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"The Impacts of Family Policies on
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The Impacts of Family Policies on Labor Supply, Fertility, and Social Welfare*

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Abstract

We quantitatively examine the impacts of family policies on labor supply, fertility, and social welfare in a heterogeneous agent overlapping-generations (OLG) economy. We extend a standard incomplete-market OLG model with married and single households by incorporating parental decisions on the number of children, child care, education spending, and time allocation between market work, parental care, and leisure. We use this extended model to examine the possible impacts of four major family policies: child subsidies, child care subsidies, education subsidies, and income tax deductions for dependent children. The results of all four policies suggest a tradeoff between fertility rates and female labor supply, although the individual effects of each policy on households and the macroeconomy differ significantly. Child care subsidies raise female labor supply but lower fertility rates. By contrast, child subsidies, education subsidies, and income tax deductions reduce female labor supply but raise fertility rates. Child care subsidies improve overall welfare the most among the four policies. This is because increased labor supply and a decrease in the number of children raise the consumption level in the long run, while lowering policy costs.

Keywords: Family policies, child care, fertility, household decisions

JEL classification: D10, E62, H31, J13

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1 Introduction

Governments in many countries provide significant financial support for households with children. Average public spending on family benefits among OECD countries was 2.3% of GDP in 2017.[1] This type of support is provided through various means such as child benefits, child care subsidies, and education subsidies. The purpose of these policies is to promote childhood development, raise the fertility rate, increase parental labor supply, and balance work and family lives. Although there is a large body of the literature on the implications of such policies, few studies have examined the overall macroeconomic or welfare effects of each family policy and discussed the tradeoffs between the policy goals in a single framework.

In this study, we quantitatively analyze the possible effects of major family policies on the number of children, household labor supply, child care, and education spending of individual households, as well as their overall effects on the macroeconomy and social welfare, by extending a heterogeneous agent overlapping-generations (OLG) model with married and single households. This model features parental time allocation between market work, parental care, and leisure; the heterogeneity of labor productivity and education type within a married couple; and the quantity–quality tradeoff in fertility decisions.

Households in the model economy are heterogeneous in terms of marital status, the number of children, age, education type, labor productivity, and asset holdings. Married households jointly determine the number of children at the beginning of adulthood. Both married and single households determine their consumption, labor supply, child care hours, and education spending on children during each period. Labor productivity changes depending on age, education type, hours worked, and idiosyncratic shocks. The fertility decisions in the model feature the quantity–quality tradeoff and opportunity cost of parental care, which allows the model to reproduce plausible fertility patterns.

We then calibrate the extended OLG model to the U.S. economy and choose some parameters to match several targets, including the number of children, maternal labor supply, and child care hours. The resulting model successfully reproduces labor supply patterns according to marital status, gender, education level, and the presence of dependent children. The model also reproduces labor income over lifecycles and fertility patterns according to maternal education type. Finally, we use the extended model to study the individual effects of expanding each

[1]Public spending on family benefits refers to public support exclusively for families and children such as child payments and allowances, parental leave benefits, and child care support. The data source is the OECD Family Database.

of the four major family policies: child subsidies, child care subsidies, education subsidies, and income tax deductions for dependent children. Each change is assumed to be a financed consumption tax.

The quantitative results suggest a tradeoff between female labor supply and fertility rates, which are common goals of family policies. The results also imply that each family policy has different impacts on individual households and the macroeconomy. The effects on labor supply are greater among low-educated mothers under all four family policies. However, the effects on welfare and fertility rates differ significantly depending on the family policy type.

Child subsidies and income tax deductions increase fertility rates and reduce female labor supply and education spending on children. The effects of these two policies on the overall fertility rate are similar, but their effects on individual fertility rates are not. Child subsidies tend to raise the fertility rates of females with low levels of education, whereas income tax deductions tend to raise those of females with high levels of education. Income tax deductions lower the marginal cost of having children for highly educated couples more strongly as a result of progressive income taxation.

Education subsidies raise fertility rates, reduce female labor supply, and increase education spending on children. The effect of this policy on fertility is smaller than that of the above two policies because education subsidies indirectly lower child-related costs by reducing private education costs, whereas the above two policies directly lower costs. These three policies, which lead to higher fertility rates, worsen the welfare of married couples with highly educated wives. The increased number of children also depresses consumption levels through a decrease in female labor supply, which is prominent among households with highly educated females. Income tax deductions worsen overall welfare the most as a result of their regressive effects and higher consumption tax rates.

Child care subsidies lower fertility rates and increase female labor supply and education spending. Fertility rates fall, especially among couples with low-educated wives because these households use paid child care less than others in the benchmark economy. Child care subsidies improve the overall welfare level the most among the four policies because the increase in female labor supply raises the consumption level and lowers policy costs.

This paper relates to several strands of the literature. The first strand is the empirical literature that examines the effects of child-related policies on maternal labor supply and fertility. Many studies have found a positive effect of child care subsidies on maternal labor supply and the range of their estimated child care cost elasticity of maternal labor supply is relatively large. In Nollenberger and Rodríguez-Planas (2015), the estimated child care cost elasticity of employment is –0.61. This study examined a reform in Spain that led to a sizable expansion of full-time public child care. In Carta and Rizzica (2018), the elasticities of employment are –0.15 with respect to the price of private nurseries and –0.18 with respect to the price of public nurseries based on a reform in Italy that extended access to highly subsidized child care. Other studies that find a positive effect of child care subsidies on maternal labor supply include Bauernschuster and Schlotter (2015), Andresen and Havnes (2019), and Müller and Wrohlich (2020). Consistent with the empirical literature, our study indicates that child care subsidies increase the labor supply of married and single women with small children. Regarding other policies, Sánchez-Mangas and Sánchez-Marcos (2008) and Azmat and González (2010) found positive effects of cash benefits or tax breaks with working requirements on maternal labor supply, whereas González (2013) and Ang (2015) found a negative effect of universal cash benefits.

Many studies have identified positive effects of cash benefits on fertility with varying magnitudes. Milligan (2005) examined the Allowance for Newborn Children in Quebec that paid up to C$8,000 (approximately US$5,800) and found that the fertility of those eligible for the new program increased by 12% and the fertility of those eligible for the maximum benefit increased by 25%. González (2013) examined a universal child benefit of €2,500 (approximately US$3,900) introduced in Spain in 2007 and found that the annual number of births increased by approximately 6%. Our model also predicts a positive effect of child subsidies on fertility and that an increase in the universal child subsidy of $8,400 increases the total fertility rate by approximately 7%. Other studies that have identified a positive effect of cash benefits include Cohen, Dehejia and Romanov (2013), Malak, Rahman and Yip (2019), and Lyssiotou (2021). However, the impacts of other policies on fertility are mixed.[2]

The second strand is the theoretical literature that analyzes fertility decisions dating back to Becker (1960) and Becker and Lewis (1973). Recent examples include Hazan and Zoabi (2015), Vogl (2016), and Bar, Hazan, Leukhina, Weiss and Zoabi (2018). Following the literature, in our model, the parental care of children requires time and parents derive utility from the number of children and their quality, where quality refers to the amount of education given to children.

[2]Some studies such as Baughman and Dickert-Conlin (2003), Rindfuss, Guilkey, Morgan and Kravdal (2010), Mörk, Sjögren and Svaleryd (2013), and Bauernschuster, Hener and Rainer (2016) have found a positive effect of child care subsidies or tax breaks on fertility, whereas other studies such as Hank and Kreyenfeld (2003) and Baughman and Dickert-Conlin (2009) have found no statistically significant effect or negative effects on fertility.

In this setting, our model features a quantity–quality tradeoff and opportunity cost of maternal time, which can be partially lowered by the marketization of child care, as described in Hazan and Zoabi (2015) and Bar et al. (2018). These features allow for reproducing plausible fertility patterns according to female educational attainment. Unlike previous studies, we abstract the effects of education on initial labor productivity or education levels and the distributions in future generations. Education in our model is treated as parental consumption to make a model with rich heterogeneity tractable.

This study is most closely related to the recent macroeconomics literature that quantitatively examines the impacts of family policies. The macro public finance literature includes Fehr and Ujhelyiova (2013), Bick (2016), Guner, Kaygusuz and Ventura (2020), and Zhou (2021).[3]

Bick (2016) evaluated two policy reforms that expanded subsidized child care in Germany in a lifecycle model with discrete choices regarding fertility, labor supply, and child care arrangements. The results indicated that both reforms failed to increase fertility rates because of the taxes imposed on financial reforms. Our results also imply a negative effect of child care subsidies on fertility, but the mechanism differs. The subsidies raise the paid child care cost of low-educated mothers, who use little paid child care in the benchmark economy. Furthermore, subsidies increase education spending through income effects, which increases the marginal cost of having children.

Zhou (2021) examined the impacts of cash benefits for childbirth in a heterogeneous agent OLG framework with a quantity–quality tradeoff and rich demographic structure. In their model, agents supply labor inelastically, except when they have small children. The results indicated that benefits raise average welfare because the old-age dependency ratio decreases. In our study, even after considering the benefits of demographic change, cash benefits worsen the welfare of households with highly educated women. Decreased labor supply due to cash benefits prevents labor productivity growth and subsequently reduces labor supply, thereby reducing the consumption level.

Fehr and Ujhelyiova (2013) examined the impacts of family policies on female labor supply and fertility decisions in an OLG economy consisting of married couples. In their model, both

[3]Other recent macroeconomics literature quantitatively examining the impact of family policies includes Adda, Dustmann and Stevens (2017), Garcia-Moran and Kuehn (2017), and Bastani, Blomquist and Micheletto (2020). Adda et al. (2017) estimated a lifecycle model with labor supply, fertility, and occupational choices, and analyzed the impacts of pronatalist transfer. Garcia-Moran and Kuehn (2017) developed a model with residence choice, fertility, and employment decisions, and analyzed the effects of child care subsidies. Bastani et al. (2020) theoretically and quantitatively evaluated child care subsidies in a Mirrleesian optimal tax framework with child care quality.

adult members of a household have the same skill level and parents derive utility from the number of children, and their quality is abstracted.

Guner et al. (2020) studied the impacts of family policies on household labor supply and welfare in an environment with rich heterogeneity, keeping fertility decisions and child care demand exogenously determined. Households in their model are heterogeneous with respect to marital status, the education levels of household members, the number of children, access to informal care, and child care costs. Our quantitative exercise implies that endogenizing fertility decisions amplifies the effects on female labor supply, which qualitatively changes the welfare implications for some subgroups.

Compared with previous studies, our study examines the effects of family policies more comprehensively. Specifically, we examine the policy effects on parental decisions regarding fertility and time allocation and social welfare in a dynamic general equilibrium model. Furthermore, by introducing heterogeneity into several dimensions, the policy effects by household type are examined closely. Our model is mainly characterized by parental time allocation, a quantity–quality tradeoff in fertility decisions, and the heterogeneity of labor productivity and education type within a married household. These features impact the policy implications described in the literature and allow us to provide a rich analysis of the impacts of family policies.

The remainder of this paper is organized as follows. Section 2 presents the extended OLG model. Section 3 provides some intuition regarding the policy effects with a one-period model. Section 4 presents the calibration of the proposed model and describes the benchmark economy. Section 5 quantitatively demonstrates the possible effects of the four family policy changes. Section 6 concludes.

2 Model

The economy consists of heterogeneous OLG households, identical firms, and the government. Firms produce consumption goods with capital and labor, and have constant returns to scale technology. The government imposes taxes on labor income, capital income, and consumption, and spends the resulting funds on social security payments, their own consumption, and family policies.

2.1 Households

Households are heterogeneous with respect to age, $j = 1, \ldots, J$, marital status, the number of children, $n \in \mathbf{N}$, assets, $a \in \mathbf{A}$, the education types of the husband and wife, $(\theta_1, \theta_2) \in \Theta \times \Theta$, and the labor productivities of the husband and wife, $(z_1, z_2) \in \mathbf{Z} \times \mathbf{Z}$, where $\mathbf{N} \subset R_+$, $\mathbf{A} \subset R_+$, $\Theta = \{0, Low, High\}$, and $\mathbf{Z} \subset R_+$ are finite sets. There are two education types called low ($\theta_i = Low$) and high ($\theta_i = High$). $\theta_i = 0$ and $z_i = 0$ denote the absence of a spouse. Agents are ex-ante heterogeneous regarding education type, labor productivity, and marital status, and enter the economy with no assets. Education type is invariant throughout life. They face three shocks: a mortality shock, a divorce shock, and a shock to labor productivity. They work from $j = 1$ to $j = J_R - 1$, where J_R denotes the retirement age, and receive pension benefits on and after J_R, the amount of which depends on labor productivity at the time of retirement.

At the beginning of their life, married couples choose the number of children that will be born in their lifetime, n, which is invariant throughout their life, except in the case of the divorce shock. Then, all agents determine the consumption per adult c, assets in the next period a', and labor supply h. If they have children, they also determine the time of using paid child care services per child d and educational demand per child e, given factor prices and government policies: $\Omega = \left\{\tau^p, \tau^c, \tau^I(I, h, d, e; \mathbf{s}), ss(j, z), GC\right\}$, where τ^p is the payroll tax rate, τ^c is the consumption tax rate, $\tau^I(\cdot)$ is an income tax function, $ss(\cdot)$ is a social security payment function, and GC is government consumption. The government implements family policies through an income tax break. Therefore, the income tax depends on the labor supply of the caregiver, child care demand, and education demand, in addition to the total household income I and tax filing status, which is determined by the household state variables $\mathbf{s}$. The social security function $ss(j, z)$ is zero before retirement, namely $j \leq J_R - 1$.

The time endowment per period is normalized to one. A small child requires a certain amount of time $\gamma_s \in (0, 1)$. Parents can take care of their children or purchase child care services. Suppose the cost of paid child care is proportional to the number of children, but parents can take care of their children with less time as a result of economies of scale. The total time required for maternal care m is given by $(\gamma_s - d)\mu(n_s)$, where d is the time using paid child care per small child and $\mu(n_s)$ is a maternal time-cost scale that depends on the number of small children n_s. It satisfies $\mu(0) = 0$, $\mu'(n_s) \geq 0$, and $\mu''(n_s) \leq 0$. Parents pay for their children's education when child care is no longer required. The number of small children and older children in each age group is exogenously defined depending on the parental age and number of children born in a lifetime (i.e., $n_s = n_s(j,n)$ and $n_o = n_o(j,n)$).

First, we describe the problems of married and single households given the number of children. We then describe fertility decisions.

2.1.1 Household problems

Married households. Suppose that a married couple consists of a man and woman of the same age. They obtain disutility from labor h and parental care m, and utility from consumption c, the number of children n, and amount of education per child (quality of children) e if they have children. Married couples decide to maximize the sum of both members' utility. At the beginning of j years, a fraction σ_{j-1} of married couples divorce and their assets are equally divided between the husband and wife, while wives take all children if they have children.[4] Once married couples divorce, they remain single for the remainder of their lives.

The state variables of a household are $\mathbf{s} \equiv (j, a, z_1, z_2; \theta_1, \theta_2, n)$. We use the subscript 1 for the husband's variables and subscript 2 for the wife's variables. For single women (men), z_1 (z_2) and θ_1 (θ_2) are zero. The value function of a married couple with state $\mathbf{s}$, $V^c(\mathbf{s};\Omega)$, is

$$V^c(\mathbf{s};\Omega) = \max_{c,a',d,e,h_1,h_2} u\Big(c, e, h_1, h_2, m_1, m_2; \mathbf{s}\Big) + \beta\Bigg\{\phi_j^2(1-\sigma_j)V^c(\mathbf{s}';\Omega) + \phi_j^2\sigma_j\Big(V^s(\mathbf{s}_1';\Omega) + V^s(\mathbf{s}_2';\Omega)\Big) + \phi_j(1-\phi_j)\Big(V^s(\tilde{\mathbf{s}}_1';\Omega) + V^s(\tilde{\mathbf{s}}_2';\Omega)\Big)\Bigg\}, \quad (1)$$

subject to time constraints

$$h_i + m_i \leq 1 \quad \text{for } i = 1,2, \quad (2)$$

$$m_1 = 0, \quad m_2 = (\gamma_s - d)\mu\left(n_s(j,n)\right). \quad (3)$$

The law of motion of the state variables is defined as

$$a' = (1+r)\,a + \sum_{i=1}^{2}\Big\{(1-\tau^p)wz_ih_i \cdot \mathbf{1}_{[j<J_R]} + ss(j,z_i) + bq \cdot \mathbf{1}_{[\theta_i \neq 0]}\Big\} - p_d dn_s(j,n) - p_e en_o(j,n) - \tau^I\Big(ra + w(z_1h_1 + z_2h_2), h_2, d, e; \mathbf{s}\Big) - (1+\tau^c)\psi(\mathbf{s})c, \quad (4)$$

$$z_i' = z_i + z_i\big(g^z(j,\theta_i,h_i) + \varepsilon\big) \cdot \mathbf{1}_{[j<J_R]} \quad \text{for } i = 1,2, \quad (5)$$

[4] The proportion of children living with their father only is very low, especially small children (i.e., below 5% for children under nine years; Current Population Survey 2021).

$$\mathbf{s}' = \left(j+1, a', z_1', z_2'; \theta_1, \theta_2, n\right), \tag{6}$$

$$\mathbf{s}_1' = \left(j+1, a'/2, z_1', 0; \theta_1, 0, 0\right), \quad \tilde{\mathbf{s}}_1' = \left(j+1, a', z_1', 0; \theta_1, 0, n\right), \tag{7}$$

$$\mathbf{s}_2' = \left(j+1, a'/2, 0, z_2'; 0, \theta_2, n\right), \quad \tilde{\mathbf{s}}_2' = \left(j+1, a', 0, z_2'; 0, \theta_2, n\right), \tag{8}$$

where ϕ_j is the survival rate from j to $j+1$, $\mathbf{1}_{[j<J_R]}$ is an indicator function that takes a value of one if $j < J_R$ and a value of zero otherwise, bq is the accidental bequest per capita, p_d is the hourly fee for child care services, p_e is the education price per unit, and $\psi(\mathbf{s})$ denotes the inverse of each parent's share of household consumption, which depends on the number of adult members and children. When all household members die, leaving their assets, the government collects and distributes them to all surviving household members.

(5) implies that during working periods, labor productivity changes depending on current labor productivity, education type, age, and hours worked, in addition to idiosyncratic shocks ε. Labor productivity at retirement is considered as a state variable of the household. Regarding the state variables in the next period, $\mathbf{s}'$ denotes the state variable when a couple is neither divorced nor bereaved, $\mathbf{s}_i'$ denotes the variables of the husband ($i = 1$) and wife ($i = 2$) when divorced, and $\tilde{\mathbf{s}}_i'$ denotes those of the husband ($i = 1$) and wife ($i = 2$) when bereaved.

Single households. This problem is a simplified version of the problem described above. The value function of a single man or woman ($i = 1, 2$) with a state $\mathbf{s}$, which is denoted as $V^s(\mathbf{s})$, is

$$V^s(\mathbf{s}; \Omega) = \max_{c, a', d, e, h_i} u\Big(c, e, h_1, h_2, m_1, m_2; \mathbf{s}\Big) + \beta\phi_j V^s(\mathbf{s}'; \Omega), \tag{9}$$

subject to (2), (4), (5), (6),

$$m_i = (\gamma_s - d)\mu\left(n_s(j, n)\right), \quad h_{-i} = m_{-i} = 0, \tag{10}$$

where $-i$ denotes an index other than i.

Fertility decisions. At the beginning of $j = 1$, married couples decide the number of children they will have to maximize their expected lifetime utility. They derive utility from the number of children and education spending on children. Therefore, the fertility decision problem is defined as

$$n = \arg\max_{n \geq 0} E\Big[V^c\left(1, a, z_1, z_2; \theta_1, \theta_2, n; \Omega\right)\Big]. \tag{11}$$

By solving the problems described above, we obtain the decision rules, $n\ (a, z_1, z_2, \theta_1, \theta_2; \Omega)$, $c\ (\mathbf{s}; \Omega)$, $h_1\ (\mathbf{s}; \Omega)$, $h_2\ (\mathbf{s}; \Omega)$, $m_1\ (\mathbf{s}; \Omega)$, $m_2\ (\mathbf{s}; \Omega)$, $d\ (\mathbf{s}; \Omega)$, and $e\ (\mathbf{s}; \Omega)$, where $\mathbf{s} \equiv (j,\ a, z_1, z_2; \theta_1, \theta_2, n)$ is the individual state. We can also obtain $a'(\mathbf{s}; \Omega)$ and $z'(\mathbf{s}, \varepsilon; \Omega)$ by substituting the decision rules into the budget constraint and law of motion of labor productivity.

2.1.2 Distribution of households

Let $g(j, a, z_1, z_2, \theta_1, \theta_2, n)$ denote the population density with state $(j, a, z_1, z_2, \theta_1, \theta_2, n)$ and $G(j, a, z_1, z_2, \theta_1, \theta_2, n)$ be the corresponding cumulative distribution. The distribution of individuals with $j = 1$ is calculated using fertility decision rules and the initial demographic distribution (i.e., initial distributions of marital status, education type, and labor productivity by gender and marital status). We do not consider the effects of education spending on the initial distributions of education type and labor productivity in future generations and these are assumed to be fixed to make the model tractable. The law of motion of the distribution is then given as follows:

when both θ_1 and θ_2 are non-zero (married couples),

$$\begin{aligned} &g(j+1, a', z_1', z_2', \theta_1, \theta_2, n) \\ &\quad = \frac{\phi_j^2(1-\sigma_j)}{1+g^n} \int\limits_{\mathbf{A}\times\mathbf{Z}^2} \mathbf{1}_{[a'=a'(\mathbf{s};\Omega)]} \prod_{i=1}^{2} \mathbf{1}_{[z_i'=z_i'(\mathbf{s},\varepsilon_i;\Omega)]} dG(j, a, z_1, z_2, \theta_1, \theta_2, n). \end{aligned} \tag{12}$$

Also, when $z_2' = \theta_2 = 0$ (single males),

$$\begin{aligned} g(j+1, a', z_1', 0, \theta_1, 0, n) &= \frac{\phi_j}{1+g^n} \int\limits_{\mathbf{A}\times\mathbf{Z}} \mathbf{1}_{[a'=a'(\mathbf{s};\Omega)]} \cdot \mathbf{1}_{[z_1'=z_1'(\mathbf{s},\varepsilon_1;\Omega)]} dG(j, a, z_1, 0, \theta_1, 0, n) \\ &+ \frac{\phi_j^2\sigma_j}{2(1+g^n)} \int\limits_{\mathbf{A}\times\mathbf{Z}^2\times\Theta\times\mathbf{N}} \mathbf{1}_{[2a'=a'(\mathbf{s};\Omega)]} \cdot \mathbf{1}_{[z_1'=z_1'(\mathbf{s},\varepsilon_1;\Omega)]} \cdot \mathbf{1}_{[n=0]} dG(j, a, z_1, z_2, \theta_1, \theta_2, \tilde{n}) \\ &+ \frac{\phi_j(1-\phi_j)}{2(1+g^n)} \int\limits_{\mathbf{A}\times\mathbf{Z}^2\times\Theta} \mathbf{1}_{[a'=a'(\mathbf{s};\Omega)]} \cdot \mathbf{1}_{[z_1'=z_1'(\mathbf{s},\varepsilon_1;\Omega)]} dG(j, a, z_1, z_2, \theta_1, \theta_2, n). \end{aligned} \tag{13}$$

Finally, when $z_1' = \theta_1 = 0$ (single females),

$$\begin{aligned} g(j+1, a', 0, z_2', 0, \theta_2, n) &= \frac{\phi_j}{(1+g^n)} \int\limits_{\mathbf{A}\times\mathbf{Z}} \mathbf{1}_{[a'=a'(\mathbf{s};\Omega)]} \cdot \mathbf{1}_{[z_2'=z_2'(\mathbf{s},\varepsilon_2;\Omega)]} dG(j, a, 0, z_2, 0, \theta_2, n) \\ &+ \frac{\phi_j^2\sigma_j}{2(1+g^n)} \int\limits_{\mathbf{A}\times\mathbf{Z}^2\times\Theta} \mathbf{1}_{[2a'=a'(\mathbf{s};\Omega)]} \cdot \mathbf{1}_{[z_2'=z_2'(\mathbf{s},\varepsilon_2;\Omega)]} dG(j, a, z_1, z_2, \theta_1, \theta_2, n) \end{aligned}$$

$$+\frac{\phi_j(1-\phi_j)}{2(1+g^n)}\int_{\mathbf{A}\times\mathbf{Z}^2\times\Theta}\mathbf{1}_{[a'=a'(\mathbf{s};\Omega)]}\cdot\mathbf{1}_{[z_2'=z_2'(\mathbf{s},\varepsilon_2;\Omega)]}dG(j,a,z_1,z_2,\theta_1,\theta_2,n), \tag{14}$$

where g^n denotes the population growth rate. We assume that the population structure is stationary and that children born to parents of the same generation enter the economy simultaneously when the parental age is j_c. Then, the population growth rate is calculated as

$$g^n=\left\{\frac{\int_{\mathbf{A}\times\mathbf{Z}^2\times\Theta^2\times\mathbf{N}}n(a,z_1,z_2,\theta_1,\theta_2;\Omega)dG(1,a,z_1,z_2,\theta_1,\theta_2,n)}{\int_{\mathbf{A}\times\mathbf{Z}^2\times\Theta^2\times\mathbf{N}}dG(1,a,z_1,z_2,\theta_1,\theta_2,n)}\right\}^{\frac{1}{j_c}}-1, \tag{15}$$

where the curly brackets represent the ratio of children to parents per generation.

Then, the total assets and labor supply in the efficiency unit are calculated as

$$K=N\int_{\mathbf{J}\times\mathbf{A}\times\mathbf{Z}^2\times\Theta^2\times\mathbf{N}}adG(j,a,z_1,z_2,\theta_1,\theta_2,n)\quad\text{and} \tag{16}$$

$$L=N\int_{\mathbf{J}\times\mathbf{A}\times\mathbf{Z}^2\times\Theta^2\times\mathbf{N}}\Big(z_1h_1(\mathbf{s};\Omega)+z_2h_2(\mathbf{s};\Omega)\Big)dG(j,a,z_1,z_2,\theta_1,\theta_2,n), \tag{17}$$

where N is the total population. The total population of individuals with $j=1$ is normalized to one.

$$N=1+\sum_{j=1}^{J-1}\frac{\prod_{k=1}^{j}\phi_k}{(1+g^n)^j}. \tag{18}$$

2.2 Firms

Firms produce consumption goods with labor and capital and have constant returns to scale technology $F(K,L)$. They determine the input required to maximize their profits. Factor markets are assumed to be competitive. Therefore, the factor prices are $w=F_L(K,L)$ and $r=F_K(K,L)-\delta$, where δ is the capital depreciation rate.

2.3 Government

The government collects taxes and spends the acquired funds on social security payments, family policies, and their own consumption. The government's budget constraint is then given by

$$GC+SS=T_p+T_c+T_I\ \text{ and }\ SS=T_p, \tag{19}$$

where SS is the total social security payment, and T_p, T_c, and T_I are the total revenues from the payroll tax, consumption tax, and income tax, respectively. We also assume that the pension replacement rate is determined such that payroll tax revenue is equal to the total payment for social security. The social security function is assumed to be

$$ss(j,z) = xz \quad \text{if } j \geq J_R \text{ and 0 otherwise,} \tag{20}$$

where x denotes the replacement rate. Then, the total social security payment and tax revenue are calculated as

$$SS = N \int_{\mathbf{J}\times\mathbf{A}\times\mathbf{Z}^2\times\Theta^2\times\mathbf{N}} \Big(ss(j,z_1) + ss(j,z_2)\Big) dG(j,a,z_1,z_2,\theta_1,\theta_2,n), \tag{21}$$

$$T_p = \tau^p wL, \tag{22}$$

$$T_c = N \int_{\mathbf{J}\times\mathbf{A}\times\mathbf{Z}^2\times\Theta^2\times\mathbf{N}} \tau^c \psi(\mathbf{s}) c(\mathbf{s};\Omega) dG(j,a,z_1,z_2,\theta_1,\theta_2,n), \quad \text{and} \tag{23}$$

$$T_I = N \int_{\mathbf{J}\times\mathbf{A}\times\mathbf{Z}^2\times\Theta^2\times\mathbf{N}} \tau^I \Big(ra + w\left(z_1 h_1(\mathbf{s};\Omega) + z_2 h_2(\mathbf{s};\Omega)\right), h_{2-\mathbf{1}_{[\theta_2=0]}}(\mathbf{s};\Omega), d(\mathbf{s};\Omega), e(\mathbf{s};\Omega); \mathbf{s}\Big)$$
$$dG(j,a,z_1,z_2,\theta_1,\theta_2,n), \tag{24}$$

where $h_{2-\mathbf{1}_{[\theta_2=0]}}$ denotes the hours worked by individuals caring for their children. Additionally, the accidental bequest per capita is given by

$$bq = \int_{\mathbf{J}\times\mathbf{A}\times\mathbf{Z}^2\times\Theta^2\times\mathbf{N}} a'(\mathbf{s};\Omega)\left(1 - \phi_j \cdot \mathbf{1}_{[\theta_1>0]}\right)\left(1 - \phi_j \cdot \mathbf{1}_{[\theta_2>0]}\right) dG(j,a,z_1,z_2,\theta_1,\theta_2,n). \tag{25}$$

2.4 Equilibrium

The recursive equilibrium of this model comprises the decision rules of households, $\{n(a,z_1,z_2,\theta_1,\theta_2;\Omega), c(\mathbf{s};\Omega), h_1(\mathbf{s};\Omega), h_2(\mathbf{s};\Omega), m_1(\mathbf{s};\Omega), m_2(\mathbf{s};\Omega), d(\mathbf{s};\Omega), e(\mathbf{s};\Omega)\}$, where $\mathbf{s} \equiv (j,a,z_1,z_2;\theta_1,\theta_2,n)$ denotes the individual household state; the distributions of households, $G(j,a,z_1,z_2,\theta_1,\theta_2,n)$; the factor prices, (r,w); and the government policy rule, $\Omega \equiv \{\tau^p,\tau^c,\tau^I,ss,GC\}$ such that

1. households solve the optimization problem (1)–(11),

2. household distributions change according to (12)–(14),

3. firms solve their profit maximization problem,

4. the labor and capital markets clear, and

5. the government's policy rule satisfies (19)–(24).

3 A simple one-period model

In this section, by using a one-period model, we first demonstrate how maternal time allocation and the marginal cost of having children vary with the labor productivities of household members. Next, we examine the impact of family policies on time allocation and marginal costs. In the simple model, a married couple chooses the wife's time allocation and education demand given the number of children and the husband's time allocation. This problem can be written as

$$\max_{e,h_2,m_2} \; U(c, h_2, m_2, n, e) \tag{26}$$

$$\text{subject to} \quad c\psi(n) = w\left(z_1 + z_2 h_2\right) - n\left(p_d d + p_e e\right), \tag{27}$$

$$h_2 + m_2 \leq 1, \; \text{ where } m_2 = (\gamma_s - d)\,\mu(n). \tag{28}$$

The utility function is assumed to be

$$U(c, h_2, m_2, n, e) = \ln c - \frac{H(h_2, m_2)^{1+\gamma}}{1+\gamma}\chi + \lambda_n \ln n + \lambda_e \ln\left(e + \underline{e}\right), \tag{29}$$

where $H(h_2, m_2) \equiv \left(h_2^{1+\omega} + \delta_m m_2^{1+\omega}\right)^{\frac{1}{1+\omega}}$ is the composite disutility of non-leisure time, $\delta_m \in (0, 1)$ captures the degree of child care disutility, and $\underline{e} > 0$ is the minimum education level. Suppose that $0 < \omega < \gamma$, indicating that $U_{h_2 m_2} < 0$ and $U_{m_2 h_2} < 0$. The first-order conditions are then given as follows:

$$[h_2]: \quad \frac{w z_2}{c\psi(n)} = h_2^{\omega} H^{\gamma-\omega} + \lambda_T, \tag{30}$$

$$[m_2]: \quad \frac{n p_d}{c\psi(n)\mu(n)} \geq \delta_m m_2^{\omega} H^{\gamma-\omega} + \lambda_T, \; \text{ where the equality holds when } \; m < \gamma_s \mu(n), \tag{31}$$

$$[e]: \quad \frac{n p_e}{c\psi(n)} \geq \frac{\lambda_e}{e + \underline{e}}, \; \text{ where the equality holds when } \; e > 0, \tag{32}$$

where λ_T is the Lagrange multiplier for the time constraint, $h_2 + m_2 \leq 1$.

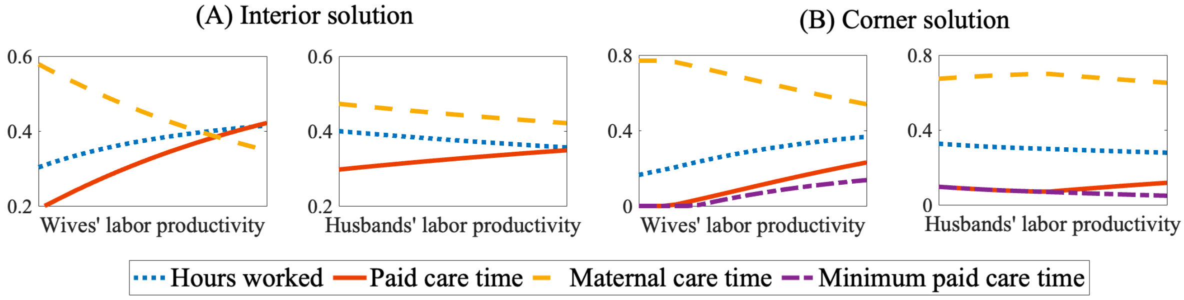


Figure 1: Maternal time allocation

3.1 Maternal time allocation

Interior solution case: First, we consider the interior solutions, $m_2 + h_2 < 1$ and $d > 0$. In this case, $\lambda_T = 0$ and the equality in (31) holds. (30) and (31) yield the ratio of h_2 to m_2 as

$$\frac{h_2}{m_2} = \left(\frac{wz_2}{\tilde{p}_d}\right)^{\frac{1}{\omega}}, \quad \text{where } \tilde{p}_d \equiv \frac{p_d n}{\delta_m \mu(n)}. \tag{33}$$

Here, $\tilde{p}_d$ denotes the real price of paid child care, reflecting the number of children and disutility of child care. (33) implies that higher wages lead to a higher ratio of hours worked (lower ratio of maternal care time), and expensive paid child care and many children lead to a lower ratio of hours worked (higher ratio of maternal care time). Intuitively, higher wages for wives lead to higher labor supply, which raises the disutility of maternal care. Expensive paid child care increases maternal care, which in turn increases the disutility of labor. Finally, total spending on paid child care, $np_d d$, is linear in n, but maternal care time, $(\gamma_s - d)\mu(n)$, is concave with it. Therefore, mothers with more children care for their children by themselves more and work outside less.

The husband's labor productivity affects maternal time allocation through an income effect. A higher z_1 leads to higher consumption levels, which lowers the wife's labor supply and maternal care time. Panel (A) in Figure 1 illustrates the optimal time allocation when the time constraints are not binding. Additionally, substituting (33) into (30) and (31) explicitly yields the optimal maternal time allocation:

$$h_2 = \left\{\frac{1}{\chi} \cdot \frac{wz_2}{c\psi(n)} \left[1 + \delta_n \left(\frac{\tilde{p}_d}{wz_2}\right)^{\frac{1+\omega}{\omega}}\right]^{\frac{\omega-\gamma}{1+\omega}}\right\}^{\frac{1}{\gamma}}, \; m_2 = \left\{\frac{1}{\chi} \frac{\tilde{p}_d}{c\psi(n)} \left[\left(\frac{wz_2}{\tilde{p}_d}\right)^{\frac{1+\omega}{\omega}} + \delta_n\right]^{\frac{\omega-\gamma}{1+\omega}}\right\}^{\frac{1}{\gamma}}. \tag{34}$$

Corner solution case: We consider the corner solutions, $h_2 + m_2 = 1$ or $d = 0$. In this case,

$$d = \underline{d} \equiv \max\left(0, \gamma_s + \frac{h_2 - 1}{\mu(n)}\right) \quad \text{and} \quad m = \overline{m} \equiv \min\left(1 - h_2, \gamma_s \mu(n)\right). \tag{35}$$

This corresponds to a situation in which mothers use minimal paid child care and care for their children as much as possible because the marginal cost of paid child care exceeds its marginal benefit. The minimum necessary paid child care time given the number of children and hours worked, $\underline{d}$, increases with maternal hours worked. Then, the optimal time allocation is as shown in Panel (B) in Figure 1. When $h_2 + m_2 = 1$, the husband's higher labor productivity increases care time and reduces labor supply because the disutility weight of labor is assumed to be larger than that of child care, $\delta_m \in (0, 1)$.

3.2 Marginal cost of having children

We then examine how the marginal cost of having children in terms of the marginal utility of consumption varies with the labor productivities of household members, z_1 and z_2. The increased cost of having an additional child can be decomposed into four factors:

$$-U_c \frac{\partial c}{\partial n} - U_m \frac{\partial m_2}{\partial n} = \underbrace{\frac{\psi'(n)}{\psi(n)}}_{\substack{\text{consumption}\\ \text{cost}}} + \underbrace{\frac{p_d d}{c\psi(n)}}_{\substack{\text{paid care}\\ \text{cost}}} + \underbrace{\max\left(0, \frac{\lambda_e}{n} - \frac{e p_e}{c\psi(n)}\right)}_{\text{education cost}} - \underbrace{\frac{\mu'(n) m_2}{\mu(n)} U_m}_{\text{maternal care cost}}. \tag{36}$$

The marginal cost from the consumption share depends on only n as a result of the log-utility. The marginal cost of education increases with consumption, thereby increasing in z_1 and z_2.

The maternal care cost is the product of the increased care time, $\mu'(n)(\gamma_s - d) = \mu'(n) m_2 / \mu(n)$, and the disutility from parental care, $-U_m$. Therefore, a higher z_2 lowers the maternal care cost as long as $d > 0$ because even when $d = 0$ and $m = \gamma_s \mu(n)$, an increase in hours worked increases the utility cost. A higher z_1 also lowers the cost, except when $d = \underline{d}$.

The paid child care cost increases with z_2 because paid child care demand increases more than consumption with a rise in z_2. As implied in Figure 1, paid child care demand is more sensitive to z_2 than z_1. This is because a higher z_2 lowers the paid child care cost measured by the marginal utility of consumption and raises the price of maternal labor relative to paid child care, $w z_2 / \tilde{p}_d$, whereas a higher z_1 only has the first effect. However, paid child care costs are not monotonic with z_1. When $d = \underline{d}(> 0)$ or z_2 is high, the marginal cost decreases with z_1.

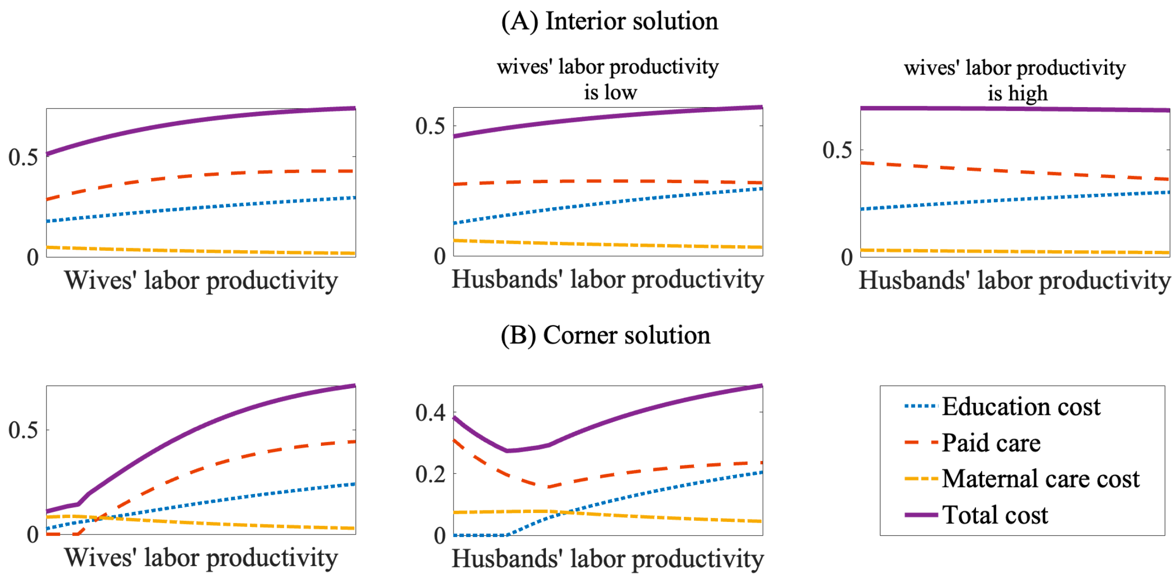


Figure 2: Marginal cost by the labor productivities of husbands and wives

In the latter case, because paid child care demand is already high as a result of a high z_2, it increases less with a rise in z_1. Additionally, when z_2 is low and $d > \underline{d}$, paid child care costs increase with z_1. In this case, h_2 and d are small and the increase in d caused by the increase in z_1 is larger than that in c. These patterns can be derived from (28) and (34) as follows:

$$\frac{\partial d}{\partial c} > 0, \ \frac{\partial^2 d}{\partial c^2} < 0, \ \text{and} \ \frac{\partial^2 d}{\partial c \partial z_2} < 0. \tag{37}$$

From the discussion above, the resulting marginal costs are as shown in Figure 2. A higher z_2 leads to higher costs of education and paid care, while lowering the maternal care cost as long as $d > 0$. Then, the total marginal cost increases in z_2 (left panels in Figure 2), implying that the number of children decreases in z_2. However, the total marginal cost is not monotonic with z_1. When $d = \underline{d}$, a higher z_1 lowers demand for paid care; hence, the total marginal cost decreases (right panel in Figure 2B). When z_2 is low and $d > \underline{d}$, a higher z_1 leads to higher education and paid care costs. As a result, the marginal cost increases (center panel in Figure 2A). Finally, when z_2 is high, the increase in education costs and decrease in paid and maternal care costs cancel each other out (right panel in Figure 2A), leading to an ambiguous total marginal cost.

3.3 Policy implications

Finally, we describe how time allocation and the marginal cost of having children respond to the introduction of family policies. Here, we consider only interior solutions and ignore tax increases associated with the introduction of family policies. The child subsidy per child is denoted as cs, and then the marginal cost, excluding the consumption share cost, is

$$\underbrace{\frac{p_d d}{c\psi(n)}}_{\text{paid care cost}} + \underbrace{\max\left(0, \frac{\lambda_e}{n} - \frac{e p_e}{c\psi(n)}\right)}_{\text{education cost}} - \underbrace{\frac{\mu'(n) m_2}{\mu(n)} U_m}_{\text{maternal care cost}} - \underbrace{\frac{cs}{c\psi(n)}}_{\text{child subsidy}}. \tag{38}$$

In addition, (34) leads the equations below:

$$\frac{\partial h_2}{\partial c} < 0,\ \frac{\partial^2 h_2}{\partial c^2} > 0,\ \frac{\partial m_2}{\partial c} < 0,\ \frac{\partial^2 m_2}{\partial c^2} > 0, \quad \text{and} \tag{39}$$

$$\frac{\partial h_2}{\partial p_d} < 0,\ \frac{\partial^2 h_2}{\partial p_d \partial c} > 0,\ \frac{\partial m_2}{\partial p_d} > 0,\ \frac{\partial^2 m_2}{\partial p_d \partial c} < 0. \tag{40}$$

Child subsidy: As implied by (39), the increase in c due to the child subsidy lowers h_2 and m_2 and raises d, and the effects are larger among low-income households. Subsequently, the subsidy mitigates maternal care costs, as well as the marginal costs of having children directly, which is captured by the last term in (38). Additionally, the subsidy raises education costs through an income effect, and the effects on paid child care are ambiguous because both d and c increase. Figure 3 presents the changes in the total marginal cost and paid child care cost. Although the subsidy raises the marginal paid care cost among low-income households, it lowers the resulting total marginal cost, which implies a positive effect on fertility rates.

Child care subsidy: The decrease in p_d causes both the price of paid care relative to labor, $\tilde{p}_d/wz_2$, and the care cost measured by the marginal utility of consumption, $\tilde{p}_d/c$, to be lower. Then, h_2 and d increase, m_2 decreases, and e may increase through the income effect as a result of the higher h_2. Therefore, child care subsidies lower maternal care costs and increase education costs, and their effects on paid care costs are ambiguous as before. As implied by (40), the effects on time allocation are greater for low-income households. Therefore, the paid care demand of households with low z_1 and z_2 responds sensitively and their marginal paid care costs increase, whereas the costs of those with high z_1 and z_2 decrease, as shown in Figure 3. The resulting effects on fertility rates are unclear.

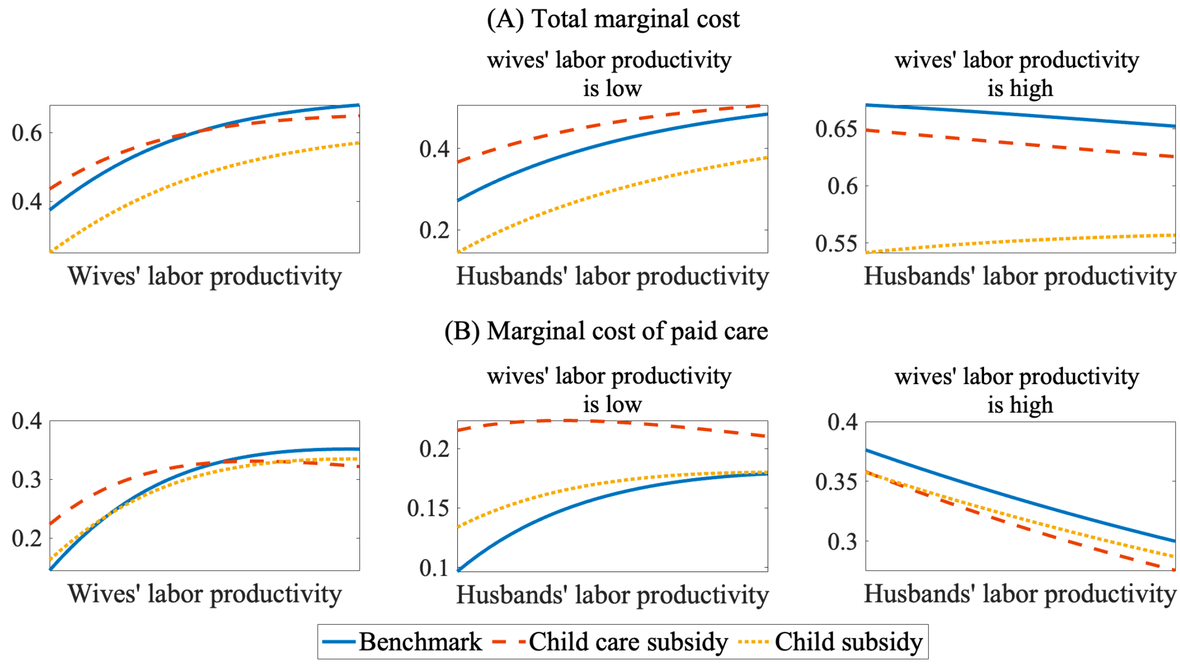


Figure 3: The impacts of family policies on the marginal cost of having children

4 Benchmark economy

4.1 Calibration

This subsection describes the parameter settings and initial distributions in the model economy. The model is calibrated to the U.S. economy around 2019. One unit in the model corresponds to \$100,000 and one period is set to three years. $j = 1$ corresponds to 25 to 27 years old, and the last period and retirement period are set to $J = 22$ (88 to 90 years old) and $J_R = 14$ (64 to 66 years old), respectively. Table 1 summarizes the parameter values, and some parameters are jointly calibrated to match target moments. We describe the details below.

Preference. The time discount factor β is set to match the target capital-output ratio.[5] The utility functions are assumed to be

$$u\Big(c, e, h_1, h_2, m_1, m_2; \mathbf{s}\Big) = \left(\mathbf{1}_{[\theta_1 \neq 0]} + \mathbf{1}_{[\theta_2 \neq 0]}\right) \ln c - \frac{H(h_1, m_1)^{1+\gamma}}{1+\gamma}\chi - \frac{H(h_2, m_2)^{1+\gamma}}{1+\gamma}\chi$$
$$+ \lambda_n \ln(n + \underline{n}) + \mathbf{1}_{[n_o > 0]} \cdot \lambda_e \ln(e + \underline{e}),$$

[5]The annual capital-output ratio is calculated as the ratio of the net stock of fixed assets to GDP using data from 1990 to 2021, and the average ratio during this period was 2.93. One period in the model is three years. Therefore, our target capital-output ratio is $2.93/3 \approx 0.97$.

Table 1: Parameter values

Parameter	Value	Source or target
Preference		
Discount factor (β)	0.92	K/Y
Inverse of Frisch elasticity (γ)	3	Literature estimate
Disutility weight of labor (χ)	47	Labor supply of married male
Utility weight of education (λ_e)	0.3	Education expenditure to total consumption
Utility weight of children (λ_n)	0.85	Total fertility rate
Tolerance of childless ($\underline{n}$)	3	Fertility rate of high school-educated females
Minimum education level ($\underline{e}$)	1.28	Public spending on education
Production		
Capital share (α)	0.3	Literature estimate
Depreciation rate (δ)	0.16	K/Y and target interest rate
Total factor productivity (A)	1.29	K/Y and target wage rate
Labor productivity growth (g^z)	see Fig 4	Growth rate of mean earnings
Demographic		
Survival rate (ϕ_j)	see text	Social Security Program Data
Divorce rate (σ_j)	see text	Distribution of marital status at aged 50
Child-related parameters		
Hourly fee for child care (p_d)	1.68	Hourly out-of-pocket expense
Education per unit (p_e)	0.1	Normalization
Time cost per small child (γ_s)	0.8	Labor supply of mothers
Maternal time-cost scale (ι)	0.05	Average hours of maternal care
Disutility weight of child care (δ_m)	0.22	Usage of non-parental care
Disutility weight of labor (δ_h)	1.37	
Elasticity b/w labor and care (ω)	1	
Government policy		
Consumption tax rate (τ^c)	0.05	
Payroll tax (τ^p)	0.124	Total tax rate for social security in the U.S.
Income tax function (τ^I)	see Table 2	U.S. income tax schedule in 2019

$$
\text{where } H(h_i, m_i) \equiv \begin{cases} h_i & \text{if } m_i = 0 \\ \left(\delta_h h_i^{1+\omega} + \delta_m m_i^{1+\omega}\right)^{\frac{1}{1+\omega}} & \text{if } m_i > 0 \end{cases}. \tag{41}
$$

Following the literature such as de la Croix and Doepke (2003) and Vogl (2016), we assume a log-linear utility function for the number and quality of children. Except for women with small children, γ corresponds to the inverse of the Frisch elasticity. ω denotes the inverse of the elasticity of substitution between hours worked and maternal care time, which we set to one. $\underline{n}$ is a parameter that governs the tolerance for childlessness and $\underline{e}$ denotes the minimum education level. $\underline{e}$ is calibrated to match U.S. school system spending on instructional salaries per pupil (2020 Annual Survey of School System Finances). The calibration for $(\chi, \delta_h, \delta_m, \lambda_n, \lambda_e, \underline{n})$ is described below.

Demographic. To calibrate the divorce rate and initial distribution of marital status, we target women aged 50 (Social Security Program Data from 2010 to 2021) because after approximately 50 years, the proportion of divorced individuals does not increase significantly, whereas the proportion of widowed individuals begins to increase. The average proportions of never-married and now-married individuals at age 50 are 9.5% and 69.3%, respectively, which yields

$$\int_{\mathbf{A}\times\mathbf{Z}^2\times\Theta^2\times\mathbf{N}} dG(1,a,z_1,0,\theta_1,0,n) = \int_{\mathbf{A}\times\mathbf{Z}^2\times\Theta^2\times\mathbf{N}} dG(1,a,0,z_2,0,\theta_2,n) = \frac{0.095}{2}\int_{\mathbf{A}\times\mathbf{Z}^2\times\Theta^2\times\mathbf{N}} dG(1,a,z_1,z_2,\theta_1,\theta_2,n).$$

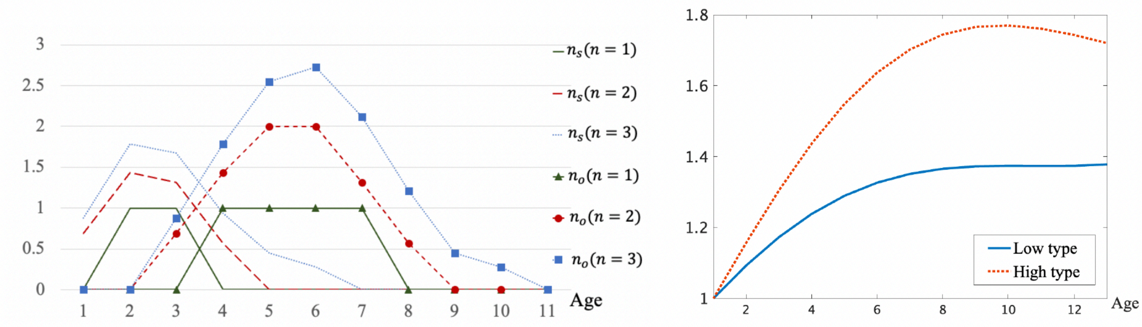
The divorce rates from $j = 1$ to 8 are set such that the proportion of now-married individuals is 69.3% at $j = 9$ and they are constant. This yields $\sigma_{j-1} = 0.032$ for $j = 1 \cdots 8$. After $j = 9$, the divorce rate is set to zero. Survival rates are set to one until $j = 11$ to prevent parents from dying before their children grow to adulthood. ϕ_j for $j = 12, \cdots J-1$ is set to be consistent with the average death probability in the 2018 Social Security Program Data, and ϕ_J is set to zero.

Suppose that women can have at most three children. Small children refer to those within two periods of birth, whereas older children refer to those in the third to sixth periods of birth. n takes values from 0 to 3, and for simplicity, the numbers of small children and older children conditional on n are assumed to be uniquely determined and calculated as

$$n_i(j,n) = (1-\xi)n_i(j,n') + \xi n_i(j,n'+1) \quad \text{for } i = s,o,$$

where n' is the largest integer that is less than n and $\xi \equiv n - n'$. $n_i(j,0)$ is set to zero for all j and $i = s,o$, and $n_i(j,n = 1,2,3)$ for $i = s,o$ are calibrated as follows. First, the timing of childbearing is exogenously determined such that the duration of having children increases with n. A woman with $n = 3$ gives birth between $j = 1$ and 5, a woman with $n = 2$ gives birth between $j = 1$ and 3, and a woman with $n = 1$ has one child at $j = 2$. Then, the number of children born in each period with $n = 2,3$ is calibrated by targeting the female distribution of the number of children ever born by age group (CPS 2018; see the Appendix for further details). The resulting number of children is presented in Figure 4(A). The age of parents whose children enter the economy, j_c, is also set to 11.

Labor productivity and education type. We consider two education types, namely *Low* and *High*. *Low* corresponds to individuals with less than a bachelor's degree and *High* corresponds to those with a bachelor's degree or higher. The initial distributions of education type and



(A) Number of children conditioned on n (B) Labor productivity growth

Figure 4: Calibration for the benchmark economy

labor productivity for single men and women are determined from the average distributions of men and women among those aged 30 to 50 years from the Current Population Survey, Annual Social and Economic Supplement (CPS ASEC) 2021. For married couples, the distribution is calibrated to match the distribution of educational attainment among married couples aged 30 to 50 years using the 2019 American Community Survey Public Use Microdata Sample.

The average initial labor productivities of men and women are calibrated using the mean earnings of a full-time worker aged 25 to 29 years by educational attainment (CPS ASEC 2021).[6] Tables A.1 and A.2 in the Appendix present the resulting distributions and average initial labor productivities.

The change in labor productivity is assumed to be

$$z' = z + z\big(g^z(j,\theta,h) + \varepsilon\big)\cdot \mathbf{1}_{[j<J_R]}, \tag{42}$$

$$\text{where} \quad g^z(j,\theta,h) = \tilde{g}^z(j,\theta)\,\big(\mathbf{1}_{[\tilde{g}^z\geq 0]}\cdot h/\bar{h} + \mathbf{1}_{[\tilde{g}^z<0]}\big)\,. \tag{43}$$

For the sake of simplicity, we do not consider the effect of hours worked on the depreciation of labor productivity and assume that labor productivity depreciates depending only on age and education level. Standard hours worked $\bar{h}$ are set to match average hours worked with a value of 0.35.[7] Suppose that ε takes values of 0.1 or -0.1 with even probabilities. The average growth rate $\tilde{g}^z(j,\theta)$ is calibrated to match the growth rate of mean earnings of a full-time worker with education type θ by age group (CPS ASEC 2021). Figure 4(B) presents the average labor

[6] The mean wage data for those aged 25 to 29 years with professional and doctorate degrees are not available. Therefore, the mean wages for those aged 25 to 34 years are used instead.

[7] This value is calculated using the average weekly hours of all employees in the private sector (Current Employment Statistics Survey 2021)

productivity growth in the benchmark economy of those who work $\bar{h}$ in every period, where the labor productivity in $j = 1$ is normalized to one.

Child costs. According to the Early Childhood Program Participation Survey 2019, the average hourly out-of-pocket expense of center-based care was \$8.22 in 2019. Based on this value, the hourly fee for child care p_d is set to 1.68.[8] The education price per unit p_e is normalized to 0.1. The maternal time cost scale $\mu(n_s)$ is assumed to be $\mu(n_s) = n_s^{\iota}$ if $n_s > 0$, otherwise 0. The calibration for γ_s and ι is described below.

According to the OECD-modified scale, the consumption scale of an additional adult member is 0.5 and that of each child is 0.3. Using this, we set the inverse of the parents' fraction of household consumption as

$$\psi(\mathbf{s}) = \begin{cases} 1 + 0.3(n_s + n_o)^{\varphi} & \text{if } \theta_1 = 0 \text{ or } \theta_2 = 0 \\ 1.5 + 0.3(n_s + n_o)^{\varphi} & \text{if } \theta_1 \neq 0 \text{ and } \theta_2 \neq 0 \end{cases},$$

where we assume an additional cost from having more children, namely $0.3(n_s + n_o)^{\varphi - 1}$ and set φ to 1.3. Including $\varphi > 1$ is necessary for the marginal cost of having children to be increasing in n. It shoule be noted that $\psi(\cdot)$ does not directly affect consumption decisions because we assume a logarithmic utility function with respect to consumption.[9]

Production. The production function is assumed to be a Cobb–Douglas function, $Y = AK^{\alpha}L^{1-\alpha}$, where $\alpha = 0.3$ and the wage rate in the benchmark economy is set to one. Total factor productivity A is calibrated using the capital-output ratio and wage rate, yielding $A = 1.29$. The depreciation rate δ is also calibrated to be consistent with our target interest rate of 14.42% and set to 0.16. We calculate our target interest rate as the average ratio of personal income receipts on assets to the net stock of fixed assets from 1990 to 2021.

Government policy. In the benchmark economy, $\tau^p = 0.124$ and $\tau^c = 0.05$. The income tax is calibrated to be in line with the U.S. income tax schedule in 2019. Taxable income is given

[8] One unit of time in the model is three years, excluding meals and sleep time. Therefore, one unit of time equals $14h \times 365 \times 3 = 15,330h$. In the benchmark economy, a 25% child care subsidy is introduced, resulting in $p_d = 15,330 \times 8.22/(0.75 \times 10^5) \approx 1.68$.

[9] Therefore, φ affects only fertility decisions and we can freely set its value as long as the marginal cost is increasing because we choose λ_n and $\underline{n}$ to match the total fertility rate and fertility rate by female education type given φ.

Table 2: Income tax schedule

		Tax bracket (i)						
	Filing status (fs)	1	2	3	4	5	6	7
Lower limit (b^i_{fs})	*Single*	0	0.291	1.184	2.526	4.821	6.123	15.309
	Head of household	0	0.415	1.585	2.526	4.821	6.123	15.309
	Filing jointly	0	0.582	2.368	5.052	9.643	12.246	18.370
Marginal tax rate (τ^i_{fs})	All	0.1	0.12	0.22	0.24	0.32	0.35	0.37

by $I - TD(\mathbf{s})$, where I is total income, $ra + w\sum_{i=1}^{2} z_i h_i$, and $TD(\mathbf{s})$ is the tax deductions for individuals with state $\mathbf{s}$. Then, the income tax is calculated as

$$\begin{aligned}\tau^I(I,h,d,e;\mathbf{s}) = \max&\left[\sum_{i=1}^{7}\max\left(\min\left(I - TD(\mathbf{s}), b^{i+1}_{fs}\right) - b^i_{fs}, 0\right)\tau^i_{fs} - tc^N(I,h,d,e;\mathbf{s}), 0\right] \\ &- tc^R(I,h,d,e;\mathbf{s}), \qquad (44)\end{aligned}$$

where fs is the tax filing status (*Single*, *Head of household*, or *Filing jointly*), b^i_{fs} is the lower limit of taxable income bracket i, τ^i_{fs} is the corresponding tax rate, and $tc^N(\cdot)$ and $tc^R(\cdot)$ are the non-refundable and refundable tax credit functions, respectively, as described below. The filing status of married couples is *Filing jointly*, that of single individuals with no children is *Single*, and that of single individuals with children is *Head of household*. b^i_{fs} and τ^i_{fs} are listed in Table 2 and the standard deductions for *Single*, *Head of household*, and *Filing jointly* are set to 0.3660, 0.5505, and 0.7320, respectively.

We model the following U.S. major family policies in 2019 in a simple form: the Child Tax Credit (CTC), Child and Dependent Care Credit (CDCC), and American Opportunity Tax Credit (AOTC).[10] The information below is obtained from the official website of the IRS (https://www.irs.gov).

- CTC: This corresponds to child subsidies. It provides a tax credit for each qualifying child under the age of 17 years. In 2019, the maximum amount per child was \$2,000, and up to \$1,400 could be refunded for each child. According to the White House, nearly all families with children qualify, although some income limitations are imposed. Therefore, we model the CTC as a universal fully-refundable child subsidy and the amount per child in model units is 0.06.

[10] Lifetime Learning Credit is also a tax credit for education expenses, but we model only the AOTC since households cannot take more than one education benefit for the same student and the same expense.

- CDCC: This corresponds to child care subsidies. It provides a non-refundable tax credit for child care expenses to work or look for work. The amount of the credit is a percentage of the work-related expenses paid to care providers, and the percentage depends on gross household income. The percentage starts at 35% and declines with household income to a minimum rate of 20%. The maximum qualifying expenses are \$3,000 when there is one child and \$6,000 with two or more qualifying children. We then model the CDCC as a non-refundable tax credit for child care expenses during parental work and the percentage is assumed to be uniformly 25%.

- AOTC: This corresponds to education subsidies. This provides a tax credit for education expenses, pays for the first four years of higher education, and provides a maximum annual credit of \$2,500 per eligible student. The amount of the credit is 100% of the first \$2,000. It is refundable up to 40% of credit and imposes income limitations. We model the AOTC as a universal fully refundable education tax credit. We introduce the education subsidies of four years evenly across the entire period of older children.

The resulting $tc^N(\cdot)$ and $tc^R(\cdot)$ in the benchmark economy are

$$tc^N(I,h,d,e;\mathbf{s}) = \min\Big\{\tau^d p_d n_s(j,n)\left[(d-h)\cdot\mathbf{1}_{[h-d\geq 0]}+h\right], \max\left(n_s(j,n),\ 2\right)\bar{tc}_d\Big\}, \quad (45)$$

$$tc^R(I,h,d,e;\mathbf{s}) = \min\Big(p_e e,\ \bar{tc}_e\Big)n_o(j,n) + cs\Big(n_s(j,n)+n_o(j,n)\Big), \quad (46)$$

where $\tau^d = 0.25$, $\bar{tc}_d = 0.02$, $\bar{tc}_e = 0.02$, and $cs = 0.06$. These denote the subsidy rate for child care, maximum credit for child care expenses, maximum credit for education expenses, and child subsidy per child, respectively.

Others. Finally, eight parameters, $(\gamma_s, \delta_h, \delta_m, \iota, \chi, \lambda_n, \underline{n}, \lambda_e)$, are set jointly to match the seven targets, as listed in Table 3. Average labor supply is calculated as the product of the labor participation rate and average hours worked. "Average hours of maternal care" refers to the average hours per day for women with children under the age of 6 years spent caring for children as a primary or secondary activity.

Table 3: Data targets

Target	Target value	Source
Education spending ratio to total consumption	0.02	Bureau of Labor Statistics 2014-21
Average labor supply (weekly)		
female with children under 6	25h	American Community Survey 2019
married male	40h	
Average hours of maternal care (daily)	9h	American Time Use Survey 2021
Proportion of two parents using non-parental care	0.58	Early Childhood Program Participation Survey 2019
Fertility rate		
total	1.70	National Vital Statistics 2019
less than a bachelor's degree	2.07	

Table 4: Labor supply, non-parental care usage, and fertility patterns

	Data		Model	
	Married	Single	Married	Single
Average labor supply				
Male	0.41	0.33	0.41	0.39
Female	0.28	0.31	0.33	0.37
with children under 18	0.27	0.31	0.31	0.36
with children under 6	0.24	0.28	0.25	0.28
less than a bachelor's degree	0.20	0.27	0.21	0.25
a bachelor's degree or more	0.29	0.36	0.30	0.33
Non-parental care usage (%)	58	65	57	69
	Data		Model	
Total fertility rate	1.70		1.70	
less than a bachelor's degree	2.07		2.08	
a bachelor's degree or more	1.33		1.33	

Notes: Actual labor supply corresponds to the average for individuals 25 to 60 years old and calculated using the 2019 American Community Survey Public Use Microdata Sample. Non-parental care usage in the model corresponds to the proportion of those who use paid child care for at least half a day per week.

4.2 Benchmark model

In this subsection, we validate our benchmark model using moments that are not used as targets for the calibration. Table 4 lists average labor supply, non-parental care usage, and fertility rates by household type.[11] One can see that the labor supply and non-parental care

[11] Non-parental care usage in the data corresponds to the percentage of children from birth through age five and not yet in kindergarten participating in at least one weekly non-parental care arrangement such as relative care, non-relative care, and center-based care (The Early Childhood Program Participation Survey 2019). The

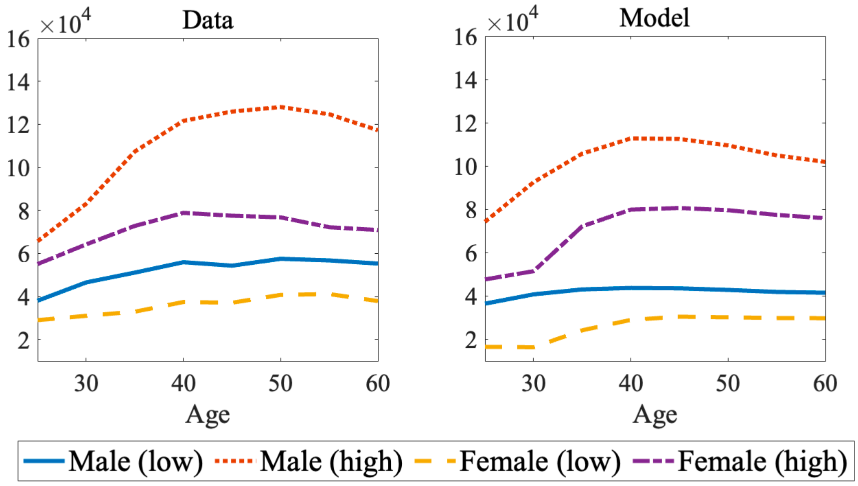


Figure 5: Labor income over the lifecycle by gender and education type

usage patterns in the benchmark model are generally consistent with the data, especially the labor supply patterns of married females, even though we use the labor supply of married males and females with small children and child care usage of married couples as the targets for the calibration. Figure 5 presents the average labor incomes by age, gender, and education type in the model economy and data (CPS ASEC 2021). The model patterns match the data relatively well, while we do not distinguish between genders with respect to the change in labor productivity. Because we abstract many welfare programs in the model, labor participation becomes higher than the actual values, especially among single households. Therefore, the labor supply of single individuals is larger and the average earnings of low-educated women are lower.

Table 5 presents the optimal number of children in the benchmark economy by the husband and wife's education levels and this pattern is consistent with the results in the one-period model. The number of children decreases with wives' labor productivity, but not monotonic with husbands' labor productivity. The table also shows that the model can replicate plausible fertility patterns by female education level.

fertility rates by maternal education type are the results of our calculations based on the fertility rates by maternal educational attainment (National Vital Statistics Reports 2021) and the distribution of educational attainment of women aged 15 to 45 years (ACS PUMS 2019).

Table 5: The number of children by education type in the benchmark economy

	Wife education level						
Husband education level	HS-	HS	SC	AD	BA	BA+	Total
HS-	2.91	2.46	2.38	2.31	1.54	1.40	2.27
HS	2.77	2.39	2.32	2.26	1.55	1.43	1.92
SC	2.70	2.37	2.30	2.24	1.55	1.43	1.87
AD	2.63	2.32	2.26	2.20	1.55	1.42	1.74
BA	2.22	2.00	1.96	1.93	1.51	1.39	1.45
BA+	2.07	1.88	1.84	1.82	1.47	1.34	1.34
Total	2.49	2.04	2.04	1.91	1.37	1.26	1.70
Data	2.79	2.05	1.80	1.31	1.28	1.43	1.70

Notes: HS-, HS, SC, AS, BA, and BA+ refer to those whose educational attainment is less than high school, high school, some college, associate's degree, bachelor's degree, and more than bachelor's degree, respectively. "Total" refers to the average number of children of those with that education level, including single individuals.

5 Policy experiments

Finally, we conduct four policy experiments by extending the current policies of child subsidies, child care subsidies, education subsidies, and tax deductions. For each experiment, we compute the results for two cases: the fertility pattern is fixed at the benchmark level and endogenous. To standardize the scale of the policy experiments, transfer amounts are set such that the government's additional spending on each policy is the same when the fertility pattern is fixed at the benchmark level. For tax deductions, a change in income tax revenue is considered as spending on the policy. The resulting scale of each policy is as follows: an increase in child subsidies of \$1,400, a rise in the child care subsidy rate of 10 p.p. and double maximum child care expense for the credit, a double maximum education expense for the credit, and a deduction per child of \$9,800. In all the simulations, the government consumption per capita is fixed in the benchmark model and a consumption tax covers the additional cost. Table 6 summarizes the results. Table 7 presents the impacts on labor supply and child care time. Tables 8 and 9 show the impacts on the fertility rate and welfare by education type, respectively. Welfare is measured using the consumption equivalence (CE) of newborn households. Specifically, this is the rate of change in consumption for the expected value after a divorce shock at $j = 1$ to be equal in the benchmark economy and in the new steady state. For example, CE=0.05 indicates that a 5% lifetime increase in consumption is required to achieve the same expected value in

Table 6: Policy experiment results

	Policy type / Fertility pattern							
	child subsidy		child care subsidy		education subsidy		income tax deduction	
	fixed	endo	fixed	endo	fixed	endo	fixed	endo
Change from baseline (%)								
Capital stock per capita	0.1	–0.5	0.4	0.6	–0.0	–0.5	0.5	–0.3
Labor supply per capita	–0.2	1.3	0.2	–0.8	–0.1	0.6	–0.1	1.3
per worker	–0.2	–0.8	0.2	0.4	–0.1	–0.5	–0.1	–0.7
Output per capita	–0.1	0.8	0.2	–0.4	–0.1	0.3	0.1	0.8
Wage rate	0.1	–0.5	0.1	0.4	0.0	–0.3	0.2	–0.5
Interest rate	–0.3	2.8	–0.3	–2.1	–0.1	1.6	–0.8	2.5
Consumption per capita	–0.0	0.5	0.5	0.1	0.0	0.2	0.2	0.6
per young	0.1	–0.6	0.4	0.7	–0.0	–0.4	0.4	–0.4
per old	–0.4	1.8	–0.2	–1.4	–0.3	0.6	–0.3	1.7
Child care (mean $p_d d$)	0.0	–1.6	22.6	25.0	0.3	–0.3	0.7	–0.5
Education (mean $p_e e$)	0.6	–4.3	–0.3	2.7	6.6	4.1	0.8	–3.9
New fertility rate	1.70	1.82	1.70	1.64	1.70	1.76	1.70	1.82
Consumption tax rate (p.p.)	0.4	0.5	0.3	0.3	0.4	0.4	0.3	0.5
Welfare (CE)	0.43	0.08	0.60	0.78	0.31	0.07	0.45	–0.03
Single male	–0.27	–0.25	–0.21	–0.16	–0.23	–0.24	–0.20	–0.22
Single female	0.08	0.05	–0.09	–0.01	0.25	–0.08	–0.17	–0.33
without children	–0.21	–0.22	–0.15	–0.13	–0.18	–0.21	–0.16	–0.21
with children	0.99	0.90	0.09	0.36	1.62	0.34	–0.20	–0.71
Married couple	0.50	0.11	0.70	0.91	0.36	0.10	0.53	0.01

Notes: "fixed" corresponds to the case where the fertility pattern by labor productivity and education level is fixed at the benchmark economy and "endo" corresponds to the case where fertility decisions are endogenous given the new policies.

the benchmark economy after the policy is introduced. The welfare of a "single female with children" refers to that of single females who underwent the divorce shock at $j = 1$ and the welfare of a "single female with no children" refers to that of initially single females.

5.1 Child subsidy

The new child subsidy per child cs is set to 0.074, which equates to an increase of \$1,400 per child during the period. When fertility decisions are fixed, a universal child subsidy reduces the labor supply of women with children through an income effect, especially that of single less educated mothers (Table 7). As shown in the one-period model, the effects of universal child subsidies on labor supply are larger among low-income households. The effect on maternal care time is negative in most cases, but when the time constraint binds, the income effects reduce maternal labor supply and increase maternal care time. Additionally, when the number

Table 7: Average labor supply and maternal care time, percentage change from the baseline

			Policy type / Fertility pattern							
			child subsidy		child care subsidy		education subsidy		income tax deduction	
			fixed	endo	fixed	endo	fixed	endo	fixed	endo
Hours worked										
Single male			0.0	–0.0	–0.0	–0.1	–0.0	–0.0	–0.1	–0.1
Single female			–0.2	–0.6	0.1	0.4	–0.1	–0.3	–0.1	–0.4
with small children			–1.2	–1.1	1.9	3.7	–0.0	–0.3	–0.7	–0.7
	low-education		–1.9	–0.9	1.0	2.9	0.2	–0.3	–1.7	–1.3
	high-education		–0.3	–0.4	3.1	3.3	–0.2	–0.4	0.6	0.5
with older children only			–0.3	–0.7	–0.1	0.1	–0.3	–0.3	0.2	–0.1
Married male			–0.2	0.0	–0.3	–0.4	–0.1	0.0	–0.1	0.1
Married female			–0.1	–1.5	1.0	1.7	–0.1	–0.7	–0.1	–1.3
with small children			–0.2	–0.7	4.1	4.8	–0.1	–0.2	0.0	–0.2
	low-education	$\theta_1 = L$	–0.4	–0.9	7.3	8.0	0.0	–0.4	–0.2	–0.6
	high-education	$\theta_1 = L$	–0.2	–0.4	0.4	0.7	–0.2	–0.3	0.1	–0.1
	low-education	$\theta_1 = H$	–0.3	–0.7	5.8	5.6	–0.2	–0.4	0.1	–0.2
	high-education	$\theta_1 = H$	–0.1	–0.4	1.0	1.1	–0.1	–0.3	0.3	–0.1
with older children only			–0.1	–0.6	0.1	0.3	–0.1	–0.3	–0.1	–0.5
Maternal care time										
Single	low-education		0.5	–1.6	–0.8	–0.2	–0.3	–0.6	0.3	–1.2
Single	high-education		–0.1	–0.5	–3.5	–3.8	–0.1	–0.4	–0.7	–1.2
Married	low-education	$\theta_1 = L$	–0.0	–1.0	–3.9	–3.2	–0.1	–0.3	0.0	–0.8
Married	high-education	$\theta_1 = L$	–0.1	–0.3	–0.7	–1.0	–0.0	–0.2	–0.4	–0.6
Married	low-education	$\theta_1 = H$	–0.1	–0.5	–5.1	–5.0	–0.1	–0.4	–0.3	–0.7
Married	high-education	$\theta_1 = H$	–0.1	0.2	–1.6	–1.8	0.0	0.1	–0.5	–0.1

Notes: L and H denote the low- and high-education types, respectively.

of children is fixed, education spending per child increases.

As implied in the one-period model, the universal child subsidies reduce the resulting marginal cost of having children while raising the education cost and paid child care cost of low-income households. Therefore, the effect on fertility is positive and the positive effect is greater among households with less educated wives (Table 8). Because the subsidy per child is constant, regardless of household income, and the total amount depends on the number of children, households with low income and low-educated women benefit more from the subsidies.

The increase in the working population due to the higher fertility rates reduces the wage level and raises the interest rate. The increase in the number of children and decrease in the wage rate reduce the price of labor relative to paid child care, $wz_2/\tilde{p}_d$, which depresses the labor supply of women with small children more. Meanwhile, the effects on paid child care demand and maternal care time are ambiguous because the income effect associated with child

Table 8: Fertility effects by education type, percentage change from the baseline

(θ_1, θ_2)	Policy type: child subsidy	child care subsidy	education subsidy	income tax deduction
(L, L)	7.86	–5.55	2.27	6.07
(L, H)	6.20	–0.65	4.47	6.62
(H, L)	6.33	–4.58	4.01	6.51
(H, H)	5.63	–0.93	4.50	7.63

Table 9: Welfare effects by household type

Household type		Policy type / Fertility pattern: child subsidy, fixed	child subsidy, endo	child care subsidy, fixed	child care subsidy, endo	education subsidy, fixed	education subsidy, endo	income tax deduction, fixed	income tax deduction, endo
Single male	L	–0.30	–0.24	–0.23	–0.18	–0.24	–0.22	–0.23	–0.20
	H	–0.22	–0.26	–0.17	–0.15	–0.22	–0.26	–0.15	–0.24
Single female									
with no children	L	–0.24	–0.24	–0.18	–0.17	–0.21	–0.23	–0.20	–0.23
	H	–0.17	–0.19	–0.11	–0.09	–0.16	–0.19	–0.11	–0.18
with children	L	1.37	1.83	–1.41	–1.10	2.94	0.91	–0.74	–0.74
	H	0.63	–0.02	1.55	1.78	0.34	–0.22	0.33	–0.68
Married couple (θ_1, θ_2)	(L, L)	0.94	0.69	0.72	1.01	0.67	0.54	0.73	0.42
	(L, H)	0.29	–0.46	1.05	1.26	0.25	–0.31	0.35	–0.54
	(H, L)	0.38	0.18	0.52	0.75	0.27	0.14	0.47	0.17
	(H, H)	0.13	–0.32	0.58	0.67	0.07	–0.25	0.41	–0.26

subsidies and the substitution effect associated with lower wages and a larger number of children have opposite effects. Because income effects are smaller among high-income households, their parental care time increases slightly. An increased number of children also reduces education spending per child.

When the fertility pattern is fixed, the effects on welfare are mostly positive and the effects are larger among low-income households with children (see Table 9). However, in the case of endogenous fertility, welfare gains decrease due to higher fertility and consumption tax rates. An increase in the fertility rate leads to a further decline in labor supply, which in turn causes higher policy costs and a lower consumption level of the working population, while raising the replacement rate of the pension benefit, thereby increasing the consumption level of households with older people. This is prominent among households with highly educated women, resulting in increased welfare losses. Family policies typically worsen the welfare of single agents with no children because they receive fewer benefits and face higher consumption tax rates.

5.2 Child care subsidy

The new child care subsidy rate τ^d is set to 0.35 and the maximum child care expense for credit $\bar{tc}_d$ is doubled. The decline in paid child care price increases paid child care demand and maternal labor supply because it reduces the price of paid child care relative to labor, $\tilde{p}_d/wz_2$. The effects are larger among low-educated mothers, who work less and care more for their children by themselves in the benchmark economy. However, the subsidy is not refundable. Therefore, those with low income tax payments may benefit only slightly. For this reason, the effects on single low-educated mothers are not as significant.

An increase in female labor supply fosters the growth of labor productivity. As a result, the subsidy increases female labor supply not only when they have small children but also afterward. Increased female labor supply slightly reduces the labor supply of married men through an income effect and results in lower additional policy costs. Therefore, the new consumption tax rate becomes lower than that of other policies in the case of endogenous fertility.

The effects on fertility are negative. Child care subsidies increase paid child care demand overall, but the impact on the cost measured by the marginal utility of consumption, $p_d du_c$, is non-uniform, as shown in the one-period model. The subsidies raise the paid child care cost for households that use paid child care less in the benchmark economy, whereas the cost decreases for households that already use paid child care to some degree in the benchmark economy (i.e., households with high labor productivity). Additionally, child care subsidies may raise education spending per child due to increased maternal labor supply. These two effects reduce the fertility rate and this negative effect is larger among households with low-educated wives, whose paid child care demand is small in the benchmark economy. A decreased number of children increases female labor supply, paid child care demand, and education demand.

The impacts on welfare are mostly positive among those with children, but the subsidy worsens the welfare of single low-educated mothers because their benefit from the non-refundable tax credit is small. In particular, child care subsidies improve the welfare of single high-educated mothers and married couples with low-educated husbands and high-educated wives. Such couples already use paid child care to some extent in the benchmark economy, so the subsidy lowers their paid child care costs and raises their consumption levels through an increase in maternal labor supply.

5.3 Education subsidy

The new maximum education expense for credit $\bar{tc}_e$ is set to 0.04, which is twice the baseline level. The subsidies increase education spending per child, but their effects on the macroeconomy are small when fertility decisions are fixed. The subsidy provides a fully refundable tax credit for 100% of education expenses up to the maximum deductible expenses, thereby lowering the private education costs per child. Therefore, the effect on fertility is positive but smaller than that of child subsidies and tax deductions. This is because child subsidies and tax deductions lower the marginal cost of having children directly, whereas education subsidies reduce this cost indirectly by reducing private education costs. The positive effect is smaller among couples with less educated wives because the education spending of households with low income or many children is small and the marginal cost of having children does not decrease as significantly. The increase in the number of children depresses female labor supply through a decline in the price of labor relative to paid child care, $wz_2/\tilde{p}_d$, and reduces the increase in education demand per child.

Similar to child subsidies, the effects on welfare are mostly positive in the fixed case, but the welfare gain shrinks due to the higher fertility rate in the endogenous case. The positive effect on welfare is greater among low-income households, whose utility from the quality of children is lower in the benchmark economy. Additionally, the welfare loss for households with highly educated wives is smaller than that for child subsidies because the decrease in female labor supply is smaller.

5.4 Income tax deduction

The tax deduction per dependent child is set to 0.098, which equates to a deduction of $9,800 per child. The taxable income is given by $I -$ standard deduction $- 0.098 \times (n_s + n_o)$. When the fertility decision is fixed, tax deductions increase the labor supply of high-educated mothers and reduce that of low-educated mothers. This is because the deduction has two opposite effects on labor supply: an income effect resulting from lower income tax and a substitution effect resulting from lower marginal tax rates. The income effect is larger among low-income households. In the fixed case, demand for paid care and education per child increase through the income effect.

The deduction raises the fertility rate because it benefits all working households with children and lowers the cost of having children directly. The total size of this effect is similar to that of child subsidies, but the effects by education type significantly differ. The effect of tax deductions on fertility is larger among high-income households, whereas that of child subsidies is larger among low-income households. This is because high-income households, whose income tax and marginal tax rate are higher, benefit from tax deductions. A larger number of children and lower wage levels reduce maternal labor supply, but a decline in marginal tax rates lowers the negative effect on labor supply.

The effects on welfare are mostly positive in the fixed fertility case, but in the endogenous fertility case, the effects become smaller and the overall welfare level becomes worse. Similar to the case of child subsidies, the welfare of households with highly educated women decreases because in such households, the declines in female labor supply and consumption level stemming from an increase in children are significant. By contrast, low-income households receive fewer benefits than child subsidies as a result of progressive income taxation. Therefore, tax deductions worsen overall welfare.

6 Concluding remarks

We quantitatively analyze the possible effects of family policies on household labor supply, fertility decisions, and social welfare by extending a heterogeneous agent OLG model. Our study provides rich policy implications compared with previous studies by endogenizing parental time allocation and fertility decisions and introducing heterogeneity in education type and labor productivity within married couples. Our model reproduces plausible female labor supply patterns by marital status, the presence of children, and education type and fertility patterns by maternal education type. We then introduce four major family policies, namely child subsidies, child care subsidies, education subsidies, and income tax deductions for children, into the benchmark economy by extending current policies in the U.S.

The results imply the following. First, there is a tradeoff between fertility rates and female labor supply, which are typical goals of family policies. Child care subsidies increase female labor supply but depress fertility rates, whereas the other three policies, namely child subsidies, education subsidies, and income tax deductions, reduce female labor supply but raise fertility rates. Second, child care subsidies improve overall welfare the most. This is because increased labor supply raises the consumption level and lowers policy costs. Third, each family policy has different effects on individual households as well as on the macroeconomy. The effects on labor supply are larger among low-educated mothers in all four policies. The effects of child care subsidies and child subsidies on fertility are larger among couples with low-educated wives, whereas those of education subsidies and tax deductions are larger among couples with high-educated wives. Regarding welfare effects, the three policies that lead to higher fertility rates worsen the welfare levels of households with highly educated women. In particular, child subsidies are the most welfare-worsening policy for highly educated couples, although they are the most welfare-improving policy for single low-educated mothers.

The model presented in this paper makes a simple assumption about education spending on children to make the model tractable. The initial distribution of labor productivity or education type is constant and education spending does not affect children's education type or labor productivity. Relaxing this assumption and considering the impact of education spending, parental education or labor productivity on children's education would provide richer insights into the impact of family policies.

A Appendix

In this Appendix, we initially present the first-order conditions for married households. Next, we describe the details of the calibration of labor productivity and the number of children conditional on n.

A.1 Optimization of married households

The assets in the next period for $j = 1$ to $J - 1$ are given by

$$-\frac{u_c}{\psi(\mathbf{s})(1+\tau^c)} + \beta E\left[\mathbf{V}'_a\right] \leq 0, \tag{A.1}$$

where

$$E\left[\mathbf{V}'_a\right] = \phi_j^2(1-\sigma_j)V_a^c\left(\mathbf{s}';\Omega\right) + \sum_{i=1}^{2}\left\{\frac{\phi_j^2\sigma_j}{2}V_a^s\left(\mathbf{s}'_i;\Omega\right) + \phi_j(1-\phi_j)V_a^s\left(\tilde{\mathbf{s}}'_i;\Omega\right)\right\}, \tag{A.2}$$

$$V_a^M(\cdot) = \frac{u_c}{\psi(\mathbf{s})(1+\tau^c)}\left(1 + r - \tau_a^I\right) \quad \text{for } M = s, c, \tag{A.3}$$

and the equality holds when $a' > 0$. The subscripts represent partial derivatives. The husband's labor supply for $j = 1$ to $J_R - 1$ is

$$\frac{u_c\left((1-\tau^p)wz_1 \cdot \mathbf{1}_{[j<J_R]} - \tau^I_{h_1}\right)}{\psi(\mathbf{s})(1+\tau^c)} + u_{h_1} + \beta\frac{\partial z_1'}{\partial h_1}E\left[\mathbf{V}'_{\mathbf{z_1}}\right] = 0, \tag{A.4}$$

where

$$E\left[\mathbf{V}'_{\mathbf{z_1}}\right] = \phi_j^2(1-\sigma_j)V^c_{z_1}(\mathbf{s}';\Omega) + \phi_j^2\sigma_j V^s_{z_1}\left(\mathbf{s}'_1;\Omega\right) + \phi_j(1-\phi_j)V^s_{z_1}\left(\tilde{\mathbf{s}}'_1;\Omega\right). \tag{A.5}$$

The wife's labor supply for $j = 1$ to $J_R - 1$ is

$$\frac{u_c\left((1-\tau^p)wz_2 \cdot \mathbf{1}_{[j<J_R]} - \tau^I_{h_2}\right)}{\psi(\mathbf{s})(1+\tau^c)} + u_{h_2} - \lambda_T + \beta\frac{\partial z_2'}{\partial h_2}E\left[\mathbf{V}'_{\mathbf{z_2}}\right] = 0, \tag{A.6}$$

where

$$E\left[\mathbf{V}'_{\mathbf{z_2}}\right] = \phi_j^2(1-\sigma_j)V^c_{z_2}(\mathbf{s}';\Omega) + \phi_j^2\sigma_j V^s_{z_2}\left(\mathbf{s}'_2;\Omega\right) + \phi_j(1-\phi_j)V^s_{z_2}\left(\tilde{\mathbf{s}}'_2;\Omega\right), \tag{A.7}$$

where λ_T is the Lagrange multiplier for the time constraint, $h_2 + m_2 \leq 1$. The marginal value of labor productivity is given by

$$V^M_{z_i}(\cdot) = \frac{u_c\left((1-\tau^p)wh_i \cdot \mathbf{1}_{[j<J_R]} + ss_{z_i} - \tau^I_{z_i}\right)}{\psi(\mathbf{s})(1+\tau^c)} + \beta\frac{\partial z_i'}{\partial z_i}E\left[\mathbf{V}'_{\mathbf{z_i}}\right] \text{ for } M = s, c. \tag{A.8}$$

Maternal care time is

$$\frac{u_c}{\psi(\mathbf{s})(1+\tau^c)}\left(\frac{n_s(j,n)p_d}{\mu(n)} - \tau^I_{m_2}\right) + u_{m_2} - \lambda_T \geq 0 \tag{A.9}$$

and the equality holds when $m < \gamma_s\mu(n)$. Education spending when $n_o > 0$ is

$$-\frac{u_c}{\psi(\mathbf{s})(1+\tau^c)}\left(n_o(j,n)p_e + \tau^I_e\right) + u_e \leq 0, \tag{A.10}$$

where the equality holds when $e > 0$. Finally, the marginal value of having children is

$$\begin{aligned} V^c_n(\cdot) = &-\frac{u_c}{\psi(\mathbf{s})(1+\tau^c)}\left\{\left(p_d d - \tau^I_{n_s}\right)\frac{\partial n_s(j,n)}{\partial n} + \left(p_e e - \tau^I_{n_o}\right)\frac{\partial n_o(j,n)}{\partial n} + \frac{c}{\psi(\mathbf{s})}\frac{\partial\psi(\mathbf{s})}{\partial n}\right\} \\ &+ u_{m_2}(\gamma_s - d)\mu'\left(n_s(j,n)\right)\frac{\partial n_s(j,n)}{\partial n} + u_n + \beta E\left[\mathbf{V}'_{\mathbf{n}}\right], \end{aligned} \tag{A.11}$$

where

$$E\left[\mathbf{V}'_{\mathbf{n}}\right] = (1-\sigma_j)V^c_n(\mathbf{s}';\Omega) + \sigma_j V^s_n\left(\mathbf{s}'_2;\Omega\right) + \phi_j(1-\phi_j)\sum_{i=1}^{2}V^s_n(\tilde{\mathbf{s}}'_i;\Omega). \tag{A.12}$$

Table A.1: Initial labor productivity and distribution

	Average initial value		Initial distribution (single)	
Educational attainment	Male	Female	Male	Female
Less than high school	1.9	1.7	0.09	0.07
High school	2.5	2.0	0.28	0.20
Some college credit, no degree	2.8	2.2	0.14	0.13
Associate's degree	3.0	2.2	0.09	0.11
Bachelor's degree	4.1	3.3	0.24	0.27
Master's degree	5.0	3.7	0.09	0.14
Professional degree	5.6	6.5	0.01	0.01
Doctorate degree	6.4	5.7	0.02	0.02

Table A.2: Initial distribution among married couples (%)

	Wife							
Husband	HS-	HS	SC	AS	BA	MA	PR	DC
HS-	3.59	1.91	1.14	0.48	0.51	0.17	0.03	0.02
HS	1.69	7.75	4.50	2.45	3.20	1.17	0.13	0.09
SC	0.74	2.75	5.59	2.46	4.38	1.87	0.22	0.16
AS	0.25	1.20	1.73	1.94	2.50	1.02	0.12	0.09
BA	0.37	1.59	2.60	2.09	12.42	5.66	0.83	0.61
MA	0.11	0.46	0.75	0.63	4.50	4.73	0.64	0.50
PR	0.03	0.07	0.15	0.12	1.00	0.82	0.90	0.13
DC	0.01	0.04	0.09	0.08	0.66	0.82	0.14	0.60

Notes: HS-, HS, SC, AS, BA, MA, PR, and DC refer to those whose educational attainment is less than high school, high school, some college no degree, associate's degree, bachelor's degree, master's degree, professional degree, and doctorate.

A.2 Calibration of the benchmark economy

In this subsection, we present the initial distribution of labor productivity in the benchmark economy and then describe how we calibrate the number of children conditional on n.

Initial distribution: Tables A.1 and A.2 present the average labor productivity by educational attainment and initial distributions of educational attainment among singles and married couples. The distribution among married couples is calculated based on the distribution of educational attainment for married couples aged 30 to 50 years from the 2019 American Community Survey Public Use Microdata Sample.

Number of children conditional on n: The number of children born in each period conditional on $n = 1, 2, 3$ is calibrated as follows. As described in the text, the timing of childbearing conditional on n is given exogenously. By using data on the distribution of the number of children born by age group, we calculate the average probability that the Nth child is born

Table A.3: Timing of childbearing by birth order

	Age				
	1	2	3	4	5
$\tilde{x}(1, j)$	0.4147	0.5853	0	0	0
$\tilde{x}(2, j)$	0.2712	0.1630	0.5658	0	0
$\tilde{x}(3, j)$	0.1918	0.1634	0.1950	0.1748	0.2750

at age j, which is denoted as $x(N, j)$. We assume that fertility patterns are invariant among age groups. Subsequently, the endpoints of $x(N, j)$ are adjusted to fit the childbearing timing conditional on n. This adjusted probability is denoted as $\tilde{x}(N, j)$. For example, for $\tilde{x}(2, 3)$, the adjusted probability of having a second child in $j = 3$ is calculated as $\sum_{j=3}^{J} x(2, j)$ because a woman with $n = 2$ is assumed to give birth between $j = 1$ and 3. The resulting $\tilde{x}(N, j)$ values are listed in Table A.3. Finally, the number of children born in j conditional on n is calculated as $\sum_{N=1}^{n} \tilde{x}(N, j)$.

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