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A heterogeneous-firm model of trade and growth with country-specific credit constraints*

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Abstract

This study constructs a two-country endogenous growth model with heterogeneous firms and asymmetric countries, where the asymmetry lies in the degree of financial frictions. The tradable intermediate goods sector consists of heterogeneous firms and requires specific goods for entry. These goods are produced by heterogeneous entrepreneurs facing credit constraints due to financial frictions. Using this framework, we derive the following results analytically. First, a permanent credit crunch in one country facilitates the exit of intermediate goods firms in that country; meanwhile, it decreases the profitability of exports of the other country's intermediate goods firms, causing exporters to switch to selling their goods domestically. Second, under no international lending and borrowing, the credit crunch reduces the growth rates of both countries not only in the long run but also during the transition to a new balanced growth path. We also compare the long-run effects under such a financial autarky and financial integration.

JEL classification: F12; F43; O16; O41
Keywords: Endogenous growth; Heterogeneous firms; Asymmetric countries; Financial frictions; Country-specific credit crunch

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1 Introduction

Since Melitz (2003), it has become standard in international trade theory to focus on firm heterogeneity in both theoretical and quantitative analyses. In addition, as international trade is considered to be an important source of economic growth, many studies combine the Melitz and endogenous growth models to analyze the effect of trade-induced firm selection—an important feature in the Melitz model—on the long-run growth rate. However, most analyses assume that countries are symmetric. This limitation of symmetric countries may not only be a theoretical problem, but also a potential obstacle to understanding actual economic phenomena. For example, during the 2008–09 financial crisis, turmoil in the U.S. financial markets rapidly spread to other countries, causing a sharp contraction in real activity in the world economy. Notably, the aftermath of such a country-specific financial shock in the U.S. was not limited to a short-run recession. For example, the average growth rate in the U.S. for the decade 1997–2007 was 2.06%, while that for the following decade was 0.66%. Even when we exclude the two years of the Great Recession and change the sample decade to 2009–2019, the average growth rate was still 1.51%. In the 20 countries in the Euro area, the average growth rate was 1.95% for 1997–2007 and 1.15% for 2009–2019.[1]

Furthermore, during the financial crisis, especially in 2008Q4 and 2009Q1, the volume of international trade declined more significantly than the decline in GDP.[2] This suggests that trade linkages may have played an important role in the transmission of the adverse effects of the U.S. financial markets to the rest of the world. Accordingly, many studies have analyzed the relationship between this world-wide trade collapse and the prolonged recessions. However, most of these studies use quantitative business cycle models extended to open economies. Thus, while the U.S. financial crisis has affected global economic activity not only in the short run but also in the long run through international trade, economic growth models to analyze this impact have not yet been fully developed.

Against this background, this study examines how country-specific conditions in financial markets affect global economic growth. To this end, we incorporate financial frictions into a simple two-country, variety-expansion model of endogenous growth. As in the standard variety-expansion model, the intermediate good is produced from the final good. This intermediate goods sector has firm heterogeneity as in the Melitz model. Besides these sectors, each country has a sector comprising heterogeneous entrepreneurs. They produce the non-tradable good used for domestic intermediate goods firms' fixed inputs for entry and serving domestic and export markets. We call this good as a "knowledge good" for convenience.

[1] The data are taken from OECD Statistics (https://stats.oecd.org/). In Germany, France, and the UK, the average growth rates for 1997–2007 were 1.69%, 1.75%, and 2.19%, respectively, while those for 2009—2019 were 1.63%, 0.95%, and 1.31%, respectively.

[2] See, for example, Figure 1 in Bems et al. (2013).

As we shall see below, several studies have already combined the Melitz and variety-expansion models. Some studies have also introduced financial frictions into the static Melitz model. However, regarding the former, we have to consider forward-looking decisions regarding firm entry and exit. This is because firms need to compare the fixed cost of entry with the sum of the discounted present value of the profits that will accrue thereafter. Simultaneously introducing both firm heterogeneity and this forward-looking decision into the model makes it difficult to obtain analytical results. Therefore, with few exceptions, most existing studies assume that countries are symmetric. Regarding the latter, exporting firms are often assumed to face more severe credit constraints than domestically oriented firms (see, for example, Chaney, 2016). While this is certainly a realistic assumption, extending such a model to a dynamic one would require the explicit consideration of forward-looking decisions of these firms into the model. Thus, analytically obtaining clear results would still be difficult.

To analytically obtain clear-cut results, we make two assumptions. First, we assume that at every point in time, they must pay a fixed entry cost and then draw their productivity. With this assumption, decisions regarding entry and exit by intermediate goods firms at a specific date are no longer based on an infinite horizon, but determined by comparing expected profits with fixed costs at that date. This assumption is along the same lines as Naito (2017a). Second, we assume that the agents facing credit constraints are the entrepreneurs producing the knowledge good, rather than the intermediate goods firms. Most static Melitz models assume labor as the only fundamental factor of production, and assume that firms need labor as fixed inputs for entry and serving each market. In reality, however, firms' entry often requires specialized knowledge, such as marketing research. This type of good is often supplied by small high-tech companies, like unicorns, who are often credit constrained. We incorporate this fact into the model. Specifically, in our model, entrepreneurs use the final good to invest in physical capital and then produce the knowledge good from it. Although they can borrow funds from external sources for investment, they face credit constraints. In addition, the marginal productivity of capital across entrepreneurs is heterogenous. As shown by Moll (2014) and subsequent studies, the entrepreneurs are endogenously divided into those who are productive enough to actively invest in capital, and those who are not, and hence, lend their net worth to other active entrepreneurs.

Hence, our model incorporates distinct productivity cutoffs: one for the exit of intermediate goods firms, another for their export activities, and a further one for entrepreneurs' investments. Within this framework, we consider a scenario in which credit constraints tighten in one country (referred to as a "credit crunch" hereafter). This decrease in credit availability reduces entrepreneurs' borrowing capacity within that country and triggers the entry of less productive entrepreneurs who would have remained inactive in the absence of the credit crunch. Through this selection process of entrepreneurs, the credit crunch exacerbates the overall productivity of aggregate capital required for producing the knowledge good. As a result, this decline in productivity contributes to an increase in the price of the knowledge good in that country. Since this good is used for the fixed inputs of the intermediate goods firms, the credit crunch facilitates these firms' exit after drawing their productivity. Thus, even if entrepreneurs directly face credit constraints, the credit crunch on these agents generates a firm selection in the tradable goods sector. Such a propagation of firm selection governs the effects of a credit crunch on the world economy.

We at first characterize the equilibrium assuming no international lending and borrowing; that is, there is only the trade in the intermediate goods. After showing the existence and global stability of the balanced growth path (BGP) where two countries grow at the same rate, we examine the effects of a permanent credit crunch in one country on its and the other country's firm selections and economic growth. We analytically obtain the following two results. First, the permanent credit crunch in one country has an asymmetric effect across countries regarding the cutoffs of intermediate goods firms. Specifically, it facilitates the exit of intermediate goods firms in the country experiencing the credit crunch; meanwhile, it decreases the profitability of exports of the other country's intermediate goods firms, causing exporters to switch to selling their goods domestically. Second, due to this country-specific credit crunch, both countries experience lower growth rates not only on the new BGP but also while transitioning to it compared to the BGP before the credit crunch.

The intuition behind these two results is as follows. For convenience, let us refer to the two countries as the home and foreign countries as in the main text, and assume that a credit crunch occurs in the home country. As already mentioned, the credit crunch worsens the productivity of aggregate capital to produce the knowledge good, which in turn promotes the exit of intermediate goods firms in the home country. Thus, immediately after the credit crunch, the home country experiences lower growth relative to that in the foreign country. From the perspective of foreign intermediate goods firms, however, this means less demand for their goods in the export market. Therefore, exporting goods becomes less profitable for them, while selling goods to the domestic market becomes more profitable. The productivity cutoff for domestic activities then decreases, while that for export activities increases in the foreign country. Since this decreases the demand for the knowledge good in the foreign country, the real price of this good drops there. This has decreasing effects on the income flows of entrepreneurs, and hence, the accumulation of their net worth. Thus, growth deteriorates in the foreign country as well. Therefore, the effects on the activity of the tradable goods producing sector are asymmetric, while the effects on the growth rates of the two countries are similar.

Several studies analyzing the relationship between trade and growth do not assume international lending and borrowing. If the two countries are symmetric in all aspects, balanced trade would be achieved even if we introduced lending and borrowing between the two countries. Therefore, the simplification of no international financial transactions would be justified assuming symmetric countries. However, if, as in this study, the two countries are asymmetric, we must examine how the impact of the country-specific credit crunch on the world economy changes under such transactions. When financial markets are integrated in both countries, if entrepreneurs in one country become unable to borrow funds, those funds will flow to the other country. Thus, the credit crunch in one country can also affect the selection of entrepreneurs in the other country. Within this framework, we analytically derive the BGP by focusing on the situation that the two countries are initially symmetric. For the more general case, we conduct the numerical analysis. In both cases, we find that the effects of the country-specific credit crunch on the BGP growth rate and cutoffs of intermediate goods firms in both countries are not significantly different from those in the absence of international lending and borrowing. International asset transactions are usually considered as an international transmission channel for financial shocks, such as financial crises. Our results imply that even if the two countries quit international financial transactions, trade in goods still serves as a large enough international transmitter of a country-specific financial shock. Thus, this result provides a new insight into the international spillovers of financial shocks.

Related Literature. This study is related to three streams of literature. First, this study is closely connected to the studies building on dynamic trade models with endogenous growth and heterogeneous firms. Baldwin and Rebert-Nicoud (2008) are among the first to posit the endogenous growth version of Melitz (2003). They introduce firm heterogeneity into a model with variety expansion and two symmetric countries, and show that whether trade opening enhances higher growth depends on the degree of international R&D spillovers. Ourens (2016) reexamines the welfare effect of trade opening in Baldwin and Rebert-Nicoud's symmetric two-country model. Sampson (2016) incorporates technology diffusion into the multi-country model, where the productivity of new entrants is drawn from the distribution of the productivity of incumbent firms. The author shows that trade opening always raises welfare through faster growth and static welfare gains. However, the author still assumes symmetric countries. Haruyama and Zhao (2017) embeds firm heterogeneity in a two-country Schumpeterian growth model characterized by creative destruction, while assuming that the two countries are symmetric.[3] By contrast, Naito (2017a) introduces a Melitz-type firm heterogeneity and international asymmetry in several aspects into an AK growth model. With this framework, the author shows that a unilateral decline in iceberg trade costs in one country raises the growth rates of all countries for all times and always improves their welfare. In addition, Naito (2017b) introduces the international asymmetry in trade costs into Baldwin and Rebert-Nicoud (2008), and examines the growth and welfare effects of trade liberalization in the form of its unilateral decline.[4] Thus,

[3]Several studies relax the assumption of symmetric countries in dynamic Melitz models while abstracting the mechanism of endogenous growth. Examples of such studies include Bonfiglioli et al. (2019) and Brooks and Dovis (2020).

[4]While Naito (2017b) uses the knowledge-driven specification of R&D activities, Naito (2019) employs the lab-equipment specification. The author shows that under the lab-equipment specification, unilateral trade liberalization always enhances the long-run growth and welfare of both countries. Replacing the iceberg trade costs by tariffs in Naito

the source of asymmetry in these studies is the international difference in trade costs; credit market imperfections are not considered.

Second, our study is related to a growing body of literature that examines the effect of financial market imperfections on international trade. Here, we focus on studies on trade combining financial frictions and Melitz-type firm heterogeneity.[5] A typical study is Chaney (2016), who constructs a static two-country model in which firms are subject to financial constraints on fixed costs to enter the export markets: the payment for the fixed costs must be smaller than the profits earned in the domestic market plus the value of assets held by the firms. Hence, the cutoff condition on the export decision of firms depends not only on their production efficiency but also on their levels of wealth. Given this setting, the author shows that financial deepening may lower the misallocation in the export markets. In a similar vein, Feenstra et al. (2014) also analyze a static two-country model, while Manova (2013) considers a multi-country model with financial frictions.[6] Notably, the aforementioned studies construct static models. In contrast, Kohn et al. (2016) explore a dynamic, small-open economy model with international lending and borrowing. The authors construct a model without capital in which a part of working capital (wage payments) is subject to collateral constraints.[7] Kohn et al. (2020) introduce capital into their earlier study. Specifically, firms use their capital stocks as collateral so that their borrowings are limited by the levels of capital holdings. The authors conduct numerical analysis on the stationary equilibrium of their model, and find that a stronger financial constraint reduces export and the cutoff level of efficiency to enter the export market rises with the net worth of firms.[8]

Finally, our study is partly related to the recent literature analyzing the international transmission of shocks in international business cycle models. The strong synchronization of business cycles among countries during the financial crisis has given rise to a literature elucidating the mechanism by which financial crises in one country propagate to other countries and synchronization of business cycles occurs. Examples of such studies include Devereux and Yetman (2010), Devereux and Sutherland

(2019), Naito (2021) examines whether the optimal tariff can be zero.

[5]If we include the literature using other trade models, many studies introduce financial frictions. See, for example, Matsuyama (2005) and Antrás and Caballero (2009).

[6]Feenstra et al. (2014) assume that firms borrow a part of their payments on variable and fixed costs from a monopolistic bank. The financial contracts between firms and the bank are conducted under asymmetric information. The authors also study the empirical validity of their theoretical model using a data set of Chinese firms and conclude that their theoretical results do not diverge from the data. Besides the theoretical discussion, Manova (2013) conducts an empirical study using data on 105 countries from 1985 to 1995 and finds that the analytical results match the data.

[7]The authors calibrate their model based on micro data from Chile and find that the model with financial friction outperforms the one without friction.

[8]See also Leibovci (2021). Kohn et al. (2022) present a useful overview of the studies on financial frictions and trade.

(2011), Kollmann et al. (2011), Dedola and Lombardo (2012), Perri and Quadrini (2018), and Yao (2019). Devereux and Yetman (2010) analyze how the magnitude and persistence of the international transmission of productivity shocks in one country is affected by financial frictions in that country. The other works mainly focus on the international propagation of financial shocks through financial linkages across countries. Dedola and Lombardo (2012) and Yao (2019) employ a two-tradable-good framework, and examine the role of the terms of trade in the transmission of financial shocks across countries. In addition, several studies analyze the collapse of international trade during the 2008–2009 global financial crisis. The most celebrated study is Eaton et al. (2016), who incorporate the trade structure of Eaton and Kortum (2002) into a dynamic general equilibrium model to quantify the impacts of various shocks on the world economy. However, all these studies are based on quantitative business cycle models. Therefore, the implications for the long-run growth rate, which is our primary concern, is outside the scope of their analysis.

In summary, the first stream of the literature does not consider financial frictions, whereas the second does not incorporate the mechanism of endogenous growth. The third stream overlooks the effect of financial shocks on the long-run growth rate. In this sense, our study can complement their contributions.

Organization of the paper. The rest of this paper is organized as follows. Section 2 sets up the model and characterizes the autarky equilibrium as the benchmark for the main analysis. Section 3 derives the two-country equilibrium with balanced trade and examines the effects of a credit crunch in one country on both countries' firm selections and growth rates. Section 4 extends the model to include international lending and borrowing and examine how the results obtained in Section 3 is robust against this extension. Section 5 discusses how the results change if each of the underlying assumptions and restrictions is removed. Section 6 presents the conclusions of this study.

2 Model

Time is continuous and indexed by $t \in [0, \infty)$, although we omit this subscript unless doing so would cause confusion. The world economy comprises two countries, called the home and foreign countries. We attach the asterisk to foreign variables and parameters, except in cases where they take the same values as those in the home country. Each country has four different types of agents: (i) a representative firm producing the single non-tradable final good used for consumption and capital accumulation; (ii) heterogeneous firms producing tradable intermediate goods under monopolistic competition with endogenous entry and exit; (iii) heterogeneous entrepreneurs producing the specific good used for the fixed inputs of intermediate goods firms; and (iv) a representative worker who supplies labor to the final good firms. Each country has two primary factors, labor and capital, neither of which moves internationally. Thus, the two countries are interrelated through trade in the intermediate goods. To focus on this trade, we assume for the moment that no other international transactions exist. In Section 4, we relax this assumption and consider the world economy when financial markets are also integrated.

2.1 Final good sector

The final good is produced from differentiated intermediate goods and labor under constant returns to scale and perfect competition.

$$Y = \frac{1}{\alpha} L^{1-\alpha} \left(\int_{\omega \in \Omega} x(\omega)^{\alpha} d\omega \right),$$

where Y is the output, L is the demand for labor, Ω is the set of available varieties of intermediate goods, $x(\omega)$ is the demand for variety $\omega \in \Omega$, and $\alpha \in (0,1)$ is the cost share of intermediate goods. The elasticity of substitution between any pairs of varieties is $1-\alpha$. We include the term $1/\alpha$ for notational simplicity.[9]

Let P denote the final good's price, w denote the wage rate, and $p(\omega)$ denote the demand price of variety ω. Profit maximization under perfect competition yields the following demand functions:

$$\begin{aligned} &L = (1-\alpha)PY/w, \\ &x(\omega) = p(\omega)^{-1/(1-\alpha)} L P^{1/(1-\alpha)}, \omega \in \Omega. \end{aligned}$$

In the foreign country,

$$\begin{aligned} &L^* = (1-\alpha)P^* Y^*/w^*, \\ &x^*(\omega) = p^*(\omega)^{-1/(1-\alpha)} L^* P^{*1/(1-\alpha)}, \omega \in \Omega^*. \end{aligned}$$

2.2 Intermediate good sector

We introduce firm heterogeneity into the standard two-country variety-expansion model à la Rivera-Batiz and Romer (1991) and modify the way of firm entry. Each firm uses the final good as the variable input. In addition, it requires three types of fixed inputs: (i) a fixed input for its entry; (ii) one for serving its domestic market; and (iii) one for serving the export market. We explain the behavior of entrepreneurs who produce this input in Section 2.3. As noted in Section 1, we name this input "knowledge good" for convenience, following Baldwin and Robert-Nicoud (2008), Ourens (2016), and Naito (2017b, 2019, 2021).

We follow Naito (2017a) to assume that at every point in time, firms must pay a fixed entry cost and then draw their productivity. Let $f_E > 0$ denote the exogenous amount of the knowledge good required for the entry, P_K denote the price of the knowledge good, and $G(\varphi)$ denote the cumulative distribution function of φ. Therefore, the fixed entry cost at time t is given by $P_{K,t} f_E$. Given the

[9] See, for example, Acemoglu (2009: Ch.13).

realization of φ, firms decide their options among exiting, serving only the domestic market, and serving both the domestic and foreign markets. Let $f_D > 0$ and $f_X > 0$ denote the fixed input of the knowledge good to serve the domestic and foreign markets, respectively. As in much of the trade literature, export is subject to trade costs: if the foreign final good firm wants to use one unit of an intermediate good produced in the home country, it must import $\tau \geq 1$ units of this good.

We drop the notation ω, as it is sufficient to identify each firm with its location and productivity level, φ. Let $y_D(\varphi)$ and $p_D(\varphi)$ denote the output and supply price for the domestic market, respectively. Similarly, let $y_X(\varphi)$ and $p_X(\varphi)$ denote the output and supply price for the export market, respectively. That is, $x^* = y_X/\tau$ by definition, and the demand and supply prices are related so that $p^* = \tau p_X$. Hereafter, we let $j(= D, X)$ denote the index of the market served by the intermediate goods firms. In this stage, the firm maximizes its profit in market j:

$$\pi_j(\varphi) \equiv \left(p_j(\varphi) - \frac{P}{\varphi} \right) y_j(\varphi) - P_K f_j, \quad j = D, X;$$

subject to the following demand conditions:

$$y_D(\varphi) = p_D(\varphi)^{-1/(1-\alpha)} L P^{1/(1-\alpha)}, \tag{1}$$

$$\frac{y_X(\varphi)}{\tau} = (\tau p_X(\varphi))^{-1/(1-\alpha)} L^* P^{*1/(1-\alpha)}. \tag{2}$$

Due to profit maximization, the supply price in market j is

$$p_j(\varphi) = \frac{P}{\alpha\varphi}. \tag{3}$$

Then, the resulting profit in market j is $\pi_j(\varphi) = P\left[(1-\alpha)y_j(\varphi)/(\alpha\varphi)\right] - P_K f_j$.

Let φ_j denote the cutoff level of productivity in market j defined in the same manner as Melitz (2003): $\pi_j(\varphi_j) = 0$. The output of a firm whose productivity is just the cutoff productivity is expressed as

$$y_j(\varphi_j) = \frac{\alpha\varphi_j}{1-\alpha} f_j q, \tag{4}$$

where $q \equiv P_K/P$ denote the real price of the knowledge good, namely, the price of this good in terms of the domestic final good.

From (1)–(3), we can express the output $y_j(\varphi)$ for any φ by using the cutoff productivity φ_j:

$$y_j(\varphi) = (\varphi/\varphi_j)^{1/(1-\alpha)} y_j(\varphi_j),$$

which is a well-known result in the Melitz model. Substituting this result into $\pi_j(\varphi)$ and using (4) yields

$$\pi_j(\varphi) = P_K f_j \left[(\varphi/\varphi_j)^{\alpha/(1-\alpha)} - 1 \right].$$

The term $(\varphi/\varphi_j)^{\alpha/(1-\alpha)}$ can be interpreted as the firm productivity relative to the exit cutoff in market j. We let $H(\varphi_j)$ denote its aggregate over the firms surviving in market j:

$$H(\varphi_j) \equiv \int_{\varphi \geq \varphi_j} (\varphi/\varphi_j)^{\alpha/(1-\alpha)}\, dG(\varphi),$$
$$H'(\varphi_j) = -\frac{\alpha}{1-\alpha}\frac{H(\varphi_j)}{\varphi_j} - G'(\varphi_j) < 0.$$

The free entry condition requires that the fixed entry cost is equal to the sum of the expected net profits over all markets. Since firms pay the fixed entry cost and draw their productivity at each point in time, the free entry condition is

$$P_K f_E = \int_{\varphi \geq \varphi_D} \pi_D(\varphi) dG(\varphi) + \int_{\varphi \geq \varphi_X} \pi_X(\varphi) dG(\varphi).$$

Following the literature, we let

$$\Pi(\varphi_j) \equiv \int_{\varphi \geq \varphi_j} \left[(\varphi/\varphi_j)^{\alpha/(1-\alpha)} - 1\right] dG(\varphi) = H(\varphi_j) - (1 - G(\varphi_j)),$$
$$\Pi'(\varphi_j) = -\frac{\alpha}{1-\alpha}\frac{H(\varphi_j)}{\varphi_j} < 0.$$

From the definition of cutoff φ_j, the expected profit for the home firm in market j, $\int_{\varphi_j}^{\infty} \pi_j(\varphi) dG(\varphi)$, is rewritten as $\Pi(\varphi_j) f_j P_K$. Therefore, the free entry condition is reduced to

$$f_E = \sum_{j=D,X} f_j \Pi(\varphi_j). \tag{5}$$

Let M^e denote the measure of entrants endogenously determined as discussed below. Then, the measure of surviving firms in market j is given by $(1 - G(\varphi_j))M^e$. Accordingly, the measure of Ω is given by the sum of $(1 - G(\varphi_d))M^e$ (i.e., measure of domestically produced varieties) and $(1 - G(\varphi_x^*))M^{e*}$ (i.e., measure of imported varieties). Note that since entry and exit occur at each point in time, the intermediate goods firms earn no profits on aggregate.[10]

We assume that the fixed inputs (f_E, f_D, f_X) and distribution function (G) are the same between the two countries, whereas the trade cost τ^* can be different from τ. Let $q^* \equiv P_K^*/P^*$ denote the real price of the knowledge good in the foreign country. For the foreign intermediate goods firms,

[10] The ex-post profits of productive firms are given by $\left[\int_{\varphi \geq \varphi_D}^{\infty} \pi_D(\varphi) dG(\varphi) + \int_{\varphi \geq \varphi_X}^{\infty} \pi_X(\varphi) dG(\varphi)\right] M^e - P_K f_E (1 - G(\varphi_D))M^e$ on aggregate. With the free entry condition, this is rewritten as $P_K f_E G(\varphi_D) M^e$, which is equal to the sum of ex-post negative profits of exiting firms.

the equations corresponding to (1)–(5) are given as follows:

$$
\begin{aligned}
p_D^*(\varphi) &= P^* L^{*1-\alpha} y_D^*(\varphi)^{\alpha-1}, \\
\tau^* p_X^*(\varphi) &= P L^{1-\alpha} (y_X^*(\varphi)/\tau^*)^{\alpha-1}, \\
p_j^*(\varphi) &= \frac{P^*}{\alpha\varphi},\ j = D, X, \\
y_j^*(\varphi_j^*) &= \frac{\alpha\varphi_j^*}{1-\alpha} f_j q^*,\ j = D, X, \\
f_E &= \sum_{j=D,X} f_j \Pi(\varphi_j^*).
\end{aligned}
$$

2.3 Entrepreneurs

Entrepreneurs comprises a continuum of heterogeneous agents with unit mass, indexed by $i \in [0, 1]$. An entrepreneur has the following expected utility:

$$
\mathbb{E}_0 \left[\int_0^\infty e^{-\rho t} \ln c_{i,t} dt \right],
$$

where c is the consumption and $\rho > 0$ is the subjective discount rate. Each entrepreneur produces the knowledge good by using physical capital under constant returns to scale technology and perfect competition:

$$
y_{Ki,t} = z_{i,t} k_{i,t},
$$

where y_K is the output of the knowledge good, k is the physical capital, and z is the marginal productivity of capital. Here, we assume that the productivity z is independent and identically distributed (iid) across agents as well as over time. At each point in time, z is drawn from a stationary distribution, where the cumulative distribution function is expressed as $F(z)$.

The budget constraint in terms of the domestic final good is given by

$$
(q_t z_{i,t} k_{i,t} - r_t^b b_{i,t}) dt + db_{i,t} = (c_{i,t} + \iota_{i,t}) dt,
$$

where b is the debt in terms of the domestic final good, r^b is the real interest rate, and ι is the investment for physical capital. Physical capital changes according to $dk_{i,t} = (\iota_{i,t} - \delta k_{i,t}) dt$.[11]

Each entrepreneur's borrowing is subject to the following credit constraint:

$$
b_{i,t} \leq \left(1 - \frac{1}{\theta}\right) k_{i,t}, \quad \theta \geq 1.
$$

This inequality implies that at most a fraction $1 - 1/\theta \in [0, 1)$ of capital can be financed by external funds. By varying θ, we can trace all degrees of financial frictions. The case of $\theta = 1$ results in $1 - 1/\theta = 0$, which implies that the entrepreneur's investment must be self-financed. The case

[11] Since debts and capital at individual-level can be discontinuous through time as shown below, we do not differentiate them with respect to time here.

of $\theta \to \infty$ results in $1 - 1/\theta \to 1$, which implies that entrepreneurs can finance all their capital investment through external financing; that is, there is no financial friction. Let $a_i \equiv k_i - b_i$ denote an entrepreneur's net worth. The credit constraint can be rewritten as

$$k_{i,t} \leq \theta a_{i,t}. \tag{6}$$

Thus, θ also represents the maximum ratio of leverage. Using net worth, we can arrange the budget constraint as

$$da_{i,t} = \left[r_t^b a_{i,t} + \left(q_t z_{i,t} - \delta - r_t^b \right) k_{i,t} - c_{i,t} \right] dt. \tag{7}$$

At each point in time, differences in entrepreneurs' behaviors are attributed to heterogeneities in a and z. Therefore, we drop the subscript i for the moment and use (a, z) to identify individuals. The maximization problem is given by

$$\rho V_t(a,z)dt = \max \left\{ (\ln c)dt + \mathbb{E}_t \left[dV_t(a,z) \right] \right\},$$

subject to (6) and (7), where $V_t(a, z)$ is the value function. Following Moll (2014) and Itskhoki and Moll (2019), we assume that at time t, the entrepreneur can determine the amount of investment after observing its productivity. Then, the optimization problem is essentially the same as the problem in their model.

First, at each point in time, the entrepreneur determines k to maximize the net profit, $(qz-\delta-r^b)k$, subject to the credit constraint (6) while taking a and z as given. Therefore, the optimal choice is given by

$$k_t = \begin{cases} 0 & \text{if } z_t < \widetilde{z}_t, \\ \theta a_t & \text{if } z_t \geq \widetilde{z}_t, \end{cases}$$

where $\widetilde{z}$ denotes the productivity cutoff for entrepreneurs to survive and produce the knowledge good:

$$\widetilde{z}_t \equiv \frac{r_t^b + \delta}{q_t}.$$

The budget constraint is rewritten as $da_t = (R_t(z_t)a_t - c_t)dt$, where

$$R_t(z_t) = \begin{cases} r_t^b & \text{if } z_t < \widetilde{z}_t, \\ r_t^b + \theta q_t \left(z_t - \widetilde{z}_t \right) & \text{if } z_t \geq \widetilde{z}_t. \end{cases}$$

Second, the entrepreneur chooses the time paths of consumption and net worth to maximize the lifetime utility subject to the budget constraint. Since instantaneous utility is a logarithmic function, this problem yields the following results:

$$c_t = \rho a_t, \quad da_t = (R_t(z_t) - \rho)a_t dt,$$

the derivations of which are given in Appendix A.1.

Let us define $A \equiv \int_0^1 a_i di$, $K_t \equiv \int_0^1 k_i di$, $B \equiv \int_0^1 b_i di$, and $Y_K \equiv \int_0^1 y_{Ki} di$. In this model, each entrepreneur's net worth at date t, $a_{i,t}$, has been already determined at that date; then, the productivity z_t^i is determined as the iid shock. Therefore, they are independent from each other. Using this fact, we obtain

$$K_t = \theta(1 - F(\widetilde{z}_t))A_t,$$
$$B_t = K_t - A_t = [\theta(1 - F(\widetilde{z}_t)) - 1]A_t.$$

The aggregate output of the knowledge good is given by

$$Y_{K,t} = Z(\widetilde{z}_t)K_t,$$

where $Z(\widetilde{z}) \equiv \int_{z \geq \widetilde{z}} z dF(z)/(1 - F(\widetilde{z}))$ represents the average productivity of capital conditional on active entrepreneurs. Thus, the knowledge good is produced by an AK technology. However, in this study, the average productivity is endogenous and affected by financial frictions through its effect on the entrepreneurs' cutoff, as first pointed out by Moll (2014).

Let us define dA_t as $dA_t \equiv \int da_{i,t} di$. Using $da_{i,t} = (R_t(z_{i,t}) - \rho)a_{i,t}dt$, and the fact that a_i and z_i are independent from each other, we obtain

$$dA_t = A_t \left[r_t^b + \theta(1 - F(\widetilde{z}_t)) \left(q_t Z(\widetilde{z}_t) - \delta - r_t^b \right) - \rho \right] dt.$$

The right-hand side does not contain any stochastic components. Hereafter, let a dot over a variable denote the time derivative of this variable; for example, $\dot{A}_t \equiv dA_t/dt$. Using $r_t^b + \delta = q_t \widetilde{z}$, we can obtain the dynamic equation of A as

$$\dot{A}_t/A_t = q_t \widetilde{z}_t \left(1 + \theta \Psi(\widetilde{z}_t)\right) - \delta - \rho, \tag{8}$$

where $\Psi(\widetilde{z})$ is defined as

$$\Psi(\widetilde{z}) \equiv (1 - F(\widetilde{z}_t)) \left(\frac{Z(\widetilde{z})}{\widetilde{z}} - 1 \right) = \int_{z \geq \widetilde{z}} (z/\widetilde{z} - 1) dF(z),$$
$$\Psi'(\widetilde{z}) = \int_{z \geq \widetilde{z}} -z/(\widetilde{z})^2 dF(z) = -\frac{\Psi(\widetilde{z}) + 1 - F(\widetilde{z})}{\widetilde{z}} < 0.$$

In (8), $q\widetilde{z}\left(1 + \theta\Psi(\widetilde{z})\right) - \delta$ corresponds to the entrepreneurs' rate of return on their aggregate net worth. As shown by the definition of $R(z)$, for any entrepreneur, there is a guaranteed rate of return of $r^b (= q\widetilde{z} - \delta)$. Thus, $\theta q \widetilde{z} \Psi(\widetilde{z})$ represents the excess from the guaranteed rate of return.

In this study, we focus on the international difference in the degree of financial frictions. For this purpose, we assume that the distribution functions of z are the same between the two countries. Thus, the entrepreneurs' productivity cutoff in the foreign country $\widetilde{z}^*$ is given by $\widetilde{z}^* = (r^{b*} + \delta)/q^*$, where r^{b*} is the real interest rate of lending and borrowing in the foreign country. The behavior of foreign entrepreneurs is summarized as follows:

$$K^* = \theta^*(1 - F(\widetilde{z}^*))A^*,$$
$$B^* = [\theta^*(1 - F(\widetilde{z}^*)) - 1]A^*,$$
$$Y_K^* = Z(\widetilde{z}_t^*)K^*,$$
$$\dot{A}^*/A^* = q^*\widetilde{z}^* (1 + \theta^*\Psi(\widetilde{z}^*)) - \delta - \rho.$$

2.4 Workers

There is a continuum of homogeneous workers with the constant mass of $L > 0$. They are myopic and do not save. Each worker chooses consumption c_w and labor supply l to solve the following optimization problem at each moment of time:

$$\max_{c_{w,t}, l_t} \ln c_{w,t} - \frac{l_t^{1+\nu}}{1+\nu},$$

subject to the budget constraint $w_t l_t = P_t c_{w,t}$. The optimal choice yields $l_t = 1$ and $c_{w,t} = w_t/P_t$ for all t. Then, the aggregate supply of labor always equals the population L and the aggregate consumption of workers is given by $w_t L/P_t$.[12]

2.5 Autarky equilibrium

We characterize the autarky equilibrium as the benchmark for the main analysis. We focus on the home country and choose the final good as the numeraire: $P_t = 1$ for all t. Substituting (3) and (4) for $j = D$ into (1), we obtain the following relationship between the real price of the knowledge good $q(= P_K)$ and productivity cutoff φ_D:

$$q_t = \chi L(\varphi_{D,t})^{\alpha/(1-\alpha)}, \tag{9}$$

where $\chi \equiv \alpha^{\alpha/(1-\alpha)}(1-\alpha)/f_D > 0$. If the price of the knowledge good is high relative to that of the final good, the fixed cost to serve the domestic market is high. This lowers the profit for the intermediate goods firms, and hence, the cutoff φ_D increases. For this reason, (9) represents the positive relationship between q and φ_D.

Without international trade, the free entry condition (5) is replaced by

$$f_E = f_D\Pi(\varphi_{D,t}).$$

[12]If workers can save, each worker's optimization problem is given by

$$\max \int_0^\infty e^{-\rho_w t}\left(\ln c_{w,t} - \frac{l_t^{1+\nu}}{1+\nu}\right) dt,$$

subject to $\dot{a}_{w,t} = r_t^b a_{w,t} + (w_t/P_t)l_t - c_{w,t}$, where $\rho_w > 0$ is the discount rate applied to workers and a_w denotes their asset holding. As pointed out by Moll (2014), if the workers' discount rate is sufficiently high so that $\rho_w > r_t$ and if they face a tight borrowing constraint such that they cannot borrow (i.e., $a_{w,t} > 0$), the workers' optimal choice yields $c_{w,t} = (w_t/P_t)l_t$ in the long run.

Thus, under autarky, the intermediate goods firms' cutoff is always constant, denoted by φ_D^a. Equation (9) then determines the price of the knowledge good as $q^a \equiv \chi L(\varphi_D^a)^{\alpha/(1-\alpha)}$.

In addition, under no international financial transactions, $A = K$ holds. From the definition of B, it follows that $\theta(1 - F(\widetilde{z}_t)) - 1 = 0$, which implies that the entrepreneurs' cutoff productivity is given by

$$\widetilde{z}_t = \overline{z}(\theta) \equiv F^{-1}(1 - 1/\theta). \tag{10}$$

As the value of θ increases, the cutoff for the entrepreneurs to produce the knowledge good increases:

$$\frac{d\overline{z}(\theta)}{d\theta} = \frac{1 - F(\overline{z}(\theta))}{\theta F'(\overline{z}(\theta))} > 0.$$

Substituting these results into (8), we can show that the growth rate of net worth is always constant:

$$\frac{\dot{A}_t}{A_t} = g^a \equiv q^a \mathcal{Z}(\theta) - \delta - \rho, \tag{11}$$

where g^a represents the growth rate under autarky and

$$\mathcal{Z}(\theta) \equiv \overline{z}(\theta)(1 + \theta\Psi(\overline{z}(\theta))).$$

Under autarky, the knowledge good is used as the intermediate goods firms' fixed inputs for entry into and serving the domestic market. The market-clearing condition of the knowledge good is given by

$$Y_{K,t} = M_t^e f_E + M_t^e (1 - G(\varphi_D^a) f_D.$$

Since $B_t = 0$ for all t, K_t always equals A_t. Therefore, the mass of entrants grows at the same rate as the net worth from the initial date:

$$M_t^e = \frac{Y_{K,t}}{f_E + f_D(1 - G(\varphi_D^a))} = \frac{Z(\overline{z}(\theta))A_t}{f_E + f_D(1 - G(\varphi_D^a))}.$$

The output of the final good under autarky is given by

$$Y_t = \frac{L^{1-\alpha} M_t^e}{\alpha} \int_{\varphi \geq \varphi_{D,t}} y_{D,t}(\varphi)^\alpha dG(\varphi) = \frac{L^{1-\alpha} M_t^e}{\alpha} H(\varphi_D^a) \left(y_D(\varphi_D^a)\right)^\alpha .$$

Thus, the output of the final good always grows through the expansion of the variety of intermediate goods. The results are summarized as:

Lemma 1. *Under autarky, the economy is on the BGP from the initial time.*

From (11), we obtain the growth effect of an increase in θ:

$$\frac{dg^a}{d\theta} = q^a \left[\overline{z}\Psi(\overline{z}) + \left(1 + \theta\Psi(\overline{z}) + \overline{z}\theta\Psi'(\overline{z})\right) \frac{d\overline{z}(\theta)}{d\theta} \right].$$

On the right-hand side, the first term is the positive direct effect of relaxing the credit constraint; it allows active entrepreneurs to earn more. The second term is the effect of relaxing credit constraints by selecting only more productive entrepreneurs. Within the parentheses of the second term, $1+\theta\Psi$ is the positive growth effect induced by the increase in the marginal productivity of aggregate capital, whereas $\overline{z}\theta\Psi'$ is the negative growth effect induced by the decrease in the entrepreneurs' rate of return, which in turn is caused by the decrease in the mass of active entrepreneurs. However, from the definition of Ψ, we can show the following result:

$$1+\theta\Psi(\overline{z})+\theta\overline{z}\Psi'(\overline{z}) = 1+\theta\Psi(\overline{z})-\theta(\Psi(\overline{z})+1-F(\overline{z})) = 1-\theta(1-F(\overline{z})) = 0,$$

where the last equality holds because of $B=0$. Thus, only the positive direct effect exists:

$$\frac{dg^a}{d\theta} = q^a\overline{z}\Psi(\overline{z}) > 0.$$

This leads to the next lemma:

Lemma 2. *Under autarky, relaxing the credit constraint (i.e., an increase in θ) increases the BGP growth rate.*

3 Equilibrium with trade

We continue to define the home country's final good as the numeraire. As we have already stated, for the moment, we assume no international lending and borrowing between the home and foreign countries. Hereafter, we refer to such a situation as "financial autarky." Since balanced trade is always achieved in this situation, we use financial autarky and balanced trade interchangeably.

We focus on the cross-country asymmetry in the degree of financial frictions. Therefore, we hereafter assume that the labor size and trade costs are symmetric across the two countries:

$$L = L^*, \quad \tau = \tau^*.$$

In Section 5.3, we investigate the case of asymmetric trade costs.

3.1 The intermediate goods firms' cutoffs given P^*

Following Melitz (2003) and subsequent studies, we assume the following inequality:

$$T \equiv \tau\left(\frac{f_X}{f_D}\right)^{(1-\alpha)/\alpha} > 1.$$

From (1)–(4), we obtain the following equation showing how φ_D and φ_X are related:

$$\frac{\varphi_{X,t}}{\varphi_{D,t}} = TP_t^{*-1/\alpha}. \tag{12}$$

Thus, the difference between the two cutoffs arises not only from the existence of the trade barrier $T>1$, but also from the possibility of $P^* \neq 1$. The intuition is explained as follows. As is apparent

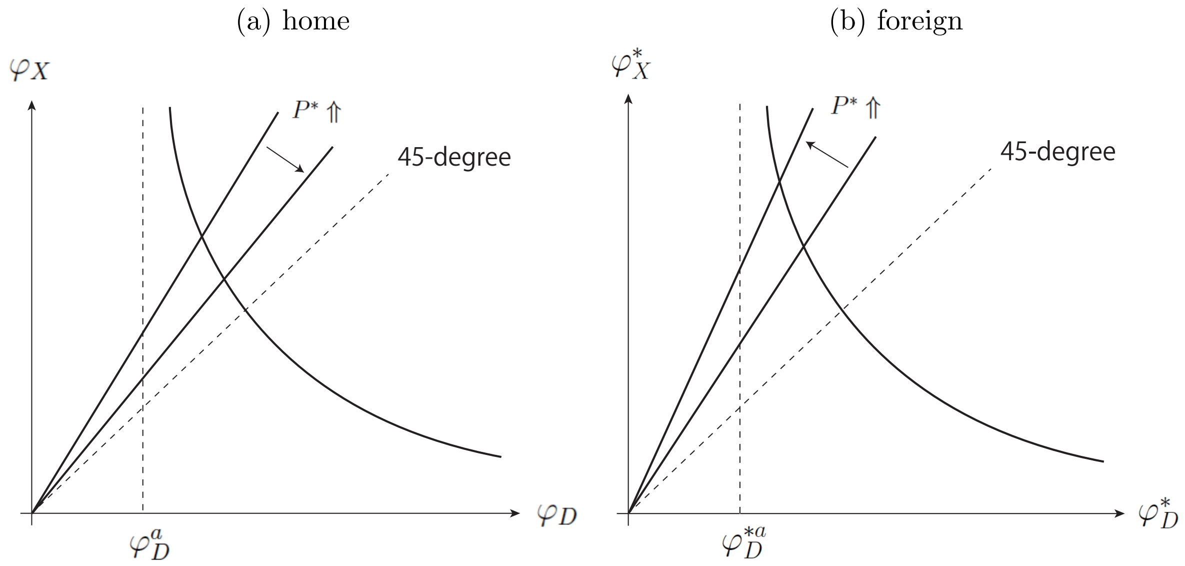

Figure 1: The intermediate goods firms' cutoffs given P^*

from (1) and (2), in this model, the higher the final good's price in a country, the greater the demand for intermediate goods in that country. For the intermediate goods firms in the home country, a larger value of P^* means a larger demand for their products in the export market. Consequently, these firms find exporting to be profitable relative to supplying it domestically, which decreases φ_X/φ_D. Given P^*, the two cutoffs φ_D and φ_X are determined from (5) and (12). Panel (a) of Figure 1 depicts how they are determined. The free entry condition (5) represents the negative relationship between φ_D and φ_X. Moreover, if φ_X is sufficiently large, (5) is reduced to $f_E = f_D \Pi(\varphi_D)$, which means that $\varphi_D = \varphi_D^a$ if φ_X is sufficiently large. Thus, as this figure shows, (5) and (12) uniquely determine φ_D and φ_X.

We can obtain the cutoffs for the foreign intermediate goods firms by following the same procedure as the home ones. We obtain

$$\frac{\varphi^*_{X,t}}{\varphi^*_{D,t}} = T P_t^{*1/\alpha}. \tag{13}$$

Note that for the foreign intermediate goods firms, a larger value of P^* means that they find it more profitable to supply the goods to the domestic market rather than the export market. Panel (b) of Figure 1 depicts how φ_D^* and φ_X^* are determined, and how a change in P^* affects them. We can summarize these results as the following lemma:

Lemma 3. *Given P_t^*, the productivity cutoffs for the intermediate goods firms are uniquely determined, and $\varphi_D'(P_t^*) > 0$, $\varphi_X'(P_t^*) < 0$, ${\varphi_D^*}'(P_t^*) < 0$, and ${\varphi_X^*}'(P_t^*) > 0$.*

Following the literature building on heterogeneous-firm trade models, we focus on the situation where not all domestic firms are able to export: $\varphi_X > \varphi_D$ and $\varphi_X^* > \varphi_D^*$. From (12) and (13), these inequalities are reduced to the following inequalities that P^* must satisfy in the equilibrium:

$$P^*_{\min} \leq P^* \leq P^*_{\max},$$

where $P^*_{\min} \equiv T^{-\alpha} < 1$ and $P^*_{\max} \equiv T^{\alpha} > 1$.[13] Applying Lemma 3 to (9), we obtain the real price of the knowledge good in each country as a function of P^*:

$$q_t = q(P^*_t) \equiv \chi L \left(\varphi_D(P^*_t)\right)^{\alpha/(1-\alpha)}, \quad q'(P^*) > 0, \tag{14}$$

$$q^*_t = q^*(P^*_t) \equiv \chi L \left(\varphi^*_D(P^*_t)\right)^{\alpha/(1-\alpha)}, \quad q^{*\prime}(P^*) < 0. \tag{15}$$

Note that q and q^* coincide only when $\varphi_D = \varphi^*_D$, that is, when $P^* = 1$ is realized in the equilibrium.

3.2 Market-clearing conditions

The market-clearing condition for the final good in the home country is

$$Y_t = C_t + \dot{K}_t + \delta K_t + M^e_t \sum_{j=D,X} \int_{\varphi \geq \varphi_{j,t}} y_{j,t}(\varphi)/\varphi dG(\varphi).$$

where $C \equiv \rho A + wL/P$. The market equilibrium condition for the knowledge good is given by

$$Y_{K,t} = M^e_t \left(f_E + \sum_{j=D,X} f_j(1 - G(\varphi_{j,t})) \right). \tag{16}$$

The equation representing the balance of payment in the home country is given by the following equation:[14]

$$\dot{B}_t = r^b_t B_t - (EX_t - IM_t)/P_t, \tag{17}$$

where EX and IM are the home country's exports and imports, respectively:

$$EX \equiv M^e \int_{\varphi \geq \varphi_X} p_X(\varphi) y_X(\varphi) dG(\varphi),$$

$$IM = EX^* \equiv M^{e*} \int_{\varphi \geq \varphi^*_X} p^*_X(\varphi) y^*_X(\varphi) dG(\varphi).$$

Since $B_t = 0$ for all t, (17) implies that trade is always balanced:

$$EX_t = IM_t. \tag{18}$$

The market-clearing conditions in the foreign country are analogously given. From Walras' law, the equation of balanced trade in the foreign country, $EX^*_t = IM^*_t$, is redundant.

[13] Since $T > 1$, $P^*_{\max} > P^*_{\min}$ is automatically satisfied.

[14] Recall that B represents the aggregate real debts in the home country.

3.3 Equilibrium characterization

Since no international financial transactions happen, $B_t = 0$ and $B_t^* = 0$ hold in the equilibrium. Hence, the entrepreneur's cutoff is determined in the same way as the case of complete autarky. Namely, the cutoff in the home country is determined by $\widetilde{z}_t = \overline{z}(\theta)$, and that in the foreign country is determined by $\widetilde{z}_t^* = \overline{z}(\theta^*)$.

We first derive the dynamical system of the economy. Let m^* denote the real wealth of the foreign country relative to that in the home country: $m^* \equiv A^*/A$. Its dynamic equation is given by

$$\frac{\dot{m}_t^*}{m_t^*} \equiv \frac{\dot{A}_t^*}{A_t^*} - \frac{\dot{A}_t}{A_t} = q^*(P_t^*)\mathcal{Z}(\theta^*) - q(P_t^*)\mathcal{Z}(\theta). \tag{19}$$

The other equation constituting the dynamical system is derived from the equation of balanced trade, (18). Let μ denote the share of the export sales in the total sales of the home country's intermediate goods firms:

$$\mu_t \equiv \frac{\int_{\varphi \geq \varphi_{X,t}} p_X(\varphi) y_X(\varphi) dG(\varphi)}{\sum_{j=D,X} \int_{\varphi \geq \varphi_{j,t}} p_j(\varphi) y_j(\varphi) dG(\varphi)}.$$

Using (3), (4), and the definition of $H(\varphi_j)$, we can express μ as a function of P^*:

$$\mu_t = \mu(P_t^*) \equiv \frac{f_X H(\varphi_X(P_t^*))}{\sum_{j=D,X} f_j H(\varphi_j(P_t^*))}, \;\; \mu'(P^*) > 0, \tag{20}$$

where the sign of $\mu'(P^*)$ comes from Lemma 3 and $H'(\varphi_j) < 0$.[15] This result is intuitive. If P^* becomes larger, it means that the export market becomes more profitable for the home country's intermediate goods firms. Therefore, the revenue share of the export sales increases in that country. Analogously, we let μ^* denote the revenue share of the export sales of the foreign country's intermediate goods firms, which is obtained by just attaching the asterisk to φ_D and φ_X in (20). From Lemma 3 and $H'(\varphi_j^*) < 0$, $\mu^{*\prime}(P^*) < 0$.[16]

Using $\mu(P^*)$ and $\mu^*(P^*)$, we can express EX and IM more simply. Specifically, we obtain the following lemma:

Lemma 4. *The home country's exports and imports are*

$$EX_t = \frac{q(P_t^*)\theta A_t \int_{z \geq \widetilde{z}_t} z dF(z)}{1-\alpha} \mu(P_t^*), \tag{21}$$

$$IM_t = \frac{q^*(P_t^*) P_t^* \theta^* A_t^* \int_{z \geq \widetilde{z}_t^*} z dF(z)}{1-\alpha} \mu^*(P_t^*). \tag{22}$$

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[15] $\mu'(P^*)$ is given by

$$\mu'(P^*) = \frac{f_X \left[H'(\varphi_x)\varphi_X'(P^*) f_D H(\varphi_D) - H(\varphi_X) f_D H'(\varphi_D)\varphi_D'(P^*)\right]}{\left(\sum_{j=D,X} f_j H(\varphi_j(P^*))\right)^2}.$$

Since $\varphi_D' > 0$ and $\varphi_X' < 0$ from Lemma 3 and $H'(\varphi_j) < 0$, we can show that $\mu'(P^*) > 0$.

[16] Since $\varphi_D^{*\prime} < 0$, and $\varphi_X^{*\prime} > 0$ from Lemma 3 and $H'(\varphi_j) < 0$, we can show that $\mu^{*\prime}(P^*) < 0$.

Proof. See Appendix A.2. □

Lemma 4 itself is valid whether the financial market is under autarky or completely integrated. Under financial autarky, $\widetilde{z}_t = \overline{z}(\theta)$ and $\widetilde{z}^*_t = \overline{z}(\theta^*)$. Therefore, substituting (21) and (22) into (18), we obtain the following relationship between P^* and m^*:

$$m^*_t = \frac{\mu(P^*_t)q\left(P^*_t\right)}{\mu^*(P^*_t)q^*(P^*_t)P^*_t}\frac{\theta\int_{z\geq\overline{z}(\theta)} zdF(z)}{\theta^*\int_{z\geq\overline{z}(\theta^*)} zdF(z)}. \tag{23}$$

Equations (19) and (23) constitute the autonomous dynamical system of the economy with international trade. From (19) and $\dot{m}^* = 0$, we can obtain

$$q(P^*)\mathcal{Z}(\theta) = q^*(P^*)\mathcal{Z}(\theta^*). \tag{24}$$

If this equation holds, then $\dot{A}/A = \dot{A}^*/A^*$. Hence, both countries' assets grow at the same rate. Since we have already shown $q'(P^*) > 0$ and $q^{*\prime}(P^*) < 0$, we can obtain the following result:

Proposition 1. *If $q(P^*_{\min})\mathcal{Z}(\theta) < q^*(P^*_{\min})\mathcal{Z}(\theta^*)$ and $q(P^*_{\max})\mathcal{Z}(\theta) > q^*(P^*_{\max})\mathcal{Z}(\theta^*)$, there exists a final good's price in the foreign country that solves (24).*

Henceforth, let P^{*tr} denote the final good's price that solves (24), where "*tr*" indicates the BGP with trade. From (23), we obtain the BGP value of m^*:

$$m^* = m^{*tr} \equiv \frac{\mu(P^{*tr})q\left(P^{*tr}\right)}{\mu^*(P^{*tr})q^*(P^{*tr})P^{tr*}}\frac{\theta\int_{z\geq\overline{z}(\theta)} zdF(z)}{\theta^*\int_{z\geq\overline{z}(\theta^*)} zdF(z)}.$$

Are the two inequalities in Proposition 1 valid? For example, assume that $\varphi \geq 1$ and follows the Pareto distribution with the shape parameter given by $\kappa > 0$:

$$G(\varphi) = 1 - \varphi^{-\kappa}. \tag{25}$$

To make $\Pi(\varphi_j)$ well-defined, we assume $\kappa > \alpha/(1-\alpha)$. In this case, $q(P^*)$ and $q^*(P^*)$ are given as follows:[17]

$$q(P^*) = \chi L\left[h\left(f_D + f_X P^{*-\kappa/\alpha}_{\max}P^{*\kappa/\alpha}\right)/f_E\right]^{\frac{\alpha}{\kappa(1-\alpha)}},$$
$$q^*(P^*) = \chi L\left[h\left(f_D + f_X P^{*\kappa/\alpha}_{\min}P^{*-\kappa/\alpha}\right)/f_E\right]^{\frac{\alpha}{\kappa(1-\alpha)}}.$$

[17] $\Pi(\varphi_j)$ is calculated as $\Pi(\varphi_j) = h\varphi_j^{-\kappa}$, where $h \equiv \alpha/[\kappa(1-\alpha)-\alpha] > 0$. Using this, we can rewrite the free entry condition (5) as $f_E/h = f_D\varphi_D^{-\kappa} + f_X\varphi_X^{-\kappa}$. Then, we can explicitly derive the productivity cutoffs as the functions of P^*:

$$\varphi_D(P^*) = \left[h\left(f_D + f_X T^{-\kappa}P^{*\kappa/\alpha}\right)/f_E\right]^{1/\kappa}, \quad \varphi_X(P^*) = \left[h\left(f_D T^{\kappa}P^{*-\kappa/\alpha} + f_X\right)/f_E\right]^{1/\kappa},$$
$$\varphi^*_D(P^*) = \left[h\left(f_D + f_X T^{-\kappa}P^{*-\kappa/\alpha}\right)/f_E\right]^{1/\kappa}, \quad \varphi^*_X(P^*) = \left[h\left(f_D T^{\kappa}P^{*\kappa/\alpha} + f_X\right)/f_E\right]^{1/\kappa}.$$

Substituting these results into (14) and (15), we obtain $q(P^*)$ and $q^*(P^*)$.

When two countries are perfectly symmetric (i.e., $\theta = \theta^*$), the two inequalities in Proposition 1 are reduced to $q(P^*_{\min}) < q^*(P^*_{\min})$ and $q(P^*_{\max}) > q^*(P^*_{\max})$. Then, we can show that both inequalities are necessarily satisfied.[18]

Next, we examine the stability of the BGP. Since $q'(P^*) > 0$ and $q^{*\prime}(P^*) < 0$, (19) implies

$$\dot{m}^*_t \gtreqless 0 \Leftrightarrow P^* \lesseqgtr P^{*tr}.$$

The variable $m^*_t \equiv A^*_t/A_t$ is a predetermined variable. Then, the necessary and sufficient condition for the global stability of m^{*tr} is $dP^*_t/dm^*_t > 0$ in (23). When φ is Pareto distributed, we can show that this condition is true.

Proposition 2. *If $G(\varphi) = 1 - \varphi^{-\kappa}$, m^{*tr} is globally stable.*

Proof. See Appendix A.3. □

Let g^{tr} denote the BGP growth rate in the equilibrium with international trade and without financial integration:

$$g^{tr} \equiv q(P^{*tr})\mathcal{Z}(\theta) - \delta - \rho. \tag{26}$$

By comparing the two BGP growth rates (11) and (26), we examine the growth effect of trade opening from autarky. We can show the following lemma:

Lemma 5. *The BGP growth rate is higher after trade opening.*

Proof. q is an increasing function of φ_D from (9). Since $\varphi_D(P^{*tr}) > \varphi^a_D$ holds from Figure 1, then $q(P^{*tr}) > q^a$. □

3.4 Effects of the country-specific credit crunch

We now examine how a permanent credit crunch occurred in one country affects both countries' firm selections and growth rates. Without any loss of generality, we focus on a decrease in θ. Under financial autarky, entrepreneurs' cutoff in each country depends solely on the degree of that country's financial frictions. Figure 2 depicts the short- and long-run effects of a change in θ on the two countries' growth rates. Suppose that the world economy is on the BGP (point E_0) at the initial date and θ permanently falls. This decreases the entrepreneurs' borrowing capacity in the home country, which in turn induces the entry of less productive entrepreneurs who were inactive if the credit crunch had not occurred. A portion of the funds flows to them. This inefficient reallocation of financial resources lowers the aggregate efficiency of entrepreneurs to produce the knowledge good. Therefore, their accumulation of net worth slows down in the home country. Thus, immediately after

[18] To understand the reason, notice that the first and second inequalities are rewritten as $f_D + f_X\,(P^*_{\min}/P^*_{\max})^{\kappa/\alpha} < f_D + f_X$ and $f_D + f_X > f_D + f_X\,(P^*_{\min}/P^*_{\max})^{\kappa/\alpha}$, respectively. Since $P^*_{\max} > P^*_{\min}$, both inequalities are satisfied.

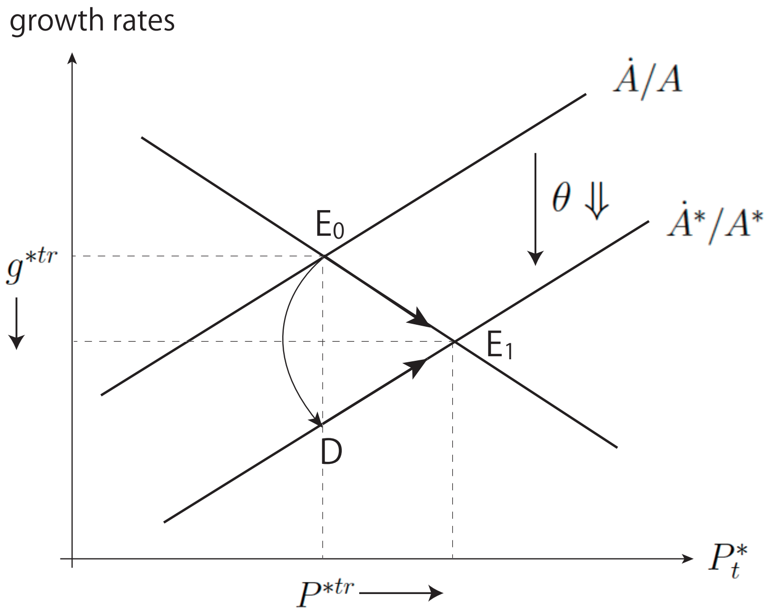

Figure 2: Effects of a decrease in θ on the growth rates

the credit crunch in the home country, its growth rate falls to point D and the foreign country begins to grow at a faster rate than the home country. This increases m^* over time.

However, since the trade balance (23) must be met, an increase in m^* over time is associated with a change in P^*. Specifically, P^* increases with m^* under the stability condition. As indicated by the arrow from E_0 to E_1 in Figure 2, an increase in P^* leads to a slowdown (recovery) in the foreign (home) country's growth rate. This effect continues until their growth rates coincide again. Eventually, the two countries arrive at the new BGP (point E_1), where the common growth rate is lower than that before the credit crunch. This result is summarized as the following proposition:

Proposition 3. *Suppose that a permanent credit crunch occurred in a country. Compared with the old BGP, both countries' growth rates decrease for all times.*

So, why does the foreign country's growth rate decrease during transition? The underlying cause lies in the firm selection in that country's intermediate goods sector. As discussed in Section 3.1, in this model, the higher the final good's price in a country, the greater the demand for intermediate goods in that country. A decrease in θ induces a decrease in $1/P_t^*$. This implies a decrease in demand for intermediate goods in the home country relative to that in the foreign country. Then, as panel (b) of Figure 1 shows, export becomes less profitable for the foreign firms. Then, the productivity cutoff $\varphi_{X,t}^*$ increases, while $\varphi_{D,t}^*$ decreases over time until the economy reaches the new BGP. Since $q_t^* = \chi L(\varphi_{D,t}^*)^{\alpha/(1-\alpha)}$, the real price of the knowledge good decreases during the transition, which causes the decrease in the foreign country's growth rate.

Thus, once the impact of a change in θ on P_t^* is revealed, Figure 1 clarifies its impacts on all productivity cutoffs for intermediate goods firms. We can state the following proposition:

Proposition 4. *Suppose that a permanent credit crunch occurred in a country.*

1. *In this country, the intermediate goods firms' productivity cutoff for domestic operations increases, while that for export decreases for all times.*
2. *In the partner country, that for domestic operations decreases, while that for export increases for all times.*

Therefore, the country-specific credit crunch has asymmetric effects on the activity of the tradable goods producing sector between the two countries, while it has similar effects on their growth rates.

4 Equilibrium with trade and financial integration

In the previous section, we focus on trade in intermediate goods as the only way of international transaction. This assumption allows for a clear-cut consideration of the effects of the credit crunch on both countries' growth rates through its impact on firm selection. However, this analysis has one shortcoming: credit tightness in one country does not affect entrepreneurs' cutoff productivity in the other country. Therefore, we extend the model to one that allows international lending and borrowing. Although physical capital is not directly mobile between the two countries, bonds issued by active entrepreneurs can be bought by the other country's lenders (i.e., inactive entrepreneurs) as well as the domestic ones.

Note that the analysis in Section 3.1 does not contain any variables or parameters concerning the financial markets. Therefore, the determination of intermediate goods firms' cutoffs given P^* discussed there is essentially the same as that under financial integration.

4.1 The BGP conditions

Since B and B^* are respectively evaluated in terms of the domestic final good, the market-clearing condition of bonds is given by

$$B_t + P_t^* B_t^* = 0. \tag{27}$$

Recall that the real interest rate of bonds in a country is defined in terms of its country's final good. Therefore, the interest parity is given by $r^b = r^{b*} + \dot{P}^*/P^*$. This is rewritten as:

$$q_t \widetilde{z}_t = q_t^* \widetilde{z}_t^* + \dot{P}_t^*/P_t^*. \tag{28}$$

The equation of balanced trade (18) is no longer necessarily valid in this case. Rather, the equation of the balance of payment (17) becomes one of the equilibrium conditions. From Walras' law, the balance of payment equation in the foreign country, $\dot{B}^* = r^{b*}B^* - (EX^* - IM^*)/P^*$, is redundant.

Here, we provide the conditions that the economy satisfies on the BGP. The equation of interest parity (28) with $\dot{P}^* = 0$ implies

$$q(P^*)\widetilde{z} = q^*(P^*)\widetilde{z}^*. \tag{29}$$

On the BGP, the growth rate of net worth is equal between the two countries: $\dot{A}/A = \dot{A}^*/A^*$. From (8) and its foreign counterpart together with (29), we obtain the following condition:

$$\theta\Psi(\widetilde{z}) = \theta^*\Psi(\widetilde{z}^*). \tag{30}$$

Using the definitions of B and B^*, we can rewrite the market-clearing condition for debts (27) as

$$\theta(1 - F(\widetilde{z})) - 1 + m^*P^*[\theta^*(1 - F(\widetilde{z}^*)) - 1] = 0. \tag{31}$$

Let $v \equiv B/A$. In Appendix A.4, we show that the dynamic equation of v is given by

$$\begin{aligned} \dot{v}_t &= \left(\rho - q(P_t^*)\widetilde{z}_t\theta\Psi(\widetilde{z}_t)\right) v_t \\ &\quad - \frac{1}{1-\alpha}\left[\mu(P_t^*)q(P_t^*)\theta\int_{z\geq\widetilde{z}_t} zdF(z) - m_t^*P_t^*\mu^*(P_t^*)q^*(P_t^*)\theta^*\int_{z\geq\widetilde{z}_t^*} zdF(z)\right]. \end{aligned} \tag{32}$$

On the BGP, B grows at the same rate as A or becomes zero. In either situation, $\dot{v} = 0$. Imposing $\dot{v} = 0$ in (32) and using $v = \theta(1 - F(\widetilde{z})) - 1$, we obtain

$$\begin{aligned} &\left(\rho - q(P^*)\widetilde{z}\theta\Psi(\widetilde{z})\right)[\theta(1 - F(\widetilde{z})) - 1] \\ &\quad = \frac{1}{1-\alpha}\left[\mu(P^*)q(P^*)\theta\int_{z\geq\widetilde{z}} zdF(z) - m^*P^*\mu^*(P^*)q^*(P^*)\theta^*\int_{z\geq\widetilde{z}^*} zdF(z)\right]. \end{aligned} \tag{33}$$

The BGP is characterized as the variables $\widetilde{z}$, $\widetilde{z}^*$, P^*, and m^* satisfying (29), (30), (31), and (33). Once these variables are determined, the other variables including the BGP growth rate can be accordingly determined.

4.2 The existence of the symmetric BGP

To analytically characterize the BGP under financial integration in this asymmetric two-country model, the following difficulty arises: trade is not necessarily balanced at zero. Specifically, the left-hand side of (33) may not be zero. Therefore, it is difficult to analytically solve (29), (30), (31), and (33) in general situations. Therefore, at first, we consider the BGP when $\theta = \theta^*$; that is, the two countries are perfectly symmetric. In this case, (30) leads to

$$\Psi(\widetilde{z}) = \Psi(\widetilde{z}^*),$$

which implies $\widetilde{z} = \widetilde{z}^*$. Substituting this result into (31) yields

$$(1 + P^*m^*)[\theta(1 - F(\widetilde{z})) - 1] = 0.$$

Since we focus on the equilibrium that A^*/A and P^* are positive finite, this equation implies $\widetilde{z} = \widetilde{z}^* = \overline{z}(\theta)$. Thus, the net foreign asset is zero on the BGP when the degree of financial frictions is the same between two countries. From (29) with $\widetilde{z} = \widetilde{z}^* = \overline{z}(\theta)$, we find that the real prices of the knowledge good are equal. From (14) and (15), $\varphi_D = \varphi_D^*$ holds. Hence,

$$P^* = 1,$$

which leads to $\mu = \mu^*$. Since $\theta(1 - F(\widetilde{z})) - 1 = 0$, (33) implies that trade is balanced in this case. Therefore, we obtain the relative wealth m^* on the BGP from (23) as

$$m^* = 1.$$

In the case of financial integration, the autonomous dynamical system no longer contains the dynamics of m_t^* alone; it also contains those of P_t^* and v_t. In Appendix A.5, we provide the autonomous dynamical system under financial integration. Furthermore, in Appendix A.6, we give the condition for the local stability of the BGP where the two countries are symmetric.

Proposition 5. *The symmetric BGP under $\theta = \theta^*$ is locally saddle-point stable if and only if $\rho < q(1)\overline{z}(\theta)\theta\Psi(\overline{z}(\theta))$.*

Proof. See Appendix A.6. □

Let g^{fi} denote the BGP growth rate under financial integration. When the two countries are symmetric,

$$g^{fi} = q(1)\overline{z}(\theta)(1 + \theta\Psi(\overline{z}(\theta))) - \delta - \rho.$$

Since $q(1)\overline{z}(\theta) - \delta$ is equal to the real interest rate, the condition for the saddle-point stability in Proposition 5 is rewritten as $g^{fi} > r^b$.[19]

4.3 Long-run effects of the country-specific credit crunch

We now consider the BGP when the degree of financial frictions is asymmetric between the two countries. Suppose that the economy is on the BGP with $\theta = \theta^*$ at the initial date and θ unilaterally decreases. We investigate the characteristics of the new BGP.

Equation (30) represents the international equalization of the rate of return on the entrepreneur's aggregate net worth. Thus, this equation represents a positive relationship between the two cutoffs

[19]Note that in our heterogeneous-agent framework, the transversality condition for each entrepreneur is satisfied even though the growth rate exceeds the interest rate. We have already verified that each entrepreneur's consumption is given by $c_t = \rho a_t$. Therefore, the transversality condition for them is satisfied as $\lim_{t\to\infty} \mathbb{E}_t[e^{-\rho t}(a_t/c_t)] = \lim_{t\to\infty} \rho^{-1}e^{-\rho t} = 0$.

$\tilde{z}$ and $\tilde{z}^*$. Totally differentiating (30) and evaluating the differential coefficient on the old BGP (i.e., $\tilde{z} = \tilde{z}^* = \overline{z}(\theta)$), we obtain

$$-\theta\Psi'(\overline{z}(\theta))(d\tilde{z} - d\tilde{z}^*) = \Psi(\overline{z}(\theta))d\theta. \tag{34}$$

Equation (34) shows that the balanced growth condition (30) gives an asymmetric effect of θ between $d\tilde{z}$ and $d\tilde{z}^*$. When θ becomes smaller, it negatively affects the growth rate in the home country. With the foreign country's borrowing capacity θ^* unchanged, the equalization of the growth rates requires higher (lower) entry entrepreneurs in the home (foreign) country.

By contrast, (31) represents the negative relationship between $\tilde{z}$ and $\tilde{z}^*$, given the other variables. Totally differentiating this equation and evaluating the differential coefficient on the old BGP, we obtain the following equation:[20]

$$\varepsilon(\theta)(d\tilde{z} + d\tilde{z}^*) = \frac{d\theta}{\theta}, \tag{35}$$

where $\varepsilon(\theta)$ is defined as:

$$\varepsilon(\theta) \equiv \frac{F'(\overline{z}(\theta))}{1 - F(\overline{z}(\theta))} > 0.$$

Equation (35) shows that the international asset market equilibrium (31) gives a symmetric effect of θ between $d\tilde{z}$ and $d\tilde{z}^*$. As a direct effect, a credit crunch in a country reduces the demand for funds by active entrepreneurs in that country. Under financial market integration, this credit crunch increases the global supply of funds relative to the global demand. However, the global asset market must be in equilibrium. Then, in both countries, less productive entrepreneurs, who were inactive if the credit crunch would not occur, become active and borrow funds to produce the knowledge good. Note that $\varepsilon(\theta)$ is the ratio of the active entrepreneurs with their productivity just given by $\overline{z}(\theta)$ to their total mass. When $\varepsilon(\theta)$ is small, a credit crunch in a country has a large impact on the entrepreneurs' productivity cutoffs.

Let $\eta(\theta) \equiv -\Psi'(\overline{z}(\theta))/\Psi(\overline{z}(\theta)) > 0$. From (34) and (35), we obtain

$$\frac{d\tilde{z}}{d\theta} = \frac{1}{2\theta}\left(\frac{1}{\varepsilon(\theta)} + \frac{1}{\eta(\theta)}\right) > 0,$$
$$\frac{d\tilde{z}^*}{d\theta} = \frac{1}{2\theta}\left(\frac{1}{\varepsilon(\theta)} - \frac{1}{\eta(\theta)}\right) \gtreqless 0 \Leftrightarrow \eta(\theta) \gtreqless \varepsilon(\theta).$$

From these equations, we can show the following proposition.

Proposition 6. *Consider the financially integrated two-country BGP with $\theta = \theta^*$ initially. With a permanent decrease only in θ, $\tilde{z}$ necessarily decreases. By contrast, $\tilde{z}^*$ increases (decreases) if $\eta(\theta) < (>)\varepsilon(\theta)$.*

Thus, a credit crunch in one country will affect the supply and demand for funds in the other country through the change in the mass of active entrepreneurs. This means that a credit crunch

[20] To obtain (35), we use $m^* = 1$, $P^* = 1$, and $(1 - F(\overline{z}(\theta))) = 1/\theta$ hold on the old BGP.

can change the net foreign asset position. Since B is defined as debts, the net foreign debts relative to the total wealth in the home country is given by $B/A \equiv v = \theta(1 - F(\widetilde{z})) - 1$. We obtain

$$\begin{aligned}\frac{dv}{d\theta} &= (1 - F(\overline{z})) - \theta F'(\overline{z})\frac{d\widetilde{z}}{d\theta} \\ &= \frac{1}{\theta} - \varepsilon(\theta)\frac{d\widetilde{z}}{d\theta} \\ &= \varepsilon(\theta)\frac{d\widetilde{z}^*}{d\theta} \gtreqless 0 \Leftrightarrow \eta(\theta) \gtreqless \varepsilon(\theta).\end{aligned}$$

Since $v = 0$ on the old BGP, this result shows the following proposition:

Proposition 7. *Consider the financially integrated two-country BGP with $\theta = \theta^*$ initially. With a permanent decrease only in θ, v becomes positive (negative) if $\eta(\theta) < (>)\varepsilon(\theta)$.*

As a direct effect, a credit crunch in the home country reduces the demand for funds in this country. Thus, under financial market integration, this is expected to make that country a creditor nation. However, this proposition shows that such an expectation is not necessarily correct. If $\eta(\theta) < \varepsilon(\theta)$, the home country becomes a debtor country due to its credit crunch.

Totally differentiating (29) and using the facts that $\widetilde{z} = \widetilde{z}^* = \overline{z}(\theta)$, $P^* = 1$, and $q(1) = q^*(1)$ hold on the old BGP, we obtain

$$\left(q'(1) - q^{*\prime}(1)\right)\overline{z}(\theta)dP^* = -q(1)(d\widetilde{z} - d\widetilde{z}^*).$$

On the left-hand side, $q'(1) - q^{*\prime}(1)$ is positive. Using (34), we obtain

$$\frac{dP^*}{d\theta} = \frac{-q(1)}{\theta\eta(\theta)\overline{z}(\theta)(q'(1) - q^{*\prime}(1))} < 0. \tag{36}$$

Thus, with respect to the change in P^*, we obtain qualitatively the same results as in the financial autarky case. This also means that the impact of credit tightness in a country on the intermediate goods firms' cutoffs is qualitatively the same as in the financial autarky case.

Finally, the effect of the unilateral change in θ on the BGP growth rate, $g^{fi} = q(P^*)\widetilde{z}(1+\theta\Psi(\widetilde{z})) - \delta - \rho$, is given by

$$\frac{dg^{fi}}{d\theta} = q(1)\overline{z}(\theta)\Psi(\overline{z}(\theta)) + q(1)\left[1 + \theta\Psi(\overline{z}(\theta)) + \overline{z}(\theta)\theta\Psi'(\overline{z}(\theta))\right]\frac{d\widetilde{z}}{d\theta} + \overline{z}(\theta)(1+\theta\Psi(\overline{z}(\theta)))q'(1)\frac{dP^*}{d\theta}. \tag{37}$$

In Section 2.5, we have already shown that $1 + \theta\Psi(\overline{z}) + \overline{z}\theta\Psi'(\overline{z}) = 0$. Therefore, the second term on the right-hand side vanishes. Using (36), the definition of η, and $1 + \theta\Psi(\overline{z}) = -\overline{z}\theta\Psi'(\overline{z})$, we can obtain

$$\frac{dg^{fi}}{d\theta} = -\frac{q^{*\prime}(1)}{q'(1) - q^{*\prime}(1)}q(1)\overline{z}(\theta)\Psi(\overline{z}(\theta)) > 0.$$

Thus, starting from the old symmetric BGP, the long-run growth effect of the credit crunch is the same as in the case of financial autarky even though the international financial transaction occurs by this shock.

4.4 Numerical examples

If $\theta \neq \theta^*$ on the BGP before the credit crunch, the second term on the right-hand side of (37) does not vanish. Are the results obtained in the previous subsection valid when $\theta \neq \theta^*$ on the old BGP? Here, we provide numerical examples.

Suppose that $G(\varphi)$ is specified as (25). In addition, assume that the entrepreneurs' productivity z satisfies $z \in [1, \beta]$ and follows the following upper-truncated Pareto distribution:

$$F(z) = \frac{1 - z^{-\gamma}}{1 - \beta^{-\gamma}},$$

where $\gamma > 1$ is the shape parameter. Then, (29) and the asset market equilibrium (30) are respectively rewritten as

$$\frac{\theta}{1-\beta^{-\gamma}}\left(\frac{\widetilde{z}^{-\gamma}}{\gamma-1} + \beta^{-\gamma}\right) = \frac{\theta^*}{1-\beta^{-\gamma}}\left(\frac{\widetilde{z}^{*-\gamma}}{\gamma-1} + \beta^{-\gamma}\right), \tag{29'}$$

$$\theta\left(\frac{\widetilde{z}^{-\gamma} - \beta^{-\gamma}}{1-\beta^{-\gamma}}\right) - 1 + m^*P^*\left[\theta^*\left(\frac{\widetilde{z}^{*-\gamma} - \beta^{-\gamma}}{1-\beta^{-\gamma}}\right) - 1\right] = 0. \tag{30'}$$

If $\beta \to \infty$, z follows a standard Pareto distribution. However, if this is the case, (29') and (30') are reduced to $\theta z^{-\gamma} = \theta^* z^{*-\gamma}$ and $\theta\widetilde{z}^{-\gamma} - 1 + m^*P^*\left[\theta^*\widetilde{z}^{*-\gamma} - 1\right] = 0$, respectively. From these equations, we obtain $(1 + m^*P^*)[\theta\widetilde{z}^{-\gamma} - 1] = 0$. That is, if z follows a standard Pareto distribution, the net foreign asset is always zero and financial autarky arises although there are international financial transactions. To avoid such a situation, we assume the upper bound for z.

We normalize the population to unity: $L = 1$. Then, in this model, 12 parameters must still to be determined: $\rho, \delta, \alpha, f_D, f_X, f_E, \kappa, \tau, \gamma, \beta, \theta$, and θ^*.[21] We set $\rho = 0.04$ and $\delta = 0.025$, both of which are standard in the literature. We borrow the value of the elasticity of substitution between varieties, given by $1/(1-\alpha)$ here, from Ghironi and Melitz (2005) and set it at 3.4. This results in $\alpha \simeq 0.71$. For the shape parameter of the distribution of φ, we set $\kappa = 5$. This is consistent with the estimation results in Balistreri et al. (2011) who estimate that κ ranges from 3.924 to 5.171.

Buera and Nicolini (2017) report that the average ratio of debts to non-financial assets for the U.S. non-financial business sector between 1997Q3 and 2007Q3 is 0.69. In our model, the debts of the active entrepreneurs can be represented as $(\theta - 1)(1 - F(\widetilde{z}_t))A_t$. Then, the ratio of the active entrepreneurs' debts to physical capital (non-financial assets) in the home country is given by

$$\frac{(\theta-1)(1-F(\widetilde{z}_t))A_t}{K_t} = \frac{\theta - 1}{\theta}.$$

We set the ratio at 0.69, which results in $\theta \simeq 3.226$. When determining the values of the parameters, we assume that the two countries are perfectly symmetric: $\theta^* = \theta$. We exogenously choose the value of γ as $\gamma = 3$. In Section 5.1, we examine the robustness of this inequality under some other values of γ.

[21] Recall that χ is defined as $\chi \equiv \alpha^{\alpha/(1-\alpha)}(1-\alpha)/f_D$ and $h \equiv \alpha/[\kappa(1-\alpha) - \alpha]$.

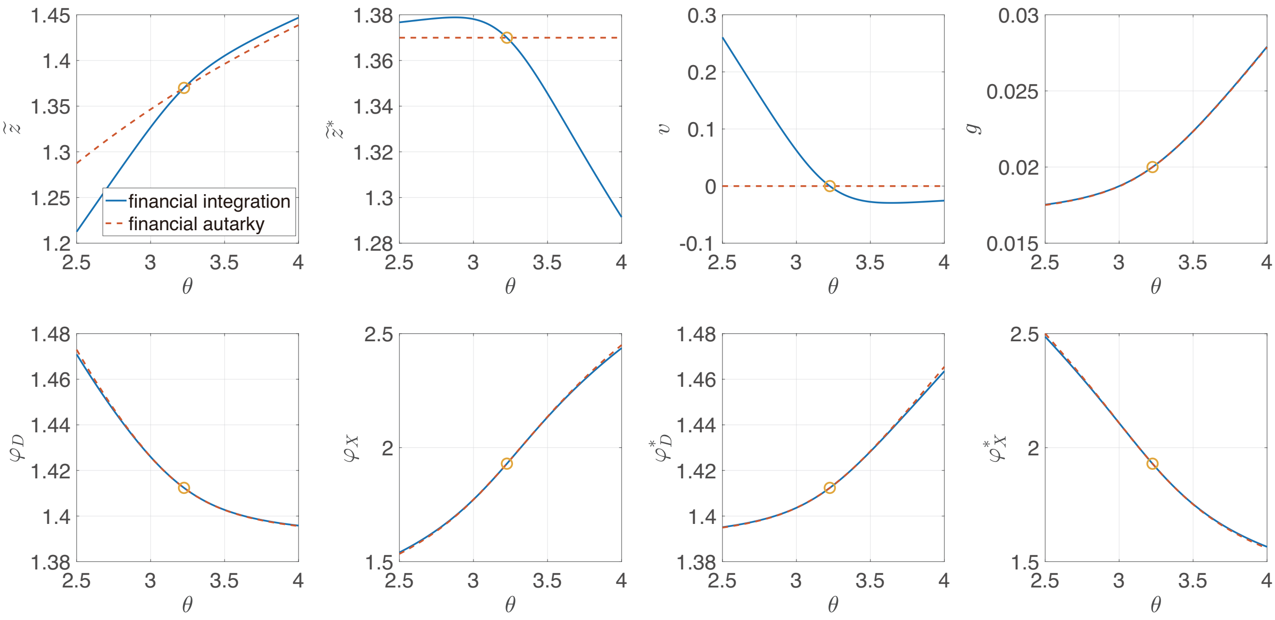

Figure 3: Effects of a permanent change in θ on key variables on the BGP

The remaining parameters still to be set are f_D, f_X, f_E, τ, and β. We normalize f_D to 10, which leads to $\chi = 0.013$. We calibrate f_X, f_E, τ, and β against the following target values with respect to the four endogenous variables. First, the BGP growth rate g^{fi} is set to 0.02. Second, the real interest rate in the home country r^b is set to 0.015.[22] Third, the fraction of exporting firms is set to 0.21, which is reported by Bernard et al. (2003, Table 1 on p.1270). Finally, the import penetration ratio, which is one minus the share of expenditure on domestic intermediate goods, is set to 0.081, which is borrowed from Sampson (2016, Table 1 on p.351). By using these target values, we obtain $f_E = 1.419$, $f_X = 4.198$, $\tau \simeq 1.962$, and $\beta \simeq 2.06$. The resulting values generate $T \equiv \tau(f_X/f_D)^{(1-\alpha)/\alpha} \simeq 1.366 > 1$. We provide the detailed derivations for these parameter values in Appendix A.7.

Then, we numerically examine how a permanent and unilateral change in θ affects some key variables on the BGP. Figure 3 shows the results. In each panel, a circle marker indicates the benchmark, while the blue line represents the variable's response to the change in θ under financial integration. The figure shows that the results of the comparative statics in the previous section are generally robust even when $\theta \neq \theta^*$. In addition, for comparison, in each panel, we depict the response under financial autarky as the red dashed line. As the first two panels straightforwardly show, the impact of the credit crunch on the productivity cutoffs for entrepreneurs significantly differ between financial integration and financial autarky. In this case, $\eta(\theta) \simeq 1.379 < \varepsilon(\theta) \simeq 3.102$. As the second panel shows, the credit crunch in the home country raises the productivity cutoff for foreign entrepreneurs, and consequently, reduces the mass of active entrepreneurs who borrow funds

[22]Since $g^{fi} > r^b$, the local saddle-point stability is guaranteed.

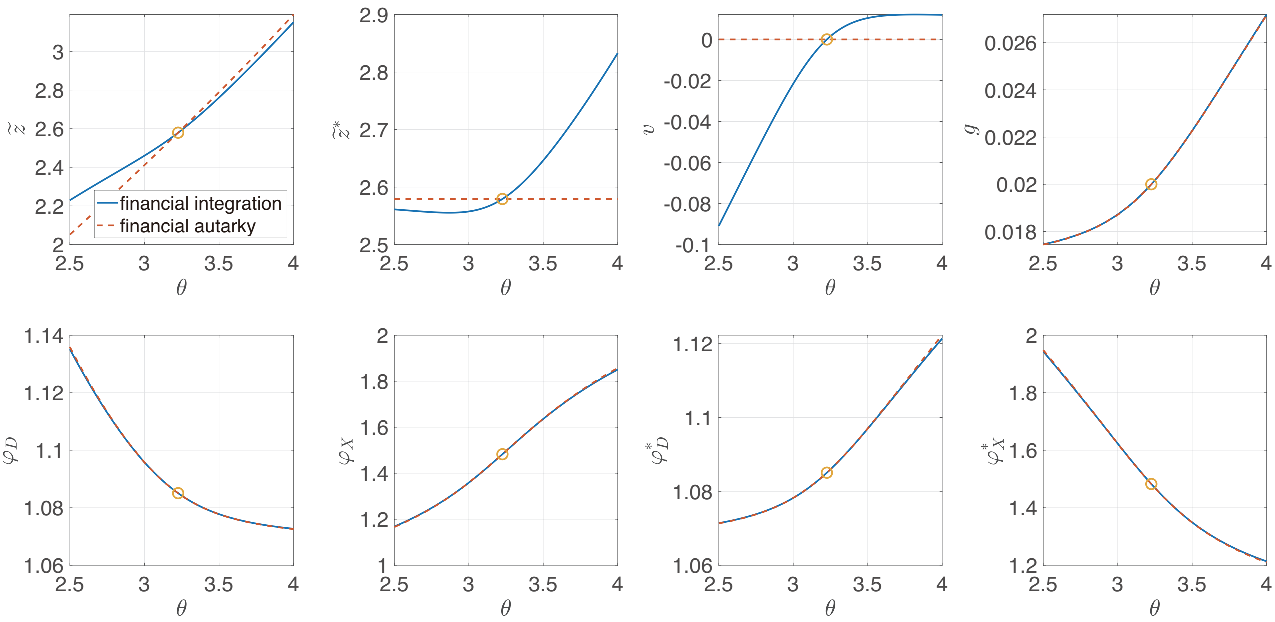

Figure 4: Sensitivity

to produce the knowledge good. Therefore, as shown in the third panel, the relative demand for funds decreases in the foreign country such that the home country (which directly experienced the credit crunch) increases its foreign debt. As shown by the red dashed line in this panel, this effect does not occur under financial autarky.

As shown in the fourth panel, however, such a difference in the impact on the entrepreneurs' cutoffs between these two cases only results in a negligible difference in the growth effects. Furthermore, as shown in the four panels in the bottom row, the effects on the cutoffs for the intermediate goods firms are almost the same between the two cases. International asset transactions are usually considered as an international propagation channel for financial shocks such as financial crises. However, the results obtained here imply that even if the two countries quit international financial transactions, trade in goods still facilitates a large enough international propagation of a country-specific financial shock.

5 Discussion

5.1 Sensitivity

In the numerical analysis of the previous section, $\eta(\theta) < \varepsilon(\theta)$ is satisfied. If we change the value of γ to 5, it follows that $\eta(\theta) \simeq 1.598 < \varepsilon(\theta) \simeq 7.189$. Then, the sign of the inequality does not change. If we change the value of γ to 1.5, it follows that $\eta(\theta) \simeq 0.7323 > \varepsilon(\theta) \simeq 0.4119$. Figure 4 depicts the numerical results of the change in θ. Comparing the second and third panels of Figures 3 and 4 shows that the results of comparative statics are reversed for $\widetilde{z}^*$ and v, which has been foreseen

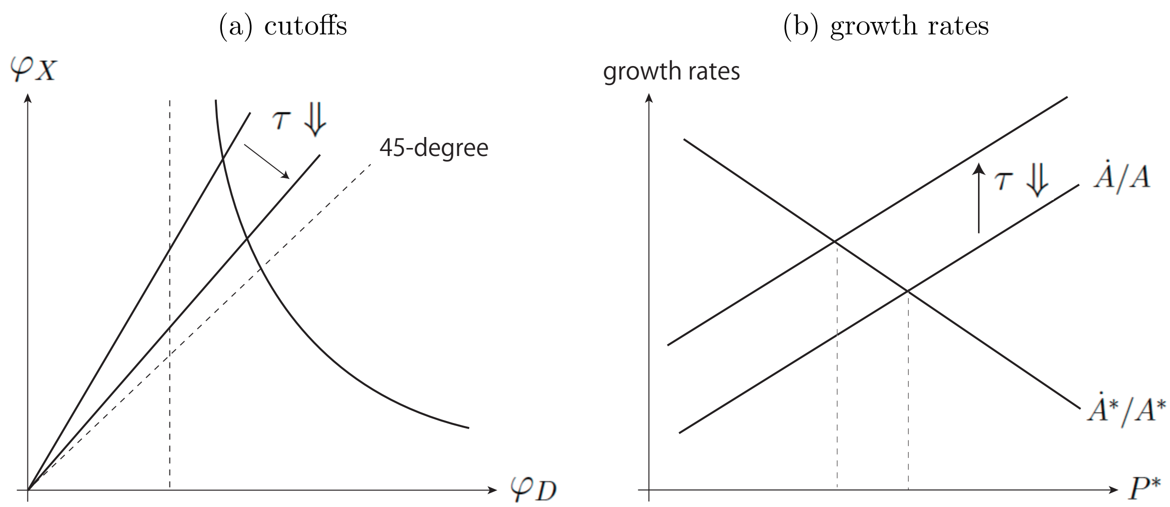

Figure 5: Effects of a unilateral decrease in τ

in Propositions 6 and 7. However, no significant differences in the two figures are observed for the other key variables.

5.2 On the non-tradability of the final good

Throughout this study, we assume that the final good produced in each country cannot be traded internationally. If we allow trade in the good, $P_t^* = 1$ for all t; this leads to $\varphi_D = \varphi_D^*$. From (14) and (15), $q(1) = q^*(1)$ holds in this case. Initially, consider the case of financial autarky as in Section 3. The condition for balanced growth under financial autarky (24) is now given by $\mathcal{Z}(\theta) = \mathcal{Z}(\theta^*)$, which is further reduced to $\theta = \theta^*$. Namely, the condition for balanced growth fails to hold unless the two countries are perfectly symmetric. Next, consider the case of financial integration as in Section 4. Since $q(1) = q^*(1)$ holds, the interest parity (29) is now given by $\tilde{z} = \tilde{z}^*$. The condition for balanced growth under financial integration (30) is reduced to $\theta = \theta^*$, which still does not hold except in the knife-edge case of perfectly symmetric countries.

Therefore, in our framework of international asymmetry in the degree of financial frictions, the non-tradability of the final good is required to obtain the BGP equilibrium.

5.3 Equilibrium with asymmetric trade costs

Finally, we introduce another source of asymmetry between the two countries. Here, we focus on the asymmetry of trade costs by assuming $\tau^* \neq \tau$. Equation (13) is rewritten as

$$\frac{\varphi_{X,t}^*}{\varphi_{D,t}^*} = T^* P_t^{*1/\alpha},$$

where $T^* \equiv \tau^* \left(f_X/f_D\right)^{(1-\alpha)/\alpha} \neq T$. Accordingly, $P_{\min}^*$ is now defined as $P_{\min}^* \equiv T^{*-\alpha}$. The intermediate goods firms' cutoffs are determined in the same way as in Section 3.1. The real prices of the knowledge goods q and q^* are determined from (14) and (15), respectively. Trade costs τ and τ^*, through their effects on the cutoffs, affect the real prices. To make it explicit, we now express q and q^* as

$$q = q(P^*; \tau), \quad \partial q(P^*; \tau)/\partial \tau < 0,$$
$$q^* = q^*(P^*; \tau^*), \quad \partial q^*(P^*; \tau^*)/\partial \tau^* < 0,$$

The signs of partial derivatives are shown as follows. Panel (a) of Figure 5 depicts how the cutoffs φ_j are determined as in the same manner as Figure 1. Under a given level of P^*, if τ falls, the export market becomes more profitable, which makes the slope of the straight line flatter. Hence, it increases φ_D, which in turn increases q.

We consider the case of financial autarky. The uniqueness of the BGP is established in Proposition 1. As in Naito (2017a), our model allows us to analyze not only worldwide trade liberalization ($d\tau < 0, d\tau^* < 0$) but also a unilateral one by a single country (e.g., $d\tau < 0, d\tau^* = 0$). Panel (b) of Figure 5 depicts how the BGP growth rate changes according to the home country's export liberalization.[23] As this figure shows, the BGP growth rate necessarily increases. We can easily extend the above analysis to the case of worldwide trade liberalization.

6 Concluding remarks

This study explores the relationship between growth, trade, and financial frictions in the context of a dynamic two-country model with heterogeneous firms and asymmetric countries. Our primary concern is to elucidate the impacts of a country-specific credit crunch on the production decisions of firms in both countries as well as on growth in the world economy. Under financial autarky, a credit crunch occurred in a country raises (lowers) the productivity cutoff of that country's intermediate goods sector for domestic (export) operation. Meanwhile, these effects are in the opposite directions for the intermediate goods sector of the trading partner. We also find that the credit crunch depresses transitional as well as long-run economic growth in both countries. If international lending and borrowing are allowed, the credit crunch affects not only the entrepreneurs' cutoff in that country but also that of entrepreneurs in the other country. This international diffusion effect cannot be observed in the case of financial autarky. However, our analysis also shows that such a difference in the impact on the entrepreneurs' cutoffs between these two cases only causes a negligible difference in the effects on the long run growth rate and cutoffs for the intermediate goods firms.

Here, we have assumed that endogenous growth is sustained by an AK technology, entrepreneurs are subject to stock-based collateral constraints, and productivity shocks are not persistent over time.

[23] Recall that τ is the shipping cost from the home country to the foreign country, while τ^* is that from the foreign country to the home country.

Nevertheless, our baseline setting is amenable to introducing endogenous growth sustained by R&D activities, flow-based borrowing constraints, and persistent productivity shocks. These extensions can enrich the analytical and numerical outcomes. Furthermore, we have not discussed policy issues. Addressing the optimal trade policy and optimal taxation in our model deserve further investigation.

Appendix

A.1 Entrepreneurs' intertemporal optimization

This maximization problem in Section 2.3 is rewritten as the following Hamilton–Jacobi–Bellman (HJB) equation:

$$\rho V_t(a,z)dt = \max\left\{(\ln c)dt + \mathbb{E}_t\left[dV_t(a,z)\right] \ : \ da = \left(R_t(z)a - c\right)dt\right\}.$$

We can solve this problem by use of "guess and verify." We guess that the value function takes the form $V_t(a,z) = \zeta(\phi_t(z) + \ln a)$ with ζ and $\phi_t(z)$ being unknown. Using this guess leads to $\mathbb{E}_t[dV_t(a,z)] = \zeta\left(\mathbb{E}_t[d\phi_t(z)] + da/a\right)$. The right-hand side of the HJB equation is rewritten as:

$$\max_c \ \ln c + \frac{\zeta(R_t(z)a - c)}{a} + \zeta\frac{1}{dt}\mathbb{E}_t\left[d\phi_t(z)\right].$$

Given ζ, the first-order condition is given by $1/c = \zeta/a$. This result must be consistent with the HJB equation. Substituting this result back into the HJB equation, we obtain

$$\rho\zeta(\phi_t(z) + \ln a) = \ln a - \ln\zeta + \zeta R_t(z) - 1 + \zeta\frac{1}{dt}\mathbb{E}_t\left[d\phi_t(z)\right],$$

the equality of which must be satisfied for all a. This implies $\zeta = 1/\rho$, which in turn yields $c_t = \rho a_t$. The dynamics of the net worth is given by $da_t = \left(R_t(z_t) - \rho\right)a_t dt$.

A.2 Proof of Lemma 4

In the main text, EX and IM are defined as

$$EX \equiv M^e \int_{\varphi\geq\varphi_X} p_X(\varphi)y_X(\varphi)dG(\varphi), \tag{A.1}$$

$$IM(=EX^*) \equiv M^{e*} \int_{\varphi\geq\varphi_X^*} p_X^*(\varphi)y_X^*(\varphi)dG(\varphi). \tag{A.2}$$

Using (3), (4), and $y_j(\varphi) = (\varphi/\varphi_j)^{1/(1-\alpha)}y_j(\varphi_j)$, we can obtain $p_X(\varphi)y_X(\varphi)$ as

$$p_X(\varphi)y_X(\varphi) = \frac{qf_X}{1-\alpha}\left(\frac{\varphi}{\varphi_X}\right)^{\alpha/(1-\alpha)}.$$

Analogously, we can obtain

$$p_X^*(\varphi)y_X^*(\varphi) = \frac{q^*f_X}{1-\alpha}\left(\frac{\varphi}{\varphi_X^*}\right)^{\alpha/(1-\alpha)}.$$

Substituting these results into (A.1) and (A.2), and using the definition of $H(\cdot)$, we obtain

$$EX = M^e\frac{qf_XH(\varphi_X)}{1-\alpha}, \tag{A.3}$$

$$IM = M^{e*}\frac{q^*P^*f_XH(\varphi_X^*)}{1-\alpha}. \tag{A.4}$$

Next, we derive M^e and M^{e*}. From (16) and $Y_K = \theta A \int_{z \geq \tilde{z}} z dF(z)$, we can obtain M^e as

$$M^e = \frac{\theta A \int_{z \geq \tilde{z}} z dF(z)}{f_E + \sum_{j=D,X} f_j (1 - G(\varphi_j))}.$$

Here, note that $\Pi(\varphi_j) = H(\varphi_j) - (1 - G(\varphi_j))$. Then, the free entry condition (5) is rewritten as

$$f_E + \sum_{j=D,X} f_j (1 - G(\varphi_j)) = \sum_{j=D,X} f_j H(\varphi_j),$$

which implies

$$M^e = \frac{\theta A \int_{z \geq \tilde{z}} z dF(z)}{\sum_{j=D,X} f_j H(\varphi_j)}.$$

Analogously, M^{e*} is given by

$$M^{e*} = \frac{\theta^* A^* \int_{z \geq \tilde{z}^*} z dF(z)}{\sum_{j=D,X} f_j H(\varphi_j^*)}.$$

Substituting the resulting M^e and M^{e*} into (A.3) and (A.4), we obtain

$$EX = \frac{q(P^*) \theta A \int_{z \geq \tilde{z}} z dF(z)}{1 - \alpha} \mu(P^*),$$
$$IM = \frac{q^*(P^*) P^* \theta^* A^* \int_{z \geq \tilde{z}^*} z dF(z)}{1 - \alpha} \mu^*(P^*).$$

A.3 Proof of Proposition 2

At first, we derive the following two equations useful for the proof. Totally differentiating (5), (12), and (13), and arranging the results, we obtain

$$\frac{P^* \varphi_D'(P^*)}{\varphi_D(P^*)} = \frac{1}{\alpha} \frac{f_X H(\varphi_X)}{\sum_{j=D,X} f_j H(\varphi_j)} = \frac{\mu}{\alpha}, \tag{A.5}$$

$$\frac{P^* \varphi_D^{*\prime}(P^*)}{\varphi_D^*(P^*)} = -\frac{\mu^*}{\alpha}. \tag{A.6}$$

Note that these equations hold without the Pareto specification of G.

Then, we show Proposition 2. Under financial autarky, the BGP is globally stable if and only if $dm^*/dP^* > 0$ holds in (23). As stated in Proposition 2, we specify G as $G(\varphi) = 1 - \varphi^{-\kappa}$. In this case, we can calculate μ as follows:[24]

$$\mu = \frac{f_X \varphi_X^{-\kappa}}{f_D \varphi_D^{-\kappa} + f_X \varphi_X^{-\kappa}} = \frac{h f_X}{f_E} \varphi_X^{-\kappa}, \tag{A.7}$$

[24]When $G(\varphi) = 1 - \varphi^{-\kappa}$, we can calculate $H(\varphi_j) = (h+1)\varphi_j^{-\kappa}$, where $h > 0$ is shown in the main text. Substituting this into (20) and using the fact that $f_D \varphi_D^{-\kappa} + f_X \varphi_X^{-\kappa} = f_E/h$ holds from the free entry condition, we obtain μ.

and we obtain μ^* as $hf_X\varphi_X^{*-\kappa}/f_E$. Using (12), (13), and (A.7), we can rewrite (23) as

$$
\begin{aligned}
m^* &= \left(\frac{\varphi_X}{\varphi_X^*}\right)^{-\kappa}\left(\frac{\varphi_D}{\varphi_D^*}\right)^{\alpha/(1-\alpha)}\frac{1}{P^*}\frac{\theta\int_{z\geq\bar{z}(\theta)}zdF(z)}{\theta^*\int_{z\geq\bar{z}(\theta^*)}zdF(z)} \\
&= \left(\frac{P^{*-1/\alpha}\varphi_D}{P^{*1/\alpha}\varphi_D^*}\right)^{-\kappa}\left(\frac{\varphi_D}{\varphi_D^*}\right)^{\alpha/(1-\alpha)}\frac{1}{P^*}\frac{\theta\int_{z\geq\bar{z}(\theta)}zdF(z)}{\theta^*\int_{z\geq\bar{z}(\theta^*)}zdF(z)} \\
&= \left(\frac{\varphi_D}{\varphi_D^*}\right)^{\alpha/(1-\alpha)-\kappa}P^{*2\kappa/\alpha-1}\frac{\theta\int_{z\geq\bar{z}(\theta)}zdF(z)}{\theta^*\int_{z\geq\bar{z}(\theta^*)}zdF(z)}.
\end{aligned}
\tag{A.8}
$$

Let a hat over a variable denote its rate of change (e.g., $\widehat{x} = dx/x$). From (A.8), we obtain

$$
\frac{\widehat{m}^*}{\widehat{P}^*} = \left(\frac{\alpha}{1-\alpha}-\kappa\right)\left(\frac{\widehat{\varphi}_D}{\widehat{P}^*}-\frac{\widehat{\varphi}_D^*}{\widehat{P}^*}\right)+\frac{2\kappa}{\alpha}-1.
$$

Using (A.5) and (A.6), we obtain

$$
\frac{\widehat{m}^*}{\widehat{P}^*} = \frac{\kappa[2-(\mu+\mu^*)]}{\alpha}+\frac{\mu+\mu^*}{1-\alpha}-1.
$$

Recall that we assume $h > 0$, that is, $\kappa > \alpha/(1-\alpha)$ for $\Pi(\varphi_j) = h\varphi_j^{-\kappa}$ to be well defined. Then, the following inequality holds:

$$
\frac{\widehat{m}^*}{\widehat{P}^*} > \frac{2}{1-\alpha}-1 > 0.
$$

Namely, when $G(\varphi) = 1-\varphi^{-\kappa}$, $dm^*/dP^* > 0$ is always satisfied. This in turn shows that the BGP under financial autarky is globally stable when $G(\varphi) = 1-\varphi^{-\kappa}$.

A.4 Derivation of (32)

Differentiating the definition of v with respect to time yields

$$
\dot{v} = \dot{B}/A - v\dot{A}/A.
$$

Using (8) and (17), we can arrange this equation as follows

$$
\begin{aligned}
\dot{v} &= r^b v - \frac{EX-IM}{A} - v\left[r^b + \theta\int_{z\geq\tilde{z}}\left(qz-\delta-r^b\right)dF(z)-\rho\right] \\
&= \left[\rho-\theta\int_{z\geq\tilde{z}}(qz-q\tilde{z})\,dF(z)\right]v - \frac{EX-IM}{A} \\
&= \Big(\rho - q(P^*)\tilde{z}\theta\Psi(\tilde{z})\Big)v - \frac{EX-IM}{A}.
\end{aligned}
\tag{A.9}
$$

In Lemma 4, we have already shown that EX and IM are expressed as (21) and (22). Therefore, the net export of the home country relative to its net worth can be expressed as

$$
\frac{EX-IM}{A} = \frac{1}{1-\alpha}\left(\mu(P^*)q(P^*)\theta\int_{z\geq\tilde{z}}zdF(z) - m^*P^*\mu^*(P^*)q^*(P^*)\theta^*\int_{z\geq\tilde{z}^*}zdF(z)\right). \tag{A.10}
$$

Substituting (A.10) into (A.9) yields (32).

A.5 The autonomous dynamical system under financial integration

Here, we derive the autonomous dynamical system. For this, it is convenient to use v instead of $\widetilde{z}$ and $\widetilde{z}^*$. From $v = \theta(1 - F(\widetilde{z})) - 1$, we can express $\widetilde{z}$ as a function of v:

$$\widetilde{z}_t = \widetilde{z}(v_t),$$

where

$$\widetilde{z}_v(v) \equiv \frac{\partial \widetilde{z}(\cdot)}{\partial v} = \frac{1}{-\theta F'(\widetilde{z})} < 0.$$

Using v, (31) can be rewritten as

$$v_t + m_t^* P_t^* [\theta^*(1 - F(\widetilde{z}_t^*)) - 1] = 0.$$

Note that this equation holds not only on but also off the BGP. This implies that $\widetilde{z}^*$ is a function of v, m^*, and P^*:

$$\widetilde{z}_t^* = \widetilde{z}^*(v_t, m_t^*, P_t^*),$$

where it follows that as long as $m^* > 0$ and $P^* > 0$,

$$\begin{aligned}
\widetilde{z}_v^*(\cdot) &\equiv \frac{\partial \widetilde{z}^*(\cdot)}{\partial v} = \frac{1}{m^* P^* \theta^* F'(\widetilde{z}^*)} > 0,\\
\widetilde{z}_{m^*}^*(\cdot) &\equiv \frac{\partial \widetilde{z}^*(\cdot)}{\partial m^*} = \frac{\theta^*(1 - F(\widetilde{z}^*)) - 1}{m^* \theta^* F'(\widetilde{z}^*)},\\
\widetilde{z}_{P^*}^*(\cdot) &\equiv \frac{\partial \widetilde{z}^*(\cdot)}{\partial P^*} = \frac{\theta^*(1 - F(\widetilde{z}^*)) - 1}{P^* \theta^* F'(\widetilde{z}^*)}.
\end{aligned}$$

We have already derived the dynamic equation of v as (32). Here, we define function Λ as

$$\Lambda(v, m^*, P^*) \equiv \frac{1}{1-\alpha}\left(\mu(P^*)q(P^*)\theta \int_{z \geq \widetilde{z}(v)} z dF(z) - m^* P^* \mu^*(P^*) q^*(P^*) \theta^* \int_{z \geq \widetilde{z}^*(v, m^*, P^*)} z dF(z)\right).$$

Then, (32) gives the dynamic equation of v, where its right-hand side now contains v, m^*, and P^*:

$$\dot{v}_t = \Big(\rho - q(P_t^*)\widetilde{z}(v_t)\theta\Psi(\widetilde{z}(v_t))\Big)v_t - \Lambda(v_t, m_t^*, P_t^*). \tag{A.11}$$

Using $\widetilde{z}_v, \widetilde{z}_v^*$, and $\widetilde{z}_{m^*}^*$, the properties of Λ are given by

$$\begin{aligned}
\Lambda_v(\cdot) &\equiv \frac{\partial \Lambda(\cdot)}{\partial v} = \frac{1}{1-\alpha}\left(-\mu q \theta \widetilde{z} F'(\widetilde{z})\widetilde{z}_v + m^* P^* \mu^* q^* \theta^* \widetilde{z}^* F'(\widetilde{z}^*)\widetilde{z}_v^*\right),\\
&= \frac{1}{1-\alpha}\left(\mu q \widetilde{z} + \mu^* q^* \widetilde{z}^*\right),\\
\Lambda_{m^*}(\cdot) &\equiv \frac{\partial \Lambda(\cdot)}{\partial m^*} = \frac{P^* \mu^* q^*}{1-\alpha}\left(-\theta^* \int_{z \geq \widetilde{z}^*} z dF(z) + m^* \theta^* \widetilde{z}^* F'(\widetilde{z}^*) z_{m^*}^*\right)\\
&= \frac{P^* \mu^* q^*}{1-\alpha}\left[-\theta^* \int_{z \geq \widetilde{z}^*} z dF(z) + \widetilde{z}^*[\theta^*(1 - F(\widetilde{z}^*)) - 1]\right].
\end{aligned}$$

The interest parity (28) provides the dynamics of the final good's price in the foreign country:

$$\frac{\dot{P}_t^*}{P_t^*} = q(P_t^*)\widetilde{z}(v_t) - q^*(P_t^*)\widetilde{z}^*(v_t, m_t^*, P_t^*). \tag{A.12}$$

From (8) and its foreign counterpart, we can express the dynamic equation of m as follows:

$$\begin{aligned}\frac{\dot{m}_t}{m_t} &\equiv \frac{\dot{A}_t^*}{A_t^*} - \frac{\dot{A}_t}{A_t} \\ &= \Gamma^*(v_t, m_t^*, P_t^*) - \Gamma(v_t, P_t^*). \end{aligned} \tag{A.13}$$

Equations (A.11)–(A.13) jointly constitute the autonomous dynamical system of the model. In (A.13), the functions Γ and Γ^* are defined as

$$\begin{aligned}&\Gamma(v, P^*) \equiv q(P^*)\widetilde{z}(v)\left[1 + \theta\Psi(\widetilde{z}(v))\right] \\ &\Gamma^*(v, m^*, P^*) \equiv q^*(P^*)\widetilde{z}^*(v, m^*, P^*)\left[1 + \theta\Psi(\widetilde{z}^*(v, m^*, P^*))\right],\end{aligned}$$

where

$$\begin{aligned}\Gamma_v(\cdot) &\equiv \frac{\partial\Gamma(\cdot)}{\partial v} = q\widetilde{z}_v\left(1 + \theta\Psi(\widetilde{z}) + \widetilde{z}\theta\Psi'(\widetilde{z})\right) = q\widetilde{z}_v[1 - \theta(1 - F(\widetilde{z}))] \\ \Gamma_{P^*}(\cdot) &\equiv \frac{\partial\Gamma(\cdot)}{\partial P^*} = q'\widetilde{z}\left(1 + \theta\Psi(\widetilde{z})\right) > 0, \\ \Gamma_v^*(\cdot) &\equiv \frac{\partial\Gamma^*(\cdot)}{\partial v} = q^*\widetilde{z}_v^*\left(1 + \theta^*\Psi(\widetilde{z}^*) + \widetilde{z}^*\theta^*\Psi'(\widetilde{z}^*)\right) = q^*\widetilde{z}_v^*\left[1 - \theta^*(1 - F(\widetilde{z}^*))\right], \\ \Gamma_{m^*}^*(\cdot) &\equiv \frac{\partial\Gamma^*(\cdot)}{\partial m^*} = q^*\widetilde{z}_{m^*}\left(1 + \theta^*\Psi(\tilde{z}^*) + \tilde{z}^*\theta^*\Psi'(\tilde{z}^*)\right) = q^*\widetilde{z}_{m^*}^*\left[1 - \theta^*(1 - F(\widetilde{z}^*))\right], \\ \Gamma_{P^*}^*(\cdot) &\equiv \frac{\partial\Gamma^*(\cdot)}{\partial P^*} = q^{*\prime}\widetilde{z}^*\left(1 + \theta^*\Psi(\tilde{z}^*)\right) + q^*\widetilde{z}_{P^*}^*\left(1 + \theta^*\Psi(\tilde{z}^*) + \widetilde{z}^*\theta^*\Psi(\widetilde{z}^*)\right) \\ &= q^{*\prime}\widetilde{z}^*\left(1 + \theta^*\Psi(\tilde{z}^*)\right) + q^*\widetilde{z}_{P^*}^*\left[1 - \theta^*(1 - F(\widetilde{z}^*))\right].\end{aligned}$$

A.6 Proof of Proposition 5

To show this proposition, we take the following three steps.

Linearization of the system. We examine the local stability of the dynamical system (A.11)–(A.13) with respect to v, m^*, and P^*. Recall the definitions of v and m^*: $v \equiv B/A$ and $m^* \equiv A^*/A$. Since $A^{(*)}$ is the aggregate wealth and B is the net foreign debts, both are predetermined variables. This implies that v_0 and m_0^* are historically given. By contrast, since the final good's price P^* is a forward-looking variable, its initial value is endogenously determined. Therefore, the system has one forward-looking and two predetermined variables.

Recall also that on the symmetric BGP, $(v, m^*, P^*) = (0, 1, 1)$. The linearization of the system around the symmetric BGP is given by

$$\begin{pmatrix} \dot{v}_t \\ \dot{m}_t^* \\ \dot{P}_t^* \end{pmatrix} = J \begin{pmatrix} v_t - 0 \\ m_t^* - 1 \\ P_t^* - 1 \end{pmatrix},$$

where the Jacobian matrix J is:

$$J = \begin{pmatrix} \rho - q\overline{z}\theta\Psi(\overline{z}) - \Lambda_v(0,1,1) & -\Lambda_{m^*}(0,1,1) & -\Lambda_{P^*}(0,1,1) \\ \Gamma_v^*(0,1,1) - \Gamma_v(0,1,1) & \Gamma_{m^*}(0,1,1) & \Gamma_{P^*}^*(0,1,1) - \Gamma_{P^*}(0,1,1) \\ q\widetilde{z}_v(0) - q^*\widetilde{z}_v^*(0,1,1) & -q^*\widetilde{z}_{m^*}^*(0,1,1) & q'\widetilde{z} - q^{*'}\widetilde{z}^* - q^*\widetilde{z}_{P^*}^*(0,1,1) \end{pmatrix}.$$

When we consider the symmetric BGP, we can obtain

$$\begin{aligned}
&\widetilde{z}_v(0) = -\widetilde{z}_v^*(0,1,1) = -\frac{1}{\theta F'(\overline{z}(\theta))} < 0, \\
&\widetilde{z}_{m^*}^*(0,1,1) = 0, \\
&\widetilde{z}_{P^*}^*(0,1,1) = 0, \\
&\Lambda_v(0,1,1) = \frac{2\mu q\overline{z}(\theta)}{1-\alpha}, \\
&\Lambda_{m^*}(0,1,1) = -\frac{\mu q\theta}{1-\alpha}\int_{z\geq\overline{z}(\theta)} z dF(z), \\
&\Gamma_v(0,1,1) = 0, \\
&\Gamma_{P^*}(0,1,1) = q'(1)\overline{z}(\theta)\left(1+\theta\Psi(\overline{z}(\theta))\right) > 0, \\
&\Gamma_v^*(0,1,1) = 0, \\
&\Gamma_{m^*}^*(0,1,1) = 0, \\
&\Gamma_{P^*}^*(0,1,1) = q^{*'}(1)\overline{z}(\theta)\left(1+\theta\Psi(\overline{z}(\theta))\right) < 0.
\end{aligned}$$

Hence, the Jacobian coefficient matrix J can be simplified as follows:

$$J = \begin{pmatrix} \rho - q\overline{z}\theta\Psi(\overline{z}) - \dfrac{2\mu q\overline{z}}{1-\alpha} & \dfrac{\mu q\theta}{1-\alpha}\int_{z\geq\overline{z}(\theta)} z dF(z) & -\Lambda_{P^*}(0,1,1) \\ 0 & 0 & (q^{*'}(1) - q'(1))\overline{z}\left(1+\theta\Psi(\overline{z})\right) \\ \dfrac{-2q}{\theta F'(\overline{z}(\theta))} & 0 & (q'(1) - q^{*'}(1))\overline{z} \end{pmatrix}.$$

The signs of the determinant and trace of J. The determinant of J is calculated as:

$$\det J = -\frac{\mu q\theta}{1-\alpha}\int_{z\geq\overline{z}(\theta)} z dF(z)(q^{*'}(1) - q'(1))\overline{z}\left(1+\theta\Psi(\overline{z})\right)\frac{2q}{\theta F'(\overline{z}(\theta))}.$$

Since $q^{*'}(P^*) < 0$ and $q'(P^*) > 0$, we can find $\det J > 0$. As for the trace of J,

$$\text{tr } J = \rho - q\overline{z}\left(\theta\Psi(\overline{z}) + \frac{2\mu}{1-\alpha} - \frac{q'(1) - q^{*'}(1)}{q}\right).$$

From (14) and (15), we obtain

$$\frac{P^* q'(P^*)}{q(P^*)} = \frac{\alpha}{1-\alpha}\frac{P^*\varphi_D'(P^*)}{\varphi_D(P^*)}, \quad \frac{P^* q^{*'}(P^*)}{q^*(P^*)} = \frac{\alpha}{1-\alpha}\frac{P^*\varphi_D^{*'}(P^*)}{\varphi_D^*(P^*)}.$$

Using these results together with (A.5) and (A.6), we obtain

$$\frac{P^* q'(P^*)}{q(P^*)} - \frac{P^* q^{*'}(P^*)}{q^*(P^*)} = \frac{\mu+\mu^*}{1-\alpha}.$$

On the symmetric BGP, $P^* = 1$ and $\mu = \mu^*$ are satisfied, which results in $(q'(1) - q^{*'}(1))/q = 2\mu/(1-\alpha)$. Therefore, the trace is given by

$$\mathrm{tr}\ J = \rho - q(1)\overline{z}(\theta)\theta\Psi(\overline{z}(\theta)).$$

Proof of Proposition 5. As the autonomous dynamical system has one forward-looking and two predetermined variables, the uniqueness of the equilibrium path converging to the BGP requires J to have one positive and two negative eigenvalues. Since $\det J > 0$ implies that J has one or three positive eigenvalues, the necessary and sufficient condition for uniqueness is $\mathrm{tr}\ J < 0$, namely, $\rho < q(1)\overline{z}(\theta)\theta\Psi(\overline{z}(\theta))$.

A.7 Determination of β, τ, f_E, and f_X

When we specify the distribution of z as $F(z) = (1 - z^{-\gamma})/(1 - \beta^{-\gamma})$, we can obtain $\Psi(\widetilde{z})$ as

$$\Psi(\widetilde{z}) = \frac{1}{1-\beta^{-\gamma}}\left(\frac{\widetilde{z}^{-\gamma}}{\gamma - 1} + \beta^{-\gamma}\right).$$

From this result and $q\widetilde{z} = r^b + \delta$, we can express the BGP growth rate as

$$g^{fi} = (r^b + \delta)\left[1 + \frac{\theta}{1-\beta^{-\gamma}}\left(\frac{\widetilde{z}^{-\gamma}}{\gamma-1} + \beta^{-\gamma}\right)\right] - \delta - \rho. \tag{A.14}$$

Since we assume $\theta^* = \theta$ when we determine the parameters, $m^* = 1$ and $P^* = 1$ hold. Therefore, the asset market equilibrium is rewritten as

$$\theta\frac{\widetilde{z}^{-\gamma} - \beta^{-\gamma}}{1 - \beta^{-\gamma}} = 1. \tag{A.15}$$

We have already determined the values of parameters ρ, δ, θ, and γ. In addition, we have already determined the target values of g^{fi} and r^b. Then, from (A.14) and (A.15), we obtain the value of β together with the value of $\widetilde{z}$.

In turn, we can obtain the value of q, since $q = (r^b + \delta)/\widetilde{z}$. Using this result and $q = \chi L\varphi_D^{\alpha/(1-\alpha)}$, we can determine φ_D.[25] When we specify the distribution of φ as $G(\varphi) = 1 - \varphi^{-\kappa}$, it follows that

$$1 - \mu = \frac{f_D H(\varphi_D)}{\sum_{j=D,X} f_j H(\varphi_j)} = \frac{hf_D}{f_E}\varphi_D^{-\kappa}. \tag{A.16}$$

Under balanced trade, μ is equal to the import penetration ratio.[26] Thus, $\mu = 0.081$. Since we have already set f_D, α, and κ, f_E is determined from (A.16). Since $P^* = 1$, φ_X is given by

$$\varphi_X = T\varphi_D. \tag{A.17}$$

[25] Recall that $L = 1$ and $\chi \equiv \alpha^{\alpha/(1-\alpha)}(1-\alpha)/f_D$.

[26] Since we assume a perfect symmetry between two countries when determining the parameters, the trade balance indeed holds.

In our model, the fraction of exporting intermediate goods firms is given by

$$\frac{1-G(\varphi_X)}{1-G(\varphi_D)} = \left(\frac{\varphi_X}{\varphi_D}\right)^{-\kappa} = T^{-\kappa},$$

which is set at 0.21. Then, T is determined as $0.21^{-1/\kappa} \simeq 1.366$. From (A.17), we can determine φ_X. When $G(\varphi) = 1 - \varphi^{-\kappa}$,

$$\mu = \frac{hf_X}{f_E}\varphi_X^{-\kappa},$$

from which f_X is determined. Finally, from the definition of T,

$$T^{-\kappa} = 0.021 = \left[\tau\left(\frac{f_X}{f_D}\right)^{(1-\alpha)/\alpha}\right]^{-\kappa}.$$

From this equation, τ is determined.

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