KIER DISCUSSION PAPER SERIES

KYOTO INSTITUTE
OF
ECONOMIC RESEARCH

Discussion Paper No.1083

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September 2022

KYOTO UNIVERSITY

KYOTO, JAPAN

Agency Problems in a Competitive Conglomerate with Production Constraints*

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September 9, 2022

Abstract

This study explores the reciprocal effects between agency problems and market competition. We develop an adverse selection model of a competing conglomerate with production constraints. The conglomerate participates as the leader in two different duopolistic markets with a Stackelberg-Cournot framework and heterogeneous goods. The conglomerate is run by its headquarters and two division managers. The agency problem arises because the market demand size is a manager's private information, which the headquarters try to elicit by a contract mechanism. We fully characterize a first and a second-best contract. When the production constraints make the first best outcome unattainable, the second-best contract is either separating or pooling, depending on the severity of the constraints. The separating second-best contract sometimes improves the ex-ante welfare in comparison to a symmetric information benchmark. The pooling second-best contract never improves the ex-ante welfare. We also find that at an intermediate level of substitutability, the second-best contract is most likely to coincide with the first-best one, and any departure from that level toward either substitutability or complementarity makes the attainment of the first-best outcome less likely.

*I would like to give special thanks to my supervisor Tadashi Sekiguchi for his advice and guidance throughout the different stages in the development of this paper. We thank Chiaki Hara and Takuya Nakaizumi and seminar participants at the Japan Association for Applied Economics Spring Meeting and Japanese Economic Association Autumn Meeting for helpful comments. This work was supported by JSPS KAKENHI [Grant Number JP20J13138]. Any opinions, findings, and conclusions or recommendations expressed in this study are those of the author and do not necessarily reflect the views of the author's organization, JSPS, or MEXT.

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JEL classification: C70, D21, D43, D82, D86, L13.

Keywords: Adverse Selection; Contract Mechanism; Multimarket Competition; Stackelberg Oligopoly; Production Constraint.

1 Introduction

Agency frameworks have been used extensively to explain the inner mechanism of conglomerates, firms that participate in several industries. The hypothesis that the conglomerate discount is caused by agency problems is of particular importance (Maksimovic & Phillips, 2007) . Broadly speaking, the conglomerate discount theory claims that the conglomerate is less than the sum of the values of its individual parts (Berger & Ofek, 1995). One argument is that conflicts of interest inside the conglomerate creates inefficiencies in capital allocation (Busenbark et al., 2017), thus causing the conglomerate discount. While the capital allocation efficiency literature is substantial, it usually neglects the conglomerate's strategic interaction with the market. Indeed, the interactions of conglomerates, such as Amazon, are not limited to those that happen within the firm, Amazon also has to deal with competitors, such as E-bay in the online retail market or Netflix in the video streaming market.

If agency problems have an effect in a conglomerate, it is reasonable to assume that the same agency problems also have an effect on the markets where the conglomerate participates, and by extension, on social welfare. Despite this, explicit policies regarding conglomerate effects related to agency problems are lacking. Indeed, only some conglomerate effects are monitored and regulated. For instance, competition authorities' concern for conglomerates effects when these have the potential to lessen competition (Markovits, 2014). In the EU "Guidelines on the assessment of non-horizontal mergers under the

Council Regulation on the control of concentrations between undertakings" (2008) , the only non-coordinated conglomerate effect listed is foreclosure. Specifically, bundling and tying practices.

This study explores the reciprocal effects between agency problems and market competition. We develop a tractable theoretical model with three defining characteristics. First, there is a conglomerate (or multi-market firm) facing oligopolistic competition. We assume that the conglomerate competes as the leader in two duopoly markets with a Stackelberg-Cournot framework with heterogeneous goods. The two demand functions are independent of each other.

Second, the conglomerate has production constraints. The conglomerate has a common pool of resources that functions as the input for both products. If the resources are low, there is an opportunity cost of producing in one market or the other.

Third, there is an agency problem inside the conglomerate. We consider an adverse selection model. We assume that a conglomerate consists of its headquarters and two division managers. The headquarters' profit depends on both divisions, while the managers' utility depends only on the performance of their own division. The headquarters decide how to allocate the resources across the divisions. The managers, using these resources, take production decisions in their respective divisions.

The headquarters do not know the value of the intercept of the demand in one of the markets, but the manager running that division does. That intercept of demand is a random variable that can take two values: high or low. Through a contract mechanism headquarters might obtain that information from the manager to allocate resources contingent on the manager's report.

The payoffs of the headquarters and the manager are different, so the manager might have incentives to lie. Depending on the state, there exist an ideal level of resources that maximizes the profit of a division. However, that ideal value might not be achieved if the resources are low enough. In this case, the motivation behind the manager's report is to obtain a level of resources as close as possible to the ideal value. Thus, the contract mechanism must offer allocations contingent on the state such that the manager receives the amount closest to the ideal value only if the truth is reported.

We first solve a benchmark with symmetric information. We find two equilibria. First, the unrestricted equilibrium, where the resources are plenty and the production constraint is not binding. Second, the restricted equilibrium, where the resources are not plenty, the production constraint is binding, and thus, there is an opportunity cost of production.

We find that the solutions of the model under asymmetric information and the benchmark are equivalent if the resources of the conglomerate are large enough, that is, the first-best contract can be achieved. Conversely, if the resources are low, the conglomerate can only induce truth-telling with the second-best contract. Depending on the level of resources, the second-best contract is separated or pooled.

A separating second-best contract is achieved if the resources are not too low. The mechanism enables headquarters to distinguish the true value of the demand and to allocate resources accordingly. With this contract, in comparison to the benchmark in any state, the conglomerate produces more in the market with asymmetric information and less in the market with the certain demand. The benchmark's high state allocation in the market with uncertain demand is too close to the low state's ideal value. Hence, the headquarters allocate more resources in the high state so that the resources are further away from the low state's ideal value. Simultaneously , more resources are also allocated in the low state to approach them to the low state's ideal value. Conversely, less resources are allocated in the market with certain demand in both states as there is an opportunity cost of producing in one market or the other.

If the resources are very low, a separating contract is too costly, and thus, the headquarters is content with a pooling contract. The headquarters know the value of the demand with the mechanism, but the resource allocation plan remains the same regardless of the information revealed. With this contract, the production is equivalent to the outcome of a model where none of the players inside the conglomerate knows the true value of the demand, so the information is useless with this scheme.

Furthermore, we analyze the ex-ante and post social welfare. We show that under certain conditions the social welfare improves in the model under asymmetric information in comparison to the symmetric information benchmark. Because there is an opportunity cost of production, the allocation distortion caused by the second-best contracts transfers surplus from one market to the other. Welfare might improve if surplus is transferred from the worst market to the best market. Welfare is more likely to increase if the goods are complements, as the surplus of the follower firms and consumers change in the same direction as the variations in the conglomerate's production.

Particular to the ex-ante welfare, we find that the separating contract sometimes improves it if the market with uncertain demand is large on average in comparison to the market with certain demand. In contrast, we find that the pooling contract never improves the ex-ante welfare.

Finally, we analyze how the degree of differentiation delimits the type of equilibrium. Under symmetric information, at higher levels of substitutability or complementarity the restricted equilibrium is more likely to occur. Under asymmetric information and when the resources are low but not too low, at higher levels of substitutability or complementarity the second-best contract is more likely to be implemented. Under asymmetric information, and when the resources are very low, at higher levels of substitutability or complementarity the pooling equilibrium is more likely to occur.

This study is organized as follows: Section 2 reviews the related literature. Section 3 specifies the model structure. Section 4 presents the symmetric information benchmark. Section 5 solves and analyzes the model. Section 6 analyzes social welfare. Section 7 analyzes the effect of the degree of differentiation on the delimitation of the type of equilibrium. Section 8 concludes the study.

2 Literature review

Our study is relevant to the literature related to resource allocation and agency problems. Especially pertinent is the literature where production constraints are explicitly considered. The theoretical model of Harris et al. (1982) shows that using a transfer pricing scheme as an allocation mechanism is cost minimizing and induces the divisions of a firm to tell the truth. A transfer pricing scheme is feasible even if the resource constraint is binding. Cachon & Lariviere (1999) consider a model where a supplier allocates a limited capacity to multiple downstream retailers. One of their main results is that the mechanism that maximizes the total profits of the retailers cannot induce truth-telling if the capacity is binding. In our model, the first-best contract can be achieved even when the production constraint is binding, if the resources are not too low. This implies that the managers tell the truth while maximizing the profit of their own divisions.

Another strand of literature about resource allocation focuses on financial resources within a conglomerate. Particularly, the corporate finance's capital allocation efficiency literature investigates whether the distribution of financial resources across divisions in a multi-market firm matches with their respective performance, that is, whether a high-prospects division receives more than a low-prospects division (Busenbark et al., 2017). Studies on efficient allocation ascertain that the firm prioritizes the most profitable endeavors over the less profitable ones (see for example Stein (1997) ,Maksimovic & Phillips (2002), Brusco & Panunzi (2005))

Opposing the theory of efficient allocation, there is literature proposing that agency problems cause inefficiency in capital allocation. In theoretical research, Rajan et al. (2000) predict that as the diversity increases, the transfers from better-opportunities divisions to worse-opportunities divisions increases. The reason is that allocating resources to the weak division improves the contribution of this division to the joint profit, increasing the strong division's incentives to invest efficiently. In Stein & Scharfstein (2000), the division managers of weak divisions engage in rent-seeking behavior, which is costly for the firm. To mitigate this behavior, the CEO can allocate capital inefficiently to the weak divisions. In Wulf (2009), the core division manager sends distorted information to the headquarters to influence the division of capital in favor of the core division and against the small division. In empirical contributions, Rajan et al. (2000) provide evidence supporting their theoretical hypothesis. In Arrfelt et al. (2013), a backward-looking logic leads to over-investment (under-investment) in low (high) expectations divisions.

We include agency problems in a conglomerate in the form of an adverse selection problem between headquarters and managers. However, the adverse selection not always cause inefficient resource allocation. Both the first and second-best contract can be achieved. The former can be interpreted as efficient and the second as inefficient. Thus, our model reconciles the theories of the efficient and inefficient allocation literature.

Literature focusing on the allocation of resources in competitive conglomerates is scarce. One of them is the study by Levinthal & Wu (2010). In their model, the authors assume two multi-market firms competing in two markets. These firms have the ability to relocate a fixed amount of resource across markets. Because the resource is finite, there is an opportunity cost in transferring the resource from one market to the other one. In equilibrium, there are more incentives to allocate resources to one market as the size of that market increases. Similarly, in our model, the headquarters allocate more resources to the greatest market under the first-best contract. However, under the second-contract, the headquarters might prioritize the worst market.

There are some other theoretical studies about competitive conglomerates, but they mainly incorporate foreclosure as the conglomerate effect. In Granier & Podesta (2010), a gas and electrical firms can price discriminate only after a merger, by selling their products in a bundle. In Tan & Yuan (2003), they study divestitures by assuming two competing conglomerates, each one supplying a group goods. Within the conglomerate, the goods are complements, while across the conglomerates the goods are substitutes.

3 The model

A conglomerate firm participates in two markets that are not related horizontally or vertically, denoted by $k \in \{C, N\}$. The division in market N is a newly acquired division, while the division market C is the core business (or original business) of the firm. The conglomerate is run by a risk-neutral headquarters, which we assume is the owner of the firm, and two managers, each one in charge of one division.

We assume that the conglomerate is competing with one standalone firm in each one of the markets. The firms are denoted by $i \in \{1, 2\}$, where the conglomerate is 1 and the standalone firm is 2. We consider a sequential quantity competition (Stackelberg-Cournot) with heterogeneous goods, where the conglomerate is the leader in both markets and standalone firms are the followers.

We assume that a representative consumer in market k and state s has a quasi-linear utility function with the form $U_k(q_{k0}, q_{k1}, q_{k2}) = q_{k0} + v_k^s(q_{k1}, q_{k2})$, where q_{k0} is the quantity of the numeraire good, q_{ki} is the output of firm i, and $v_k^s(q_{k1}, q_{k2})$ is given by:

$$v_k^s(q_{k1}, q_{k2}) = D_k^s(q_{k1} + q_{k2}) - \frac{1}{2}\left(q_{k1}^2 + 2\alpha q_{k1}q_{k2} + q_{k2}^2\right)$$

where $\alpha \in [-1, 1]$ is a constant measuring the degree of differentiation of the good. It stands that the goods are substitutes when $\alpha > 0$ and are complements when $\alpha < 0$. We assume a common α in both markets so that the only variable differentiating the markets is the intercept of the demand.

The utility function generates the following inverse demand function faced by firm i in market k:

$$P_{ki}\left(q_{ki}, q_{kj}\right) = D_k - q_{ki} - \alpha q_{kj}$$

where $j \in \{1, 2\}$ for $j \neq i$ and P_{ki} is the price of firm i in market k. In the core market, all the players know the value of the intercept, D_C, which is a positive constant. In the new market, the standalone firm and the division manager know the value of the intercept but the conglomerate's headquarters do not. However, the headquarters know that the intercept is a random variable that can take two values: high value D_N^H with probability $p_H \equiv p$ and low value D_N^L with probability $p_L \equiv 1 - p$, where $p \in (0, 1)$ is a constant. These priors are common knowledge. We assume that $D_N^H > D_N^L > 0$ and $D_C > 0$.

Although the demand functions are independent of each other, we assume that the products of both markets use a common input in their production process. For simplicity, we assume that the products are produced only with this common input. The conglomerate is endowed with a positive exogenous amount of input X, which is allocated between the divisions for them to produce their respective products. We refer to this endowment as the resources of the conglomerate. The production function for the conglomerate's product k is $q_{k1} = x_k$, where x_k is the amount of input. The total amount of input assigned to both divisions must satisfy that $x_N + x_C \leq X$. We can write this restriction in terms of quantities as $q_{N1} + q_{C1} \leq X$. We refer to the last inequality as the production–possibility constraint of the conglomerate. As for the standalone firms, we assume that in any scenario they have enough resources to operate without constraints in each one of their markets. Therefore, we ignore the production-possibility constraints of the standalone firms.

A corner solution for the conglomerate would entail producing nothing in one of the markets. As we are interested in the scenario where the conglomerate participates in both markets, we make two assumptions to guarantee interior solutions. First, for any s with $s \in \{H, L\}$:

$$X \geq \frac{(2-\alpha)}{2(2-\alpha^2)} \left| D_C - D_N^s \right| \tag{1}$$

This implies that the firm has enough resources to operate in both markets in any state. Second:

$$D_N^H - D_N^L < \frac{2D_C}{3} \tag{2}$$

This assumes that the difference between the intercepts of the demand of the new market in both states is small relative to the intercept of the core market.

The headquarters are in charge of the allocation of resources across the divisions.

Each manager is in charge of producing and supplying the good to the market in their respective division. The headquarters' payoff is the sum of the profits of both divisions minus an exogenous fixed compensation for both managers. Each manager's utility come from the fixed compensation and the profit of their own division. The latter component is explained by a preference in empire building, which in this case is interpreted as the desire to manage a profitable division.[1] Without loss of generality, we normalize the fixed compensation of both managers to zero, so the headquarters' payoff is simply the profit of the overall firm, while each manager's utility is equivalent to the profit of their own division. The reservation utility of both managers is set to be zero. If a manager does not receive at least their reservation utility, they quit.

It is possible for the headquarters to ask the manager in division N for the value of D_N. However, the maximization of the overall profit does not necessarily imply the maximization of the profit of division N, so the manager might have incentives to not report truthfully. To induce truth-telling, the headquarters establish a contract obliging the manager to produce a specific amount of output contingent on the announced value of the demand in market N.[2]

We assume that the headquarters commit to allocate enough resources to the manager to produce the agreed quantities. Headquarters do not have incentive to give to the manager in division N extra resources as they would be wasted. Headquarters might have incentives to allocate less than the necessary resources, but this strategy would

[1]Empire building is mentioned in Stein & Scharfstein (2000) as an explanation of why managers profit from their own divisions while the principal profits from all divisions, thus creating the agency problem. In Wulf (2009) and Bernardo et al. (2001) managers gain utility as their allocation of capital increases. Empire building is given as a reason for this in Bernardo et al. (2001). In our model, managers desire to maximize the profits of their own division, but they do not necessarily desire a larger allocation of resources, as the profits are decreasing for a large enough production.

[2]Managerial compensation contracts might also include an endogenous fixed payment and a profit share rule (see for example Bernardo et al. (2001)). Here, we are not interested in optimal contracts but rather in the effect of agency problems in the markets. Thus, we simplify the problem by considering output as the only the component of the contract, as its allocation is what generates the agency problem.

preclude the manager to fulfill the contract.

The sequence of events is as follows:

1. The headquarters offer a contract to the manager of division N. The contract establishes that the manager has to produce $q_N\left(D_N^s\right)$ if the reported state is s.

2. The manager of division N reports the value of the demand in N.

3. The headquarters allocate resources simultaneously to both divisions depending on the report of the manager.

4. Both managers set the output in their respective markets.

5. The standalone firms set their output in both markets after observing the quantities produced by the conglomerate.

We solve the game in the following sections. Furthermore, we concentrate on pure strategies.

4 Symmetric information benchmark

Here, it is assumed that the headquarters know the true value of the demand in market N. Given that that conglomerate has already selected $\check{q}_{k1}$, the standalone firm in market C solves the following problem:

$$\max_{q_{C2}\geq 0} \quad \left(D_C - q_{C2} - \alpha\check{q}_{C1}\right) q_{C2}$$

While the standalone firm in market N and state s solves the following problem:

$$\max_{q_{N2}^s\geq 0} \quad \left(D_N^s - q_{N2}^s - \alpha\check{q}_{N1}\right) q_{N2}^s$$

The best response functions of the standalone firms in market C and N in state s are respectively as follows:

$$q_{C2}^{B}\left(\check{q}_{C1}\right)=\frac{D_C-\alpha\check{q}_{C1}}{2}\text{ and }q_{N2}^{sB}\left(\check{q}_{N1}\right)=\frac{D_N^s-\alpha\check{q}_{N1}}{2}\tag{3}$$

The headquarters' maximization problem in state s is:

$$\begin{aligned}
&\max_{\forall k,q_{k1}^s\geq 0} \quad \left(D_C-q_{C1}^s-\alpha q_{C2}^{B}\left(q_{C1}^s\right)\right)q_{C1}^s+\left(D_N^s-q_{N1}^s-\alpha q_{N2}^{sB}\left(q_{N1}^s\right)\right)q_{N1}^s=\\
&\max_{\forall k,q_{k1}^s\geq 0} \quad \frac{1}{2}\left(\left((2-\alpha)D_C-(2-\alpha^2)q_{C1}^s\right)q_{C1}^s+\left((2-\alpha)D_N^s-(2-\alpha^2)q_{N1}^s\right)q_{N1}^s\right)\\
&\text{s.t.} \qquad q_{N1}^s+q_{C1}^s\leq X
\end{aligned}\tag{4}$$

where after the equal sign we substitute (3) into the headquarters' objective function. Whether the restriction is binding depends on the parameters of the problem. First, we assume that the resources are plenty, and hence that the restriction is not binding. We denote this solution with U. The optimal outputs of the conglomerate in market C and N in state s are as follows:

$$q_{C1}^{U}=\frac{2-\alpha}{2(2-\alpha^2)}D_C\text{ and }q_{N1}^{sU}=\frac{2-\alpha}{2(2-\alpha^2)}D_N^s$$

and the optimal outputs of the standalone firms in market C and N in state s are as follows:

$$q_{C2}^{U}=\frac{4-\alpha^2-2\alpha}{4(2-\alpha^2)}D_C\text{ and }q_{N2}^{sU}=\frac{4-\alpha^2-2\alpha}{4(2-\alpha^2)}D_N^s$$

U is an equilibrium in state s if the resources are plenty, specifically:

$$X\geq\frac{2-\alpha}{2(2-\alpha^2)}\left(D_C+D_N^s\right)=\Omega^s$$

Second, we assume that the resources are not plenty in state s, that is $X < \Omega^s$. The restriction in (4) is binding and the problem in state s can be rewritten as follows in terms of q^s_{N1}:

$$\max_{q^s_{N1}\geq 0} \; \frac{2-\alpha^2}{2}\left(\left(\frac{(2-\alpha)D_C}{2-\alpha^2} - X + q^s_{N1}\right)(X - q^s_{N1}) + \left(\frac{(2-\alpha)D^s_N}{2-\alpha^2} - q^s_{N1}\right) q^s_{N1}\right)$$

We denote this solution with R. From the first order condition (FOC) the optimal outputs of the conglomerate in state s are:

$$q^{sR}_{C1} = q^U_{C1} - \frac{\theta^{sR}}{2} \text{ and } q^{sR}_{N1} = q^{sU}_{N1} - \frac{\theta^{sR}}{2}$$

and the optimal outputs of the standalone firms in state s are:

$$q^{sR}_{C2} = q^U_{C2} + \alpha\frac{\theta^{sR}}{4} \text{ and } q^{sR}_{N2} = q^{sU}_{N2} + \alpha\frac{\theta^{sR}}{4}$$

where $\theta^{sR} = q^{sU}_{N1} + q^U_{C1} - X$ is the conglomerate's deficit in the resources needed to achieve U in state s. Thus, in R each one of the conglomerate's divisions faces a reduction in production equal to half the production deficit. Consequently, an increase in the resources increments the production in both divisions in half the increase of X ($\frac{\partial q^{sR}_{k1}}{\partial X} = \frac{1}{2}$). Given (1), it follows that $q^{sR}_{k1} \geq 0$ and $q^{sR}_{k2} \geq 0$ for all s. We summarize the results of this section in Proposition 1.

Proposition 1. *The equilibria in the symmetric information benchmark are characterized as follows:*

a) When $X \geq \Omega^H$, U is an equilibrium in both states.

b) When $\Omega^H > X \geq \Omega^L$, R is an equilibrium in the high state and U is an equilibrium *in the low state.*

b) When $X < \Omega^L$, R is an equilibrium in both states.

When the equilibrium is U, the conglomerate has plenty of resources, and thus, there is not an opportunity cost to produce in one market or the other. Each division of the conglomerate functions as a standalone firm, without considering the other market when taking decisions. As expected, in market N the conglomerate and standalone firm produce more in the high state than in the low state ($q_{Ni}^{HU} > q_{Ni}^{LU}$). Contrastingly, the outputs of both firms in market C are independent of the state.

When the equilibrium is R, the conglomerate faces an opportunity cost to produce in one market or the other as the resources are not enough to produce the optimal output in both markets. In comparison to U, with the production restriction in R, the conglomerate produces less in both markets in any state ($q_{C1}^{U} > q_{C1}^{sR}$ and $q_{N1}^{sU} > q_{N1}^{sR}$). Similar to U, in R the conglomerate produces more in market N if the state is high ($q_{N1}^{HR} > q_{N1}^{LR}$). In this case, the output in market C depends of the state. Hence, given the trade-off between market C and N, the conglomerate produces less in market C if the state is high ($q_{C1}^{LR} > q_{C1}^{HR}$).

The effect of the conglomerate's production constraint on standalone firms depends on whether the goods are substitutes or complements. If the goods are substitutes, in any state both standalone firms are better off in R than in U because their production increases ($q_{C2}^{U} < q_{C2}^{sR}$ and $q_{N2}^{sU} < q_{N2}^{sR}$). Conversely, if the goods are complements, in any state both standalone firms are worse off because their production decreases ($q_{C2}^{U} > q_{C2}^{sR}$ and $q_{N2}^{sU} > q_{N2}^{sR}$). Similar to the conglomerate, the standalone firm in market N produces more in the high state ($q_{N2}^{HR} > q_{N2}^{LR}$). This same comparison in market C depends on α. If the goods are substitutes, the standalone firm in C produces more in the high state

($q_{C2}^{HR} > q_{C2}^{LR}$). Conversely, if the goods are complements, it produces more in the low state ($q_{C2}^{HR} < q_{C2}^{LR}$). This is consistent with the observed behavior of the conglomerate in market C ($q_{C1}^{LR} > q_{C1}^{HR}$).

5 Information revelation

By the revelation principle, we can restrict our attention to only the truth-telling situations. The problem of the headquarters is:

$$\max_{\forall k, \forall s, q_{k1}^{s} \geq 0} \quad \sum_{s \in \{H,L\}} p_s \left(P_{C1}\left(q_{C1}^{s}, q_{C2}^{B}\left(q_{C1}^{s}\right)\right) q_{C1}^{s} + P_{N1}^{s}\left(q_{N1}^{s}, q_{N2}^{sB}\left(q_{N1}^{s}\right)\right) q_{N1}^{s} \right) \tag{5}$$

$$\text{s.t.} \quad \forall s, q_{N1}^{s} + q_{C1}^{s} \leq X$$

$$P_{N1}^{H}\left(q_{N1}^{H}, q_{N2}^{HB}\left(q_{N1}^{H}\right)\right) q_{N1}^{H} \geq P_{N1}^{H}\left(q_{N1}^{L}, q_{N2}^{HB}\left(q_{N1}^{L}\right)\right) q_{N1}^{L} \tag{6}$$

$$P_{N1}^{L}\left(q_{N1}^{L}, q_{N2}^{LB}\left(q_{N1}^{L}\right)\right) q_{N1}^{L} \geq P_{N1}^{L}\left(q_{N1}^{H}, q_{N2}^{LB}\left(q_{N1}^{H}\right)\right) q_{N1}^{H} \tag{7}$$

$$\forall s, P_{C1}\left(q_{C1}^{s}, q_{C2}^{B}\left(q_{C1}^{s}\right)\right) q_{C1}^{s} \geq 0 \tag{8}$$

$$\forall s, P_{N1}^{s}\left(q_{N1}^{s}, q_{N2}^{sB}\left(q_{N1}^{s}\right)\right) q_{N1}^{s} \geq 0 \tag{9}$$

Where the restrictions (6) and (7) are the high and low incentive compatibility constraints (IC), respectively. The restrictions (8) and (9) are the individual rationality constraints (IR) of the manager of division C and N, respectively. The IC constraints ensure that a truthful report is optimal for the manager. The IR constraints impose that the managers receive at least their reservation utility.[3]

The problem is simplified by substituting (3) into (5)-(9). To further simplify, we

[3] We assumed that any manager quits if their gain is not at least their reservation utility. Thus, we include the IR constraint of the manager of the division C even though there is no adverse-selection between the headquarters and that manager.

substitute q_{C1}^{U} and q_{N1}^{sU} into (5), q_{N1}^{HU} into (6), q_{N1}^{LU} into (7), q_{C1}^{U} into (8), and q_{N1}^{sU} (9). The simplified problem is as follows:

$$\max_{\forall k, \forall s, q_{k1}^{s} \geq 0} \quad \frac{2-\alpha^2}{2} \sum_{s \in \{H,L\}} p_s \left(\left(2q_{C1}^{U} - q_{C1}^{s} \right) q_{C1}^{s} + \left(2q_{N1}^{sU} - q_{N1}^{s} \right) q_{N1}^{s} \right) \tag{10}$$

$$\text{s.t.} \quad \forall s, q_{N1}^{s} + q_{C1}^{s} \leq X \tag{11}$$

$$\left(q_{N1}^{H} - q_{N1}^{L} \right) \left(2q_{N1}^{HU} - q_{N1}^{H} - q_{N1}^{L} \right) \geq 0 \tag{12}$$

$$\left(q_{N1}^{H} - q_{N1}^{L} \right) \left(q_{N1}^{H} + q_{N1}^{L} - 2q_{N1}^{LU} \right) \geq 0 \tag{13}$$

$$\forall s, 2q_{C1}^{U} \geq q_{C1}^{s} \tag{14}$$

$$\forall s, 2q_{N1}^{sU} \geq q_{N1}^{s} \tag{15}$$

5.1 First-best contract

Here, we derive the conditions on the parameters that allow to achieve the solution in the symmetric information benchmark with the contract mechanism. First, when $X \geq \Omega^{H}$, the allocation of resources is such that the division N in state s can produce q_{N1}^{sU}, achieving the maximum unconstrained profit, and thus, the ideal level of output for the manager in division N and the headquarters. Hence, the manager does not have incentives to lie, the IC constraints are satisfied and the first-best is achievable.

Second, when $\Omega^{H} > X \geq \Omega^{L}$, the allocation of resources is ideal in the low state but not in the high state. In this case the resources are not plenty enough in the high state, so the maximum unconstrained level of profit is not achieved in neither of the divisions. Because the profit of any division is quadratic and concave in the output, if the ideal output is not achievable, the preferred alternative is a level of output as close as possible to the ideal output. In this scenario, it follows that $q_{N1}^{HU} > q_{N1}^{HR} > q_{N1}^{LU}$. Here, the resources are not too low, so the deficit caused by the production constraint is not too severe to distort the high state output below the unrestricted low state output. Thus, the high state manager of division N prefers the output of the high state, so there are no incentives to lie. If the state is low, the ideal output is achieved in division N, so the manager of division N does not have incentives to lie in this state either. Therefore, the first-best is achievable in this case.

Third, when $X < \Omega^L$, the allocation of resources in any state does not allow to reach the ideal level of output in neither of the divisions. The high state manager of division N does not have incentives to lie as $q_{N1}^{HU} > q_{N1}^{HR} > q_{N1}^{LR}$. However, if the state is low, there might be incentives to lie. If q_{N1}^{HR} is closer to q_{N1}^{LU} than q_{N1}^{LR}, it is profitable for the manager to lie. In the converse case, q_{N1}^{HR} is so high that q_{N1}^{LR} is preferred. Intuitively, the manager might not lie if that results in an unprofitable overproduction.

The low state IC constraint (13) holds if $q_{N1}^{HR} + q_{N1}^{LR} \geq 2q_{N1}^{LU}$. Thus, the first-best contract is achieved if and only if:

$$X \geq \frac{2-\alpha}{4(2-\alpha^2)}\left(2D_C + 3D_N^L - D_N^H\right) = \hat{\Omega} \tag{16}$$

With (2), it follows that $\hat{\Omega} > 0$. Moreover, given that $\hat{\Omega} < \Omega^L$, (16) is not guaranteed to hold. Without (2), if the right side of the inequality in (16) is non-positive, (16) will hold as $X > 0$. Intuitively, without the assumption, the difference $D_N^H - D_N^L$ might be so large that lying when the state is low always results in overproduction. We state the main result of the first-best contract in Proposition 2.

Proposition 2. *A contract achieves the first-best outcome if and only if* $X \geq \hat{\Omega}$. *The* *equilibria in the first-best outcome are characterized as follows:*

a) When $X \geq \Omega^H$, U is an equilibrium in both states.

b) When $\Omega^H > X \geq \Omega^L$, R is an equilibrium in the high state and U is an equilibrium in the low state.

b) When $\Omega^L > X \geq \hat{\Omega}$, R is an equilibrium in both states.

5.2 Second-best contract

Suppose that $X < \hat{\Omega}$ so that the first-best contract is not achieved. Here, (13) is binding, so it follows that either $q_{N1}^H = q_{N1}^L$ or $q_{N1}^H + q_{N1}^L = 2q_{N1}^{LU}$.[4] With $q_{N1}^H = q_{N1}^L$, we obtain a pooling equilibrium candidate. With $q_{N1}^H + q_{N1}^L = 2q_{N1}^{LU}$, we get a separating equilibrium candidate. We compute the headquarters' ex-ante expected profit for each candidate and compare them to establish the existence of equilibria. We state formally these equilibria in Proposition 3.

Proposition 3. *(a) A separating equilibrium S, exists if and only if $\hat{\Omega} > X \geq \check{\Omega}$, where:*

$$\check{\Omega} = \frac{(2-\alpha)}{2(2-\alpha^2)} \left(D_C + 2D_N^L - D_N^H \right)$$

In this equilibrium, the outputs of the conglomerate are:

$$q_{N1}^{HS} = q_{N1}^{HR} + (1-p)\theta^S, \quad q_{N1}^{LS} = q_{N1}^{LR} + p\theta^S,$$

$$q_{C1}^{HS} = q_{C1}^{HR} - (1-p)\theta^S, \quad q_{C1}^{LS} = q_{C1}^{LR} - p\theta^S$$

[4] For all the proofs related to the second-best contract, see the Appendix.

where $q_{N1}^{HS} > q_{N1}^{LS}$ *always holds. The outputs of the standalone firms are:*

$$q_{N2}^{HS} = q_{N2}^{HR} - \frac{\alpha(1-p)\theta^S}{2}, \quad q_{N2}^{LS} = q_{N2}^{LR} - \frac{\alpha p \theta^S}{2},$$
$$q_{C2}^{HS} = q_{C2}^{HR} + \frac{\alpha(1-p)\theta^S}{2}, \quad q_{C2}^{LS} = q_{C2}^{HR} + \frac{\alpha p \theta^S}{2}$$

where[5]*:*

$$\theta^S = \frac{2-\alpha}{4(2-\alpha^2)}(2D_C + 3D_N^L - D_N^H) - X$$

(b) A pooling equilibrium P, exists if and only if $X \leq \check{\Omega}$*. In this equilibrium, the output of the conglomerate for any* k *is:*

$$\overline{q}_{k1}^P = pq_{k1}^{HR} + (1-p)q_{k1}^{LR}$$

Furthermore, the outputs of the standalone firms are:

$$q_{N2}^{HP} = q_{N2}^{HR} + \frac{\alpha(1-p)\theta^P}{2}, \quad q_{N2}^{LP} = q_{N2}^{LR} - \frac{\alpha p \theta^P}{2}, \quad \overline{q}_{C2}^P = pq_{C2}^{HR} + (1-p)q_{C2}^{LR}$$

where:

$$\theta^P = \frac{(2-\alpha)}{4(2-\alpha^2)}(D_N^H - D_N^L)$$

We illustrate all the equilibria found in the symmetric and asymmetric information cases for each state in Figure 1.

[5] Notice that $\theta^S > 0$ as $X < \hat{\Omega}$.

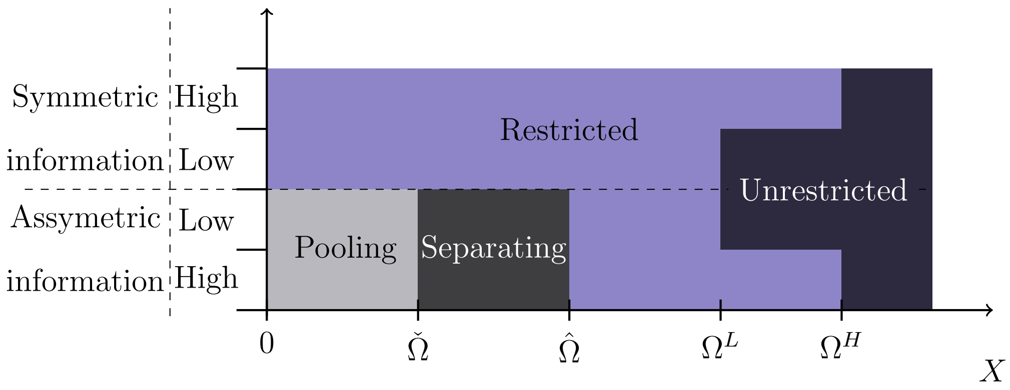

Figure 1: Existence of Equilibria.

In equilibrium $\hat{E}$, for $\hat{E} = \{P, S\}$, the restricted output of the conglomerate in state s is distorted in $\theta^{\hat{E}}$, weighted by the probability of state s not occurring. The distortion θ^S is the deficit in resources needed to achieve the first-best contract, while θ^P is the gap in the conglomerate's restricted first-best production in market N between states ($\theta^P = q_{N1}^{HR} - q_{N1}^{LR}$). The conglomerate maximizes its expected payoff in the equilibrium with the smallest distortion. Thus, the equilibrium P's condition of existence $X \leq \check{\Omega}$ is equivalent to $\theta^P \leq \theta^S$.

At $X = \check{\Omega}$, the headquarters are indifferent between the separating and pooling equilibria. Here, it holds $q_{N1}^{LU} = q_{N1}^{HR}$. Furthermore, to satisfy (13), q_{N1}^{HS} and q_{N1}^{LS} must be equally distanced from q_{N1}^{LU}. Thus, the low state output in S coincides with the output in P ($q_{N1}^{LS} = \overline{q}_{N1}^{P}$). The low state manager produces the same in either S or P. However, the headquarters are indifferent with the high state manager producing either q_{N1}^{HS} or $\overline{q}_{N1}^{P}$, because at this point both options distorts q_{N1}^{HR} in the same magnitude, but in different directions.

The mechanism in equilibrium S is such that the conglomerate produces more in market N in the high state than in the low state ($q_{N1}^{HS} > q_{N1}^{LS}$). This is desirable for the conglomerate in the sense that market N produces more (less) when the state is high (low).

In market N in any state, the conglomerate produces more in equilibrium S than in R but less than in U ($q_{N1}^{sU} > q_{N1}^{sS} > q_{N1}^{sR}$). Due to the restricted production, the increased production in market N results in a decreased production in market C ($q_{C1}^{sS} < q_{C1}^{sR}$). The headquarters prefer the outcome R, so the output in the second-best contract is as close as possible to the ones in R. In state s, the headquarters transfer θ^S weighted by the probability of state s not occurring from market C to N. The mechanism incentivizes the low state manager to tell the truth in two ways. First, it increases the manager's utility in comparison to outcome R. Second, lying when the true state is low results in overproduction, hurting the profit in market N.

Regarding the standalone firms, if the goods are substitutes, in state s the firm in market N is worse off in S in comparison to R ($q_{N2}^{sS} < q_{N2}^{sR}$) and the firm in market C is better off ($q_{C2}^{sS} > q_{C2}^{sR}$). If the goods are complements, in state s the firm in market N is better off ($q_{N2}^{sS} > q_{N2}^{sR}$) and the firm in market C is worse off ($q_{C2}^{sS} < q_{C2}^{sR}$).

When the equilibrium is P, the resources are so low that headquarters cannot implement an effective mechanism to differentiate the states in market N. Even though the mechanism induces truth-telling, P is equivalent to the outcome of a model where none of the players inside the conglomerate knows the true value of the demand, and hence, the conglomerate operates under uncertainty. Indeed, a solution where the same output $\overline{q}_{N1}$ is produced in any state can also be obtained by setting the intercept of the demand in market N equal to its expected value $pD_N^H + (1-p)\,D_N^L$.

In comparison to the outcome R, in equilibrium P the conglomerate produces less in market N if the state is high and produces more if the state is low ($q_{N1}^{HR} > \overline{q}_{N1}^{P} > q_{N1}^{LR}$), while in market C it produces more if the state is high and produces less if the state is low ($q_{C1}^{HR} < \overline{q}_{C1}^{P} < q_{C1}^{LR}$).

Comparing the standalone firms' production outputs between the outcome R and equilibrium P, it follows that if the goods are substitutes, the production in P in market N is higher if the state is high and lower if the state is low ($q_{N2}^{HP} > q_{N2}^{HR} > q_{N2}^{LR} > q_{N2}^{LP}$), while in market C the production is lower if the state is high and higher if the state is low ($q_{C2}^{HR} > \overline{q}_{C2}^{P} > q_{C2}^{LR}$). If the goods are complements, the production in P in market N is lower if the state is high and higher if the state is low ($q_{N2}^{HR} > q_{N2}^{HP} > q_{N2}^{LP} > q_{N2}^{LR}$), while in market C the production is higher if the state is high and lower if the state is low ($q_{C2}^{HR} < \overline{q}_{C2}^{P} < q_{C2}^{LR}$).

Whether the equilibrium is separating or pooling, a reduction of p approaches q_{k1}^{LS} or $\overline{q}_{k1}^{P}$ to q_{k1}^{LR}, while an increase of p approaches q_{k1}^{HS} or $\overline{q}_{k1}^{P}$ to q_{k1}^{HR}. This trade-off between the low and high states occurs because (13) is binding. As the probability of state s increases, the second-best quantities of state s become closer to the first-best quantities. Conversely, as the probability of state s decreases, the gap between the second and first-best quantities increases. Thus, as the probability of state s increases, the ex-post profit of the conglomerate improves in state s, while the ex-post profit in the other state decreases. A change in p affects the equilibrium output of the standalone firms in the same direction as the ones of the conglomerate. This is simply because of the role as followers of the standalone firms.

The resource allocation in the second-best contract can be interpreted as inefficient as the allocation in the first-best contract can improve the expected profit of the conglomerate. This meaning of inefficiency differs from the definition generally used in the literature of inefficient allocation. In those studies, allocating extra resources to weaker divisions is seen as inefficient. However, in our model the interpretation of inefficient does not depend on the relative profitability of the markets. For example, $D_N^H > D_N^L > D_C$, so that market N is undoubtedly better than market C. In equilibrium S, even though the headquarters assign more resources to the best market and less to the worst market, it is still considered inefficient in our model.

6 Social welfare

We compare the ex-ante and post total social welfare between the second-best contracts and their counterpart in symmetric information benchmark: equilibrium R. We first discuss the ex-post total social welfare. This is of interest when the policy maker knows the true value of the demand in market N. The ex-post social welfare is the sum of the ex-post producer and consumer surplus, which we define hereunder.

The ex-post total producer surplus in state s and equilibrium E for $E = \{R, P, S\}$ is the sum of the profits of both standalone firms, and the ex-post profit of the conglomerate

In equilibrium E, market k and state s the standalone firm's profit is $\pi_{k2}^{sE} = (q_{k2}^{sE})^2$, while the conglomerate's profit is $\pi_{k1}^{sE} = P_{k1}^s\left(q_{k1}^{sE}, q_{k2}^{sE}\right) q_{k1}^{sE}$. Hence, the ex-post total producer surplus in state s and equilibrium E is $\Pi^{sE} = \pi_{C1}^{sE} + \pi_{N1}^{sE} + \pi_{C2}^{sE} + \pi_{N2}^{sE}$.

The ex-post consumer surplus in market k, state s, and equilibrium E can be computed by:

$$CS_k^{sE} = v_k^s(q_{k1}^{sE}, q_{k2}^{sE}) - P_{k1}^s\left(q_{k1}^{sE}, q_{k2}^{sE}\right) q_{k1}^{sE} - P_{k2}^s\left(q_{k1}^{sE}, q_{k2}^{sE}\right) q_{k2}^{sE}$$

Therefore, the ex-post total social welfare in state s and equilibrium E is:

$$W^{sE} = CS_N^{sE} + CS_C^{sE} + \Pi^{sE} = v_N^s(q_{N1}^{sE}, q_{N2}^{sE}) + v_C^s(q_{C1}^{sE}, q_{C2}^{sE})$$

The results of the ex-post total social welfare are in Proposition 4.

Proposition 4. *a)* $W^{sP} \geq W^{sR}$ *for any* s *if and only if* [6]*:*

$$1 - p_s \leq \frac{2((1-\alpha)(7-\alpha-3\alpha^2)+1)\hat{\theta}^s}{(2-\alpha)(4-3\alpha^2)(D_N^H - D_N^L)} \tag{17}$$

where $\hat{\theta}^L = D_N^L - D_C$ *and* $\hat{\theta}^H = D_C - D_N^H$*.*

b) $W^{sS} \geq W^{sR}$ *for any* s *if and only if* [7]*:*

$$1 - p_s \leq \frac{((1-\alpha)(7-\alpha-3\alpha^2)+1)(D_N^s - D_C)}{2(4-3\alpha^2)(2-\alpha^2)\theta^S} \tag{18}$$

The contract reallocates surplus from one market to the other. The losses caused by the contract to the agents in one market is profitable for their counterparts in the other market. Thus, the asymmetry of information will improve the social welfare if the earnings in one market exceed the losses of the other.

For the consumers, they will be better-off in a specific market in the second-best contract than in R if the conglomerate overproduces in that market. Conversely, the consumers of the other market will be worse-off due to the reduction of the conglomerate's production in that market. That is, there is a group of winning consumers and a group of losing consumers.

As for the standalone firms, in any state there is always one firm that is better-off and one that is worse-off in the second-best contract equilibria in comparison to R. Thus, just like the consumers, there is one winning standalone firm and one losing firm. The winning (losing) firm will operate in the same market as the winning (losing) consumers

[6] Condition (17) is non-empty. Consider $D_C = 2$, $D_N^H = 4$, $D_N^L = 3$, $p = 0.9$, $\alpha = 0.2$ and $X = 1$. This satisfies (1), (2), $X < \check{\Omega}$, and the low state (17). Consider $D_C = 8$, $D_N^H = 6$, $D_N^L = 3$, $p = 0.1$, $\alpha = 0.2$, and $X = 3$. This satisfies (1), (2), $X < \check{\Omega}$, and the high state (17).

[7] Condition (18) is non-empty. Consider $D_C = 2$, $D_N^H = 4$, $D_N^L = 3$, $p = 0.9$, $\alpha = 0.2$, $X = 2$. This satisfies (1), (2), $\hat{\Omega} > X \geq \check{\Omega}$, and the low state (18). Consider $D_C = 4.51$, $D_N^H = 6$, $D_N^L = 3$, $p = 0.1$, $\alpha = 0.2$, and $X = 2.7$. This satisfies (1), (2), $\hat{\Omega} > X \geq \check{\Omega}$, and the high state (18).

only if the goods are complements.

Finally, while the total profit of the conglomerate in state s ($\pi_{C1}^{sE}+\pi_{N1}^{sE}$) is better in R than in the second-best contract equilibria, at the individual market level the conglomerate is always worse off in the market with underproduction but is better-off in the market with overproduction.

The overall loss of profit of the conglomerate is concordant with the conglomerate discount theory. The ex-post conglomerate discount in state s and in equilibrium $\hat{E}$, that is, the conglomerate's loss caused by the asymmetric information is $(2-\alpha^2)(1-p_s)^2(\theta^{\hat{E}})^2$. The conglomerate discount increments exponentially at increments of the distortion in production and in the probability of state s not occurring.

The denominators of the right side of (17) and (18) are positive, but depending on the parameters, the numerators can be negative, positive or zero. Any inequalities cannot hold if its respective numerator is non-positive, hence there are necessary conditions. The necessary condition for (17) in the low state is $D_N^L > D_C$, for (17) in the high state is $D_C > D_N^H$, and for (18) is $D_N^s > D_C$. The logic of the conditions is that the best market has to be the one where the conglomerate overproduces due to the information revelation scheme.

If the necessary condition holds and the right side of (17) or (18) is less than 1, then there exists a p that achieves $W^{s\hat{E}} \geq W^{sR}$. The role of the probability here is to avoid a bad scenario for the conglomerate in the second-best contract equilibria. As discussed earlier, the conglomerate is better-off as the probability approaches 0 or 1.

Now, we proceed to analyze the ex-ante total social welfare. This is relevant when the police maker does not know the true value of the demand. We compute the ex-ante total social welfare as follows:

$$EW^E = pW^{HE} + (1-p)W^{LE}$$

The results of the ex-post total social welfare are in Proposition 5.

Proposition 5. *a)* $EW^P < EW^R$ *always holds.*

b) $EW^S \geq EW^R$ *if and only if:*

$$X \geq \hat{\Omega} + \frac{(1-\alpha)(7-\alpha-3\alpha^2)+1}{4(2-\alpha^2)(4-3\alpha^2)}\left(2D_C - D_N^L - D_N^H\right) = \Omega^W \tag{19}$$

Part (a) of Proposition 5 states that the ex-ante total welfare is always worse with asymmetric information in the pooling equilibrium. The pooling contract transfers surplus from market N to market C in the high state. Conversely, it transfers surplus from C to N in the low state. The overall effect of the transfers is a reduction in the expected welfare. Intuitively, the cause of the loss of surplus is that headquarters do not use the information gained with the pooling contract.

Part (b) of Proposition 5 states that the separating second-best contract sometimes improves the ex-ante welfare. The separating equilibrium exists only if $\hat{\Omega} > X \geq \check{\Omega}$. Thus, an X exists such that (19) is satisfied and S is an equilibrium if $\hat{\Omega} > \Omega^W$. This last inequality holds if and only if $D_N^L + D_N^H > 2D_C$. In essence, if the mean of the demand of market N is greater than the demand of market C, there exists a high enough X such that the ex-ante total welfare is better in equilibrium S than in R.

A high X is required as it implies higher total production. The intuition behind $D_N^L + D_N^H > 2D_C$ is similar to the one in the ex-post analysis. In equilibrium S in any state the conglomerate overproduces in market N and underproduces in market C. Thus, the market size condition implies that the market where the conglomerate overproduces must be the best (in average).

Further, equilibrium S always welfare dominates R if $\check{\Omega} \geq \Omega^W$. That condition holds if and only if the goods are complements ($\alpha < 0$) and if $D_N^L + D_N^H > \theta^W D_C$, where:

$$\theta^W = \frac{(1-\alpha)(7-\alpha-3\alpha^2)+1}{2\alpha(\alpha-1)} \tag{20}$$

When the goods are complements, it follows $\theta^W > 2$. Thus, (20) implies that market N must be better than C in more than the average. The complementary condition is required so that the overproduction in market N benefits both the consumers and standalone firms in that market.

In the ex-ante case, the conglomerate is worse off with asymmetric information. The ex-ante conglomerate discount in equilibrium $\hat{E}$ is $(2-\alpha^2)(1-p)p(\theta^{\hat{E}})^2$. As the ex-post case, the conglomerate discount increments exponentially at increments of the distortion in production. The conglomerate discount disappears as the probability goes towards 0 or 1 because the value of D_N becomes more certain. Contrastingly, the conglomerate discount is at its highest at $p = 1/2$, when each state is equally probable.

7 Effects of the degree of differentiation on the delimitation of equilibrium

Now, we analyze the effect of the degree of differentiation on the thresholds that delimit the type of equilibrium. These thresholds are Ω^L, Ω^H, $\hat{\Omega}$, and $\check{\Omega}$ (as shown in Figure 1). All these thresholds can be written and hence be interpreted in terms of q_{C1}^U, q_{N1}^{LU} and q_{N1}^{HU}. Thus, α affects these thresholds and the unrestricted outputs in a similar manner. As the analysis of all these variables is similar, we only explicitly check q_{C1}^U. The derivative of q_{C1}^U with respect to α is:

$$\frac{\partial q_{C1}^U}{\partial \alpha} = \frac{(4\alpha - \alpha^2 - 2)D_C}{2(2 - \alpha^2)^2} \tag{21}$$

As D_C and the denominator of (21) are always positive, (21) has the same sign as $(4\alpha - \alpha^2 - 2)$. It follows that $(4\alpha - \alpha^2 - 2)$ is equal to zero when $\bar{\alpha} = 2 - \sqrt{2}$. Thus, the factor $(4\alpha - \alpha^2 - 2)$ is positive when $\alpha > \bar{\alpha}$ and negative when $\alpha < \bar{\alpha}$.

The derivatives of all Ω^L, Ω^H, $\hat{\Omega}$, $\check{\Omega}$, q_{N1}^{LU}, and q_{N1}^{HU} with respect to α also have the same sign as $(4\alpha - \alpha^2 - 2)$. We state this result in Proposition 6.

Proposition 6. *a) On the range $[-1, \bar{\alpha}]$, all q_{C1}^U, q_{N1}^{LU}, q_{N1}^{HU}, Ω^L, Ω^H, $\hat{\Omega}$, and $\hat{\Omega}$ are decreasing in α.*

b) On the range $[\bar{\alpha}, 1]$, all q_{C1}^U, q_{N1}^{LU}, q_{N1}^{HU}, Ω^L, Ω^H, $\hat{\Omega}$, and $\hat{\Omega}$ are increasing in α.

Proposition 6 implies that the type of equilibrium under symmetric information depends on α. Fix all the parameters other than X and α. There is X such that the unrestricted outcome is an equilibrium in both states for values of α close to $2 - \sqrt{2}$. The unrestricted outcome is an equilibrium in the high state and the restricted outcome is in the low state for values of α not too close but not too far from $2 - \sqrt{2}$. The restricted outcome is an equilibrium in both states for values of α quite far from $2 - \sqrt{2}$. Thus, higher levels of complementarity and substitutability increase the likelihood of the production-possibility constraint being binding.

Under asymmetric information, there is X such that the first best contract is achieved for values of α close to $2 - \sqrt{2}$. The second-best contract is most likely to coincide with the first-best one at an intermediate level of substitutability, and any departure from that level toward either substitutability or complementarity makes the attainment of the first-best outcome less likely. Further, the type of the second-best contract depends on α. The pooling equilibrium is more likely at higher levels of complementarity or substitutability.

8 Conclusion

We developed an adverse selection model of a conglomerate with restricted production participating as the leader in two duopoly markets with a Stackelberg-Cournot framework with heterogeneous goods. We derive two equilibria in the symmetric information benchmark. If the resources of the conglomerate are plenty, the production-possibility constraint is not binding and the resulting equilibrium is U. If the resources are scarce, the equilibrium is R.

The first-best contract is achievable if the resources are high enough when the information is asymmetric. Thus, U and R also exist as equilibria in this case. If the resources are low enough, only the second-best contract is possible. Here, we derive two more equilibria. If the resources are not too low, a separating equilibrium S exists. If the resources are too low, a pooling equilibrium P exists. In the S equilibrium, headquarters can allocate the resources accordingly to the manager's report. Specifically, headquarters allocate more resources to market N in the high state than in the low state. In P the production plan of the conglomerate is the same regardless of the state.

We proved that the social welfare might improve with asymmetric information depending on the parameters. The implementation of the contract mechanism leads to a reallocation of the production plan of the conglomerate, transferring surplus from one market to another. If the increase of surplus in one market exceeds the decrease of surplus in the other market, the social welfare will be better in the second-best contract. One requirement for this is that the best market must be the one with the increased surplus. Furthermore, welfare is more likely to improve if the goods are complements, because the surplus of the standalone firms and consumers will move in the same direction as the variations in the conglomerate's production.

We find that the separating contract sometimes improves the ex-ante welfare. The separating contract transfers surplus from market C to market N in any state, thus a requirement to improve the welfare is that the market N must be better in average than market C. If market N is larger than C in more than the average and the goods are complements, the separating contract always improves the ex-ante welfare. In contrast, the pooling contract never improves welfare. This contract also transfers surplus across markets, but because the information acquired is not used in a meaningful way, the overall effect is a reduction in the expected welfare.

Ideally, the policy authority should interfere when the asymmetric information hurts the social welfare. Measuring asymmetric information might be difficult in practice. However, our model predicts effects of the asymmetric information might be problematic in terms of the resources and the sizes of the markets, which are variables more easily measurable. First, agency problems are more likely to be problematic when the resources of the conglomerate are low. Second, in most cases the welfare is likely to fall if the conglomerate diversifies in markets that are smaller than its core business.

While it is unlikely that the police authority will be able to directly regulate agency problems, it can restrict the expansion of the conglomerate to new markets, preventing the creation of scenarios with agency problems. Thus, researching this kind of conglomerate effect and its policy implications is worth it even though the nature of such effect is abstract in practice. Thus, a possible extension of the model is to study a different agency framework, such as moral hazard. We leave this for future research.

We analyzed how changes in the degree of differentiation α determines the type of equilibrium. We find that the adverse selection is more likely to cause a distortion with higher levels of substitutability or complementarity. This result suggest that variables related to competition are relevant to understand phenomena within conglomerates, such as the conglomerate discount. We only examined a Stackelberg-Cournot framework, so a possible line of research is to study the inner dynamics of a conglomerate in other competitive scenarios, assuming simultaneous competition or price competition.

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Appendix

Taking the sum of (12) and (13) yields $2\left(q_{N1}^{H}-q_{N1}^{L}\right)\left(q_{N1}^{HU}-q_{N1}^{LU}\right)\geq 0$. Because $q_{N1}^{HU}>q_{N1}^{LU}$, satisfying both (12) and (13) requires $q_{N1}^{H}>q_{N1}^{L}$.

In the second-best contract at least one IC constraint is binding. When $q_{N1}^{H}=q_{N1}^{L}$ both (12) and (13) are binding. From this we get a pooling equilibrium candidate. When $q_{N1}^{H}>q_{N1}^{L}$, we prove that the optimal contract only binds (13) ($q_{N1}^{H}+q_{N1}^{L}=2q_{N1}^{LU}$). Suppose otherwise, so that optimally $q_{N1}^{H}+q_{N1}^{L}>2q_{N1}^{LU}$. Under $X<\hat{\Omega}$ it holds $q_{N1}^{HR}+q_{N1}^{LR}<2q_{N1}^{LU}$. Then, $q_{N1}^{s}>q_{N1}^{sR}$ for some s. Hence, (11) in state s is binding, so it holds $q_{C1}^{s}<q_{C1}^{sR}$. Reducing q_{N1}^{s} and increasing q_{C1}^{s} increases the profit in state s, a contradiction. Thus, there is a separating equilibrium candidate where $q_{N1}^{H}>q_{N1}^{L}$ and $q_{N1}^{H}+q_{N1}^{L}=2q_{N1}^{LU}$. We explore the pooling and separating equilibria candidates hereunder.

Pooling equilibrium candidate P

Assume $q_{N1}^{H}=q_{N1}^{L}=\overline{q}_{N}$. Clearly, this implies that $q_{C1}^{H}=q_{C1}^{L}=\overline{q}_{C}$. Hence, (11) is the same in any state. Suppose that (11) is not binding. Solving the problem yields $\overline{q}_{C}=q_{C1}^{U}$ and $\overline{q}_{N}=pq_{N1}^{HU}+(1-p)q_{N1}^{LU}>q_{N1}^{LU}$, so (11) does not hold, and thus, it must be binding.

By substituting $q_{N1}^{H}=q_{N1}^{L}=\overline{q}_{N}$ and (11) into (10), the simplified problem in terms of $\overline{q}_{N}$ is as follows:

$$\max_{\overline{q}_{N}\geq 0}\ \frac{2-\alpha^{2}}{2}\sum_{s\in\{H,L\}}p_{s}\left(\left(2q_{C1}^{U}-X+\overline{q}_{N}\right)\left(X-\overline{q}_{N}\right)+\left(2q_{N1}^{sU}-\overline{q}_{N}\right)\overline{q}_{N}\right)\tag{22}$$

From the FOC of (22), the solution candidate for market k is $\overline{q}_{k1}^{P}$. The outputs of the standalone firms in state s are q_{N2}^{sP} and $\overline{q}_{C2}^{P}$. The non-negativity conditions of q_{N2}^{sP} and

$\overline{q}^P_{C2}$ will hold if (14) and (15) are satisfied.

Now, we verify if $\overline{q}^P_{N1}$ and $\overline{q}^P_{C1}$ satisfy the remaining restrictions of the problem. First, given (1), it holds that $\overline{q}^P_{N1} \geq 0$ and $\overline{q}^P_{C1} \geq 0$. Second, because $q^U_{C1} > q^{LR}_{C1} > q^{HR}_{C1}$, (14) is satisfied as $2q^U_{C1} \geq pq^{HR}_{C1} + (1-p)q^{LR}_{C1}$ holds. Third, when $X \leq \check{\Omega}$, it holds $q^{LU}_{N1} \geq q^{HR}_{N1} > q^{LR}_{N1}$. Thus, (15) in the low state is satisfied as $2q^{LU}_{N1} \geq pq^{HR}_{N1} + (1-p)q^{LR}_{N1}$ holds. Fourth, (15) in the high state also holds as $q^{HU}_{N1} > q^{LU}_{N1}$.

Separating equilibrium candidate S

Assume $q^H_{N1} > q^L_{N1}$ and $q^H_{N1} + q^L_{N1} = 2q^{LU}_{N1}$. Suppose that (11) in the high state is not binding, thus $q^H_{C1} = q^U_{C1}$. Moreover, $q^H_{N1} > q^L_{N1}$ and $q^H_{N1} + q^L_{N1} = 2q^{LU}_{N1}$ imply that $q^H_{N1} > q^{LU}_{N1}$. Therefore, (11) in the high state does not hold, and hence, it must be binding.

Suppose that (11) in the low state is not binding. We substitute $q^H_{N1} + q^L_{N1} = 2q^{LU}_{N1}$ and (11) in the high state into (10) to obtain a problem only in terms of q^L_{N1} and q^L_{C1}. The simplified problem yields the following:

$$q^L_{N1} = \frac{(2-\alpha)(p\left(D_C + 3D^L_N - D^H_N\right) + D^L_N) - 2p(2-\alpha^2)X}{2(2-\alpha^2)(p+1)}, \quad q^L_{C1} = q^U_{C1}$$

If (11) in the low state is not binding, $q^L_{N1} + q^L_{C1} \leq X$ must hold. That condition is equivalent to:

$$X \geq \frac{(2-\alpha)((2p+1)D_C + (3p+1)D^L_N - pD^H_N)}{2(2-\alpha^2)(2p+1)}$$

which never holds when $X < \hat{\Omega}$. Therefore, (11) in the low state must be binding.

Substituting (11) in both states and $q^H_{N1} + q^L_{N1} = 2q^{LU}_{N1}$ into (10), the simplified problem in terms of q^H_{N1} is as follows:

$$\max_{q_{N1}^H \geq 0} \quad \frac{2-\alpha^2}{2}\Big[p\left(\left(2q_{C1}^U - X + q_{N1}^H\right)\left(X - q_{N1}^H\right) + \left(2q_{N1}^{HU} - q_{N1}^H\right)q_{N1}^H\right) \dots$$

$$+ (1-p)\left(\left(2q_{C1}^U - X + 2q_{N1}^{LU} - q_{N1}^H\right)\left(X - 2q_{N1}^{LU} + q_{N1}^H\right) + \left(2q_{N1}^{LU} - q_{N1}^H\right)q_{N1}^H\right)\Big] \tag{23}$$

From the FOC of (23), it follows that the solution candidate in market k and state s is q_{k1}^{sS}. The output of the standalone firms in market k and state s is q_{k2}^{sS}. Again, $q_{k2}^{sS} \geq 0$ will hold if (14) and (15) are satisfied.

Now, we verify if this candidate for the solution satisfies the remaining restrictions of the problem. We start verifying (12), which is equivalent to verify $q_{N1}^{HS} > q_{N1}^{LS}$. The previous inequality holds if and only if:

$$p < \frac{(2-\alpha)(D_N^L + D_C) - 2(2-\alpha^2)X}{4(2-\alpha^2)\theta^S} \tag{24}$$

When $X \geq \check{\Omega}$, the right side of (24) is greater or equal than 1, so it always holds. Now we verify the non-negativity constraints. Given that $q_{N1}^{HS} > q_{N1}^{LS}$, then $q_{C1}^{HS} < q_{C1}^{LS}$ because (11) is binding in any state. Thus, proving that $q_{N1}^{LS} \geq 0$ and $q_{C1}^{HS} \geq 0$ suffices to verify the non-negativity constraints. Given (1), it follows that $q_{N1}^{LS} \geq 0$. Furthermore, $q_{C1}^{HS} \geq 0$ holds if and only if:

$$p \geq \frac{(2-\alpha)(3D_N^L + D_C) - 6(2-\alpha^2)X}{4(2-\alpha^2)\theta^S} \tag{25}$$

When $X \geq \check{\Omega}$ and with (2), the right side of (25) is lower or equal than 0, so it always holds. Now, we verify (15). Because $q_{N1}^{HS} > q_{N1}^{LS} > 0$ and $q_{N1}^{HS} + q_{N1}^{LS} = 2q_{N1}^{LU}$, it follows $2q_{N1}^{LU} > q_{N1}^{HS} > q_{N1}^{LS}$. Thus, (15) is satisfied in any state. Finally, we verify (14). In the low state, (14) holds if, and only if:

$$p \geq \frac{2(2-\alpha^2)X - (2-\alpha)(D_N^L + 3D_C)}{4(2-\alpha^2)\theta^S} \tag{26}$$

As $X < \hat{\Omega}$, the right side of (26) is negative, so it always holds. Given that (14) in the low state is satisfied, (14) in the high state also holds as $q_{C1}^{LS} > q_{C1}^{HS}$.

The solution

The ex-ante expected profit of the conglomerate in equilibrium P is:

$$E\pi^P = \frac{2-\alpha^2}{2} \sum_{s \in \{H,L\}} p_s \left(\left(2q_{C1}^U - \overline{q}_{C1}^P\right) \overline{q}_{C1}^P + \left(2q_{N1}^{sU} - \overline{q}_{N1}^P\right) \overline{q}_{N1}^P \right)$$

The ex-ante expected profit of the conglomerate in equilibrium S is:

$$E\pi^S = \frac{2-\alpha^2}{2} \sum_{s \in \{H,L\}} p_s \left(\left(2q_{C1}^U - q_{C1}^{sS}\right) q_{C1}^{sS} + \left(2q_{N1}^{sU} - q_{N1}^{sS}\right) q_{N1}^{sS} \right)$$

It follows that $E\pi^S \geq E\pi^P$ if and only if:

$$X \geq \frac{(2-\alpha)}{2(2-\alpha^2)} \left(D_C + 2D_N^L - D_N^H\right) = \check{\Omega}$$

Given (2), it follows $\check{\Omega} > 0$. Moreover, because $\check{\Omega} < \hat{\Omega}$, then there exists an X such that $E\pi^S \geq E\pi^P$. Thus, in the second-best contract the equilibrium is S when $\hat{\Omega} > X \geq \check{\Omega}$, and it is P when $X \leq \check{\Omega}$.