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“Job Search Intensity and Wage Rigidity in Business Cycles”


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# Job Search Intensity and Wage Rigidity in Business Cycles*

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**Abstract**

This paper examines the job search behavior of unemployed workers over the business cycle. The paper first constructs a standard search and matching model with endogenous search efforts, wage rigidity, and a generalized matching function. Contrary to the existing literature, the proposed model generates both procyclical and countercyclical search intensity, depending on the degree of wage rigidity and the elasticity parameter of the matching function. The paper then calibrates the model to the U.S. economy and provides various impulse response analyses. The numerical exercises show that the model successfully and simultaneously reproduces countercyclical search efforts and sizable labor market fluctuations.

*JEL* Classification Numbers: E24, E32, J64.

*Key Words:* search intensity; business cycles; wage rigidity; unemployment fluctuations.

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# 1 Introduction

In search and matching models, the labor market is mainly characterized by the number of unemployed workers and the number of vacancies. The ratio of the latter to the former represents the labor market tightness, which partially explains the job-finding and job-filling rates in these economies. In reality, however, the job-finding rate also depends on the worker's job search effort or search intensity. The standard Diamond–Mortensen–Pissarides (DMP) model with endogenous search intensity predicts that search intensity is procyclical—that is, it is positively correlated with total output and thus the labor market tightness. This procyclical search intensity amplifies labor market fluctuations and appears to solve, at least partially, the "volatility puzzle" of the labor market. In contrast, some empirical studies have shown that search intensity is countercyclical; however, relatively few theoretical studies have investigated countercyclical search intensity so far.

The present paper addresses this inconsistency between the predictions of the standard model and the empirical findings and provides a comprehensive analysis of the cyclicality of search intensity. To this end, this paper extends the standard DMP model by incorporating endogenous search intensity and wage rigidity and relaxing the matching function from the Cobb-Douglas function to the CES function.

The main finding of this paper is that search intensity could be both procyclical and countercyclical, depending on the degree of wage rigidity, even when the matching function satisfies the standard properties of the labor market. In examining the cyclicality of search intensity, this paper focuses on modeling two aspects that are critical to the gain from job search: the role of search intensity in the matching function and the role of wage rigidity in determining search intensity. The former is already discussed by Shimer (2004) and Mukoyama, Patterson and Şahin (2018), and this paper similarly derives the properties of the matching function that makes search intensity countercyclical. The latter is relatively new in the literature. Incorporating wage rigidity is one of the main approaches to deal with the volatility puzzle, which was first introduced by Hall (2005). Similar to the previous study, this paper shows that wage rigidity maintains large labor market fluctuation even when search intensity is countercyclical.

In the DMP model, firm–worker pairs negotiate their wages so that the net gains from the job match are shared according to the Nash bargaining and renegotiate the wages when new information arrives. In the model, therefore, wages absorb most of the economic changes, as Shimer (2005) argues. When a negative productivity shock occurs, for example, wages are immediately adjusted to lower values, and these lower wages do not significantly reduce the incentive to post a vacancy. When wages are sticky, however, they remain relatively high for some time and make firms reluctant to post a vacancy. As a result, the unemployment rate increases, and the labor market condition worsens.

The main contributions of this paper are as follows. First, this paper presents one of the most plausible mechanisms to generate countercyclical search intensity. In this model, search intensity would be countercyclical when wages are sufficiently sticky. Intuitively, unemployed workers increase their search efforts in response to economic downturns, as relatively high wages and the low job-finding rate increase the net benefit of being employed. This paper replicates the countercyclicality of search intensity by incorporating wage rigidity, whereas previous studies replicate it by modifying the matching functions or incorporating labor–leisure choices. Second, the model generates the plausible size of the labor market fluctuation even when search intensity is countercyclical. Countercyclical search intensity dampens the labor market volatility; nevertheless, the model captures the dynamics of the labor market relatively well compared with the standard model in the presence of wage rigidity.

The rest of this paper is organized as follows: Section 2 reviews the related literature on search intensity and its cyclicality, Section 3 describes the extended DMP model and characterizes the equilibrium, Section 4 explains the steady state equilibrium and examines the formulation of the matching function, Section 5 presents the impulse response analyses of the model under various assumptions of wage rigidity and matching function, and Section 6 concludes the paper. Appendix describes how to obtain the Nash bargaining wage and explains the log-linear approximation of the model.

## 2 Related literature

This section reviews the empirical and theoretical literature on job search intensity and its cyclicality. Search intensity is the amount of effort put into the job search. Although search intensity is costly for the unemployed worker, it raises the probability of finding a new job. Pissarides (2000) states that search intensity can be thought of as a "technical change" parameter in job matching technology. The number of job matches increases, for the given labor market condition, if unemployed workers search more intensely. The natural measure of search intensity is the time spent on the job search or the number of job applications submitted.

Datasets from the Current Population Survey (CPS) and the American Time Use Survey (ATUS) are often used to measure search intensity. CPS reports the types and number of job search methods used by unemployed workers, and ATUS reports the time spent on their job search. Literature using these surveys, such as Shimer (2004), Gomme and Lkhagvasuren (2015), and Mukoyama et al. (2018), seems to show consensus on the countercyclicality of aggregate or average search intensity, although no conclusive findings have been obtained on the cyclicality of individual search intensity. Gomme and Lkhagvasuren (2015) conclude that individual search intensity is procyclical after controlling for individual characteristics and that the composition of unemployed workers changes over the business cycle and derives a countercyclical bias to the average time spent on job search. In contrast, Mukoyama et al. (2018) conclude that the change in composition significantly contributes to the cyclicality of aggregate search intensity but that search intensity is countercyclical even when unobserved heterogeneity is taken into account.

The literature that uses data other than ATUS or CPS to measure job search intensity includes Pan (2019), Faberman and Kudlyak (2019), and Bransch (2021). Pan (2019) constructs the job search index based on the internet search volume and shows that search intensity is countercyclical. Faberman and Kudlyak (2019) use data from SnagAJob, an online private job search website, and measure search intensity by the weekly number of applications sent to the engine. They find that job seekers in weak labor markets send more applications throughout the duration of their search spell; this finding is consistent with countercyclical search intensity. Bransch (2021) measures search intensity by the numbers of applications and job search methods using Dutch panel data and concludes that search intensity is countercyclical.

To sum up, much empirical literature has shown the countercyclicality of job search intensity, even when different data are used for analysis. Because these findings contradict the predictions of the standard DMP model, it is worthwhile to analyze the cyclicality of search intensity in a theoretical DMP model with search intensity.

The theoretical models can be broadly classified into two categories in the context of business cycles. The models in the first category predict procyclical search intensity. Most of the standard models with search intensity, such as that by Pissarides (2000), belong to this category. When there is a negative productivity shock, the shock reduces the incentive to post a vacancy and thus lowers the market tightness. In such a depressed labor market, unemployed workers are reluctant to search for a job partly because an additional search effort is not very useful for increasing the job-finding rate and because low wages reduce the benefit of being employed by a firm. Leduc and Liu (2020) develop a DSGE model that incorporates both search intensity and recruiting intensity and show that both intensities are procyclical.

The models in the second category predict countercyclical search intensity. To the best of our knowledge, only Shimer (2004), Mukoyama et al. (2018), and Çenesiz and Guimarães (2022) have developed such models. Unemployed workers increase their search intensity during the economic downturn because additional search intensity effectively increases the job-finding rate in the first two models and because the procyclical value of leisure makes job search cheaper in the third model. Therefore, the key to reproduce countercyclicality is how search intensity is incorporated in the matching functions or in the procyclical leisure. Contrary to these models, in the model proposed in the present paper, unemployed workers search for a job intensely during recession because wage rigidity increases the net benefit of being employed, although additional search intensity does little to increase the job-finding rate. One of the main findings of the present paper—both countercyclical search intensity and the large volatility of the labor market could be predicted at the same time—is consistent with Çenesiz and Guimarães (2022), but the channels that generate this prediction are different.

Leduc and Liu (2020) and Gertler, Huckfeldt and Trigari (2020) incorporate both wage rigidity and endogenous search intensity, as in the present paper. Note that the findings of these papers—search intensity becomes procyclical—do not contradict the findings of the present paper. In the present paper, search intensity is countercyclical when the matching function is one of CES and satisfies some standard properties but it is procyclical when the matching function is a Cobb-Douglas function, as in Leduc and Liu (2020), even if wages are rigid enough. Gertler et al. (2020) focus on the search intensity of the workers searching on-the-job, rather than unemployed workers, and assume variable search intensity only for the workers on-the-job and fixed search intensity for unemployed workers. Unlike Gertler et al. (2020), the present paper assumes variable search intensity for unemployed workers and does not consider the on-the-job search.

## 3 Model

This section extends the standard DMP model by incorporating endogenous search intensity and wage rigidity. The wage setting in the economy follows Gertler and Trigari (2009). Firms hire multiple workers and pay the same wage for both incumbent and newly hired workers. Workers and firms can negotiate to update their wages, by Nash bargaining, with probability $1 - \lambda$ in each period.

It is often argued that wages of new hires are more responsive than those of incumbent workers, e.g., Pissarides (2009). However, some literature, including Gertler and Trigari (2009), Stüber (2017), and Gertler et al. (2020), show that after controlling for composition effects, the wage cyclicality of new hires and that of existing workers are almost identical. Especially, Gertler et al. (2020) find that the procyclical upgrading of job match quality is dominant among the new hires who are job changers, and as for new hires from unemployment, there is no excess wage cyclicality compared to existing workers. Their empirical evidence justifies the assumption of wage rigidity in our model.

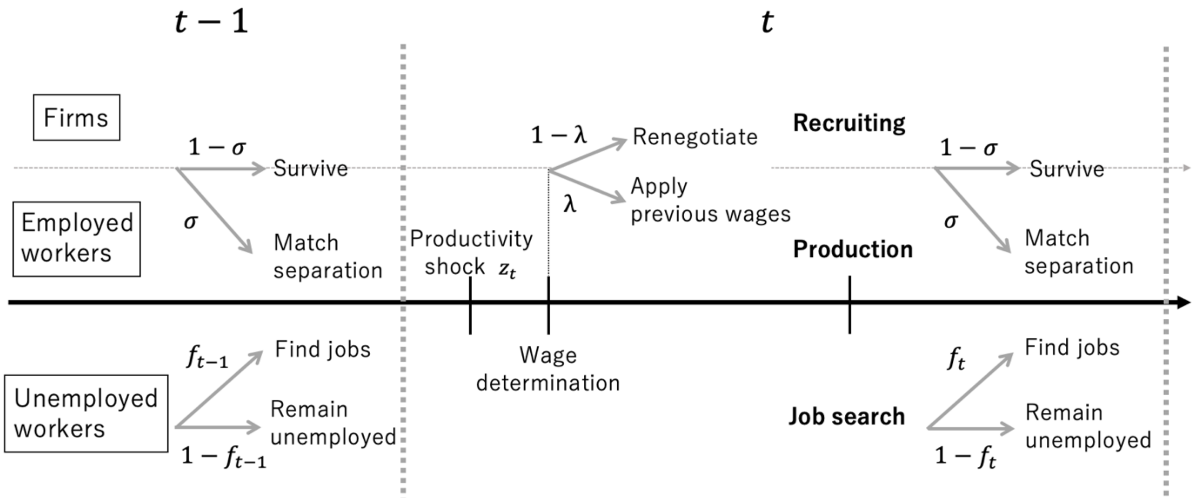


Figure 1: Summary of the events

### 3.1 Settings

Time is discrete and denoted by $t$. The economy has a continuum of workers and firms whose measures are normalized to unity. The workers comprise $n_t$ employed workers and $u_t \equiv 1 - n_t$ unemployed workers. The aggregate state of the economy is denoted by $S_t \equiv (z_t, n_t, P_t(w))$, where $z_t$ is the aggregate productivity and $P_t(w)$ is the wage distribution at the beginning of period $t$.

Figure 1 summarizes the timing of events. The workers and firms observe the aggregate productivity shock $z_t$ at the beginning of period $t$ and negotiate to update their wages with probability $1 - \lambda$ or keep the previous wages with probability $\lambda$. Then, each firm produces final goods with their workers, and the firm decides the number of job vacancies for recruiting. At the same time, each unemployed worker searches for a new job, choosing the optimal level of search intensity. At the end of period $t$, a fraction $\sigma$ of employed workers are separated, a fraction $q_t$ of job vacancies are filled, and a fraction $f_t$ of unemployed workers find new jobs.

More specifically, we assume that the probability that an unemployed worker with search intensity $s_t$ finds a job is given by $f(s_t, \theta_t)$, where $\theta_t$ denotes the market tightness, $\theta_t \equiv v_t/u_t$. Note that the job-finding rate depends on individual search intensity and market tightness and not on aggregate search intensity in this economy. This implicitly assumes that the positive and negative externalities of aggregate search efforts on individual job-finding rate offset each other. Unemployed workers are assumed to be identical; thus, aggregate search intensity also becomes $s_t$. Then, the number of new matches and the firms' job-filling rate, respectively, are given by

$$m(s_t, u_t, v_t) = f(s_t, \theta_t)u_t, \qquad q(s_t, \theta_t) = f(s_t, \theta_t)/\theta_t.$$

The specific form of the job-finding rate function or matching function will be described in Section 4.2.

### 3.2 Workers

The economy has employed and unemployed workers. Each worker has linear preference and consumes all disposable income in each period. Let $W(w_t, S_t)$ be the value function of the employed worker with wage $w_t$, and let $U(S_t)$ be the value function of the representative unemployed worker. Then, the value function of the employed worker is

$$W(w_t, S_t) = w_t + d_t - \tau_t + \beta(1-\sigma)E[\, W(w_{t+1}, S_{t+1}) \mid w_t, S_t \,] + \beta\sigma E[\, U(S_{t+1}) \mid S_t \,], \tag{1}$$

subject to

$$S_{t+1} = (z_{t+1}, n_{t+1}, P_{t+1}(w)), \tag{2}$$

$$\ln z_{t+1} = \rho \ln z_t + \varepsilon_{t+1}, \tag{3}$$

$$w_{t+1} = \begin{cases} w^*_{t+1}(S_{t+1}) & \text{with probability} \quad 1-\lambda, \\ w_t & \text{with probability} \quad \lambda, \end{cases} \tag{4}$$

where $d_t$ is the distribution of firms' profits, $\tau_t$ is the lump-sum tax, $\beta \in (0, 1)$ is the time discount factor, $\sigma$ is the job separation probability, $z_t$ is the labor productivity, $\rho \in [\,0, 1\,]$ is the auto correlation of $\ln z_t$, and $w^*_{t+1}(S_{t+1})$ is the wage determined by the Nash bargaining in period $t+1$. The optimization problem of the unemployed worker is

$$\begin{aligned} U(S_t) = \max_{s_t} \big\{ & \phi - \omega(s_t) + d_t - \tau_t \\ & + \beta f(s_t, \theta_t)\tilde{E}[\, W(w_{t+1}, S_{t+1}) \mid S_t \,] + \beta(1 - f(s_t, \theta_t))E[\, U(S_{t+1}) \mid S_t \,] \big\}, \end{aligned} \tag{5}$$

subject to (2) and (3), where $s_t$ is the search effort, $\phi$ is the unemployment benefit, $\omega(\cdot)$ is the search cost function, $f(\cdot, \cdot)$ is the job-finding rate function, and $\tilde{E}[\,\cdot\,]$ is the expected value with respect to the distribution of newly hired workers only. The job-finding rate $f(s_t, \theta_t)$ is assumed to be increasing and concave in $s_t$. The first-order condition for this problem is

$$\omega'(s_t) = \beta f_s(s_t, \theta_t)\big\{\tilde{E}[\, W(w_{t+1}, S_{t+1}) \mid S_t\,] - E[\, U(S_{t+1}) \mid S_t\,]\big\}. \tag{6}$$

The unemployed worker decides the optimal search intensity so that the marginal cost of searching a job equals the discount expected value of getting a job. Solving (6) yields the optimal search intensity $s_t(S_t)$, which is identical for all unemployed workers.

### 3.3 Firms

Firms in this economy hire workers and pay the same wages for both incumbent and newly hired workers. In each period, firms and the workers renegotiate the wages with probability $1-\lambda$ or keep the previously negotiated wages with probability $\lambda$. Each firm has its linear production technology. Let $J(w_t, \tilde{n}_t, S_t)$ be the value function of a firm with $\tilde{n}_t$ workers at wage $w_t$. Then, the optimization problem of the firm is

$$J(w_t, \tilde{n}_t, S_t) = \max_{x_t} \big\{z_t\tilde{n}_t - w_t\tilde{n}_t - \kappa(x_t)\tilde{n}_t + \beta E[\, J(w_{t+1}, \tilde{n}_{t+1}, S_{t+1}) \mid w_t, S_t\,]\big\}, \tag{7}$$

subject to (2), (3), (4), and

$$\tilde{n}_{t+1} = (1 - \sigma + q(s_t, \theta_t)x_t)\tilde{n}_t, \tag{8}$$

where $x_t$ is the number of job vacancies per worker or the vacancy rate, $\kappa(\cdot)$ is the cost function of posting vacancies per worker, and $q(\cdot)$ is the job-filling probability function.

Let $F(w_t, S_t) \equiv J(w_t, \tilde{n}_t, S_t)/\tilde{n}_t$ be the value function of the firm per worker at wage $w_t$. Then, the optimization problem of the firm is modified to

$$F(w_t, S_t) = \max_{x_t} \big\{z_t - w_t - \kappa(x_t) + \beta(1 - \sigma + q(s_t, \theta_t)x_t)E[\, F(w_{t+1}, S_{t+1}) \mid w_t, S_t\,]\big\}, \tag{9}$$

subject to (2), (3), and (4). The first-order condition for this problem is

$$\kappa'(x_t) = \beta q(s_t, \theta_t)E[\, F(w_{t+1}, S_{t+1}) \mid w_t, S_t\,]. \tag{10}$$

The firm decides the optimal vacancy rate so that the marginal cost of posting vacancy equals the expected value of hiring new workers. Solving (10) yields the optimal vacancy rate $x_t(w_t, S_t)$.

### 3.4 Wage determination

In each period $t$, a fraction $1-\lambda$ of employed workers and firms negotiate on the new wage, $w_t^*(S_t)$, by the Nash bargaining,

$$w_t^*(S_t) = \arg\max_{w_t} \left(W(w_t, S_t) - U(S_t)\right)^{\gamma} F(w_t, S_t)^{1-\gamma}, \tag{11}$$

subject to (1), (2), (3), (4), (5), and (9), where $\gamma \in [0,1]$ is the workers' bargaining power. The negotiated wage, $w_t^*(S_t)$, is the same across all workers and firms because it is independent of the current wage, $w_t$. Using (4) and (6), the surpluses of the worker and the firm, respectively, are modified to

$$\begin{aligned} W(w_t, S_t) - U(S_t) &= w_t - \phi_t + \omega(s_t(S_t)) - f(s_t(S_t), \theta_t) \frac{\omega'(s_t(S_t))}{f_s(s_t(S_t), \theta_t)} \\ &+ \beta(1-\sigma)\Big\{(1-\lambda)E\big[\, W(w_{t+1}^*(S_{t+1}), S_{t+1}) \mid S_t \,\big] + \lambda E\big[\, W(w_t, S_{t+1}) \mid S_t \,\big] - E\big[\, U(S_{t+1}) \mid S_t \,\big]\Big\}, \end{aligned} \tag{12}$$

$$\begin{aligned} F(w_t, S_t) &= z_t - w_t - \kappa(x_t(w_t, S_t)) \\ &+ \beta(1-\sigma+q(s_t,\theta_t)x_t(w_t,S_t))\Big\{(1-\lambda)E[\, F(w_{t+1}^*(S_{t+1}), S_{t+1}) \mid S_t \,] + \lambda E[\, F(w_t, S_{t+1}) \mid S_t \,]\Big\}. \end{aligned} \tag{13}$$

The first-order condition of the Nash bargaining is

$$\gamma \frac{W_w(w_t, S_t)}{W(w_t, S_t) - U(S_t)} + (1-\gamma)\frac{F_w(w_t, S_t)}{F(w_t, S_t)} = 0, \tag{14}$$

where, using the envelope theorem,

$$W_w(w_t, S_t) = 1 + \beta\lambda(1-\sigma)E[\, W_w(w_t, S_{t+1}) \mid S_t \,] \quad \text{and} \tag{15}$$

$$F_w(w_t, S_t) = -1 + \beta\lambda(1-\sigma+q(s_t,\theta_t)x_t(w_t,S_t))E[\, F_w(w_t, S_{t+1}) \mid S_t \,]. \tag{16}$$

Solving (14) yields the bargaining wage $w_t^*(S_t)$.

Now, let $\Gamma_t$ be the share of the worker's surplus in total surplus:

$$\Gamma_t \equiv \frac{W(w_t^*, S_t) - U(S_t)}{W(w_t^*, S_t) - U(S_t) + F(w_t^*, S_t)}. \tag{17}$$

Then, we can rewrite (14) as follows:

$$\Gamma_t F(w_t^*, S_t) = (1 - \Gamma_t)\left(W(w_t^*, S_t) - U(S_t)\right) \tag{18}$$

and

$$\Gamma_t = \frac{\gamma W_w(w_t^*, S_t)}{\gamma W_w(w_t^*, S_t) - (1-\gamma)F_w(w_t^*, S_t)} < \gamma. \tag{19}$$

This is the extended version of the standard bargaining wage rule. The last inequality holds because, from (15) and (16), $F_w(w_t^*, S_t)/W_w(w_t^*, S_t) < -1$ when $q(s_t, \theta_t)x_t(w_t, S_t) > 0$.

### 3.5 Distribution and aggregation

Let $P_t(w)$ be the cumulative distribution function of wages at the beginning of period $t$ such that

$$\int_W \mathrm{d}P_t(w) = 1.$$

The law of motion of the wage distribution is

$$\mathrm{d}P_{t+1}(w) = \frac{1 - \sigma + q(\theta_t)x_t(w, S_t)}{1 - \sigma + q(\theta_t)\bar{x}_t}\left((1-\lambda)\mathbf{1}_{[w=w_t^*]} + \lambda \mathrm{d}P_t(w)\right), \tag{20}$$

where $w_t^*$ is the negotiated wage, $\mathbf{1}_{[w=w_t^*]}$ is a indicator function that returns 1 if $w = w_t^*$ and 0 otherwise, and $\bar{x}_t$ is the average vacancy rate per worker,

$$\bar{x}_t = (1-\lambda)x_t(w_t^*, S_t) + \lambda \int_W x_t(w, S_t)\mathrm{d}P_t(w). \tag{21}$$

The total numbers of vacancies is

$$v_t = n_t \bar{x}_t. \tag{22}$$

The total number of unemployed and employed workers in period $t+1$ are

$$u_{t+1} = (1 - u_t)\sigma + u_t(1 - f(s_t, \theta_t)) \tag{23}$$

and

$$n_{t+1} = (1 - \sigma + q(\theta_t)\bar{x}_t)n_t, \tag{24}$$

respectively, and $n_{t+1} + u_{t+1} = 1$.

In the next period, employed workers receive the negotiated wage $w^*_{t+1}$ with probability $1 - \lambda$ and the current wage $w_t$ with probability $\lambda$. The conditional expectation of the value of employed workers at wage $w_t$ is

$$E[\,W(w_{t+1}, S_{t+1})\,|\,w_t, S_t] = \int_Z \left\{(1-\lambda)W(w^*_{t+1}, S_{t+1}) + \lambda W(w_t, S_{t+1})\right\} \mathrm{d}\Pi(z_{t+1}\,|\,z_t), \tag{25}$$

where $\Pi(z_{t+1}\,|\,z_t)$ is the conditional distribution function of $z_{t+1}$ given $z_t$. The conditional expectation of the value of all employed workers is

$$E[\,W(w_{t+1}, S_{t+1})\,|\,S_t] = \int_Z \left\{(1-\lambda)W(w^*_{t+1}, S_{t+1}) + \lambda \int_W W(w, S_{t+1})\mathrm{d}P_{t+1}(w)\right\} \mathrm{d}\Pi(z_{t+1}\,|\,z_t). \tag{26}$$

Similarly, the conditional expectation of the value of newly hired workers is

$$\tilde{E}[\,W(w_{t+1}, S_{t+1})\,|\,S_t] = \int_Z \left\{(1-\lambda)W(w^*_{t+1}, S_{t+1}) + \lambda \int_W W(w, S_{t+1})\mathrm{d}\tilde{P}_{t+1}(w)\right\} \mathrm{d}\Pi(z_{t+1}\,|\,z_t), \tag{27}$$

where the wage distribution of newly hired workers is

$$\mathrm{d}\tilde{P}_{t+1}(w) = (1-\lambda)\frac{x_t(w^*_t, S_t)}{\bar{x}_t}\mathbf{1}_{[w=w^*_t]} + \lambda\frac{x_t(w, S_t)}{\bar{x}_t}\mathrm{d}P_t(w).$$

The government imposes lump-sum tax on all workers to finance the unemployment benefits. The lump-sum tax, $\tau_t$, in each period is

$$\tau_t = \phi u_t. \tag{28}$$

The sum of the firms' profits, or the uniform dividend income, is given by

$$d_t = \left\{z_t - (1-\lambda)\left(w^*_t + \kappa\left(x_t(w^*_t, S_t)\right)\right) - \lambda \int_W \left(w + \kappa\left(x_t(w, S_t)\right)\right)\mathrm{d}P_t(w)\right\} n_t. \tag{29}$$

### 3.6 The equilibrium

The state of the economy, $S_t$, is characterized by the aggregate productivity, $z_t$, the number of employed workers, $n_t$, and the wage distribution at the beginning of period $t$, $P_t(w)$. Then, the recursive equilibrium of this economy is defined as follows.

**Definition:** The recursive equilibrium comprises time series of the value functions of employed workers, unemployed workers, and firms, $\left\{W(w_t, S_t), U(S_t), F(w_t, S_t)\right\}_{t=0}^{\infty}$; the decision rules of workers, firms, and Nash bargaining wages, $\left\{s_t(S_t), x_t(w_t, S_t), w_t^*(S_t)\right\}_{t=0}^{\infty}$; and the labor market tightness, $\left\{\theta_t(S_t)\right\}_{t=0}^{\infty}$; such that

1. the workers solve the utility maximization problem, (2), (3), and (5),
2. the firms solve the profit maximization problem, (2), (3), (4), (7), and (8),
3. wage is determined by the Nash bargaining problem, (11), (12), and (13),
4. the labor market clears.

The equilibrium is in a steady state if the distribution functions of workers are time invariant.

# 4 Steady state analysis

In this section, we first describe the steady state of this model. Next, we analyze how search intensity in the steady state responds to the changes in market tightness in the two extreme cases—the case of fixed wages ($\lambda = 1$) and the case of flexible wages ($\lambda = 0$)—relaxing the matching function form from the commonly used Cobb-Douglas function.

$\bar{X}$ denotes the steady state value of $X_t$. In the steady state, all employed workers receive the same wages, and $F(w, S)$ is the same across firms. (9) and (10) imply that $\bar{x}$ satisfies the following:

$$\begin{aligned} \kappa'(\bar{x}) &= \beta q(\bar{s}, \bar{\theta}) \left( \bar{z} - \bar{w} - \kappa(\bar{x}) + \frac{\kappa'(\bar{x}) \left(1 - \sigma + \bar{x} q(\bar{s}, \bar{\theta})\right)}{q(\bar{s}, \bar{\theta})} \right) \\ &= \frac{\beta q(\bar{s}, \bar{\theta}) \left(\bar{z} - \bar{w} - \kappa(\bar{x})\right)}{1 - \beta \left(1 - \sigma + \bar{x} q(\bar{s}, \bar{\theta})\right)}. \end{aligned} \tag{30}$$

Because all firms choose the same vacancy rate $\bar{x}$ in the steady state, the numbers of vacancies and employed workers, respectively, are

$$\bar{v} = \bar{x}\bar{n} \quad \text{and} \quad \bar{n} = (1 - \sigma + \bar{x} q(\bar{s}, \bar{\theta}))\bar{n}.$$

The latter equation leads to

$$\bar{x} q(\bar{s}, \bar{\theta}) = \sigma.$$

Using this, we can rewrite (30) as follows:

$$\kappa'(\bar{x}) = \frac{\beta q(\bar{s}, \bar{\theta}) \left(\bar{z} - \bar{w} - \kappa(\bar{x})\right)}{1 - \beta}. \tag{31}$$

(1), (5), and (6) also imply that $\bar{s}$ satisfies the following:

$$\omega'(\bar{s}) = \beta \bar{f}_s(\bar{s}, \bar{\theta}) \frac{\bar{w} - \phi + \omega(\bar{s})}{1 - \beta(1 - \sigma - f(\bar{s}, \bar{\theta}))}. \tag{32}$$

The steady state wage satisfies $\bar{\Gamma}\bar{F} = (1 - \bar{\Gamma})\left(\bar{W} - \bar{U}\right)$; then, substituting (1), (5), and (9) into this yields

$$\bar{\Gamma}\left(\bar{z} - \bar{w} - \kappa(\bar{x}) + \kappa'(\bar{x})\bar{x} + \beta(1-\sigma)\bar{F}\right) = (1 - \bar{\Gamma})\left(\bar{w} - \phi + \omega(\bar{s}) - \frac{\omega'(\bar{s}) f(\bar{s}, \bar{\theta})}{f_s(\bar{s}, \bar{\theta})} + \beta(1-\sigma)\left(\bar{W} - \bar{U}\right)\right),$$

$$\Leftrightarrow \bar{w} = \bar{\Gamma}\Big(\bar{z} - \kappa(\bar{x}) + \kappa'(\bar{x})\bar{x}\Big) + (1 - \bar{\Gamma})\left(\phi - \omega(\bar{s}) + \frac{\omega'(\bar{s}) f(\bar{s}, \bar{\theta})}{f_s(\bar{s}, \bar{\theta})}\right), \tag{33}$$

$$\text{where } \bar{\Gamma} = \frac{\gamma}{\gamma + (1-\gamma)\frac{1-\beta\lambda(1-\sigma)}{1-\beta\lambda}}. \tag{34}$$

From the above equations, the key labor market variables $(\bar{x}, \bar{s}, \bar{w}, \bar{u}, \bar{\theta})$ are characterized by the following five equations:

$$\kappa'(\bar{x}) = \frac{\beta q(\bar{s}, \bar{\theta})\left(\bar{z} - \bar{w} - \kappa(\bar{x})\right)}{1 - \beta}, \tag{31}$$

$$\omega'(\bar{s}) = \beta \bar{f}_s(\bar{s}, \bar{\theta}) \frac{\bar{w} - \phi + \omega(\bar{s})}{1 - \beta(1 - \sigma - f(\bar{s}, \bar{\theta}))}, \tag{32}$$

$$\bar{w} = \bar{\Gamma}\big(\bar{z} - \kappa(\bar{x}) + \kappa'(\bar{x})\bar{x}\big) + (1 - \bar{\Gamma})\left(\phi - \omega(\bar{s}) + \frac{\omega'(\bar{s}) f(\bar{s}, \bar{\theta})}{f_s(\bar{s}, \bar{\theta})}\right), \tag{33}$$

$$\bar{\theta}\bar{u} = \bar{x}(1 - \bar{u}), \quad \text{and} \tag{35}$$

$$f(\bar{s}, \bar{\theta})\bar{u} = \sigma(1 - \bar{u}). \tag{36}$$

Next, to get an intuition about the cyclicality of search intensity in the case of rigid wages, we examine how the search intensity in the steady state responds to the changes in market tightness when wages are fixed and flexible.

## 4.1 Search intensity in the steady state when wages are fixed

When wages are fixed, the relationship between search intensity and market tightness is characterized only by (32).

$$\omega'(\bar{s}) = \beta \bar{f}_s(\bar{s}, \bar{\theta}) \frac{\bar{w} - \phi + \omega(\bar{s})}{1 - \beta(1 - \sigma - f(\bar{s}, \bar{\theta}))}. \tag{32}$$

This leads to (we denote $\bar{f} \equiv f(\bar{s}, \bar{\theta})$)

$$\frac{d\bar{s}}{d\bar{\theta}} = -\frac{\overbrace{\beta \bar{f}_{s\theta} \frac{\bar{w} - \phi + \omega(\bar{s})}{1 - \beta(1 - \sigma - \bar{f})}}^{\equiv A} \overbrace{- \beta \bar{f}_s \frac{\beta \bar{f}_\theta (\bar{w} - \phi + \omega(\bar{s}))}{(1 - \beta(1 - \sigma - \bar{f}))^2}}^{\equiv B(<0)}}{\underbrace{C - \omega''(\bar{s})}_{<0}}, \tag{37}$$

where[1]

$$\begin{aligned} C \equiv & \beta \bar{f}_{ss} \frac{\bar{w} - \phi + \omega(\bar{s})}{1 - \beta(1 - \sigma - \bar{f})} + \beta \bar{f}_s \frac{\omega'(\bar{s})}{1 - \beta(1 - \sigma - \bar{f})} + \beta \bar{f}_s \frac{-\beta \bar{f}_s (\bar{w} - \phi + \omega(\bar{s}))}{(1 - \beta(1 - \sigma - \bar{f}))^2} \\ = & \beta \bar{f}_{ss} \frac{\bar{w} - \phi + \omega(\bar{s})}{1 - \beta(1 - \sigma - \bar{f})} + \beta \bar{f}_s \frac{\omega'(\bar{s})}{1 - \beta(1 - \sigma - \bar{f})} + \beta \bar{f}_s \frac{-\omega'(\bar{s})}{1 - \beta(1 - \sigma - \bar{f})} \\ = & \beta \bar{f}_{ss} \frac{\bar{w} - \phi + \omega(\bar{s})}{1 - \beta(1 - \sigma - \bar{f})} < 0. \end{aligned}$$

Unemployed workers determine their search intensity so that the marginal search cost equals the marginal search benefit, which is the product of the increased odds of job finding and the benefit of being employed. (37) implies that in the steady state, how search intensity responds to the changes in market tightness depends on the magnitude of $A$ relative to $B$. $A$ and $B$ reflect the direct effects of the changes in market tightness on the marginal search benefit.

$A$ represents the effect on the increased odds of finding a job owing to an additional search intensity, $f_s(s, \theta)$, and it determines the sign of $A$. How $f_s(s, \theta)$ responds to the change in $\theta$ depends on the assumption of the matching function. $f_{s\theta}(s, \theta) > 0$ means the complementary relationship between search intensity and the job-finding rate, where job search efforts become less effective in increasing the job-finding rate during recession and more effective during booms. On the contrary, $f_{s\theta}(s, \theta) < 0$ implies the substitutive relationship between search intensity and the job-finding rate, where an additional search effort does not significantly contribute to increasing the job-finding rate during booms.

$B$ represents the effect on the benefit of being employed. In the steady state, the benefit of being employed, $\bar{V} \equiv W(\bar{w}, \bar{S}) - U(\bar{S})$, becomes

$$\bar{V} = \frac{\bar{w} - \phi + \omega(\bar{s})}{1 - \beta(1 - \sigma - \bar{f})}.$$

This benefit is discounted by the discount factor $\beta$ and "the relative ease of being employed," which is given by the survival rate of employed workers, $1 - \sigma$, minus the job-finding rate of unemployed

[1]In the second equality, (32) is substituted into the third term.

workers, $f(s,\theta)$. The above equation implies that a low job-finding rate or high survival rate increases the benefit of being employed: the benefit is highly evaluated when finding a job is difficult or becoming unemployed is unlikely once workers get a job, if all else are equal. Because $f(s,\theta)$ is assumed to be increasing in $\theta$, $B$ becomes negative. Note that we do not consider the effect of $\bar{w}$ on $\bar{V}$ here.

Whether search intensity responds positively or negatively to the change in market tightness depends on which of the above two effects is dominant, and this is governed by the form of the matching function.

### 4.2 Formulation of the matching function

In this subsection, we examine how the sign of $d\bar{s}/d\bar{\theta}$, or $A+B$, varies with the formulation of the matching function when wages are fixed. We can rewrite $A+B$ as follows:

$$A + B = \frac{\beta\left(\bar{w} - \phi + \omega(\bar{s})\right)}{\left(1 - \beta(1 - \sigma - \bar{f})\right)^2} \left\{ \bar{f}_{s\theta}\left(1 - \beta(1 - \sigma - \bar{f})\right) - \beta \bar{f}_s \bar{f}_\theta \right\}, \tag{38}$$

where the first term is positive. Thus, the sign of $A+B$ is determined by

$$\bar{f}_{s\theta}\left(1 - \beta(1 - \sigma - \bar{f})\right) - \beta \bar{f}_s \bar{f}_\theta \equiv \Phi. \tag{39}$$

(39) implies that when wages are fixed, $f_{s\theta} < 0$ is sufficient for search intensity to negatively respond to the change in market tightness, whereas Mukoyama et al. (2018) and Shimer (2004) conclude that $f_{s\theta} < 0$ is necessary for search intensity to be countercyclical. We show below that $\Phi$ could be negative even under matching functions with $f_{s\theta} > 0$ and standard properties (e.g., the concavity of $u$ or $v$).

When the job-finding rate is linear in search intensity,

$$f(s,\theta) = \tilde{f}(\theta)s, \tag{40}$$

$\Phi$ is always positive because $f_{s\theta} f = f_s f_\theta$, and this yields

$$\Phi = \bar{f}_{s\theta}\left(1 - \beta(1 - \sigma)\right) = \tilde{f}'(\theta)\left(1 - \beta(1 - \sigma)\right) > 0.$$

Then, to examine the case where search intensity is nonlinear in the job-finding rate, we assume the following job-finding rate function[2]:

$$f(s,\theta) = \chi\left[\alpha s^{\psi} + (1-\alpha)\theta^{\psi}\right]^{1/\psi}, \tag{41}$$

$$\chi > 0, \text{ and } \alpha \in (0,1).$$

When unemployed workers choose the same $s$, the matching function is given by

$$m(s,u,v) = \chi\left[\alpha(su)^{\psi} + (1-\alpha)v^{\psi}\right]^{1/\psi}. \tag{42}$$

This formulation is the same as that used by Mukoyama et al. (2018); CES-type matching functions have also been used in other literature (e.g., Hagedorn and Manovskii (2008), Coşar, Guner and Tybout (2016), and Birinci, Karahan, Mercan and See (2021)). The presented CES matching function includes the Cobb-Douglas matching function, $m = \chi(su)^{\alpha}v^{1-\alpha}$, when $\psi \to 0$ and is homogenous in $u$ and $v$. The elasticity of substitution of the matching function and job-finding rate function is given by $\frac{1}{1-\psi}$, and $\psi < 1$ must be satisfied for the job-finding rate to be increasing and concave in $s$ and for the matching function to be increasing and concave in $u$ and $v$, which are desirable basic properties in a matching function, as described by Petrongolo and Pissarides (2001), Stevens (2007), and so on. However, $\psi > 1$ must be satisfied to $f_{s\theta} < 0$ because

$$f_{s\theta} = \chi\alpha(1-\alpha)(s\theta)^{\psi-1}(1-\psi)[\alpha s^{\psi} + (1-\alpha)\theta^{\psi}]^{\frac{1-2\psi}{\psi}}.$$

$\Phi$ can be rewritten as follows.

$$\Phi = \chi\alpha(1-\alpha)(s\theta)^{\psi-1}\left[\alpha s^{\psi} + (1-\alpha)\theta^{\psi}\right]^{\frac{1-2\psi}{\psi}}\Big((1-\psi)(1-\beta(1-\sigma)) - \psi\beta f\Big).$$

Figure 2A shows how $f_{s\theta}$ and $\Phi$ change depending on the value of $\psi$. In our calibration strategy, roughly when $\psi > 0.06$, $\Phi$ becomes negative. Again, note that here we assume that wages are fixed. When wages are not fixed (i.e., $0 \leq \lambda < 1$), the average wage becomes procyclical and this works in the direction that makes the benefit of being employed, $V$, procyclical. Therefore, when wages are

[2]Of course, other formulations are possible. Shimer (2004) uses "urn-ball matching function" with endogenous search intensity, where the job-finding rate is given as

$$f(s,\theta) = 1 - (1-\mu(\theta))^{s}, \text{ where } \mu(\theta) = \theta(1-e^{-1/\theta}).$$

In this formulation, $f_s$ is not a monotone function of $\theta$, and $f_s$ is decreasing in $\theta$ when the job-finding rate, $f(s,\theta)$, is sufficiently high.

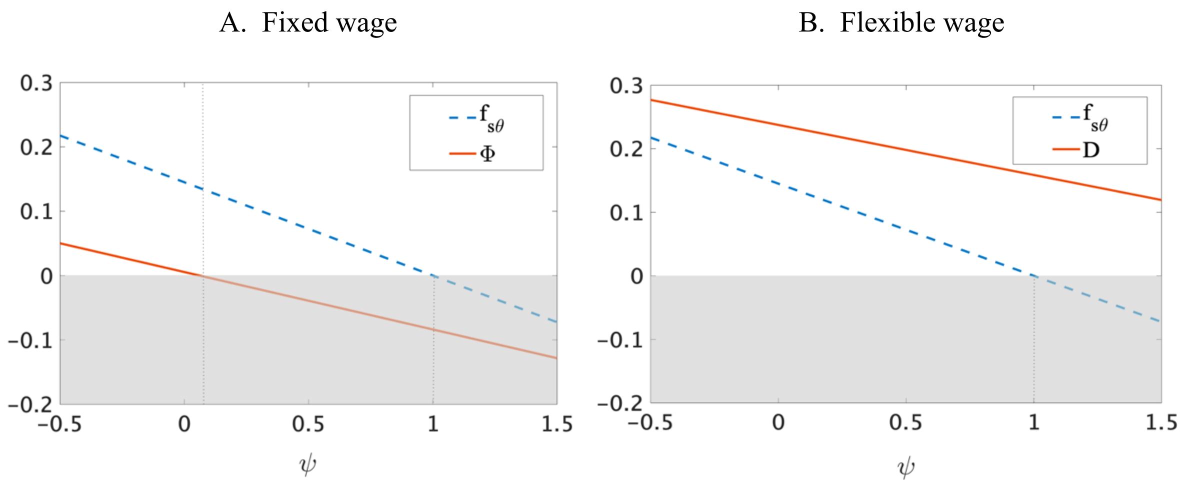


Figure 2: Responses of search intensity and $f_{s\theta}$ to the change in $\theta$

Note: The other parameters are shown in the next section, and $\bar{s}$ and $\bar{\theta}$ are set to 1. The sign of $\Phi$ and $D$ captures whether search intensity responds negatively or positively to the change in $\theta$.

not fixed, $\psi$ should be larger than the above value for search intensity to be countercyclical, but search intensity becomes countercyclical with $\psi < 1$ even when wages are not fixed, as shown in the quantitative exercises.

### 4.3 Search intensity in the steady state when wages are flexible

In this subsection, we examine the case of flexible wages using the CES matching function defined in (42). When wages are completely flexible, namely $\lambda = 0$, all employed workers receive the same renegotiated wage every period, $\Gamma_t = \gamma$, and $W(w_t^*, S_t) - U(S_t) = \tilde{E}[\,W(w_t, S_t)\,] - U(S_t)$. Thus, the wage satisfies

$$\gamma F(w_t, S_t) = (1-\gamma)\left(\tilde{E}[\,W(w_t, S_t)\,] - U(S_t)\right). \tag{43}$$

From (6), (10), and (43), the below equation holds

$$\frac{\kappa'(\bar{x})}{q(\bar{s}, \bar{\theta})} = \frac{1-\gamma}{\gamma}\frac{\omega'(\bar{s})}{f_s(\bar{s}, \bar{\theta})}. \tag{44}$$

Because $\bar{x}q(\bar{s}, \bar{\theta})$ is equal to $\sigma$, the relationship between search intensity and market tightness in the steady state is characterized by

$$\kappa'(\bar{x}) f_s(\bar{s}, \bar{\theta}) = \frac{1-\gamma}{\gamma}\omega'(\bar{s}) q(\bar{s}, \bar{\theta}), \quad \text{and}$$

$$\bar{x} = \frac{\sigma}{q(\bar{s}, \bar{\theta})}. \tag{45}$$

This leads to

$$\frac{d\bar{s}}{d\bar{\theta}} = \frac{\overbrace{\kappa'(\bar{x})\bar{f}_{s\theta} - \kappa''(\bar{x})\bar{f}_s \frac{\sigma \bar{q}_\theta}{\bar{q}^2}}^{>0} - \frac{1-\gamma}{\gamma}\omega'(\bar{s})\bar{q}_\theta}{\underbrace{\frac{1-\gamma}{\gamma}\left(\omega''(\bar{s})\bar{q} + \omega'(\bar{s})\bar{q}_s\right) - \kappa'(\bar{x})\bar{f}_{ss} + \kappa''(\bar{x})\bar{f}_s \frac{\sigma \bar{q}_s}{\bar{q}^2}}_{>0}}. \tag{46}$$

The above equation implies that $\bar{f}_{s\theta} < 0$ is not sufficient for search intensity to negatively respond to the change in market tightness, unlike the case of fixed wages. On the contrary, when $\bar{f}_{s\theta}$ is positive, $d\bar{s}/d\bar{\theta}$ always becomes positive. Figure 2B plots $\bar{f}_{s\theta}$ and $D$ for varying $\psi$, where $D$ is the numerator of the RHS in (46):

$$D \equiv \kappa'(\bar{x})\bar{f}_{s\theta} - \kappa''(\bar{x})\bar{f}_s \frac{\sigma \bar{q}_\theta}{\bar{q}^2} - \frac{1-\gamma}{\gamma}\omega'(\bar{s})\bar{q}_\theta.$$

$D < 0$ means that search intensity negatively responds to the change in market tightness. The figure implies that $\bar{f}_{s\theta}$ must be significantly negative for search intensity to be countercyclical.

# 5 Quantitative analysis

In this section, we quantitatively examine our model. First, we show how the labor market responds to a one percent negative productivity shock and how the impulse responses vary with the degree of wage rigidity or the form of matching functions. Next, we compare our model predictions with U.S. data and show that our model can reproduce both countercyclical search intensity and plausible labor market fluctuations compared to the standard models.

## 5.1 Calibration

We consider a monthly model. In our calibration, we target the steady state unemployment rate of $\bar{u} = 0.055$. Following Shimer (2005), we set $\beta$ to $0.988^{1/3}$ and $\sigma$ to 0.034; then, the job-finding rate in the steady state becomes 0.58. We also set $\alpha = 0.5$ so that the commonly used Cobb-Douglas form would be included—that is, $m_t = \chi(s_t u_t)^{0.5} v_t^{0.5}$ when $\psi \to 0$. Table 1 summarizes the calibrated and implied parameters, most of which are commonly used in the literature. We assume that the search cost function and vacancy cost function are given by $\omega(s_t) = \omega_0 s_t^2$ and $\kappa(x_t) = \kappa_0 x_t^2$, respectively, and set $\omega_0$, $\kappa_0$, and $\phi$ so that in the steady state, market tightness and search intensity are 1 and the net flow of

Table 1: The parameters for calibration

| Parameters | | Value | Sources |
|---|---|---|---|
| Calibrated parameters | | | |
| $\beta$ | discount factor | 0.996 | Shimer (2005) |
| $\sigma$ | separation rate | 0.034 | Shimer (2005) |
| $\lambda$ | wage rigidity | 11/12 | Gertler et al. (2020) |
| $\rho$ | autocorrelation of $z$ | 0.983 | Gertler et al. (2020) |
| Implied parameters | | | |
| $\kappa_0$ | slope of vacancy cost | 4.68 | |
| $\omega_0$ | slope of search cost | 0.13 | |
| $\phi$ | unemployment benefit | 0.53 | |
| $\chi$ | matching efficiency | 0.58 | |
| $\gamma$ | workers' bargaining power | 0.57 | |

unemployment is 0.4. $\gamma$ is set such that the effective bargaining power, $\Gamma$, becomes 0.5 in the steady state. We also set the degree of wage rigidity, $\lambda$, to $11/12$, following Gertler et al. (2020). This implies that the average duration of wages is one year.

### 5.2 Impulse response exercise

In this subsection, we show how the labor market in the model economy responds to one percent negative productivity shock and examine the effects of the degree of wage rigidity or the formulation of matching functions on the predicted labor market fluctuations. We assume that productivity follows the below AR(1) process and set the AR(1) parameter, $\rho$, to 0.983:

$$\ln z_{t+1} = \rho \ln z_t + \varepsilon_{t+1}, \qquad \varepsilon \sim N(0, \sigma_\varepsilon^2). \tag{47}$$

To better understand the result, we again show the two key equations below using $V(\cdot)$: the benefit of being employed and the first-order condition with respect to search intensity, respectively:

$$\omega'(s_t) = \beta f_s(s_t, \theta_t) E[\, V(\tilde{w}_{t+1}, S_{t+1}) \mid S_t \,] \quad \text{and} \tag{48}$$

$$\begin{aligned} V(\tilde{w}_t, S_t) =& \tilde{w}_t - \phi + \omega(s_t) + \beta\,(1 - \sigma - f(s_t, \theta_t))\, E[\, V(\tilde{w}_{t+1}, S_{t+1}) \mid S_t \,] \\ & + \beta(1-\sigma)\Big\{ E[\, W(w_{t+1}, S_{t+1}) \mid \tilde{w}_t, S_t \,] - \tilde{E}[\, W(w_{t+1}, S_{t+1}) \mid S_t \,] \Big\}, \end{aligned} \tag{49}$$

where $\tilde{w}$ denotes the average wage among new hires; the last term in (49) captures the difference in the expected value in period $t+1$ between new hires in period $t$ and those in period $t+1$, and this term

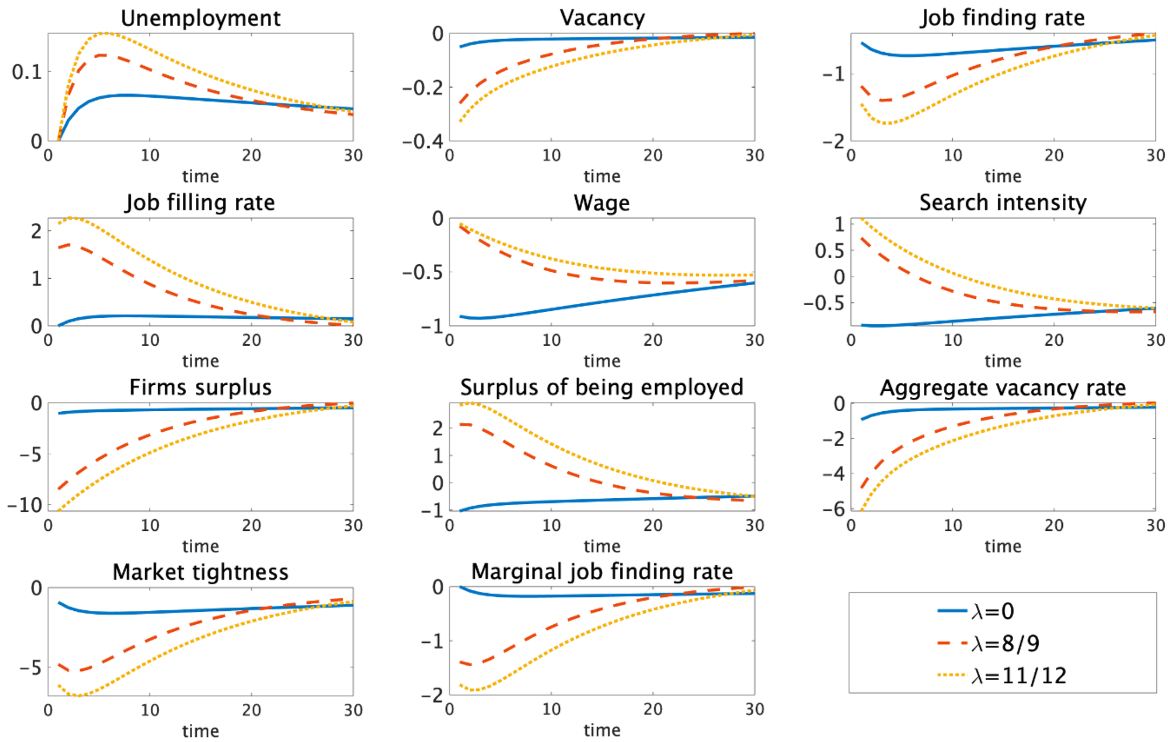


Figure 3: Effect of wage rigidity on the labor market

Note: Impulse response to one percent negative productivity shock for varying degree of wage rigidity. Unemployment, vacancy, job-finding rate, and job-filling rate are plotted in terms of their deviation from the steady state, and the others are plotted as percentage change from the steady state.

becomes 0 in the steady state.

In the below figures, unemployment rate, vacancy, job-finding rate, and job-filling rate are plotted in terms of their deviation from the steady state. For example, an increase from 3 to 6 percent of $\bar{u}$ is expressed as an increase of 3 and not $(\log 2) \times 100$. The remaining parameters are plotted in terms of the log deviation value from the steady state.

### 5.2.1 Effect of wage rigidity on labor market

First, we show how wage rigidity affects the cyclicality of search intensity and labor market fluctuations. We simulate the three cases, $\lambda = 0, 8/9$, and $11/12$.[3] $\lambda = 0$ and $8/9$ correspond to the cases where wages are renegotiated every month and three quarters, respectively. We set $\psi = 0.5$ in all three cases. Note that the wage and firms' surplus in the figures are average values. The result is shown in Figure 3, and this shows that the more rigid the wages are, the larger the labor market fluctuations are and the more unemployed workers increase their search intensity in response to the shock. The intuitive explanation

[3] In the case of $\lambda = 0$ and $\lambda = 8/9$, $\gamma$ is set to 0.5 and 0.55, respectively, so that $\Gamma$ would be 0.5 in the steady state.

of this result is as follows. When wages are sticky, they remain relatively high in response to a negative productivity shock for a certain period. Therefore, the firms' average surplus significantly decreases, making them reluctant to post vacancies; this lowers market tightness, $\theta$, and the job-finding rate, $f$. On the contrary, a relatively high wage and low job-finding rate enhance the benefit of being employed, $V$.

As mentioned in the previous section and implied by (49), $V$ is discounted by $\beta(1-\sigma-f)$: that is, low job-finding rate improves the benefit of being employed. The marginal job-finding rate, $f_s$, is increasing in $\theta$; therefore, $f_s$ decreases, as we use the CES matching function with $\psi < 1$. As (48) shows, search intensity is determined by $V$ and $f_s$. When wages are sufficiently sticky, the increase in $V$ is larger than the decrease in $f_s$, leading unemployed workers to increase their search efforts; although finding a job is getting more difficult, the benefits of being employed are sufficiently large owing to wage rigidity, which increases the total search benefit, $f_s V$. Note that the job-finding rate gradually recovers to the pre-shock level and wages reflect the actual economic condition. When the wages reflect the actual economic condition and the job-finding rate sufficiently recovers, $V$ becomes lower than that at the pre-shock level and unemployed workers reduce their search intensity to less than that at the pre-shock level. In other words, unemployed workers increase search intensity only for the periods when wages remain high; after these periods, they maintain the search intensity at a lower level than that before the shock until the productivity recovers.

### 5.2.2 Effect of matching function form on labor market

Next, we examine how the formulation of the matching function affects the cyclicality of search intensity and labor market fluctuations. We analyze three cases, $\psi = 0.8, \psi = 0.5$, and $\psi = 0$, and set $\lambda = 11/12$ in all three cases. Figure 4 shows the result. The key to this result is the marginal job-finding rate, $f_s$. The change in $f_s$ in response to the change in $\theta$, $f_{s\theta}$, can be rewritten as follows.

$$f_{s\theta} = \chi\alpha(1-\alpha)(s\theta)^{\psi-1}(1-\psi)\left[\alpha s^{\psi} + (1-\alpha)\theta^{\psi}\right]^{\frac{1-2\psi}{\psi}}, \qquad \psi < 1.$$

The above equation implies that when $\psi$ is small, the marginal job-finding rate, $f_s$, is sensitive to the change in market tightness, as also shown in the figure. In the case of $\psi = 0$, the decrease in $f_s$ is larger than the increase in $V$; thus, unemployed workers decrease their search intensity. In other words, during recession, unemployed workers are reluctant to search for a job because an additional search intensity does little to increase the job-finding rate, whereas the benefit of being employed increases owing to wage rigidity. As the figure shows, this procyclical search intensity amplifies the increase in the


Figure 4: Effect of matching function form on the labor market

Note: Impulse response to one percent negative productivity shock for varying $\psi$. Unemployment, vacancy, job-finding rate, and job-filling rate are plotted in terms of their deviation from the steady state, and the others are plotted as percentage change from the steady state.

unemployment rate or the decrease in market tightness. On the contrary, when $f_s$ is not very responsive to market tightness, as in the case of $\psi = 0.8$ and 0.5, unemployed workers increase their search intensity, and this countercyclical search intensity dampens the increase in the unemployment rate or the decrease in market tightness. As the figure shows, $\psi$ has little effect on wage or vacancy. (17) implies that the job-finding rate and the level of search intensity affect $w_t^*$, but the net effect is not large. Moreover, the average wage reflects the bargaining wage considerably slowly owing to wage rigidity; therefore, the behaviors of average wage, firms' average surplus, and vacancy are quite similar in all three cases.

### 5.2.3 Effect of endogenous search intensity on labor market

Finally, we examine the role of endogenous search intensity in explaining labor market fluctuations. For this purpose, we compare the three cases; the first is the case where search intensity is endogenous and wages are sticky (case1), the second is the case where search intensity is fixed and wages are sticky (case2), and the third is the case where search intensity is fixed and wages are flexible (case3). We make this comparison for $\psi = 0$ and $\psi = 0.8$.

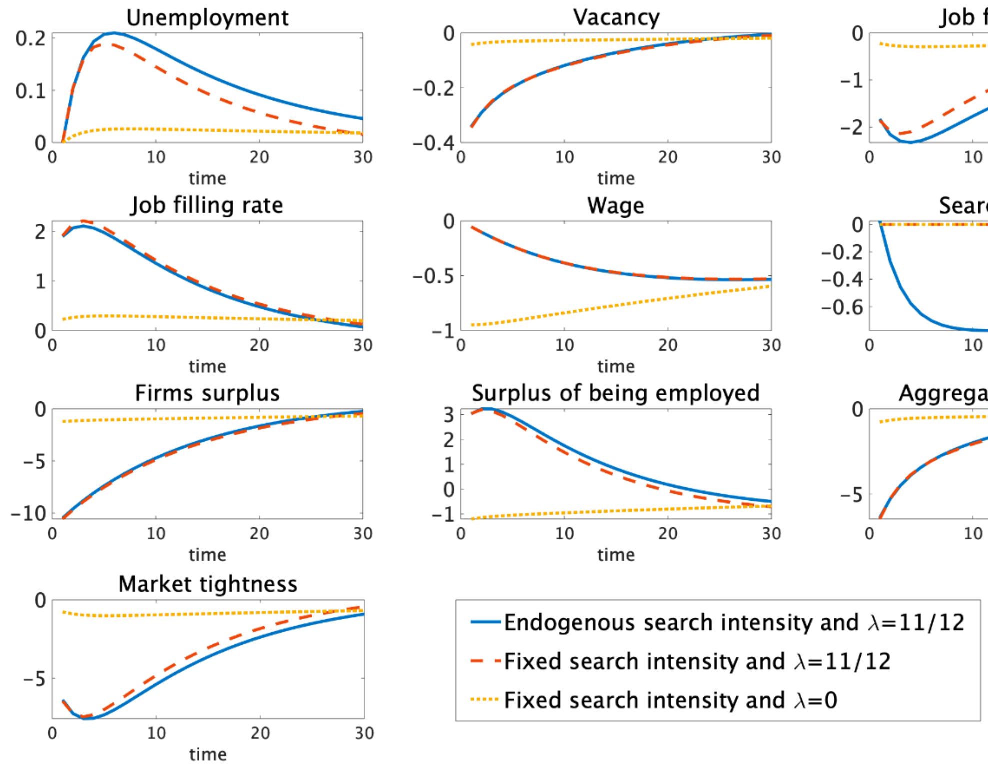


Figure 5: Comparison of the benchmark model and standard models when $\psi = 0$

Note: Impulse response to one percent negative productivity shock for the standard and benchmark models. Unemployment, vacancy, job-finding rate, and job-filling rate are plotted in terms of their deviation from the steady state, and the others are plotted as percentage change from the steady state.

In Figure 5, we set $\psi$ to 0. Comparison of case2 (dashed line) with case3 (dotted line) shows that wage rigidity significantly amplifies the labor market fluctuations; this finding is consistent with previous research such as Hall (2005) and Shimer (2005). As shown earlier, $\psi = 0$ makes search intensity procyclical. As case1 (solid line) and case2 (dashed line) show, endogenous search intensity additionally amplifies the increase in the unemployment rate or the decrease in the job-finding rate. In Figure 6, we set $\psi$ to 0.8. As before, the figure shows that wage rigidity amplifies the labor market fluctuations. Contrary to the previous case, however, when search intensity is endogenous, unemployed workers increase their search intensity in response to the negative productivity shock because of $\psi = 0.8$. For a while after the shock, endogenous search intensity dampens the labor market fluctuations, but a few moments after the shock, wages begin to reflect the economic condition, causing unemployed worker to decrease their search intensity to less than that at the pre-shock level and maintain this lower level until the productivity recovers. In the latter phase, endogenous search intensity delays the recovery of the unemployment rate or the job-finding rate. As the figure shows, even if search intensity is countercyclical, labor market fluctuations are still large compared with the standard model (case3).

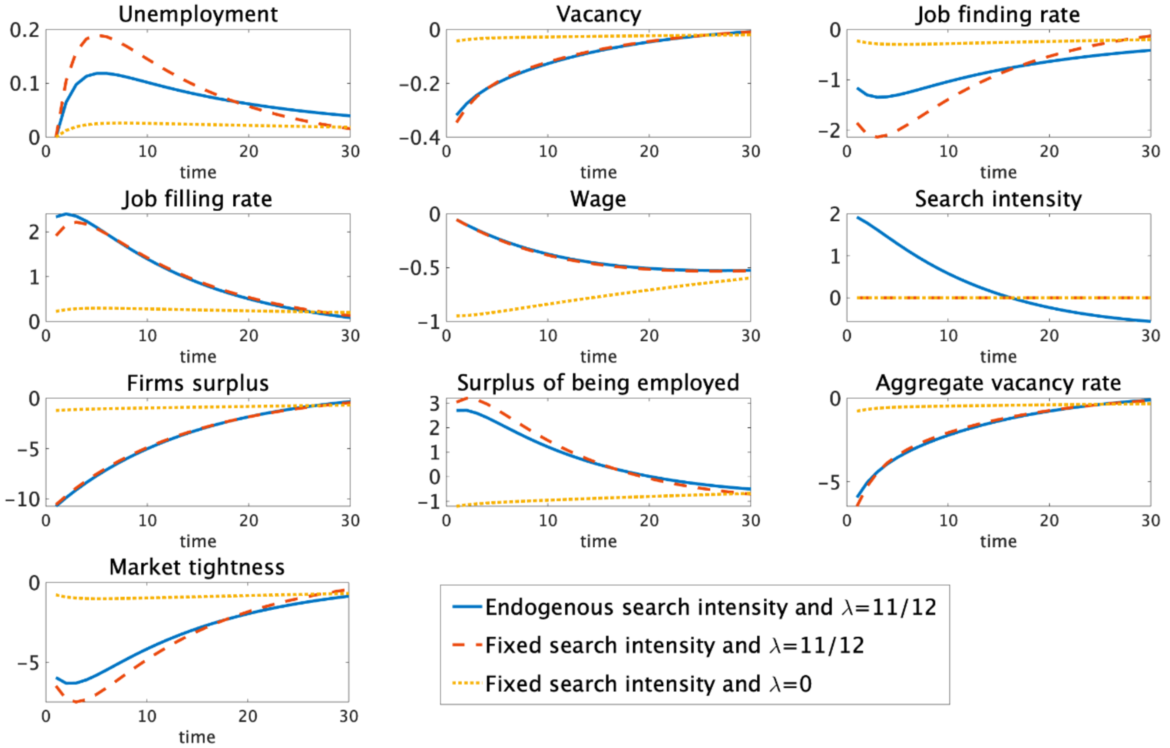


Figure 6: Comparison of the benchmark model and standard models when $\psi = 0.8$

Note: Impulse response to one percent negative productivity shock for the standard and benchmark models. Unemployment, vacancy, job-finding rate, and job-filling rate are plotted in terms of their deviation from the steady state, and the others are plotted as percentage change from the steady state.

### 5.3 Comparison with actual data

Finally, we evaluate how successful our model is in reproducing the labor market fluctuations. We compare the model predictions with monthly U.S. data from 1985 to 2019. Table 2 shows the U.S. data and the predictions on the labor market obtained using our model and the standard model, where search intensity is constant, wages are flexibly determined ($\lambda = 0$), and the matching function is Cobb-Douglas ($\psi = 0$). All variables are logged and HP filtered with smoothing parameter 14,400 for monthly frequency. As before, we assume that $z$ follows (47) and set $\sigma_\varepsilon$ to 0.007, following Gertler et al. (2020).

Table 2 shows that the assumption of wage rigidity captures the wage dynamics relatively well (the theoretical standard deviation is 0.37 and that in the data is 0.42), although it overshoots the autocorrelation. Overall, our model reproduces labor market fluctuations reasonably well compared to the standard model. As explained in the previous subsection, when $\psi = 0$, search intensity becomes procyclical (the correlation with $\theta$ is 0.62), and when $\psi = 0.5$, it becomes countercyclical (the correlation is $-0.87$). The table shows that in the case of $\psi = 0.5$, countercyclical search intensity slightly dampens the labor market fluctuations, but the model still captures the labor market dynamics well compared

Table 2: The predictions of the models and actual data (Monthly)

| | $\bar{u}$ | $\bar{v}$ | $\theta$ | $w$ |
|---|---|---|---|---|
| U.S. data | | | | |
| $std \times 100$ | 5.39 | 7.55 | 12.79 | 0.42 |
| autocorrelation | 0.89 | 0.74 | 0.87 | 0.87 |
| Model ($\lambda = 11/12$, $\psi = 0$) | | | | |
| $std \times 100$ | 4.75 | 6.57 | 10.05 | 0.37 |
| autocorrelation | 0.94 | 0.76 | 0.90 | 0.98 |
| correlation with $s$ | −0.89 | 0.30 | 0.62 | 0.83 |
| Model ($\lambda = 11/12$, $\psi = 0.5$) | | | | |
| $std \times 100$ | 3.47 | 6.40 | 8.93 | 0.36 |
| autocorrelation | 0.94 | 0.77 | 0.88 | 0.98 |
| correlation with $s$ | 0.47 | −0.95 | −0.87 | 0.23 |
| Standard model ($\lambda = 0$, $\psi = 0$) | | | | |
| $std \times 100$ | 0.54 | 0.79 | 1.21 | 1.16 |
| autocorrelation | 0.94 | 0.77 | 0.90 | 0.84 |

Note: $\bar{u}$ is the seasonally adjusted unemployment for individuals aged 16 years and over, and $\bar{v}$ is the job opening level in the non-farm sector (Job Openings and Labor Turnover Survey). $\theta$ is calculated by diving the job opening level in the non-farm sector by the unemployment level, and this value is calculated from 2001 to 2019 owing to the unavailability of data on the job opening level before 2000. Wages are measured by the average hourly earnings of a production and nonsupervisory employee in the Current Employment Statistics survey and deflated with the PCE.

to the standard model. In sum, when wages are rigid, search intensity could be both procyclical and countercyclical under the matching functions that satisfy the standard properties. Moreover, even when search intensity is countercyclical, the labor market fluctuations are large.

## 6 Concluding Remarks

This paper extends the standard DMP model by incorporating endogenous search intensity and wage rigidity to address the inconsistency between the canonical models and the empirical evidence presented in previous studies: the theoretical models predict that search intensity is procyclical, but much empirical evidence suggests that search intensity is countercyclical or acyclical. The paper proposes a new mechanism that makes search intensity countercyclical. In the model, the assumption of wage rigidity makes the net benefit of being employed as well as total marginal search benefit countercyclical, even when the marginal job-finding rate is procyclical. Further, quantitative exercises show that the labor market fluctuations of the model economy are still large even when search intensity is countercyclical.

The model makes a simple assumption about the unemployment benefit: unemployed workers are assumed to receive constant and identical income in every period. This assumption makes the benefit of being employed uniform across all unemployed workers. In reality, however, the benefit of being employed varies for each unemployed workers, depending on various factors such as the eligibility of unemployment insurance, assets holdings, and household composition (e.g., marital status and spouse's employment status). Relaxing this assumption and introducing heterogeneity in unemployment status would provide rich insights into not only the individual job search behavior but also the composition change of unemployed workers, which some literature such as Gomme and Lkhagvasuren (2015) and Mukoyama et al. (2018) argue is an important factor in explaining the fluctuations of aggregate search intensity. Therefore, future research should incorporate such heterogeneity and examine job search behavior in a more general framework.

# Appendix

## A Renegotiated wage $w_t^*$

For the firms and workers that renegotiate wages in period $t$, the expected wages in future periods are as follows:

$$\begin{aligned} E_t w_{t+1} &= (1-\lambda) E_t w_{t+1}^* + \lambda w_t^* \\ E_t w_{t+2} &= (1-\lambda) E_t w_{t+2}^* + (1-\lambda)\lambda E_t w_{t+1}^* + \lambda^2 w_t^* \\ &\vdots \\ E_t w_{t+s} &= (1-\lambda) E_t \sum_{m=1}^{s} \lambda^{s-m} w_{t+m}^* + \lambda^s w_t^*. \end{aligned} \tag{A.1}$$

The discounted sum of expected future wage, $\mathcal{W}_t$, is given as follows:

$$\begin{aligned} \mathcal{W}_t =& w_t^* + E_t \sum_{s=1}^{\infty} \beta^s (1-\sigma)^s w_{t+s} \\ =& \left(1 + \beta\lambda(1-\sigma) + \beta^2\lambda^2(1-\sigma)^2 + \beta^3\lambda^3(1-\sigma)^3 + \cdots\right) w_t^* + \\ & \beta(1-\sigma)(1-\lambda)\left(1 + \beta\lambda(1-\sigma) + \beta^2\lambda^2(1-\sigma)^2 + \cdots\right) E_t w_{t+1}^* + \\ & \vdots \\ =& \Delta^{-1} w_t^* + (1-\lambda)\Delta^{-1} E_t \sum_{m=1}^{\infty} \beta^m (1-\sigma)^m w_{t+m}^*, \end{aligned} \tag{A.2}$$

where $\Delta \equiv 1-\beta\lambda(1-\sigma)$. $H(w_t^*, S_t) \equiv W(w_t^*, S_t) - U(S_t)$ and $F(w_t^*, S_t)$ can be rewritten[4], respectively, as

$$\begin{aligned} H(w_t^*, S_t) =& \mathcal{W}_t + E_t \sum_{m=0}^{\infty} \beta^m (1-\sigma)^m \left(-h_{t+m}\right) \quad \text{and} \\ F(w_t^*, S_t) =& -\mathcal{W}_t + E_t \sum_{m=0}^{\infty} \beta^m (1-\sigma)^m \Big( z_{t+m} + \kappa_0 x_{t+m}^2 \left(w_{t+m} \mid w_t^*\right)\Big), \end{aligned}$$

[4]For simplicity, we denote $E_t\left[x_{t+m}(w_{t+m}, S_{t+m}) \mid w_t = w_t^*\right]$ by $E_t x_{t+m}(w_{t+m} \mid w_t^*)$.

where $h_{t+m} = \phi - \omega(s_{t+m}) + \omega'(s_{t+m}) f(s_{t+m}, \theta_{t+m}) / f_s(s_{t+m}, \theta_{t+m})$. Substituting these into (18), that is, $\Gamma_t F(w_t^*, S_t) = (1-\Gamma_t) H(w_t^*, S_t)$, we get

$$\mathcal{W}_t = E_t \sum_{m=0}^{\infty} \beta^m (1-\sigma)^m \left\{ \Gamma_t \left( z_{t+m} + \kappa_0 x_{t+m}^2 \left( w_{t+m} \mid w_t^* \right) \right) + (1-\Gamma_t) h_{t+m} \right\}. \tag{A.3}$$

We can rewrite the above equation as

$$\begin{aligned} \mathcal{W}_t =& \Gamma_t z_t + (1-\Gamma_t) h_t + E_t \left\{ \Gamma_t \sum_{m=0}^{\infty} \beta^m (1-\sigma)^m \kappa_0 x_{t+m}^2 \left( w_{t+m} \mid w_t^* \right) \right\} \\ &+ E_t \sum_{m=1}^{\infty} \beta^m (1-\sigma)^m \left\{ \Gamma_t z_{t+m} + (1-\Gamma_t) h_{t+m} \right\}. \end{aligned}$$

Factoring out the last terms with $\beta(1-\sigma)$ yields[5]

$$\begin{aligned} \mathcal{W}_t =& \Gamma_t z_t + (1-\Gamma_t) h_t + E_t \left\{ \Gamma_t \sum_{m=0}^{\infty} \beta^m (1-\sigma)^m \kappa_0 x_{t+m}^2 \left( w_{t+m} \mid w_t^* \right) \right\} + \beta(1-\sigma) \\ & E_t \left[ \sum_{m=0}^{\infty} \beta^m (1-\sigma)^m \left\{ \Gamma_{t+1} \left( z_{t+m+1} + \kappa_0 x_{t+m+1}^2 \left( w_{t+m+1} \mid w_{t+1}^* \right) \right) + (1-\Gamma_{t+1}) h_{t+m+1} \right. \right. \\ & \left. \left. + (\Gamma_t - \Gamma_{t+1}) \left( z_{t+m+1} - h_{t+m+1} \right) - \Gamma_{t+1} \kappa_0 x_{t+m+1}^2 \left( w_{t+m+1} \mid w_{t+1}^* \right) \right\} \right]. \end{aligned}$$

Based on $(A.3)$, the second line is equal to $E_t[\mathcal{W}_{t+1}]$, and $(A.2)$ implies $\mathcal{W}_t - \beta(1-\sigma) E_t[\mathcal{W}_{t+1}] = \Delta^{-1} w_t^* - (1-\Delta) \Delta^{-1} E_t w_{t+1}^*$. Then,

$$\begin{aligned} \Delta^{-1} w_t^* =& (1-\Delta) \Delta^{-1} E_t w_{t+1}^* + \Gamma_t \left( z_t + \kappa_0 x_t^2 (w_t^*, S_t) \right) + (1-\Gamma_t) h_t \\ &+ \kappa_0 \sum_{m=1}^{\infty} \beta^m (1-\sigma)^m E_t \left\{ \Gamma_t x_{t+m}^2 \left( w_{t+m} \mid w_t^* \right) - \Gamma_{t+1} x_{t+m}^2 \left( w_{t+m} \mid w_{t+1}^* \right) \right\} \\ &+ \beta(1-\sigma) E_t \left\{ (\Gamma_t - \Gamma_{t+1}) \sum_{m=0}^{\infty} \beta^m (1-\sigma)^m \left( z_{t+m+1} - h_{t+m+1} \right) \right\} \\ \equiv& (1-\Delta) \Delta^{-1} E_t w_{t+1}^* + w_t^N + \left( \mu_t(w_t^*) - \mu_t(w_{t+1}^*) \right) + \beta(1-\sigma) E_t \left\{ \Gamma_t Q_{t+1} - \Gamma_{t+1} Q_{t+1} \right\}, \end{aligned} \tag{A.4}$$

where

$$w_t^N = \Gamma_t \left( z_t + \kappa_0 x_t^2 (w_t^*, S_t) \right) + (1-\Gamma_t) h_t, \tag{A.5}$$

$$\mu_t(w_t^*) = \kappa_0 \sum_{m=1}^{\infty} \beta^m (1-\sigma)^m E_t \left\{ \Gamma_t x_{t+m}^2 \left( w_{t+m} \mid w_t^* \right) \right\}, \tag{A.6}$$

$$\mu_t(w_{t+1}^*) = \kappa_0 \sum_{m=1}^{\infty} \beta^m (1-\sigma)^m E_t \left\{ \Gamma_{t+1} x_{t+m}^2 \left( w_{t+m} \mid w_{t+1}^* \right) \right\}, \quad \text{and} \tag{A.7}$$

$$Q_t = \sum_{m=0}^{\infty} \beta^m (1-\sigma)^m \left( z_{t+m} - h_{t+m} \right). \tag{A.8}$$

[5]We denote $E\left[ x_{t+m}(w_{t+m}, S_{t+m}) \mid w_{t+1} = w_{t+1}^* \right]$ by $E\left[ x_{t+m} \left( w_{t+m} \mid w_{t+1}^* \right) \right]$.

# B Log-linearization

In this section, we first describe the log-linear model and then its derivations. Let ˆ denote the log deviation from the steady state.

## B.1 Log-linear model

$(-1)$ and $(+1)$ denote the previous period and the next period, respectively.

- matching function

$$\hat{m} = \frac{\alpha(\bar{s}\bar{u})^{\psi}(\hat{s}+\hat{u}) + (1-\alpha)\bar{v}^{\psi}\hat{v}}{\alpha(\bar{s}\bar{u})^{\psi} + (1-\alpha)\bar{v}^{\psi}} \tag{B.1}$$

- job-finding rate and marginal job-finding rate

$$\hat{f} = \frac{\alpha\bar{s}^{\psi}\hat{s} + (1-\alpha)\bar{\theta}^{\psi}\hat{\theta}}{\alpha\bar{s}^{\psi} + (1-\alpha)\bar{\theta}^{\psi}} \tag{B.2}$$

$$\hat{f}_s = (\psi-1)\hat{s} + \frac{(1-\psi)\left(\alpha\bar{s}^{\psi}\hat{s} + (1-\alpha)\bar{\theta}^{\psi}\hat{\theta}\right)}{\alpha\bar{s}^{\psi} + (1-\alpha)\bar{\theta}^{\psi}} \tag{B.3}$$

- job-filling rate

$$\hat{q} = \hat{m} - \hat{v} \tag{B.4}$$

- market tightness

$$\hat{\theta} = \hat{v} - \hat{u} \tag{B.5}$$

- unemployed and employed workers

$$\bar{u}\hat{u} = -\bar{n}\hat{n} \tag{B.6}$$

$$\bar{n}\hat{n} = (1-\sigma)\bar{n}\hat{n}(-1) + \bar{m}\hat{m}(-1) \tag{B.7}$$

- average vacancy

$$\hat{v} = \hat{n} + \hat{\bar{x}} \tag{B.8}$$

- average vacancy rate

$$\hat{\bar{x}} = \hat{q} + E_t\bar{F}^{-1}\Big(\bar{z}\hat{z}(+1) - \bar{w}\hat{w}(+1)\Big) + \beta E_t\hat{\bar{x}}(+1) - \beta(1-\sigma)E_t\hat{q}(+1) \tag{B.9}$$

- average wage

$$\hat{w} = (1-\lambda)\hat{w}^* + \lambda\hat{w}(-1) \tag{B.10}$$

- bargaining wage

$$\hat{w}^* = \tilde{\Delta}\hat{w}^N + (1-\tilde{\Delta})E_t\hat{w}^*(+1) + \Lambda\Big(\hat{\Gamma} - E_t\hat{\Gamma}(+1)\Big), \tag{B.11}$$

$$\text{where}\quad \tilde{\Delta} = \Big(1-\beta\lambda(1-\sigma)\Big)\left(1 + \frac{(\beta\lambda)^2\sigma(1-\sigma)\bar{\Gamma}}{1-\beta\lambda}\right)^{-1} \quad \text{and}$$

$$\Lambda = \frac{\tilde{\Delta}\beta(1-\sigma)\bar{\Gamma}\left(\bar{z} - \bar{h} + \kappa_0\bar{x}^2\right)(\bar{w})^{-1}}{1-\beta(1-\sigma)}$$

- Nash bargaining wage

$$\bar{w}\hat{w}^N = \bar{\Gamma}\left(\bar{z}\hat{z} + 2\kappa_0\bar{x}^2\hat{x}^*\right) + (1-\bar{\Gamma})\left(-2\omega_0\bar{s}^2\hat{s} + \frac{\bar{\omega}'\bar{f}}{\bar{f}_s}\left(\hat{s} + \hat{f} - \hat{f}_s\right)\right)$$

$$+\bar{\Gamma}\left(\bar{z}+\kappa_0\bar{x}^2-\phi+\omega_0\bar{s}^2-\frac{\bar{\omega}'\bar{f}}{\bar{f}_s}\right)\hat{\Gamma} \tag{B.12}$$

- bargaining power

$$\hat{\Gamma}=-(1-\bar{\Gamma})\hat{F}_w \quad \text{and} \tag{B.13}$$

$$\hat{F}_w=\beta\bar{x}\bar{q}\lambda(\hat{x}^*+\hat{q})+\beta\lambda E_t\hat{F}_w(+1)+\left\{\left(\frac{\beta\lambda}{1-\beta\lambda}\right)^2\left(\bar{w}\lambda\sigma\bar{F}^{-1}\right)\right\}\left(E_t\hat{w}^*(+1)-\hat{w}^*\right) \tag{B.14}$$

- vacancy rate for renegotiated firms

$$\begin{aligned}\hat{x}^*=\hat{q}+E_t\bar{F}^{-1}\Big(\bar{z}\hat{z}(+1)-\bar{w}\big[\hat{w}^*(+1)+\frac{\lambda}{1-\beta\lambda}(\hat{w}^*-\hat{w}^*(+1))\big]\Big)\\ -\beta(1-\sigma)E_t\hat{q}(+1)+\beta E_t\hat{x}^*(+1)\end{aligned} \tag{B.15}$$

- average value of firms

$$\bar{F}\hat{F}=\bar{z}\hat{z}-\bar{w}\hat{w}+2\kappa_0\bar{x}^2\hat{\bar{x}}+\beta(1-\sigma)\bar{F}E_t\hat{F}(+1) \tag{B.16}$$

- expected surplus of being employed

$$\bar{V}\hat{V}=\bar{w}\hat{w}+2\bar{\omega}\hat{s}+\beta(1-\sigma)\bar{V}E_t\hat{V}(+1)-\beta\bar{f}\bar{V}\left(\hat{f}+E_t\hat{V}(+1)\right) \tag{B.17}$$

- search intensity

$$\hat{s}=\hat{f}_s+E_t\hat{V}(+1) \tag{B.18}$$

## B.2 Renegotiated wage and its marginal effect

In this subsection, we derive $\hat{w}^*$ and $\hat{F_w}$, namely $(B.11)$ and $(B.14)$. First, we obtain the following equation by log-linearizing $(A.4)$.

$$\begin{aligned}\bar{w}\hat{w}_t^*=(1-\Delta)\bar{w}E_t\hat{w}_{t+1}^*+\Delta\bar{w}\hat{w}_t^N+\Delta\bar{\mu}\left(\hat{\mu}_t(w_t^*)-\hat{\mu}_t(w_{t+1}^*)\right)\\ +\Delta\beta(1-\sigma)\bar{\Gamma}\bar{Q}\left(\hat{\Gamma}_t-E_t\hat{\Gamma}_{t+1}\right),\end{aligned} \tag{B.19}$$

where

$$\Delta\beta(1-\sigma)\bar{\Gamma}\bar{Q}=\frac{\Delta\beta(1-\sigma)\bar{\Gamma}(\bar{z}-\bar{h})}{1-\beta(1-\sigma)}. \tag{B.20}$$

For log-linearizing $\mu_t(w_t^*)-\mu_t(w_{t+1}^*)$, we first calculate $\hat{x}_s\left(w_t^*\right)-\hat{x}_s\left(w_{t+1}^*\right)$, where $s\geq t+1$[6]. From (10), $x_s(w_t^*)$ and $x_s(w_{t+1}^*)$ satisfy,

$$\begin{aligned}2\kappa_0x_s(w_t^*)=\beta q_sE_s\left\{(1-\lambda)F(w_{s+1}^*,S_{s+1})+\lambda F(w_t^*,S_{s+1})\right\} \quad \text{and}\\ 2\kappa_0x_s(w_{t+1}^*)=\beta q_sE_s\left\{(1-\lambda)F(w_{s+1}^*,S_{s+1})+\lambda F(w_{t+1}^*,S_{s+1})\right\}.\end{aligned}$$

These imply

$$\begin{aligned}x_s(w_t^*)-x_s(w_{t+1}^*)=\frac{\beta\lambda q_s}{2\kappa_0}E_s\left\{F(w_t^*,S_{s+1})-F(w_{t+1}^*,S_{s+1})\right\}=\frac{\beta\lambda q_s}{2\kappa_0}\times\\ E_s\Big\{w_{t+1}^*-w_t^*+\kappa_0\left(x_{s+1}^2(w_t^*)-x_{s+1}^2(w_{t+1}^*)\right)+\frac{2(1-\sigma)\kappa_0}{q_{s+1}}\left(x_{s+1}(w_t^*)-x_{s+1}(w_{t+1}^*)\right)\Big\}.\end{aligned}$$

[6]In this subsection, we denote $x_t(w_s^*,S_t)$ by $x_t(w_s^*)$.

Log-linearizing the above yields

$$\hat{x}_s(w_t^*) - \hat{x}_s(w_{t+1}^*) = \lambda\bar{w}\bar{F}^{-1}\left(\hat{w}_{t+1}^* - \hat{w}_t^*\right) + \beta\lambda E_s\left(\hat{x}_{s+1}(w_t^*) - \hat{x}_{s+1}(w_{t+1}^*)\right)$$
$$= \frac{\lambda\bar{w}\bar{F}^{-1}}{1-\beta\lambda}\left(\hat{w}_{t+1}^* - \hat{w}_t^*\right), \tag{B.21}$$

where $\bar{F}^{-1} = \beta\bar{q}/(2\kappa_0\bar{x})$. $(A.6)$ can be rewritten as follows:

$$\frac{\mu_t\left(w_t^*\right)}{\kappa_0} = \beta(1-\sigma)E_t\left\{(1-\lambda)\Gamma_t x_{t+1}^2(w_{t+1}^*) + \lambda\Gamma_t x_{t+1}^2(w_t^*)\right\}$$
$$+\beta^2(1-\sigma)^2E_t\left\{(1-\lambda)\Gamma_t x_{t+2}^2(w_{t+2}^*) + (1-\lambda)\lambda\Gamma_t x_{t+2}^2(w_{t+1}^*) + \lambda^2\Gamma_t x_{t+2}^2(w_t^*)\right\}$$
$$+\beta^3(1-\sigma)^3E_t\left\{(1-\lambda)\Gamma_t x_{t+3}^2(w_{t+3}^*) + (1-\lambda)\lambda\Gamma_t x_{t+3}^2(w_{t+2}^*) + (1-\lambda)\lambda^2\Gamma_t \cdots\right.$$

Similarly, $(A.7)$ can be rewritten as

$$\frac{\mu_t\left(w_{t+1}^*\right)}{\kappa_0} = \beta(1-\sigma)E_t\left\{\Gamma_{t+1}x_{t+1}^2(w_{t+1}^*)\right\}$$
$$+\beta^2(1-\sigma)^2E_t\left\{(1-\lambda)\Gamma_{t+1}x_{t+2}^2(w_{t+2}^*) + \lambda\Gamma_{t+1}x_{t+2}^2(w_{t+1}^*)\right\}$$
$$+\beta^3(1-\sigma)^3E_t\left\{(1-\lambda)\Gamma_{t+1}x_{t+3}^2(w_{t+3}^*) + (1-\lambda)\lambda\Gamma_{t+1}x_{t+3}^2(w_{t+2}^*) + \lambda^2\Gamma_{t+1}x_{t+3}^2(w_{t+1}^*)\right\} + \cdots$$

Then,

$$\frac{\mu_t(w_t^*) - \mu_t(w_{t+1}^*)}{\kappa_0} = \beta(1-\sigma)E_t\left\{\tilde{\Gamma}x_{t+1}^2(w_{t+1}^*) + \lambda\Gamma_t\left(x_{t+1}^2(w_t^*) - x_{t+1}^2(w_{t+1}^*)\right)\right\}$$
$$+\beta^2(1-\sigma)^2E_t\left\{(1-\lambda)\tilde{\Gamma}x_{t+2}^2(w_{t+2}^*) + \lambda\tilde{\Gamma}x_{t+2}^2(w_{t+1}^*) + \lambda^2\Gamma_t\left(x_{t+2}^2(w_t^*) - x_{t+2}^2(w_{t+1}^*)\right)\right\}$$
$$+\beta^3(1-\sigma)^3E_t\left\{(1-\lambda)\tilde{\Gamma}x_{t+3}^2(w_{t+3}^*) + (1-\lambda)\lambda\tilde{\Gamma}x_{t+3}^2(w_{t+2}^*) + \lambda^2\tilde{\Gamma}x_{t+3}^2(w_{t+1}^*) + \cdots,\right.$$

where $\tilde{\Gamma} \equiv \Gamma_t - \Gamma_{t+1}$. By log-linearizing the above and substituting $(B.21)$ into it, we get

$$\bar{\mu}\left(\hat{\mu}_t(w_t^*) - \hat{\mu}_t(w_{t+1}^*)\right) = \frac{\beta(1-\sigma)\bar{\Gamma}\kappa_0\bar{x}^2}{1-\beta(1-\sigma)}\left(\hat{\Gamma}_t - E_t\hat{\Gamma}_{t+1}\right)$$
$$+\frac{2\bar{\Gamma}\kappa_0\bar{x}^2\beta\lambda(1-\sigma)}{1-\beta\lambda(1-\sigma)}\left\{\frac{\lambda\bar{w}\bar{F}^{-1}}{1-\beta\lambda}\left(E_t\hat{w}_{t+1}^* - \hat{w}_t^*\right)\right\}, \tag{B.22}$$

where

$$\frac{2\bar{\Gamma}\kappa_0\bar{x}^2\beta\lambda(1-\sigma)}{1-\beta\lambda(1-\sigma)}\frac{\lambda\bar{w}\bar{F}^{-1}}{1-\beta\lambda} = \Delta^{-1}\frac{(\beta\lambda)^2\sigma(1-\sigma)\bar{\Gamma}\bar{w}}{1-\beta\lambda}. \tag{B.23}$$

Substituting $(B.22)$ into $(B.19)$ yields

$$\hat{w}_t^* = (1-\Delta)E_t\hat{w}_{t+1}^* + \Delta\hat{w}_t^N + \frac{\Delta\beta(1-\sigma)\bar{\Gamma}\left(\bar{z} - \bar{h} + \kappa_0\bar{x}^2\right)(\bar{w})^{-1}}{1-\beta(1-\sigma)}\left(\hat{\Gamma}_t - E_t\hat{\Gamma}_{t+1}\right)$$
$$+\frac{(\beta\lambda)^2\sigma(1-\sigma)\bar{\Gamma}}{1-\beta\lambda}\left(E_t\hat{w}_{t+1}^* - \hat{w}_t^*\right).$$

Rearranging this, we get

$$\hat{w}_t^* = \tilde{\Delta}\hat{w}_t^N + (1-\tilde{\Delta})E_t\hat{w}_{t+1}^* + \Lambda\left(\hat{\Gamma}_t - E_t\hat{\Gamma}_{t+1}\right), \tag{B.11}$$

where

$$\tilde{\Delta} = \Delta\left(1 + \frac{(\beta\lambda)^2\sigma(1-\sigma)\bar{\Gamma}}{1-\beta\lambda}\right)^{-1} \quad \text{and}$$
$$\Lambda = \frac{\tilde{\Delta}\beta(1-\sigma)\bar{\Gamma}\left(\bar{z} - \bar{h} + \kappa_0\bar{x}^2\right)(\bar{w})^{-1}}{1-\beta(1-\sigma)}.$$

Next, we describe the derivation of $(B.14)$. (16) can be rewritten as

$$\begin{aligned} F_w(w_t^*, S_t) = &-1 + \beta\lambda(1-\sigma+q_t x_t(w_t^*))E_t[\,F_w(w_{t+1}^*, S_{t+1})\,] \\ &+ \beta\lambda(1-\sigma+q_t x_t(w_t^*))E_t\left[F_w(w_t^*, S_{t+1}) - F_w(w_{t+1}^*, S_{t+1})\right], \end{aligned}$$

where

$$\begin{aligned} F_w(w_t^*, S_{t+1}) &= -1 + \beta\lambda\Big(1-\sigma+q_{t+1}x_{t+1}(w_t^*)\Big)E_{t+1}[\,F_w(w_t^*, S_{t+2})\,] \quad \text{and} \\ F_w(w_{t+1}^*, S_{t+1}) &= -1 + \beta\lambda\Big(1-\sigma+q_{t+1}x_{t+1}(w_{t+1}^*)\Big)E_{t+1}[\,F_w(w_{t+1}^*, S_{t+2})\,]. \end{aligned}$$

Log-linearizing this yields the following:

$$\hat{F}_w(w_t^*, S_t) = \beta\lambda\bar{q}\bar{x}\left(\hat{q}_t + \hat{x}_t(w_t^*)\right) + \beta\lambda E_t\hat{F}_w(w_{t+1}^*, S_{t+1}) + \beta\lambda\Big(E_t\hat{F}_w(w_t^*, S_{t+1}) - E_t\hat{F}_w(w_{t+1}^*, S_{t+1})\Big), \tag{B.24}$$

where

$$\begin{aligned} &E_t\hat{F}_w(w_t^*, S_{t+1}) - E_t\hat{F}_w(w_{t+1}^*, S_{t+1}) \\ &= \beta\lambda\bar{q}\bar{x}\Big(E_t\hat{x}_{t+1}(w_t^*) - E_t\hat{x}_{t+1}(w_{t+1}^*)\Big) + \beta\lambda\Big(E_t\hat{F}_w(w_t^*, S_{t+2}) - E_t\hat{F}_w(w_{t+1}^*, S_{t+2})\Big). \end{aligned}$$

Using $(B.21)$, we get

$$E_t\hat{F}_w(w_t^*, S_{t+1}) - E_t\hat{F}_w(w_{t+1}^*, S_{t+1}) = \frac{\beta\lambda\bar{q}\bar{x}}{1-\beta\lambda}\left(\frac{\lambda\bar{w}\bar{F}^{-1}}{1-\beta\lambda}\left(E_t\hat{w}_{t+1}^* - \hat{w}_t^*\right)\right). \tag{B.25}$$

We get $(B.14)$ by substituting $(B.25)$ into $(B.24)$.

## B.3 Average vacancy and average wages

In this subsection, we derive $(B.9)$ and $(B.10)$. The average wages across all workers in period $t$, $\bar{w}_t$, and $t+1$, $\bar{w}_{t+1}$ and that among new hires, $\tilde{w}_{t+1}$, are respectively given by

$$\bar{w}_t = (1-\lambda)w_t^*(S_t) + \lambda\int w\,\mathrm{d}P_t(w), \tag{B.26}$$

$$\begin{aligned} E\left[\bar{w}_{t+1} \mid S_t\right] = \int_Z \Bigg\{&(1-\lambda)w_{t+1}^*(S_{t+1}) + \lambda\Big((1-\lambda)\frac{1-\sigma+q_t x_t(w_t^*, S_t)}{1-\sigma+q_t\bar{x}_t}w_t^*(S_t) \\ &+ \lambda\int_W \frac{1-\sigma+q_t x_t(w, S_t)}{1-\sigma+q_t\bar{x}_t}w\,\mathrm{d}P_t(w)\Big)\Bigg\}\mathrm{d}\Pi(z_{t+1} \mid z_t), \quad \text{and} \end{aligned} \tag{B.27}$$

$$E\left[\tilde{w}_{t+1} \mid S_t\right] = \int_Z \Bigg\{(1-\lambda) w^*_{t+1}(S_{t+1}) + \lambda\Bigg((1-\lambda)\frac{x_t(w^*_t, S_t)}{\bar{x}_t} w^*_t(S_t)$$

$$+ \lambda \int_W \frac{x_t(w, S_t)}{\bar{x}_t} w \mathrm{d}P_t(w)\Bigg)\Bigg\} \mathrm{d}\Pi(z_{t+1} \mid z_t), \tag{B.28}$$

where

$$\bar{x}_t = (1-\lambda) x_t(w^*_t, S_t) + \lambda \int_W x_t(w, S_t) \mathrm{d}P_t(w). \tag{21}$$

Log-linearizing $(B.26)$ and $(B.27)$ yields[7]

$$\hat{\bar{w}}_t = (1-\lambda)\hat{w}^*_t + \lambda \int_W \hat{w} \mathrm{d}P_t(w) \quad \text{and} \tag{B.29}$$

$$\begin{aligned} E_t \hat{\tilde{w}}_{t+1} &= (1-\lambda) E_t \hat{w}^*_{t+1} + \lambda(1-\lambda)\left(\hat{x}_t(w^*_t, S_t) + \hat{w}^*_t - \hat{\bar{x}}_t\right) + \lambda^2 \int_W \left(\hat{x}_t(w, S_t) + \hat{w} - \hat{\bar{x}}_t\right) \mathrm{d}P_t(w) \\ &= (1-\lambda) E_t \hat{w}^*_{t+1} + \lambda\left\{(1-\lambda)\hat{w}^*_t + \lambda \int_W \hat{w} \mathrm{d}P_t(w)\right\} \\ &\quad + \lambda\left\{(1-\lambda)\hat{x}_t(w^*_t, S_t) + \lambda \int_W \hat{x}_t(w, S_t) \mathrm{d}P_t(w) - \hat{\bar{x}}_t\right\}. \end{aligned} \tag{B.30}$$

By log-linearizing (21), we get

$$\hat{\bar{x}}_t = (1-\lambda)\hat{x}_t(w^*_t, S_t) + \lambda \int_W \hat{x}_t(w, S_t) \mathrm{d}P_t(w). \tag{B.31}$$

Thus, the following equation is obtained by substituting $(B.29)$ and $(B.31)$ into $(B.30)$:

$$E_t \hat{\tilde{w}}_{t+1} = (1-\lambda) E_t \hat{w}^*_{t+1} + \lambda \hat{\bar{w}}_t. \tag{B.32}$$

Similarly, log-linearizing $(B.28)$, the expected wage among new hires becomes

$$E_t \hat{\tilde{w}}_{t+1} = (1-\lambda) E_t \hat{w}^*_{t+1} + \lambda \hat{\bar{w}}_t. \tag{B.33}$$

This implies that to a first-order approximation, the average wage among new hires is the same as that among all workers.

Next, we derive $(B.9)$. From (10), the vacancy rate in period $t$ of the firm paying $w$ satisfies

$$2\kappa_0 x_t(w, S_t) = \beta q_t E_t\left[z_{t+1} - \left((1-\lambda) w^*_{t+1} + \lambda w\right) + \kappa_0 x^2_{t+1}(w_{t+1}, S_{t+1}) + \frac{2(1-\sigma)\kappa_0 x_{t+1}(w_{t+1}, S_{t+1})}{q_{t+1}}\right],$$

$$\text{where} \quad w_{t+1} = \begin{cases} w^*_{t+1} & \text{with probability } 1-\lambda \\ w & \text{with probability } \lambda \end{cases}. \tag{B.34}$$

Log-linearizing this around the steady state, we get

$$\hat{x}_t(w, S_t) = \hat{q}_t + F^{-1} E_t\left(\bar{z}\hat{z}_{t+1} - (1-\lambda)\bar{w}\hat{w}^*_{t+1} - \lambda \bar{w}\hat{w}\right) + \beta E_t \hat{x}_{t+1}(w_{t+1}, S_{t+1}) - \beta(1-\sigma) E_t \hat{q}_{t+1}. \tag{B.35}$$

Substituting this into $(B.31)$:

$$\hat{\bar{x}}_t = \hat{q}_t + F^{-1} E_t\left[\bar{z}\hat{z}_{t+1} - \bar{w}\left\{(1-\lambda)\hat{w}^*_{t+1} + \lambda\left((1-\lambda)\hat{w}^*_t + \lambda \int_W \hat{w} \mathrm{d}P_t(w)\right)\right\}\right] - \beta(1-\sigma) E_t \hat{q}_{t+1}$$

[7] Let $w_t$ denote the average wage among all workers.

$$+\beta E_t\left[(1-\lambda)\hat{x}_{t+1}(w^*_{t+1},S_{t+1})+\lambda\Big((1-\lambda)\hat{x}_{t+1}(w^*_t,S_{t+1})+\lambda\int_W \hat{x}_{t+1}(w,S_{t+1})\mathrm{d}P_t(w)\Big)\right]$$
$$=\hat{q}_t+F^{-1}E_t\left(\bar{z}\hat{z}_{t+1}-\bar{w}\hat{\bar{w}}_{t+1}\right)+\beta E_t\hat{\bar{x}}_{t+1}-\beta(1-\sigma)E_t\hat{q}_{t+1}. \tag{B.36}$$

### B.4 Vacancy rate of renegotiated firms

Finally, we derive the vacancy rate of renegotiated firms, $x_t(w^*_t,S_t)$. Using $(B.35)$, we get the following equation.

$$\hat{x}_t(w^*_t,S_t)=\hat{q}_t+F^{-1}E_t\Big(\bar{z}\hat{z}_{t+1}-\bar{w}\left(\hat{w}^*_{t+1}+\lambda\left(\hat{w}^*_t-\hat{w}^*_{t+1}\right)\right)\Big)-\beta(1-\sigma)E_t\hat{q}_{t+1}$$
$$+\beta E_t\hat{x}_{t+1}(w^*_{t+1},S_{t+1})+\beta E_t\Big((1-\lambda)\hat{x}_{t+1}(w^*_{t+1},S_{t+1})+\lambda\hat{x}_{t+1}(w^*_t,S_{t+1})-\hat{x}_{t+1}(w^*_{t+1},S_{t+1})\Big). \tag{B.37}$$

Using $(B.21)$, the last term can be rewritten as

$$\beta\lambda E_t\Big(\hat{x}_{t+1}(w^*_t,S_{t+1})-\hat{x}_{t+1}(w^*_{t+1},S_{t+1})\Big)=\beta\lambda\frac{\lambda\bar{w}F^{-1}}{1-\beta\lambda}\left(E_t\hat{w}^*_{t+1}-\hat{w}^*_t\right). \tag{B.38}$$

Substituting $(B.38)$ into $(B.37)$, we get

$$\hat{x}_t(w^*_t,S_t)=\hat{q}_t+F^{-1}E_t\left(\bar{z}\hat{z}_{t+1}-\bar{w}\left\{\hat{w}^*_{t+1}+\frac{\lambda}{1-\beta\lambda}\left(\hat{w}^*_t-\hat{w}^*_{t+1}\right)\right\}\right)$$
$$-\beta(1-\sigma)E_t\hat{q}_{t+1}+\beta E_t\hat{x}_{t+1}(w^*_{t+1},S_{t+1}). \tag{B.15}$$

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