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“Unemployment, Fiscal Competition, and the Composition of Public Expenditure

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Unemployment, Fiscal Competition, and the Composition of Public Expenditure

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Abstract

This paper investigates the efficiency of equilibrium policies and public expenditure composition under labor market imperfection in fiscal competition model. The sources of the inefficiency for supplying public goods and inputs with capital tax are the employment-stimulus and fund-raising effects of public inputs and fiscal and unemployment-exporting externalities. Our main findings are explained as follows. First, if public expenditure is financed by capital and lump-sum taxes, public goods are efficiently provided while public inputs are overprovided in the first-best sense because jurisdictional governments seek to attract capital for creating employment and tax revenue. However, public inputs are efficiently provided in the second-best sense. After that, we focus on financing by capital tax. If the capital tax is solely available, public goods are undersupplied in the second-best sense as with previous studies. In contrast, public inputs can be either undersupplied and oversupplied in the second-best sense, depending on positive effects of public input on employment and tax revenue through attracting capital.

Keywords: Fiscal Competition; Unemployment; public inputs; public goods.

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1. Introduction

Fiscal competition has been widely observed and attracted much public attention, backgrounding international/interregional competition aim to increase employment by attracting investment. In reality, OECD countries have executed the numerous tax reforms have sought to encourage investment, especially through decreased tax on business and subsidizing capital, and therefore many of the countries that lowered their corporate income tax rates did so through multi-year cuts (OECD 2017). Such tax policies have been recognized as tax competition, while public services for business have been expanded like expenditure competition.\(^1\) Focusing on multiple aspects of fiscal competition under employment creation, this paper aims to clarify equilibrium outcomes of fiscal competition and the composition of public expenditure.

Keen and Marchand (1997) pioneeringly examined the composition of public expenditure considering public goods for residents as amenity goods and public inputs for business as infrastructure. Jurisdictional governments use public inputs, instead of public goods, as the instruments to attract capital because public inputs directly attract capital through increasing marginal product of capital. Hence, public goods are relatively undersupplied while public inputs are relatively oversupplied in their composition. They conclude that changing the public expenditure composition improves social welfare even if the tax revenue cannot be increased. However, their analysis is based on Zodrow and Mieszkowski (1986)'s stability condition (hereafter, ZM stability condition), which has been criticized in its economic rationality (e.g., Noiset 1995).\(^2\)

Matsumoto (2000) extended Keen and Marchand (1997) model by incorporating interregional mobility of labor and assumed that public inputs were as the creation of atmosphere type (Meade 1952) to evade the difficulty of ZM stability condition. If the labor is complementary to capital in production, increased supply of public goods in a region induces capital inflow to the region through labor inflow. This effect possibly overweighs capital-attracting effect of public inputs. Then, the jurisdictional governments do not necessarily have incentives to use public inputs for attracting capital.

\(^1\) Some empirical studies examined and found the evidence on this issue (e.g., Bénassy-Quéré et al. 2007; Hauptmeier et al. 2012).
\(^2\) Noiset (1995) points out that ZM condition directly assume the dominance of the capital-outflow effect of increased capital tax to the capital-inflow effect of increased public inputs.
Therefore, it is shown that the inefficiency of the public expenditure composition may be solved by the coexistence of mobile capital and labor.

Labor supply is a key of determining the efficiency of supplying public goods and inputs and the expenditure composition as Matsumoto (1998, 2000) shows the role of mobile labor. Along with the literature, unemployment should be considered from not only theoretical but also realistic viewpoint, backgrounder fiscal competition to create jobs. To illustrate the relationship between employment, tax, and expenditure policies, the previous studies considered fixed wage model (e.g., Ogawa et al. 2006a, 2006b), search model (e.g., Sato 2009), and labor union model (e.g., Eichner and Upmann 2012). A common key feature of labor market imperfections is unemployment-exporting externality (see Ogawa et al. 2006a; Sato 2009). Depending on a degree of complementary/substitutability between capital and labor, tax policy affects the amount of employment through capital flow.4

The purpose and theoretical framework of this paper are parallel to the existing literature on fiscal competition (e.g., Zodrow and Mieszkowski 1986; Wilson 1986; Keen and Marchand 1997; Matsumoto 2000; Ogawa et al. 2006a, 2006b). In particular, our analytical framework is basically along with Zodrow and Mieszkowski (1986) and Keen and Marchand (1997), except for the labor market imperfection by adopting fixed wage model based on Ogawa et al. (2006a). The approach includes policy instruments such as taxes on capital and labor, lump-sum tax, expenditure for public goods supply, and for public inputs supply. To address the aim of this paper, we derive equilibrium policies, equilibrium tax rate on capital (and on labor) and the equilibrium supply levels of public goods, and characterize the efficiency of equilibrium outcomes including public expenditure composition.

The constrained economy with fixed wage needs appropriate criteria to evaluate the efficiency of public goods and inputs provision. The first-best policy which maximize regional welfare under perfect labor market, the marginal utility of public goods and marginal products of public inputs equal their physical cost, leads to the highest welfare level. However, it might not be replicated excepts for

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3 Aloi et al. (2009), Exbrayat et al. (2012), Ogawa et al. (2016)
4 Numerous studies have found significant effects of taxes on employment (e.g., Bettendorf et al. 2009; Feld and Kirchgassner 2002; Felix 2009; Harden and Hoyt 2003; Zirgulis and Sarapovas 2017).
5 Aronsson and Wehke (2008) examined the related issues how welfare was impacted by tax coordination with public intermediate and consumption goods under wage bargaining. In contrast, our focus is to characterize equilibrium policies rather than the outcome of policy coordination.
some extreme cases. Therefore, the second-best policy formulated by Gillet and Pauser (2018) is our criterion for evaluating the efficiency of public goods and inputs supply. Furthermore, the first-best policy is meaningful to compare the results in the constrained economy with the unconstrained perfect labor market economy.

The main findings of the present study are summarized as follows. First, we focus on financing public expenditure by capital and lump sum taxes. With capital and lump-sum taxes, public inputs are efficiently provided in the second-best sense while they are overprovided in the first-best sense, and public goods are efficiently provided in both senses. We also find that the equilibrium tax rate on capital is negative. In the other words, capital must be subsidized. The public expenditure composition exhibits overweighted expenditure for public inputs. The conclusion about the public expenditure composition is similar to that of Keen and Marchand (1997). However, the equilibrium tax rate and supply levels of public inputs differ from their findings. The jurisdictional governments have incentive to use public inputs to create employment through directly attracting capital. Therefore, public inputs are supplied over its efficient level that its marginal productivity equals its marginal cost. Focusing on only public input provision, Gillet and Pauser (2018) show the same result under fixed wage. Our result implies that fund-raising effect of public inputs improve the inefficiency of overprovision though it never offset the employment-stimulus effect which causes public inputs overprovision.

If the capital tax is solely available, we demonstrate that public inputs can be either oversupplied or undersupplied in the second-best sense (are oversupplied in the first-best sense) while public goods are inefficiently undersupplied in the first-best and second-best sense. Keen and Marchand (1997) and Matsumoto (2000) show that the expenditure level of public inputs does not excess its efficient level. However, the expenditure level of public inputs can be over its efficient level if employment-stimulus and fund-raising effects of public inputs are sufficiently large. The positive impact of public inputs on immobile labor significantly influences the efficiency of public goods and inputs provision because the unemployment is native issue. Unlike Matsumoto (2000) with mobile labor, regarding the expenditure composition, a marginal shift of tax revenue from public inputs supply to public goods supply always improve regional welfare and Keen and Marchand (1997)’s result will be revived under

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6 Using labor tax, it is possible in certain cases (e.g., Eichner and Upmann 2012; Gillet and Pauser 2018).
imperfect labor market with immobile labor.

Labour tax

The remainder of this paper is organized as follows. Section 2 explains our analytical framework and provides preliminary analyses for proceeding sections. Section 3 derives non-cooperative equilibrium policies with/without a lump-sum tax and characterizes the equilibrium policies and the composition of public expenditure. Section 4 extends our basic model incorporating labor tax and conducts equilibrium analyses as same as those developed in Sections 2 and 3. Finally, Section 5 concludes this paper.

2. The basic model

This section describes the basic setup of our theoretical model. The model is based on Keen and Marchand (1997). The economy consists of a large number of identical regions. The population of the residents in each region is normalized to unity. The residents possess capital and land, which are equally shared by them, and they supply one unit of labor if they are employed. Capital freely moves across regions while land and labor are immobile.

In each region, a homogenous good is producible using capital, labor, land, and public inputs. Let \( Y \) be output of the homogenous good. The production technology is formulated as

\[
Y = F(K, L, Z, B),
\]

where \( K, L, Z, \) and \( B \) are the capital, labor, land, and public inputs, respectively. \( F \) is (at least) continuously twice differentiable function and increasing in all inputs.\(^7\) We assume that all inputs are complementary each other. Following Matsumoto (1998, 2000), we also assume that the production function satisfies a linear homogeneity with respect to private inputs.\(^8\) Hence, we have

\[
F_x \equiv \frac{\partial F}{\partial x} > 0, F_{xx} \equiv \frac{\partial^2 F}{\partial x^2} < 0, F_{xy} \equiv \frac{\partial^2 F}{\partial x \partial y} > 0,
\]

\( x, y = K, L, Z, B \) and \( x \neq y. \)

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\(^7\) Ogawa et al. (2006) and Kikuchi and Tamai (2019) consider \( F_{KL} < 0 \) as well as \( F_{KL} \geq 0. \)

\(^8\) This assumption corresponds to the creation of atmosphere type (Meade 1952).
Governments. Keen and Marchand (1997) considered the taxes on capital, labor, and rent. Instead of the rent tax, we incorporate a lump-sum tax in our basic model. Each jurisdictional government provides public goods for residents and public inputs for business and finance their supply costs by capital tax on business and lump-sum tax on residents. We assume that one unit of homogenous goods is convertible to one unit of public goods/inputs. Hence, the budget equation of the government satisfies

$$ tK + h = G + B, $$

where $t$ is the capital tax rate, $h$ is the lump-sum tax, and $G$ is the public goods for residents.

Firms. Given that competitive firms in each region face the capital taxation, they pay the following rent:

$$ \pi = F(K, L, Z, B) - wL - (r + t)K, $$

where $\pi$ is the rent, $w$ is the wage rate, and $r$ is the interest rate. Following Ogawa et al. (2006a), we assume $w = \bar{w}$ and $Z = \bar{Z}$. Note that $\bar{w}$ is inefficiently higher than its competitive rate. Then, profit maximization of competitive firms leads to

$$ F_L(K, L, \bar{Z}, B) = \bar{w}, \quad (2) $$

$$ F_K(K, L, \bar{Z}, B) = r + t. \quad (3) $$

$F_{LL}F_{KK} - F_{KL}^2 \equiv D > 0$ is required for firms’ profit maximization.

Equations (2) and (3) yield $L = L(t, B)$ and $K = K(t, B)$. Using Equations (4), and (5), we obtain

$$ \frac{\partial K}{\partial t} = \frac{F_{LL}}{D} < 0, \quad \frac{\partial L}{\partial t} = -\frac{F_{LK}}{D} < 0, $$

$$ \frac{\partial K}{\partial B} = \frac{F_{LK}F_{LB} - F_{LL}F_{KB}}{D} > 0, \quad \frac{\partial L}{\partial B} = \frac{F_{LK}F_{KB} - F_{KK}F_{LB}}{D} > 0. $$

Equation (4) indicates that a rise in $t$ decreases regional capital input because the net return of capital is decreased by the increased tax on capital. Given that labor is complementary to capital, decreased capital negatively impacts the amount of employment. In contrast, Equation (5) shows that an increase in $B$ has positive effects on capital and employment. Larger public inputs lead to larger marginal productivity of each input and therefore enhances using such inputs.
Residents. Two types of residents exist in each region: One is the employed and the other is the unemployed. The employed earns their income from not only capital and land but also labor while the unemployed obtain their earnings from capital and land. All of them have to pay lump-sum taxes regardless of their income levels. Therefore, the budget equations of residents are given by

\[ C_e = r \sigma K + \pi + \bar{w} - h, \]
\[ C_u = r \sigma K + \pi - h, \]

where \( C_e \) is the employed resident’s private consumption, \( C_u \) is the unemployed resident’s private consumption, \( K \) is the economy-wide capital stock, and \( \sigma \) is the share of the capital owned by the resident to whole capital stock \((0 < \sigma < 1)\).

The residents’ utility function is formulated as

\[ U_i(C_i, G) = C_i + v(G), \]

where \( i = e, u \) and \( v(G) \) is concave and increasing in \( G \) \((v'(G) > 0 > v''(G))\). Inserting the residents’ budget equations into the utility function yields

\[ U_e(C_e, G) = C_e + v(G) = r \sigma K + \pi + \bar{w} - h + v(G), \]
\[ U_u(C_u, G) = C_u + v(G) = r \sigma K + \pi - h + v(G). \]

Equilibrium policies. Suppose that the regional welfare function is Benthamite welfare function:

\[ W = L \cdot U_e(C_e, G) + (1 - L) \cdot U_u(C_u, G). \]

Let the policy vector be \( p = (t, h, G, B) \). With all instruments, the jurisdictional government’s optimization problem is formulated as

\[ \max_p W = L \cdot U_e(C_e, G) + (1 - L) \cdot U_u(C_u, G) \]

subject to (1)-(5). The corresponding Lagrange function becomes

\[ L = r \sigma K + F(K(t, B), L(t, B), Z, B) - (r + t)K(t, B) - h + v(G) + \lambda [tK(t, B) + h - G - B], \]

where \( \lambda \) denotes the Lagrange multiplier. The first-order conditions are

\[ \frac{\partial L}{\partial t} = F_t \frac{\partial L}{\partial t} - K + \lambda \left[ K + t \frac{\partial K}{\partial t} \right] = 0, \]  
(6)

\[ \frac{\partial L}{\partial h} = -1 + \lambda = 0 \Leftrightarrow \lambda = 1, \]  
(7)

\[ \frac{\partial L}{\partial G} = v'(G) - \lambda = 0 \Leftrightarrow \lambda = v'(G), \]  
(8)
\[ \frac{\partial L}{\partial B} = F_L \frac{\partial L}{\partial B} + F_B + \lambda \left[ t \frac{\partial K}{\partial B} - 1 \right] = 0, \]  
\hspace{1cm} (9) \]

and the government’s budget equation (1). In the next section, we will solve the system of first-order conditions and characterize the solutions.

**Assumption 1.**

\[ 0 < \epsilon_K \equiv -\frac{t \partial K}{K \partial t} < 1. \]

**Symmetric equilibrium system of Equations (2) and (3).** Preparing the proceeding analyses to characterize the symmetric equilibrium policies and their outcomes, we consider an equilibrium system of Equations (2) and (3) with symmetric regions. In the equilibrium, \( K = \sigma \bar{K} \) and \( Z = \bar{Z} \) hold. Let \( K^* \equiv \sigma \bar{K} \). Equation (2) and (3) derive

\[ L = L^*(B), r = r^*(t, B), \]

where

\[ \frac{dL^*(B)}{dB} = -\frac{F_{LB}(K^*, L, \bar{Z}, B)}{F_{LL}(K^*, L, \bar{Z}, B)} > 0, \]  
\hspace{1cm} (10) \]

\[ \frac{\partial r^*(t, B)}{dt} = -1, \frac{\partial r^*(t, B)}{dB} = F_{KB}(K^*, L, \bar{Z}, B) + F_{LK}(K^*, L, \bar{Z}, B) \frac{dL^*(B)}{dB} > 0. \]  
\hspace{1cm} (11) \]

Equations (10) and (11) are similar to those derived in Ogawa et al. (2006b) and Gillet and Pauser (2018). Except for causing readers’ confusion, we simply express \( L^* \) and \( r^* \) as \( L^*(B) \) and \( r^*(t, B) \), respectively.

**The first-best and second-best policies.** Considering that this economy is constrained by Equation (2), we have to set appropriate optimal criterion. Naturally, the first-best outcome is an equilibrium unconstrained by Equation (2). Then, we can easily arrive at

**Lemma 1.** First best policy satisfies \( v'(G) = 1 \) and \( F_B = 1. \)
Note that $L = 1$ holds for the first-best equilibrium. The optimal conditions in Lemma 1 are straightforward; marginal benefit of public goods and marginal product of public inputs are equal to the physical cost (i.e., their marginal cost). The first-best equilibrium is one of benchmarks in compared with the perfect labor market. However, the first-best outcome cannot be attainable in the constrained economy.

Following Gillet and Pauser (2018), we consider the second-best policy which is derived from the following social welfare maximization problem:

$$\max_{K, L, G, B} \sum W$$

subject to (2) and

$$\sum K = \bar{K}.$$ 

Solving the above optimization problem, we obtain the following lemma (See Appendix A the proof of Lemma 2):

**Lemma 2.** The second-best policy satisfies

$$v'(G) = 1$$ and $$F_B = 1 - \Omega < 1,$$

where

$$\Omega \equiv F_L \frac{dL^*(B)}{dB}.$$ 

Lemma 2 implies that marginal product of public inputs in the second-best is less than its marginal cost because decreasing unemployment improves welfare under labor market imperfection. The second-best policy described in Lemma 2 will be our criterion for evaluating efficiency though we will sometimes refer the first-best policy to compare our results to those under perfect labor market such as Keen and Marchand (1997) and Matsumoto (2000).

### 3. Equilibrium analysis
This section examines non-cooperative equilibrium policies when each jurisdictional government maximize regional welfare subject to (1)-(5) using available policy instruments. Focusing on capital and lump-sum taxes, we consider three regimes of financing by (i) capital and lump-sum taxes and solely by (ii) capital tax.

3.1. Financing public expenditure by capital and lump-sum taxes

We now examine the equilibrium policies if (i) capital and lump-sum taxes are available for financing public expenditure. Lemmas 2 and Equations (2), (6), and (7) lead to the following proposition:

**Proposition 1.** Suppose that capital and lump-sum taxes are available. Then, the equilibrium tax and expenditure policy must satisfy

\[
v'(G) = 1, \quad (12)
\]

\[
F_B = 1 - \Omega < 1, \quad (13)
\]

\[
t = -\omega < 0, \quad (14)
\]

\[
tK + h = G + B,
\]

\[
\omega \equiv -\frac{F_{Lr}}{F_{rL}} > 0.
\]

Therefore, public goods and inputs are optimally provided in the second-best sense.

(Proof) Equations (2), (6), and (7) lead to Equation (14):

\[
t = -\frac{\partial L}{\partial t} = -\omega.
\]

Using Equations (7) and (8), we obtain Equation (12). Equation (12) and Lemma 2 show that the public goods for residents are efficiently supplied in the second-best senses.

Equations (2), (7), (9), and (14) give
Equation (13) and Lemma 2 show that the marginal product of public inputs is equal to that of the second-best policy. $\blacksquare$

Note that $\omega$ denotes a degree of unemployment-exporting externality measured by capital-inflow effect of decreased capital tax. Equation (14) implies that capital must be subsidized depending on unemployment-exporting externality relative to fiscal externality (see Ogawa et al. 2006a). In particular, larger unemployment-exporting externality leads to larger subsidy rate for capital. From Equation (1), the lump-sum tax must be positive to finance public expenditure including capital subsidy. Regarding public goods provision, it does not affect the production side and therefore capital and labor inputs. When the lump-sum tax is available, there is no distortion to provide the public goods.

In contrast, we arrive at the different outcome regarding the public inputs. The public inputs add some distortional effects because it affects capital and labor through its productivity effect. An increase in $B$ raises the marginal product of capital. Then, capital will be gathered in the region with increased public inputs. Capital inflow to the region has two effects: one is stimulating employment (i.e., increasing the benefit of public inputs or equivalently decreasing relative cost of public inputs) and the other is increasing supply cost of public inputs through increasing capital subsidy expenditure. The former effect overweighs the latter effect. Therefore, the jurisdictional governments have an incentive to use public inputs to increase welfare through stimulating employment.

Equation (13) is the extended version of the formula derived in Gillet and Pauser (2018) who analyze fiscal competition model of public inputs with two asymmetric regions. With public goods, the equilibrium tax rate is not zero. Negative tax rate on capital implies that attracting capital increases the public expenditure for capital subsidy. The coexistence of public goods and inputs generates such effect and makes the difference from the equilibrium policy under fiscal competition with only public inputs. However, with the lump-sum tax, the second-best equilibrium is attainable. Therefore, the outcome is consistent with that of Gillet and Pauser (2018) when capital and lump-sum taxes are available.
If we consider perfect labor market to compare our results to Keen and Marchand (1997), then public inputs no longer have employment effects (i.e., $\partial L/\partial B = \partial L/\partial t = 0$). Using this condition and Equation (14) yield $t = 0$ (e.g., Zodrow and Mieszkowski 1986). Hence, under perfect labor market, we obtain $F_B = 1$ from Equation (14), $\partial L/\partial B = 0$, and $t = 0$ (or Lemma 1). These conditions are the same of Lemma 1. In the economy with perfect labor market, the supply levels of public goods and inputs are determined at the level that marginal benefit of public goods or marginal product of public inputs equals to marginal rate of transformation. However, separating from perfect labor market, the economy has unemployment-exporting externality in addition to fiscal externality. The unemployment-exporting externality induce the jurisdictional government to use public inputs for increasing employment. Therefore, the marginal product of public inputs exceeds the marginal rate of transformation. These results are summarized as the following remark:

**Remark 1.** If public expenditure is financed by capital and lump-sum taxes, public goods are efficiently supplied in the first-best sense while public inputs are oversupplied in the first-best sense.

### 3.2. Financing public expenditure by capital tax

We consider the equilibrium policies if (ii) jurisdictional governments finance their expenditure by solely capital tax (i.e., $t > 0$ and $h = 0$). Without the lump-sum tax, Equation (7) no longer holds while Equations (6), (8), and (9) are alive. Lemma 2 and Equations (2), (6), (8), (9), and $h = 0$ derive the following proposition:

**Proposition 2.** Suppose that capital tax is solely available. Then, equilibrium tax and expenditure policy must satisfy

$$v'(G) = \frac{t + \omega \epsilon_K}{(1 - \epsilon_K)t} \equiv \phi > 1,$$

(15)

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This means that public inputs are overprovided in compared with the first-best (flexible wage rate) equilibrium.
\[ F_B = \phi \left[ 1 - t \frac{\partial K}{\partial B} \right] - \bar{w} \frac{\partial L}{\partial B} \] 

Therefore, public goods are inefficiently undersupplied and public inputs can be either undersupplied or oversupplied in the second-best sense.

(Proof) Regarding the supply condition for public foods, Equations (2), (6), and (8) yield

\[ v'(G) = \frac{K - \bar{w} \frac{\partial L}{\partial t}}{K + t \frac{\partial K}{\partial t}} = \frac{t + \omega \varepsilon_K}{(1 - \varepsilon_K)t} = \phi. \]

The denominator in Equation (15) must be positive by Assumption 1.

Using Equations (2), (6), (9), and (15) provide

\[ F_B = \phi \left[ 1 - t \frac{\partial K}{\partial B} \right] - \bar{w} \frac{\partial L}{\partial B}. \]

Comparing the above to \( 1 - \Omega \), we obtain Equation (16). 

Equation (15) demonstrates that the marginal cost of financing public expenditure for public goods is larger than the marginal cost of public goods that is equal to unity. Hence, the public goods are inefficiently underprovided. The marginal cost of financing public inputs depends on the marginal cost of financing public goods and employment-stimulus and fund-raising effects of public inputs.

Given that the marginal cost of financing public goods, \( \phi \), is larger than unity and the other two terms are negative in Equation (16), the marginal cost of financing public inputs could be positive can be either larger or smaller than the second-best marginal cost of public inputs. As shown in Equation (16), if \( \phi \) is sufficiently large in compared with employment-stimulus effect (\( \partial L/\partial B \)) relative to fund raising effect (\( t \partial K/\partial B \)), the marginal cost of financing public inputs is larger (smaller) than the second-best marginal cost of public inputs.

Under-provision of public goods are consistent with previous studies (e.g., Zodrow and Mieszkowski 1986; Keen and Marchand 1997). However, the presence of unemployment-exporting externality strengthens inefficiency through its positive externality in addition to fiscal (positive)
externality; Unemployment-exporting externality increases a degree of undersupply of the public goods in compared with an equilibrium without such externality (Ogawa et al. 2006a).

Our new finding is that public inputs can be either overprovided or underprovided depending on the overall externality effect. This result is contrasted with that derived by Keen and Marchand (1997) and Matsumoto (2000). Without unemployment, the sources of inefficiency are fiscal externality and fund-raising effect of public input. If the former externality effect dominates the latter effect, the public inputs are underprovided as same as the public goods. Indeed, considering the perfect labor market, Equation (14) becomes the following one (See Appendix B):

\[
F_B = \phi \left[ 1 - \frac{\partial K}{\partial B} \right] > 1 \iff \frac{\partial L}{\partial t} = \frac{\partial L}{\partial B} = 0.
\]

The marginal cost of funding public inputs under perfect labor market depends on fiscal externality and fund-raising effect of public inputs. If and only if the capital-outflow effect of increased capital tax caused by fiscal externality dominates over the fund-raising effect of public inputs through capital-inflow caused by increased public inputs, the public inputs are underprovided. However, if there is unemployment, the government has an incentive to use public inputs for creating employment. Based on Lemma 1 and Proposition 2, the discussed results are summarized as follows:

**Remark 2.** If public expenditure is financed by capital tax, public goods are undersupplied in the first-best sense while public inputs are oversupplied in the first-best sense.

We now turn to the composition of public expenditure. Keeping tax rates constant \((dt = dh = 0)\), by Equation (1), a small change in the expenditure composition must be

\[
-dB = dG.
\]

Equation (17) holds regardless of tax types. In symmetric equilibrium, a small change in the expenditure for public goods satisfying Equation (17) affects regional welfare:\(^{10}\)

\[
dW = \bar{w}dL + F_B dB + v'(G)dG.
\]

Note that positive and negative effects on capital income through \(dr\) are offset each other in

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\(^{10}\) Total differentiation of regional welfare function gives
equilibrium. Using Equations (17) and (18) and Proposition 2, we obtain the following result:

**Proposition 3.** Suppose that the government expenditure is financed by capital tax. Starting at an equilibrium that tax revenue is constant, a small increase in $G$ financed by a small decrease in $B$ improves regional welfare.

(Proof) Inserting Equations (10) and (17) into the above equation yields

$$\frac{dW}{dG} = v'(G) - \left(F_B + \tilde{w} \frac{dL^*}{dB}\right) = v'(G) - (F_B + \Omega).$$

Evaluating $dW/dG$ at the public expenditure levels of Equation (15) and (16) provides

$$\frac{dW}{dG} = \left(\phi - \left[(1 - t) \frac{\partial K}{\partial B}\phi - \tilde{w} \frac{\partial L}{\partial B} + \tilde{w} \frac{dL^*}{dB}\right]\right) = t \frac{\partial K}{\partial B} \phi + \tilde{w} \left[\frac{\partial L}{\partial B} - \frac{dL^*}{dB}\right]
= (\phi t + \omega) \frac{\partial K}{\partial B} > 0.$$

Keeping tax revenue, a small change in the expenditure composition from public input to public goods raises utility from public goods and raises marginal productivity of public inputs. Even though the additional increase in public goods decreases the marginal utility of public goods, increasing utility through increased marginal productivity of public inputs decreased marginal utility of public goods. Therefore, recompositing public expenditure improves regional welfare. Keen and Marchand (1997) shows that public inputs expenditure is too much spent than public goods expenditure under perfect labor market while Matsumoto (2000) shows that the expenditure share of public inputs is not necessarily larger than its efficient level under perfect labor market with mobile labor.

Proposition 3 in this case is parallel to the result of Keen and Marchand (1997). However, the degree of excess expenditure for public inputs in our model is larger than them because the mechanism behind our result differs from perfect labor market. As shown in Proposition 2, the unemployment-exporting externality worsens the inefficiency of public goods and public inputs (i.e., the degree of public goods undersupply and of public inputs oversupply). Therefore, the disparity of two expenditures under
imperfect labor market is larger than that under perfect labor market. Strong incentive to stimulate employment brings about this sort of inefficiency of public expenditure composition.

4. Further analysis: labor tax

In the previous sections, we have excluded using labor tax because labor tax has distortionary effects under labor market imperfection. However, the distortionary effects of capital and labor taxes may be offset each other. Hence, this section extends the basic model by incorporating a labor tax. With labor tax, jurisdictional government’s budget equation (1) is replaced as

\[ tK + \tau L + h = G + B, \]  

where \( \tau \) denotes the labor tax rate. Furthermore, Equation (2) becomes

\[ F_L(K, L, Z, B) = \bar{w} + \tau. \]  

The system of Equations (3) and (20) leads to \( L = L(t, \tau, B) \) and \( K = K(t, \tau, B) \). Indeed, total differentiation of Equations (3) and (20) yields

\[ \frac{\partial K}{\partial \tau} = -\frac{F_{LK}}{D} < 0, \quad \frac{\partial L}{\partial \tau} = \frac{F_{KL}}{D} < 0. \]  

When (iii) capital, labor, and lump-sum taxes, are available, the capital and labor taxes might be negative. However, based on Equation (20) and \( L \leq 1 \), the lower limit exists for the labor tax rate \( \tau \). Naturally, the lower limit must set

\[ \bar{w} + \tau = F_L = w^*, \]

where \( w^* \) is the competitive wage rate.

Let \( q = (t, \tau, h, G, B) \). The equilibrium policies are derived from the solutions to

\[ \max_q W = L \cdot U^e(C^e, G) + (1 - L) \cdot U^u(C^u, G) \]

subject to Equations (3)-(5), (19)-(21), and \( F_L \geq w^* \).\(^{11}\)

The corresponding Lagrange function becomes

\(^{11}\) By Equation (20), the competitive wage depends on public inputs in principle. However, the competitive wage is determined at the public inputs level when jurisdictional governments make their decision. Hence, the competitive wage level is referred as \( w^* = F_L(K^*, 1, Z, B^*) = w^*(B^*) \). Hence, \( w^* \) should be treated as given.
\begin{align*}
\mathcal{L} &= r\sigma K + F(K(t, \tau, B), L(t, \tau, B), Z, B) - (r + t)K(t, \tau, B) - \tau L(t, \tau, B) - h + v(G) \\
&\quad + \lambda [tK(t, \tau, B) + \tau L(t, \tau, B) + h - G - B] \\
&\quad + \zeta [F_L(K(t, \tau, B), L(t, \tau, B), Z, B) - w^*],
\end{align*}

where $\lambda$ is Lagrange multiplier and $\zeta$ is Kuhn-Tucker multiplier.

The first-order conditions are Equations (7), (8),

\begin{align*}
\frac{\partial \mathcal{L}}{\partial t} &= F_L \frac{\partial L}{\partial t} - K - \tau \frac{\partial L}{\partial t} + \lambda \left[ K + t \frac{\partial K}{\partial \tau} + \tau \frac{\partial L}{\partial \tau} \right] + \zeta \left[ F_{LL} \frac{\partial L}{\partial \tau} + F_{LK} \frac{\partial K}{\partial \tau} \right] = 0, \quad (22) \\
\frac{\partial \mathcal{L}}{\partial \tau} &= F_L \frac{\partial L}{\partial \tau} - L - \tau \frac{\partial L}{\partial \tau} + \lambda \left[ t \frac{\partial K}{\partial \tau} + L + \tau \frac{\partial L}{\partial \tau} \right] + \zeta \left[ F_{LL} \frac{\partial L}{\partial \tau} + F_{LK} \frac{\partial K}{\partial \tau} \right] = 0, \quad (23) \\
\frac{\partial \mathcal{L}}{\partial B} &= F_L \frac{\partial L}{\partial B} - \tau \frac{\partial L}{\partial B} + F_B + \lambda \left[ t \frac{\partial K}{\partial B} + \tau \frac{\partial L}{\partial B} - 1 \right] + \zeta \left[ F_{LL} \frac{\partial L}{\partial B} + F_{LB} \frac{\partial L}{\partial B} + F_{LB} \right] = 0, \quad (24)
\end{align*}

complementary slackness condition, and the government’s budget equation (19).

Lemma 1 and Equations (7), (8), (22), (23), and (24) yield the following proposition (see Appendix C for Proposition 4):

**Proposition 4.** Suppose that capital, labor, and lump-sum taxes are available. Then, the equilibrium tax and expenditure policy must satisfy

\begin{align*}
v'(G) &= 1, \quad (12) \\
F_B &= 1 - \Omega^* < 1, \quad (25) \\
t &= -\omega^* < 0, \quad (26) \\
\tau &= -(\bar{w} - w^*) < 0, \quad (27) \\
tK + \tau L + h &= G + B,
\end{align*}

\begin{align*}
\omega^* &\equiv -w^* \frac{F_{LK}}{F_{LL}} > 0 \text{ and } \Omega^* \equiv -w^* \frac{F_{LB}}{F_{LL}} > 0.
\end{align*}

Therefore, public goods for residents are efficiently provided and public inputs are always overprovided in the first-best sense.

Equations (26) and (27) indicate negative equilibrium tax rates on capital and labor. Equations (20)
and (27) leads to $w^{\ast} = F_L$. Hence, full employment is attainable. Proposition 4 indicates that using both capital and labor taxes with lump-sum tax causes inefficiency of providing public inputs through the second-order effect of lower limit of wage rate. Without labor tax, jurisdictional governments efficiently provide public goods and inputs in the second-best sense using capital and lump-sum taxes appropriately. Negative capital tax (i.e., capital subsidy) internalizes fiscal and unemployment-exporting externalities and lump-sum tax finances all expenditures for capital subsidy and providing public goods and inputs. However, things go wrong if the jurisdictional governments use labor taxes as well as capital and lump-sum taxes. Labor tax has additional distortionary (second-order) effect through the lower limit of wage rate because one additional constraint is required for using labor tax. This is contrasted with Eichner and Upmann (2012) and Gillet and Pauser (2018), who show the optimality of using labor tax.

We now turn to the composition of public expenditure. Equation (18) cannot be used for comparing the equilibrium policy of Proposition 4 with the first-best policy, Instead of Equation (18), we obtain

$$d\hat{W} = [v'(G) - F_B] dG,$$

where $\hat{W}$ denotes the welfare function under full employment. Equations (13), (25), and (28) derive

$$\frac{d\hat{W}}{dG} = \Omega^{\ast} > 0.$$

Therefore, cutting public inputs expenditure improves regional welfare. Negative labor tax (27) removes unemployment (i.e., employment-stimulus effect of public inputs). Therefore, reducing excess expenditure for public inputs increases regional welfare at full employment equilibrium. This implies that unemployment arises again and casts the question that the equilibrium policy of Proposition 4 may generate better outcome compared to that of Proposition 2.

To verify the welfare effect of these two policies, we use the following specified production function:

$$Y = AK^\alpha L^\beta Z^{1-\alpha-\beta} B^\gamma.$$  \hfill (29)

Then, the labor demand function under symmetric equilibrium is

$$L = \left(\frac{\beta \bar{A} \gamma}{\bar{w}}\right)^{\frac{1}{1-\beta}},$$  \hfill (30)
where $\tilde{A} \equiv A(\sigma K)^{a-\alpha-\beta}$. Inserting Equation (30) into Equation (29) leads to

$$Y = \tilde{A} \left( \frac{\beta \tilde{A}}{\tilde{w}} \right)^{\frac{\beta}{1-\beta}} B^{Y_B}. $$

We assume $\gamma < 1 - \beta$ to ensure the concavity of the production function with respect to $B$.

Equations (13), (25), (29) and (30) yield

$$B^* = \left( \frac{\gamma \tilde{A}}{1-\beta} \right)^{\frac{1}{1-\gamma}}, \quad (31)$$

$$B^* = \left( \frac{\gamma \tilde{A}^{1-\gamma} \left( \frac{1}{\tilde{w}} \right)^{\frac{Y_B}{1-\gamma}}}{1-\beta} \right)^{\frac{1-\beta}{1-\gamma}}. \quad (32)$$

Equation (31) and (32) shows

$$w^* = \tilde{w} \iff B^* = B^*, \quad (33)$$

where

$$\beta \tilde{A}^{1-\gamma} \left( \frac{Y}{1-\beta} \right)^{\frac{Y_B}{1-\gamma}} = w^*. $$

Differentiation of Equation (32) with respect to $\tilde{w}$ yields

$$\frac{dB^*}{d\tilde{w}} < 0 \text{ for } \gamma < 1 - \beta. \quad (34)$$

Using Equations (33) and (34) and $w^* < \tilde{w}$, we obtain

$$B^* > B^*. \quad (35)$$

Note that $B^*$ is greater than the first best supply level of public inputs:

$$B^* > \left( \gamma \tilde{A} \right)^{\frac{1}{1-\gamma}}. $$

Equations (29), (30), and (35) derive $L(B^*) < 1$ and $Y^* < Y^*$. Social welfare functions become

$$W^* = Y^* - B^* - G^* + v(G^*), $$

$$W^* = Y^* - B^* - G^* + v(G^*), $$

where $G^* \equiv v'^{-1}(1)$. Hence, we have

$$W^* \equiv W^* \iff Y^* - B^* \equiv Y^* - B^* \iff \frac{Y^* - Y^*}{B^* - B^*} \equiv 1.$$
Table 1. Equilibrium values of public inputs, employment, and income

<table>
<thead>
<tr>
<th>Capital and lump-sum taxes</th>
<th>Capital, labor, and lump-sum taxes</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B^* = 0.420$</td>
<td>$B^* = 0.396$</td>
</tr>
<tr>
<td>$L^* = 1$</td>
<td>$L(B^*) = 0.923$</td>
</tr>
<tr>
<td>$Y^* = 0.841$</td>
<td>$Y^* = 0.792$</td>
</tr>
<tr>
<td>$Y^* - B^* = 0.421$</td>
<td>$Y^* - B^* = 0.396$</td>
</tr>
</tbody>
</table>

To verify the relationship, we use numerical analysis. We set the parameters as $\alpha = 0.2$, $\beta = 0.6$, $\gamma = 0.2$, and $A = 1$. For capital, land, and full employment level, $\sigma R = 1$, $Z = 1$, and $L^* = 1$ are used. Table 1 reports the calculated values of key equilibrium variables. When capital and lump-sum taxes are available, the unemployment rate is 7.7%. Public inputs and employment levels are less than those in case of three tax instruments. Hence, the total income in case of two tax instruments is also less than that in case of three tax instruments. These results show $Y^* - B^* > Y^* - B^*$. Consequently, we arrive at $W^* > W^*$.

5. Conclusion

This paper considered the efficiency of equilibrium policies and public expenditure composition under labor market imperfection in fiscal competition model. Based on Keen and Marchand (1997) and Ogawa et al. (2006a), the sources of the inefficiency for supplying public goods and inputs with capital tax are the employment-stimulus and fund-raising effects of public inputs and fiscal and unemployment-exporting externalities. With labor tax, the second-order effect occurs as the additional inefficiency source.

Our main findings are explained as follows. First, if (i) public expenditure is financed by capital and lump-sum taxes, public goods are efficiently provided while public inputs are overprovided in the first-best sense because jurisdictional governments seek to attract capital for creating employment and
tax revenue. However, public inputs are efficiently provided in the second-best sense. Therefore, the public expenditure composition exhibits no excessive expenditure for public inputs in the second-best sense. After that, we focus on financing (ii) by capital tax. If the capital tax is solely available, public goods are undersupplied in the second-best sense as with previous studies. In contrast, public inputs can be either undersupplied and oversupplied in the second-best sense, depending on positive effects of public input on employment and tax revenue through attracting capital.

Incorporating labor tax into the basic model are also studies to ensure robustness of our results. We demonstrate that the public inputs are overprovided in compared with the first-best level when the capital, labor, and lump-sum taxes are all available. The first-best policy is derived from the full employment under perfect competitive labor market. However, the equilibrium with fixed wage involves the second-order effects of the tax instruments. Hence, the government policy cannot be the first-best even if the capital, labor, and lump-sum taxes are all available. Furthermore, our numerical analysis shows that the welfare level is improved if the labor tax is available in addition to capital and lump-sum taxes. This implies that the government policy with the capital, labor, and lump-sum taxes is the second-best and that with the capital and lump-sum taxes is the third-best.

The results derived in this paper show that the presence of unemployment affects the efficiency of supplying public goods and public inputs and they are contrasted with previous studies under perfect labor market. The jurisdictional governments value public inputs as the instruments to attract capital and to create employment through its marginal productivity effect though they do not public goods as just amenity. If either of capital and lump-sum tax is lacked, the jurisdictional governments cannot replicate the second-best equilibrium and it causes excess tax revenue and oversupply of public inputs.

Finally, we mention two possible extensions. First extension is incorporating fiscal system of tax transfer and central government into our basic model. Our analysis is focused on only horizontal government competition; however, we should tackle the issues in correcting devices of inefficiency and vertical fiscal competition in reality. Second extension is considering the asymmetric regions with respect to endowments of capital, labor, and land. This issue relates to inequality and the first extension (i.e., the importance of tax transfer system). These features are essential for future research. Our findings will provide analytical basis for the extensions.
Appendix

A. Proof of Lemma 2

The Lagrange function is formulated as

\[ \tilde{\mathcal{L}} = \sum [F(K, L, Z, B) - G - B + v(G)] + \mu [\tilde{K} - \sum K] + \chi [F_L(K, L, Z, B) - \bar{w}], \]

where \( \mu \) and \( \chi \) are Lagrange multipliers.

The first-order conditions are

\[ \frac{\partial \mathcal{L}}{\partial K} = F_K - \mu + \chi F_{LK} = 0, \quad (A1) \]
\[ \frac{\partial \mathcal{L}}{\partial L} = F_L + \chi F_{LL} = 0 \quad \Leftrightarrow \quad \chi = - \frac{F_L}{F_{LL}}, \quad (A2) \]
\[ \frac{\partial \mathcal{L}}{\partial G} = v'(G) - 1 = 0, \quad (A3) \]
\[ \frac{\partial \mathcal{L}}{\partial B} = F_B - 1 + \chi F_{LB} = 0, \quad (A4) \]

Equation (A3) derives Equation (13). Equations (A2), and (A4) lead to

\[ F_B = 1 - \chi F_{LB} = 1 + F_L \frac{F_{LB}}{F_{LL}} < 1. \]

Equations (A1), and (A2) provides

\[ F_K = \mu - \chi F_{LK} = \mu + F_L \frac{F_{LK}}{F_{LL}} \quad (A5) \]

for all regions. Symmetric equilibrium condition and Equation (A5) give the shadow price of capital.

B. Marginal product of public inputs provision under perfect labor market

Note that Euler theorem leads to
\[ F_B(K, L, Z, B) = F_{KB}K + F_{LB}L + F_{ZB}Z. \]

Then, we obtain

\[
F_B - 1 = \frac{1 - t \frac{\partial K}{\partial B}}{1 + t \frac{\partial K}{K \partial t}} - 1 = \frac{t \frac{\partial K}{\partial B} + 1 \frac{\partial K}{K \partial t}}{1 + t \frac{\partial K}{K \partial t}}
\]

\[
= -\frac{t}{K} \frac{-F_{LB}F_{LZ}Z - F_{Lk}[F_B - F_{ZB}Z - 1]}{1 + t \frac{\partial K}{K \partial t}}
\]

\[
= \frac{t}{F_{KK}K} \frac{F_B - F_{ZB}Z - 1}{1 + \frac{t}{F_{KK}K}}.
\]

Solving the above equation with respect to \((F_B - 1)\), we obtain

\[
F_B - 1 = -t \frac{F_{ZB}Z}{F_{KK}K} > 0.
\]

C. Proposition 4

(iii) Capital, labor, and lump-sum taxes. Equation (7) leads to \(\lambda = 1\). Complementary condition is \(\zeta[F_L - w^*]\). We first consider \(F_L > w^*\) and \(\zeta = 0\). Inserting \(\lambda = 1\) and \(\zeta = 0\) into Equations (22) and (23) provides

\[
F_L \frac{\partial L}{\partial t} + t \frac{\partial K}{\partial t} = 0, F_L \frac{\partial L}{\partial \tau} + t \frac{\partial K}{\partial \tau} = 0.
\]

Solving these two equations with respect to \(F_L\) and \(\tau\), we obtain

\[
\left(\frac{\partial K \partial L}{\partial t \partial \tau} - \frac{\partial K \partial L}{\partial \tau \partial \tau}\right) t = \left(\frac{F_{LL}F_{KK} - F_{Lk}^2}{D^2}\right) t = \frac{t}{D} = 0,
\]

\[
\left(\frac{\partial K \partial L}{\partial t \partial \tau} - \frac{\partial K \partial L}{\partial \tau \partial \tau}\right) F_L = \left(\frac{F_{LL}F_{KK} - F_{Lk}^2}{D^2}\right) F_L = \frac{F_L}{D} = 0.
\]
Equation (A6) contradicts $F_L > w^*> 0$. 

We next consider $F_L = w^*$ and $\zeta > 0$. $F_L = w^*$ and Equation (20) derive $\bar{w} + \tau = w^*$. Hence, we obtain $\tau = -(\bar{w} - w^*) < 0$. Using $\lambda = 1$, $F_L = w^*$, and Equations (22) and (23) yields

$$F_L \frac{\partial L}{\partial t} + \frac{\partial K}{\partial t} + \zeta \left[ F_{LL} \frac{\partial L}{\partial t} + F_{LK} \frac{\partial K}{\partial t} \right] = 0, \quad (A7)$$

$$F_L \frac{\partial L}{\partial \tau} + \frac{\partial K}{\partial \tau} + \zeta \left[ F_{LL} \frac{\partial L}{\partial \tau} + F_{LK} \frac{\partial K}{\partial \tau} \right] = 0, \quad (A8)$$

$$F_L \frac{\partial L}{\partial B} + F_B + \frac{\partial K}{\partial B} - 1 + \zeta \left[ F_{LL} \frac{\partial L}{\partial B} + F_{LK} \frac{\partial K}{\partial B} + F_{LB} \right] = 0, \quad (A9)$$

Note that the following is true:

$$F_{LL} \frac{\partial L}{\partial t} + F_{LK} \frac{\partial K}{\partial t} = -\frac{F_{LL} F_{LK}}{D} + \frac{F_{L,L} F_{LL}}{D} = 0.$$ 

Solving (A7) and (A8) with respect to $t$ and $\zeta$ derives

$$t = F_L \frac{F_{LK}}{F_{LL}} = w^* \frac{F_{LK}}{F_{LL}} < 0 \text{ and } \zeta = -\frac{F_L}{F_{LL}} = -\frac{w^*}{F_{LL}} > 0. \quad (A10)$$

Equations (A9) and (A10) with $F_L = w^*$ give

$$F_B = 1 - F_L \frac{\partial L}{\partial B} - F_{LL} \frac{F_{LK}}{F_{LL}} \frac{\partial K}{\partial B} + F_L \left[ F_{LL} \frac{\partial L}{\partial B} + F_{LK} \frac{\partial K}{\partial B} + F_{LB} \right]$$

$$= 1 + F_L \frac{F_{LB}}{F_{LL}} = 1 + w^* \frac{F_{LB}}{F_{LL}} < 1.$$
References


Kikuchi, Y. and T. Tamai (2019), Tax competition, unemployment, and intergovernmental transfers,


