_w, C_w), \text{ where } \frac{\partial p}{\partial K_w} = \frac{1}{Z_{hf}(2k_i(p) - K_w)} (> 0), \ \frac{\partial p}{\partial C_w} = \frac{1}{Z_{hf} C_w} (> 0), \tag{18}$$

We note that Z_{hf} is given by

$$Z_{hf} \equiv \frac{f_c'(k_c(p)) k_c'(p)}{f_c(k_c(p))} + \frac{k_i'(p)(K_w - 2k_c(p))}{(2k_i(p) - K_w)(k_i(p) - k_c(p))} + \frac{k_c'(p)}{k_i(p) - k_c(p)} (> 0). \tag{19}$$

Note that Z_{hf} depends on the global price and aggregate capital. Since $k_x(p) = k_x^*(p)$ $(x = i, c)$, both countries face the same Z_{hf} value. From (18), $Z_{hf} C_w$ can be explained as the static marginal consumption effect on p, while $Z_{hf}(2k_i - K_w)$ represents the static capital effect on p.

Finally, utilizing (16*b*), we derive the global-capital accumulation equation. By replacing $Y^i(K,p) + Y^i(K^*,p)$ in (16*b*) with $(1 - L(K,p))\, f_i(k_i(p)) + (1 - L^*(K^*,p))\, f_i(k_i(p))$ and substituting (18) into (16*b*), we can modify the global market condition for the investment goods:

$$G(K_w, C_w) = \dot{K}_w + \delta K_w. \tag{20a}$$

The function $G(K_w, C_w)$ is represented by:

$$G(K_w, C_w) \equiv \underbrace{\frac{K_w - 2k_c(p(K_w, C_w))}{k_i(p(K_w, C_w)) - k_c(p(K_w, C_w))}}_{(1-L)+(1-L^*)} f_i(k_i(p(K_w, C_w))). \tag{20b}$$

We can differentiate $G(K_w, C_w)$ with respect to each variable:[4]

$$G_{K_w}\left(=\frac{\partial G(K_w,C_w)}{\partial K_w}\right)=\frac{f_i(k_i(p))}{k_i(p)-k_c(p)}+\frac{\partial p}{\partial K_w}B_{hf}(>0),\;\; G_{C_w}\left(=\frac{\partial G(K_w,C_w)}{\partial C_w}\right)=\frac{\partial p}{\partial C_w}B_{hf}(<0),$$

where B_{hf} is as follows:

$$B_{hf}\equiv\frac{f_i(k_i(p))}{(k_i(p)-k_c(p))^2}\left[k_c'(p)(K_w-2k_i(p))+k_i'(p)(K_w-2k_c(p))\left(\frac{(k_i(p)-k_c(p))f_i'(k_i(p))}{f_i(k_i(p))}-1\right)\right](<0).$$

Note that we utilize the notation p instead of $p(K_w, C_w)$ to conserve space.

2.4 The Aggregate Consumption Path

To obtain the dynamic equation of aggregate consumption, we derive the dynamic equation of relative price from (17):

$$\frac{\dot{p}}{p}=\frac{1}{Z_{hf}}\left(\frac{\dot{C}_w}{C_w}+\frac{K_w}{2k_i(p)-K_w}\frac{\dot{K}_w}{K_w}\right). \tag{21}$$

Turning back to the household optimization conditions in ($12a$) and ($12b$), we obtain:

$$\dot{C}_j=\omega_j\left(r(p)-\delta-\rho-\frac{\dot{p}}{p}\right)\left(=\frac{\dot{C}_j^*\omega_j}{\omega_j^*}\right),\;\text{ where }\;\omega_j=-\frac{u'(C_j)}{u''(C_j)}(>0),\;\;\omega_j^*=-\frac{u'(C_j^*)}{u''(C_j^*)}(>0). \tag{22a}$$

It should be noted that the dynamic equations of private consumption may exhibit different levels of inverse absolute risk aversion (ARA), suggesting that households might respond diversely to variations in interest rates, depreciation rates, time preference rates, and the dynamic movement of relative prices. Aggregating Equation ($22a$) for all households in the home country, we find:

$$\dot{C}=\Omega\left(r(p)-\delta-\rho-\frac{\dot{p}}{p}\right)\left(=\frac{\dot{C}^*\Omega}{\Omega^*}\right),\;\text{ where }\;\Omega=\int_0^1\omega_j dj(>0),\quad \Omega^*=\int_0^1\omega_j^* dj(>0). \tag{22b}$$

When the utility function is homothetic, ω_j remains constant or linear with respect to private consumption, ensuring that Ω is independent of the distribution of private consumption. For example, with a constant relative risk aversion (CRRA) utility function, $u(C_j)=\frac{C_j^{1-\gamma}}{1-\gamma}$,

[4]Specifically, G_{K_w} can be rewritten as:

$$G_{K_w}=\frac{f_i(k_i(p))k_c'(p)}{Z_{hf}(k_i(p)-k_c(p))}\left\{\frac{f_c'(k_c(p))p}{f_c(k_c(p))}+\frac{p(K_w-2k_c(p))k_i'(p)f_i'(k_i(p))}{(2k_i(p)-K_w)k_c'(p)f_i(k_i(p))}\right\}(>0).$$

we observe $\omega_j = \frac{C_j}{\gamma}$ so that $\Omega = \frac{C}{\gamma}$. This implies that the elasticities of intertemporal substitution (EIS), such as $\frac{\omega_j}{C_j}$ and $\frac{\Omega}{C}$, are homogeneous among all households. In contrast, under the non-homothetic utility function, ω_j is non-linear in private consumption, resulting in Ω being contingent on the distribution of individuals' consumption. Therefore, (22b) demonstrates that the average individual responds to changes in interest rates depending on the initial distributions of individuals' capital stock.

Using (21) and (22b), we obtain the dynamic equation of aggregate consumption in the global economy:

$$\dot{C}_w = \Delta \left(r(p(K_w, C_w)) - \delta - \rho - \frac{G(K_w, C_w) - \delta K_w}{Z_{hf}(2k_i(p(K_w, C_w)) - K_w)} \right). \tag{22c}$$

where

$$\Delta = \frac{Z_{hf} C_w (\Omega + \Omega^*)}{Z_{hf} C_w + \Omega + \Omega^*}. \tag{22d}$$

Finally, regarding the growth paths of personal consumption and total consumption in both countries, from (22a) and (22b), the following relation holds:[5]

$$\frac{\dot{C}_w}{\Omega + \Omega^*} = \frac{\dot{C}}{\Omega} = \frac{\dot{C}^*}{\Omega^*} = \frac{\dot{C}_j}{\omega_j} = \frac{\dot{C}_j^*}{\omega_j^*}. \tag{23}$$

2.5 Steady State of the Global Economy

We demonstrate the existence of the stationary state in the global economy, where the steady-state values of each variable are denoted by the upper bar. The equations $\dot{K}_w = 0$ in (20a) and $\dot{C}_w = 0$ in (22c) are as follows:

$$\delta \bar{K}_w = G(\bar{K}_w, \bar{C}_w), \tag{24a}$$

$$r(\bar{p}(\bar{K}_w, \bar{C}_w)) = \delta + \rho. \tag{24b}$$

Then, we obtain the following proposition.

[5] For instance, substituting (21) into (22a) and (22b), and moreover, substituting (20a) and (22c) into these equations, we can obtain the dynamic equations of consumption in the home country:

$$\dot{C} = \frac{Z_{hf} C_w \Omega}{Z_{hf} C_w + \Omega + \Omega^*} \left(r(p(K_w, C_w)) - \delta - \rho - \frac{G(K_w, C_w) - \delta K_w}{Z_{hf}(2k_i(p(K_w, C_w)) - K_w)} \right),$$

$$\dot{C}_j = \frac{Z_{hf} C_w \omega_j}{Z_{hf} C_w + \Omega + \Omega^*} \left(r(p(K_w, C_w)) - \delta - \rho - \frac{G(K_w, C_w) - \delta K_w}{Z_{hf}(2k_i(p(K_w, C_w)) - K_w)} \right).$$

These equations satisfy (23).

Proposition 1 *The equilibrium of the global economy in its steady state is uniquely determined and exhibits saddle-path stability.*

Proof. See Appendix A. ■

It is noteworthy that the steady-state values of aggregate consumption $\bar{C}_w$ and capital $\bar{K}_w$ and the relative price $\bar{p}$ are uniquely determined regardless of initial conditions such as preference parameters and initial capital levels. However, the steady-state values of individual consumption and capital $\bar{K}_j$ and $\bar{C}_j$ and domestic capital and consumption levels $\bar{K}$, $\bar{K}^*$, $\bar{C}$, and $\bar{C}^*$ depend on initial conditions.

Denoting the stable root of the global economy as $\lambda(<0)$, according to Proposition 1, we obtain:[6]

$$2\lambda = G_{K_w} - \delta + \frac{\partial \dot{C}_w}{\partial C_w} - \left(\left(G_{K_w} - \delta + \frac{\partial \dot{C}_w}{\partial C_w} \right)^2 - 4 \left((G_{K_w} - \delta) \frac{\partial \dot{C}_w}{\partial C_w} - G_{C_w} \frac{\partial \dot{C}_w}{\partial K_w} \right) \right)^{\frac{1}{2}} (<0), \tag{25}$$

where

$$\frac{\partial \dot{C}_w}{\partial C_w} = \bar{\Delta} \left(r'(\bar{p}) \frac{\partial p}{\partial C_w} - \frac{G_{C_w}}{Z_{hf}(2\bar{k}_i - \bar{K}_w)} \right) (>0),$$

$$\frac{\partial \dot{C}_w}{\partial K_w} = \bar{\Delta} \left(r'(\bar{p}) \frac{\partial p}{\partial K_w} - \frac{G_{K_w} - \delta}{Z_{hf}(2\bar{k}_i - \bar{K}_w)} \right) \left(\gtreqless 0 \right).$$

When the utility function is homothetic, the speeds of convergence remain unaffected by initial conditions due to the homogeneity of EIS across agents. Conversely, under a non-homothetic utility function, the non-linear nature of the inverse of ARA allows initial conditions to influence private consumption and, consequently, EIS. Therefore, initial conditions impact the convergence rate, with higher convergence rates associated with greater values of Ω.[7]

On the saddle path of the global economy, the relationship between consumption and capital is expressed as:[8]

$$C_w - \bar{C}_w = \frac{\rho - \lambda}{\bar{p}(\bar{K}_w, \bar{C}_w)} (K_w - \bar{K}_w), \tag{26}$$

[6] For details on the determinant's sign, see Appendix A.

[7] See Appendix B.

[8] See Appendix C.

where the stable root, λ has a negative sign. Utilizing (26), the approximated behavior of aggregate capital in the global economy is given by:

$$\dot{K}_w = \lambda(K_w - \bar{K}_w). \tag{27}$$

Finally, each relationship between capital in the global economy and private consumption (or, aggregate consumption) is:

$$\frac{C_w - \bar{C}_w}{\bar{\Omega} + \bar{\Omega}^*} = \frac{C - \bar{C}}{\bar{\Omega}} = \frac{C^* - \bar{C}^*}{\bar{\Omega}^*} = \frac{C_j - \bar{C}_j}{\bar{\omega}_j} = \frac{C_j^* - \bar{C}_j^*}{\bar{\omega}_j^*}. \tag{28}$$

This relationship is identical to that of the (23) around the steady state.

3 Relative Wealth and Wealth Distribution

We investigate wealth distribution in our two-country global economy. Throughout, we simplify expressions of functions and variables where there is no risk of confusion. For instance, production and price functions are denoted as f_c and p, respectively, and functions in the steady state are represented as $\bar{f}_c$ and $\bar{p}$.

When each country is populated by a mass 1 of infinitely-lived agents and consists of two countries, $K_w/2$ represents the average level of per capital capital in the global economy. We then denote relative wealth and consumption in the home and foreign countries as follows: $\tilde{K} \equiv \frac{K}{K_w/2}$, $\tilde{K}^* \equiv \frac{K^*}{K_w/2}$, $\tilde{C} \equiv \frac{C}{C_w/2}$, and $\tilde{C}^* \equiv \frac{C^*}{C_w/2}$, respectively.

To characterize the dynamic motion of relative wealth, we first show the relationship between the aggregate consumption, capital, and price. We now use (21), which can be rewritten as:

$$p - \bar{p} = \frac{1}{\bar{Z}_{hf}} \left(\frac{C_w - \bar{C}_w}{\bar{C}_w} + \frac{K_w - \bar{K}_w}{2\bar{k}_i - \bar{K}_w} \right), \tag{29}$$

where $2\bar{k}_i > \bar{K}_w$ and $\bar{Z}_{hf} > 0$. Noting that sign $\{C_w - \bar{C}_w\}$ = sign $\{K_w - \bar{K}_w\}$ in (26), from (29) we obtain the following lemma.

Lemma 1 *The relationships between the price, aggregate global consumption, and aggregate global capital are given by:*

$$sign\{p - \bar{p}\} = sign\{C_w - \bar{C}_w\} = sign\{K_w - \bar{K}_w\}. \tag{30}$$

To show the dynamic motion of the relative wealth of $\tilde{K}$, we differentiate $\tilde{K}$ with respect to time:

$$\dot{\tilde{K}} = \frac{\dot{K}}{K_w/2} - \tilde{K}\frac{\dot{K}_w}{K_w}. \tag{31}$$

Substituting (13a) and (13b) into (31), we derive

$$\dot{\tilde{K}} = \frac{2w}{K_w}\left(1-\tilde{K}\right) + \frac{pC_w}{K_w}\left(\tilde{K}-\tilde{C}\right), \tag{32}$$

Using (1), Equation (32) can be rewritten as

$$\dot{\tilde{K}} = \frac{p}{K_w}\left\{2(f_c - k_c f_c')\left(1-\tilde{K}\right) + C_w\left(\tilde{K}-\tilde{C}\right)\right\}, \tag{33}$$

where Equation $\dot{\tilde{K}} = 0$ is held in the steady state:

$$\bar{C}_w\left(\bar{\tilde{K}} - \bar{\tilde{C}}\right) = 2(\bar{f}_c - \bar{k}_c \bar{f}_c')\left(\bar{\tilde{K}} - 1\right). \tag{34}$$

Linearly approximating Equation (33) around the steady state, we derive [9]

$$\dot{\tilde{K}} = \rho(\tilde{K} - \bar{\tilde{K}}) + 2\left(\bar{\tilde{K}} - 1\right)\frac{\bar{w}}{\bar{K}_w}\Bigg\{\underbrace{\left(1 - \frac{w'(\bar{p})\bar{p}}{w(\bar{p})}\right)\frac{p-\bar{p}}{\bar{p}} + \frac{C_w - \bar{C}_w}{\bar{C}_w}}_{(\#1)}\Bigg\} - \frac{2\bar{p}\bar{C}}{\bar{K}_w}\left(\frac{C-\bar{C}}{\bar{C}} - \frac{C_w - \bar{C}_w}{\bar{C}_w}\right). \tag{35}$$

Here, (1) indicates that the price elasticity of wages exceeds unity:

$$\frac{\bar{p}w'(\bar{p})}{w(\bar{p})} > 1. \tag{36}$$

Notice that $w'(\bar{p}) = \bar{f}_c - \bar{k}_c \bar{f}_c' - \bar{p}\bar{k}_c \bar{f}_c'' \bar{k}_c' (> 0)$. Because $\text{sign}\left\{p - \bar{p}\right\} = \text{sign}\left\{C_w - \bar{C}_w\right\}$ in (30), (36) means that the sign of (#1) in (35) may be positive or negative. If the growth rate of global consumption is low such that $\left(\frac{w'(\bar{p})\bar{p}}{w(\bar{p})} - 1\right)\frac{\bar{p}-p}{\bar{p}} > \frac{\bar{C}_w - C_w}{\bar{C}_w} (> 0)$, then (#1) has a positive sign. Alternatively, if the opposite relationship is applicable, then (#1) has a negative sign.

3.1 Homothetic Utility Function

We now assume that the utility function is homothetic. In this case, EIS are constant and equal for all individuals. Therefore, from (28), we obtain

$$\frac{C_w - \bar{C}_w}{\bar{C}_w} = \frac{C - \bar{C}}{\bar{C}} = \frac{C^* - \bar{C}^*}{\bar{C}^*}. \tag{37}$$

[9] Specifically, we obtain the following:

$$\dot{\tilde{K}} = \rho(\tilde{K} - \bar{\tilde{K}}) + 2\left(\bar{\tilde{K}} - 1\right)\frac{\bar{p}}{\bar{K}_w}\left\{\bar{k}_c \bar{f}_c'' \bar{k}_c'(p - \bar{p}) + \frac{\bar{w}}{\bar{p}}\frac{C_w - \bar{C}_w}{\bar{C}_w}\right\} - \frac{2\bar{p}\bar{C}}{\bar{K}_w}\left(\frac{C-\bar{C}}{\bar{C}} - \frac{C_w - \bar{C}_w}{\bar{C}_w}\right).$$

Substituting $w'(\bar{p})$ into the above expression yields (35).

For instance, employing the CRRA utility function, which is a typical homothetic utility function, yields $\frac{\Omega+\Omega^*}{C+C^*} = \frac{\Omega}{C} = \frac{\Omega^*}{C^*} = \frac{1}{\gamma}$ where γ stands as a constant parameter denoting the inverse EIS. This equation directly implies (37).

Substituting (37) into (35), we derive

$$\dot{\tilde{K}} = \rho(\tilde{K} - \bar{\tilde{K}}) + 2\left(\bar{\tilde{K}} - 1\right)\frac{\bar{w}}{\bar{K}_w}\left\{\left(1 - \frac{w'(\bar{p})\bar{p}}{w(\bar{p})}\right)\frac{p-\bar{p}}{\bar{p}} + \frac{C_w - \bar{C}_w}{\bar{C}_w}\right\}. \tag{38}$$

By substituting (26) and (29) into (38), we obtain:

$$\tilde{K}(t) - 1 = \left(\bar{\tilde{K}} - 1\right)\left[1 + D_{hf}(\bar{K}_w - K_w^0)e^{\lambda t}\right], \tag{39a}$$

where the variable D_{hf} is given by:

$$D_{hf} = \frac{2\bar{w}}{\bar{p}(\rho-\lambda)\bar{K}_w}\left\{\left(1 - \frac{w'(\bar{p})\bar{p}}{w(\bar{p})}\right)\frac{1}{Z_{hf}}\left(\frac{\rho-\lambda}{\bar{C}_w\bar{p}} + \frac{1}{2\bar{k}_i - \bar{K}_w}\right) + \frac{\rho-\lambda}{\bar{C}_w}\right\}. \tag{39b}$$

Note that D_{hf} is common between home and foreign countries.

Setting $t = 0$ in (39a), we derive

$$\bar{\tilde{K}} - 1 = \frac{\tilde{K}^0 - 1}{1 + D_{hf}(\bar{K}_w - K_w^0)}. \tag{39c}$$

Proposition 2 *Assume that the home and foreign countries have identical production structures and all individuals have identical and homothetic utility functions. When both countries start with the same amount of aggregate capital in the initial period (i.e., $K^0 = K^{*,0}$), the levels of aggregate capital in the two countries remain the same over time, regardless of the initial distribution of capital stock.*

Proof. In (39c), substitute $\tilde{K}^0 = 1$ so that $\bar{\tilde{K}} = 1$, which implies $\tilde{K}(t) = 1$ in (39a). ■

The intuition behind Proposition 2 is as follows. If the home country initially holds half of the aggregate global capital, then $\tilde{K}^0 = 1$ (i.e., $K^0 = \frac{K_w^0}{2}$). Consequently, the initial level of aggregate capital in the foreign country equals that in the home country, that is, $K^0 = K^{*,0} = \frac{K_w^0}{2}$. Under these conditions, Proposition 2 demonstrates that the levels of aggregate capital in the home and foreign countries remain identical over time. This result is attributed to the homothetic and identical utility functions for all individuals and the symmetric production structures in both countries. Therefore, in the long run, the global economy maintains a capital balance, where $\bar{K} = \bar{K}^* = \frac{\bar{K}_w}{2}$.

Next, consider a scenario where the two countries have different amounts of aggregate capital initially. Specifically, suppose $K^0 \neq \frac{K_w^0}{2}$ and $K^{*,0} \neq \frac{K_w^0}{2}$, meaning $K^0 \neq K^{*,0}$. Assume that $\bar{K}_w > K_w^0$. The key to characterizing the dynamic motion of relative wealth is the wage elasticity of the price. For instance, if the wage elasticity of the price is close to unity (i.e., a 1% increase in the price leads to approximately a 1% increase in the wage), the dynamic behavior of the price is negligible for characterizing the relative wealth of $\tilde{K}$. In this case, the variable D_{hf} has a positive sign, indicating that an initially capital-wealthy country remains wealthier than an initially capital-poor one over time. This outcome is plausible given that the initially capital-wealthy country has better investment opportunities, assuming symmetry between the countries. Conversely, if the wage elasticity of the price is significantly above unity, the sign of variable D_{hf} could be negative. In this scenario, an initially capital-wealthy country might become poorer than an initially capital-poor one in the long run, contrary to expectations. Consequently, under homothetic preferences, the dynamic HO theorem of comparative advantage generally holds, but there may still remain the case in which the long-run trade pattern would not be determined by the initial holdings of capital and labor alone.

3.2 Non-homothetic Utility Function

We assume that all individuals have a non-homothetic and identical utility function. In this case, we cannot derive the equation (37). Additionally, based on the definitions of Ω and Ω^* in (22b), EIS $\frac{\bar{\Omega}}{\bar{C}}$ and $\frac{\bar{\Omega}^*}{\bar{C}^*}$ are not constant parameters. This implies that the initial distribution of capital stock within a country affects its elasticity. Consequently, even if the initial levels of aggregate capital in the home and foreign countries are the same, the difference in the initial distribution of capital stock results in different EIS.

Therefore, since $\frac{C-\bar{C}}{\bar{C}} \neq \frac{C_w-\bar{C}_w}{\bar{C}_w}$, the equation in (35) can be rewritten as:

$$\tilde{K}(t) - 1 = \left(\bar{\tilde{K}} - 1\right)\left[1 + D_{hf}(\bar{K}_w - K_w^0)e^{\lambda t}\right] - \frac{2\bar{C}}{\bar{\Omega} + \bar{\Omega}^*}\left(\frac{\bar{\Omega}}{\bar{C}} - \frac{\bar{\Omega} + \bar{\Omega}^*}{\bar{C}_w}\right)\frac{\bar{K}_w - K_w^0}{\bar{K}_w}e^{\lambda t}, \quad (40)$$

Substituting $t = 0$ into (40), we obtain

$$\bar{\tilde{K}} - 1 = \frac{\tilde{K}^0 - 1 + \frac{2\bar{C}}{\bar{\Omega}+\bar{\Omega}^*}\left(\frac{\bar{\Omega}}{\bar{C}} - \frac{\bar{\Omega}+\bar{\Omega}^*}{\bar{C}_w}\right)\frac{\bar{K}_w - K_w^0}{\bar{K}_w}}{1 + D_{hf}(\bar{K}_w - K_w^0)}. \quad (41)$$

Then, we obtain the following main result:

Proposition 3 *Assume that home and foreign countries have identical production structures and all individuals have identical and non-homothetic utility functions. Even if both countries have the same initial aggregate capital (i.e., $K^0 = K^{*,0}$), differences in the initial distribution of capital stock between the two countries will result in different levels of aggregate capital over time.*

Proof. Assume that home and foreign countries have the same initial aggregate capital, so $\tilde{K}^0 = 1$. In this case, (41) can be rewritten as

$$\bar{\tilde{K}} - 1 = \frac{\frac{2\bar{C}}{\bar{\Omega}+\bar{\Omega}^*}\left(\frac{\bar{\Omega}}{\bar{C}} - \frac{\bar{\Omega}+\bar{\Omega}^*}{\bar{C}_w}\right)\frac{\bar{K}_w - K_w^0}{\bar{K}_w}}{1 + D_{hf}(\bar{K}_w - K_w^0)}. \tag{42}$$

Due to the non-homothetic nature of the utility function, the condition $\frac{\bar{\Omega}}{\bar{C}} \neq \frac{\bar{\Omega}+\bar{\Omega}^*}{\bar{C}_w}$ indicates that the equality $\bar{\tilde{K}} = 1$ in (42) is not upheld. As a result, from (40), we find that $\tilde{K}(t) = 1$ is not kept. ■

In contrast to Proposition 2, even if an equal-capital global economy starts at the initial period under the symmetric production structure, the non-homothetic utility function leads to diverging levels of aggregate capital over time. For instance, assuming $\bar{K}_w > K_w^0$ and $\tilde{K}^0 = 1$ under $D_{hf} > 0$, if $\frac{\bar{\Omega}}{\bar{C}} > \frac{\bar{\Omega}+\bar{\Omega}^*}{\bar{C}_w}$, then $\bar{\tilde{K}} > 1$ in (41). This is intuitive: with EIS in the home country surpassing the average EIS globally, individuals in the home country respond more rapidly to interest rate changes compared to those in the foreign country. Consequently, when $\bar{K}_w > K_w^0$, we observe $\bar{K} > \bar{K}^*$. This discrepancy elucidates the trade patterns: with $\bar{K} > \bar{K}^*$, we find $\bar{L}_c < \bar{L}_c^*$ in (6), and hence, $\bar{L}_i > \bar{L}_i^*$ from (4). Consequently, we find that $\bar{K}_c < \bar{K}_c^*$ and $\bar{K}_i > \bar{K}_i^*$ because it holds that $\frac{K_i}{L_i} = \frac{K_i^*}{L_i^*}$ and $\frac{K_c}{L_c} = \frac{K_c^*}{L_c^*}$ under $k_i(p) = k_i^*(p)$ and $k_c(p) = k_c^*(p)$. Essentially, the home country directs more capital and labor towards the investment goods sector, while the foreign country prioritizes consumption goods. Conversely, when $\frac{\bar{\Omega}}{\bar{C}} < \frac{\bar{\Omega}+\bar{\Omega}^*}{\bar{C}_w}$, the aggregate capital level in the home country is lower than that in the foreign country. These findings offer intuitive insights.

However, if the wage elasticity of price exceeds unity, D_{hf} might turn negative, reversing the aforementioned explanation.

4 Numerical Analysis

4.1 Purpose and Discussion: Non-homothetic Utility Function

In previous research, international economists have explored the underlying structures that influence trade between symmetric countries. One significant finding from prior studies is that when both home and foreign countries have the identical production structures and all individuals have identical utility functions, the difference in the initial sizes of aggregate capital between these countries is necessary to initiate trade. However, our main results, as outlined in Propositions 2 and 3, challenge this notion. These propositions demonstrate that the initial difference in aggregate capital between countries is not a prerequisite for trade initiation. Specifically, even if two countries start with the same amount of aggregate capital, variations in the non-homotheticity of households and differences in individuals' capital endowments lead to capital inequality between the two countries, thereby prompting trade.

The key factor driving trade initiation is the presence of non-homothetic utility functions, which result in varying responsiveness to changes in interest rates among households. While Propositions 2 and 3 contribute to advancing our understanding of trade structure in international economics, further exploration is required regarding the conditions of non-homothetic utility functions. In this section, we focus on the concavity or convexity of the inverse of ARA in the non-homothetic utility function to characterize both global and domestic inequalities in numerical examples. Specifically, we conduct numerical analyses by varying the initial levels of domestic and individual capital across a wide range. For this purpose, we utilize the non-homothetic utility function outlined in Bertola et al. (2006, Section 2):

$$u'(C_j) = (C_j^{\beta} - \underline{c})^{-\eta}, \quad \omega_j = \frac{C_j\left(1 - \underline{c}C_j^{-\beta}\right)}{\eta\beta}, \quad \frac{\Omega}{C} = \frac{1}{\eta\beta}\left(1 - \frac{\underline{c}}{C}\int_0^1 C_j^{1-\beta} dj\right) \tag{43}$$

where β, η and $\underline{c}$ are constants. Specifically, β and η represent degrees of risk aversion. To maintain positive marginal utilities of consumption, the non-homothetic utility function must satisfy $C_j^{\beta} > \underline{c}$ where $\underline{c}$ represents the subsistence level. The parameters of the non-homothetic utility function in (43) are as follows:[10]

$$\underline{c} = 0.5, \quad \beta = \{0.5, 1.5\}, \quad \eta = 0.25. \tag{44}$$

[10]We confirmed that the figures of $\eta = 0.75$ are nearly all the same as those of $\eta = 0.25$.

The presence of two cases for parameter β generates the concavity and convexity of the inverse of ARA of the non-homothetic utility function.

$$\frac{\partial \omega_j}{\partial C_j} = \frac{1-(1-\beta)\underline{c}C_j^{-\beta}}{\eta\beta}(>0), \quad \frac{\partial^2 \omega_j}{\partial C_j^2} = \frac{(1-\beta)\underline{c}C_j^{-\beta-1}}{\eta}. \tag{45}$$

In our simulations, it holds that $\frac{\partial \omega_j}{\partial C_j} > 0$ in all cases. Alternatively, we can confirm that $\frac{\partial^2 \omega_j}{\partial C_j^2} > 0$ when $\beta = 0.5$, while $\frac{\partial^2 \omega_j}{\partial C_j^2} < 0$ when $\beta = 1.5$. In sum, the inverse of ARA increases with private consumption convexity when $\beta = 0.5$, whereas it increases with private consumption concavely when $\beta = 1.5$.

We now mention the relationship between EIS and the inverse of ARA. Suppose the inverse of ARA is convex with respect to private consumption. In this scenario, higher levels of private consumption lead to a convex increase in the inverse of ARA. Consequently, the increase in ω_j outpaces the increase in C_j, resulting in a higher $\frac{\omega_j}{C_j}$ ratio, which corresponds to high EIS. Therefore, affluent individuals, who consume more, respond more sensitively to changes in interest rates, and initially rich individuals become significantly wealthier over time.

Conversely, if the inverse of ARA is concave with respect to private consumption, higher levels of consumption lead to a concave increase in the inverse of ARA. In this case, $\frac{\omega_j}{C_j}$ decreases because the increase in ω_j is less than the increase in C_j. This implies lower EIS, resulting in lower sensitivity to changes in interest rates among affluent individuals; thus, they become less wealthy over time.

4.2 Specification and parameters

We now assume that the production functions in two sectors follow the Cobb-Douglas form $f_i(k_i) = A_i k_i^{\alpha_i}$ and $f_c(k_c) = A_c k_c^{\alpha_c}$ where $A_i(>0)$, $A_c(>0)$, $\alpha_i \in (0,1)$, and $\alpha_c \in (0,1)$ are constant parameters. In this case, we specify the key variable, $\bar{D}_{hf}$, of characterizing the relative wealth between two countries, where we consider the steady-state equilibrium that satisfies (24a) and (24b).

First, the specification of the Cobb-Douglas production function leads to the steady-state level of Z_{hf} in (19) as follows:[11]

$$\bar{Z}_{hf} = \frac{1}{\bar{p}} \times \underbrace{\frac{1}{\alpha_i - \alpha_c}\left\{\alpha_c + \frac{\alpha_c(1-\alpha_i)}{\alpha_i - \alpha_c}\left(1 + \frac{\alpha_c(1-\alpha_i)\delta}{(1-\alpha_c)(\rho + (1-\alpha_i)\delta)}\right)\right\}}_{\equiv M(>0)} \equiv \frac{M}{\bar{p}}(>0), \quad \text{(46a)}$$

where M is fixed and consists of the production and depreciation parameters.

Next, using (46a), we find that the steady-state level of D_{hf} in (39b) is given by:

$$\bar{D}_{hf} = \frac{1}{\bar{K}_w} \times \underbrace{\frac{(1-\alpha_i)\left\{(1-\alpha_c)\rho + \delta(1-\alpha_i)\right\}(\rho+\delta)\delta}{\left\{\rho + \delta(1-\alpha_i)\right\} M(\alpha_i - \alpha_c)^2(\rho - \lambda)}\left\{\underbrace{\alpha_c}_{(\#2A)} - \frac{\delta + \lambda}{\delta}\frac{(1-\alpha_i)\delta + (1-\alpha_c)\rho}{(1-\alpha_i)(\delta+\rho)}\right\}}_{\equiv \xi(\lambda)(\lesseqgtr 0)} \equiv \frac{\xi(\bar{\lambda})}{\bar{K}_w}. \quad \text{(46b)}$$

where

$$\xi'(\lambda) = \underbrace{\frac{(1-\alpha_c)\rho + \delta(1-\alpha_i)}{\rho + \delta(1-\alpha_i)}\frac{\delta(1-\alpha_i)(\rho+\delta)}{M(\alpha_i - \alpha_c)^2(\rho - \lambda)}}_{(+)}\left\{\underbrace{\alpha_c}_{(\#2B)} - \frac{(1-\alpha_c)\rho + \delta(1-\alpha_i)}{\delta(1-\alpha_i)(\rho - \lambda)}\right\}(\lesseqgtr 0). \quad \text{(46c)}$$

By specifying the Cobb-Douglas production function, we find that the variable $\bar{D}_{hf}$ simplifies $\frac{\xi(\lambda)}{\bar{K}_w}$ where the steady-state level of global aggregate capital can be uniquely determined by (24a) and (24b). It is important to note that ($\#2A$) and ($\#2B$) correspond to the wage elasticity of the price. Using the Cobb-Douglas production function, we find that

$$\frac{w'(p)p}{w(p)} - 1 = \frac{\alpha_c}{\alpha_i - \alpha_c}(>0). \quad \text{(47)}$$

Therefore, if the value of α_c is sufficiently close to zero, the wage elasticity of the price is close to 1. As a result, as confirmed in (#1) of (35), the effect of price changes on the dynamic movement of relative wealth becomes quietly limited.

Subsequently, the equation in (41) is denoted as

$$\bar{\tilde{K}} - 1 = \frac{\frac{2\bar{C}}{\bar{\Omega}+\bar{\Omega}^*}\left(\frac{\bar{\Omega}}{\bar{C}} - \frac{\bar{\Omega}+\bar{\Omega}^*}{\bar{C}_w}\right)\frac{\bar{K}_w - K_w^0}{\bar{K}_w}}{1 + \xi(\bar{\lambda})\frac{(\bar{K}_w - K_w^0)}{\bar{K}_w}}, \quad \text{(48a)}$$

where we assume that the initial levels of aggregate capital in the home and foreign countries are the same:

$$K^0 = K^{*,0} = \frac{K_w^0}{2}. \quad \text{(48b)}$$

[11] See Appendix D for the detailed derivation.

The basic parameters we use are as follows:

$$A_i = 0.5, \quad A_c = 1, \quad \alpha_i = 0.5, \quad \alpha_c = 0.25, \quad \delta = 0.04, \quad \rho = 0.05. \tag{49}$$

A time preference rate of 5% and a depreciation rate of 4% are commonly used in this field. Given these parameters, the steady-state interest rate, $\bar{r}$, is 9%. The choice of $\alpha_i = 0.5$ indicates that 50% of output in the investment sector accrues to private capital, while $\alpha_c = 0.25$ indicates that 25% of output in the consumption sector accrues to private capital. The production parameters A_i and A_c are set such that $A_i < A_c$ to satisfy the inequality (15).

With these parameters, the steady-state values are as follows:

$$\begin{aligned} \bar{r} &= 0.09, \quad \bar{w} = 0.44, \quad \bar{k}_i = \bar{k}_i^* = 2.37, \\ \bar{k}_c &= \bar{k}_c^* = 1.46, \quad \bar{p} = 0.53, \\ \bar{K}_w &= 3.18, \quad \bar{C}_w = 3.54, \quad \bar{L}_i + \bar{L}_i^* = 0.28. \end{aligned} \tag{50}$$

These steady-state levels are independent of the initial capital distribution and do not respond to the specification of the utility function.

Without loss of generality, we specify the home and foreign households as two types: $j = 1, 2$. We assume that the global economy is initially poorer than its steady state, that is, $K_w^0 < \bar{K}_w$ where we assume (48b). That is, the home and foreign countries have the same initial total capital, while their domestic distribution may differ. Specifically, we consider a scenario where $K_1^0 < K_2^0$, and $K_1^{*0} < K_2^{*0}$. That is, households 1 in both countries have less initial capital than households 2 in each country. Furthermore, we assume that capital is relatively more evenly distributed among households in the home country compared with those in the foreign country. Formally, this implies $\left|K_1^0 - K_2^0\right| < \left|K_1^{0*} - K_2^{0*}\right|$.

We assume $K_w^0 < \bar{K}_w$ and denote the initial state with $K_w^0 = \epsilon \bar{K}_w$ where ϵ, increments by 0.05 from 0.05 to 0.95. The initial states of the two agents in each country are given by:

$$K_1^0 = \mu K^0, \quad K_2^0 = (1-\mu) K^0, \quad K_1^{*0} = 0.01 K^{*0}, \quad K_2^{*0} = 0.99 K^{*0}, \tag{51}$$

where μ increments by 0.05 from 0.05 to 0.45. This setup indicates that households 1 in both countries are relatively poorer than households 2 in terms of initial capital holdings, and the foreign country has a more unequal distribution of capital. In the foreign country, households

1 start with only 1% of the domestic capital, while households 2 start with 99%. In contrast, in the home country, μ varies between 0.05 and 0.45, allowing for different patterns of initial capital distribution. For example, in (51), when $\mu = 0.45$, the home country is initially more equal than when $\mu = 0.05 - 0.4$, the least equal scenario in our discussion.

Finally, let us consider the trade pattern in our simulation. Given that $\bar{k}_i > \bar{k}_c > 0$ and $\bar{k}_i^* > \bar{k}_c^* > 0$ in (50), our simulation indicates that both countries produce both consumption and investment goods at the steady state. Additionally, since $\bar{K}_w > K_w^0$ under $\epsilon \in [0.05, 0.45]$, according to Lemma 1, we find that $\bar{p} > p^0(> 0)$, implying that $\bar{k}_i > k_i^0(> 0)$ and $\bar{k}_c > k_c^0(> 0)$ from (1) and (3).

Given that our economy has only one stable root, capital stocks such as k_i and k_c move monotonically towards the steady state. Consequently, in our simulation, the economy does not fully specialize in producing only one type of good over time but rather produces both consumption and investment goods.

Furthermore, as detailed in Section 3.2., if $\bar{K} > \bar{K}^*$, then it holds that $\bar{L}_c < \bar{L}_c^*$, $\bar{L}_i > \bar{L}_i^*$, $\bar{K}_c < \bar{K}_c^*$, and $\bar{K}_i > \bar{K}_i^*$. Thus, in this scenario, the home country uses more capital and labor in the investment goods sector, whereas the foreign country uses more capital and labor in the consumption goods sector. If $\bar{K} < \bar{K}^*$, this relationship is reversed.

Based on the above specification, our simulations confirm the following in all cases: (i) The marginal utilities in (43) always have a positive sign; (ii) It always holds that $D_{hf} > 0$; (iii) The initial level of private consumption is positive; (iv) The economy remains incompletely specialized over time.

4.3 Inequality in the global economy

Figures 1 and 2 illustrate the long-run inequality in the global economy with two different values of β. The horizontal axis in these figures represents the ratio of the initial level of global capital to the steady-state level.: $\epsilon = K_w^0/\bar{K}_w$, which ranges from 0.05 to 0.95. The vertical axis, on the other hand, represents the ratio of households 1's initial capital holdings to total domestic capital holdings: $\mu = K_1^0/K^0$. Since we assume that households 1 are poorer than households 2, μ takes values between 0.05 and 0.45.

When $\beta = 0.5$, we denote the steady state where the foreign country is wealthier than the home country with "+" in Figure 1(a). That is, "+" represents $\bar{K} < \bar{K}^*$. When this relationship is reversed, we use a triangle "▲" to denote $\bar{K} > \bar{K}^*$. Figure 1(b) shows the aggregate EIS on the same $\epsilon - \mu$ plane, which is closely related to the inequality between home and foreign countries. Here, "+" represents the scenario where $\frac{\bar{\Omega}}{\bar{C}} < \frac{\bar{\Omega}+\bar{\Omega}_w}{\bar{C}_w}$, while "▲" represents the opposite scenario where $\frac{\bar{\Omega}}{\bar{C}} > \frac{\bar{\Omega}+\bar{\Omega}_w}{\bar{C}_w}$. Starting from the initial condition where $\epsilon < 1$ and $\mu < 0.5$, regardless of how small ϵ and μ are, the home country's capital eventually becomes less than the foreign country's capital $\bar{K} < \bar{K}^*$ marked as "+". Simultaneously, the home country's EIS are always smaller than the global EIS : $\frac{\bar{\Omega}}{\bar{C}} < \frac{\bar{\Omega}+\bar{\Omega}_w}{\bar{C}_w}$, marked as "+" as well.

When $\beta = 0.5$, which implies the inverse of ARA is convex in consumption, the initially more-unequal economy (the foreign country) ends up with a larger inverse level of ARA than the initially more-equal economy. This means that in the long run, the initially more-unequal economy will possess a greater capital stock.

[**Figure 1 around here.**]

However, when $\beta = 1.5$, the distribution outcomes may vary because the inverse of ARA is concave in consumption. Still we use the same symbols "+", and "▲" to represent the cases of $\bar{K} < \bar{K}^*$, and $\bar{K} > \bar{K}^*$ respectively. The rows of "▲" in Figure 2(a) can be shown when $\epsilon = K_w^0/\bar{K}_w$ is sufficiently small, particular when $\epsilon \leq 0.25$. That is, if the initial world economy is very far from the steady state, then it is possible for the home economy to have the long-run level of aggregate capital stock more than the foreign country. That is, $\bar{K} > \bar{K}^*$ is realized. This result is accompanied by the home EIS more than the world-average EIS, $\frac{\bar{\Omega}}{\bar{C}} > \frac{\bar{\Omega}+\bar{\Omega}_w}{\bar{C}_w}$, which is shown with the same "▲" symbol in Figure 2(b). When the initial world economy is very far from the steady state, the concavity of the inverse of ARA allows the initially more-equal economy (the home country) to possess the capital stock more than the initially more-unequal economy.

However, if the initial state of the global economy is not significantly distant from the steady state, for instance, when $\epsilon \geq 0.30$, then the initially equal home country cannot surpass the initially unequal foreign country in terms of capital stock: $\bar{K} < \bar{K}^*$ always holds. Figure 2(b) presents the home EIS and the global-average EIS in a similar manner. When $\epsilon \geq 0.30$, indicating that the initial state of the global economy is not far from the steady-state equilibrium, for all μ and $\epsilon \geq 0.30$, the home country consistently exhibits smaller EIS compared to the global-average EIS. That is, $\frac{\bar{\Omega}}{\bar{C}} < \frac{\bar{\Omega}+\bar{\Omega}_w}{\bar{C}_w}$. We maintain the use of the same symbols "+", and "▲" to denote scenarios where $\frac{\bar{\Omega}}{\bar{C}} < \frac{\bar{\Omega}+\bar{\Omega}_w}{\bar{C}_w}$, and $\frac{\bar{\Omega}}{\bar{C}} > \frac{\bar{\Omega}+\bar{\Omega}_w}{\bar{C}_w}$, respectively.

[Figure 2 around here.]

Intuitively, when the inverse of ARA is convex under $\beta = 0.5$, an initially more-unequal economy (the foreign country) possesses a greater capital stock compared to an initially less-unequal economy (the home country). This outcome can be attributed to the presence of very affluent households 2 in the foreign country, which hold $K_2^{*,0} = 0.99K^{*,0}$. Affluent households respond more sensitively to changes in interest rates. Since households 2 possess significant wealth and can afford a large amount of consumption, the assumption that the inverse of ARA convexly increases allows for a high level of inverse ARA under (43) and (45). Consequently, the foreign EIS become large, surpassing the home EIS. Figure 1(b) supports this intuition, showing that the sum of the aggregate EIS in the foreign country is larger than that in the home country.

However, when the inverse of ARA is concave under $\beta = 1.5$, some markers display a triangle symbol, as shown in Figure 2(a). This symbol appears when the initial economy is distant from the steady state. In other words, when the inverse of ARA is concave, an initially more-unequal economy (the foreign country) may possess a smaller capital stock than the other country. The very affluent households 2 in the foreign country become less sensitive to changes in interest rates under the concavity of the inverse of ARA. Thus, the sum of the aggregate EIS in the initially less-unequal economy (the home country) may be greater than that in the initially more-unequal economy (the foreign country), corresponding to the triangle markers in Figure 2(b).

4.4 Domestic Inequality

In addition to examining global inequality, we also focus on domestic inequality within home and foreign countries, as shown in Figures 3 and 4. Figures 3(a) and 4(a) illustrate domestic inequality, while Figures 3(b) and 4(b) depict individual EIS. Figures 3(c) and 4(c) verify whether there is a reversal in individual capital stock. It should be noted that Figures 3(a)–(c) correspond to the scenario where $\beta = 0.5$, while Figures 4(a)–(c) represent $\beta = 1.5$.

Similar to the previous subsection, we analyze the $\epsilon - \mu$ plane, where $\epsilon \equiv K_w^0/\bar{K}_w$ denotes the initial global capital ratio, and $\mu = K_1^0/K^0$ represents the capital share of households 1 in the home country. The trends observed in Figures 3(a)–(c) closely resemble those in Figures 4(a)–(c). Therefore, we focus on the scenario where $\beta = 0.5$ in Figures 3(a)–(c). We redefine the symbols representing domestic inequality at home and abroad as follows:

Symbol	Domestic inequality
x	H and F expansion $\lvert K_1^0 - K_2^0\rvert < \lvert \bar{K}_1 - \bar{K}_2\rvert$ and $\lvert K_1^{*0} - K_2^{*0}\rvert < \lvert \bar{K}_1^* - \bar{K}_2^*\rvert$
+	H expansion and F contraction $\lvert K_1^0 - K_2^0\rvert < \lvert \bar{K}_1 - \bar{K}_2\rvert$ and $\lvert K_1^{*0} - K_2^{*0}\rvert > \lvert \bar{K}_1^* - \bar{K}_2^*\rvert$
▲	H contraction and F expansion $\lvert K_1^0 - K_2^0\rvert > \lvert \bar{K}_1 - \bar{K}_2\rvert$ and $\lvert K_1^{*0} - K_2^{*0}\rvert < \lvert \bar{K}_1^* - \bar{K}_2^*\rvert$
o	H and F contraction $\lvert K_1^0 - K_2^0\rvert > \lvert \bar{K}_1 - \bar{K}_2\rvert$ and $\lvert K_1^{*0} - K_2^{*0}\rvert > \lvert \bar{K}_1^* - \bar{K}_2^*\rvert$

Figures 3(a) and 4(a) show similar patterns. Specifically, when $\beta = 0.5$, Figure 3(a) indicates that, except for cases where $\epsilon \in (0.3, 0.5)$, the $x-$ markers appear, indicating the expansion of domestic inequalities in both countries. However, when $\epsilon \in (0.3, 0.5)$ in Figure 3(a), plus or circle markers are observed. For instance, when $\epsilon = 0.45$, if household 1 in the home country is sufficiently impoverished such that $\mu \leq 0.15$, circle markers are visible, indicating an improvement in domestic inequality in both countries. Conversely, if household 1 in the home country is relatively less impoverished with $\mu \geq 0.20$, only domestic inequality in the home country expands, while that in the foreign country contracts in the long run. A similar trend is observed in Figure 4(a) when $\beta = 1.5$. Therefore, unlike the inequality in the global economy discussed in the previous subsection, the long-run domestic distribution in home and foreign countries cannot be fully explained solely by the inverse of ARA.

Figures 3(b) and 4(b) elucidate why domestic inequality expands in regions close to and far from the steady state by associating it with households' EIS, while Figures 3(c) and 4(c) illustrate who is rich at the steady state.

First, in Figure 3(b), we observe that when $\epsilon \geq 0.55$, $x-$ markers appear, whereas for $\epsilon \leq 0.4$, $o-$ markers appear. This indicates that when the initial economy is close to the steady state, which corresponds to the scenario where $\epsilon \geq 0.55$, $\frac{\bar{\omega}_1}{\bar{C}_1} < \frac{\bar{\omega}_2}{\bar{C}_2}$ and $\frac{\bar{\omega}_1^*}{\bar{C}_1^*} < \frac{\bar{\omega}_2^*}{\bar{C}_2^*}$. Essentially, initially affluent households 2 in both countries exhibit higher EIS compared to initially poor households 1. This outcome aligns intuitively with the notion that initially affluent households tend to save more, thereby accruing more wealth over time and widening domestic inequality. Conversely, for $\epsilon \leq 0.4$, the results are reversed. That is, $\frac{\bar{\omega}_1}{\bar{C}_1} > \frac{\bar{\omega}_2}{\bar{C}_2}$ and $\frac{\bar{\omega}_1^*}{\bar{C}_1^*} > \frac{\bar{\omega}_2^*}{\bar{C}_2^*}$, indicating that initially poor households in both countries exhibit larger EIS and consequently save more. However, one might question why initially poor households have large EIS and why domestic inequality expands rather than contracts. Figure 3(c) addresses this concern, with the symbol "x" appearing when $\epsilon \leq 0.4$, indicating $\bar{K}_1 > \bar{K}_2$ and $\bar{K}_1^* > \bar{K}_2^*$. In summary, in the left region of Figures 3(a)–(c), where the initial economy is distant from the steady state ($\epsilon \leq 0.4$), a scenario unfolds wherein the trajectory towards the steady state takes a long time. Consequently, the initial jump of consumption by initially poor households 1 is further curbed, leading to high EIS. This consumption behavior continues on a long journey towards the steady state. As initially poor households save more compared to initially affluent households 2, a reversal in capital stock between households 1 and 2 occurs during the transition to the steady state. Consequently, the disparity in capital stock amplifies, culminating in greater inequality in the steady state compared to the initial state.

[Figure 3 around here.]

Similarly, the case of $\beta = 1.5$ is shown in Figure 4.

[Figure 4 around here.]

5 Conclusion

Using a two-country dynamic model, this paper examines the role of preferences in the distribution of wealth in a global economy and among domestic households. When all the household in the world economy have an identical and homothetic utility function, the initially equal capital in two countries lead to a capital-balanced world in the long run in the sense that $\bar{K} = \bar{K}^* = \frac{\bar{K}_w}{2}$. Instead, when the utility function is non-homothetic, this result may not be held due to the different EIS in both countries. Furthermore, in the numerical examples, when focusing on the concavity and convexity of the inverse of ARA, we find the various patterns of global and domestic inequalities. We show, for example, that the initially poor can catch up with the initially rich when preference is non-homothetic.

Appendices

Appendix A

Notice that the steady-state value of relative price is uniquely determined in $(24b)$. Therefore, totally differentiating (17) given price, we can show that

$$\bar{C}_w = C_w(\bar{K}_w), \tag{A.1}$$

where

$$\frac{\partial \bar{C}_w}{\partial \bar{K}_w} = -\frac{f_c}{\bar{k}_i - \bar{k}_c}(< 0), \quad \lim_{\bar{K}_w \to 0} \bar{C}_w = \frac{2\bar{k}_i f_c}{\bar{k}_i - \bar{k}_c}(> 0), \quad \lim_{\bar{K}_w \to \infty} \bar{C}_w = -\infty(< 0).$$

Next, substituting $(A.1)$ into $\dot{K}_w = 0$ yields:

$$\Psi(\bar{K}_w) \equiv G(\bar{K}_w, \bar{C}_w(\bar{K}_w)) - \delta \bar{K}_w = 0. \tag{A.2}$$

where[12]

$$\Psi'(\bar{K}_w) = \frac{\bar{f}_i}{\bar{k}_i - \bar{k}_c} - \delta = \frac{2\bar{f}_i \bar{k}_c}{\bar{K}_w(\bar{k}_i - \bar{k}_c)}(> 0), \quad \lim_{\bar{K}_w \to 0} \Psi(\bar{K}_w) = -\frac{2\bar{k}_c f_i}{\bar{k}_i - \bar{k}_c}(< 0).$$

Since the level of Ψ monotonically increases with $\bar{K}_w$, we obtain the uniquely determined value of aggregate capital in the steady state. Moreover, when the steady-state value of capital, $\bar{K}_w$, is substituted into $(A.1)$, we confirm that the steady-state value of aggregate consumption is uniquely determined.

As for the stability analysis of steady state, the determinant of our system in $(20a)$ and $(22c)$ is given by:

$$\begin{aligned} \text{Det} &= \frac{\partial \dot{C}_w}{\partial C_w}\frac{\partial \dot{K}_w}{\partial K_w} - \frac{\partial \dot{C}_w}{\partial K_w}\frac{\partial \dot{K}_w}{\partial C_w}, \\ &= \bar{\Delta}\left(r'(\bar{p})\frac{\partial p}{\partial C_w} - \frac{G_{C_w}}{Z_{hf}(2\bar{k}_i - \bar{K}_w)}\right)(G_{K_w} - \delta) - \bar{\Delta}\left(r'(\bar{p})\frac{\partial p}{\partial K_w} - \frac{G_{K_w} - \delta}{\bar{Z}_{hf}(2\bar{k}_i - \bar{K}_w)}\right)G_{C_w}, \end{aligned} \tag{A.3}$$

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[12] As for $\Psi'(\bar{K}_w) > 0$, using $\dot{K}_w = 0$, we find that

$$\frac{f_i}{\bar{k}_i - \bar{k}_c} - \delta = \frac{f_i}{\bar{k}_i - \bar{k}_c} - \frac{G(\bar{K}_w, \bar{C}_w)}{\bar{K}_w} = \frac{2f_i\bar{k}_c}{\bar{K}_w(\bar{k}_i - \bar{k}_c)} > 0.$$

Arranging for (A.3), we find that

$$\begin{aligned}\text{Det} &= \frac{\bar{\Delta}r'(\bar{p})\bar{p}}{\bar{Z}_{hf}(2\bar{k}_i - \bar{K}_w)}\left(\frac{(2\bar{k}_i - \bar{K}_w)(G_{K_w} - \delta)}{\bar{C}_w} - G_{C_w}\right), \\ &= \frac{\bar{\Delta}r'(\bar{p})\bar{p}}{\bar{Z}_{hf}(2\bar{k}_i - \bar{K}_w)}\left[\frac{2\bar{k}_i - \bar{K}_w}{\bar{C}_w}\left(\frac{\bar{f}_i}{\bar{k}_i - \bar{k}_c} + \frac{\partial p}{\partial K_w}\Theta - \delta\right) - \frac{\partial p}{\partial C_w}\Theta\right], \end{aligned} \tag{A.4}$$

where $\bar{\Theta} \equiv \frac{\bar{f}_i}{(\bar{k}_i - \bar{k}_c)^2}\left[k_c'(\bar{p})(\bar{K}_w - 2\bar{k}_i) + \bar{k}_i'(\bar{p})(\bar{K}_w - 2\bar{k}_c)\left(\frac{(\bar{k}_i - \bar{k}_c)\bar{f}_i'}{\bar{f}_i} - 1\right)\right] (< 0)$.

Finally, as $\frac{2\bar{k}_i - \bar{K}_w}{\bar{C}_w}\frac{\partial p}{\partial K_w} = \frac{\partial p}{\partial C_w}$, we obtain

$$\text{Det} = \frac{\bar{\Delta}r'(\bar{p})\bar{p}}{\bar{Z}_{hf}(2\bar{k}_i - \bar{K}_w)}\left[\frac{2\bar{k}_i - \bar{K}_w}{\bar{C}_w}\left(\frac{\bar{f}_i}{\bar{k}_i - \bar{k}_c} - \delta\right)\right] (< 0) \tag{A.5}$$

where we utilize $r'(\bar{p}) < 0$ and $\frac{\bar{f}_i}{\bar{k}_i - \bar{k}_c} > \delta$ in the footnote 12.

Appendix B

Differentiating (22d) with respect to the variable Ω, we observe that

$$\frac{\partial \Delta}{\partial \Omega} = \left(\frac{\bar{C}_w \bar{Z}_{hf}}{\bar{Z}_{hf}\bar{C}_w + \bar{\Omega} + \bar{\Omega}^*}\right)^2 (> 0). \tag{B.1}$$

In what follows, we rewrite (25) as follows:

$$2\lambda = T - (T^2 - 4D)^{\frac{1}{2}} (< 0), \tag{B.2}$$

where T and D correspond to the trace and determinant of our system, respectively, in (25):

$$T = G_{K_w} - \delta + \frac{\partial \dot{C}_w}{\partial C_w}, \quad D = (G_{K_w} - \delta)\frac{\partial \dot{C}_w}{\partial C_w} - G_{C_w}\frac{\partial \dot{C}_w}{\partial K_w} \tag{B.3}$$

Subsequently, we find

$$2\frac{\partial \lambda}{\partial \Delta} = \underbrace{(T^2 - 4D)^{-\frac{1}{2}}}_{(+)}\underbrace{\left((T^2 - 4D)^{\frac{1}{2}} - T\right)}_{(+)}\frac{\partial T}{\partial \Delta} + \underbrace{2(T^2 - 4D)^{-\frac{1}{2}}}_{(+)}\frac{\partial D}{\partial \Delta} (< 0), \tag{B.4}$$

where

$$\frac{\partial T}{\partial \Delta} = r'\frac{\partial p}{\partial C_w} - \frac{G_{C_w}}{Z_{hf}(2k_i - K_w)} (< 0), \quad \frac{\partial D}{\partial \Delta} = \frac{\bar{p}r'}{Z_{hf}\bar{C}_w}\left(\frac{f_i}{\bar{k}_i - \bar{k}_c} - \delta\right) (< 0).$$

Taking account of (B.1) and (B.4), we find that $\frac{\partial \lambda}{\partial \Omega} = \frac{1}{2}\frac{\partial \lambda}{\partial \Delta}\frac{\partial \Delta}{\partial \Omega} < 0$. Since the variable λ has a negative sign, the negative sign of $\frac{\partial \lambda}{\partial \Omega}$ means that an increase in the variable Ω leads to a faster speed of convergence.

Appendix C

To derive (26), we first show that the following equation holds:

$$r'(\bar{p})\bar{K}_w + 2w'(\bar{p}) - \bar{C}_w = 0. \tag{C.1}$$

where using (1) yields:

$$r'(\bar{p}) = f_c'(k_c(\bar{p})) + \bar{p}\bar{f}_c''\bar{k}_c'(\bar{p}), \quad w'(\bar{p}) = f_c(k_c(\bar{p})) - k_c(\bar{p})f_c''(k_c(\bar{p}))k_c'(\bar{p}). \tag{C.2}$$

Substituting (C.2) and $\dot{K}_w = 0$ in (13b) into (C.1), we can rewrite the right-hand side of (C.1) as follows:

$$r'(\bar{p})\bar{K}_w + 2w'(\bar{p}) - \bar{C}_w = (f_c'(k_c(\bar{p})) + \bar{p}f_c''(k_c(\bar{p}))k_c'(\bar{p}))\bar{K}_w - 2\bar{p}k_c'(k_c(\bar{p}))k_c(\bar{p})f_c''(k_c(\bar{p})) - \frac{\rho\bar{K}_w}{\bar{p}},$$

$$= \frac{\delta\bar{K}_w}{\bar{p}} + \bar{p}f_c''(k_c(\bar{p}))k_c'(\bar{p})(\bar{K}_w - 2k_c(\bar{p})), \tag{C.3}$$

where we utilize $f_c'(\bar{k}_c) = \frac{\bar{r}}{\bar{p}}$. Furthermore, substituting (3) into $k'(\bar{p})$ in (C.3), we find

$$\text{(C.3)} = \frac{\delta\bar{K}_w}{\bar{p}} - \frac{1}{\bar{p}}\frac{\bar{K}_w - 2\bar{k}_c}{\bar{k}_i - \bar{k}_c}f_i(k_i(\bar{p})) = 0, \tag{C.4}$$

where we use (20a). Therefore, the equation in (C.1) holds.

In the following, linearly approximating (13b) around the steady state, we obtain the following:

$$\dot{K}_w = \rho(K_w - \bar{K}_w) + \left(r'(\bar{p}) + 2w'(\bar{p}) - \bar{C}_w\right)(p - \bar{p}) - \bar{p}(C_w - \bar{C}_w). \tag{C.5}$$

Substituting (27) into (C.5), we derive

$$\lambda(K_w - \bar{K}_w) = \rho(K_w - \bar{K}_w) - \bar{p}(C_w - \bar{C}_w), \tag{C.6}$$

where we use (C.1). Finally, by rearranging (C.6), we obtain (26).

Appendix D

In this appendix, we derive (46a) and (46b) using the Cobb-Douglas production functions. First, from (1) and (3), we can easily show that

$$\frac{k_c(p)}{k_i(p)} = \frac{k_c'(p)}{k_i'(p)} = \frac{\alpha_c(1-\alpha_i)}{\alpha_i(1-\alpha_c)}, \quad \frac{k_c'(p)}{k_c(p)} = \frac{1}{p(\alpha_i - \alpha_c)}. \tag{D.1}$$

In addition, we demonstrate that the wage elasticity of the price is fixed as follows:

$$\frac{w'(p)p}{w(p)} = 1 + \frac{\alpha_c}{\alpha_i - \alpha_c}(> 0). \tag{D.2}$$

Since our concern is to confirm the steady-state levels of $\bar{D}_{hf}$ and $\bar{Z}_{hf}$, we hereafter assume that the economy is in the steady state. In that case, using (1) and (24b), we find that

$$\frac{\bar{f}_i}{\bar{k}_i} = \frac{\rho + \delta}{\alpha_i}, \quad \frac{\bar{f}_c}{\bar{k}_c} = \frac{\rho + \delta}{\alpha_c \bar{p}}. \tag{D.3}$$

and, from the equation $\dot{K}_w = 0$ in (20a), we find that [13]

$$\frac{\bar{K}_w}{\bar{k}_i} = \frac{2(\rho + \delta)\alpha_c(1 - \alpha_i)}{\alpha_i(1 - \alpha_c)(\rho + \delta) - \delta\alpha_i(\alpha_i - \alpha_c)}(> 0). \tag{D.4}$$

We now derive (46a). Utilizing (19), we find that[14]

$$\bar{Z}_{hf} = \frac{\alpha_i(1 - \alpha_c)}{\alpha_c(1 - \alpha_i)} \frac{\bar{k}'_c}{\bar{k}_c} \left\{ \frac{\bar{k}'_c}{\bar{k}'_i} \frac{\bar{f}'_c \bar{k}_c}{\bar{f}_c} + \left(\frac{\frac{\bar{K}_w}{\bar{k}_i} - \frac{2\bar{k}_c}{\bar{k}_i}}{2 - \frac{\bar{K}_w}{\bar{k}_i}} + \frac{\bar{k}'_c}{\bar{k}'_i} \right) \frac{1}{\frac{\bar{k}_i}{\bar{k}_c} - 1} \right\}. \tag{D.5}$$

where we use (D.1). Furthermore, substituting (D.1), we can derive (46a).

Next, we derive (46b). Substituting (D.2) into (39b), we can obtain:

$$D_{hf} = \frac{2\bar{w}}{\bar{p}\bar{K}_w(\rho - \bar{\lambda})} \left\{ -\frac{\alpha_c}{\alpha_i - \alpha_c} \frac{1}{M} \left(\frac{\rho - \bar{\lambda}}{\bar{C}_w} + \frac{\bar{p}}{2\bar{k}_i - \bar{K}_w} \right) + \frac{\rho - \bar{\lambda}}{\bar{C}_w} \right\}. \tag{D.6}$$

Furthermore, by substituting (17) into (D.6) and rearranging it, we find that[15]

$$D_{hf} = \frac{2\bar{w}}{\bar{K}_w(\rho - \bar{\lambda})(2\bar{k}_i - \bar{K}_w)} \left\{ \frac{(\rho - \bar{\lambda})(\alpha_i - \alpha_c)}{(1 - \alpha_i)(\rho + \delta)} \left(1 - \frac{\alpha_c}{M(\alpha_i - \alpha_c)} \right) - \frac{\alpha_c}{M(\alpha_i - \alpha_c)} \right\}. \tag{D.7}$$

Finally, using (1), (24b), and (D.3), we obtain (46b).

[13]Specifically, $\dot{K}_w = 0$ equation can be rewritten as

$$\frac{\bar{K}_w}{\bar{k}_i} \left(\frac{\bar{f}_i}{\bar{k}_i - \bar{k}_c} - \delta \right) = \frac{2\bar{k}_c \bar{f}_i}{\bar{k}_i(\bar{k}_i - \bar{k}_c)}, \Rightarrow \frac{\bar{K}_w}{\bar{k}_i} = \frac{2}{\frac{\bar{k}_i}{\bar{k}_c} - \frac{\delta \bar{k}_i}{\bar{f}_i}\left(\frac{\bar{k}_i}{\bar{k}_c} - 1\right)}.$$

When we use (D.1) and (D.3), we can lead to (D.4).

[14]Using (D.1) and (D.4), we find that $\frac{\frac{\bar{K}_w}{\bar{k}_i} - \frac{2\bar{k}_c}{\bar{k}_i}}{2 - \frac{\bar{K}_w}{\bar{k}_i}} = \frac{\alpha_c(1-\alpha_i)\delta}{(1-\alpha_c)\{\rho + (1-\alpha_i)\delta\}}$.

[15]As for $\frac{2\bar{w}}{\bar{K}_w(\rho - \bar{\lambda})(2\bar{k}_i - \bar{K}_w)}$ in (D.6), we can rewrite it as follows:

$$\frac{2\bar{w}}{\bar{K}_w(\rho - \bar{\lambda})(2\bar{k}_i - \bar{K}_w)} = \frac{2(\rho + \delta)\left(\frac{1}{\alpha_i} - 1\right)}{(\rho - \lambda)\left(2 - \frac{\bar{K}_w}{\bar{k}_i}\right)}.$$

References

[1] Becker, R.A., 1980. On the Long-Run Steady State in a Simple Dynamic Model of Equilibrium with Heterogeneous Households. *Q. J. Econ.* 95, 375—382.
`https://doi.org/10.2307/1885506`

[2] Bertola, G., Foellmi, R., Zweimuelle, J., 2006. *Income Distribution in Macroeconomic Models*, Princeton University Press.

[3] Bond, E.W., Iwasa, K., and Nishimura, K., 2011. Dynamic Two Country Heckscher–Ohlin Model with Non-homothetic Preferences. *Econ. Theory* 48, 171–204.
`https://doi.org/10.1007/s00199-010-0572-8`

[4] Bond, E.W., Iwasa, K., Nishimura, K., 2013. Poverty traps and inferior goods in a dynamic Heckscher-Ohlin model. Macroecon. Dyn. 17, 1227-1251.
`https://doi.org/10.1017/S136510051200003X`

[5] Bourguignon, F., 1980. Pareto Superiority of Unegalitarian Equilibria in Stiglitz's Model of Wealth Distribution with Convex Saving Function. *Econometrica* 49, 1469-1475.
`https://doi.org/10.2307/1911412`

[6] Caliendo, L., 2011. On the Dynamics of the Heckscher-Ohlin Theory. unpublished manuscript.

[7] Caselli, F., Ventura, J., 2000. A Representative Consumer Theory of Distribution. *Am. Econ. Rev.* 90, 909-926.
`DOI:10.1257/aer.90.4.909`

[8] Chatterjee, S., 1994. Transitional Dynamics and the Distribution of Wealth in a Neoclassical Growth Model. *J. Public Econ.* 54, 97-119.
`https://doi.org/10.1016/0047-2727(94)90072-8`

[9] Chen, Z., 1992. Long-run Equilibria in a Dynamic Heckscher–Ohlin Model. *Can. J. Econ.* 25, 923–943.
`https://doi.org/10.2307/135772`

[10] Chen, Y.-C., Turnovsky, S., 2010. Growth and Inequality in a Small Open Economy. *J. Macroecon.*, 32,497-514.
https://doi.org/10.1016/j.jmacro.2009.11.005

[11] Cremers, E.T., 1997. Capital Markets and Dimension in Neoclassical Models of Growth and Trade. *J. Int. Econ.* 43, 155-172.
https://doi.org/10.1016/S0022-1996(96)01462-6

[12] Foellmi, R., 2011. Inequality and Aggregate Savings in the Neoclassical Growth Model. Working Paper, Institute for Empirical Research in Economics, University of Zurich.

[13] García-Peñalosa, C., Turnovsky, S., 2006. Growth and Income Inequality: a Canonical Model. *Econ. Theory* 28, 25-49.
https://www.jstor.org/stable/25056055

[14] García-Peñalosa, C., Turnovsky, S., 2009. The Dynamics of Wealth Inequality in a Simple Ramsey Model: a Note on the Tole of Production Flexibility. *Macroecon. Dyn.* 13, 250-262.
doi:10.1017/S1365100508070508

[15] Hu, Y., Kemp, M., Shimomura, K., 2009. A Two-Country Dynamic Heckscher–Ohlin Model with Physical and Human Capital Accumulation. *Econ. Theory* 41, 67–84.
https://www.jstor.org/stable/40283050

[16] Hunter, L., 1991. The Contribution of Nonhomothetic Preferences to Trade. *J. Int. Econ.* 30, 345–358.
https://doi.org/10.1016/0022-1996(91)90027-4

[17] Mino, K., Nakamoto, Y., 2015. Heterogeneous Conformism and Wealth Distribution. *Econ. Theory* 62, 689–717.
https://doi.org/10.1007/s00199-015-0923-6

[18] Nakamoto, Y. Mino, K., 2012. Consumption Externalities and Equilibrium Dynamics with Heterogeneous Agents. *Math. Soc. Sci.* 64, 225-233.
https://doi.org/10.1016/j.mathsocsci.2012.04.001

[19] Nishimura, K., Shimomura, K., 2002. Trade and Indeterminacy in a Dynamic General Equilibrium Model. *J. Econ. Theory* 105, 249-259.
https://doi.org/10.1006/jeth.2001.2892

[20] Oniki, H., Uzawa, H., 1965. Patterns of Trade and Investment in a Dynamic Model of International Trade. *Rev. Econ. Stud.*, 32, 15-38.
https://doi.org/10.2307/2296328

[21] Ramsey, F., (1928), A Mathematical Theory of Saving, *Economic Journal* 38, 543–559.
https://doi.org/10.2307/2224098

[22] Schmidt-Hebbel, K., Servén, S., 2000. Does Income Inequality Raise Aggregate Saving? *J. Dev. Econ.* 61. 417-446.
https://doi.org/10.1016/S0304-3878(00)00063-8

[23] Stiglitz, J.E., 1970. Factor Price Equalization in a Dynamic Economy. *J. Political Econ.* 78, 456–488.
https://www.jstor.org/stable/1829645

[24] Stiglitz, J.E., 1969. Distribution of Income and Wealth Among Individuals. *Econometrica* 37, 382-397.
https://doi.org/10.2307/1912788

[25] Sorger, G., 2002. On the Long-Run Distribution of Capital in the Ramsey Model. *J. Econ. Theory* 105, 226-243.
https://doi.org/10.1006/jeth.2001.2841

Figure 1: The inequality in the world economy and the aggregate elasticity of intertemporal substitution where $\beta = 0.5$

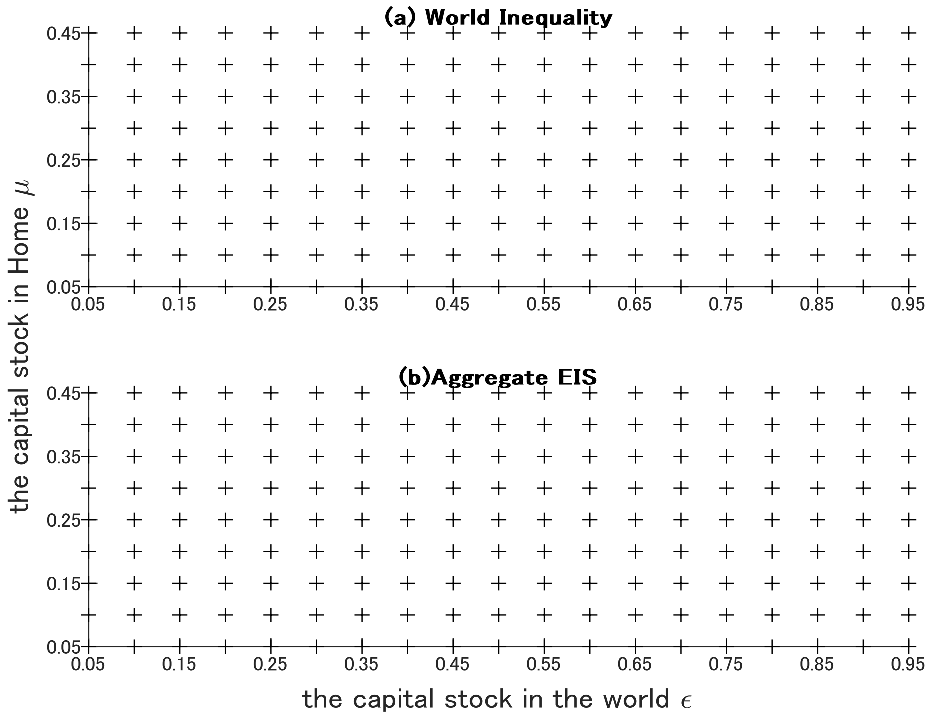

Table: Explanation of Markers in Figure 1

Figure	"+" (plus)	"▲" (triangle)
1(a)	$\bar{K}^* > \bar{K}$	$\bar{K}^* < \bar{K}$
1(b)	$\frac{\bar{\Omega}}{\bar{C}} < \frac{\bar{\Omega}+\bar{\Omega}^*}{\bar{C}_w}$	$\frac{\bar{\Omega}}{\bar{C}} > \frac{\bar{\Omega}+\bar{\Omega}^*}{\bar{C}_w}$

Explanation of Markers in Figure 1:

Figure 1(a) shows that $\bar{K}^* > \bar{K}$ in the plus marker and $\bar{K}^* < \bar{K}$ in the triangle marker.

Figure 1(b) shows that $\frac{\bar{\Omega}}{\bar{C}} < \frac{\bar{\Omega}+\bar{\Omega}^*}{\bar{C}_w}$ in the plus marker and $\frac{\bar{\Omega}}{\bar{C}} > \frac{\bar{\Omega}+\bar{\Omega}^*}{\bar{C}_w}$ in the triangle marker.

Figure 2: The inequality in the world economy and the aggregate elasticity of intertemporal substitution where $\beta = 1.5$

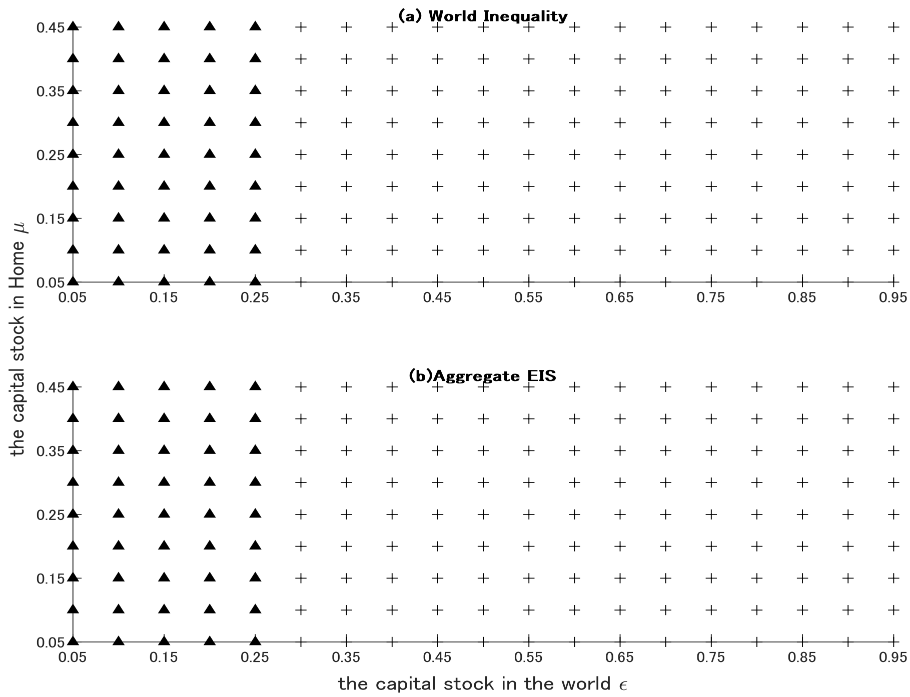

Table: Explanation of Markers in Figure 2

Figure	"+" (plus)	"▲" (triangle)
2(a)	$\bar{K}^* > \bar{K}$	$\bar{K}^* < \bar{K}$
2(b)	$\frac{\bar{\Omega}}{\bar{C}} < \frac{\bar{\Omega}+\bar{\Omega}^*}{\bar{C}_w}$	$\frac{\bar{\Omega}}{\bar{C}} > \frac{\bar{\Omega}+\bar{\Omega}^*}{\bar{C}_w}$

Explanation of Markers in Figure 2

Figure 2(a) shows that $\bar{K}^* > \bar{K}$ in the plus marker and $\bar{K}^* < \bar{K}$ in the triangle marker.
Figure 2(b) shows that $\frac{\bar{\Omega}}{\bar{C}} < \frac{\bar{\Omega}+\bar{\Omega}^*}{\bar{C}_w}$ in the plus marker and $\frac{\bar{\Omega}}{\bar{C}} > \frac{\bar{\Omega}+\bar{\Omega}^*}{\bar{C}_w}$ in the triangle marker.

Figure 3: The inequality in the domestic economy and the individual elasticity of intertemporal substitution where $\beta = 0.5$

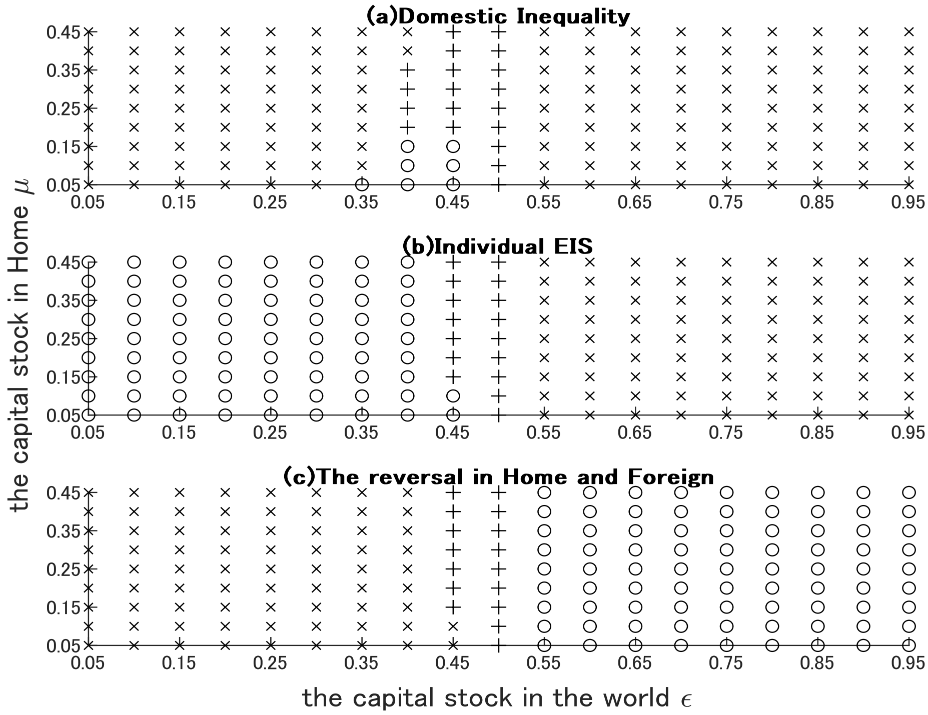

Table: Explanation of Markers in Figure 3

Figure		"x"	"+" (plus)	"▲" (triangle)	"o" (round)
3(a)	Home	$\lvert\bar{K}_1 - \bar{K}_2\rvert > \lvert K_1^0 - K_2^0\rvert$	$\lvert\bar{K}_1 - \bar{K}_2\rvert > \lvert K_1^0 - K_2^0\rvert$	$\lvert\bar{K}_1 - \bar{K}_2\rvert < \lvert K_1^0 - K_2^0\rvert$	$\lvert\bar{K}_1 - \bar{K}_2\rvert < \lvert K_1^0 - K_2^0\rvert$
	Foreign	$\lvert\bar{K}_1^* - \bar{K}_2^*\rvert > \lvert K_1^{*,0} - K_2^{*,0}\rvert$	$\lvert\bar{K}_1^* - \bar{K}_2^*\rvert < \lvert K_1^{*,0} - K_2^{*,0}\rvert$	$\lvert\bar{K}_1^* - \bar{K}_2^*\rvert > \lvert K_1^{*,0} - K_2^{*,0}\rvert$	$\lvert\bar{K}_1^* - \bar{K}_2^*\rvert < \lvert K_1^{*,0} - K_2^{*,0}\rvert$
3(b)	Home	$\frac{\bar{\omega}_2}{\bar{C}_2} > \frac{\bar{\omega}_1}{\bar{C}_1}$	$\frac{\bar{\omega}_2}{\bar{C}_2} > \frac{\bar{\omega}_1}{\bar{C}_1}$	$\frac{\bar{\omega}_2}{\bar{C}_2} < \frac{\bar{\omega}_1}{\bar{C}_1}$	$\frac{\bar{\omega}_2}{\bar{C}_2} < \frac{\bar{\omega}_1}{\bar{C}_1}$
	Foreign	$\frac{\bar{\omega}_2^*}{\bar{C}_2} > \frac{\bar{\omega}_1^*}{\bar{C}_1}$	$\frac{\bar{\omega}_2^*}{\bar{C}_2} < \frac{\bar{\omega}_1^*}{\bar{C}_1}$	$\frac{\bar{\omega}_2^*}{\bar{C}_2} > \frac{\bar{\omega}_1^*}{\bar{C}_1}$	$\frac{\bar{\omega}_2^*}{\bar{C}_2} < \frac{\bar{\omega}_1^*}{\bar{C}_1}$
3(c)	Home	$\bar{K}_1 > \bar{K}_2$	$\bar{K}_1 < \bar{K}_2$	$\bar{K}_1 > \bar{K}_2$	$\bar{K}_1 < \bar{K}_2$
	Foreign	$\bar{K}_1^* > \bar{K}_2^*$	$\bar{K}_1^* > \bar{K}_2^*$	$\bar{K}_1^* < \bar{K}_2^*$	$\bar{K}_1^* < \bar{K}_2^*$

Explanation of Markers in Figure 3

Figure 3(a) shows that $|\bar{K}_1 - \bar{K}_2| > |K_1^0 - K_2^0|$ and $|\bar{K}_1^* - \bar{K}_2^*| > |K_1^{*,0} - K_2^{*,0}|$ in the $x-$marker, that $|\bar{K}_1 - \bar{K}_2| > |K_1^0 - K_2^0|$ and $|\bar{K}_1^* - \bar{K}_2^*| < |K_1^{*,0} - K_2^{*,0}|$ in the plus marker, that $|\bar{K}_1 - \bar{K}_2| < |K_1^0 - K_2^0|$ and $|\bar{K}_1^* - \bar{K}_2^*| > |K_1^{*,0} - K_2^{*,0}|$ in the triangle marker, and that $|\bar{K}_1 - \bar{K}_2| < |K_1^0 - K_2^0|$ and $|\bar{K}_1^* - \bar{K}_2^*| < |K_1^{*,0} - K_2^{*,0}|$ in the round marker.

Figure 3(b) shows that $\frac{\bar{\omega}_2}{\bar{C}_2} > \frac{\bar{\omega}_1}{\bar{C}_1}$ and $\frac{\bar{\omega}_2^*}{\bar{C}_2} > \frac{\bar{\omega}_1^*}{\bar{C}_1}$ in the $x-$marker, that $\frac{\bar{\omega}_2}{\bar{C}_2} > \frac{\bar{\omega}_1}{\bar{C}_1}$ and $\frac{\bar{\omega}_2^*}{\bar{C}_2} < \frac{\bar{\omega}_1^*}{\bar{C}_1}$ in the plus marker, that $\frac{\bar{\omega}_2}{\bar{C}_2} < \frac{\bar{\omega}_1}{\bar{C}_1}$ and $\frac{\bar{\omega}_2^*}{\bar{C}_2} > \frac{\bar{\omega}_1^*}{\bar{C}_1}$ in the triangle marker, and that $\frac{\bar{\omega}_2}{\bar{C}_2} < \frac{\bar{\omega}_1}{\bar{C}_1}$ and $\frac{\bar{\omega}_2^*}{\bar{C}_2} < \frac{\bar{\omega}_1^*}{\bar{C}_1}$ in the round marker.

Figure 3(c) shows that $\bar{K}_1 > \bar{K}_2$ and $\bar{K}_1^* > \bar{K}_2^*$ in the $x-$marker, that $\bar{K}_1 < \bar{K}_2$ and $\bar{K}_1^* > \bar{K}_2^*$ in the plus marker, that $\bar{K}_1 > \bar{K}_2$ and $\bar{K}_1^* < \bar{K}_2^*$ in the triangle marker, and that $\bar{K}_1 < \bar{K}_2$ and $\bar{K}_1^* < \bar{K}_2^*$ in the round marker.

Figure 4: The inequality in the domestic economy and the individual elasticity of intertemporal substitution where $\beta = 1.5$

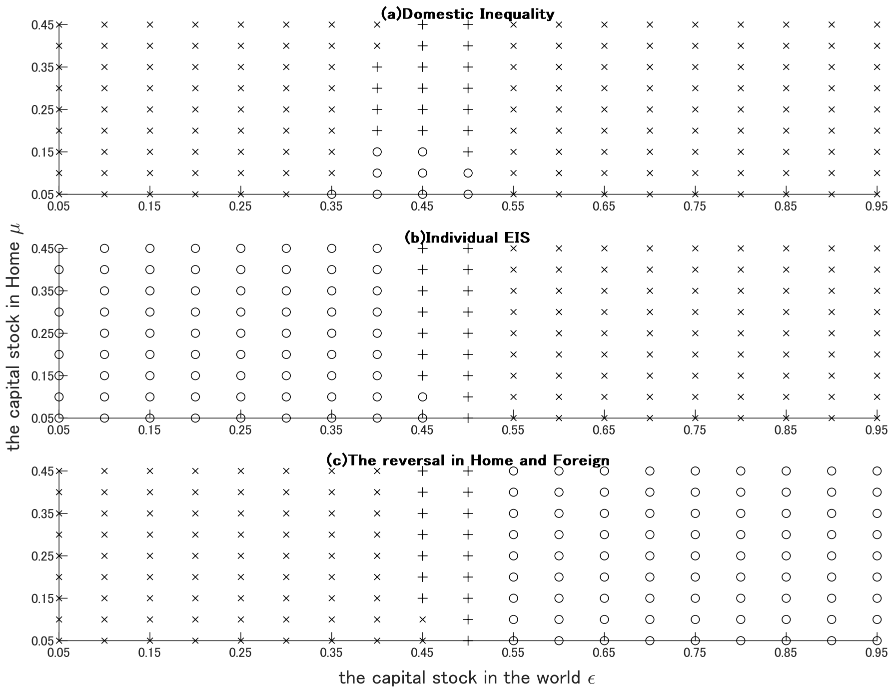

Table: Explanation of Markers in Figure 4

Figure		"x"	"+" (plus)	"▲" (triangle)	"o" (round)
4(a)	Home	$\lvert\bar{K}_1-\bar{K}_2\rvert > \lvert K_1^0-K_2^0\rvert$	$\lvert\bar{K}_1-\bar{K}_2\rvert > \lvert K_1^0-K_2^0\rvert$	$\lvert\bar{K}_1-\bar{K}_2\rvert < \lvert K_1^0-K_2^0\rvert$	$\lvert\bar{K}_1-\bar{K}_2\rvert < \lvert K_1^0-K_2^0\rvert$
	Foreign	$\lvert\bar{K}_1^*-\bar{K}_2^*\rvert > \lvert K_1^{*,0}-K_2^{*,0}\rvert$	$\lvert\bar{K}_1^*-\bar{K}_2^*\rvert < \lvert K_1^{*,0}-K_2^{*,0}\rvert$	$\lvert\bar{K}_1^*-\bar{K}_2^*\rvert > \lvert K_1^{*,0}-K_2^{*,0}\rvert$	$\lvert\bar{K}_1^*-\bar{K}_2^*\rvert < \lvert K_1^{*,0}-K_2^{*,0}\rvert$
4(b)	Home	$\frac{\bar{\omega}_2}{\bar{C}_2} > \frac{\bar{\omega}_1}{\bar{C}_1}$	$\frac{\bar{\omega}_2}{\bar{C}_2} > \frac{\bar{\omega}_1}{\bar{C}_1}$	$\frac{\bar{\omega}_2}{\bar{C}_2} < \frac{\bar{\omega}_1}{\bar{C}_1}$	$\frac{\bar{\omega}_2}{\bar{C}_2} < \frac{\bar{\omega}_1}{\bar{C}_1}$
	Foreign	$\frac{\bar{\omega}_2^*}{\bar{C}_2^*} > \frac{\bar{\omega}_1^*}{\bar{C}_1^*}$	$\frac{\bar{\omega}_2^*}{\bar{C}_2^*} < \frac{\bar{\omega}_1^*}{\bar{C}_1^*}$	$\frac{\bar{\omega}_2^*}{\bar{C}_2^*} > \frac{\bar{\omega}_1^*}{\bar{C}_1^*}$	$\frac{\bar{\omega}_2^*}{\bar{C}_2^*} < \frac{\bar{\omega}_1^*}{\bar{C}_1^*}$
4(c)	Home	$\bar{K}_1 > \bar{K}_2$	$\bar{K}_1 < \bar{K}_2$	$\bar{K}_1 > \bar{K}_2$	$\bar{K}_1 < \bar{K}_2$
	Foreign	$\bar{K}_1^* > \bar{K}_2^*$	$\bar{K}_1^* > \bar{K}_2^*$	$\bar{K}_1^* < \bar{K}_2^*$	$\bar{K}_1^* < \bar{K}_2^*$

Explanation of Markers in Figure 4

Figure 4(a) shows that $|\bar{K}_1-\bar{K}_2| > |K_1^0-K_2^0|$ and $|\bar{K}_1^*-\bar{K}_2^*| > |K_1^{*,0}-K_2^{*,0}|$ in the $x-$marker, that $|\bar{K}_1-\bar{K}_2| > |K_1^0-K_2^0|$ and $|\bar{K}_1^*-\bar{K}_2^*| < |K_1^{*,0}-K_2^{*,0}|$ in the plus marker, that $|\bar{K}_1-\bar{K}_2| < |K_1^0-K_2^0|$ and $|\bar{K}_1^*-\bar{K}_2^*| > |K_1^{*,0}-K_2^{*,0}|$ in the triangle marker, and that $|\bar{K}_1-\bar{K}_2| < |K_1^0-K_2^0|$ and $|\bar{K}_1^*-\bar{K}_2^*| < |K_1^{*,0}-K_2^{*,0}|$ in the round marker.

Figure 4(b) shows that $\frac{\bar{\omega}_2}{\bar{C}_2} > \frac{\bar{\omega}_1}{\bar{C}_1}$ and $\frac{\bar{\omega}_2^*}{\bar{C}_2^*} > \frac{\bar{\omega}_1^*}{\bar{C}_1^*}$ in the $x-$marker, that $\frac{\bar{\omega}_2}{\bar{C}_2} > \frac{\bar{\omega}_1}{\bar{C}_1}$ and $\frac{\bar{\omega}_2^*}{\bar{C}_2^*} < \frac{\bar{\omega}_1^*}{\bar{C}_1^*}$ in the plus marker, that $\frac{\bar{\omega}_2}{\bar{C}_2} < \frac{\bar{\omega}_1}{\bar{C}_1}$ and $\frac{\bar{\omega}_2^*}{\bar{C}_2^*} > \frac{\bar{\omega}_1^*}{\bar{C}_1^*}$ in the triangle marker, and that $\frac{\bar{\omega}_2}{\bar{C}_2} < \frac{\bar{\omega}_1}{\bar{C}_1}$ and $\frac{\bar{\omega}_2^*}{\bar{C}_2^*} < \frac{\bar{\omega}_1^*}{\bar{C}_1^*}$ in the round marker.

Figure 4(c) shows that $\bar{K}_1 > \bar{K}_2$ and $\bar{K}_1^* > \bar{K}_2^*$ in the $x-$marker, that $\bar{K}_1 < \bar{K}_2$ and $\bar{K}_1^* > \bar{K}_2^*$ in the plus marker, that $\bar{K}_1 > \bar{K}_2$ and $\bar{K}_1^* < \bar{K}_2^*$ in the triangle marker, and that $\bar{K}_1 < \bar{K}_2$ and $\bar{K}_1^* < \bar{K}_2^*$ in the round marker.