KIER DISCUSSION PAPER SERIES

KYOTO INSTITUTE
OF
ECONOMIC RESEARCH

Discussion Paper No.1071

"Precautionary Saving against Correlation
under Risk and Ambiguity"

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January 2022

KYOTO UNIVERSITY

KYOTO, JAPAN

Precautionary Saving against Correlation under Risk and Ambiguity*

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This Version: January 11, 2022

Abstract

This paper considers precautionary saving against the correlation between two risky attributes (wealth and health) and investigates how the correlation affects optimal savings under multivariate preferences. The signs of higher-order cross derivatives play a key role in determining the direction of precautionary saving against such correlation. Mixed correlation averse (seeking) individuals increase (decrease) savings in response to increases in correlation. Furthermore, we introduce ambiguity to the correlation and investigate how ambiguity affects the amount of optimal savings. The analyses enable us to deepen our understanding of saving behavior under multivariate preferences in the presence of correlation.

Key Words: Mixed correlation aversion (seekingness), Multivariate preferences, Smooth ambiguity model, Stochastic dominance

*We are grateful to Chiaki Hara, Nobuyuki Hanaki, Takashi Kunimoto, Takehito Masuda, Hitoshi Sadakane, Tadashi Sekiguchi and thank participants in seminars held at Kyoto and Osaka Universities for their comments and suggestions. Of course, all remaining errors are our sole responsibility. This research is financially supported by JSPS KAKENHI Grant Numbers 20K01745, 17K03806, 20H01507, 20H01519, 21K01393, and the Joint Research Program of KIER.

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1 Introduction

How much to save and consume are essential household financial decisions. Uncertainty is one reason why households save, and this is commonly referred to as the precautionary saving motive.[1] In the literature on precautionary saving, one-dimensional risk has mainly been investigated. However, saving decisions often face multidimensional risks. For example, both the riskiness of the wealth levels and health conditions of individuals affect saving decisions because both are important in our daily lives.

The analyses of multivariate preferences provide clues for understanding our saving behaviors in the presence of multidimensional risks. Moreover, it is plausible to assume that the level of wealth and the condition of health are closely related. This implies that the role of correlation between the wealth level and the health condition should be incorporated into analyses of precautionary saving decisions. Motivated by the above recognition, the purpose of this paper is to analyze precautionary saving against the correlation between two risky attributes.

In most cases, it is more difficult to quantify the correlation between two risky attributes than the risk of an individual attribute due to the lack of experience and insufficient observations. We acknowledge this difficulty by introducing ambiguity about the correlation between two risky attributes. One of the purposes of this paper is to extend the analyses of precautionary saving to include correlation with ambiguity and to investigate how correlation with ambiguity affects the amount of savings.

Since the seminal paper by Ellsberg (1961), the importance of ambiguity has been recognized in the literature. The developments of Choquet expected utility and maxmin expected utility have enabled us to investigate behaviors and decisions under ambiguity (see Gilboa and Schmeidler (1989) and Schmeidler (1989) for axiomatizations). This paper adopts the smooth ambiguity model in Klibanoff

[1] Uncertainty is used as an umbrella term that includes both risk and ambiguity. Risk is the situation where uncertainty can be measured by a unique probability and ambiguity is the situation where it cannot.

et al. (2005, 2009). The model can differentiate the decision makers' attitudes toward ambiguity from their perception of ambiguity.[2] Furthermore, the model is more tractable than most of the models analyzing ambiguity. The usefulness of the smooth ambiguity model has been widely recognized, for example, an application to the portfolio problem by Gollier (2011).

Precautionary saving has been investigated extensively in the literature. It dates back to Leland (1968) that compares future labor income under risk with that under certainty. Since Leland (1968), numerous studies have used theoretical and empirical approaches.[3] Kimball (1990) is a milestone in research on precautionary saving and characterizes the notion of prudence that is related to the convexity of marginal utility. Most studies on precautionary saving have focused on investigating situations where households face only risky future incomes. This paper advances this research field by examining the effects of correlation between two risky attributes on optimal savings in a framework of multivariate preferences. In addition, we analyze the situation in which there are multiple possible realizations of correlation by applying the notion of ambiguity.

This paper makes the following three contributions to the literature on precautionary saving and multivariate preferences. First, we provide a framework for analyzing the effects of correlation between two risky attributes on precautionary saving under multivariate preferences. Second, we demonstrate how the signs of successive cross derivatives of a utility function affect precautionary saving toward correlation. The condition is called mixed correlation aversion (seekingness) which is a bivariate extension of mixed risk aversion (seekingness) by Caballé and Pomansky (1996) in a univariate framework. Mixed correlation aversion (seekingness) generalizes correlation aversion (seekingness) with a positive (negative) sign of a cross derivative. Mixed correlation averse (seeking) individuals increase (decrease) their precaution-

[2] Many decision models designed to capture ambiguity aversion are proposed with axiomatic foundations and are applied to various problems that appear in economics and finance. Machina and Siniscalchi (2014) provide a review of canonical models.

[3] See Baidrdi et al. (2020) for a review of the theoretical studies and Liguilde et al. (2019) for the empirical studies.

ary savings in response to an increase in correlation. Third, we consider a situation where there are many possible correlations because of the difficulty of quantifying the relationship between multiple attributes compared with one-dimensional uncertainty itself. This situation is captured by the notion of ambiguity. We determine the conditions under which ambiguous correlations make decision makers increase their precautionary savings by more than risky correlations. These conditions are determined by the combination of risk and ambiguity attitudes.

The organization of this paper is as follows. Section 2 presents the literature review. Section 3 presents a basic framework for analyzing the effects of correlation between two risky attributes on optimal savings under risk. Section 4 introduces the notion of stochastic dominance. Section 5 analyzes how correlation affects optimal savings under risk. Section 6 discusses the notion of being mixed correlation averse (seeking). Section 7 introduces ambiguity into the correlation between two risky attributes and shows that whether optimal savings increase or not depends on attitudes toward both risk and ambiguity. Section 8 relates the experimental results and empirical observations in the existing literature to our theoretical results. Section 9 concludes the paper.

2 Literature review

This paper lies at the intersection of the literature on multivariate preferences and precautionary saving. As in Eeckhoudt et al. (2007), we restrict our analyses to bivariate preferences by fixing all but two of the attributes. The first attribute is the level of wealth, and the second attribute is the condition of health. Even though we could use other variables as the second attribute, for example, other's level of wealth or the condition of the environment, we adopt the condition of health because of its importance in household financial decisions and its use in many studies.

It is difficult to interpret the signs of higher-order cross derivatives. Eeckhoudt et al. (2007) provide their systematic characterization by applying a combination of two bads, sure reduction and zero-mean risk.[4] They relate preferences for specific combinations of the two bads to the signs of successive higher-order cross derivatives of a utility function.[5] Jokung (2011) applies stochastic dominance relations to express a preference for the good attribute and the bad one, and shows that the signs of cross derivatives of a utility function can be related to the preference for the combination of good and bad. The signs also play a significant role in our analyses.

Most studies on precautionary saving have focused on settings in which individuals face risky future incomes in a one-dimensional situation. This suggests two directions for advancing research on precautionary saving. One direction is to compare one risky situation with another risky situation. Another direction is to shed light on precautionary saving motives in multidimensional settings. Eeckhoudt and Schlesinger (2008) are a seminal paper on the first direction and demonstrate how risky shifts in future income affect optimal savings based on the notion of changes in higher-order stochastic dominance. We compare two different risky situations as in Eeckhoudt and Schlesinger (2008), but the difference between their paper and ours is our inclusion of correlation between two risky attributes and multivariate preferences.

Courbage and Rey (2007) analyze precautionary saving in a multidimensional setting and investigate conditions under which individuals increase their savings by more when there is risk in the second attribute. Whereas Courbage and Rey (2007) compare two different situations in which health conditions are either certain or risky, we compare two different situations in which health conditions are both risky but differ in correlation against income risks. Even though Courbage and Rey (2007) consider a situation in which future income and health risks are correlated,

[4] This characterization is an extension of Eeckhdout and Schlesinger (2006) in one dimension. Even though the arguments are closely related between univariate and multivariate preferences, we only provide a review by Eeckhoudt and Schlesinger (2014) instead of referring to individual studies of univariate preferences.

[5] Eeckhoudt et al. (2007) show that (i) an individual is correlation averse if and only if $u_{(1,1)}(x, y) \leq 0$ for all x, y, (ii) an individual is cross-prudent in health if and only if $u_{(2,1)}(x, y) \geq 0$ for all x, y, (iii) an individual is cross-prudent in wealth if and only if $u_{(1,2)}(x, y) \geq 0$ for all x, y, and (iv) an individual is cross-temperate if and only if $u_{(2,2)}(x, y) \leq 0$ for all x, y, where $u(x, y)$ is a bivariate utility function of wealth x and health y, where $u_{(i,j)}(x, y)$ stands for $\partial^{i+j}u/\partial x^i \partial y^j$.

this paper adopts a simpler representation of the correlation based on Doherty and Schlesinger (1990). This representation enables us to examine precautionary saving against the correlation. In addition, we analyze the situation in which there are multiple possible realizations of correlation by applying the notion of ambiguity.

Some studies investigate how ambiguity in future income affects saving decisions in a univariate framework. Gierlinger and Gollier (2017) and Osaki and Schlesinger (2014) determine conditions under which individuals increase their savings to protect against ambiguity in future income compared with the case of risk.[6] Following Osaki and Schlesinger (2014), ambiguity aversion distorts the relative weights on worse priors and the preference for consumption timing. This distortion also occurs in our analyses. Unlike Osaki and Schlesinger (2014), we introduce ambiguity into the correlation between the two risky attributes, not future income.

3 Optimal savings in the presence of correlation

In this section, we provide a basic framework for analyzing the effects of correlation between two attributes on optimal savings under risk. Let us consider a simple dynamic model with two dates, $t = 0$ and $t = 1$. An individual enjoys lifetime time-separable utility from two attributes $(x, y) \in X \times Y \subseteq \mathbb{R}^2_+$. The first attribute is a financial variable and the second is a nonfinancial variable. For the sake of exposition, we assume that the financial variable is wealth level and the nonfinancial variable is health condition. Other examples of nonfinancial variables are other's wealth level and environmental conditions. The analysis can be applied to nonfinancial variables, which can be measured numerically.

Let us denote a bivariate utility function $u : X \times Y \to \mathbb{R}$. We denote $u_{(1,0)}(x, y)$ as $\partial u/\partial x$, $u_{(0,1)}(x, y)$ as $\partial u/\partial y$ and $u_{(1,1)}(x, y)$ as $\partial^2 u/\partial x \partial y$. The same notation can be used for the function $u_{(i,j)}(x, y)$, which stands for $\partial^{i+j} u/\partial x^i \partial y^j$. We assume that all higher-order partial and cross derivatives exist if necessary for the analysis.

[6] Gierlinger and Gollier (2017) consider the term structure of interest rates under ambiguous consumption growth. As in Berger (2014), their results can be translated in terms of precautionary saving against ambiguity.

The utility function $u(x, y)$ is increasing and concave in both wealth and health, $u_{(1,0)}(x, y) \geq 0$, $u_{(0,1)}(x, y) \geq 0$, and $u_{(2,0)}(x, y) \leq 0$, $u_{(0,2)}(x, y) \leq 0$. The concavity means risk aversion for both wealth and health. We do not impose any restriction on the sign of $u_{(1,1)}(x, y)$. The signs of the higher-order cross derivatives play a crucial role in our analyses.

The individual faces a future income risk and future health risk. Two types of risks, which are called "good" and "bad," are involved in both the future income and health risks. Regarding the future income risk, the random variables $\tilde{\epsilon}_B$ and $\tilde{\epsilon}_G$ occur with probability p and $1-p$. Regarding the future health risk, the random variables $\tilde{\delta}_B$ and $\tilde{\delta}_G$ occur with probability q and $1 - q$. We assume that all future risks, $\tilde{\epsilon}_G$, $\tilde{\epsilon}_B$, $\tilde{\delta}_G$, and $\tilde{\delta}_B$, are mutually independent. The terms "good" and "bad" indicate that both good wealth and health risks are preferred, that is, $E[u(x + \tilde{\epsilon}_G, y)] \geq E[u(x + \tilde{\epsilon}_B, y)]$ and $E[u(x, y + \tilde{\delta}_G)] \geq E[u(x, y + \tilde{\delta}_B)]$ for all x, y. As in the next section, we rank good and bad by stochastic dominance relations.

There are four possible combinations of future income and health risks. The following are the combinations and their probabilities:

- $\tilde{\epsilon}_B$ and $\tilde{\delta}_B$ with probability kpq;
- $\tilde{\epsilon}_G$ and $\tilde{\delta}_B$ with probability $(1 - kp)q$;
- $\tilde{\epsilon}_B$ and $\tilde{\delta}_G$ with probability $p(1 - kq)$;
- $\tilde{\epsilon}_G$ and $\tilde{\delta}_G$ with probability $1 - p - q + kpq$.

We can calculate the probability of the future income and health risks, for example, the probability of the simultaneous occurrence of bad future income and health risk is kpq. Other probabilities of the future income and health risks can be calculated similarly. A positive value of k is chosen so that all probabilities are nonnegative and less than unity. This value of k captures the correlation between the future income risk and the health risk. When the value of k is unity, the future income risk and the health risk are independent. Indeed, if $k = 1$, the probability occuring $\tilde{\epsilon}_B$

and $\tilde{\delta}_B$ is equal to pq, which is the multiplication of each probability. A value of k greater (less) than unity indicates a positive (negative) correlation. The correlation increases in k.

The individual earns the sure income w and is endowed with the sure health condition h in both $t = 0$ and $t = 1$. In addition to the sure income and health, the individual faces the income and health risks at $t = 1$. The individual must decide a level of saving at $t = 0$ to maximize the lifetime utility. The savings are invested at the risk-free rate of interest. Negative savings are the amount of borrowing from future income for current consumption. We assume that the risk-free rate of interest is zero and the individual is not impatient, the latter of which means no time-discounting. Because of this simplified setting, we can focus on the effect of risk on saving decisions. The individual determines the level of saving to maximize lifetime time-separable utility from wealth and health:

$$\begin{aligned}
&\max_s U(s) \\
=& \, u(w-s,h) + v(s,k) \\
=& \, u(w-s,h) + kpqE[u(w+s+\tilde{\epsilon}_B, h+\tilde{\delta}_B)] + (1-kp)qE[u(w+s+\tilde{\epsilon}_G, h+\tilde{\delta}_B)] \\
&+ p(1-kq)E[u(w+s+\tilde{\epsilon}_B, h+\tilde{\delta}_G)] + (1-p-q+kpq)E[u(w+s+\tilde{\epsilon}_G, h+\tilde{\delta}_G)].
\end{aligned} \tag{1}$$

Here, $v(s,k)$ denotes the expected utility at $t = 1$ given s and k.

The first-order condition for (1) is

$$\begin{aligned}
&U'(s) \\
=& -u'(w-s^*,h) + v_s(x^*,k) \\
=& -u'(w-s^*,h) + kpqE[u_{(1,0)}(w+s^*+\tilde{\epsilon}_B, h+\tilde{\delta}_B)] + (1-kp)qE[u_{(1,0)}(w+s^*+\tilde{\epsilon}_G, h+\tilde{\delta}_B)] \\
&+ p(1-kq)E[u_{(1,0)}(w+s^*+\tilde{\epsilon}_B, h+\tilde{\delta}_G)] + (1-p-q+kpq)E[u_{(1,0)}(w+s^*+\tilde{\epsilon}_G, h+\tilde{\delta}_G)] = 0.
\end{aligned} \tag{2}$$

Because $U(s)$ is concave by $u_{(2,0)} \leq 0$, the second-order condition for a maximum is satisfied. For simplicity, we assume that the optimal amount of savings is interior, $-w \leq s^* \leq w$, and is unique throughout the paper.

4 Stochastic dominance

In this section, we introduce the notion of stochastic dominance to represent the "good" or "bad" future income risk and health risk. Stochastic dominance is a partial order to compare two random variables. Let us consider two random variables $\tilde{x}$ and $\tilde{y}$ with the cumulative distribution functions F and G, which are defined over a bounded support $[a, b]$. We note that the notation of $\tilde{x}$ and $\tilde{y}$ is used for the exposition of stochastic dominance in this section.

The distribution function F dominates the distribution function G in the sense of first-order stochastic dominance (FSD) if $F(z) \leq G(z)$ for all $z \in [a, b]$. If the random variables $\tilde{x}$ and $\tilde{y}$ have the distribution functions F and G, and F dominates G in the sense of FSD, it is said that a random variable $\tilde{x}$ dominates $\tilde{y}$ in the sense of FSD. The same goes for other notions of stochastic dominance. Applying FSD to the future income risk, the individual with $u_{(1,0)} \geq 0$ prefers the good future income risk $\tilde{\epsilon}_G$ to the bad one $\tilde{\epsilon}_B$. Formally, the following two conditions are equivalent:

- $\tilde{\epsilon}_G$ dominates $\tilde{\epsilon}_B$ in the sense of FSD;
- $E[u(w + \tilde{\epsilon}_G, h)] \geq E[u(w + \tilde{\epsilon}_B, h)]$ for $u_{(1,0)} \geq 0$.

The same argument can be applied to the future health risk. The individual with $u_{(0,1)} \geq 0$ prefers the good future health risk $\tilde{\delta}_G$ to the bad one $\tilde{\delta}_B$, that is, $E[u(w, h + \tilde{\delta}_G)] \geq E[u(x, h + \tilde{\delta}_B)]$ for $u_{(0,1)} \geq 0$ when $\tilde{\delta}_G$ dominates $\tilde{\delta}_B$ in the sense of FSD.

For the distribution functions F and G on $[a, b]$, let us define $F_1(z) = F(z)$ and $G_1(z) = G(z)$, and define $F_n(z) = \int_a^z F_{n-1}(t)dt$ and $G_n(z) = \int_a^z G_{n-1}(t)dt$ for all $z \in [a, b]$ and for all $n = 2, 3, \ldots, N$. Following Jean (1980) and Ingersoll (1987), the distribution function F dominates the distribution function G in the sense of Nth-order stochastic dominance (NSD) if $F_N(z) \leq G_N(z)$ for all $z \in [a, b]$ and

$F_n(b) \leq G_n(b)$ for all $n = 1, 2, \ldots, N-1$. The following result is known in the literature, for example, see Ingersoll (1987).

- $\tilde{\epsilon}_G$ dominates $\tilde{\epsilon}_B$ in the sense of NSD;
- $E[u(w+\tilde{\epsilon}_G, h)] \geq E[u(w+\tilde{\epsilon}_B, h)]$ for any function u such that $(-1)^{n+1}u_{(n,0)} \geq 0$ for all $n = 1, 2, \ldots, N$.

The individual prefers the future good income risk to the bad one, which are ranked according to second-order stochastic dominance because we assume that $u_{(1,0)} \geq 0$ and $u_{(2,0)} \leq 0$. For the third-order stochastic dominance, we need to assume $u_{(3,0)} \geq 0$ in addition to $u_{(1,0)} \geq 0$ and $u_{(2,0)} \leq 0$ so that the individual prefers the good future income risk to the bad one. Because the positive third-order derivative represents prudence in the univariate utility framework, we refer to $u_{(3,0)} \geq 0$ as prudence for wealth. The same argument can be applied to the future health risk. The following two conditions are equivalent:

- $\tilde{\delta}_G$ dominates $\tilde{\delta}_B$ in the sense of NSD;
- $E[u(w, h+\tilde{\delta}_G)] \geq E[u(w, h+\tilde{\delta}_B)]$ for any function u such that $(-1)^{n+1}u_{(0,n)} \geq 0$ for all $n = 1, 2, \ldots, N$.

Following the terminology coined by Caballé and Pomansky (1996), the second condition represents mixed risk aversion in wealth and health, respectively. In other words, individuals are called mixed risk averse if the signs of successive derivatives of utility functions have alternate signs, with all positive odd derivatives and all negative even derivatives. As shown by Brockett and Golden (1987), the utility functions commonly adopted in economics have the property of having all positive odd derivatives and all negative even derivatives.[7]

[7] A real-valued function $u(x)$ on $(0, \infty)$ is *complete monotone* if its derivatives $u^n(x)$ of all orders exist and $(-1)^n u^n(x) \geq 0$ for all $x > 0$ and all $n = 0, 1, 2, \ldots$. A real-valued, continuous utility function u defined on $[0, \infty)$ exhibits *mixed risk aversion* if it has a completely monotone first derivative on $(0, \infty)$ (i.e., $(-1)^n u^{n+1}(x) \geq 0$ for all $x > 0$ and for all $n = 0, 1, 2, \ldots$) and $u(0) = 0$. As also pointed out by Pratt and Zeckhauser (1987), the majority of utility functions analyzed in applied work have completely monotone first derivatives. For example, if a class of utility functions u is the class of hyperbolic absolute risk aversion with $-u''(x)/u'(x) = 1/(a + bx)$ for $a > 0$ and $b > 0$, then they are mixed risk averse. See Caballé and Pomansky (1996, p.490) for details.

We introduce another stochastic dominance relation referred to as an increase in N-th degree risk. As a special case of NSD, Ekern (1980) proposes that the distribution function G has more Nth-degree risk than the distribution function F if $F_N(z) \leq G_N(z)$ for all $z \in [a, b]$ and $F_n(b) = G_n(b)$ for all $n = 1, 2, \ldots, N$. This indicates that the first $(N-1)$th-moments of F and G coincide. Ekern (1980) shows that the following conditions are equivalent:

- $\tilde{\delta}_B$ has more Nth-degree risk than $\tilde{\delta}_G$;
- $E[u(w + \tilde{\epsilon}_G, h)] \geq E[u(w + \tilde{\epsilon}_B, h)]$ for $(-1)^{N+1}u_{(N,0)} \geq 0$.

The same argument can be applied to the case of health risk. We provide some examples from the literature. An increase in risk by Rothschild and Stiglitz (1970) corresponds to a second-degree increase in risk. Rothschild and Stiglitz (1970) show that any increase in risk can be obtained by a sequence of mean-preserving spreads. This implies that two random variables with the same means are ranked by an increase in risk. Risk averse individuals dislike any increases in risk. An increase in downside risk introduced by Menezes et al. (1980) corresponds to a third-degree increase in risk. This implies that two random variables with the same means and the same variances are ranked by an increase in downside risk. Prudent individuals dislike any increases in downside risk.

5 Precautionary saving against correlation under risk

In this section, we examine how correlation affects optimal savings under risk. In our setup, this means that we examine how optimal savings change in k, which is the parameter representing correlation. Before stating the main result, we present the following lemma.

Lemma 1. *Let $f(\epsilon, \delta)$ be the payoff function and define*

$$\begin{aligned} E[f(\tilde{\epsilon}, \tilde{\delta})] &= kpqE[f(\tilde{\epsilon}_B, \tilde{\delta}_B)] + (1-kp)qE[f(\tilde{\epsilon}_G, \tilde{\delta}_B)] \\ &\quad + p(1-kq)E[f(\tilde{\epsilon}_B, \tilde{\delta}_G)] + (1-p-q+kpq)E[f(\tilde{\epsilon}_G, \tilde{\delta}_G)]. \end{aligned}$$

Suppose that

- *$\tilde{\epsilon}_G$ dominates $\tilde{\epsilon}_B$ in the sense of NSD;*
- *$\tilde{\delta}_G$ dominates $\tilde{\delta}_B$ in the sense of MSD.*[8]

If $(-1)^{n+m} f_{(n,m)} \geq (\leq) 0$ for all $n = 1, 2, \ldots, N$ and $m = 1, 2, \ldots, M$, then $E[f(\tilde{\epsilon}, \tilde{\delta})]$ increases (decreases) in k.

Proof We will prove the case that $E[f(\tilde{\epsilon}, \tilde{\delta})]$ increases in k, because the opposite case can be proven in a similar way.

By a simple calculation, we have the following:

$$\frac{\partial E[f(\tilde{\epsilon}, \tilde{\delta})]}{\partial k} = pq \left(E[f(\tilde{\epsilon}_B, \tilde{\delta}_B)] - E[f(\tilde{\epsilon}_G, \tilde{\delta}_B)] - E[f(\tilde{\epsilon}_B, \tilde{\delta}_G)] + E[f(\tilde{\epsilon}_G, \tilde{\delta}_G)] \right).$$

This leads to the following:

$$\operatorname{sgn}\left(\frac{\partial E[f(\tilde{\epsilon}, \tilde{\delta})]}{\partial k} \right) \geq 0 \Leftrightarrow E[f(\tilde{\epsilon}_G, \tilde{\delta}_G)] - E[f(\tilde{\epsilon}_G, \tilde{\delta}_B)] \geq E[f(\tilde{\epsilon}_B, \tilde{\delta}_G)] - E[f(\tilde{\epsilon}_B, \tilde{\delta}_B)]. \tag{3}$$

When

$$(-1)^{n+1} \frac{\partial^n \{E[f(\epsilon, \tilde{\delta}_G)] - E[f(\epsilon, \tilde{\delta}_B])\}}{\partial \epsilon^n} \geq 0 \text{ for all } n = 1, 2, \ldots, N, \tag{4}$$

we have that

$$E[f(\tilde{\epsilon}_G, \tilde{\delta}_G)] - E[f(\tilde{\epsilon}_G, \tilde{\delta}_B)] \geq E[f(\tilde{\epsilon}_B, \tilde{\delta}_G)] - E[f(\tilde{\epsilon}_B, \tilde{\delta}_B)]$$

[8] Recall that NSD and MSD denote Nth-order stochastic dominance and Mth-order stochastic dominance, respectively.

because $\tilde{\epsilon}_G$ dominates $\tilde{\epsilon}_B$ in the sense of NSD. We can prove (3) by determining the condition in which (4) holds. The condition (4) can be rewritten as follows:

$$(-1)^{n+1}E[f_{(n,0)}(\epsilon,\tilde{\delta}_G)] \geq (-1)^{n+1}E[f_{(n,0)}(\epsilon,\tilde{\delta}_B)] \text{ for all } n=1,2,\ldots,N. \quad (5)$$

Inequality (5) holds for

$$(-1)^{m+1}\frac{\partial^m\{(-1)^{n+1}f_{(n,0)}(\epsilon,\delta)\}}{\partial\delta^m} = (-1)^{n+m+2}f_{(n,m)}(\epsilon,\delta) = (-1)^{n+m}f_{(n,m)}(\epsilon,\delta) \geq 0$$

for all $n=1,2,\ldots,N$ and $m=1,2,\ldots,M$, because $\tilde{\delta}_G$ dominates $\tilde{\delta}_B$ in the sense of MSD. Thus, the proof is complete. (Q.E.D.)

A similar result can be obtained for Nth-degree risk.

Lemma 2. *For the payoff function $f(\epsilon,\delta)$, define*

$$\begin{aligned} E[f(\tilde{\epsilon},\tilde{\delta})] &= kpqE[f(\tilde{\epsilon}_B,\tilde{\delta}_B)] + (1-kp)qE[f(\tilde{\epsilon}_G,\tilde{\delta}_B)] \\ &\quad + p(1-kq)E[f(\tilde{\epsilon}_B,\tilde{\delta}_G)] + (1-p-q+kpq)E[f(\tilde{\epsilon}_G,\tilde{\delta}_G)]. \end{aligned}$$

Suppose that

- *$\tilde{\epsilon}_B$ has more Nth-degree risk than $\tilde{\epsilon}_G$;*
- *$\tilde{\delta}_B$ has more Mth-degree risk than $\tilde{\delta}_G$.*

If $(-1)^{N+M}f_{(N,M)} \geq (\leq) 0$, then $E[f(\tilde{\epsilon},\tilde{\delta})]$ is increasing (decreasing) in k.

Assume that future income risk is ranked by NSD and future health risk is ranked by MSD, that is,

- $\tilde{\epsilon}_G$ dominates $\tilde{\epsilon}_B$ in the sense of NSD;
- $\tilde{\delta}_G$ dominates $\tilde{\delta}_B$ in the sense of MSD.

It follows from Lemma 1 that

$$
\begin{aligned}
&v_s(s,k)\\
=&kpqE[u_{(1,0)}(w+s+\tilde{\epsilon}_B,h+\tilde{\delta}_B)]+(1-kp)qE[u_{(1,0)}(w+s+\tilde{\epsilon}_G,h+\tilde{\delta}_B)]\\
&+p(1-kq)E[u_{(1,0)}(w+s+\tilde{\epsilon}_B,h+\tilde{\delta}_G)]+(1-p-q+kpq)E[u_{(1,0)}(w+s+\tilde{\epsilon}_G,h+\tilde{\delta}_G)].
\end{aligned}
\tag{6}
$$

increases (decreases) in k for $(-1)^{n+m}u_{(n+1,m)}(x,y) \geq (\leq)0$ for all $n = 1, 2, \ldots, N$ and $m = 1, 2, \ldots, M$. Here, recall that v represents the expected utility at $t = 1$ given s and k.

Let us consider an individual whose correlation between future income risk and health risk is represented by k. We consider two cases where the correlation is low or high, denoted by k_L and k_H with $k_L \leq k_H$ and all other things equal. Note that the correlation increases in k. We denote s_L and s_H as optimal savings under k_L and k_H, respectively. Suppose that $(-1)^{n+m}u_{(n+1,m)}(x,y) \geq (\leq)0$ for all $n = 1, 2, \ldots, N$ and $m = 1, 2, \ldots, M$. We obtain the following inequality:

$$
\begin{aligned}
V_s(s_L,k_L) &= -u'(w-s_L,h)+v_s(s_L,k_L)=0\\
&\leq(\geq)-u'(w-s_L,h)+v_s(s_L,k_H)=V_s(s_L,k_H)\\
&\Leftrightarrow s_L \leq (\geq) s_H
\end{aligned}
\tag{7}
$$

The inequality follows from Lemma 1 where $f(\epsilon, \delta)$ is set to $u_{(1,0)}(w + s + \epsilon, h + \delta)$. Now, we can summarize the above argument into the following proposition. We refer to individuals as being mixed correlation averse (seeking) if the following holds:

$$(-1)^{n+m}u_{(n+1,m)}(x,y) \geq (\leq)0 \text{ for } n = 1,2,\ldots,N \text{ and } m = 1,2,\ldots,M.$$

Proposition 1. *Suppose that the future income risk is ranked by NSD and the future health risk is ranked by MSD, that is,*

- $\tilde{\epsilon}_G$ *dominates* $\tilde{\epsilon}_B$ *in the sense of NSD;*

- *$\tilde{\delta}_G$ dominates $\tilde{\delta}_B$ in the sense of MSD.*

If the following condition holds,

$$(-1)^{n+m}u_{(n+1,m)}(x,y) \geq (\leq) 0 \textit{ for } n = 1,2,\ldots,N \textit{ and } m = 1,2,\ldots,M,$$

then optimal savings increase (decrease) in k.

Let us consider the special case of $N = M = 1$. In this case, if $u_{(2,1)}(x,y) \geq (\leq)0$, then optimal savings increase in k. Following the terminology by Eeckhoudt et al. (2007), individuals are called cross prudent (imprudent) if $u_{(2,1)}(x,y) \geq (\leq)0$. We provide an interpretation of the condition on the sign of the cross derivatives and provide the intuition of this proposition in the next section.

6 Signs of cross derivatives

Following Eeckhoudt et al. (2007), we provide the intuition of the result in Proposition 1 by relating the signs of the cross derivatives to the precautionary saving against correlation. In the following analyses, we investigate individual's preferences for a 50–50 lottery. Let $[A, B]$ denote a lottery that pays either A or B with a probability of one-half. Cross prudence (imprudence) is characterized as a type of lottery preference.[9] An individual is cross prudent (imprudent) if the lottery $[(w+\tilde{\epsilon}, h), (w, h-c)]$ is preferred to the lottery $[(w, h), (w+\tilde{\epsilon}, h-c)]$ (the lottery $[(w, h), (w+\tilde{\epsilon}, h-c)]$ is preferred to the lottery $[(w+\tilde{\epsilon}, h), (w, h-c)]$) such that $h - c > 0$. Each outcome of the lottery occurs with equal probability. The first outcome is wealth level and the second outcome is health status. $\tilde{\epsilon}$ is zero-mean risk in wealth and $-c$ is sure loss in health. $\tilde{\epsilon}$ is considered to be "bad" compared with 0

[9] Strictly speaking, this definition corresponds to cross prudence (imprudence) *in health*. Cross prudence (imprudence) in wealth can be defined in a similar way. An individual is cross prudent (imprudent) in wealth if the lottery $[(w, h+\tilde{\delta}), (w-k, h)]$ is preferred to the lottery $[(w, h), (w-k, h+\tilde{\delta})]$ (the lottery $[(w, h), (w-k, h+\tilde{\delta})]$ is preferred to the lottery $[(w, h+\tilde{\delta}), (w-k, h)]$) such that $w - k > 0$. Eeckhoudt et al. (2007, p.120, Proposition 1) show that an individual is cross prudent (imprudent) in health if and only if $u_{(2,1)} \geq (\leq)0$ and that an individual is cross prudent (imprudent) in wealth if and only if $u_{(1,2)} \geq (\leq)0$. Because only the results about cross prudence (imprudence) in health appear in the following analyses, we omit the phrase "in health."

because $\tilde{\epsilon}$ is preferred to 0 by $u_{(2,0)} \leq 0$. Cross-prudent (imprudent) individuals prefer to accept noise risk $\tilde{\delta}$ to wealth in good (bad) health outcomes. Cross prudence (imprudence) displays a type of preference to receive one of the two bads for certain rather than both two bads or nothing (both two bads or nothing rather than one of the two bads for certain). This is referred to as a type of preference for combining good with bad (good) following Eeckhoudt and Schlesinger (2006) and Eeckhoudt et al. (2007).

Next, we see the intuition as to why cross-prudent individuals increase their savings as k increases. Cross-prudent individuals dislike occurring bad future income and health risks simultaneously. Because the simultaneous occurrence increases in k, cross-prudent individuals increase their savings to protect against the simultaneous occurrence of the two bad future risks. This leads to positive precautionary saving against correlation for cross-prudent individuals. The opposite intuition can be applied for cross-imprudent individuals.

Epstein and Tanny (1980) propose the notion of correlation aversion. An individual is correlation averse (seeking) if the lottery $[(w-k,h),(w,h-c)]$ is preferred to the lottery $[(w,h),(w-k,h-c)]$ (the lottery $[(w,h),(w-k,h-c)]$ is preferred to the lottery $[(w-k,h),(w,h-c)]$). Here, sure losses $-k$ and $-c$ are considered to be "bad." Eeckhoudt et al. (2007, p.120, Proposition 1) show that an individual is correlation averse (seeking) if and only if $u_{(1,1)} \leq (\geq) 0$.[10] Correlation aversion (seekingness) is a type of preference for combining good with bad (good with good). We can characterize higher-order versions of correlation aversion (seekingness), which are called mixed correlation aversion (seekingness). Cross prudence (imprudence) can be viewed as the third-order version of correlation aversion (seekingness).

Let us consider two risks $\tilde{\epsilon}_G$ and $\tilde{\epsilon}_B$ in wealth and assume that $\tilde{\epsilon}_G$ dominates $\tilde{\epsilon}_B$ in the sense of NSD. Then, individuals with $(-1)^{n+1}u_{(n,0)} \geq 0$ for $n = 1, 2, \ldots, N$ prefer $\tilde{\epsilon}_G$ to $\tilde{\epsilon}_B$, that is, $E[u(w+\tilde{\epsilon}_G,h)] \geq E[u(w+\tilde{\epsilon}_B,h)]$ for all w and h. The two random variables $\tilde{\epsilon}_G$ and $\tilde{\epsilon}_B$ can be viewed as good and bad, respectively. Let us

[10] See also Richard (1975, p.14, Theorem 1) and Epstein and Tanny (1980, p.20).

also consider two risks $\tilde{\delta}_G$ and $\tilde{\delta}_B$ in health and that $\tilde{\delta}_G$ dominates $\tilde{\delta}_B$ in the sense of MSD. Then, individuals with $(-1)^{m+1}u_{(0,m)} \geq 0$ for $m = 1, 2, \ldots, M$ prefer $\tilde{\delta}_G$ to $\tilde{\delta}_B$, that is, $E[u(w, h + \tilde{\delta}_G)] \geq E[u(w, h + \tilde{\delta}_B)]$ for all w and h. The two variables $\tilde{\delta}_G$ and $\tilde{\delta}_B$ can be viewed as good and bad, respectively.

An individual is called mixed correlation averse (seeking) if[11]

$$(-1)^{n+m+1}u_{(n,m)}(x, y) \geq (\leq) 0 \text{ for } n = 1, 2, \ldots, N \text{ and } m = 1, 2, \ldots, M.$$

Jokung (2011, p.449, Theorem 3) shows that if individuals are mixed correlation averse (seeking), then the lottery $[(w+\tilde{\epsilon}_G, h+\tilde{\delta}_B), (w+\tilde{\epsilon}_B, h+\tilde{\delta}_G)]$ is preferred to the lottery $[(w+\tilde{\epsilon}_G, h+\tilde{\delta}_G), (w+\tilde{\epsilon}_B, h+\tilde{\delta}_B)]$ (the lottery $[(w+\tilde{\epsilon}_G, h+\tilde{\delta}_G), (w+\tilde{\epsilon}_B, h+\tilde{\delta}_B)]$ is preferred to the lottery $[(w + \tilde{\epsilon}_G, h + \tilde{\delta}_B), (w + \tilde{\epsilon}_B, h + \tilde{\delta}_G)]$).[12]

As in correlation aversion (seekingness), mixed correlation aversion (seekingness) is a type of preference for combining good with bad (good). The condition in our proposition is mixed correlation aversion (seekingness) excluding correlation aversion (seekingness). We obtain the intuition of the result by the analogy of the case $N = M = 1$. Because the simultaneous occurrence of the two bad future risks increases in k, mixed correlation-averse individuals increase their savings to protect against the simultaneous occurrence. The opposite argument can be applied to mixed correlation-seeking individuals.

7 Ambiguous correlation

In sections above, we considered the case of risk, in the sense that the correlation is uniquely identified. In this section, we introduce ambiguity into correlation

[11] Although this property was first proposed by Jokung (2011), in this paper, it might be better called "mixed correlation averse (seeking)" based on the notion of being "mixed risk averse" by Caballé and Pomansky (1996).

[12] By introducing the notion of increasing concave order, Jokung (2011, p.449, Theorem 3) extends Eeckhoudt et al. (2009, p.997, Theorem 3) to the bivariate case. Eeckhoudt et al. (2009, p.997, Theorem 3) show that if $\tilde{X}$ dominates $\tilde{Y}$ in the sense of NSD and $\tilde{Z}$ dominates $\tilde{T}$ in the sense of MSD, then $[\tilde{X} + \tilde{T}, \tilde{Y} + \tilde{Z}]$ dominates $[\tilde{X} + \tilde{Z}, \tilde{Y} + \tilde{T}]$ in the sense of $(N + M)$th-order stochastic dominance.

through the parameter k. We assume that a plausible set of possible k is the set $\{k_1, k_2, \ldots, k_\Theta\}$. Without loss of generality, k_θ is arranged in ascending order, $k_1 < k_2 < \ldots < k_\Theta$. The individual attaches subjective probability q_θ to the parameter of k_θ for $\theta = 1, 2, \ldots, \Theta$. We assume that the individual follows the recursive version of the smooth ambiguity model by Klibanoff et al. (2005, 2009). Define an increasing and concave second-order utility function ϕ whose variable takes expected utility. The function ϕ is assumed to be thrice differentiable. The concavity of ϕ captures ambiguity aversion. Given s, the objective function is written as

$$V(s) = u(w - s, h) + \phi^{-1}(\sum_{\theta=1}^{\Theta} q_\theta \phi(v(s, k)). \tag{8}$$

Recall that $v(s, k)$ is the expected utility at $t = 1$ given s and k, that is,

$$\begin{aligned} & v(s, k) \\ = & \, kpqE[u(w + s + \tilde{\epsilon}_B, h + \tilde{\delta}_B)] + (1 - kp)qE[u(w + s + \tilde{\epsilon}_G, h + \tilde{\delta}_B)] \\ & + p(1 - kq)E[u(w + s + \tilde{\epsilon}_B, h + \tilde{\delta}_G)] + (1 - p - q + kpq)E[u(w + s + \tilde{\epsilon}_G, h + \tilde{\delta}_G)]. \end{aligned}$$

The first-order condition for (8) is

$$V'(s^*) = -u'(w - s^*, h) + \sum_{\theta=1}^{\Theta} q_\theta \frac{\phi'(v(s^*, k_\theta))}{\phi'(\phi^{-1}(\sum_\theta q_\theta \phi(v(s^*, k_\theta))))} v_s(s^*, k_\theta) = 0. \tag{9}$$

The second-order condition is easily verified by the concavity of u and ϕ.

We define k_O by $k_O = \sum_\theta q_\theta k_\theta$. The level of optimal savings is denoted s^O under k_O. To examine the effect of ambiguous correlation on optimal savings, we evaluate

(9) at s^O as follows:

$$
\begin{aligned}
V'(s^O) &= -u'(w-s^O,h) + \sum_\theta q_\theta \frac{\phi'(v(s^O,k_\theta))}{\phi'(\phi^{-1}(\sum_\theta q_\theta \phi(v(s^O,k_\theta))))} v_s(s^O,k_\theta) \\
&= -u'(w-s^O,h) + \frac{\sum_\theta q_\theta \phi'(v(s^O,k_\theta))}{\phi'(\phi^{-1}(\sum_\theta q_\theta \phi(v(s^O,k_\theta))))} \sum_\theta q_\theta v_s(s^O,k_\theta) \qquad (10) \\
&\quad + \frac{Cov(\phi'(v(s^O,k_\theta)), v_s(s^O,k_\theta))}{\phi'(\phi^{-1}(\sum_\theta q_\theta \phi(v(s^O,k_\theta))))}
\end{aligned}
$$

The second equality is obtained by applying $Cov(\tilde{x},\tilde{y}) = E[\tilde{x}\tilde{y}] - E[\tilde{x}]E[\tilde{y}]$ for all random variables $\tilde{x}$ and $\tilde{y}$.

The second term of (10) is called the timing of uncertainty effect by Osaki and Schlesinger (2014). Because the probability is linear in expected utility theory,

$$\sum_\theta q_\theta v_s(s^O,k_\theta) = v_s(s^O,k_O)(= -u'(w-s^O,h)).$$

When

$$\beta(s^O) = \frac{\sum_\theta q_\theta \phi'(v(s^O,k_\theta))}{\phi'(\phi^{-1}(\sum_\theta q_\theta \phi(v(s^O,k_\theta))))} \geq (<)1,$$

the effect implies that savings should increase (decrease) because the individual places more (less) weight on the future.

Let $\lambda(z) \equiv -\phi''(z)/\phi'(z)$ denote the coefficient of absolute ambiguity aversion defined by Klibanoff et al. (2005). If $\lambda(z)$ is a decreasing function in z, then the function ϕ exhibits *decreasing absolute ambiguity aversion* (DAAA). Following Osaki and Schlesinger (2014), it holds that $\beta(s^O) \geq 1$ if and only if ϕ exhibits DAAA.[13] The proof can be found in Appendix. We need to determine the sign of the covariance in the third term of (10) to see whether ambiguous correlation increases the amount of savings.

We assume for now that the future income risk is ordered by NSD and the future health risk is ordered by MSD. Applying Lemma 1 to $v(s,k)$ and $v_s(s,k)$, we have

[13] Note that $\beta(s^O) < 1$ if and only if ϕ exhibits strictly increasing absolute ambiguity aversion, which defines that $\lambda(z)$ is a strictly increasing function in z.

the following Lemma.

Lemma 3. *Suppose that*

- $\tilde{\epsilon}_G$ *dominates* $\tilde{\epsilon}_B$ *in the sense of NSD;*
- $\tilde{\delta}_G$ *dominates* $\tilde{\delta}_B$ *in the sense of MSD.*

When the individual is mixed correlation averse (seeking), that is, $(-1)^{n+m}u_{(n,m)} \geq (\leq) 0$ *for all* $n = 1, 2, \ldots, N$ *and* $m = 1, 2, \ldots, M$*, then* $v(s, k)$ *is increasing (decreasing) in* k *and* $v_s(s, k)$ *is decreasing (increasing) in* k*.*

Assume that u exhibits mixed correlation aversion or seeking. From Lemma 3, the signs of $\partial v(s^O, k)/\partial k$ and $\partial v_s(s^O, k)/\partial k$ are different. Because $\phi'(\cdot)$ is a decreasing function, the covariance is positive when u exhibits mixed correlation aversion or seeking. Combining the above arguments, we have the following proposition.

Proposition 2. *Suppose that the future income risk is ranked by NSD and the future health risk is ranked by MSD, that is,*

- $\tilde{\epsilon}_G$ *dominates* $\tilde{\epsilon}_B$ *in the sense of NSD;*
- $\tilde{\delta}_G$ *dominates* $\tilde{\delta}_B$ *in the sense of MSD.*

If ϕ *exhibits DAAA and* u *exhibits mixed correlation aversion or seeking, then ambiguous correlation raises the optimal amount of savings.*

Even though decreasing absolute risk aversion is a reasonable property, there are few experimental or empirical studies that examine the property of absolute ambiguity aversion to our knowledge. Berger and Bosetti (2020) are an exception and found experimental evidence on DAAA. We need additional evidence to conclude that absolute ambiguity aversion is either decreasing or increasing.

We provide the intuition for Proposition 2 under DAAA. Assume that $v(s^O, k_\theta)$ is decreasing in θ without loss of generality. Ambiguity averse individuals place more weight on higher θ, which is a worse correlation in the sense that expected utility is lower. When individuals increase their precautionary savings in the face of worse correlation, $v_s(s^O, k_\theta)$ is increasing in θ. Combining the above argument, ambiguous correlation raises savings to protect against the worse correlation. This case holds for mixed correlation aversion. The opposite case where $v(s^O, k_\theta)$ is increasing in θ and $v_s(s^O, k_\theta)$ is decreasing in θ holds for mixed correlation seeking.

The same result can be obtained for the case where the future income risk and the future health risk are replaced with Nth-degree risk and Mth-degree risk. First, we show the following lemma corresponding to Lemma 3.

Lemma 4. *Suppose that*

- $\tilde{\epsilon}_B$ *increases* $\tilde{\epsilon}_G$ *in* N*th-degree risk;*
- $\tilde{\delta}_B$ *increases* $\tilde{\delta}_G$ *in* M*th-degree risk.*

When $(-1)^{N+M} u_{(N,M)} \geq (\leq) 0$*,* $v(s, k)$ *is increasing (decreasing) in* k *and* $v_s(s, k)$ *is decreasing (increasing) in* k*.*

The same argument leads to the following proposition corresponding to Proposition 2.

Proposition 3. *Suppose that the future income risk is ranked by* N*th-degree risk and the future health risk is ranked by* M*th-degree risk, that is,*

- $\tilde{\epsilon}_B$ *increases* $\tilde{\epsilon}_G$ *in* N*th-degree risk;*
- $\tilde{\delta}_B$ *increases* $\tilde{\delta}_G$ *in* M*th-degree risk.*

If ϕ *exhibits DAAA and* u *satisfies* $(-1)^{N+M} u_{(N,M)} \geq (\leq) 0$ *and* $(-1)^{N+M} u_{(N+1,M)} \geq (\leq) 0$*, then ambiguous correlation raises the optimal amount of savings.*

We note that mixed correlation aversion (seeking) is a sufficient condition for $(-1)^{N+M} u_{(N,M)} \geq (\leq) 0$ and $(-1)^{N+M} u_{(N+1,M)} \geq (\leq) 0$ to hold when $N + M$ is odd (even). The intuition of Proposition 2 can be applied to Proposition 3.

8 Linkage between actual observations and theoretical results

There are two types of actual observations that can be related to our theoretical results:

- experimental observations about the signs of cross derivatives of multivariate preferences;
- empirical observations of household financial decisions in the presence of multivariate risks.

We consider the case of risk in the sense that correlation is uniquely identified. Based on the framework of Eeckhoudt and Schlesinger (2005), numerous studies have conducted an experimental analysis to test higher-order risk aversion in a one-dimension framework since Deck and Schlesinger (2010).[14] However, few studies test higher-order risk attitudes of multivariate preferences. Even though it is difficult to draw their concluding evidence, Attema et al. (2019) assume that the first attribute is the level of wealth and the second is the length of longevity which is viewed as a proxy of the condition of health. We apply the framework of Attema et al. (2019) in our analysis.[15] For ease of exposition, we consider a special case of $N = M = 1$, that is, both future income and health risks are ranked by the FSD. Proposition 1 in this paper states that the amount of optimal savings increases (decreases) in correlation if an individual is cross prudent (imprudent). Attema et al. (2019) observe both cross prudence and cross imprudence. From their experimental observations, we can conclude that the optimal amount of savings can increase or decrease in correlation. As a realistic case, let us assume that the wealth level and health status are positively correlated, $k > 1$. In this case, the optimal amount of savings is higher (lower) than

[14] Instead of referring to individual studies, we cite the review by Trautmann and van de Kuilen (2015).

[15] Ebert and van de Kuilen (2015) are another experimental study on this topic. They take the first attribute as the level of current wealth, and the second as time, the level of future wealth and the other's wealth

in the case of independence, $k = 1$, when an individual is cross prudent (imprudent).

Next, we consider the case of ambiguity in the sense that multiple correlations are perceived. As in the previous section, we assume that an individual is ambiguity averse and exhibits decreasing absolute ambiguity aversion. As in the case of risk, we assume that both the future income and health risks are ranked by FSD. From our Proposition 2, ambiguous correlation raises the optimal amount of savings if an individual is correlation averse and cross prudent or correlation seeking and cross imprudent in wealth and health. Attema et al. (2019) observe that most subjects exhibit correlation aversion in the gain domain and correlation seekingness in the loss domain. However, Attema et al. (2019) observe both cross prudence and imprudence in both the gain and loss domains. For example, when an individual exhibits correlation aversion and cross prudence, ambiguous correlation raises the optimal amount of savings. Following the experimental evidence of Attema et al. (2019), when health status is considered in the gain domain, ambiguous correlation raises the optimal amount of savings if an individual is cross prudent. However, for cross-imprudent individuals, we cannot obtain a definitive prediction, and ambiguous correlation might lower the optimal amount of savings.[16]

Finally, we discuss the literature that empirically investigates the effect of health conditions on household financial decisions. Rosen and Wu (2004) observe that poor health status leads to safer investment choices. These observations are consistent with cross prudence, which means positive precautionary savings against correlation. The subject of this paper, precautionary savings against correlation, is closely related to the literature on the role of the public health system in household financial decisions because it affects the correlation between wealth and health. The coverage rate of medical expenses depends on the public health system and its expansion can separate the riskiness of wealth and health to some extent, which means a decrease in correlation in our context. Atella et al. (2012) investigate the influence of current health conditions and future health risk on households' investments in risky assets

[16]Note that $Cov(\phi'(v(s^O, k_\theta)), v_s(s^O, k_\theta))$ is negative for correlation-averse and cross-imprudent individuals.

and empirically find that households are more willing to invest in risky assets when health risk is mitigated by a highly protective national health care system. Naranjo and Gameren (2016) find that the population of Mexicans aged from 50 to 75 years accumulate precautionary savings for fear that the future of the social security system is uncertain. Ayyagarari and He (2017) find that the reduction in prescription drug spending by the introduction of Medicare Part D in the US in 2006 increased risky investments. As a recent study related to Ayyagarari and He (2017), Christelis et al. (2020) empirically show that Medicare eligibility has quantitatively and statistically significant effects on stockholding for households with college degrees. It is observed that the public health system affects precautionary saving. Chou et al. (2003) observe that the introduction of public health insurance reduced the amount of precautionary savings in Taiwan. Jappelli et al. (2007) find that the amount of precautionary savings is higher in districts with lower health care quality in Italy. The former corresponds to a decrease in correlation between wealth and health, and the latter corresponds to an increase in correlation. These empirical results are consistent with cross prudence in our theoretical results, which is also observed in households' portfolio choices in the literature.

9 Conclusion

This paper provided a framework for analyzing the effect of correlation among risky attributes in multivariate preferences with higher-order risk changes on the optimal amount of savings. We investigated not only the situation in which a unique correlation is identified but also the situation in which multiple correlations are perceived. Under the risky situation, we showed that whether the optimal amount of savings increases or not depends on the signs of the derivatives of the utility functions. Furthermore, in the ambiguous situation, we showed that whether the optimal amount of savings increases or not depends on not only the signs of the derivatives of the utility functions (being mixed correlation averse (seeking)) but also the coefficient of absolute ambiguity aversion.

Future research should address several issues. As we mentioned above, it should be emphasized that experimental evidence is insufficient, and more studies are necessary to obtain firm observations on the properties of higher-order risk attitudes of multivariate preferences. From a theoretical perspective, an extension to multiperiod models is worth investigating. The results can be embedded into multiple-period models in a straightforward manner under the same settings. For example, we can investigate the situation where the value and the uncertainty of correlation change over time in multiperiod frameworks.

Appendix

We present the proof that shows that decreasing absolute ambiguity aversion is equivalent to $\beta(s) \geq 1$. A complete proof of this claim can be found in Osaki and Schlesinger (2014). We define the ambiguity premium π_A and ambiguity precautionary premium ψ_A, which are analogous to risk, as follows:

$$\sum_{\theta} q_\theta \phi(v(s, k_\theta)) = \phi[v(s, k_O) - \pi_A],$$

$$\sum_{\theta} q_\theta \phi'(v(s, k_\theta)) = \phi'[v(s, k_O) - \psi_A].$$

We can therefore rewrite $\beta(s)$ as

$$\beta(s) = \frac{\phi'[v(s, k_O) - \psi_A]}{\phi'[v(s, k_O) - \pi_A]}.$$

Here, we note that $\phi'[\phi^{-1}\phi[v(s, k_O) - \pi_A]] = \phi'[v(s, k_O) - \pi_A]$. Because $\phi'(\cdot)$ is a decreasing function because of ambiguity aversion, we have

$$\beta(s) \geq 1 \Leftrightarrow \psi_A \geq \pi_A.$$

Thus, we obtain that decreasing absolute ambiguity aversion is equivalent to $\psi_A \geq \pi_A$, which is an analogous property of decreasing absolute risk aversion in expected utility.

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