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Time-varying ambiguity shocks and business cycles*

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Abstract

This study investigates how ambiguity has driven output and inflation in the U.S. over the past 70 years. We adopt the recently developed techniques that disentangle ambiguity from risk and assess the responses of output and inflation to ambiguity shocks. We observe that an increase in ambiguity led to an increase in output during high inflation periods, indicating the ambiguity lover behavior. We also uncover that ambiguity and risk estimated by realized volatility have the opposite impacts on business cycles, which is consistent with the prevailing asset pricing literature.

Keywords: Ambiguity, Risk premiums, Uncertainty, TVP-VAR
JEL Codes: E32, E44

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1 Introduction

It is difficult to predict what will happen in the future. For example, the randomness of asset returns is one of the most important elements in portfolio selection problems and asset pricing, as it makes it troublesome for investors to determine how much and which stocks should be invested. It is also hard to pin down the probability distributions of asset returns. When a decision maker does not have a unique probability distribution of random variables, we say that the decision maker perceives ambiguity in random variables. The importance of the distinction between risk and ambiguity has long been recognized in economics, particularly in decision theory, since Knight (1921).[1] This paper sheds light on the role of ambiguity and empirically estimates the effects of such ambiguity on output and inflation in the U.S. post-World War II era. Although "uncertainty" is often referred to as risk or ambiguity, we employ the notion of uncertainty in a broader sense; that is, uncertainty includes the notions of risk and ambiguity and entails all situations except for deterministic cases.

Axiomatizations of rational behaviors under ambiguity have been proposed to overcome Ellsberg's paradox (1961), which experimentally shows that individuals' behaviors cannot be explained by the standard expected utility theory. Gilboa and Schmeidler (1989) propose the Maxmin expected utility theory (MEU) and Schmeidler (1989) introduces the Choquet expected utility theory (CEU) to provide rigorous behavioral foundations of decision makers under ambiguity. Klibanoff et al. (2005) present the smooth ambiguity model that can differentiate decision makers' attitudes toward ambiguity from their perception of ambiguity.[2] The applications of MEU, CEU, and the smooth ambiguity model to portfolio choice and asset pricing highlight the importance of ambiguity (Dow and Werlang, 1992; Epstein and Wang, 1994; Cao et al., 2005; Gollier, 2011). However, empirical studies have only recently begun in this area owing to the difficulty of estimating ambiguity.

Recently, a series of works based on the theoretical model by Izhakian (2020)

[1] In decision theory, risk is the situation in which individuals have a unique probability. However, in this paper, risk indicates the volatility of random variables.

[2] For a textbook presentation, see, for example, Gilboa (2009), and also Nishimura and Ozaki (2017).

disentangle ambiguity from risk and evaluate how these two elements are linked to asset prices (Izhakian and Yermack, 2017; Brenner and Izhakian, 2018; Augustin and Izhakian, 2020). These studies present empirical evidence that risk and ambiguity have opposite impacts on early exercises of employee options, stock market expected returns, and prices of credit default swaps. However, in contrast to these asset pricing contexts, how ambiguity drives macroeconomic output and inflation over a longer period remains an open question. The previous literature uncovers that risk plays an important role in business cycles (Bloom et al. 2009; Basu and Bundick, 2017). Our first contribution is that we investigate whether ambiguity has different impacts on output and inflation in comparison with risk.

This paper adopts the approach proposed by Brenner and Izhakian (2018) and extract ambiguity from the market risk premium.[3] Estimating ambiguity using other risk premiums including size, value, and momentum, is our second contribution.[4] These risk premiums reflect investors' expectations and sentiment since many investors invest in money based on these investment styles (Barberis and Shleifer, 2003; Stambaugh and Yuan, 2016). The previous literature demonstrates that these risk premiums contain future information about economic growth (Liew and Vassalou, 2000) and function as proxies of investment opportunities (Petkova, 2006). These results support the view of Fama and French (1995) and Petkova and Zhang (2005) who highlight that three- and four-factor models are interpreted by a risk-based story. We extend this literature by estimating ambiguity from these risk premiums and explore whether it contains different information from the market risk premium ambiguity.

Our third contribution is that we deploy a time-varying parameter vector autoregressive model with stochastic volatility (TVP-VAR-SV) to evaluate the time-varying relationships between ambiguity shocks and macroeconomic variables (e.g., Primiceri, 2005; Galí and Gambetti, 2009; Baumeister and Peersman, 2013). The estimation period in this study exceeds 70 years, and it is reasonable to consider that the underlying economic structure has changed. For instance, Del Negro et al.

[3]We refer to this ambiguity as *market risk premium ambiguity* when we need to distinguish it from ambiguity obtained from other risk premiums. See Table A2.

[4]Fama and French (1993) and Carhart (1997) reveal that including these factors in the Capital Asset Pricing Model (CAPM) reduces the pricing errors of cross-sectional asset pricing models.

(2019) suggest that demographic trends lower real interest rates and Clarida et al. (2000) and Bernanke (2020) highlight that monetary policies play an important role in stabilizing the economy. New information technologies lead to improvements in inventory management (Kahn et al., 2002) and financial innovations that allow firms and households to mitigate their financial constraints (Dynan et al., 2006). To reflect these structural changes, the TVP-VAR-SV model is employed.

Our main findings are fourfold. First, the responses of output and inflation to ambiguity shocks vary over time. An increase in ambiguity led to an increase in output during the high inflation period in the 1970s and the 1980s, and this is consistent with the ambiguity lover behavior in Brenner and Izhakian (2018). This suggests that economic agents prefer ambiguity with an increase in the probability of unfavorable outcomes, such as high inflation.

Second, the responses of inflation to ambiguity shocks were negative in the 1950s. The inflation rate in the 1950s was relatively high and an increase in ambiguity was a positive signal for economic agents, and hence the high value of ambiguity led to low inflation. However, the response of inflation was unclear in the 1970s and the 1980s, indicating that positive ambiguity shocks were not sufficient in reducing the high inflation during this period.

Third, ambiguity estimated from the size, value, and momentum risk premiums is weakly linked to output and inflation, suggesting that these risk premiums contain information about future economic growth, but do not capture information about the entire market.

Finally, our estimated ambiguity differs from stock market volatility. Our realized and mixed data sampling (MIDAS) volatilities show that positive volatility shocks reduced output during the high inflation period in the 1970s and the 1980s.[5] Moreover, we deploy the uncertainty measure proposed by Jurado et al. (2015) and find that positive uncertainty shocks reduce output and raise inflation during the high inflation period. These indicate that ambiguity and risk have the opposite impacts on business cycles.

The rest of this paper is organized as follows: Section 2 reviews the related liter-

[5]The MIDAS model is proposed by Ghysels et al. (2005).

ature, Section 3 describes our theoretical background and methodologies, Section 4 explains the dataset, Section 5 presents our main empirical results, Section 6 conducts further analysis, and Section 7 concludes.

2 Literature Review

Since the pioneering works by Gilboa and Schmeidler (1989) and Schmeidler (1989) that provide an answer to the Ellsberg paradox, we have recognized the role of ambiguity through the applications of CEU and MEU. Dow and Werlang (1992) explain the existence of portfolio inertia (or no trade). Portfolio inertia means price ranges over which investors neither buy nor sell stocks. Arrow (1965) demonstrates that portfolio inertia cannot be explained under the standard expected utility theory. Under a representative agent model, Epstein and Wang (1994) show that equilibrium prices are characterized by Euler inequalities, in stark contrast to Lucas (1978) who presents that the equilibrium price is uniquely determined by Euler equality. This implies that ambiguity leads to the indeterminacy of equilibria.

From the empirical viewpoint, previous studies have clarified the importance of ambiguity. Epstein and Schneider (2008) investigate the effects of bad and good news on investors' behaviors, and provide the empirical result that investors overvalue negative information and undervalue positive information under ambiguity. Analyzing the U.S. stock market and accounting data, Kelsey et al. (2010) find that negative momentum is greater than positive momentum in terms of the magnitude and persistence of portfolio returns, which indicates that ambiguity plays a part in such asymmetric patterns. Driouchi et al. (2018) analyze the behavior of U.S. index put option holders during the pre-crisis and credit crunch period in 2006–2008 and find evidence of ambiguity in the U.S. index options market during the credit crunch period. Ilut and Schneider (2014) consider shocks to confidence about future total factor productivity (TFP) as changes in ambiguity, and empirically show that TFP and confidence shocks play a role in explaining business cycle fluctuations.[6]

Risk has been intensively investigated by the literature and plays a key role in

[6]See Bianchi et al. (2018) for further analysis

economic growth. It triggers a sudden drop in aggregate outputs and then induces a strong recovery in economic activities, which impacts firms' decisions about new investments. The recent economic literature proposes measures of risk and assesses how they influence business cycles. For instance, realized and implied volatilities on the stock market are widely employed by previous studies (e.g., Bloom et al. 2009; Byrne et al., 2013; Basu and Bundick, 2017). Several studies decompose volatility into good and bad volatilities. Patton and Sheppard (2015) adopt high frequency data of stock prices and uncover that negative volatility is more strongly associated with future volatility. Segal et al. (2015) estimate good and bad volatilities from macroeconomic data and find that they have opposite impacts on economic growth. Berger et al. (2020) distinguish realized risk from forward looking risk, and find that the former drives economic activities. Several other studies use a large number of macroeconomic variables and deploy factor models in estimating risks (Jurado et al., 2015; Carriero et al., 2018).

TVP-VAR-SV models are employed to investigate whether response of shocks is time-varying, which is crucial in exploring economic relationships over a longer horizon. These include the response of interest rates to inflation shocks (Primiceri, 2005), the response of output to nontechnology shocks (Galí and Gambetti, 2009), the response of GDP to real price of crude oil shocks (Baumeister and Peersman, 2013), the response of inflation in the U.K. to monetary policy shocks (Ellis et al., 2014), the response of GDP to financial condition shocks (Abbate, et al., 2016), and the response of food production to food supply shocks (Peersman et al., 2021).

3 Methodologies

3.1 Measuring ambiguity

In this paper, we use the measure of ambiguity, $\mho^2$, proposed by Izhakian (2020), as explained below. This measure allows us to separate risk, attitudes toward risk, and attitudes toward ambiguity, and to obtain the degree of ambiguity. This provides the empirical value of ambiguity, which has been widely adopted in the literature; see Izhakian and Yermack (2017), Brenner and Izhakian (2018), Augustin and Izhakian

(2020), and Izhakian et al. (2022).[7]

Let S be a state space, let $\mathcal{E}$ be a σ-algebra on S, and let $P : \mathcal{E} \to [0,1]$ be a probability measure. Let $(S, \mathcal{E}, P)$ be a probability space, let $r : S \to X$ be a random variable (measurable function) on S, where $X \subseteq \mathbb{R}$ denotes an interval of outcomes (consequences) in $\mathbb{R}$ that contains the interval $[0,1]$. Let $\mathcal{P}$ be a set of probability measures on $(S, \mathcal{E})$ and let $P_r(x) = P(\{s \in S | r(s) \le x\})$ denote the cumulative probability (cumulative distribution function). Izhakina (2020) defines the expected cumulative probability of $x \in X$ over $\mathcal{P}$ by

$$E[P_r(x)] = \int_{\mathcal{P}} P_r(x) d\xi,$$

and the variance of the cumulative probability of $x \in X$ over $\mathcal{P}$ by[8]

$$\mathrm{Var}[P_r(x)] = \int_{\mathcal{P}} (P_r(x) - E[P_r(x)])^2 d\xi.$$

The measure of ambiguity is defined as

$$\mho^2[r] \equiv \int_X E[P_r(x)] \mathrm{Var}[P_r(x)] dx, \tag{1}$$

which represents the weighted average of the variances of the probabilities. In the following analysis, random variable r is considered to be the rates of returns of stock prices and $X = \mathbb{R}$.

We divide the range of daily returns, from -6% to $+6\%$, into ten intervals with each width 1%, and add two intervals, $(-\infty, -6\%]$ and $(+6\%, \infty)$, which yields 12 intervals. We discretize the return distributions into 12 bins $B_i = (r_{i-1}, r_i]$ of equal sizes for $i = 0, 1, \ldots, 11$, that is, $B_0 = (r_{-1}, r_0] = (-\infty, -6\%]$, $B_1 = (r_0, r_1] = (-6\%, -5\%], \ldots, B_{11} = (r_{10}, r_{11}] = (6\%, \infty)$. We compute the mean and the variance of the probabilities for each interval, $P(B_i)$ for $i = 0, 1, \ldots, 11$. Because some bins may not have return realizations, it is not easy to compute

[7] Fu et al. (2023) point out that there are some caveats to adopting $\mho^2$ as a measure of ambiguity, but Kostopoulos et al. (2022) also present empirical evidence that an alternative measure of ambiguity obtained from the expected volatility in the implied volatility and $\mho^2$ generate similar patterns. See Online Appendix for details.

[8] Similarly, Izhakian (2020) defines the expected probability density over $\mathcal{P}$ by $E[\varphi_r(x)] = \int_{\mathcal{P}} \varphi_r(x) d\xi$ and the variance of the probability density over $\mathcal{P}$ by $\mathrm{Var}[\varphi_r(x)] = \int_{\mathcal{P}} (\varphi_r(x) - E[\varphi_r(x)])^2 d\xi$, where $\varphi_r(x) = P(\{s \in S | r(s) = x\})$ is the probability density.

the probabilities. Therefore, similar to Augustin and Izakian (2020) and Brenner and Izhakian (2018), we alternatively assume a normal distribution, that is, $P[B_i] = [\Phi(r_i; \mu, \sigma) - \Phi(r_{i-1}; \mu, \sigma)]$, where $\Phi(\cdot)$ denotes the cumulative normal distribution function. Note that $P(B_0) = \Phi(r_0; \mu, \sigma) - \Phi(r_{-1}; \mu, \sigma) = \Phi(r_0; \mu, \sigma)$ and $P(B_{11}) = \Phi(r_{11}; \mu, \sigma) - \Phi(r_{10}; \mu, \sigma) = 1 - \Phi(r_{10}; \mu, \sigma)$. Note that μ and σ are estimated at a monthly frequency using daily data.

We estimate the degree of ambiguity of each month based on the following discrete form of Equation (1):

$$\begin{aligned}
& \mho^2[r] \\
= \; & E[\Phi(r_0; \mu, \sigma)]\mathrm{Var}[\Phi(r_0; \mu, \sigma)] \\
+ \; & \sum_{i=1}^{10} E[\Phi(r_i; \mu, \sigma) - \Phi(r_{i-1}; \mu, \sigma)]\mathrm{Var}[\Phi(r_i; \mu, \sigma) - \Phi(r_{i-1}; \mu, \sigma)] \\
+ \; & E[1 - \Phi(r_{10}; \mu, \sigma)]\mathrm{Var}[1 - \Phi(r_{10}; \mu, \sigma)]. \qquad (2)
\end{aligned}$$

Note that $\Phi(r_{-1}; \mu, \sigma) = 0$ and $\Phi(r_{11}; \mu, \sigma) = 1$. The right-hand side of Equation (2) does not include a scaling factor in Brenner and Izhakian (2018), since our interval size is larger than that of Brenner and Izhakian (2018). We confirm that this change does not impact our main results since the degree of ambiguity obtained by Brenner and Izhakian (2018) and our method in Equation (2) exhibit similar fluctuations, as reported in Figure A11.

3.2 Time-varying parameter vector autoregressive (TVP-VAR) model

The TVP-VAR model is an extension of the standard VAR model and allows us to obtain time-varying parameters. In this study, we employ the TVP-VAR to capture the time-varying relationships between ambiguity and business cycles. This model includes time variation of the coefficients and the multi-variate SV (Primiceri, 2005). Ellis et al. (2014) and Peersman et al. (2021) highlight that the SV captures nonlinearities in the simultaneous relations between the variables in the system. The previous literature presents empirical evidence that the effects of inflation, the real oil price, and the food price vary over time (Primiceri, 2005; Baumeister and Peersman, 2013; Peersman et al., 2021). We focus on a period of more than 70 years and assume that the economic structure has changed due to new information technologies, financial innovations, and low interest rates (Kahn et al. 2002; Dynan et al., 2006; Del Negro et al., 2019). Taken together, we consider that the effects of ambiguity vary over time.

We follow Primiceri (2005) and describe a basic TVP-VAR model as follows:

$$A_t Y_t = \sum_{i=1}^{k} \Psi_{i,t} Y_{t-i} + u_t, i = 1, ..., k, t = k+1, ..., T \tag{3}$$
$$u_t \sim \mathcal{N}(0, \Sigma_t),$$

where Y_t is a $n \times 1$ vector of variables. In this study, we employ output (Δy), inflation (Δp), and ambiguity ($\mho^2$); that is, $n = 3$. The TVP-VAR model has many parameters, and we restrict the number of variables (e.g., Peersman et al., 2021). A_t is the $n \times n$ simultaneous relations; $\Psi_{i,t}$ is the $n \times n$ matrix of coefficients; u_t represents a $n \times 1$ vector of error terms and follows the $\mathcal{N}(0, \Sigma_t)$ distribution; Σ_t is the $n \times n$ variance-covariance matrix of error terms. The number of lags in the VAR is denoted as k and we adopt $k = 2$ based on the Bayesian information criterion (BIC).

We then multiply both sides of Equation (3) by A_t^{-1} to obtain a reduced form representation of the TVP-VAR model:

$$Y_t = \sum_{i=1}^{k} B_{i,t} Y_{t-i} + A_t^{-1} V_t \epsilon_t \tag{4}$$
$$\epsilon_t \sim \mathcal{N}(0, I_n),$$

where the coefficient matrix $B_{i,t} = A_t^{-1} \Psi_{it}$; ϵ_t is the $n \times 1$ residual vector and follows the $\mathcal{N}(0, I_n)$ distribution. The variance-covariance matrix Σ_t in Equation (3) is decomposed as $\Sigma_t = A_t^{-1} V_t V_t' (A_t^{-1})'$ where V_t is the $n \times n$ diagonal matrix of standard deviations and written as:

$$V_t = \begin{pmatrix} \sigma_{1,t} & 0 & \cdots & 0 \\ 0 & \sigma_{2,t} & \cdots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \sigma_{n,t} \end{pmatrix}$$

The simultaneous relations A_t is assumed to be a lower triangular matrix as follows:

$$A_t = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ a_{21,t} & 1 & \cdots & 0 \\ \vdots & \ddots & \ddots & 0 \\ a_{n1,t} & \cdots & a_{nn-1,t} & 1 \end{pmatrix}$$

The coefficient matrix B_t, the simultaneous relations A_t and the standard deviation matrix V_t are all time-varying matrices. According to Primiceri (2005), these are all assumed to follow a random walk process as follows:

$$B_t = B_{t-1} + \xi_t^B$$
$$a_t = a_{t-1} + \xi_t^a$$
$$h_t = h_{t-1} + \xi_t^h$$
$$\begin{pmatrix} \epsilon_t \\ \xi_t^B \\ \xi_t^a \\ \xi_t^h \end{pmatrix} \sim \mathcal{N}\left(0, \begin{pmatrix} I_n & 0 & 0 & 0 \\ 0 & Q & 0 & 0 \\ 0 & 0 & S & 0 \\ 0 & 0 & 0 & W \end{pmatrix}\right),$$

where a_t is the element of the lower-triangular matrix A_t, and $h_t = \log \sigma_t^2$ where σ_t is the diagonal element of V_t.

Following Primiceri (2005), Koop and Korobilis (2010), and Ellis et al. (2014), we employ Bayesian methods for estimation. We implement the Gibbs sampler that evaluates the posterior distributions of the parameters. This entire algorithm is executed 30,000 times, with the first 10,000 draws discarded as a burn-in.

Our identification scheme follows Bernanke et al. (2005) and assumes that a slow-moving variable does not have a contemporaneous impact on a fast-moving variable. Our variable order of the base model is: output (Δy), inflation (Δp), and ambiguity ($\mho^2$). A change in output is relatively slow while that in ambiguity is fast, since it is based on stock market information. Bekaert et al. (2013) employ the uncertainty variable obtained from the stock market and adopt the same identification scheme. In the robustness section, we present that our results do not depend on the identification scheme.

4 Data

We adopt a market risk premium that is calculated as the excess return on the value-weighted return of Center for Research in Security Prices (CRSP) firms (e.g., Fama and French, 1993). We employ the market risk premium at a daily frequency and estimate ambiguity at a monthly frequency. Although Brenner and Izhakian (2018) and Augustin and Izhakian (2020) use intraday data, this paper deploys daily data, which enables us to investigate the effects of ambiguity on output and inflation in the U.S. over a long period of time.

We also use size, value, and momentum premiums which are calculated as spread returns at a daily frequency (Fama and French, 1993; Carhart, 1997). We construct six value-weighted portfolios based on firm size and book-to-market ratio. The size premium is calculated as the average spread return between three small and three large groups. The value premium is obtained as the average spread return between two value and two growth groups. We also deploy the momentum premium that is calculated as the average spread return between two high prior and two low prior return groups. The prior return is measured from the last 12 to two months but we skip the most recent month to avoid the reversal effect (Carhart, 1997). These data are downloaded from Kenneth French's website. Liew and Vassalou (2000) demonstrate that the size and value premiums contain information to predict future economic growth. Petkova (2006) adopts financial variables and shows that these premiums are linked to investment opportunity sets. The momentum premium is generated by heterogeneous exposures to past winner and past loser firms (Johnson, 2002; Liu and Zhang, 2008). These results suggest that the size, value, and momentum premiums differ from the market risk premium, and hence the ambiguity obtained from these risk premiums also has different impacts on business cycles.

We employ the industrial production index for the U.S. to capture output at a monthly frequency (Bekaert et al., 2013; Jurado et al., 2015). We measure inflation by adopting the consumer price index (CPI) for all items (Bekaert et al., 2013; Jurado et al., 2015). Following Bekaert et al., (2013), we employ the log difference of these variables and evaluate growth rates. We denote the growth rate of output Δy, and that of inflation Δp. The data are obtained from the Federal Reserve Bank of St. Louis. Our data period extends from January 1947 to December 2021, and the starting point depends on the availability of the monthly CPI.[9]

5 Empirical results

5.1 Estimated ambiguity

We begin our discussion with the fluctuations of ambiguity and Figure 1 depicts time series changes in ambiguity. The upper panel indicates ambiguity (left y-axis) and National Bureau of Economic Research (NBER) recessions, and the lower panel demonstrates ambiguity (left y-axis) and the market risk premium (right y-axis) that is calculated using the excess return of all CRSP firms. We employ the market risk premium at daily frequency and obtain monthly ambiguity. We observe a relatively high average value of ambiguity in the 1950s and the 1960s. The upper panel illustrates that the average value of ambiguity tends to be low during recessions. Brenner and Izhakian (2018) find that the average value of ambiguity becomes high when market volatility is relatively low. The lower panel focuses on longer-term results than those in Brenner and Izhakian (2018), but we can observe the same pattern.

5.2 Time-varying impulse responses

Next, we move onto the results of the TVP-VAR. Figure 2 demonstrates the time-varying impulse responses of output to ambiguity shocks. The results include periods from one- to four-months. We present the median responses with 68% bootstrap confidence bands, which is standard in the literature (e.g., Primiceri, 2005; Baumeister and Peersman, 2013). First, we focus on the two-month result in the upper right panel of Figure 2, since the one-month result does not show a clear pattern, indicating that the ambiguity shocks have lagged effects. We note that one standard deviation of the ambiguity shocks raises output, whereas there is clear time variation. The confidence intervals of the responses are above zero between the middle of the 1970s and the beginning of the 1990s, suggesting that high ambiguity is positive for

[9]See Tables A1 and A2 for details.

output. This period includes high inflation triggered by oil price shocks (e.g., Hamilton, 1983; Edelstein and Kilian, 2009). An increase in ambiguity suggests that high inflation does not last, which is positive for aggregate output. The median response of output achieved the highest value (0.43%) around 1979 when the U.S. experienced the highest inflation in our sample period.[10]

Our results are consistent with the ambiguity lover behavior. Brenner and Izhakian (2018) report that when the probability of unfavorable outcomes increases, agents prefer ambiguity in the stock market. In our situation, unfavorable outcomes correspond to the high inflation period. In contrast, Ilut and Schneider (2014) adopt confidence about TFP and reveal that confidence shocks lower output, which is opposite to our results. They deploy a different approach and consider that an increase in ambiguity leads to more cautious behavior of economic agents, and this accounts for difference in results. Note that the positive responses of output to ambiguity shocks differ from the responses of that to uncertainty shocks. Basu and Bundick (2017) and Berger et al. (2020) present that stock market return volatility has negative impacts on output. Combining these results, our findings suggest that ambiguity contains different information from risk estimated by stock market return volatility.

We also observe the time-varying response of inflation to ambiguity shocks in Figure 3. The upper left panel shows that the responses at the one-month were negative from the 1940s to the middle of the 1960s. The median response in 1950 was the lowest (-0.08%), but it reached around zero in 1966. The inflation rate in the 1950s was relatively high and an increase in ambiguity was a positive signal for economic agents, and hence the high value of ambiguity led to low inflation. The response of inflation was unclear in the 1970s and the 1980s, which suggests that positive ambiguity shocks were not sufficient in reducing high inflation and that tight monetary policy was needed, as reported by the literature (Clarida et al., 2000; Mavroeidis, 2010).

[10]In addition, the lower panels in Figure 2 illustrate that the positive ambiguity effects persist at the three- and four-months. The results at the eight- and 12-months are reported in Figure A4 and the impacts became marginal.

5.3 Ambiguity of other risk premiums

Having found the significant ambiguity impacts, this section estimates ambiguity from other risk premiums. We focus on size, value, and momentum premiums proposed by Fama and French (1993) and Carhart (1997), since the risk premiums are available at daily frequency. Liew and Vassalou (2000) uncover that the size and value premiums contain information about future economic growth, and Vassalou (2003) reports that they are linked to future news about economic growth. Petkova (2006) and Bali and Engle (2010) address that they are associated with investment opportunity sets and are considered risk premiums. Moreover, the literature reports the strong relationship between the momentum risk premium and business cycles since past winner firms have high exposure to business cycles compared with past loser firms (e.g., Johnson, 2002; Liu and Zhang, 2008).

Figure 4 demonstrates the responses of output and inflation to the size ambiguity shocks. The upper two panels in Figure 4 reveal that the positive size ambiguity shocks lead to an increase in output, except for the 1950s, and that these impacts increase gradually. The lower two panels display that the positive size ambiguity shocks raise inflation from the middle of the 1980s. The literature reports that the size risk premiums decayed since many investors employed the size factor.[11] Figure A1 also illustrates that the average size ambiguity became lower after the 1980s. These are related to the results that output and inflation responses are more sensitive to changes in size ambiguity. However, the median values of responses are much smaller than those to the market risk premium ambiguity shocks in Figures 2 and 3.

Figures A7 and A8 present the responses of output to the value and momentum ambiguity shocks. We can see that the responses differ from the results in Figures 2 and 3 and that the median values are small, which suggests that ambiguity of the size, value and momentum risk premiums contains different information from ambiguity of the market risk premium. The size and value premiums contain information about future economic growth, but do not capture information about the entire market. For instance, not all investors focus on the size and value factors and some investors

[11] See Asness et al. (2018) and references therein.

employ only a specific investment style (Cronqvist et al., 2015). Therefore, ambiguity of these premiums has relatively smaller impacts on output and inflation compared with the ambiguity of the market risk premium.

6 Further analysis

6.1 Different identification scheme

The empirical results of impulse responses depend on the chosen identification scheme. In our main results, we adopted a recursive identification scheme and assumed that slow-moving variables did not have any contemporaneous impacts on fast-moving variables (e.g., Bernanke et al., 2005). This section investigates whether our main results are sensitive to variable order. Identification II uses the following order: ambiguity, inflation, and output, and Identification III employs the following order: output, ambiguity, and inflation. Figures A5 and A6 present empirical evidence that the time-varying impulse responses of output and inflation are robust against the change in variable order.

6.2 Responses to volatility shocks

This section investigates how the effects of ambiguity shocks differ from those of volatility shocks. Izhakian and Yermack (2017) and Brenner and Izhakian (2018) present empirical evidence that volatility and ambiguity have the opposite impacts on equity risk premiums and option values. We employ the following three measures of volatility and replace ambiguity with one of them. First, we estimate monthly realized volatility (VOL_{RV}) using daily excess returns as in Bloom (2009) and Byrne et al. (2013). Second, we deploy a MIDAS model and estimate monthly volatility (VOL_{MIDAS}) using daily realized volatility. Ghysels et al. (2005) address that the size of rolling window size is crucial for volatility forecast and the MIDAS model provides effects of the optimal window size. Finally, we adopt total macroeconomic uncertainty (UNC) proposed by Jurado et al. (2015). This measure is obtained as the forecast error variance of macro economic indicators and the forecast model includes latent common factors and stochastic volatility.

Figure 5 shows the time-varying responses of output and inflation to the volatility shocks. The upper two panels show that the volatility shock lowers output. This is consistent with the results reported by Basu and Bundick (2017) who employ implied volatility and estimate a constant VAR model. We uncover that the volatility effects were greater during the high inflation period in the 1970s and 1980s. These findings are opposite to the effects of the ambiguity shocks reported by the previous section, and linked to the findings reported by Brenner and Izhakian (2018) and Izhakian and Yermack (2018) who focus on asset prices. Figure A10 displays that we observe a similar pattern using volatility estimated by the MIDAS model. Figure A9 demonstrates that the total macroeconomic uncertainty shock reduced output for the entire period and raised inflation at the end of the 1970s.

In summary, we observe that the effects of the ambiguity shocks are not captured by the volatility shocks.

6.3 Excluding effects of stochastic volatility

In this section, we explore the responses of output and inflation to ambiguity shocks using a constant VAR model. The TVP-VAR model includes many parameters and stochastic volatility. This complexity of the model may affect our estimation results. In particular, stochastic volatility is strongly linked to risk in modern finance theory (e.g., Harvey et al., 1994) and generates dynamic interactions across macroeconomic variables (Mumtaz and Zanetti, 2013). To deal with these concerns, we follow Giannone et al. (2015) and employ the constant Bayesian VAR (BVAR) model. We split the data into two periods: Period I covers February 1947 to December 1989, and Period II extends from January 1990 to December 2021.

Figure 6 presents the impulse response results obtained by the BVAR. The upper and lower left panels demonstrate that the response of output was positive and that of inflation was negative in Period I. The output result is consistent with the results in Figure 2, indicating that the positive ambiguity shocks led to an increase in output in the 1970s and 1980s. The negative inflation result is linked to the results in Figure 3, which illustrates that the positive ambiguity shocks lowered inflation in the 1950s.

The right two panels in Figure 6 also show the consistent results in Figures 2 and 3. The ambiguity shocks have less impact on output and inflation after 1990. The possible reasons for these weak ambiguity effects are that forward guidance of the Federal Reserve Board (FRB) plays an important role (e.g., Bernanke, 2020). For instance, Gürkaynak et al. (2005) uncover that Federal Open Market Committee statements have persistent impacts on long-term interest rates. Campbell et al. (2012) report that forward guidance that is tied with action by the FRB is informative for economic agents in reducing economic uncertainty.

7 Conclusion

This paper conducts an empirical assessment of how ambiguity is associated with output and inflation for the U.S. during the post-World War II era. Estimating ambiguity has long been considered a difficult task, while we adopt the recently developed method proposed by Izhakian (2020) and Brenner and Izhakian (2018) who disentangle ambiguity from risk. This paper estimates ambiguity from the market, size, value, and momentum risk premiums at a monthly frequency over the past 70 years.

It is observed that the ambiguity obtained from the market risk premium has time-varying impacts on output and inflation. The positive ambiguity shocks raised output between the middle of the 1970s and the beginning of the 1990s. An increase in ambiguity was positive during the high inflation period because high ambiguity indicated that the high inflation period was not persistent. In contrast, it is found that market volatility shocks reduced output in the same period, which is consistent with the asset pricing findings reported by Izhakian and Yermack (2017), Brenner and Izhakian (2018), and Augustin and Izhakian (2020). Our results provide useful information for policymakers. Focusing solely on market volatility may lead to an overestimation of the negative impacts on output, as ambiguity occasionally has positive effects on output. Moreover, the impacts of ambiguity shocks on output and inflation vary over time. Therefore, policymakers should update their understanding of the impact driven by ambiguity when assessing effective policies to address changes in output and inflation.

It is also uncovered that ambiguity estimated from the size, value, and momentum risk premiums has weaker impacts on output and inflation than that obtained from the market risk premium. This is because the size, value, and momentum risk premiums do not capture information about the entire market, since some investors do not change their investment style (Cronqvist et al., 2015). Investigating the information content of ambiguity obtained by these premiums is a task for future work.

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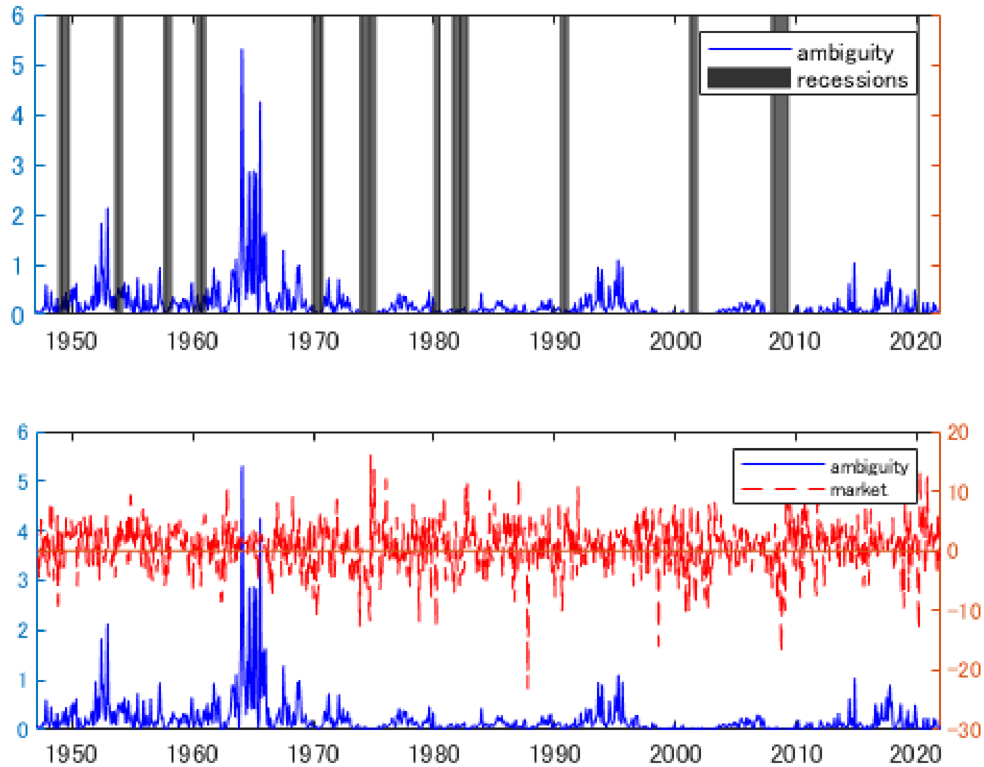

Figure 1. Ambiguity of the market risk premium

Notes: This figure presents the ambiguity of the market risk premium, which is obtained by Equation (2). The upper panel indicates ambiguity (left y-axis) and NBER recessions and the lower panel demonstrates ambiguity (left y-axis) and the market risk premium (right y-axis, %) that is calculated by the excess return of all CRSP firms.

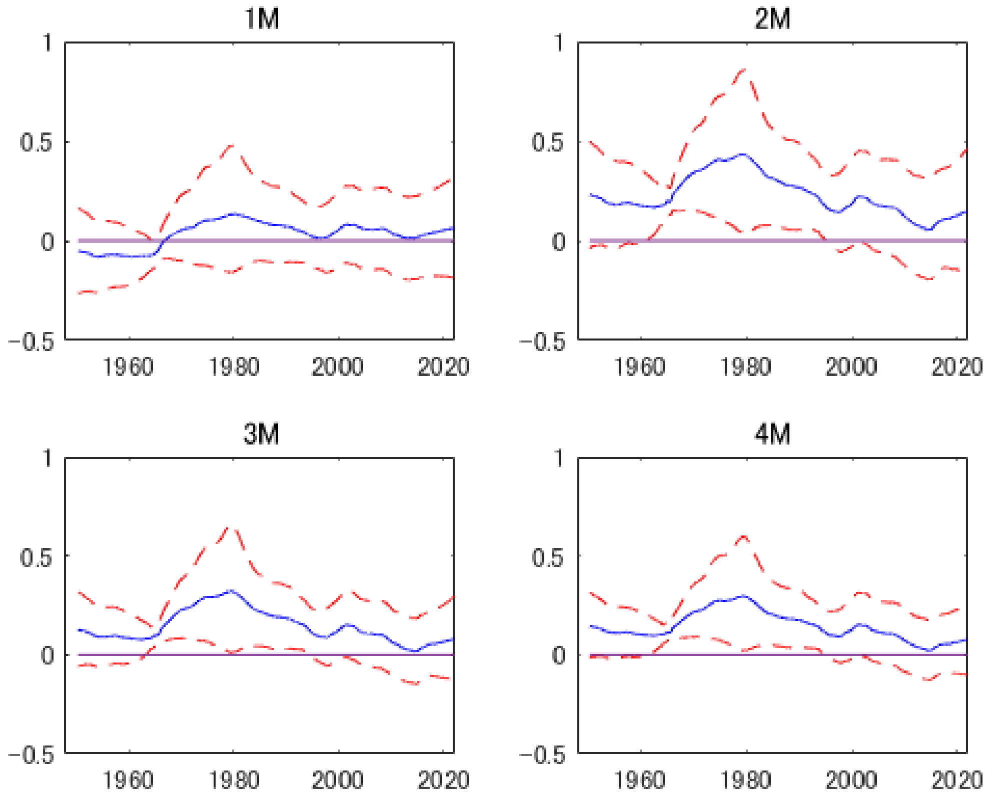

Figure 2. Impulse responses of output

Notes: This figure plots the median impulse responses of output to one standard deviation of the ambiguity shocks with 68% error bands. The TVP-VAR model employs the following variables: output (Δy), inflation (Δp), and ambiguity ($\mho^2$). The caption of each panel demonstrates the time horizon for the response.

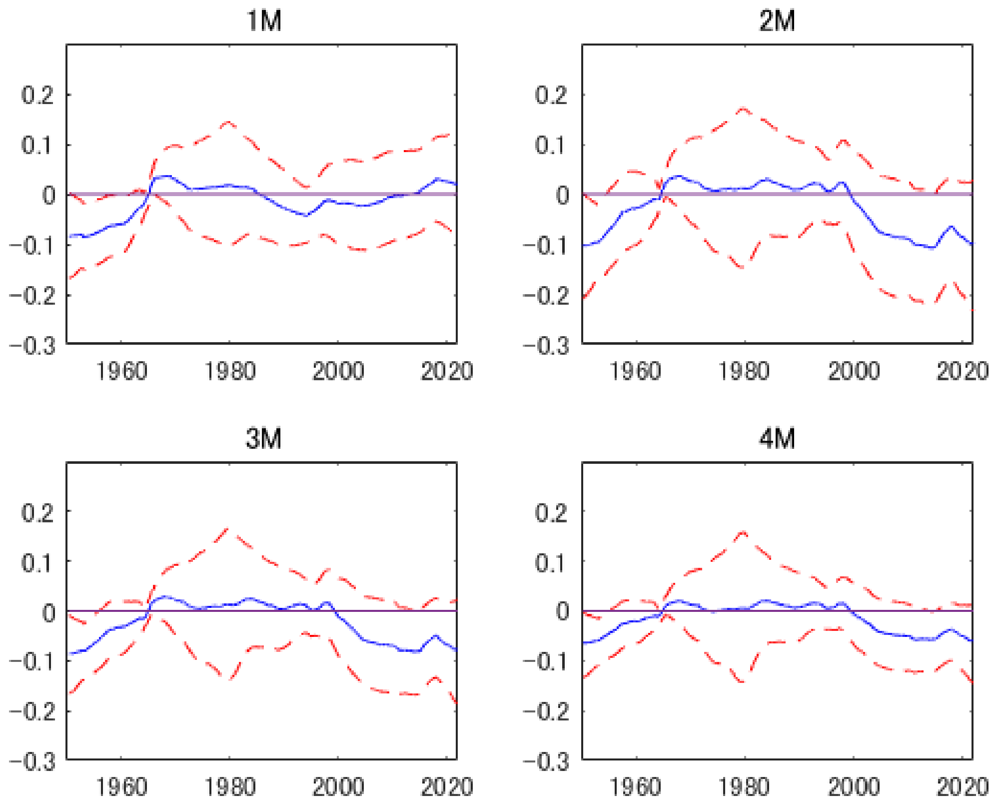

Figure 3. Impulse responses of inflation

Notes: This figure plots the median impulse responses of inflation to one standard deviation of the ambiguity shocks with 68% error bands. The TVP-VAR model employs the following variables: output (Δy), inflation (Δp), and ambiguity ($\mho^2$). The caption of each panel demonstrates the time horizon for the response.

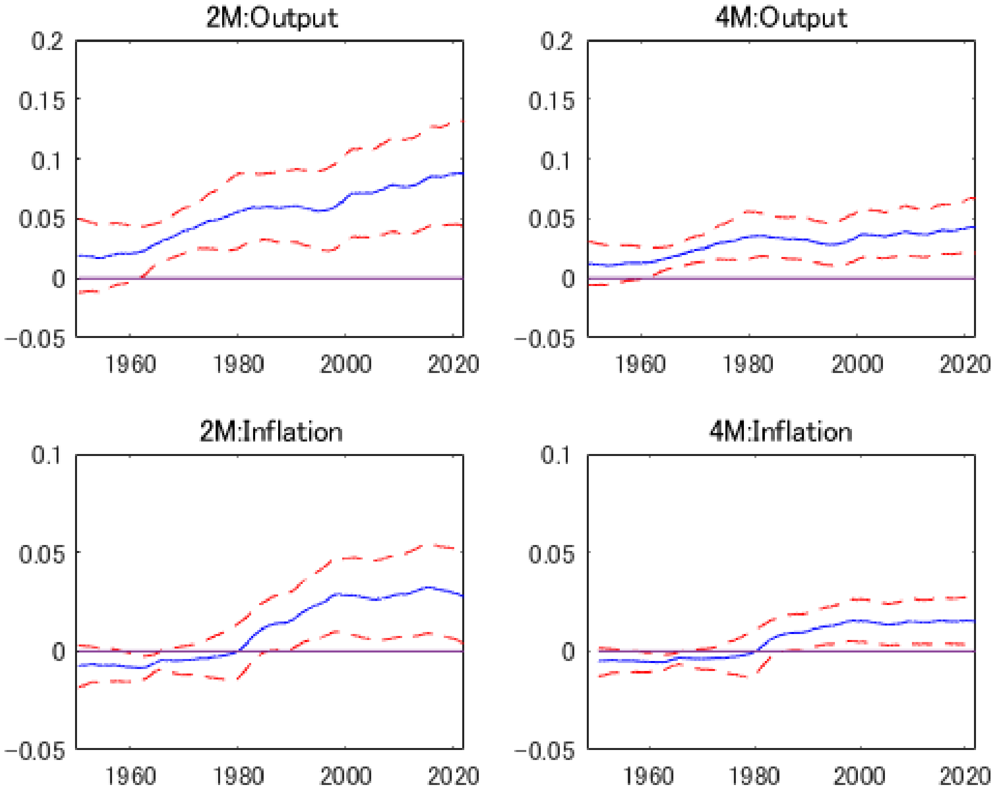

Figure 4. Impulse responses to size ambiguity shocks

Notes: This figure plots the median impulse responses of output and inflation to one standard deviation of the size ambiguity shocks with 68% error bands. The TVP-VAR model employs the following variables: output (Δy), inflation (Δp), and size ambiguity ($\mho^2_{\text{size}}$). The caption of each panel demonstrates the response and the time horizon.

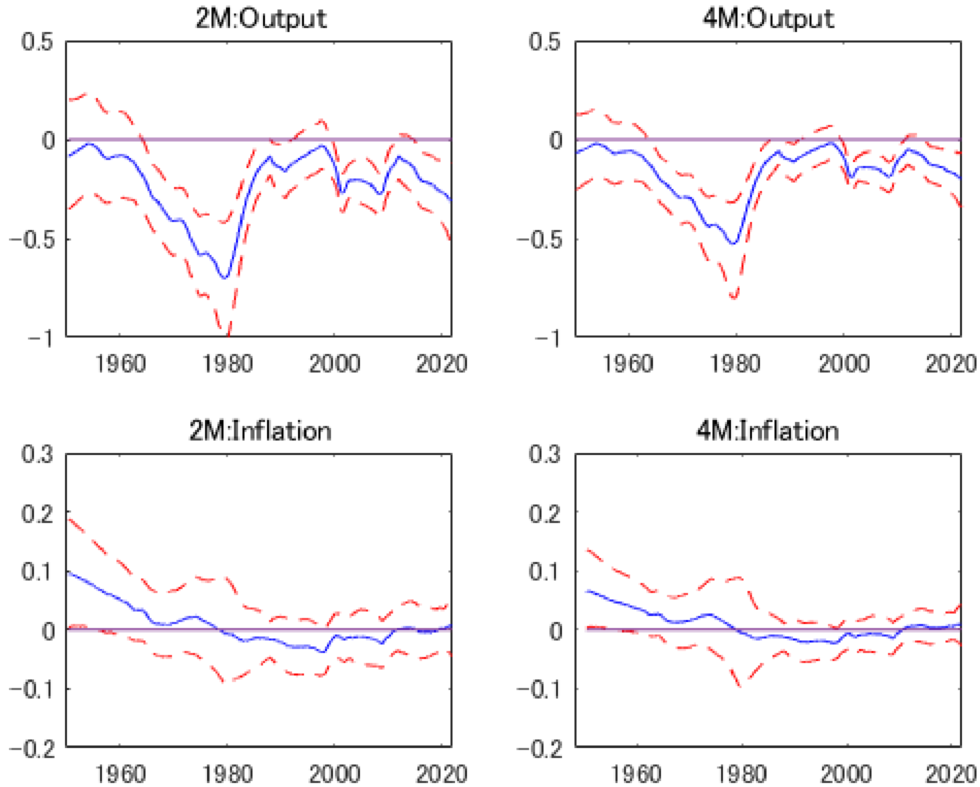

Figure 5. Impulse responses to realized volatility shocks

Notes: This figure plots the median impulse responses of output and inflation to one standard deviation of the realized volatility of shocks with 68% error bands. The realized volatility is calculated using the daily excess returns of the CRSP value-weighted index. The TVP-VAR model employs the following variables: output (Δy), inflation (Δp), and realized volatility (VOL_{RV}). The caption of each panel demonstrates the response and the time horizon.

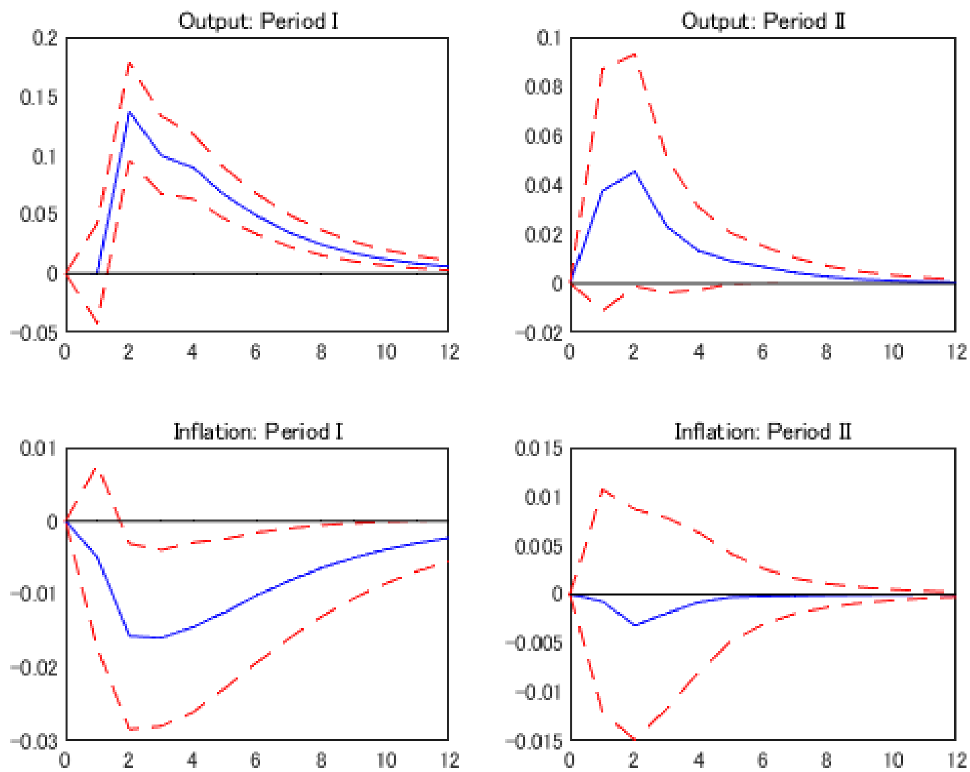

Figure 6. Impulse responses obtained by the constant BVAR model

Notes: This figure plots the median impulse responses of output and inflation to one standard deviation of the ambiguity shocks with 68% error bands. We employ the constant BVAR model. The BVAR model employs the following variables: output (Δy), inflation (Δp), and ambiguity ($\mho^2$). Period I is from February 1947 to December 1989 and Period II is from January 1990 to December 2021. The BVAR model is estimated based on the method of Giannone et al. (2015).

Time-varying ambiguity shocks and business cycles

August 16, 2023

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List of contents

A Estimation of time-varying parameter vector autoregressive (TVP-VAR) model

A.1 Priors

We follow Primiceri (2005), Koop and Korobilis (2010), and Baumeister and Peersman (2013) and the prior for the coefficients B_t, the simultaneous relations A_t, and the log of variances h_t are set to

$$
\begin{aligned}
B_0 &\sim \mathcal{N}(B_{OLS},\, 4 \cdot Var(B_{OLS})) \\
A_0 &\sim \mathcal{N}(A_{OLS},\, 4 \cdot Var(A_{OLS})) \\
h_0 &\sim \mathcal{N}(h_{OLS},\, 4 \cdot I_k),
\end{aligned}
$$

where the subscript OLS indicates the point estimates obtained by ordinary least squares (OLS) and Var denotes the covariance matrix.

The prior for the hyper-parameters are set to the inverse-Wishart distributions as:

$$
\begin{aligned}
Q_0 &\sim \mathcal{IW}(k_q^2 \cdot \tau \cdot Var(B_{OLS}), \tau) \\
W_0 &\sim \mathcal{IW}(k_w^2 \cdot 3 \cdot I_3, 2) \\
S_{1,0} &\sim \mathcal{IW}(k_s^2 \cdot 2 \cdot Var(A_{1,OLS}), 2) \\
S_{2,0} &\sim \mathcal{IW}(k_s^2 \cdot 3 \cdot Var(A_{2,OLS}), 3),
\end{aligned}
$$

where S_1 and S_2 indicate the two blocks of S, $\tau = 12$ denotes the sample size using the constant OLS, $k_q = 0.01$, $k_w = 1$, and $k_s = 0.1$.

A.2 Estimation

We implement the Gibbs sampler that evaluates the posterior distributions of the four blocks of parameters: the coefficients B^T; the simultaneous relations A^T; the standard deviations V^T, where the superscript T refers to the whole sample; and the hyper-parameters M that contain Q, W, S_1, and S_2.[1]

The estimation procedure is summarized as follows:

Step 1. Initialize B^T, A^T, V^T, and M by setting up training sample prior.

Step 2. Draw B^T from the conditional posterior distribution $p(B|Y^T, A^T, V^T, M)$.

Step 3. Draw A^T from the conditional posterior distribution $p(A|Y^T, B^T, V^T, M)$.

Step 4. Draw V^T from the conditional posterior distribution $p(V|Y^T, B^T, A^T, M)$.

Step 5. Draw M by sampling Q, W, S_1, and S_2 from $p(V|Y^T, B^T, A^T, V^T) = p(Q|Y^T, B^T, A^T, V^T) \cdot p(W|Y^T, B^T, A^T, V^T) \cdot p(S_1|Y^T, B^T, A^T, V^T) \cdot p(S_2|Y^T, B^T, A^T, V^T)$, since Q, W, S_1, and S_2 are independent of the other blocks.

Step 6. Go to Step 2.

[1]See Primiceri (2005) and Baumeister and Peersman (2013) for details.

B Measure of Ambiguity

This paper employs a measure of ambiguity, $\mho^2$, proposed by Izhakian (2020). As Fu et al. (2023) point out, some caveats are in order. Although Izhakian (2020) shows that the ordering based on $\mho^2$ can represent the decision maker's preference order, Fu et al. (2023) cast doubt on the validity of Izhakian's (2020) main theorems, in particular, Theorems 5, 6, and 7 that relate the decision maker's preference order to the measure of ambiguity $\mho^2$. Despite these problems, we adopt $\mho^2$ as a measure of ambiguity for two reasons. First, as discussed by Fu et al. (2023, p. 3), "While the strong connection between preferences under ambiguity and $\mho^2$ seems lost, $\mho^2$ may well measure ambiguity–just like there are many other measures of risk besides the variance", this measure is worth adopting. Second, Kostopoulos et al. (2022), employing the V-VSTOXX as a measure of ambiguity, adopt $\mho^2$ as an alternative measure and obtain similar results.

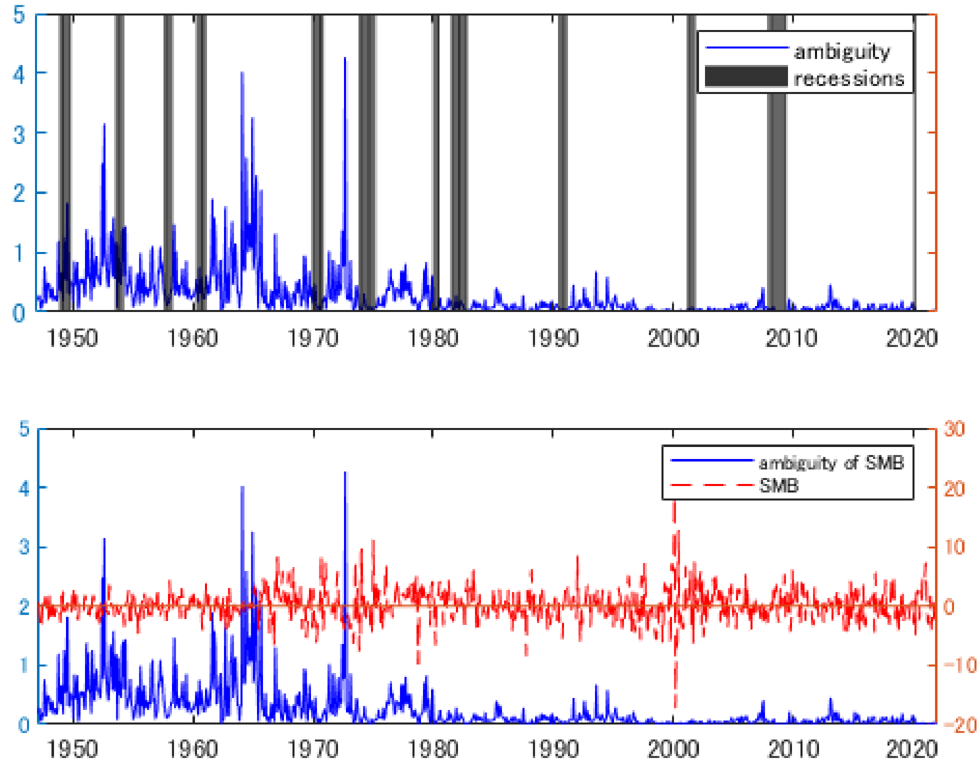
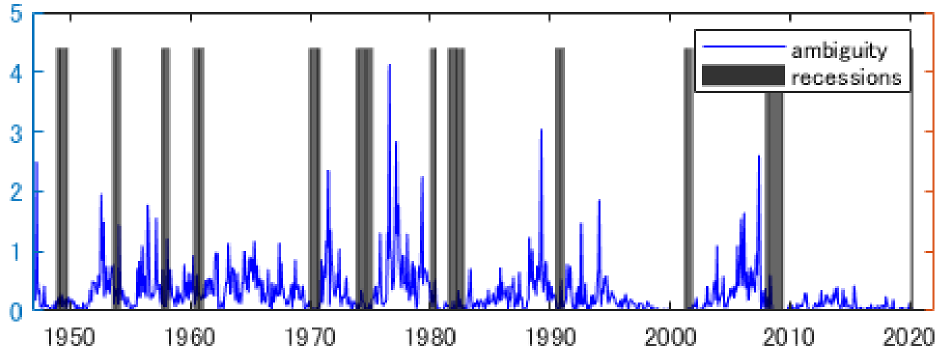


Figure A1. Ambiguity of the size premium

Notes: This figure presents ambiguity of the size premium, which is obtained by Equation (2). The upper panel indicates ambiguity (left y-axis) and NBER recessions and the lower panel demonstrates ambiguity (left y-axis) and the size premium (right y-axis, %).



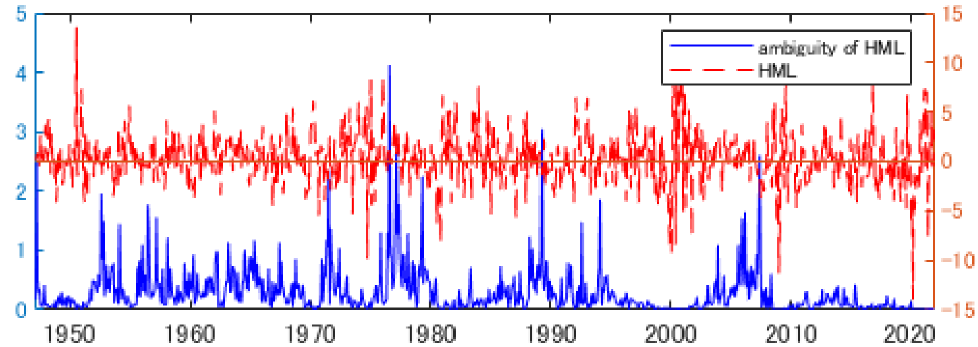


Figure A2. Ambiguity of the value premium

Notes: This figure presents ambiguity of the value premium, which is obtained by Equation (2). The upper panel indicates ambiguity (left y-axis) and NBER recessions and the lower panel demonstrates ambiguity (left y-axis) and the value premium (right y-axis, %).

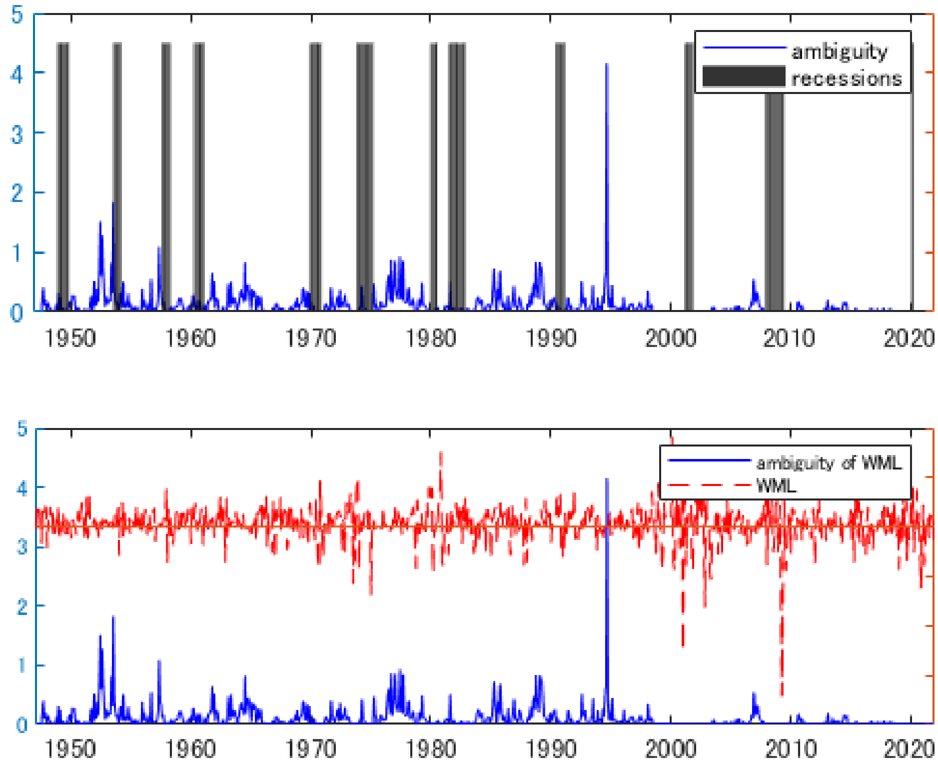


Figure A3. Ambiguity of the momentum premium

Notes: This figure presents ambiguity of the momentum premium, which is obtained by Equation (2). The upper panel indicates ambiguity (left y-axis) and NBER recessions and the lower panel demonstrates ambiguity (left y-axis) and the momentum premium (right y-axis, %).

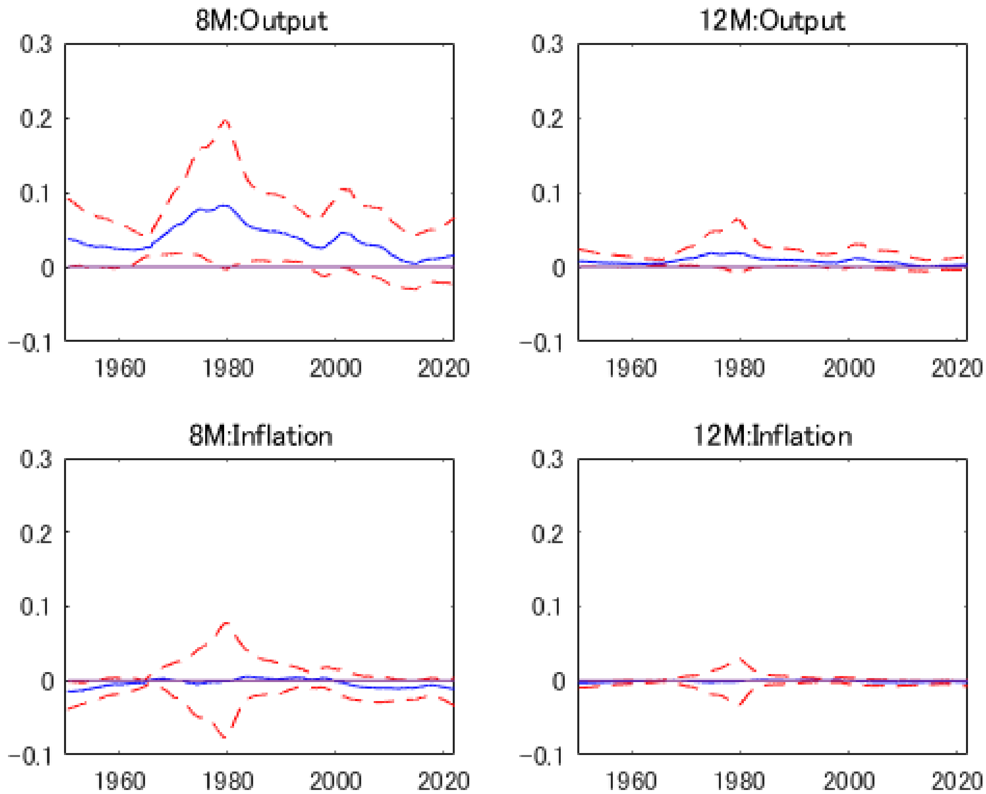


Figure A4. Impulse responses to ambiguity: 8- and 12-months ahead

Notes: This figure plots the median impulse responses of output and inflation to one standard deviation of the ambiguity shocks with 68% error bands. The TVP-VAR model employs the following variables: output (Δy), inflation (Δp), and ambiguity ($\mho^2$). The caption of each panel demonstrates the response and the time horizon.

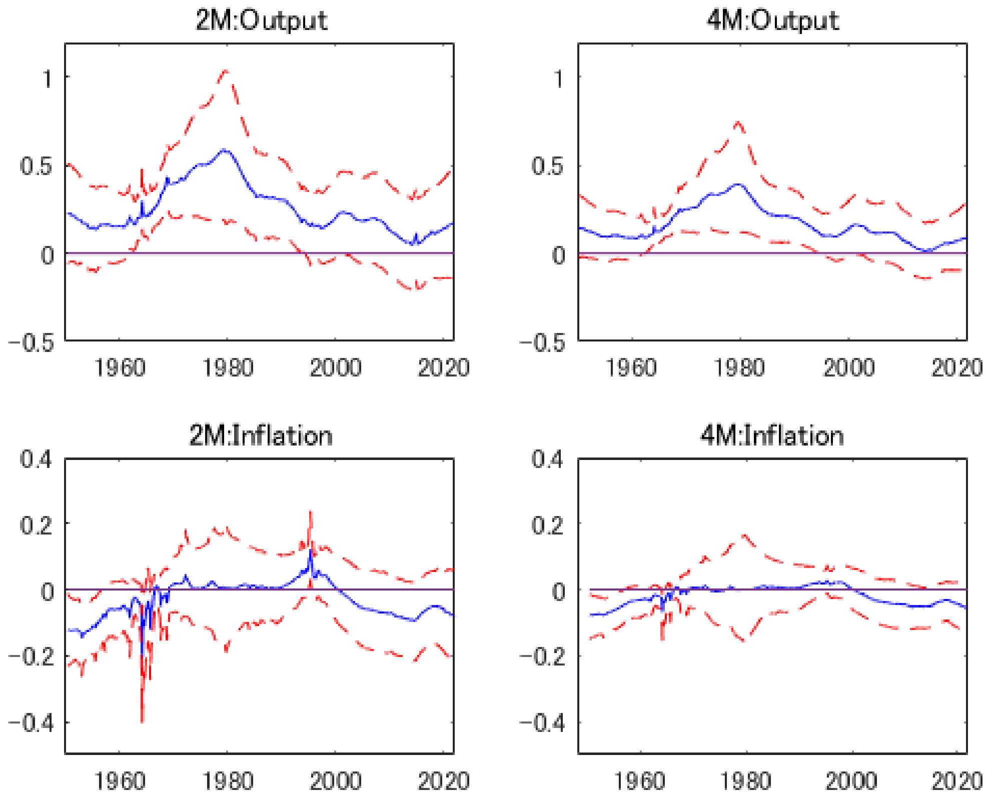


Figure A5. Impulse responses to ambiguity: Identification II

Notes: This figure plots the median impulse responses of output and inflation to one standard deviation of the ambiguity shocks with 68% error bands. The TVP-VAR model employs the following order: ambiguity ($\mho^2$), inflation (Δp), and output (Δy). The caption of each panel demonstrates the response and the time horizon.

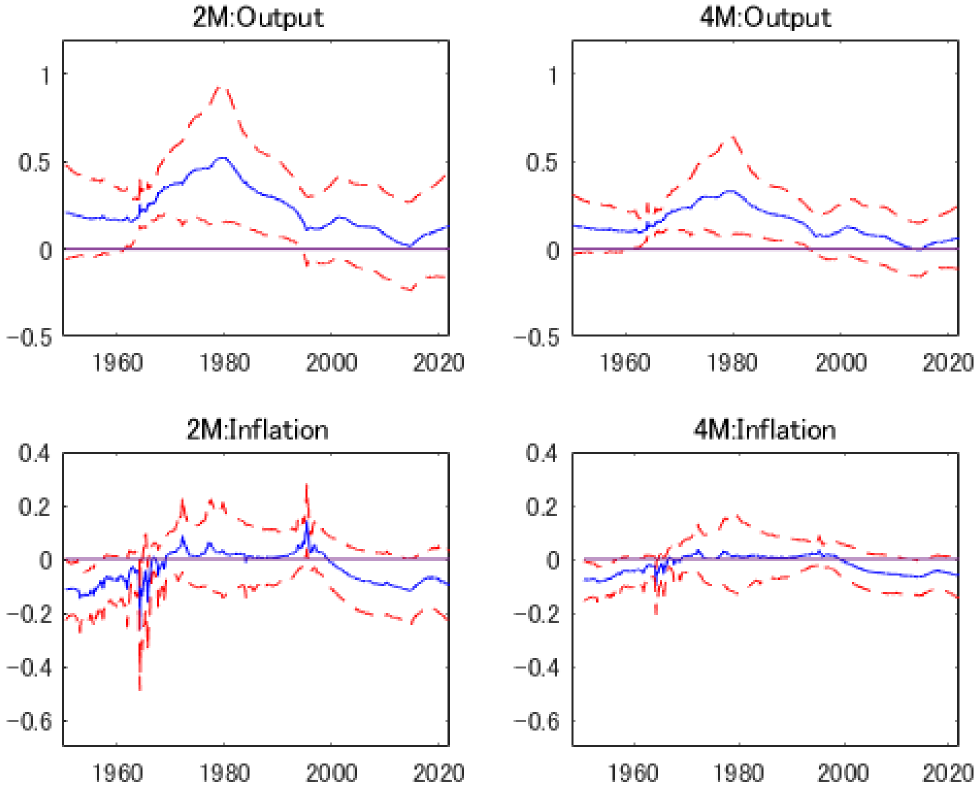


Figure A6. Impulse responses to ambiguity: Identification III

Notes: This figure plots the median impulse responses of output and inflation to one standard deviation of the ambiguity shocks with 68% error bands. The TVP-VAR model employs the following order: output (Δy), ambiguity ($\mho^2$), and inflation (Δp). The caption of each panel demonstrates the response and the time horizon.

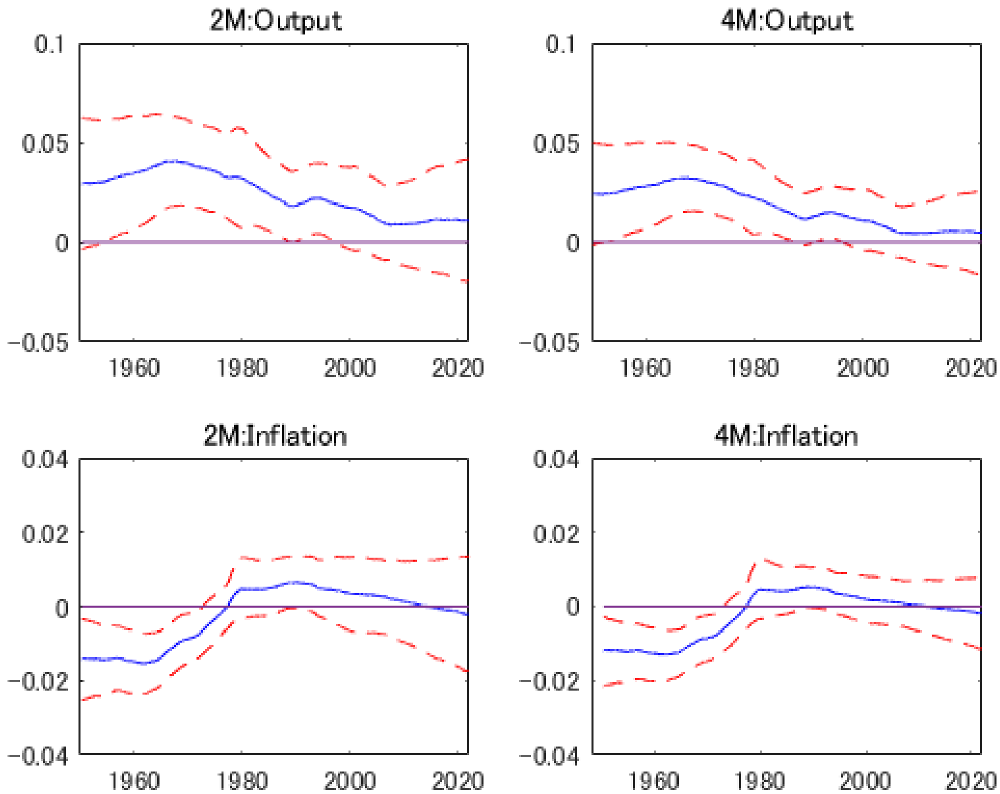


Figure A7. Impulse responses to value ambiguity shocks

Notes: This figure plots the median impulse responses of output and inflation to one standard deviation of the value ambiguity shocks with 68% error bands. The TVP-VAR model employs the following variables: output (Δy), inflation (Δp), and value ambiguity ($\mho^2_{\text{value}}$). The caption of each panel demonstrates the response and the time horizon.

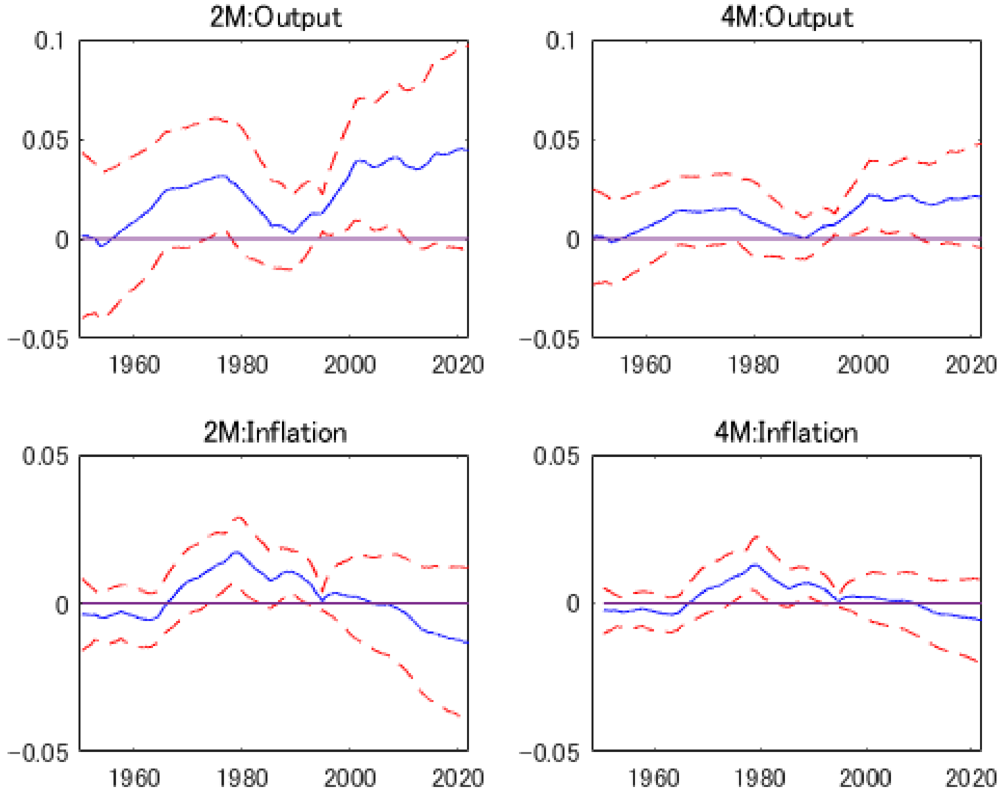


Figure A8. Impulse responses to momentum ambiguity shocks

Notes: This figure plots the median impulse responses of output and inflation to one standard deviation of the momentum ambiguity shocks with 68% error bands. The TVP-VAR model employs the following variables: output (Δy), inflation (Δp), and momentum ambiguity ($\mho^2_{\text{mom}}$). The caption of each panel demonstrates the response and the time horizon.

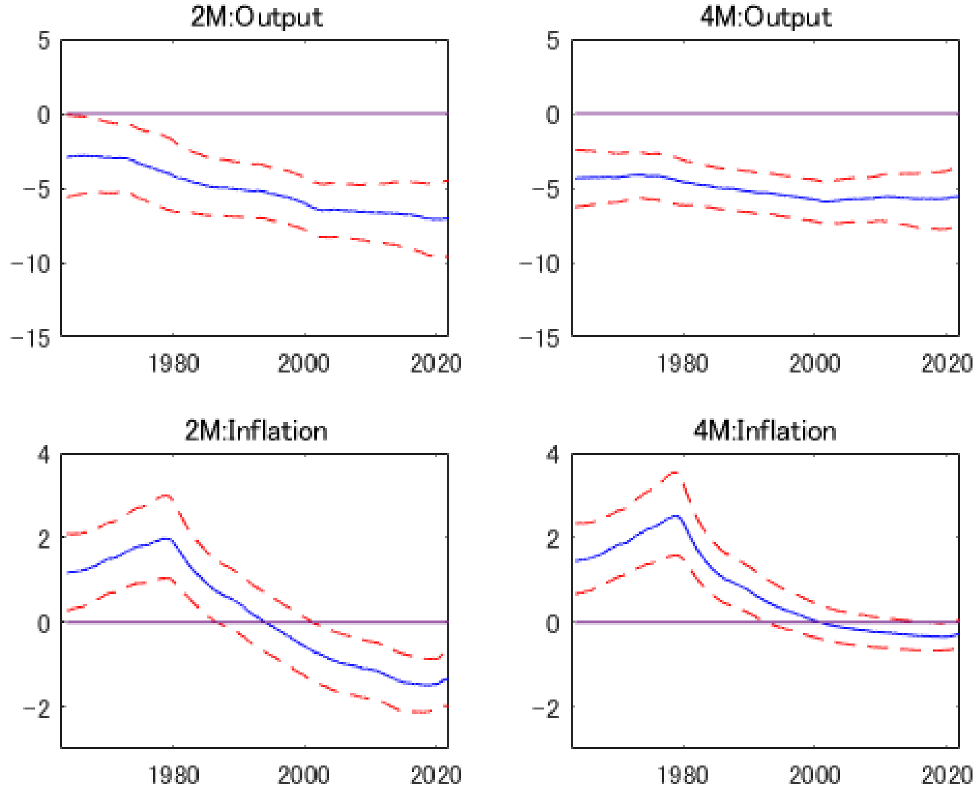


Figure A9. Impulse responses to macro uncertainty shocks

Notes: This figure plots the median impulse responses of output and inflation to one standard deviation of the total macro uncertainty index (Jurado et al., 2015) of shocks with 68% error bands. The TVP-VAR model employs the following variables: output (Δy), inflation (Δp), and macro uncertainty (UNC). The caption of each panel demonstrates the response and the time horizon.

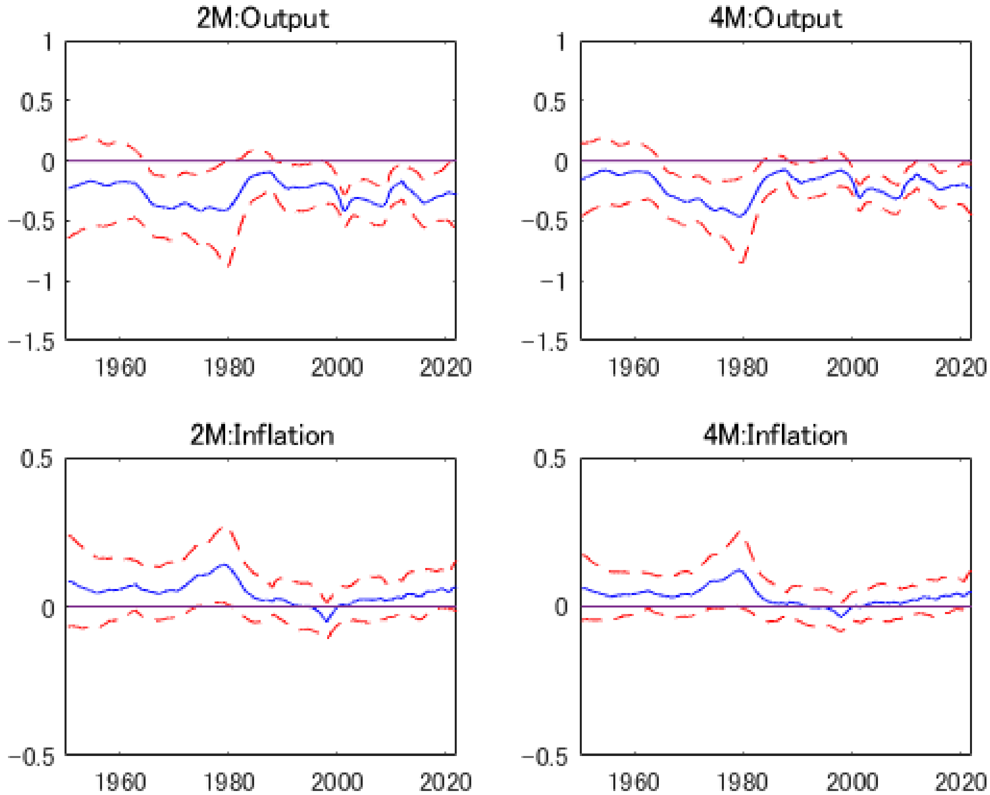


Figure A10. Impulse responses to MIDAS volatility shocks

Notes: This figure plots the median impulse responses of output and inflation to one standard deviation of the MIDAS volatility shocks with 68% error bands. The TVP-VAR model employs the following variables: output (Δy), inflation (Δp), and MIDAS volatility (VOL_{MIDAS}). The caption of each panel demonstrates the response and the time horizon.

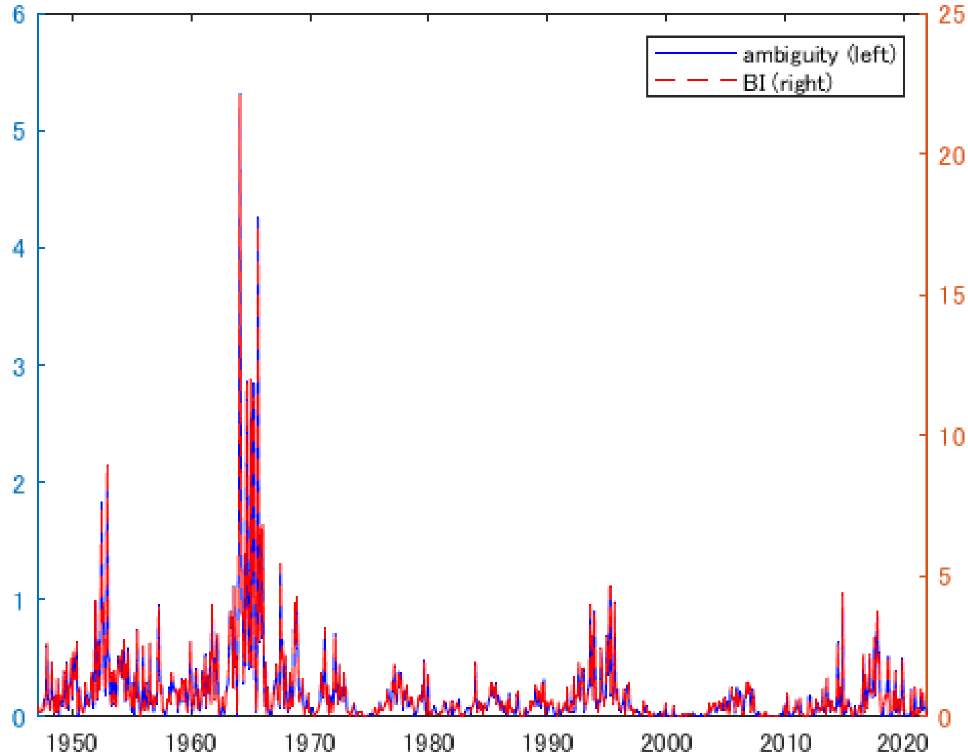


Figure A11. Comparison of ambiguity

Notes: This figure presents the comparison of ambiguity. Ambiguity (left) is obtained by Equation (2) and BI (right) is estimated by the original procedures in Brenner and Izhakian (2018) using daily data.

Table A1: Descriptive statistics

Panel A	Daily				
	Mean	Std.Dev.	Max	Min	Obs.
Market risk premiums	0.032	0.945	11.350	−17.440	19053
Size premiums	0.003	0.490	6.240	−11.630	19053
Value premiums	0.014	0.520	6.740	−5.000	19053
Momentum premiums	0.029	0.695	7.120	−14.370	19053
Panel B	**Monthly**				
	Mean	Std.Dev.	Max	Min	Obs.
Market risk premiums	0.683	4.277	16.100	−23.240	899
Size premiums	0.122	2.806	21.420	−17.230	899
Value premiums	0.289	2.749	13.560	−13.970	899
Momentum premiums	0.690	3.880	18.200	−34.300	899
$\mho^2$	0.200	0.373	5.315	0.000	899
$\mho^2_{size}$	1.304	2.058	21.330	0.000	899
$\mho^2_{value}$	1.415	1.920	20.640	0.002	899
$\mho^2_{mom}$	1.097	2.189	41.614	0.000	899
Δy	0.223	1.092	6.233	−14.152	899
Δp	0.286	0.342	1.945	−1.786	899
VOL_{RV}	0.800	0.505	5.845	0.174	899
VOL_{MIDAS}	0.800	0.332	3.946	0.295	899
UNC	0.648	0.107	1.268	0.523	738

Notes: This table reports the means, standard deviations, maximums, minimums, and the number of observations. Following Brenner and Izhakian (2018), $\mho^2$, $\mho^2_{size}$, $\mho^2_{value}$, and $\mho^2_{mom}$ are obtained by Equation (2). VOL_{RV} is monthly realized volatility using daily excess returns. VOL_{MIDAS} is estimated by the MIDAS model (Ghysels et al., 2005). UNC is total macroeconomic uncertainty proposed by Jurado et al. (2015).

Table A2: Data sources

	Definition	First Difference	Source
Market risk premiums	The excess return on the value weighted return of CRSP firms	No	Kenneth French's website
Size premiums	The average spread return between three small and three large groups	No	Kenneth French's website
Value premiums	The average spread return between two value and two growth groups	No	Kenneth French's website
Momentum premiums	The average spread return between two high prior and two low prior return groups	No	Kenneth French's website
y	Log of industrial production index	Yes	Federal Reserve Bank of St. Louis
p	Log of consumer price index for all items in U.S. city average	Yes	Federal Reserve Bank of St. Louis
$\mho^2$	Ambiguity obtained from the market risk premiums	No	Our estimation by Equation (2)
$\mho^2_{size}$	Ambiguity obtained from the size premiums	No	Authors' estimation by Equation (2)
$\mho^2_{value}$	Ambiguity obtained from the value premiums	No	Authors' estimation by Equation (2)
$\mho^2_{mom}$	Ambiguity obtained from the momentum premiums	No	Authors' estimation by Equation (2)
VOL_{RV}	Monthly realized volatility	No	Authors' estimation
VOL_{MIDAS}	Monthly volatility estimated by the MIDAS model (Ghysels et al., 2005)	No	Authors' estimation
UNC	Total macroeconomic uncertainty (Jurado et al., 2015)	No	Sydney Ludvigson's web site

Notes: This table shows the data definition and source.