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Non-financial Correlated Background Uncertainty"

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Cross Risk Apportionment and Non-financial Correlated Background Uncertainty*

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Abstract

This paper considers a portfolio problem with one safe asset and one risky asset in the presence of background risk. We assume that the background risk is a non-financial variable and it is correlated to financial risk. The aim of this paper is to investigate the effect of correlation on portfolio choices. While we find that an increase in correlation lowers (raises) the expected utility for mixed correlation averse (seeking) individuals, contrary to intuition, it does not necessarily reduce (increase) the investment in the risky asset. We determine the conditions needed to reduce (increase) the investment and find that these conditions can be related to cross risk apportionment, which is the type of preferences for the combination of good and bad. We also introduce ambiguity into the correlation and investigate its effects on the portfolio choices.

JEL classification numbers: D81, D91, G11

Key Words: Ambiguity, Bivariate Utility Function, Linear Payoff, Mixed Correlation Aversion (Seekingness), Background Uncertainty, Portfolio Choice

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1 Introduction

People need to make decisions in the face of multidimensional consequences that are risky, and also interdependent.[1] Let us illustrate such a situation with two examples. To start, imagine the situation where you are considering purchasing a new health insurance policy. In this case, you need to consider not only your future financial conditions but also your future health status. Now imagine that you are the general manager of a multinational firm in the East Asian region, where you are concerned about the profits from other areas, such as the North American region, which may affect the management strategy of your region.[2] These illustrative examples highlight the need to incorporate multidimensional and interdependent risks into a model and to investigate the effects of their interdependence on decision making.

The above examples are included in the linear payoff model of Dionne et al. (1993) in the presence of non-financial background risk.[3] Here, background risk is a type of risk that is exogenous and cannot be controlled. For clarity, we specify the linear payoff model as a portfolio problem choosing between one safe asset and one risky asset in the presence of a health risk.[4] The reasons for choosing the above specification are: (i) the portfolio problem is representative of the linear payoff problem, and (ii) empirical analysis is making progress on the properties of the bivariate utility function of wealth level and health status. The research question

[1] This analysis focuses on bivariate consequences, but can also be applied to higher-order dimensions when all variables are exogenous except for two variables.

[2] He and Ng (1998) and Allayannis and Ihrig (2001) empirically find that correlations between returns and exchange rates may be positive or negative, depending on the company's characteristics. He and Ng (1998) investigate Japanese multinational firms and demonstrate a positive correlation between firm returns and exchange rates, while Allayannis and Ihrig (2001) analyze US manufacturing industries and find a negative correlation.

[3] The former is like the analysis of the demand for insurance by Mossin (1968), where the bivariate variables are wealth level and health status. The latter is like the analysis of production under price risk by Sandmo (1971), where the bivariate variables are the profits of one region and another region.

[4] This model is justifiable as a portfolio problem of a representative investor from empirical observations of a positive relationship between health status and macroeconomic conditions. Empirical results show that the relationship between macroeconomic conditions and the infection status of diseases is related to macroeconomic health status. See Bloom et al. (2022) for a review.

of this paper is how correlation between financial and health risks affects portfolio choices.

Background risk has been extensively studied in the literature under the assumption of independence from endogenous risks. For the portfolio problem, the effects of background risk are analyzed in the univariate setting by Weil (1992) and the bivariate setting by Crainich et al. (2017). Apart from specific contexts, the analysis of background risk can be made using the derived utility function by Kihlstrom et al. (1981) and Nachman (1982). This approach requires the assumption of independence.

Independent background risk contradicts reality in many situations. This leads to the need to relax the independence assumption and introduce dependent background risk. This analysis is not a straightforward extension because the standard approach using derived utility function cannot be applied. The notion of stochastic dependence by Lehmann (1966) is widely used to describe dependent background risk in the literature. This stochastic dependence and its variants are stronger than correlation because these notions imply correlation (Li, 2011). Some interesting results are obtained under dependent background risk. For example, the investment in the risky asset is not undertaken, even with zero excess returns. This relates to the stock market participation puzzle by Mankiw and Zeldes (1991) and Haliassos and Bertaut (1995).

Due to its analytical difficulty, we have obtained few results on dependent background risk compared to independent background risk. Typical previous studies have found conditions for positive or negative investment in a risky asset with zero mean returns in the presence of dependent background risk. We also note that previous studies analyzed the effects of stochastic dependence compared to those of independence. This means that much room for research remains to be done. In this paper, we adopt a different description of stochastic dependence and find conditions under which changes in stochastic dependence affect portfolio choices, not comparison with independence.

This paper describes stochastic dependence following Doherty and Schlesinger (1983) that modelled the relationship between insurable and contract nonperformance risks. This description can capture correlation through a single parameter. Moreover, in addition to facilitating the analysis, this description has several advantages: (i) the structure of cross risk apportionment is embedded into the model to analyze an increase in correlation,[5] (ii) the effect of correlation on portfolio choices can be investigated, and (iii) ambiguity can be incorporated into correlation.

This paper employs the term uncertainty in a broad sense, whereby uncertainty is an umbrella term that includes the notions of risk and ambiguity.[6] Whereas ambiguity about asset returns in a univariate framework has been analyzed in the framework of portfolio problem,[7] ambiguity about correlation in a bivariate framework remains to be investigated.[8] In general, it is difficult to pin down the correlation between two variables, for example, financial and health risks. We overcome this difficulty through ambiguous correlation and investigate its effects on the portfolio problem in a bivariate framework. By adopting the smooth ambiguity model by Klibanoff et al. (2005), which is the primary ambiguity model used in past applications, we are able to differentiate between ambiguity and ambiguity attitude.[9]

The contribution of our paper is threefold. First, we provide a framework for analyzing the effects of correlation between financial and health background risks on portfolio choices under a bivariate setting. We use this framework to determine the conditions under which an increase in correlation monotonically affects the optimal choices. Second, we relate these conditions to cross risk apportionment. Finally, we introduce ambiguity in the correlation and show that ambiguous correlation

[5]The notion of risk apportionment is introduced by Eeckhoudt and Schlesinger (2006), and is characterized by a specific pattern of the preference for a combination of "good" and "bad". To apply to bivariate utility functions, "cross" is added to the beginning of the term.

[6]Since seminal works by Gilboa and Schmeidler (1989) and Schmeidler (1989), the effects of ambiguity on decision-making have been extensively investigated. For textbook explanations, see Gilboa (2009).

[7]See Guidolin and Rinaldi (2013) for a review.

[8]Jiang et al. (2022) tackle ambiguous correlation in the portfolio problem. Using the mean-variance approach, they show that portfolio concentration occurs under the max–min expected utility in Gilboa and Schmeidler (1989).

[9]For applications of the smooth ambiguity model to finance, see Gollier (2011).

decreases the optimal investment in the risky asset.

The organization of this paper is as follows. Section 2 provides a literature review, Section 3 introduces a formal model, and Section 4 investigates the effects of correlation on portfolio choices. Section 5 relates the conditions determined in Section 4 to cross risk apportionment, which is a specific type of preference for the combination of good and bad. Section 6 introduces ambiguity into correlation and investigates its effects on portfolio choices. Section 7 presents higher-order generalizations of the results in previous sections. Section 8 considers correlation seeking and mixed correlation seeking individuals. Section 9 concludes.

2 Literature review

The extant literature has investigated the effects of independent background risk. For example, as early references for specific contexts in addition to Weil (1992), we refer to Eeckhoudt and Kimball (1992) for insurance demand and Bryis et al. (1993) for hedging behavior. Apart from specific contexts, Gollier and Pratt (1996) introduce the notion of risk vulnerability under which the derived utility function is more risk averse than the underlying utility function. That is, background risk leads to more risk averse behaviors under risk vulnerability. Malevergne and Rey (2009) apply the notion of risk vulnerability to the bivariate utility function and term it as cross risk vulnerability. Even though these previous works have obtained fruitful results under independent background risk, as discussed earlier, the results cannot necessarily be obtained under dependent background risk.

There have been several attempts to investigate the effects of dependent background risk. The main idea of dependent background risk draws on that of stochastic dependence by Lehmann (1966) in the statistical literature and Wright (1987) for economic applications. Tsetlin and Winkler (2005), Li (2011), and Chiu (2020) align with this description. Unlike independent background risk, these studies reveal the possibility that investment is not made even with positive excess returns. Even though their findings have some interest in particular problems, we face difficulty in obtaining clear comparative static predictions within this framework. For example, previous studies have not obtained the conditions under which an increase in stochastic dependence leads to more risk averse behaviors.

This paper adopts a different notion of dependence as introduced by Doherty and Schlesinger (1983, 1990) to describe dependent background risk. This dependence is widely adopted in various models, for example, index insurance (Clarke, 2016) comorbidity risk (Bleichrodt et al., 2003; Fujii and Osaki, 2019), and precautionary savings (Courbage et al., 2020; Asano and Osaki, 2022). However, to our knowledge, the introduction of this type of dependence is novel for portfolio problems. Using this dependence, the present paper determines the conditions under which an increase in correlation raises or lowers the investment in the risky assets. We also investigate the effect of ambiguous correlation on portfolio choices.

The signs of the cross derivatives of the bivariate utility functions play a crucial role in determining the effects of correlated background uncertainty. In a univariate framework, Caballé and Pomansky (1996) provide the notion of mixed risk aversion (seekingness). Eeckhoudt and Schlesinger (2006) characterize the notion of mixed risk aversion (seekingness) by the preference for the combination of good and bad as constructed using two building blocks: a sure reduction and a zero-mean risk. This idea is extended to the bivariate utility setting by Eeckhoudt et al. (2007) that characterize the notions of correlation aversion and cross prudence. In a univariate setting, Eeckhdout et al. (2009) adopt a stochastic dominance relation to rank good and bad, and characterize mixed risk aversion (seekingness). This framework is extended to the bivariate setting by Jokung (2011). These works present useful characterizations of individuals' attitudes toward risk and correlation using the signs of the utility functions. A few studies also investigate the signs of the cross derivatives in experimental settings using the framework in Eeckhoudt et al. (2009), e.g., Attema et al., (2019).

3 The model

In this section, we provide a framework for analyzing the effects of the correlation between two risks, namely, financial risk and non-financial background risk, on portfolio choices. As discussed earlier, for convenience, we interpret background risk as a health risk. We suppose that an individual faces a portfolio problem with one safe asset and one risky asset in the presence of a health risk. We consider a static model in which there are two dates, $t = 0$ and $t = 1$. The individual makes a portfolio choice at $t = 0$, and its return is realized at $t = 1$. Utility is determined by wealth level and health status $(w, h) \in W \times H \subseteq \mathbb{R}^2_+$. Let $u : W \times H \to \mathbb{R}$ denote a bivariate utility function. We denote $u_{(1,0)}(w, h)$ as $\partial u/\partial w$, $u_{(0,1)}(w, h)$ as $\partial u/\partial h$, and $u_{(1,1)}(w, h)$ as $\partial^2 u(w, h)/\partial w \partial h$. Similarly, $u_{(i,j)}(w, h)$ stands for $\partial^{i+j} u(w, h)/\partial w^i \partial h^j$. All higher-order partial and cross derivatives are assumed to exist if necessary for the analysis. We impose the standard assumptions on the utility function for wealth level and health status, that is, $u(w, h)$ is increasing and concave in wealth level w and health status h. We consider the two cases of the signs of cross derivatives, $u_{(1,1)}(w, h) \leq 0$ and $u_{(1,1)}(w, h) \geq 0$, which capture individuals' correlation aversion and correlation seekingness, respectively.[10] Because the analysis of correlation seekingness is the same as correlation aversion, we focus on the analysis of correlation aversion and present the results for correlation seekingness in Section 8. Note that we do not make any claims about the validity of correlation aversion and seekingness. Indeed, both cases are possible. It then depends on situations whether correlation aversion or seekingness is compelling based on observations from empirical studies.

The individual has initial wealth w_0 to invest it in either safe or risky assets. The net return of the safe asset is r_f. There are two possible returns of the risky asset: "good", $\tilde{r}_G$, and "bad", $\tilde{r}_B$ with probability p and $1 - p$. The excess returns

[10] Eeckhoudt et al. (2007) show that the notions of correlation aversion and correlation seekingness are captured by the signs of the cross derivatives of a bivariate utility function. See also Epstein and Tanny (1980). In empirical studies, we obtain mixed evidence of correlation aversion and correlation seekingness (for example, Edwards, 2008; Sloan et al., 1990; Viscusi and Evans, 1990). We note that our analytical method can be applied to both correlation aversion and correlation seekingness cases.

are denoted by $\tilde{x}_G = \tilde{r}_G - r_f$ and $\tilde{x}_B = \tilde{r}_B - r_f$. We omit "excess" for the remainder of this analysis because only excess returns are considered. We assume that these random variables are defined over the same compact support $[\underline{x}, \overline{x}]$ with $\underline{x} < 0 < \overline{x}$. The two types of risks are also involved in health status: "good", $\tilde{h}_G$, and "bad", $\tilde{h}_B$ with probability q and $1 - q$, which are defined over the same compact support $[\underline{h}, \overline{h}]$ with $\underline{h} < \overline{h}$. We use the terms "good" and "bad" in the ranking of expected utility; that is, good risk leads to higher expected utility than bad risk.

We introduce the stochastic dependence between risky return and health risks. There are four combinations: two types of risky returns and two types of health risks,

- $\tilde{x}_G$ and $\tilde{h}_G$ with probability kpq;
- $\tilde{x}_B$ and $\tilde{h}_G$ with probability $(1 - kp)q$;
- $\tilde{x}_G$ and $\tilde{h}_B$ with probability $p(1 - kq)$;
- $\tilde{x}_B$ and $\tilde{h}_B$ with probability $1 - p - q + kpq$.

We calculate the probability of risky returns and health risks such that, for example, the good risky return occurs with probability $p = kpq + p(1 - kq)$. Other probabilities of the risky returns and health risks can be calculated similarly. A value of k, taking a positive value, is chosen so that all probabilities are strictly positive and the sum of all probabilities is unity. This k captures the stochastic dependence between the risky return and the health risk. When the value of k is unity, the risky returns and health risks are independent. Indeed, if $k = 1$, then the probability that $\tilde{x}_G$ and $\tilde{h}_G$ occur simultaneously is equal to pq. A value of k greater (less) than unity indicates a positive (negative) correlation. Because correlation increases in k, correlation and k have a one-to-one relation. Thus, we can treat k to identify correlation.

Under this setting, the individual determines how much to invest in the risky and safe assets, which is denoted by $(\alpha, w_0 - \alpha)$, where w_0 denotes the individual's initial wealth. The value of this portfolio at $t = 1$ is written as

$$\begin{aligned} \alpha(1+\tilde{r}) + (w_0 - \alpha)(1+r_f) &= w_0(1+r_f) + \alpha(\tilde{r} - r_f) \\ &= w_1 + \alpha\tilde{x}, \end{aligned}$$

where $\tilde{r}$ denotes the random return of the risky asset, $w_1 = w_0(1+r_f)$, and $\tilde{x} = \tilde{r} - r_f$. Then, the individual determines the investment in the risky asset α to maximize the following expected utility:

$$\begin{aligned} &V(\alpha) \\ &= kpqE[u(w_1 + \alpha\tilde{x}_G, \tilde{h}_G)] + p(1-kq)E[u(w_1 + \alpha\tilde{x}_G, \tilde{h}_B)] \\ &+ (1-kp)qE[u(w_1 + \alpha\tilde{x}_B, \tilde{h}_G)] + (1-p-q+kpq)E[u(w_1 + \alpha\tilde{x}_B, \tilde{h}_B)]. \end{aligned} \tag{1}$$

The first-order condition for (1) is

$$\begin{aligned} &V'(\alpha^*) \\ &= kpqE[\tilde{x}_G u_{(1,0)}(w_1 + \alpha^*\tilde{x}_G, \tilde{h}_G)] + p(1-kq)E[\tilde{x}_G u_{(1,0)}(w_1 + \alpha^*\tilde{x}_G, \tilde{h}_B)] \\ &+ (1-kp)qE[\tilde{x}_B u_{(1,0)}(w_1 + \alpha^*\tilde{x}_B, \tilde{h}_G)] + (1-p-q+kpq)E[\tilde{x}_B u_{(1,0)}(w_1 + \alpha^*\tilde{x}_B, \tilde{h}_B)] = 0. \end{aligned} \tag{2}$$

Because $V(\alpha)$ is concave by $u_{(2,0)}(w,h) \leq 0$, the second-order condition for a maximum is satisfied. For simplicity, we assume that the optimal investment in the risky asset is interior, $0 < \alpha^* < w_0$, and is unique throughout the paper. We sometimes omit the asterisk when there is no possibility of confusion and use the notation $\alpha(k)$ to represent the optimal investment explicitly under the correlation of k.

Let us consider that correlation increases by Δk from k to $k+\Delta k$. This increases the probability of the combination of $\tilde{x}_G$ and $\tilde{h}_G$ and that of $\tilde{x}_B$ and $\tilde{h}_B$ by $\Delta k \times pq$. However, it also decreases the probability of the combination of $\tilde{x}_G$ and $\tilde{h}_B$ and that of $\tilde{x}_B$ and $\tilde{h}_G$ by $\Delta k \times pq$. In other words, an increase in correlation enhances the possibility of the combination of good and good,[11] but reduces the possibility of the combination of good and bad, which suggests that the structure of risk apportionment by Eeckhoudt and Schleisinger (2006) is incorporated into the effects of correlation on portfolio choices.

We can confirm this structure with the marginal expected utility for k. By differentiating (1) with respect to k, it follows that

$$\begin{aligned}\frac{\partial V(\alpha)}{\partial k} &= pq\{E[u(w_1+\alpha\tilde{x}_G,\tilde{h}_G)]+E[u(w_1+\alpha\tilde{x}_B,\tilde{h}_B)] \\ &- (E[u(w_1+\alpha\tilde{x}_G,\tilde{h}_B)]+E[u(w_1+\alpha\tilde{x}_B,\tilde{h}_G)])\}.\end{aligned}$$

The effects of an increase in correlation can be classified into two parts: the former two terms represent gains in utility, and the latter two terms represent losses in utility or disutility. The former is the combination of good and good (and bad and bad), and the latter is the combination of good and bad (bad and good).

4 Correlation and portfolio choice

We suppose that good and bad risks are ranked via first-order stochastic dominance (FSD). In the remainder of this paper, the cumulative distribution functions of good and bad risks are denoted by G and B, respectively. Distribution function $G_{\tilde{x}}$ dominates $B_{\tilde{x}}$ in the sense of FSD if $G_{\tilde{x}}(x) \leq B_{\tilde{x}}(x)$ for all $x \in [\underline{x}, \overline{x}]$. The subscript of a distribution function is used to specify a random variable and it can be omitted with no confusion. We also say that random variable $\tilde{x}_G$ dominates random variable $\tilde{x}_B$ in the sense of FSD. For all $\alpha \in (0, w_0)$ and $h \in [\underline{h}, \overline{h}]$,

$$E[u(w_1+\alpha\tilde{x}_G,h)] \geq E[u(w_1+\alpha\tilde{x}_B,h)]$$

[11] Following the literature, we simply say the combination of good and good instead of bad and bad in the remainder of the paper. Similarly, we apply the same convention for good and bad by omitting bad and good.

for all utility function satisfying $u_{(1,0)}(w,h) \geq 0$ when $\tilde{x}_G$ dominates $\tilde{x}_B$ in the sense of FSD. Because good risky returns lead to higher expected utility than bad risky returns for individuals with $u_{(1,0)}(w,h) \geq 0$, FSD can identify good and bad risky returns. Similarly, good health risk $\tilde{h}_G$ dominates bad health risk $\tilde{h}_B$ in the sense of FSD. Individuals with $u_{(0,1)}(w,h) \geq 0$ then obtain higher expected utility from good health risk than bad health risk.

Before starting the analysis, we prepare the following lemma. The proof can be found in Appendix.

Lemma 1. *Let $\pi(w,h)$ be the payoff function and define*

$$\begin{aligned} E[\pi(\tilde{w},\tilde{h})] = kpqE[\pi(\tilde{w}_G,\tilde{h}_G)] + (1-kpq)E[\pi(\tilde{w}_G,\tilde{h}_B)] \\ + p(1-kq)E[\pi(\tilde{w}_B,\tilde{h}_G)] + (1-p-q+kpq)E[\pi(\tilde{w}_B,\tilde{h}_B)]. \end{aligned}$$

Suppose that

- *$\tilde{w}_G$ dominates $\tilde{w}_B$ in the sense of FSD;*
- *$\tilde{h}_G$ dominates $\tilde{h}_B$ in the sense of FSD.*

The following two conditions are also equivalent:
(i) $\pi_{(1,1)}(w,h) \leq 0$.
(ii) $E[\pi(\tilde{w},\tilde{h})]$ is decreasing in k.

By replacing π with u in Lemma 1, we ascertain that the expected utility for correlation averse individuals decreases in correlation, that is, $\partial V(\alpha)/\partial k \leq 0$ for all $\alpha \in (0,w)$. The result is summarized as the following proposition.

Proposition 1. *The following two conditions are equivalent:*
(i) Individuals are correlation averse.
(ii) An increase in correlation lowers expected utility.

Proposition 1 is a result consistent with the notion of correlation aversion. By reducing investment in the risky asset, individuals can mitigate the effect of correlation because the safe asset does not correlate with background health risk. In the case of correlation aversion, we expect that an increase in correlation reduces investment in the risky asset, $\partial\alpha(k)/\partial k \leq 0$, because an increase in correlation lowers expected utility.

This intuition is only correct in the limited case where correlation averse individuals are *cross prudent neutral*, which is defined by $u_{(2,1)}(w,h) = 0$. We have that

$$\begin{aligned}
&\frac{\partial V'(\alpha)}{\partial k} \leq 0 \\
&\Leftrightarrow E[\tilde{x}_G(u_{(1,0)}(w_1+\alpha\tilde{x}_G,\tilde{h}_G) - u_{(1,0)}(w_1+\alpha\tilde{x}_G,\tilde{h}_B))] \\
&\leq E[\tilde{x}_B(u_{(1,0)}(w_1+\alpha\tilde{x}_B,\tilde{h}_G) - u_{(1,0)}(w_1+\alpha\tilde{x}_B,\tilde{h}_B))]. \qquad (3)
\end{aligned}$$

Because $\tilde{x}_G$ dominates $\tilde{x}_B$ in the sense of FSD, (3) holds under the following condition:

$$E[x(u_{(1,0)}(w_1+\alpha x,\tilde{h}_G) - u_{(1,0)}(w_1+\alpha x,\tilde{h}_B))]$$

is decreasing in x, that is,

$$\begin{aligned}
&\frac{\partial}{\partial x}E[x(u_{(1,0)}(w_1+\alpha x,\tilde{h}_G) - u_{(1,0)}(w_1+\alpha x,\tilde{h}_B))] \leq 0 \\
&\Leftrightarrow E[u_{(1,0)}(w_1+\alpha x,\tilde{h}_G)] + E[\alpha x u_{(2,0)}(w_1+\alpha x,\tilde{h}_G)] \\
&\leq E[u_{(1,0)}(w_1+\alpha x,\tilde{h}_B)] + E[\alpha x u_{(2,0)}(w_1+\alpha x,\tilde{h}_B)].
\end{aligned}$$

By $u_{(2,1)}(w,h) = 0$, we have that

$$E[\alpha x u_{(2,0)}(w_1+\alpha x,\tilde{h}_G)] = E[\alpha x u_{(2,0)}(w_1+\alpha x,\tilde{h}_B)].$$

This leads to the following:

$$E[u_{(1,0)}(w_1+\alpha x,\tilde{h}_G)] \leq E[u_{(1,0)}(w_1+\alpha x,\tilde{h}_B)] \Leftrightarrow u_{(1,1)}(w,h) \leq 0.$$

We have confirmed that an increase in correlation reduces investment in the risky asset for correlation averse individuals who are also cross prudent neutral.

From the above analysis, we know that correlation aversion is not enough to obtain the intuitive comparative static results, and we can guess that the sign of $u_{(2,1)}(w,h)$ plays a crucial role in obtaining the intuitive results. We say that individuals are *cross prudent* if $u_{(2,1)}(w,h) \geq 0$, and are *cross imprudent* if $u_{(2,1)}(w,h) \leq 0$.[12]

We introduce the two intensity measures of cross prudence to obtain definitive comparative static results without assuming cross prudent neutrality.

Definition 1. We call the following two measures *the degree of partial cross prudence* and *the degree of relative cross prudence*, respectively.

- the degree of partial cross prudence:

$$-\frac{xu_{(2,1)}(w+x,h)}{u_{(1,1)}(w+x,h)};$$

- the degree of relative cross prudence:

$$-\frac{wu_{(2,1)}(w,h)}{u_{(1,1)}(w,h)}.$$

The terms "partial" and "relative" are based on the terminology in a univariate setting, partial risk aversion (Menezes and Hanson, 1970) and relative risk aversion (Pratt, 1964).[13]

Because it follows from (2) that $V'(\alpha) = E[\pi(\tilde{x},\tilde{h})]$ by setting $\pi(x,h) = xu_{(1,0)}(w_1 + \alpha x, h)$ and applying Lemma 1, we obtain that $V'(\alpha)$ decreases in k if and only if

$$\pi_{(1,1)}(x,h) \leq 0 \Leftrightarrow -\frac{\alpha(k)xu_{(2,1)}(w_1+\alpha(k)x,h)}{u_{(1,1)}(w_1+\alpha(k)x,h)} \leq 1. \tag{4}$$

Using the terminology of Definition 1, the condition can be said that the partial cross prudence is less than unity. Because $V'(\alpha)$ decreases in k, we know that the

[12] In Eeckhoudt et al. (2007), individuals with $u_{(2,1)}(w,h) \geq 0$ is are referred to as cross prudent in health to distinguish them from individuals with $u_{(1,2)}(w,h) \geq 0$, who are cross prudent in wealth. Because $u_{(2,1)}(w,h)$ only appears in this paper, we can omit "health" without confusion.

[13] These intensity measures are used in the context of the consumption–saving model by Courbage et al. (2022).

investment in the risky asset decreases in k, which is obtained by the following:

$$V'(\alpha(k);k) = 0 \Rightarrow V'(\alpha(k);k') \leq 0$$

for $k \geq k'$. The value k (k') after the semicolon means that correlation is equal to k (k').

Assuming cross prudence, $u_{(2,1)}(w,h) \geq 0$, a sufficient condition for (4) is that

$$-\frac{wu_{(2,1)}(w,h)}{u_{(1,1)}(w,h)} \leq 1. \tag{5}$$

This condition means that the degree of relative cross prudence is less than unity. We summarize the above argument as the following proposition and provide interpretations of these conditions in the next section.

Proposition 2. *Suppose that individuals are correlation averse. Then,*
(i) the degree of partial cross prudence is less than unity if and only if an increase in correlation reduces the investment in the risky asset, and
(ii) if individuals are cross prudent and the degree of relative cross prudence is less than unity, then an increase in correlation reduces the investment in the risky asset.

From (i) in Proposition 2, when partial cross prudence is more than unity, we reach the counterintuitive result: correlation averse individuals increase their investment in the risky asset with higher correlation. When individuals are correlation averse and cross prudence, partial cross prudence is always negative for negative returns. In this case, the condition cannot be satisfied. We should note that partial cross prudence depends on the investment in the risky asset, which is an endogenous variable, but relative cross prudence does not depend on it.

As discussed in Section 3, an increase in correlation has a greater likelihood of providing the combination of good and good than good and bad. The combination of good and good is undesirable for correlation averse individuals because an increase in correlation lowers the expected utility as in Proposition 1.[14] There are two effects

[14] We relate the preference for combining good with bad (good) to the signs of the cross derivatives

concerning the FSD shift of risky returns: the substitution effect and the wealth effects. In a univariate setting, it is known that the wealth effect dominates the substitution effect when the degree of relative risk aversion is less than unity (e.g., Fishburn and Porter, 1976). Correlation averse individuals should then reduce their investment in the risky asset in response to an increase in correlation to avoid the undesirable combination when the wealth effect dominates the substitution effect. To determine the condition, we need to incorporate the effect of health risks. Combining these two effects, the condition is determined by comparing the degrees of relative or partial cross prudence with the threshold value being equal to unity.

Example 1. As a concrete example, we consider the following Cobb–Douglas utility function:

$$u(w,h) = \frac{(w^{\psi}h^{1-\psi})^{1-\gamma}}{1-\gamma}.$$

Here, ψ takes a value in $[0,1]$ for $u_{(1,0)}(w,h), u_{(0,1)}(w,h) \geq 0$ and $u_{(2,0)}(w,h), u_{(0,2)}(w,h) \leq 0$. The sign of $u_{(1,1)}(w,h)$ depends on the parameter value of γ. When $\gamma = 1$, we have that $u_{(1,1)}(w,h) = 0$, that is, $\gamma = 1$ is the threshold to determine the signs of $u_{(1,1)}(w,h)$. We have that $u_{(1,1)}(w,h) > (<)0$ for $\gamma < (>)1$, which represents correlation seekingness (aversion). For $\gamma < 1$, it holds that

$$-\frac{wu_{(2,1)}(w,h)}{u_{(1,1)}(w,h)} = 1-(1-\gamma)\psi \leq 1.$$

Thus, from Proposition 8 in Section 8, we conclude that an increase in correlation increases the investment in the risky asset for investors who have the Cobb–Douglas utility function with $\gamma < 1$. For $\gamma > 1$, it holds that

$$-\frac{wu_{(2,1)}(w,h)}{u_{(1,1)}(w,h)} = 1-(1-\gamma)\psi \geq 1. \tag{6}$$

For correlation averse individuals, we cannot obtain clear comparative statics results. We note that an increase in correlation lowers the investment in the risky asset even

of the utility function in the following section.

though (6) holds but (4) is satisfied.

It is known that the sign of the investment in the risky asset is the same as that of the risky asset return in a univariate setting, that is, individuals invest in a positive amount of risky assets when their expected returns are positive. This result can be extended to the portfolio problem in the presence of independent background risk. For our setting, $k = 1$ corresponds to independent background risk. We have that

$$\begin{aligned} V'(0) &= E[\tilde{y}u_{(1,0)}(w_1, \tilde{z})] \\ &= E[\tilde{y}]E[u_{(1,0)}(w_1, \tilde{z})], \end{aligned} \tag{7}$$

where $\tilde{y} = p\tilde{x}_G + (1-p)\tilde{x}_B$ and $\tilde{z} = q\tilde{h}_G + (1-q)\tilde{h}_B$.

Let us consider a risky asset $\tilde{y}$ whose expected return is equal to zero, $E[\tilde{y}] = pE[\tilde{x}_G] + (1-p)E[\tilde{x}_B] = 0$. When k is more than unity, correlation averse individuals make negative investments in the risky asset when (5) holds by Proposition 2 and short selling is allowed. When short selling is not allowed, the negative investment means that individuals do not hold the risky asset and hold only the safe asset. By a continuity argument, this means that correlation averse individuals may make negative investments in risky assets with positive expectations. This observation can be related to the stock market participation puzzle by Mankiw and Zeldes (1991) and Haliassos and Bertaut (1995). The puzzle involves empirical observations that many households do not own any equities and hold only safe assets despite positive excess returns.

5 Cross risk apportionment

Eeckhoudt and Schlesinger (2006) introduce a notion of risk apportionment as particular types of preferences between two lotteries. This preference can be related to the signs of higher-order derivatives of a utility function in a univariate setting within an expected utility framework. A sure reduction and noise risk are two building blocks used to construct the pairs of lotteries to characterize risk apportionment. By comparing zero, these two building blocks are bad for $u'(w) \geq 0$ and $u''(w) \leq 0$.[15] The two lotteries are constructed by the combination of good and bad: combining good with bad and good with good. Risk apportionment can be classified into two types: the preferences for combining good with bad and good with good. Eeckhoudt and Schlesinger (2006) show that the former (good and bad) is related to $(-1)^{n+1}u^n(w) \geq 0$[16] and Crainich et al. (2013) show that the latter (good and good) is related to $u^n(w) \geq 0$. In this section, we show that all the second conditions in Proposition 2 are related to the particular type of preference for the combination of good and bad in a bivariate setting. In the present paper, this type of preference is called *cross risk apportionment*, following the term in a univariate setting. As in risk apportionment, cross risk apportionment can be classified into two types: the preference for combining good with bad or good with good. Cross risk apportionment can be considered a consistent preference because it exhibits the preference for the same pattern of the lottery preference.

First, we consider the two conditions in Proposition 2: correlation aversion and cross prudence. Let $(w, h) \in \mathbb{R}^2_+$ denote a nonnegative vector of wealth level and health status, and let $[A; B]$ denote a lottery where outcomes A and B occur with probability one-half. For positive constants k and c with $k < w$ and $c < h$, we define two lotteries: $A_{(1,1)} = [(w, h); (w - k, h - c)]$ and $B_{(1,1)} = [(w - k, h); (w, h - c)]$.[17] Similarly, for a positive constant c and a noise risk $\tilde{\epsilon}$, we define two lotteries: $A_{(2,1)} = [(w, h); (w + \tilde{\epsilon}, h - c)]$ and $B_{(2,1)} = [(w + \tilde{\epsilon}, h); (w, h - c)]$. The lotteries have the same pattern for the combination of good and bad. Lotteries $A_{(1,1)}$ and $A_{(2,1)}$ are constructed by combining good with good. Conversely, lotteries $B_{(1,1)}$ and $B_{(2,1)}$

[15] A sure reduction $-k$ with $k > 0$ lowers utility for every increasing utility function because $u(w-k) \leq u(w)$ given intimal wealth w. Now recall that the term of good and bad is related to the ranking of expected utility. Comparing $[-k]$ with $[0]$, we can say that $[-k]$ is bad and $[0]$ is good. Similarly, we can say that a noise risk $[\tilde{\varepsilon}]$ is bad for every concave utility function by comparing $[0]$.

[16] We denote $u^n(w) = \partial^n u(w)/\partial w^n$.

[17] Because this pair of lotteries is used to characterize the sign of $u_{(1,1)}(w, h)$, the subscript of, for example, $A_{(1,1)}$, corresponds to the subscript of utility function $u_{(1,1)}(w, h)$. Similar notations are applied to other lottery pairs.

are constructed by combining good with bad. Following Eeckhoudt et al. (2007), particular types of lottery preferences between the above pairs can be related to the signs of the cross derivatives of a bivariate utility function.

Result 1 (Eeckhoudt et al. (2007))**.** *Eeckhoudt et al. (2007) show the following:*
(i) Individuals are correlation averse, $u_{(1,1)}(w,h) \leq 0$ *if and only if they prefer* $B_{(1,1)}$ *over* $A_{(1,1)}$*.*
(ii) Individuals are cross prudent, $u_{(2,1)}(w,h) \geq 0$ *if and only if they prefer* $B_{(2,1)}$ *over* $A_{(2,1)}$*.*

Next, we consider another condition in Proposition 2: the degree of relative cross prudence is less than unity. We introduce a multiplicative version of lottery pairs. For $0 < k < 1$ and $0 < r < 1$, we define the two lotteries: $\mathcal{A}_{(2,1)} = [(w,h);(w(1-k)(1-r),h)]$ and $\mathcal{B}_{(2,1)} = [(w(1-k),h);(w(1-r),h)]$. The lotteries are the same as Eeckhoudt et al. (2009a), except that health status is included. Next, we consider preferences between two lotteries for marginal utility in health, $u_{(0,1)}(w,h)$, to incorporate the effect of health risks into a bivariate utility function. This idea is in the same spirit as Kimball (1990), where the precautionary premium is defined for marginal utility. In other words, precautionary effects against health risks are incorporated into the lottery preferences. Applying an argument of Eeckhoudt et al. (2009a), we obtain the following proposition.

Proposition 3. *When individuals are correlation averse, the following conditions are equivalent:*
(i) The degree of relative cross prudence is less than unity.
(ii) Lottery $\mathcal{B}_{(2,1)}$ *is preferred to* $\mathcal{A}_{(2,1)}$*.*

Proof. See Appendix. □

In Proposition 2, there are three conditions for obtaining intuitive comparative static results:

- correlation aversion, cross prudence, and the degree of relative cross prudence being less than unity;

These conditions are related to cross risk apportionment that corresponds to the preference for combining good with bad. In other words, the second conditions in Proposition 2 are consistent with the preference for the combination of good and bad.

6 Ambiguous correlation

In general, understanding the relationship of stochastic dependence between financial and health risks is a more difficult task when compared with grasping the financial and health risks themselves. Applying to our context, it is more difficult to determine correlation between risky returns and health risks compared with figuring out risky returns and health risks themselves. To capture this difficulty, we introduce ambiguity into correlation through the parameter k. We assume that a plausible set of possible k is the set $\{k_1, k_2, \ldots, k_\Theta\}$. Without loss of generality, k_θ is arranged in ascending order, $k_1 < k_2 < \ldots < k_\Theta$. The individual attaches subjective probability μ_θ to the parameter of k_θ for $\theta = 1, 2, \ldots, \Theta$. We assume that the individual employs the smooth ambiguity model in Klibanoff et al. (2005). Define a concave and strictly increasing function ϕ whose variable takes expected utility. The function ϕ is assumed to be twice differentiable. The concavity of ϕ captures ambiguity aversion. The convexity and linearity of ϕ correspond to ambiguity seeking and ambiguity neutral attitudes, respectively.

Given α, the objective function is written as

$$V(\alpha) = \sum_{\theta=1}^{\Theta} \mu_\theta \phi(U(\alpha, k_\theta)). \tag{8}$$

Here, $U(\alpha, k)$ is the expected utility given α and k, that is, for $\theta = 1, 2, \ldots, n$,

$$\begin{aligned} U(\alpha, k_\theta) = {} & kpqE[u(w_1 + \alpha\tilde{x}_G, \tilde{h}_G)] + p(1-kq)E[u(w_1 + \alpha\tilde{x}_G, \tilde{h}_B)] \\ & + (1-kp)qE[u(w_1 + \alpha\tilde{x}_B, \tilde{h}_G)] + (1-p-q+kpq)E[u(w_1 + \alpha\tilde{x}_B, \tilde{h}_B)]. \end{aligned}$$

As in the previous sections, good and bad are ranked via FSD, that is, $\tilde{x}_G$ dominates $\tilde{x}_G$ in the sense of FSD, and $\tilde{h}_G$ dominates $\tilde{h}_G$ in the sense of FSD,

The first-order condition for (8) is

$$V'(\alpha^*) = \sum_{\theta=1}^{\Theta} \mu_\theta \phi'(U(\alpha^*, k_\theta))U_\alpha(\alpha^*, k) = 0. \tag{9}$$

The second-order condition is easily verified by the concavity of u and ϕ.

We define k_O by $k_O = \sum_\theta q_\theta k_\theta$ and find $i \in \{1, 2, \ldots, n\}$ such that $k_i \leq k_O \leq k_{i+1}$. The level of the investment in the risky asset is denoted by α^O under k_O. Because linear ϕ corresponds to expected utility, we have

$$U_\alpha(\alpha_O, k_O) = \sum_{\theta=1}^{n} \mu_\theta U(\alpha_O, k_\theta) = 0.$$

We assume that u exhibits correlation aversion. From Proposition 1, $U(\alpha, k)$ decreases in k. This leads to the following:

- $U(\alpha_O, k_\theta) \leq U(\alpha_O, k_O) \Leftrightarrow \phi'(U(\alpha_O, k_\theta)) \leq \phi'(U(\alpha_O, k_O))$ for $\theta = 1, \ldots, i$, and
- $U(\alpha_O, k_\theta) \geq U(\alpha_O, k_O) \Leftrightarrow \phi'(U(\alpha_O, k_\theta)) \geq \phi'(U(\alpha_O, k_O))$ for $\theta = i+1, \ldots, n$.

From Proposition 2, the investment in the risky asset decreases in k when the individual is correlation averse, cross prudent, and

$$-\frac{wu_{(2,1)}(w,h)}{u_{(1,1)}(w,h)} \leq 1.$$

This leads to the following:

- $U_\alpha(\alpha_O, k_\theta) \leq 0$ for $\theta = 1, \ldots, i$, and
- $U_\alpha(\alpha_O, k_\theta) \geq 0$ for $\theta = i+1, \ldots n$.

From the above two relations, we have the following:

- $\phi'(U(\alpha_O, k_\theta))U_\alpha(\alpha_O, k_\theta) \leq \phi'(U(\alpha_O, k_O))U_\alpha(\alpha_O, k_O) \leq 0$ for $\theta = 1, \ldots, i$, and

- $0 \leq \phi'(U(\alpha_O, k_\theta))U_\alpha(\alpha_O, k_\theta) \leq \phi'(U(\alpha_O, k_O))U_\alpha(\alpha_O, k_O)$ for $\theta = i+1, \ldots n$.

Finally, we obtain the following:

$$\begin{aligned} V'(\alpha^*) &= \sum_{\theta=1}^{i} \mu_\theta \phi'(U(\alpha_O, k_\theta))U_\alpha(\alpha_O, k_\theta) + \sum_{\theta=i+1}^{n} \mu_\theta \phi'(U(\alpha_O, k_\theta))U_\alpha(\alpha_O, k_\theta) \\ &\leq \sum_{\theta=1}^{i} \mu_\theta \phi'(U(\alpha_O, k_O))U_\alpha(\alpha_O, k_O) + \sum_{\theta=1}^{n} \mu_\theta \phi'(U(\alpha_O, k_O))U_\alpha(\alpha_O, k_O) \qquad (10) \\ &= \phi'(U(\alpha_O, k_O) \sum_{\theta=1}^{n} U_\alpha(\alpha_O, k_\theta) = 0. \end{aligned}$$

Inequality (10) shows that ambiguous correlation reduces the investment in the risky asset. We summarize the above argument into the following proposition.

Proposition 4. *Suppose that risky return and health risk are ranked by FSD. If u exhibits correlation aversion, cross prudence, and*

$$-\frac{wu_{(2,1)}(w,h)}{u_{(1,1)}(w,h)} \leq 1,$$

then ambiguous correlation reduces the investment in the risky asset.

Ambiguity aversion puts more weight on worse expected utilities. For correlation averse individuals, worse expected utilities correspond to higher correlation. From Proposition 2, the conditions on u guarantee that the demand for the risky asset is lower when individuals face higher correlation. Combining the above arguments, ambiguous correlation reduces the investment in the risky asset.

7 Higher-order generalization

This section considers higher-order stochastic dominance relations to rank good and bad risks. The basic idea can be shared with the case of FSD, and the analysis can be extended from FSD to higher-order stochastic dominance relations. We consider two types of higher-order stochastic dominance, N-th order stochastic dominance (NSD, e.g., Ingersoll, 1987) and an N-th degree increase in risk (Ekern, 1980). Let $G_{\tilde{w}}$ and $B_{\tilde{w}}$ denote two cumulative distribution functions of good and bad wealth risks, $\tilde{w}_G$ and $\tilde{w}_B$, that are defined on the same compact support, $[\underline{w}, \overline{w}]$. Let us define $G^1_{\tilde{w}}(w) = G_{\tilde{w}}(w)$ and $G^{n+1}_{\tilde{w}}(w) = \int_{\underline{w}}^{w} G^n_{\tilde{w}}(z)dz$ for $n = 1, 2, \ldots, N$ and $B^1_{\tilde{w}}(w) = B_{\tilde{w}}(w)$ and $B^{n+1}_{\tilde{w}}(w) = \int_{\underline{w}}^{w} B^n_{\tilde{w}}(z)dz$ for $n = 1, 2, \ldots, N$. Distribution function $G_{\tilde{w}}$ dominates distribution function $B_{\tilde{w}}$ in the sense of NSD if $G^N_{\tilde{w}}(w) \leq B^N_{\tilde{w}}(w)$ for all $w \in [\underline{w}, \overline{w}]$ and $G^n_{\tilde{w}}(\overline{w}) \leq B^n_{\tilde{w}}(\overline{w})$ for $n = 1, 2, \ldots, N-1$. As in Section 4, we also say that $\tilde{w}_G$ dominances $\tilde{w}_B$ in the sense of NSD when random variables $\tilde{w}_G$ and $\tilde{w}_B$ have the distribution functions $G_{\tilde{w}}$ and $B_{\tilde{w}}$, respectively. Similarly, we can define NSD for health risks $\tilde{h}_G$ and $\tilde{h}_B$ whose distribution functions are $G_{\tilde{h}}$ and $B_{\tilde{h}}$, respectively.

We assume that the good risky return $\tilde{x}_G$ dominates the bad risky return $\tilde{x}_B$ in the sense of NSD, and that the good health risk $\tilde{h}_G$ dominates the bad health risk $\tilde{h}_B$ in the sense of MSD. We also assume that individuals are mixed risk averse in both wealth and health, that is, $(-1)^{i+1}u_{(i,0)}(w, h) \geq 0$ for $i = 1, 2, \ldots, N$ and $(-1)^{j+1}u_{(0,j)}(w, h) \geq 0$ for $j = 1, 2, \ldots, M$.[18] By this assumption, stochastic dominance relations of wealth and health risks can be related to the ranking of expected utility. Thus, good risky return (health risk) leads to higher expected utility than does bad risky return (health risk). We say that individuals are mixed correlation averse if

$$(-1)^{i+j+1}u_{(i,j)}(w, h) \geq 0 \text{ for } i = 1, 2, \ldots, N \text{ and } j = 1, 2, \ldots, M.$$

We define the two intensity measures for mixed correlation aversion as follows.

Definition 2. We call the following two measures *the degree of partial mixed correlation aversion* and *the degree of relative mixed correlation aversion*, respectively.

[18] The term for mixed risk aversion is borrowed from Caballé and Pomansky (1996) in a univariate setting. The property that $(-1)^{n+1}u^n(w) \geq 0$ is also called complete monotone.

- the degree of partial mixed correlation aversion in (n, m):

$$-\frac{xu_{(n+1,m)}(w+x,h)}{u_{(n,m)}(w+x,h)}.$$

- the degree of relative mixed correlation aversion in (n, m):

$$-\frac{wu_{(n+1,m)}(w,h)}{u_{(n,m)}(w,h)}.$$

These measures are higher-order versions of the degrees of partial and relative cross prudence. From the above preparation, we provide the following results about correlation and portfolio choices.

Proposition 5. *Suppose that good risky return dominates bad risky return in the sense of NSD and good health risk dominates bad health risk in the sense of MSD, and individuals are mixed correlation averse. Then,*
(i) the degree of partial mixed correlation aversion in (n, m) *is less than* n *for* $n = 1, 2, \ldots, N$ *and* $m = 1, 2, \ldots, M$ *if and only if an increase in correlation reduces the investment in the risky asset, and*
(ii) if the degree of relative mixed correlation aversion in (n, m) *is less than* n *for* $n = 1, 2, \ldots, N$ *and* $m = 1, 2, \ldots, M$*, then an increase in correlation reduces the investment in the risky asset.*

Proof. See Appendix. □

Next, we consider whether N-th and M-th degree increases in risk introduced by Ekern (1980) can be related to the signs of the higher-order cross derivatives of the utility function. Random variable $\tilde{w}_G$ is an N-th degree riskier than random variable $\tilde{w}_G$ if $G^N_{\tilde{w}}(w) \leq B^N_{\tilde{w}}(w)$ for all $w \in [\underline{w}, \overline{w}]$ and $G^n_{\tilde{w}}(\overline{w}) = B^n_{\tilde{w}}(\overline{w})$ for $n = 1, 2, \ldots, N-1$. The latter condition means that the first $N-1$ moments have the same values. Individuals with $(-1)^{N+1}u_{(N,0)}(w, h) \geq 0$ prefer $\tilde{w}_G$ over $\tilde{w}_B$. Similarly, we can define good and bad health risks in which $\tilde{h}_G$ is an M-th degree riskier than $\tilde{h}_G$. For example, $N = 2$ corresponds to the increase in risk introduced by Rothschild and

Stiglitz (1970). It is constructed using a series of mean-preserving spreads. Good risky return (health risk) leads to higher expected utility than does bad risky return (health risk) when individuals are averse to wealth (health) risk.[19]

From Proposition 5, we can obtain the following corollary.

Corollary 1. *Suppose that bad risky return is an N-th degree riskier than good risky return and bad health risk is an M-th degree riskier than good health risk, and that $(-1)^{N+M+1}u_{(N,M)}(w,h) \geq 0$. Then,*
(i) the degree of partial mixed correlation aversion in (N,M) is less than N if and only if an increase in correlation reduces the investment in the risky asset, and
(ii) if the degree of relative mixed correlation aversion in (N,M) is less than N, then an increase in correlation reduces the investment in the risky asset.

All the conditions in Proposition 2 can be related to cross risk apportionment and the specific type of lottery preferences for the combination of good and bad. Following Jokung (2011), we relate the signs of the higher-order cross derivatives to cross risk apportionment. Suppose that $\tilde{w}_G$ dominates $\tilde{w}_B$ in the sense of NSD and $\tilde{h}_G$ dominates $\tilde{h}_B$ in the sense of MSD. Let us consider two lotteries: $A_{(N,M)} = [(\tilde{w}_G, \tilde{h}_G); (\tilde{w}_B, \tilde{w}_B)]$ and $B_{(N,M)} = [(\tilde{w}_G, \tilde{h}_B); (\tilde{w}_B, \tilde{w}_G)]$. The lottery $A_{(N,M)}$ is constructed by combining good with bad, and the lottery $B_{(N,M)}$ is constructed by combining good with good. Using the lottery preference, mixed correlation averse individuals prefer $B_{(N,M)}$ to $A_{(N,M)}$. The following result is a natural extension of Eeckhdout et al. (2009) from the univariate to bivariate setting.

Result 2 (Jokung (2011))**.** *Individuals are mixed correlation averse if and only if $(-1)^{i+j+1}u_{(i,j)}(w,h) \geq 0$ for $i = 1,2,\ldots,N$ and $j = 1,2,\ldots,M$.*

For $N = M = 1$, the condition degenerates into correlation aversion.

[19] $N = 3$ corresponds to the increase in downside risk in Menezes et al. (1980) that is ranked by skewness with the same means and variances. The preference for an increase in downside risk can be related to the positive third derivative of the utility function and is called prudence. $N = 4$ corresponds to the increase in outer risk by Menezes and Wang (2005) that is ranked by kurtosis with the same means, variances, and skewnesses. The preference for an increase in outer risk can be related to the negative fourth derivative of the utility function and is called temperance.

Finally, we consider the condition on the degree of relative mixed risk aversion. We now apply the lottery preference to $u_{(0,m)}(w,h)$ for $m = 1,2,\ldots,M$ to incorporate the effect of changes in health risks. Let us consider two lotteries that are multiplicative versions of the above lotteries: $\mathcal{A}_{(N,M)} = [(w(1+\tilde{\epsilon}_G)(1+\tilde{\delta}_B),h);(w(1+\tilde{\epsilon}_B)(1+\tilde{\delta}_G),h)]$ and $\mathcal{B}_{(N,M)} = [(w(1+\tilde{\epsilon}_G)(1+\tilde{\delta}_G),h);(w(1+\tilde{\epsilon}_B)(1+\tilde{\delta}_B),h)]$. Here, random variables $\tilde{\epsilon}_G$ and $\tilde{\epsilon}_B$ are defined over the same compact support $[-1,\overline{\epsilon}]$, and random variables $\tilde{\delta}_G$ and $\tilde{\delta}_B$ are defined over the same compact support $[-1,\overline{\delta}]$. $\tilde{\epsilon}_G$ and $\tilde{\epsilon}_B$ are ranked via NSD, and $\tilde{\delta}_G$ and $\tilde{\delta}_B$ are ranked via FSD. The four risks ($\tilde{\epsilon}_G$, $\tilde{\epsilon}_B$, $\tilde{\delta}_G$ and $\tilde{\delta}_B$)in the lotteries $\mathcal{A}_{(N,M)}$ and $\mathcal{B}_{(N,M)}$ are mutually independent. The following characterizations can be viewed as an extension of Eeckhdout et al. (2009) from a univariate setting with additive lotteries to a bivariate setting with multiplicative lotteries.[20] The proof can be found in Appendix.

Proposition 6. *When individuals are mixed correlation averse, the following two conditions are equivalent:*
(i) The degree of relative mixed correlation aversion of (n,m) *is less than* n *for* $n = 1,2,\ldots,N$ *and* $m = 1,2,\ldots,M$.
(ii) Lottery $\mathcal{B}_{(N,M)}$ *is preferred to* $\mathcal{A}_{(N,M)}$.

8 Correlation seekingness

Up until now, we focused on the analysis of the correlation averse or mixed correlation averse individuals. This section provides the results for correlation seeking individuals parallel to those of correlation averse individuals. Therefore, we only present the results and omit their proofs and interpretations.

Note that correlation seeking individuals and cross imprudence individuals are captured by $u_{(1,1)}(w,h) \geq 0$ and $u_{(2,1)}(w,h) \geq 0$, respectively. In this section, good and bad are ranked via FSD, that is, $\tilde{x}_G$ dominates $\tilde{x}_B$ in the sense of FSD, and

[20] Wang and Li (2010) provide a higher-order generalization of Eeckhoudt et al. (2009a) following Eeckhoudt and Schlesinger (2006). They construct the lottery pairs using the combination of a sure reduction and independent risks.

$\tilde{h}_G$ dominates $\tilde{h}_B$ in the sense of FSD. To obtain the first two results, we use the following lemma, which is a symmetric version of Lemma 1.

Lemma 2. *Let consider $\pi(w,h)$ in Lemma 1. Suppose that*

- *$\tilde{w}_G$ dominates $\tilde{w}_B$ in the sense of FSD;*
- *$\tilde{h}_G$ dominates $\tilde{h}_B$ in the sense of FSD.*

The following two conditions are also equivalent:
(i) $\pi_{(1,1)}(w,h) \geq 0$.
(ii) $E[\pi(\tilde{w},\tilde{h})]$ is increasing in k.

Applying Lemma 2, Proposition 7 is obtained by setting $\pi(x,h) = u(w+\alpha x, h)$ and Proposition 8 is obtained by setting $\pi(x,h) = xu(w+\alpha x, h)$.

Proposition 7. *The following two conditions are also equivalent:*
(i) Individuals are correlation seeking.
(ii) An increase in correlation raises expected utility.

Proposition 8. *Suppose that individuals are correlation seeking. Then,*
(i) the degree of partial cross prudence is less than unity if and only if an increase in correlation increases the investment in the risky asset, and
(ii) if individuals are cross imprudent and the degree of relative cross prudence is less than unity, then an increase in correlation increases the investment in the risky asset.

Note that the conditions in Proposition 2 are the same as those in Proposition 8. This is because the signs of $u_{(1,1)}(w,h)$ differ between correlation aversion and correlation seekingness. Because the sign is reversed twice, comparing correlation aversion and correlation seekingness results in the same conditions appearing in Propositions 2 and 8.

Applying a similar argument in Section 5, we have the following proposition.

Proposition 9. *Suppose that the risky return and health risk are ranked by FSD. If u exhibits correlation seekingness and cross imprudence, and*

$$-\frac{wu_{(2,1)}(w,h)}{u_{(1,1)}(w,h)} \leq 1,$$

then ambiguous correlation decreases the investment in the risky asset.

The conditions in Proposition 8 can be related to the lottery preference.

Result 3 (Eeckhoudt et al. (2007))**.** *Eeckhoudt et al. (2007) show the following:*
(i) Individuals are correlation seeking, $u_{(1,1)}(w,h) \geq 0$, if and only if they prefer $A_{(1,1)}$ over $B_{(1,1)}$.
(ii) Individuals are cross imprudent, $u_{(2,1)}(w,h) \leq 0$, if and only if they prefer $A_{(2,1)}$ over $B_{(2,1)}$.

The following proposition corresponds to Proposition 3.

Proposition 10. *When individuals are correlation seeking, the following conditions are equivalent:*
(i) The degree of relative cross prudence is less than unity.
(ii) Lottery $\mathcal{A}_{(2,1)}$ is preferred to $\mathcal{B}_{(2,1)}$.

We note that the signs of $u_{(1,1)}(w,h)$ differ between correlation aversion and seekingness, and that the first condition in Proposition 3 and 10 is the same, that is, the degree of relative cross prudence is less than unity.

In Proposition 8, there are three conditions for obtaining intuitive comparative static results:

- correlation seekingness, cross imprudence, and the degree of relative cross prudence being less than unity.

Combining Result 3 and Proposition 10, these conditions are related to cross risk apportionment, which corresponds to the preference for combining good with good. In other words, the conditions in Proposition 8 are consistent with the preference for the combination of good and good.

As in FSD, we can obtain the corresponding results of higher-order stochastic dominance for mixed correlation seeking individuals. Thus, we omit these to avoid repetition.

9 Conclusion

This paper considered the portfolio problem with one risk-free asset and one risky asset in the presence of background risk. We investigated the effects of correlation on portfolio choices. While increases in correlation lower (raise) expected utility for mixed correlation averse (seeking) individuals, contrary to intuition, they do not necessarily reduce (an increase) the investment in the risky asset. We determined the conditions for a reduction (increase) in the investment. The conditions can be related to the preference for the combination of good and bad. Furthermore, we introduced ambiguity into correlation and investigated its effects on the portfolio choices.

There are several directions for future research that extend the analysis of this paper. First, because the linear payoff problem has versability, the analysis can be applied to specific problems, such as the demand for insurance and a firm's production under price risk, by incorporating characteristics specific to each problem. Second, it is interesting to consider health risk as controllable. In this paper, health risk is treated as background risk. However, it may be controlled by introducing, for example, insurance and prevention activities. Third, we can consider other forms of stochastic dependence beyond the specific form used in this paper. Notions such as copula, majorization, and supermodular, are promising candidates to investigate the effects of interdependence in more general settings as in Meyer and Strulovici (2015). Using these notions, our analysis may be extended to interdependence among general multivariate variables. This should be left for future research.

We should mention that experimental studies are worth investigating. Only a few studies have experimentally investigated higher-order risk preferences based on bivariate utility functions (e.g., Attema et al., 2019). These studies are context-free and use the framework of Eeckhoudt et al. (2007). However, to the best of our knowledge, there is no experimental investigation of these under specific contexts. In addition to our results, experimental studies would illuminate the importance of higher-order risk preferences and the effects of correlation in multidimensional frameworks.

Appendix

Proof of Lemma 1. We show that (i) implies (ii). By straightforward calculation, we have

$$\begin{aligned} &\operatorname{sgn}\left(\frac{\partial}{\partial k}E[\pi(\tilde{w},\tilde{h})]\right) \leq 0 \\ &\Leftrightarrow E[\pi(\tilde{w}_G,\tilde{h}_G)] - E[\pi(\tilde{w}_G,\tilde{h}_B)] \leq E[\pi(\tilde{w}_B,\tilde{h}_G)] - E[\pi(\tilde{w}_B,\tilde{h}_B)]. \end{aligned} \tag{11}$$

Because $\tilde{w}_G$ dominates $\tilde{w}_B$ in the sense of FSD, (11) holds if $E[\pi(w,\tilde{h}_G)]-E[\pi(w,\tilde{h}_B)]$ decreases in w, which is equivalent to $E[\pi_{(1,0)}(w,\tilde{h}_G)] \leq E[\pi_{(1,0)}(w,\tilde{h}_B)]$. Again applying the property of FSD, we have

$$E[\pi_{(1,0)}(w,\tilde{h}_G)] \leq E[\pi_{(1,0)}(w,\tilde{h}_B)] \Leftrightarrow \pi_{(1,1)}(w,h) \leq 0.$$

We conclude the proof that (i) implies (ii).

Next, we show by contradiction that (ii) implies (i). Assume that $\pi_{(1,1)}(w^o,h^o) > 0$ for the neighborhood of w^o and h^o. Let us consider that $\tilde{w}_G$ and $\tilde{w}_B$ that are identical except for the neighborhood of w^o and h^o. In this case, we have that $E[\pi(\tilde{w},\tilde{h})]$ increases in k. Because this is a contradiction, we complete the proof. □

Proof of Proposition 3. We only provide a proof that (ii) implies (i). The opposite can be easily proven by contradiction, and it can be omitted.

Because we assume that $\mathcal{B}_{(2,1)} \succeq \mathcal{A}_{(2,1)}$, we have

$$\begin{aligned} &\frac{1}{2}u_{(0,1)}(w(1-r),h) + \frac{1}{2}u_{(0,1)}(w(1-k),h) \geq \frac{1}{2}u_{(0,1)}(w,h) + \frac{1}{2}u_{(0,1)}(w(1-r)(1-k),h) \\ \Leftrightarrow\ & u_{(0,1)}(w(1-k),h) - u_{(0,1)}(w,h) \geq u_{(0,1)}(w(1-r)(1-k),h) - u_{(0,1)}(w(1-r),h). \end{aligned} \tag{12}$$

From (12), it holds that $u_{(0,1)}(w(1-r)(1-k),h) - u_{(0,1)}(w(1-r),h)$ decreases in r, that is,

$$\begin{aligned}
& \frac{\partial}{\partial r} u_{(0,1)}(w(1-r)(1-k),h) - u_{(0,1)}(w(1-r),h) \leq 0 \\
\Leftrightarrow\quad & (1-k)u_{(1,1)}(w(1-k)(1-r),h) \geq u_{(1,1)}(w(1-r),h). \qquad (13)
\end{aligned}$$

From (13), it holds that $u_{(1,1)}(w(1-r)(1-k),h)$ increases in k, that is,

$$\begin{aligned}
& \frac{\partial}{\partial k} u_{(1,1)}(w(1-r)(1-k),h) \geq 0 \\
\Leftrightarrow\quad & -u_{(1,1)}(w(1-k)(1-r),h) - w(1-k)(1-r)u_{(2,1)}(w(1-k)(1-r),h) \geq 0 \\
\Leftrightarrow\quad & -\frac{w(1-k)(1-r)u_{(2,1)}(w(1-k)(1-r),h)}{u_{(1,1)}(w(1-k)(1-r),h)} \leq 1. \qquad (14)
\end{aligned}$$

This completes the proof. The proof is almost the same as Eeckhoudt et al. (2009a), but the inequality is reversed in the last step of (14) because of $u_{(1,1)} \leq 0$ by correlation aversion. □

To show Proposition 5, we need the following lemma, which is a generalization of Lemma 1.

Lemma 3. *Suppose that*

- $\tilde{w}_G$ *dominates* $\tilde{w}_B$ *in the sense of NSD;*
- $\tilde{h}_G$ *dominates* $\tilde{h}_B$ *in the sense of MSD.*

The following two conditions are equivalent:
(i) $(-1)^{n+m}\pi_{(1,1)}(w,h) \leq 0$.
(ii) $E[\pi(\tilde{w},\tilde{h})]$ *is decreasing in* k.

Proof of Lemma 3. To show that (i) implies (ii), as in the proof of Lemma 1, we need to prove that

$$E[\pi(\tilde{w}_G,\tilde{h}_G)] - E[\pi(\tilde{w}_G,\tilde{h}_B)] \leq E[\pi(\tilde{w}_B,\tilde{h}_G)] - E[\pi(\tilde{w}_B,\tilde{h}_B)]. \qquad (15)$$

Because $\tilde{w}_G$ dominates $\tilde{w}_B$ in the sense of NSD, (15) holds if

$$\begin{aligned} & (-1)^{n+1}\frac{\partial^n}{\partial w^n}\left\{E[\pi(w,\tilde{h}_G)] - E[\pi(w,\tilde{h}_B)]\right\} \leq 0 \\ \Leftrightarrow \quad & (-1)^{n+1}E[\pi_{(n,0)}(w,\tilde{h}_G)] \leq (-1)^{n+1}E[\pi_{(n,0)}(w,\tilde{h}_B)]. \end{aligned}$$

Applying the property of MSD, we have

$$\begin{aligned} & (-1)^{m+1}\frac{\partial^m}{\partial h^m}\left\{(-1)^{n+1}\pi_{(n,0)}(w,h)\right\} \leq 0 \\ \Leftrightarrow \quad & (-1)^{(n+m)}\pi_{(n,m)}(w,h) \leq 0. \end{aligned}$$

Similar to the proof of Lemma 1, we can show that (ii) implies (i). □

Proof of Proposition 5. Applying Lemma 3 to $\pi(x,h) = xu_{(1,0)}(w+\alpha x,h)$, we can obtain the condition. There are two cases where $n+m$ is an even or odd number. Note that $\pi_{(n,m)}(w,h) = n\alpha^{n-1}u_{(n,m)}(w+\alpha x,h) + \alpha^n xu_{(n+1,m)}(w+\alpha x,h)$. For the case of $n+m$ being even numbers, we have the following:

$$\begin{aligned} & \pi_{(n,m)}(x,h) \leq 0 \\ \Leftrightarrow \quad & n\alpha^{n-1}u_{(n,m)}(w+\alpha x,h) + \alpha^n xu_{(n+1,m)}(w+\alpha x,h) \leq 0 \\ \Leftrightarrow \quad & nu_{(n,m)}(w+\alpha x,h) + \alpha xu_{(n+1,m)}(w+\alpha x,h) \leq 0 \\ \Leftrightarrow \quad & -\frac{\alpha xu_{(n+1,m)}(w+\alpha x,h)}{u_{(n,m)}(w+\alpha x,h)} \leq n, \end{aligned}$$

where the last equivalence holds because $(-1)^{n+m+1}u_{(n,m)}$ is positive by mixed correlation aversion, and thus $u_{(n,m)}$ is negative in the case of $n+m$ being even numbers. Because $u_{(n+1,m)}(w,h) \geq 0$ for $n+m$ being an even number, we have

$$-\frac{\alpha xu_{(n+1,m)}(w+\alpha x,h)}{u_{(n,m)}(w+\alpha x,h)} \leq -\frac{(w+\alpha x)u_{(n+1,m)}(w+\alpha x,h)}{u_{(n,m)}(w+\alpha x,h)}.$$

Thus, if the right-hand-side of the inequality is less than n, that is, the degree of relative mixed correlation aversion in (n,m) is less than n, then the degree of partial mixed correlation aversion in (n, m) is less than n. Thus, an increase in correlation reduces the investment in the risky asset. We complete the proof for the case where $n+m$ is an even number. For the case of $(n+m)$ being odd numbers, we can apply a similar argument. $\square$

Proof of Proposition 6. As in the proof of Proposition 3, we only provide a proof that (ii) implies (i). Because $\mathcal{B}_{(n,m)} \succeq \mathcal{A}_{(n,m)}$, it follows that

$$\begin{aligned}
&\frac{1}{2}E[u_{(0,m)}(w(1+\tilde{\epsilon}_G)(1+\tilde{\delta}_B),h)] + \frac{1}{2}E[u_{(0,m)}(w(1+\tilde{\epsilon}_B)(1+\tilde{\delta}_G),h)] \\
&\geq \frac{1}{2}E[u_{(0,m)}(w(1+\tilde{\epsilon}_G)(1+\tilde{\delta}_G),h)] + \frac{1}{2}E[u_{(0,m)}(w(1+\tilde{\epsilon}_B)(1+\tilde{\delta}_B),h)] \\
\Leftrightarrow\quad &E[u_{(0,m)}(w(1+\tilde{\epsilon}_G)(1+\tilde{\delta}_B),h)] - E[u_{(0,m)}(w(1+\tilde{\epsilon}_G)(1+\tilde{\delta}_G),h)] \\
&\geq E[u_{(0,m)}(w(1+\tilde{\epsilon}_B)(1+\tilde{\delta}_B),h)] - E[u_{(0,m)}(w(1+\tilde{\epsilon}_B)(1+\tilde{\delta}_G),h)]. \quad (16)
\end{aligned}$$

By NSD, (16) holds when

$$\begin{aligned}
&\frac{\partial^n}{\partial \epsilon^n}\{E[u_{(0,m)}(w(1+\epsilon)(1+\tilde{\delta}_B),h)] - E[u_{(0,m)}(w(1+\epsilon)(1+\tilde{\delta}_G),h)]\} \geq 0 \\
&\Leftrightarrow E[w^n(1+\tilde{\delta}_G)^n u_{(n,m)}(w(1+\epsilon)(1+\tilde{\delta}_G),h)] \geq E[w^n(1+\tilde{\delta}_B)^n u_{(n,m)}(w(1+\epsilon)(1+\tilde{\delta}_B),h)]. \quad (17)
\end{aligned}$$

By FSD, (17) is equivalent to the following:

$$\begin{aligned}
&\frac{\partial}{\partial \delta}\{w^n(1+\delta)^n u_{(n,m)}(w(1+\epsilon)(1+\delta),h)] \geq 0 \\
&\Leftrightarrow nw^n(1+\delta^{n-1})u_{(n,m)}(w(1+\epsilon)(1+\delta),h) - w^{n+1}(1+\delta^n)(1+\epsilon)u_{(n+1,m)}(w(1+\epsilon)(1+\delta),h)] \geq 0 \quad (18)
\end{aligned}$$

(18) is equivalent to

$$-\frac{w(1+\epsilon)(1+\delta)u_{(n+1,m)}(w(1+\epsilon)(1+\delta),h)}{u_{(n,m)}(w(1+\epsilon)(1+\delta),h)} \leq n.$$

This completes the proof. $\square$

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