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"Theoretical Analysis of University Research and Teaching
in the Presence of External Research Funding"

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Theoretical Analysis of University Research and Teaching in the Presence of External Research Funding

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Abstract

This paper theoretically investigates how university research and teaching activities interact to generate research output and student enrollment under a setting in which a university earns tuition revenue and obtains external research funding. The main analytical finding is that while research funding can increase both research output and student enrollment when the tuition fee is fixed and university capacity is not fully used ("multiplier effect"), student enrollment is crowded out when a university operates at full capacity ("crowding-out effect"). In particular, this paper shows that when a tuition fee is controlled to maximize tuition revenue, a marginal amount of research funding never positively affect student enrollment due to the emergence of a "binary divide" among universities, namely, multiple equilibria generating a "large university" or a "small college."

Keywords: University research and teaching; Research output; Student enrollment; Research funding

JEL classification: I23; I28; O39

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1 Introduction

Most universities conduct research to acquire universal knowledge and teach students to enhance human capital, and these two activities exactly define what universities are. [1] Although several economics studies on universities have focused on the mechanisms of knowledge creation, the effects of knowledge diffusion from universities to industries, and the measures of promoting university research, [2] not many studies have investigated the complex interactions between research and teaching activities. Meanwhile, we recognize that research grant and tuition fee policies are highly likely to affect the achievement of both research output and student enrollment generated by universities through their research and teaching activities.

It is widely believed among developed countries that increased public research funding serves to produce more research output. Based on cross-sectional data of US universities and higher educational institutions (hereafter, "universities" as a whole) during 2011, Figure 1 exhibits a strong positive correlation between the total number of doctorates awarded (that is used as a proxy of research output) and federally funded general research and development (R&D) to both public and private universities. In addition, it appears from Figure 2 that total student enrollment is also positively correlated with R&D expenditure of universities and that large universities with many students enrolled have a propensity to invest more in R&D than small universities.

Certainly, we should note that these findings rely on simple correlation analysis and only indicate average tendencies across universities rather than any causal relationship. However, these facts motivate us to further probe the detailed interactions between university research and teaching activities and their effects on research output and student enrollment. More precisely, the intriguing thing is to address how and under what circumstances research output and student enrollment increase or decrease. In particular, this involves theoretically investigating how research output and student enrollment respond to a change in policy measures such as external research funding and tuition fee setting could be a major issue.

[1] Various definitions of universities have been presented. Haskins (1957) finds that modern universities have their roots in encouraging researchers to study disciplines to seek truth and knowledge. On the other hand, Mill (1867) indicates in his famed speech the importance of university education and the acquisition of specialist knowledge there.

[2] Representative empirical works of university knowledge diffusion include those of Jaffe (1989), Henderson, Jaffe, and Trajtenberg (1998), and Lach and Schankerman (2008). In addition, Foray and Lissoni (2010) comprehensively survey the topics of university research and its knowledge creation.

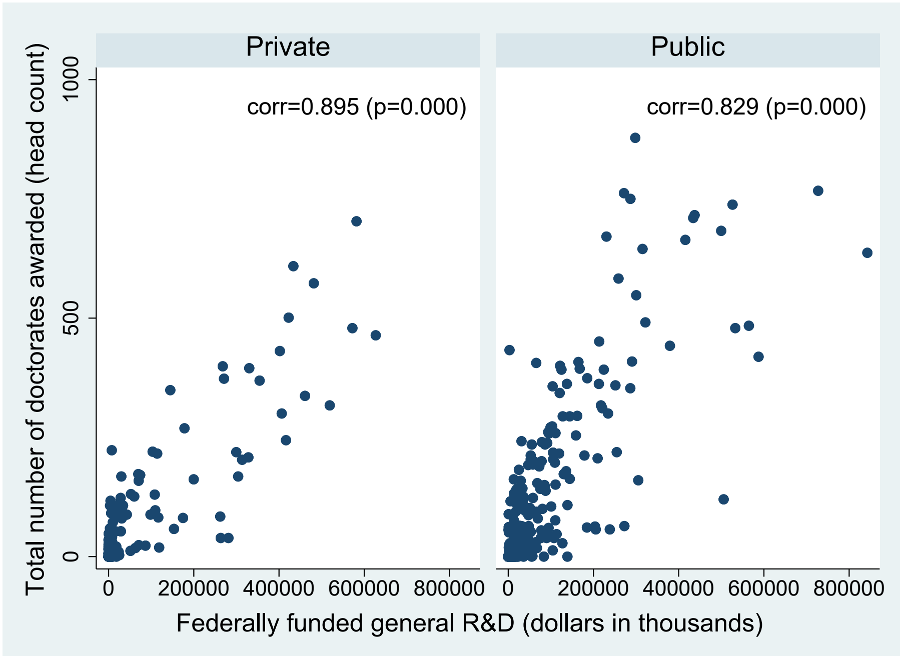

FIGURE 1. US federally funded general R&D vs. total number of doctorates awarded in 2011 (Source: National Center for Science Engineering Statistics).

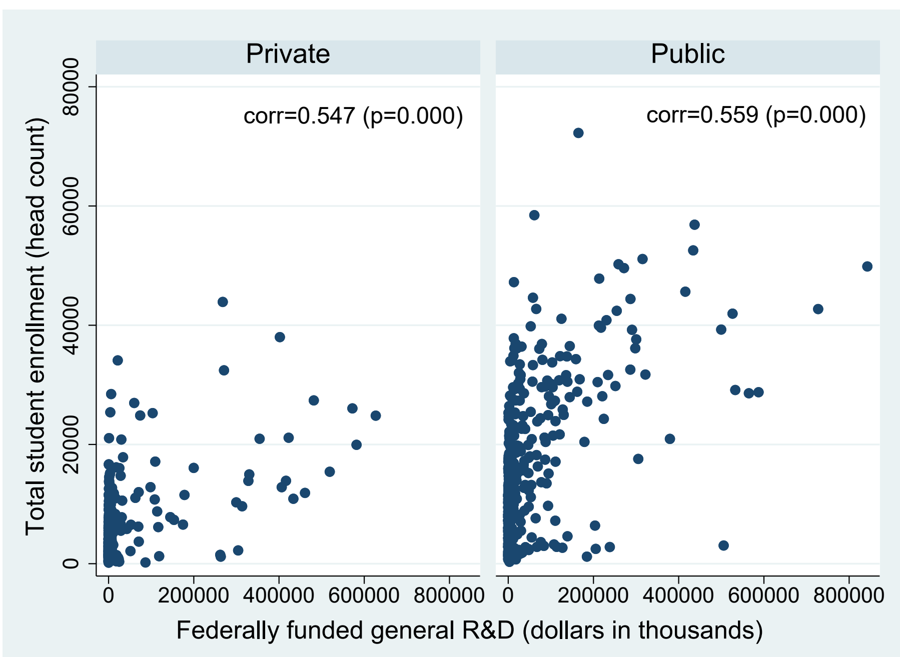

FIGURE 2. US federally funded general R&D vs. total student enrollment in 2011 (Source: National Center for Science Engineering Statistics).

Some authors have addressed the issue of multitasking universities that face tension between research and teaching activities. Del Rey (2001) analyzes a model in which two competing universities conduct both research and teaching activities and are financed by the government. She shows that depending on the parameters, such as the financing scheme, teaching efficiency, and relative weights of research activity, the model generates multiple equilibria in which universities conduct only teaching (or research) and teach selective (or mass) students. In addition, De Fraja and Iossa (2002), supposing that the prestige of universities relies on the number of students enrolled and their research outcomes, derive several equilibrium configurations associated with varying student mobility costs.

Although these studies regard research activity as a "residual" of universities' total capacities, Beath, Poyago-Theotoky, and Ulph (2012) treat research as a trade-off with teaching. By allowing universities to voluntarily choose the quality level of research and teaching, Beath, Poyago-Theotoky, and Ulph (2012) demonstrate that when a government funding system is used as a tool to control university research incentives, a variety of university cultures may emerge, such as research-oriented and teaching-oriented universities. De Fraja and Valbonesi (2012) compare research and teaching distributions among universities according to university management policies – the unregulated private provision policy versus the government intervention policy. They argue that while the former policy inefficiently allows the spreading of research across all universities, the latter system can efficiently concentrate research and teaching on fewer, more productive universities.

From a different perspective, Gautier and Wauthy (2007) investigate an incentive problem within a university that needs to govern research and teaching conducted by its individual departments and redistribute an aggregate tuition revenue. The authors posit that the university evaluates its departments based solely on their research output and find a trade-off problem. In other words, research activity can be increased due to yardstick competition caused by this assessment policy, while teaching activities can be decreased due to free-riding by departments that cannot appropriate their own tuition revenues. Thus, they pointed out that both activities can be promoted when the departments are integrated into a multiunit institution with natural complementarity between research and teaching activities, both activities.

Based on these existing studies, this paper constructs a microfounded university-student model with a trade-off between university research and teaching activities. The baseline model assumes that a single university intends to maximize its academic prestige from research output given fixed external research funding while conducting both research and teaching activities. Teaching is not assumed to be the university's ultimate goal, although it potentially contributes to its research budget through tuition revenue. Instead, by evaluating teaching offered by a university, innumerable students decide whether to attend the university if their benefits exceed their costs. Therefore, a university needs to draw a fine balance between research and teaching activities to earn tuition revenue that can also be exploited as a research resource.

The capacity constraint, which limits a university's total activities irrespective of its research funding monies, carries a critical meaning in this modeling. This capacity can be regarded as a limitation of "ability" inherent to a university. Due to this constraint, a university is compelled to allocate its limited capacity to research and teaching activities appropriately. Since it is usually difficult to enhance university capacity in the short run, these two activities are jointly limited at some level. Therefore, if a university intends to increase its research activity, it may need to decrease its teaching activity instead (that is, a trade-off relation). In the long run, improvement can occur that strengthen university capacity, but we do not explicitly consider such a long-run effect and treat the capacity constraint as exogenous.

The findings of this paper are summarized in what follows. In the first place, the element of substitutability between research and teaching activities can be critical, in that one activity can increase the cost of the other activity. If such substitutability is strong enough, student enrollment and research output can be reduced in response to an increase in external research funds. This seemingly paradoxical argument is deliberately demonstrated in a general model as a likely scenario.

Subsequently, assuming that the degree of substitutability is zero for analytical simplicity, this paper illustrates that the results depend not only on whether the capacity of a university is fully utilized but also on whether a tuition fee is fixed or controlled. More precisely, in the case of a fixed tuition fee, while research funding can increase both research output and student enrollment when the university capacity is not fully used ("multiplier effect"), student enrollment is crowded out when a university operates at full capacity ("crowding-out effect"). This former result is not surprising because research funding allows teaching activity when the capacity constraint is slack. However, if the capacity constraint is binding, research funding reduces teaching activity because a university favors more research output to obtain a higher payoff.

This paper also reveals that when a government controls a tuition fee to maximize tuition revenue, a marginal amount of research funding is never expected to positively affect student enrollment due to the emergence of a a "binary divide" among a universities. This binary divide means multiple equilibria depending on capacity size; while a "large university" operates at full capacity, a "small college" conducts marginal research and teaching activities. The mechanism is briefly described as follows. When a tuition fee is a controlled variable set for maximizing tuition revenue, it is optimal that the tuition fee rises in parallel with teaching activity because the decrease in student enrollment can be compensated by enhancing teaching activity. But since the positive effect on tuition revenue is relatively modest for low-level teaching activity, the payoff of the university will decline as teaching activity is augmented to some point and would then increase beyond that point (mathematically, a saddle point). For this reason, if the capacity is small (large), it is rational for a university to select its teaching activity at the minimal (maximal) level.

While sharing some similarities with earlier works, this study differs on some points. First, although the model includes only a single university in the baseline model in contrast to other studies (Del Rey, 2001; De Fraja and Iossa, 2002; De Fraja and Valbonesi, 2012), we obtain some new findings regarding the effect of research funding on both research output and student enrollment. [3] Second, this study allows students to endogenously make their own decisions in a multistage game as to whether they attend a university considering the level of teaching so that their decisions affect the research and teaching activities of a university. Third, this study focuses on a tuition fee that is a key source of university revenue, while other studies have not conducted such a thorough investigation. It is noticeable to distinguish the case where a tuition fee is exogenously fixed from the case where it is endogenously controlled to maximize tuition revenue, which illustrates that the implications for providing research funding can be entirely different between them. Finally, this study explicitly considers the capacity limitation of a university to undertake research and teaching activities. It is made clear that this capacity limitation influences the action taken by a university in conjunction with the above tuition fee schemes.

Meanwhile, this paper omits some important aspects to which other authors have drawn attention. While Beath, Poyago-Theotoky, and Ulph (2012) and Gautier and Wauthy (2007) relate the distribution of research funds to university research productivities, this paper does not consider the productivity issue because the model posits a single university. Likewise, this paper does not assume competition among universities seeking research funding. However, since it is obvious that research funding should be preferentially allocated to the most research- productive universities under many general scenarios, this analysis does not address the allocation problem of research funding. Rather, we focus exclusively on a more simplistic analysis of the interplays between university research and teaching activities in the presence of external research funding.

The reminder of this paper is as follows. Section 2 outlines a model structure and describes the decisions of a university and students, and Section 3 derives a fundamental theoretical result. Section 4 assumes a controlled tuition fee and compares the results with those derived from a fixed tuition fee. Sections 3 and 4 first introduce a general model framework and subsequently present illustrative cases. Section 5 makes concluding remarks. All the mathematical proofs are compiled in Section 6.

[3] The previous working paper (Ambashi, 2019) extends the model in which multiple universities compete in seeking students through their teaching activity in the same jurisdiction and compares the results with a single university.

2 Basic Model Outline

The objective of this section is to formulate a general university-student model before deriving the theoretical results based on the specific parameterized model described in Sections 3 and 4. This general model is expected to provide a favorable outlook of the theoretical results throughout this paper.

2.1 Players

In a particular jurisdiction, there exists a single university and numerous prospective students. The details of how each player behaves in this model are described below.

2.1.1 University

To conduct its activities, a university needs to input positive research and teaching efforts, $r > 0$ and $t > 0$, respectively, represented as $\boldsymbol{e} = (r, t)$. The research and teaching efforts can be interpreted as activity levels of a university, for example, improving research environments and training teaching staff, respectively.

A university is assumed to be constrained by a finite capacity, $\overline{a} > 0$, the level of which is defined by $r + t \leq \overline{a}$ for any r and t. [4] In other words, since the capacity exogenously specifies an upper bound of the total effort, it is also regarded as an inherent ability of a university in the short run.. [5] Hereafter, we particularly focus on the short-run framework, in which the capacity limitation is fixed. In this theoretical analysis, the model intends to derive conditions of a capacity scale required to produce desired research and teaching activities. Rightfully, this implies that the improvement in a capacity matters in the long-run framework.

Research output, R, is determined by both research effort and the total budget the university makes readily available for research activity. This relation is represented by $R = R(r, b)$, where b is the research budget. Suppose that the research effort is separable from the research budget and that the research output function is determined by the simple product of the two: $R = rb$. Although the research output function can be assumed to exhibit diminishing returns to scale for a research budget, this allows us to greatly simplify the following analyses without losing the essence of the discussion. Or the capacity limitation, $\overline{a}$, can be viewed as supplementing the assumption of diminishing returns to scale.

[4]As defined in the cost function later, it is also possible to state that the cost becomes infinite beyond the capacity level, $\overline{a}$.

[5]Irrespective of whether monetary resources are abundant, it is difficult to immediately enhance a university capacity in the short run, for example, by constructing new campuses or hiring more highly qualified university faculty members, which are all likely to reinforce the intrinsic ability of a university.

The budget, b, consists not only of external research funding but also of tuition revenue. It is denoted by $b = F + sn$, where $F > 0$ is research funding allocation, s is a tuition fee per student, and n is the number of students enrolled. Accordingly, other things being equal, a higher student enrollment could raise research output through an increase in the research budget of a university.

Research and teaching activities inevitably involve costs, such as establishing experimental instruments in research labs and hiring professional teaching staff. The cost function is represented by $C = C(r, t)$ with $\frac{\partial C}{\partial r} > 0$ and $\frac{\partial C}{\partial t} > 0$, which is typically assumed continuous and higher-order differentiable at any point. It is postulated that $C(r, t)$ is a strictly convex function: $\frac{\partial^2 C}{\partial r^2} > 0$, $\frac{\partial^2 C}{\partial t^2} > 0$, and $(\frac{\partial^2 C}{\partial r^2})(\frac{\partial^2 C}{\partial t^2}) - (\frac{\partial^2 C}{\partial r \partial t})^2 > 0$. This condition is that the Hessian matrix of $C(r, t)$ is a positive definite. It is also assumed that $\frac{\partial^3 C}{\partial r^3} = \frac{\partial^3 C}{\partial t^3} = 0$. The sign of the cross-derivative for research and teaching efforts, $\frac{\partial^2 C}{\partial r \partial t}$, is not obvious, depending on whether the effort is a substitute, complement, or independent. If the efforts are a substitute (complement) in terms of the cost function, we can maintain that $\frac{\partial^2 C}{\partial r \partial t} > 0 (< 0)$. That is, research and teaching activities being reciprocal substitutes (complements) suggests that an increase in one activity increases (or reduces) the marginal cost of the other so that negative (positive) externalities exist between them.

The payoff of a university is assumed to be determined by the value of its research output minus the costs of its efforts:

$$U(r, t) = R(r, b) - C(r, t) = rb - C(r, t), \quad \text{where } b = F + sn. \tag{1}$$

It is reasonable to postulate that the revenue must exceed the cost of research and teaching efforts, and therefore, $U(r, t) \geq 0$ must be guaranteed.

As De Fraja and Valbonesi (2012) highlight, the ultimate goal of universities is assumed to achieve so-called academic “prestige" by producing research output, but not teaching outcome. Our modeling assumption is also based up their idea. [6] Yet this assumption does not necessarily mean that the university underrates teaching activity. Rather, it is contemplated that a university views teaching activity as indirectly affecting its prestige by increasing its research budget to be used for research activity. Based on this simple model, a university maximizes the payoff function given by Equation (1) for both research and teaching efforts, r and t, respectively.

[6] In the US evaluation system of higher education, universities are provided with strong incentives to undertake outstanding research, because they heavily depend on external research funds for a great part of their research budgets. In this system, if a university does not produce satisfactory research output, it cannot obtain any research funding and faces difficulties in continuing research activity.

2.1.2 Students

The decision to enroll made by students rests on the teaching effort, t, of the university. The reason for this is, for example, that if students accumulate a sufficient amount of human capital through quality university teaching, they can gain an advantage in obtaining better jobs after graduation.

Since a mobility cost, $k > 0$, and a tuition fee, $s > 0$, are also highly likely to affect the student decisions, student enrollment, $n = n(t, k, s)$, is assumed. Note that we do not have to literally interpret k as indicating some physical distance between the location of a particular student and the university. Rather, it could be that k represents the difficulties of entrance examinations or of getting caught up with their studies after gaining admittance, both of which impose some kind of psychological burden on students. Moreover, a tuition fee is normally expected to negatively affect student enrollment. If a university's tuition fee rises, students will cease admission or choose to attend another university outside the jurisdiction.

Finally, assuming that $n(t, k, s)$ is continuous and higher-order differentiable at any points, we postulate $\frac{\partial n}{\partial t} \geq 0$, $\frac{\partial n}{\partial k} \leq 0$, $\frac{\partial n}{\partial s} \leq 0$, $\frac{\partial^2 n}{\partial t^2} \leq 0$, $\frac{\partial^2 n}{\partial t \partial k} \leq 0$, and $\frac{\partial^2 n}{\partial t \partial s} \leq 0$.

2.1.3 Financing agency

It is assumed that a governmental financing agency allocates a constant amount of research funding, $F > 0$, to a university. In a single-shot research model as considered here, there are no links in this model between consequential research output and a future research funding. We have to also take note that since only a single university exists in this analysis, there is no allocation problem with research funding.

2.2 Timing of the model

The model framework is described based on a multi-stage game. The timing of the model as follows.

0. A financing agency allocates a research funding, F, to a university.
1. A university chooses its research and teaching efforts, $\boldsymbol{e} = (r, t)$, respectively.
2. Students choose whether to attend a university.
3. The payoffs to the university and students are realized.

The game is solved by backward induction to find a subgame-perfect equilibrium.

2.3 Equilibrium solution

A university maximizes its payoff given that student enrollment is obtained in Stage 2:

$$\max_{r,t} U(r,t) = rb - C(r,t) \text{ subject to } b = F + sn(t,k,s). \tag{2}$$

From Equation (2), the payoff of a university is abbreviated into $U(r,t) = r[F + sn(t,k,s)] - C(r,t)$. On the assumption of a positive interior solution ($\boldsymbol{e}^* = (r^*, t^*) > \mathbf{0}$ and $r^* + t^* < \bar{a}$ are satisfied), the first-order condition is formulated as follows:

$$\frac{\partial U(r,t)}{\partial r} = F + sn(t,k,s) - \frac{\partial C(r,t)}{\partial r} = 0, \tag{3}$$

$$\frac{\partial U(r,t)}{\partial t} = rs\left[\frac{\partial n(t,k,s)}{\partial t}\right] - \frac{\partial C(r,t)}{\partial t} = 0. \tag{4}$$

$\boldsymbol{e}^* = (r^*, t^*)$ satisfies both Equations (3) and (4). From here on, the variables of the functions are abbreviated for descriptive simplicity. To secure a global maximum solution, we confirm whether the second-order condition is satisfied using a Hessian matrix of $U(r,t)$:

$$\tilde{U} = \begin{bmatrix} \frac{\partial^2 U}{\partial r^2} & \frac{\partial^2 U}{\partial r \partial t} \\ \frac{\partial^2 U}{\partial t \partial r} & \frac{\partial^2 U}{\partial t^2} \end{bmatrix} = \begin{bmatrix} -\frac{\partial^2 C}{\partial r^2} & s(\frac{\partial n}{\partial t}) - \frac{\partial^2 C}{\partial r \partial t} \\ s(\frac{\partial n}{\partial t}) - \frac{\partial^2 C}{\partial r \partial t} & -\frac{\partial^2 C}{\partial t^2} \end{bmatrix}.$$

From the assumption of the cost function, we obtain $-\frac{\partial^2 C}{\partial r^2} < 0$, $-\frac{\partial^2 C}{\partial t^2} < 0$. If it is posited that the determinant of $\tilde{U}$ is positive like $|\tilde{U}| = (\frac{\partial^2 C}{\partial r^2})(\frac{\partial^2 C}{\partial t^2}) - [\frac{\partial^2 C}{\partial r \partial t} - s(\frac{\partial n}{\partial t})]^2 > 0$, the payoff function is strictly concave. Hence, $\boldsymbol{e}^* = (r^*, t^*)$ induces a global maximum. The equilibrium student enrollment and research output are defined as $n^* = n(t^*, k, s)$ and $R^* = r^*(F + sn^*)$, respectively. To observe a change in the endogenous variables in an interior point, we suppose that student underenrollment occurs (that is, some students do not apply for admission to a university).

2.4 Comparative statics of a research fund

An interesting undertaking is to analyze the effect of research funding on research output and student enrollment when substitutability (or complementarity) exists. From this standpoint, let us direct our attention to examining comparative statics of research output and student enrollment with respect to an increase in research funding. [7] We take the derivatives on both sides of Equations (3) and (4) by F, and obtain the following matrix notation:

[7] See Subsection 6.2 for the comparative statics with respect to a mobility cost and tuition fee.

$$\begin{bmatrix} \frac{\partial^2 C}{\partial r^2} & \frac{\partial^2 C}{\partial r \partial t} - s(\frac{\partial n}{\partial t}) \\ \frac{\partial^2 C}{\partial r \partial t} - s(\frac{\partial n}{\partial t}) & \frac{\partial^2 C}{\partial t^2} - r^* s(\frac{\partial^2 n}{\partial t^2}) \end{bmatrix} \begin{bmatrix} \frac{\partial r^*}{\partial F} \\ \frac{\partial t^*}{\partial F} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}. \tag{5}$$

Let us denote the first matrix of Equation (5) as A_F. Its determinant is calculated as: $|A_F| = (\frac{\partial^2 C}{\partial r^2})[\frac{\partial^2 C}{\partial t^2} - r^* s(\frac{\partial^2 n}{\partial t^2})] - [\frac{\partial^2 C}{\partial r \partial t} - s(\frac{\partial n}{\partial t})]^2$. In this analysis, while $|A_F| > 0$ is assumed, it is always satisfied when $\frac{\partial^2 C}{\partial r \partial t} > 0$ holds. Using Cramer's rule, we can obtain the following solution of the simultaneous equations:

$$\frac{\partial r^*}{\partial F} = \frac{\begin{vmatrix} 1 & \frac{\partial^2 C}{\partial r \partial t} - s(\frac{\partial n}{\partial t}) \\ 0 & \frac{\partial^2 C}{\partial t^2} - r^* s(\frac{\partial^2 n}{\partial t^2}) \end{vmatrix}}{|A_F|} = \frac{1}{|A_F|}\left[\frac{\partial^2 C}{\partial t^2} - r^* s\left(\frac{\partial^2 n}{\partial t^2}\right)\right] > 0, \tag{6}$$

$$\frac{\partial t^*}{\partial F} = \frac{\begin{vmatrix} \frac{\partial^2 C}{\partial r^2} & 1 \\ \frac{\partial^2 C}{\partial r \partial t} - s(\frac{\partial n}{\partial t}) & 0 \end{vmatrix}}{|A_F|} = \frac{1}{|A_F|}\left[s\left(\frac{\partial n}{\partial t}\right) - \frac{\partial^2 C}{\partial r \partial t}\right]. \tag{7}$$

Increased research funding always produces more research effort (Equation [6]). In addition, if research and teaching activities are complements ($\frac{\partial^2 C}{\partial r \partial t} < 0$), we necessarily obtain $\frac{\partial t^*}{\partial F} > 0$ (Equation [7]). If substitutability is strong enough such that $\frac{\partial^2 C}{\partial r \partial t} > s(\frac{\partial n}{\partial t})$, we derive $\frac{\partial t^*}{\partial F} < 0$. Importantly, this negative effect on teaching effort may reduce not only student enrollment but also research output in extreme cases. Proposition 1 summarizes the results of comparative statics regarding a change in research funding.

Proposition 1 With respect to the effect of increased research funding, F, on research and teaching activities, we can obtain the following:
(1) $\frac{\partial r^*}{\partial F} > 0$ holds for any $\frac{\partial^2 C}{\partial r \partial t}$ (irrespective of substitutability or complementarity);
(2-i) $\frac{\partial t^*}{\partial F} > 0$, $\frac{\partial n^*}{\partial F} > 0$, and $\frac{\partial R^*}{\partial F} > 0$ for $\frac{\partial^2 C}{\partial r \partial t} < s(\frac{\partial n}{\partial t})$; (2-ii) $\frac{\partial t^*}{\partial F} < 0$ and $\frac{\partial n^*}{\partial F} < 0$ for $\frac{\partial^2 C}{\partial r \partial t} > s(\frac{\partial n}{\partial t})$; and
(3) $\frac{\partial R^*}{\partial F} < 0$ for $\frac{\partial^2 C}{\partial r \partial t} > s(\frac{\partial n}{\partial t}) + \Omega$, where $\Omega = \frac{|A_F| r^* + [\frac{\partial^2 C}{\partial t^2} - r^* s(\frac{\partial^2 n}{\partial t^2})](F + sn^*)}{r^* s(\frac{\partial n}{\partial t})} > 0$.

Increased research funding entices a university to generate greater research effort due to the enriched research budget (Proposition 1[1]). Because of this additional research budget, a university normally finds it more profitable to devote more teaching effort, too, if substitutability is small enough (Proposition 1[2-i]). It is also expected that as student enrollment increases, the research budget gets larger due to increased tuition revenue. As a result, a university is likely to produce more research output.

However, the effects of research funding on teaching effort and student enrollment are not uniform according to substitutability between research and teaching activities. If substitutability is strong enough to bring about additional unwanted costs, teaching effort is decreased by contraries and followed by a decline in student enrollment (Proposition 1[2-ii]). Furthermore, the result of Proposition 1(3) is paradoxical and much more controversial; increased research funding may lead to a decrease in research output, which entirely contradicts the common notion. The intuition is explained as follows. When research and teaching activities are reciprocally strong substitutes, a decrease in teaching effort leads to reduced student enrollment. Since tuition revenue earned from students is also greatly reduced, less research output may be produced due to the smaller research budget appropriated for research activity. Consequently, it can be theoretically demonstrated that for strong substitutability, increased research funding may decrease a university's research output. Proposition 1(3) is a seemingly paradoxical result, yet the theory is quite indicative.

3 Modeling of Illustrative Case

3.1 Analysis when substitutability exists

In Section 3, we investigate an illustrative case to derive explicit solutions for the model by parameterizing the formulations. The following parameterization is a mere benchmark; however, since it satisfies an important qualitative nature, the parameterization certainly allows us to illustrate the concrete behavior of the model.

As in the previous section, the research output function is defined as $R = rb$, where $b = F + sn$. We suppose that the cost function takes the form $C(r,t) = \frac{r^2}{2} + \frac{t^2}{2} + \varepsilon rt$. This choice encompasses research and teaching efforts to affect cost through the interaction term, εrt. The element ε represents the substitutability (or complementarity) between university research and teaching efforts in terms of the cost function. More precisely, while research and teaching efforts are mutually a substitute for $\varepsilon > 0$, it is mutually a complement for $\varepsilon < 0$. This cost function satisfies the previous conditions assumed in Section 2: $\frac{\partial C}{\partial r} = r > 0$, $\frac{\partial C}{\partial t} = t > 0$, $\frac{\partial^2 C}{\partial r^2} = \frac{\partial^2 C}{\partial t^2} = 1 > 0$, and $\frac{\partial^3 C}{\partial r^3} = \frac{\partial^3 C}{\partial t^3} = 0$. But $(\frac{\partial^2 C}{\partial r^2})(\frac{\partial^2 C}{\partial t^2}) - (\frac{\partial^2 C}{\partial r \partial t})^2 = 1 - \varepsilon^2 > 0$ for $-1 < \varepsilon < 1$ is also required for the convexity of $C(r,t)$ (that is, ε needs to be bounded).

Next, in the student market, students are assumed to be evenly distributed over a horizontal line, the length of which is normalized to 1. The university is located at the middle point ($\frac{1}{2}$) of this line. In Hotelling's model (Hotelling, 1929), players such as firms or shops determine their locations to differentiate their products from others. By way of contrast, our model postulates a fixed university location at the middle point, which implies that a university is assumed to be situated in a balanced place within the jurisdiction. Since it is not easy for a university to move physically in the short run, the assumption of a fixed university also appears reasonable.

Thus, we formulate the utility function of a student located at $x < \frac{1}{2}$, such as

$$u = t - s - k\left(\frac{1}{2} - x\right). \tag{8}$$

Whether the mobility cost is linear or nonlinear is critical when the university chooses its location, but it does not affect the nature of the analysis in this fixed location model. Equation (8) assumes that the tuition fee linearly affects the utility of students.

Assuming without loss of generality that the outside option other than enrolling at the university in this jurisdiction gives each student zero utility ($u = 0$), we can find a particular $\hat{x}$ who is indifferent between enrolling and not. This condition satisfies $t - s - k\left(\frac{1}{2} - \hat{x}\right) = 0$, which implies $\hat{x} = \frac{1}{2} + \frac{s-t}{k}$. It can be easily shown that because of the symmetric characteristics, $\hat{x} = \frac{1}{2} + \frac{t-s}{k}$ for students who are located at $x > \frac{1}{2}$. From these, a total student enrollment can be represented as $n = n(t, k, s) = \frac{2(t-s)}{k}$.

Since $n \in [0, 1]$ is assumed, t must be bounded such that $t \in [s, s + \frac{k}{2}]$. The condition, $t > s$, implies that the teaching value should be larger than the tuition fee for a university to obtain a positive student enrollment. By contrast, if $t \leq s$, a university cannot obtain any students ($n = 0$). In addition, even if teaching effort is excessive to the point that $t \geq s + \frac{k}{2}$, the intake of students cannot be higher than 1 ($n = 1$). As expected, $\frac{\partial n}{\partial t} = \frac{2}{k} > 0$, $\frac{\partial n}{\partial k} = -\frac{2(t-s)}{k^2} \leq 0$, $\frac{\partial n}{\partial s} = -\frac{2}{k} < 0$, $\frac{\partial^2 n}{\partial t} = 0$, $\frac{\partial n^2}{\partial t \partial k} = -\frac{1}{k^2} < 0$, and $\frac{\partial^2 n}{\partial t \partial s} = 0$ are confirmed, which also satisfies the previous assumptions.

Based on this setting, a university maximizes its payoff given Stage 2 (the decision of students):

$$\max_{r,t} U(r, t) = rb - \left(\frac{r^2}{2} + \frac{t^2}{2} + \varepsilon rt\right) \text{ s.t. } b = F + sn \text{ and } n = \frac{2(t-s)}{k}. \tag{9}$$

From Equation (9), the payoff function of a university is abbreviated into $U(r, t) = r[F + \frac{2s(t-s)}{k}] - (\frac{r^2}{2} + \frac{t^2}{2} + \varepsilon rt)$. The first-order condition of maximizing $U(r, t)$ with respect to r and t is formulated as follows:

$$\frac{\partial U}{\partial r} = b - r - \varepsilon t = 0, \tag{10}$$

$$\frac{\partial U}{\partial t} = r\left(\frac{2s}{k}\right) - t - \varepsilon r = 0. \tag{11}$$

Let us define the solution of Equations (10) and (11) as $\hat{\boldsymbol{e}} = (\hat{r}, \hat{t})$. Although the calculated result for $\hat{r}$ and $\hat{t}$ may be strictly negative, research and teaching efforts cannot be negative in principle. We are interested only in the case where both types of efforts are positive. One reason for this assumption can be supported by the idea that a financing agency usually provides the minimum research funding needed to enable a positive level of research and teaching activities. Meanwhile, we have to examine the second-order condition of the maximization problem. Generally, the payoff function must be a strictly concave function to have a global maximum. For strict concavity, $(1-\varepsilon^2)k^2 + 4ks\varepsilon - 4s^2 < 0$, that is, $\varepsilon \in (-1 + \frac{2s}{k}, 1 + \frac{2s}{k})$ is required, which suggests that ε must be bounded both upward and downward. [8] Under a normal condition, it is expected that the substitutability (complementarity) cannot diverge to infinity and thus falls within a finite range. The analysis proceeds assuring that this second-order condition is always satisfied.

Lemma 1 The solution, $\hat{e} = (\hat{r}, \hat{t})$, has a closed form such that:

$$\hat{r} = \frac{k(kF - 2s^2)}{(1-\varepsilon^2)k^2 + 4ks\varepsilon - 4s^2}, \tag{12}$$

$$\hat{t} = \frac{(2s - k\varepsilon)(kF - 2s^2)}{(1-\varepsilon^2)k^2 + 4ks\varepsilon - 4s^2}. \tag{13}$$

When both conditions $F > \frac{2s^2}{k}$ and $\varepsilon \in (-1 + \frac{2s}{k}, \frac{2s}{k})$ are satisfied, $\hat{e} = (\hat{r}, \hat{t})$ is a positive interior equilibrium solution with $\hat{r} > 0$ and $\hat{t} > 0$.

The first condition of Lemma 1 is that research funding is sufficiently large compared with a tuition fee (discounted by a mobility cost). This implies that unless research funding is extremely small, a university can exert a positive research effort. Moreover, the second condition for the substitutability stipulates a bounded range that is narrower upward than that assumed before ($\varepsilon < 1 + \frac{2s}{k}$). [9]

Our interest largely lies in a positive interior solution, in which a university undertakes strictly positive research and teaching efforts. For the solution presented in Equations (12) and (13), do research and teaching efforts increase as the substitutability between these activities becomes smaller? Do research output and student enrollment increase, too, in tandem with a change in research and teaching efforts? The answer is absolutely “yes" and they all increase. The following proposition describes this result.

Proposition 2 Consider a positive interior solution, $r^* = \hat{r} > 0$ and $t^* = \hat{t} > 0$. The smaller the substitutability between research and teaching efforts in terms of the cost function, the more effort a university dedicates to both activities, and thereby, research output and student enrollment also increase. In other words, $\frac{\partial r^*}{\partial \varepsilon} < 0$, $\frac{\partial t^*}{\partial \varepsilon} < 0$, $\frac{\partial R^*}{\partial \varepsilon} < 0$, and $\frac{\partial n^*}{\partial \varepsilon} < 0$ hold.

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[8] The second derivatives of the function $U(r,t)$ are as follows : $\frac{\partial^2 U}{\partial r^2} = \frac{\partial^2 U}{\partial t^2} = -1$ and $\frac{\partial^2 U}{\partial r \partial t} = \frac{\partial^2 U}{\partial t \partial r} = \frac{2s}{k} - \varepsilon$. Hence, the second-order condition is calculated as $(\frac{\partial^2 U}{\partial r^2})(\frac{\partial^2 U}{\partial t^2}) - (\frac{\partial^2 U}{\partial r \partial t})^2 > 0$. This inequality expression can be simplified into $k^2\varepsilon^2 - 4ks\varepsilon + 4s^2 - k^2 < 0$. Solving it for ε, we obtain $-1 + \frac{2s}{k} < \varepsilon < 1 + \frac{2s}{k}$. Assuming the cost function, $C(r,t)$, is strictly concave, we can further confine the range of ε to $\varepsilon \in (-1 + \frac{2s}{k}, 1)$.

[9] When $\frac{2s}{k} > 1$ holds, we can rewrite the condition as $\varepsilon \in (-1 + \frac{2s}{k}, 1)$.

Indeed, Proposition 1 is not a surprising result. Nevertheless, the size of ε is important from the viewpoint of university policies and management. ε tends to be positive in most universities except some top-ranked general universities where many high-achieving research students study. In such universities, better-educated students help faculty members produce high-quality research output – for example, as co-authors. But they may be rare. Many university faculty members, especially in recent years, have found it more difficult to strike a fine balance between research and teaching activities as the demand for teaching responsibility grows. Hence, it is much more essential that policymakers or university officials design institutional arrangements of universities toward reducing the substitutability between research and teaching activities given limited resources. [10]

Supplementary note

Let us continue to consider a strictly positive interior solution, $r^* = \hat{r} = \frac{k(kF-2s^2)}{(1-\varepsilon^2)k^2+4ks\varepsilon-4s^2} > 0$ and $t^* = \hat{t} = \frac{(2s-k\varepsilon)(kF-2s^2)}{(1-\varepsilon^2)k^2+4ks\varepsilon-4s^2} > 0$. When it comes to the effect of research funding on research and teaching activities in this illustrative modeling, $\frac{\partial r^*}{\partial F} = \frac{k^2}{(1-\varepsilon^2)k^2+4ks\varepsilon-4s^2} > 0$ and $\frac{\partial t^*}{\partial F} = \frac{k(2s-k\varepsilon)}{(1-\varepsilon^2)k^2+4ks\varepsilon-4s^2} > 0$ are assured for $F > \frac{2s^2}{k}$ and $\varepsilon \in (-1+\frac{2s}{k}, \frac{2s}{k})$. By using the same demonstration with Proposition 1, we can also demonstrate that $\frac{\partial n^*}{\partial F} > 0$ and $\frac{\partial R^*}{\partial F} > 0$. This suggests that in the range of a positive interior solution, increased research funding positively affects both research output and student enrollment so long as research funding is large and substitutability is not strong. It is noticeable that an extreme case does not appear in which increased research funding reduces research output and student enrollment for strong substitutability, as Proposition 1(3) refers to this possibility. In this sense, this illustrative model indicates a normal research and teaching environment.

3.2 Analysis when substitutability is zero

The following examination formulates a specific case when the substitutability between research and teaching activities is zero (i.e., $\varepsilon = 0$), which means they are independent. This simplification helps us elicit precise effects of parameters change of research funding and a tuition fee on university research and teaching activities.

In later analyses, the theoretical results are further extended as we add new assumptions and constraints compared with the basic results. More precisely, a technical assumption is

[10] Demski and Zimmerman (2000), who succinctly examine the question on "research versus teaching" in the academic community, acknowledge that they could be substitutes in the short run because the time academic staff can devote to research is limited by teaching obligations. On the other hand, the authors maintain that they could be mutually complementary activities in the long run when research motivations of academic staff are frequently stirred by class notes, exams, student inquiries, and other activities. The authors argue in their conclusion that academic staff should be encouraged to better exploit teaching opportunities to generate more research output.

made about the allocation of research funding F, contingent on teaching effort to avoid the complexity of the analytical solution:

Assumption 1 A financing agency allocates no research funding ($F = 0$) to a university when a university enrolls zero students ($n = 0$) through a deficient teaching effort. [11]

This assumption is solely technical and intended to eliminate the case wherein a university can obtain a higher payoff by concentrating only on research activity and not enrolling any new students. Since a minimum student enrollment through teaching activity can also be viewed as an important mission in addition to research activity that most universities are required to fulfill, it is possible that a university does not qualify to receive any research funding if not students have been enrolled.

By substituting $\varepsilon = 0$ into Equations (12) and (13), we derive the following expressions, respectively: $\hat{r} = \frac{k(kF-2s^2)}{k^2-4s^2}$ and $\hat{t} = \frac{2s(kF-2s^2)}{k^2-4s^2}$. Indeed, $k^2 - 4s^2 > 0$ needs to be assumed to satisfy the second-order condition. By solving for s, we obtain $s < \frac{k}{2}$. In addition, considering that $\hat{r} > 0$ and $\hat{t} > 0$, we suppose that F is relatively larger compared with s, that is, $F > \frac{2s^2}{k}$. These two conditions imply that a tuition fee must not be extremely high, which appears to be supported by the fact that governments carefully regulate it to keep it low in support of the students' welfare.

According to the condition regarding student enrollment, $n \in [0, 1]$, we can derive Lemma 2 that describes the equilibrium solutions, $\boldsymbol{e}^* = (r^*, t^*)$.

Lemma 2 Let us denote the closed form of $\hat{n}$ and $\hat{R}$ as: $\hat{n} = \frac{2s(2F-k)}{k^2-4s^2}$ and $\hat{R} = \hat{r}^2 = \left[\frac{k(kF-2s^2)}{k^2-4s^2}\right]^2$. Suppose that $r^* + t^* < \bar{a}$ is satisfied at an equilibrium (that is, the capacity has slack). Then, the equilibrium solutions, $e^* = (r^*, t^*)$, n^*, and R^*, are as follows:
(1) **University closure**: $\boldsymbol{e}^* = (0, 0)$, $n^* = 0$, and $R^* = 0$ for $\frac{2s^2}{k} < F < s$;
(2) **Minimum teaching activity**: $\boldsymbol{e}^* = (F + \delta_r, s + \delta_t) \approx (F, s)$, $n^* = \delta_n \approx 0$, and $R^* = (F + \delta_r)^2 = F^2 + \delta_R \approx F^2$ for $s < F \leq \frac{k}{2}$;
(3) **Underenrollment**: $\boldsymbol{e}^* = (\hat{r}, \hat{t})$, $n^* = \hat{n} \in (0, 1)$, and $R^* = \hat{R}$ for $\frac{k}{2} < F < \frac{k}{2} + \frac{k^2-4s^2}{4s}$; and
(4) **Full enrollment**: $\boldsymbol{e}^* = (s + F, s + \frac{k}{2})$, $n^* = 1$, and $R^* = (s + F)^2$ for $F \geq \frac{k}{2} + \frac{k^2-4s^2}{4s}$,
where δ_i with $i = r, t, n$, and R is a positive infinitesimal value.

Lemma 2 demonstrates that the equilibrium solutions can differ because the range of student enrollment is confined to $n \in [0, 1]$. Indeed, external research funding is truly used for research activity and helps enhance the university's incentive to conduct teaching activity. From Assumption 1, if research funding is quite small, a university cannot afford to

[11] A slightly different assumption can be made such that a financing agency allocates no research funding when a university enrolls students less than $\underline{n} > 0$. Although this kind of alternative does not change the intuition of the analysis, the simple assumption that has been already defined in the text is employed.

conduct any research and teaching activities (Lemma 2[1]: university closure) or conducts them only at minimum levels that assure a small student enrollment (Lemma 2[2]: minimum teaching activity). Notably, because research output is discontinuous between zero and F^2, the amount of research funding distributed by a financing agency is critical for a university to engage in research activity. When producing more research output as well as acquiring a significantly positive student enrollment, a certain amount of research funding needs to be allocated to a university (Lemma 2[3]: underenrollment). By contrast, even if a university obtains a much larger monetary resource, it cannot increase student enrollment more than 1 (Lemma 2[4]: full enrollment).

3.3 Comparative statics in a simplified setting

We analyze comparative statics in the equilibrium solutions, especially when $\varepsilon = 0$. This postulate helps us elicit the impact of the parameters, in particular, external research funding on research and teaching activities.

Nonbinding capacity constraint

Let us consider the case where the capacity constraint is not binding (i.e., $r^*+t^* < \overline{a}$). Namely, a university has a certain affordable capacity to commit efforts for research and teaching activities. Proposition 3 answers how research funding, F, affects the equilibrium solutions, $e^* = (r^*, t^*)$, n^*, and R^*, which are defined in Lemma 2.

Proposition 3 Suppose that $r^* + t^* < \overline{a}$ at the equilibrium solutions defined in Lemma 2. The comparative statics with respect to research funding indicates:
(1) With respect to research effort and research output: (i) $\frac{\partial r^*}{\partial F} > 0$ and $\frac{\partial R^*}{\partial F} > 0$ for $F > s$; and (ii) $\frac{\partial r^*}{\partial F} = \frac{\partial R^*}{\partial F} = 0$ for $\frac{2s^2}{k} < F < s$.
(2) With respect to teaching effort and student enrollment: (i) $\frac{\partial t^*}{\partial F} > 0$ and $\frac{\partial n^*}{\partial F} > 0$ for $\frac{k}{2} < F < \frac{k}{2} + \frac{k^2-4s^2}{4s}$; and (ii) $\frac{\partial t^*}{\partial F} = \frac{\partial n^*}{\partial F} = 0$ for $\frac{2s^2}{k} < F \leq \frac{k}{2}$ and $F \geq \frac{k}{2} + \frac{k^2-4s^2}{4s}$.
(3) When research funding changes, $\frac{dr^*}{dt^*} > 1$ for $\frac{k}{2} < F < \frac{k}{2} + \frac{k^2-4s^2}{4s}$.

Clearly, in the absence of the substitutability, increased research funding can spur research output (Proposition 3[1-i]) except when a university entirely shuts down its research and teaching activities (Proposition 3[1-ii]). In particular, at the equilibrium of $\boldsymbol{e}^* = (\hat{r}, \hat{t})$, Proposition 1(2-i) can immediately lead to $\frac{\partial R^*}{\partial F} > 0$ by setting $\frac{\partial^2 C}{\partial r \partial t} = 0$. This simple result may provide the support for the claim that devoting more resources into universities can stimulate research activity. As evidence, since most recent empirical studies have observed a positive correlation between R&D investments financed from the outside and research output in universities, the result endorses a frequently observed common finding.

[12] However, this result is highly dependent on the specific assumption of the absence of the substitutability. As we already examined in Proposition 1, the strong substitutability may cause a decrease in research output, although such situation would not be considered prevailing across universities.

In addition, as Proposition 1(2-i) suggests, research funding increases teaching activity as well with underenrollment (Proposition 3[2-i]). The intuition is as follows. At the beginning, a university can afford to devote more effort to research activity owing to increased research funding. At the same time, it becomes more profitable to dedicate some efforts toward teaching activity than research activity because the marginal payoff obtained from research activity has dropped. In the next stage, since the utilities of students are fostered by the improved teaching effort, student enrollment is also expected to increase, and thus, marginal students decide to enroll at a university. This substantial increase in student enrollment contributes to enriching the budget of a university through tuition revenue. This is how research and teaching activities indirectly interact with each other not relying on their substitutability. In short, the so-called "multiplier effect" is in force between research and teaching efforts in response to an increase in research funding.

This intuition is much easier to understand by looking at Figure 3 that depicts the response functions: $r(t) = (\frac{2s}{k})t + \frac{kF-2s^2}{k}$ and $t(r) = (\frac{2s}{k})r$. The intersection of the two lines denoted by point A represents an initial equilibrium solution (r^*, t^*). Note that when F increases, $r(t)$ shifts outward (right-hand side). If research funding is increased and teaching effort is kept constant at the level of t^*, the combination of research and teaching efforts moves to point B, and r^* also increases to $\check{r}$. But since t^* is no longer optimal at point B, the equilibrium solution ends up at point C, where further increases in both research and teaching efforts occur (r^{**}, t^{**}). Moreover, since the slope of $r(t)$ ($t(r)$) is larger (smaller) than 1, we can see that more increased effort is diverted to research rather than teaching activities (Proposition 3[3]).

[12]Using a database of 18 US research universities, Payne and Siow (2003) find that an increase of 1 million US dollar in a federal research funding to a university generates 10 more articles and 0.2 more patents and argue that increasing research funds produces more research output. With a particular focus on the Canadian nanotechnology field, Beaudry and Allaoui (2012) conclude that a greater amount of public funds certainly produces more research output of individual academics as represented by the number of scientific articles. Furthermore, based on the panel data of Japanese universities, Yonetani, Ikeuchi, and Kuwahara (2013) discover that intramural expenditure of R&D funds received from external sources has a positive correlation with articles published by researchers at both national and private universities.

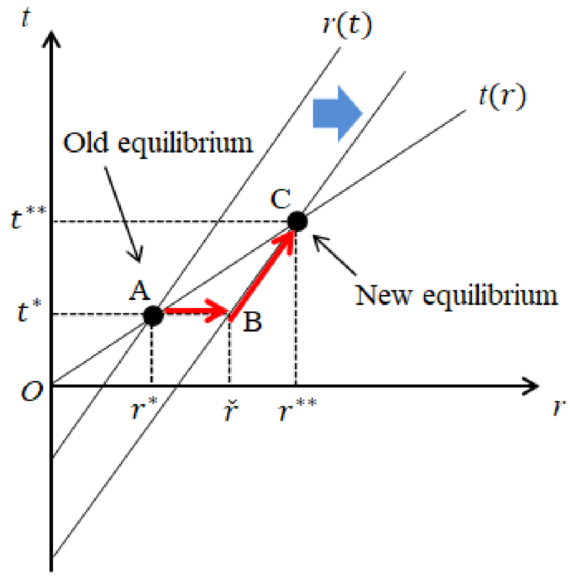

FIGURE 3. Effect of an increase in F on research and teaching efforts.

Continuously focusing on the interior equilibrium solution, $\boldsymbol{e}^* = (\hat{r}, \hat{t})$, we derive the comparative statics regarding k (mobility cost) and s (tuition fee).

Proposition 4 Consider the interior equilibrium solutions, $\boldsymbol{e}^* = (\hat{r}, \hat{t})$, $n^* = \hat{n}$, and $R^* = \hat{R}$. The comparative statics with respect to k and s indicates:
(1) $\frac{\partial r^*}{\partial k} < 0$, $\frac{\partial t^*}{\partial k} < 0$, $\frac{\partial n^*}{\partial k} < 0$, and $\frac{\partial R^*}{\partial k} < 0$; and
(2) $\frac{\partial r^*}{\partial s} > 0$, $\frac{\partial t^*}{\partial s} > 0$, $\frac{\partial n^*}{\partial s} > 0$, and $\frac{\partial R^*}{\partial s} > 0$.

What is noteworthy is that the effects of a mobility cost and a tuition fee operate in a different direction in this specific illustrative case, although they are similar in that both of them lower the utilities of students. More precisely, whereas a rise in a mobility cost causes a reduction in both university research and teaching activities, a rise in a tuition fee gives a university an incentive to improve these two activities.

In fact, since a higher mobility cost definitely decreases student enrollment, it reduces the budget of a university, which also culminates in a decrease in research output. Like the mechanism that works in a mobility cost, a rise in a tuition fee actually decreases student enrollment at an initial stage. Nevertheless, a university may still be able to increase tuition revenue as a whole. A university is incentivized to make more teaching effort by an increase in a tuition fee, as it can earn more tuition revenue per student. In association with such increased teaching effort, the contribution to tuition revenue from an intramarginal population of students is large compared with the loss from marginal students who do not apply. [13] In turn, this positive effect on the research budget can strengthen

[13]This result stems from the fact that the student enrollment function, $\hat{n}$, defined at the equilibrium is

the incentive of a university for research effort. Accordingly, despite the negative effect on student enrollment, a university is expected to achieve higher research output and student enrollment than before the tuition fee is increased.

From the abovementioned result, we may be tempted to reach a hasty conclusion that the higher we set a tuition fee, the more we can expect research output and student enrollment to increase. But this is not always true for the reason that the tuition fee, s, is restricted by the condition, $F > \frac{2s^2}{k}$ and $s < \frac{k}{2}$, which requires that the tuition fee must be kept sufficiently low. It is therefore impossible to arbitrarily increase the tuition fee to increase research and teaching activities.

Supplementary note

Subsection 6.3 illustrates that the signs of comparative statics can be changed by the substitutability between research and teaching activities based on the general model as defined in Section 2. If we present an assumption that the substitutability is zero, it can be demonstrated that a rise in mobility cost k negatively affects research and teaching activities. On the other hand, it is also revealed that the effect of a rise in a tuition fee is not necessarily decisive depending on other parameters, even when the substitutability is zero.

Binding capacity constraint

Next, let us consider the case where a university fully exerts its capacity, that is, the capacity constraint is binding. Hereupon, we mainly focus on underenrollment, $n^* \in (0, 1)$, described by Lemma 2 (3), which is most common in real university-student markets. The following proposition points to a clear-cut opposite conclusion from Proposition 3 regarding the effects on teaching effort and student enrollment.

Proposition 5 Suppose that the capacity constraint of a university is binding as $\hat{r} + \hat{t} > \bar{a}$. At the binding equilibrium solution $e^* = (r^*, t^*)$ with $r^* + t^* = \bar{a}$, we obtain $\frac{\partial r^*}{\partial F} > 0$, $\frac{\partial t^*}{\partial F} < 0$, $\frac{\partial n^*}{\partial F} < 0$, and $\frac{\partial R^*}{\partial F} > 0$ with respect to an increase in research funding.

The mechanism behind Proposition 5 is quite straightforward. Enhancing the budget enables a university to engage in more research activity. Nevertheless, since the efforts have already reached the maximum level, teaching effort is reduced, and student enrollment is thus certain to decline. When capacity is fully exerted, increased research funding allocated by a financing agency ends up crowding out teaching activity and thereby student

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inelastic with respect to a tuition fee in our illustrative model. In checking the tuition fee elasticity of the student enrollment function, $\hat{n} = \frac{2s(2F-k)}{k^2-4s^2}$, it is sufficient to examine the sign of $\hat{n} + s(\frac{\partial \hat{n}}{\partial s})$; if $\hat{n} + s(\frac{\partial \hat{n}}{\partial s}) < 0$ (> 0), then $\hat{n}$ is elastic (inelastic). We obtain $\hat{n} + s(\frac{\partial \hat{n}}{\partial s}) = \frac{2s(2F-k)}{k^2-4s^2} + s[\frac{2(2F-k)(k^2-4s^2)+16s^2(2F-k)}{(k^2-4s^2)^2}] = \frac{4k^2 s(2F-k)}{(k^2-4s^2)^2} > 0$ under the presumed assumption of $F > \frac{k}{2}$. Hence, the student enrollment function, $n^* = \hat{n}$, is inelastic for a tuition fee.

enrollment. In sharp contrast with the multiplier effect, we can name this polar change the "crowding-out effect." From this, we can see that if a capacity constraint is introduced into the model, a decrease in teaching effort may be caused even in the absence of the substitutability between research and teaching activities.

4 Tuition Fee as Control Variable

Now suppose that a tuition fee is no longer exogenous, but an endogenously controlled variable set to maximize tuition revenue. A government may intend to control tuition fees of universities because, in doing so, it can save on research spending that is distributed to universities. Or universities may be allowed to freely determine their tuition fees to maximize their payoffs. As explained below, these two interpretations are mathematically equivalent from an analytical viewpoint.

The timing of the model is slightly modified to include a government's decision in Stage 1.5; the government determines a tuition fee, s, of a university between Stages 1 and 2.

4.1 Analysis of general case

In the first place, what is considered is the government problem of finding an optimal tuition fee, s^*, that maximizes university tuition revenue. Letting E denote this revenue, we define the maximization problem such that: $\max_s E = sn(t, k, s)$. On this problem, the first-order condition for s is rendered by

$$\frac{\partial E}{\partial s} = n + s\left(\frac{\partial n}{\partial s}\right) = 0. \tag{14}$$

Solving Equation (14) by s, we obtain $s = s(t; k)$ as a function of t. If we assume instead that a university is allowed to choose an optimal tuition fee by itself, the first-order condition of maximizing $U = r[F + sn(t, k, s)] - C(r, t)$ is given by $r[n + s(\frac{\partial n}{\partial s})] = 0$. By positing $r > 0$, we obtain the same condition as above.

To check whether the solution has a global maximum, we derive the second-order condition: $\frac{\partial^2 E}{\partial s^2} = 2(\frac{\partial n}{\partial s}) + s(\frac{\partial^2 n}{\partial s^2}) < 0$. We can see that unless $n(t, k, s)$ is a strong convex function against s (i.e., $\frac{\partial^2 n}{\partial s^2} > 0$), this condition is not violated. But we hereafter proceed by assuming that $\frac{\partial^2 E}{\partial s^2} < 0$ is satisfied at $s = s(t; k)$. [14]

[14] In an illustrative case discussed in Section 3, since $n = \frac{2(t-s)}{k}$, the second-order condition is always satisfied.

If we take a derivative on both sides of Equation (14) by t, we obtain

$$\frac{\partial n}{\partial t} + 2\left(\frac{\partial n}{\partial s}\right)\left(\frac{\partial s}{\partial t}\right) + s\left(\frac{\partial^2 n}{\partial s \partial t}\right) + s\left(\frac{\partial^2 n}{\partial s \partial t}\right) = 0 \Longleftrightarrow \underbrace{\left[2\left(\frac{\partial n}{\partial s}\right) + s\left(\frac{\partial^2 n}{\partial s^2}\right)\right]}_{\text{negative}}\left(\frac{\partial s}{\partial t}\right) = -\left[\frac{\partial n}{\partial t} + s\left(\frac{\partial^2 n}{\partial t \partial s}\right)\right].$$

If we also suppose $\frac{\partial^2 n}{\partial t \partial s}$=0, that is, there exist no cross-terms between t and s in the function of $n(t, k, s)$, we can derive $\frac{\partial s}{\partial t} > 0$ from the assumption.

By using an optimal tuition fee, $s = s(t; k)$, we redefine the student enrollment function as $n(t, k, s) = n(t, k, s(t; k)) = \tilde{n}(t; k)$. Based on these settings, we confirm in what follows the Hessian matrix of $U(r, t) = r[F + s(t; k)\tilde{n}(t; k)] - C(r, t)$. To this end, let us consider the first-order condition for maximizing $U(r, t)$:

$$\frac{\partial U}{\partial r} = F + s\tilde{n} - \frac{\partial C}{\partial r} = 0, \tag{15}$$

$$\frac{\partial U}{\partial t} = r\left[\tilde{n}\left(\frac{\partial s}{\partial t}\right) + s\left(\frac{\partial \tilde{n}}{\partial t}\right)\right] - \frac{\partial C}{\partial t} = 0. \tag{16}$$

From Equations (15) and (16), we can find an equilibrium solution, $e^* = (r^*, t^*)$, and an optimal tuition fee, $s^* = s(t^*; k)$.

The second derivatives of $U(r, t)$ are as follows:

$$\begin{aligned}
\frac{\partial^2 U}{\partial r^2} &= -\frac{\partial^2 C}{\partial r^2} < 0, \\
\frac{\partial^2 U}{\partial t^2} &= r\left[\tilde{n}\left(\frac{\partial^2 s}{\partial t^2}\right) + s\left(\frac{\partial^2 \tilde{n}}{\partial t^2}\right) + 2\left(\frac{\partial s}{\partial t}\right)\left(\frac{\partial \tilde{n}}{\partial t}\right)\right] - \frac{\partial^2 C}{\partial t^2}, \\
\frac{\partial^2 U}{\partial r \partial t} &= \tilde{n}\left(\frac{\partial s}{\partial t}\right) + s\left(\frac{\partial \tilde{n}}{\partial t}\right) - \frac{\partial^2 C}{\partial r \partial t}.
\end{aligned}$$

Hence, the Hessian matrix, $\tilde{U}$, is specified as

$$\tilde{U} = \begin{bmatrix} -\frac{\partial^2 C}{\partial r^2} & \tilde{n}\left(\frac{\partial s}{\partial t}\right) + s\left(\frac{\partial \tilde{n}}{\partial t}\right) - \frac{\partial^2 C}{\partial r \partial t} \\ \tilde{n}\left(\frac{\partial s}{\partial t}\right) + s\left(\frac{\partial \tilde{n}}{\partial t}\right) - \frac{\partial^2 C}{\partial r \partial t} & r\left[\tilde{n}\left(\frac{\partial^2 s}{\partial t^2}\right) + s\left(\frac{\partial^2 \tilde{n}}{\partial t^2}\right) + 2\left(\frac{\partial s}{\partial t}\right)\left(\frac{\partial \tilde{n}}{\partial t}\right)\right] - \frac{\partial^2 C}{\partial t^2} \end{bmatrix}.$$

We obtain the determinant of $\tilde{U}$ as follows:

$$|\tilde{U}| = \underbrace{\frac{\partial^2 C}{\partial r^2}}_{\text{positive}}\left[\frac{\partial^2 C}{\partial t^2} - r\left[\tilde{n}\left(\frac{\partial^2 s}{\partial t^2}\right) + s\left(\frac{\partial^2 \tilde{n}}{\partial t^2}\right) + 2\left(\frac{\partial s}{\partial t}\right)\left(\frac{\partial \tilde{n}}{\partial t}\right)\right]\right] - \underbrace{\left[\tilde{n}\left(\frac{\partial s}{\partial t}\right) + s\left(\frac{\partial \tilde{n}}{\partial t}\right) - \frac{\partial^2 C}{\partial r \partial t}\right]^2}_{\text{positive}}. \tag{17}$$

It is necessary that $\frac{\partial^2 U}{\partial t^2} = r\left[\tilde{n}\left(\frac{\partial^2 s}{\partial t^2}\right) + s\left(\frac{\partial^2 \tilde{n}}{\partial t^2}\right) + 2\left(\frac{\partial s}{\partial t}\right)\left(\frac{\partial \tilde{n}}{\partial t}\right)\right] - \frac{\partial^2 C}{\partial t^2} < 0$ holds for $U(r, t)$ to indicate a global maximum. But the sign of $|\tilde{U}|$ in Equation (17) is indecisive. Thus, while $U(r,t)$ has a global maximum at $\boldsymbol{e}^* = (r^*, t^*)$ and $s^* = s(t^*; k)$ for $|\tilde{U}| > 0$, a saddle point emerges for $|\tilde{U}| < 0$. For more details, see Subsection 6.2.

4.2 Analysis of illustrative case

Let us revert to the illustrative case formulated in Section 4 to derive the explicit equilibrium solutions. We continue to assume $\frac{\partial^2 C}{\partial r \partial t} = \varepsilon = 0$ for analytical simplicity. The maximization problem of university tuition revenue is defined such that: $\max_s E = sn = \frac{2s(t-s)}{k}$. The optimal solution is $s = \frac{t}{2}$ and the maximum value of E is $E = \frac{t^2}{2k}$.

Substituting the optimal s back into the university payoff function, we obtain

$$U(r,t) = r\left(F + \frac{t^2}{2k}\right) - \left(\frac{r^2}{2} + \frac{t^2}{2}\right) = Fr - \frac{r^2}{2} + \left(\frac{r-k}{2k}\right)t^2. \tag{18}$$

The first-order conditions with respect to r and t of Equation (18) are as follows:

$$\frac{\partial U}{\partial r} = F - r + \frac{t^2}{2k} = 0, \tag{19}$$

$$\frac{\partial U}{\partial t} = \left(\frac{r-k}{k}\right)t = 0. \tag{20}$$

Equations (19) and (20) provide two possible values that induce $\frac{\partial U}{\partial r} = \frac{\partial U}{\partial t} = 0$: that is, $(r,t) = (F, 0)$ and $(k, \sqrt{2k(k-F)})$ for $F < k$. These two points are depicted as Points A and S, respectively, in Figure 4. Let us focus on the point, $\tilde{\boldsymbol{e}} = (\tilde{r}, \tilde{t}) = (k, \sqrt{2k(k-F)})$ assuming $F < k$. By solving Equation (19) with respect to t, we derive $t = \sqrt{2k(r-F)}$ with $r > F$. Focusing on the term $(\frac{r-k}{2k})t^2$ of Equation (18), we see that the larger (smaller) the t for $r > k$ ($r < k$), the higher the payoff of a university, and t is irrelevant to the payoff for $r = k$. This suggests that $\tilde{\boldsymbol{e}} = (k, \sqrt{2k(k-F)})$ with $F < k$ is a saddle point. In the range of $F < k$, we can also see that $(r,t) = (F, 0)$ achieves a local maximum. Moreover, when $F > k$ holds, $(r,t) = (F, 0)$ becomes a saddle point ($\tilde{\boldsymbol{e}} = (k, \sqrt{2k(k-F)})$ disappears for $F > k$).

We make the following assumption about the research activity conducted by a university:

Assumption 2 A university is required to choose an optimal amount of research effort for any amount of given teaching effort.

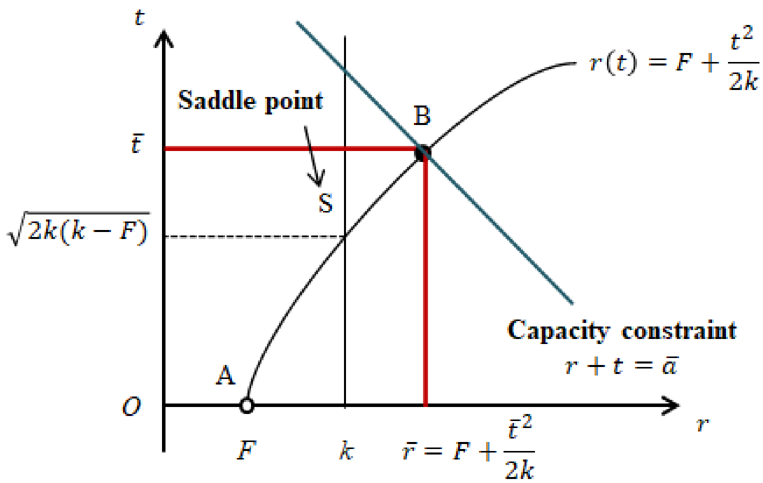

FIGURE 4. Reaction curve of $r(t)$ and the capacity constraint.

Assumption 2 indicates that $r(t) = F + \frac{t^2}{2k}$ (Equation [19]) applies to any $t > 0$, which is critical for the equilibrium solution derived in Proposition 6 with a binding capacity constraint. [15] This postulate could be justifiable on the ground that a government often intends to maintain the level of research effort conducted by a university or that faculties of a university are reluctant to be forced to decrease research effort.

Recall again that research and teaching efforts are bounded by its capacity, $r + t \leq \bar{a}$. Hence, a corner solution, $\bar{e} = (\bar{r}, \bar{t}) > 0$, that satisfies both $r + t = \bar{a}$ and $r = F + \frac{t^2}{2k}$ can be an equilibrium solution that achieves a maximum university payoff because more active research and teaching efforts can generate a higher payoff for a university. Calculating these two simultaneous equations, we obtain $\bar{r} = \bar{a} + k - \sqrt{k(k + 2\bar{a} - 2F)}$ and $\bar{t} = -k + \sqrt{k(k + 2\bar{a} - 2F)}$ (where $k + 2\bar{a} - 2F > 0 \Leftrightarrow \bar{a} > F - \frac{k}{2}$ is assumed). Hereafter, $\bar{t}$ is conveniently used instead of $\bar{a}$ to denote a corner solution for descriptive simplicity.

Given that $\bar{e} = (\bar{r}, \bar{t}) = (F + \frac{\bar{t}^2}{2k}, \bar{t})$ is an equilibrium solution, we can also represent $\bar{s} = \frac{\bar{t}}{2}$, $\bar{n} = \frac{2(\bar{t} - \bar{s})}{k} = \frac{\bar{t}}{k}$, and $\bar{R} = \bar{r}(F + \bar{s}\,\bar{n}) = (F + \frac{\bar{t}^2}{2k})^2$, respectively. Additionally, we continue to assume that even if full capacity is attained, underenrollment, $n^* \in (0, 1)$, still exists. Based on these derivations, we lead to Proposition 6 that describes the equilibrium solutions.

Proposition 6 Suppose underenrollment, $n^* \in (0, 1)$. When a tuition fee is a control variable, the equilibrium solutions, $\boldsymbol{e}^* = (r^*, t^*)$, n^*, s^*, and R^*, are as follows:

[15] This assumption is analytically needed to explicitly identify an equilibrium solution and eliminate the case where a university obtains a higher payoff by decreasing research effort and increasing teaching effort in the left region from $r(t) = F + \frac{t^2}{2k}$. Considering a total differential of Equation (18) provides $dU = (F + \frac{t^2}{2k})dr + (\frac{rt}{k})dt - (rdr + tdt) = (F + \frac{t^2}{2k} - r)dr + [\frac{(r-k)t}{k}]dt$, where $dr < 0$ and $dt > 0$. If we evaluate the effect of a minute change (first-order approximation) in r and t along the line of $r(t) = F + \frac{t^2}{2k}$, we obtain $dU = [\frac{(r-k)t}{k}]dt > 0$ for $r > k$ because $F + \frac{t^2}{2k} - r = 0$. Therefore, a university can increase its payoff by marginally decreasing r with the capacity constraint being binded.

(1) With respect to $F < k$, (1-i) $\boldsymbol{e}^* = \overline{\boldsymbol{e}} = (F + \frac{\bar{t}^2}{2k}, \bar{t})$, $s^* = \bar{s} = \frac{\bar{t}}{2}$, $n^* = \bar{n} = \frac{\bar{t}}{k}$, and $R^* = \overline{R} = (F + \frac{\bar{t}^2}{2k})^2$ for $\bar{t} > 2\sqrt{k(k-F)}$ (**large university**); and (1-ii) $\boldsymbol{e}^* = \boldsymbol{e}^0 = (F + \delta_r, \delta_t) \approx (F, 0)$, $s^* = s^0 \approx 0$, $n^* = n^0 \approx 0$, and $R^* = R^0 \approx F^2$ for $\bar{t} < 2\sqrt{k(k-F)}$ (**small college**),
(2) With respect to $F > k$, $\boldsymbol{e}^* = \overline{\boldsymbol{e}}$, $s^* = \bar{s}$, $n^* = \bar{n}$, and $R^* = \overline{R}$ for any $\bar{t}$.

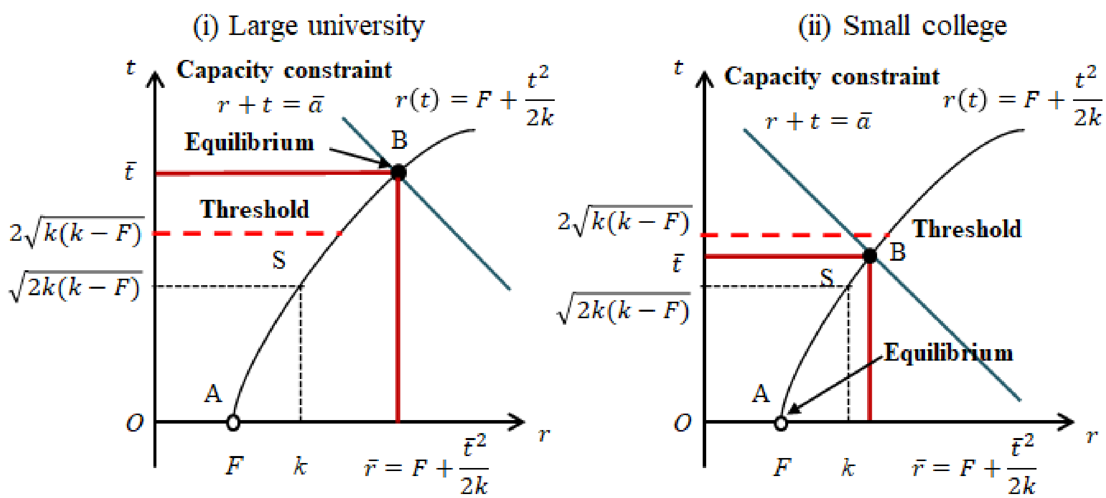

FIGURE 5. Equilibrium of a large university and a small college.

Figure 5(i) and 5(ii) illustrate the two polar equilibrium solutions of a “large university" and a “small college" demonstrated by Proposition 6(1-i) and 6(1-ii), respectively. Under the assumption of $F < k$, research and teaching efforts are made by using a maximum capacity, $\bar{a}$, associated with the equilibrium solution, $\boldsymbol{e}^* = \overline{\boldsymbol{e}} = (F + \frac{\bar{t}^2}{2k}, \bar{t})$, only if the capacity is sufficiently large ($\bar{t} > 2\sqrt{k(k-F)}$). Put simply, this pattern is the case with a large university that can afford to become involved in several activities. In this case, Point B in Figure 5(i) indicates the equilibrium solution. However, there is another possibility that if a small college with a small capacity ($\bar{t} < 2\sqrt{k(k-F)}$) operates, a university will choose a minimum combination of research and teaching efforts approximated by $\boldsymbol{e}^* = \boldsymbol{e}^0 = (F + \delta_r, \delta_t) \approx (F, 0)$, as shown at Point A in Figure 7(ii). [16]

Why does a university prefer to make minimum efforts? The reason is intuitive, as follows – if the potential capacity is small enough, the university finds it difficult to benefit from “economies of scale" in research and teaching efforts. An increase in efforts reduces the payoff from the beginning up until the saddle point, $\tilde{\mathbf{e}} = (k, \sqrt{2k(k-F)})$ as shown in Point S, but in turn, they are likely to improve the payoff past the point since a university can impose a higher tuition fee on more present and incoming students. In such a situation,

[16] The infinitesimally small teaching effort, δ_t, at the equilibrium may seem a bit extreme. However, if Assumption 1 regarding a minimum student enrollment (more than just zero) is modified as highlighted in Footnote 11, we can derive an equilibrium teaching effort that is of significantly positive value, $\underline{t} \in (\delta_t, \bar{t})$, but not an infinitesimal one. In such an interpretation, a university may be termed as a small college.

a small college with small capacity cannot benefit from exploiting its capacity to the fullest before it reaches a point over which increased effort can provide a higher payoff to that small college.

Meanwhile, when research funding is large enough to satisfy $F > k$, the payoff of a university becomes larger as efforts increase along with the function, $r = r(t) = F + \frac{t^2}{2k}$. As a result, since the saddle point appears at $(F, 0)$ as shown in Point A, exerting its efforts to full capacity is always optimal for a university. As is demonstrated, the assumption of $F > k$ ensures that all the payoffs located on $r = r(t)$ are always strictly positive because the minimum payoff at $\boldsymbol{e}^0 = (F + \delta_r, \delta_t)$ is given by $U(F + \delta_r, \delta_t) \approx \frac{F^2}{2} > 0$.

The important point made here is that even being in a monopolist position over the student market, a small college may place its smallest amount of research and teaching efforts below its potential capacity. Therefore, the strong support extended toward using research funding by a government can be justified especially for a small college, to make a university choose an effort level that exploits full capacity, thereby, shifting to an equilibrium creating higher research output and student enrollment.

Proposition 7 regarding comparative statics of research funding exhibits a contrasting result to that was shown in Proposition 3.

Proposition 7 The comparative statics with respect to F for the equilibrium solutions led by Proposition 6 indicates that:
(1) $\frac{\partial r^*}{\partial F} > 0$, $\frac{\partial t^*}{\partial F} < 0$, $\frac{\partial s^*}{\partial F} < 0$, $\frac{\partial n^*}{\partial F} < 0$, and $\frac{\partial R^*}{\partial F} > 0$ for Proposition 6(1-i) and 6(2); and
(2) $\frac{\partial r^*}{\partial F} > 0$, $\frac{\partial t^*}{\partial F} = 0$, $\frac{\partial s^*}{\partial F} = 0$, $\frac{\partial n^*}{\partial F} = 0$, and $\frac{\partial R^*}{\partial F} > 0$ for Proposition 6(1-ii).

The result of Proposition 7(1) regarding the crowding-out effect is the same mechanism in force as Proposition 4. In addition, Proposition 7(2) reveals that when a university makes minimum effort below its potential capacity, research funding has a "nil" effect on teaching effort and student enrollment while it positively affects research effort and research output. More precisely, research funding does not change any teaching activity and resultant student enrollment of a small college that has already selected minimum efforts. Consolidating all matters discussed, when a tuition fee is controlled to maximize tuition revenue, a marginal amount of research funding may decrease student enrollment or, at best, be wholly ineffective for increasing student enrollment.

Finally, the discussion facing a government would be whether a change from an existing tuition fee system is relevant. In what follows, we probe the conditions of when research output and student enrollment increase under a controlled tuition fee, accompanied with the change in the tuition fee scheme from a fixed tuition fee.

Proposition 8 Suppose that $\hat{e} = (\hat{r}, \hat{t}) = (\frac{k(kF-2s^2)}{k^2-4s^2}, \frac{2s(kF-2s^2)}{k^2-4s^2})$ (with $\hat{r} > 0$, $\hat{t} > 0$, and $\hat{r} + \hat{t} < \overline{a}$), $\hat{n} = \frac{2s(2F-k)}{k^2-4s^2} \in (0,1)$, and $\hat{R} = [\frac{k(kF-2s^2)}{k^2-4s^2}]^2$ have been initially achieved as a positive interior

equilibrium solution under the scheme of fixed tuition. If the tuition fee scheme has been changed into the controlled tuition fee scheme, then:
(1) The maximum effort, $\overline{e} = (\overline{r}, \overline{t})$ with $\overline{r} + \overline{t} = \overline{a}$, $\overline{r} > \hat{r}$, and $\overline{t} > \hat{t}$ can be achieved by: (1-i) a large capacity $\overline{a}$ that satisfies $\overline{t} > 2\sqrt{k(k-F)}$ and $\frac{k}{2} < F < k$; or (1-ii) large research funding that satisfies $F > k$ with $s < \frac{(\sqrt{5}-1)k}{4} \approx 0.309k$;
(2) With respect to student enrollment, $\overline{n} > \hat{n}$ holds for $\overline{t} > \frac{2ks(2F-k)}{k^2-4s^2} = k\hat{n} > \hat{t}$;
(3) With respect to research output, $\overline{R} > \hat{R}$ holds if (1) is the case; and
(4) There can exist particular F and s that induce $\overline{R} > \hat{R}$ and $\overline{n} < \hat{n}$.

It is noticeable that when the condition, $\overline{t} > \hat{t}$, is postulated, $\overline{r} > \hat{r}$ is also satisfied by the construction. With this condition in mind, Proposition 8(1) maintains that when a tuition fee is initially fixed, there may be some room to increase both research and teaching efforts by applying a flexibly controlled tuition fee that maximizes tuition revenue. Specifically, if a university does not operate at full capacity under the fixed tuition fee scheme, it is possible to encourage it to exert more of its capacity. But some additional conditions are necessary for an increase in research and teaching efforts, as indicated by Proposition 8(1-i) and 8(1-ii) that can be immediately led by Proposition 6.

First, the potential capacity of a university must be large enough ($\overline{t} > 2\sqrt{k(k-F)}$: large university) to achieve the maximum efforts, $\overline{e} = (\overline{r}, \overline{t})$, when research funding is small ($F < k$). Otherwise, if the capacity is small ($\overline{t} < 2\sqrt{k(k-F)}$: small college), a university prefers to choose the minimum efforts, $e^0 = (F + \delta_r, \delta_t) \approx (F, 0)$. Second, large research funding ($F > k$) enables a university to exert maximum research and teaching efforts irrespective of its potential capacity. In this instance, the tuition fee level set under the scheme of a fixed tuition fee has to satisfy $s < \frac{k(\sqrt{5}-1)}{4} \approx 0.309k$, which is stricter than $s < \frac{k}{2} = 0.5k$, to guarantee the initial equilibrium solution, $\hat{e} = (\hat{r}, \hat{t})$, in underenrollment.

From Proposition 8(2)-(4), while more research output can be produced if research effort is enhanced along with a change in the tuition fee scheme (Proposition 8[3]), student enrollment cannot be necessarily increased from the initial equilibrium solution, $\hat{n} = \frac{2s(2F-k)}{k^2-4s^2}$ (Proposition 8[2]). The reason for the latter is as follows. Now that a university can freely establish an optimal tuition fee that is adjusted to satisfy $s^* = \frac{t^*}{2}$ under the controlled tuition fee scheme, this tuition fee is likely to rise in tandem with improved teaching effort. Although an increase in teaching effort raises the utility of students, an increase in the tuition fee reduces it in the opposite manner. As exhibited in the utility function of students in Equation (8), the net effect of an increase in teaching effort on the student utility is $\Delta t^* - \Delta s^* = \frac{\Delta t^*}{2} < \Delta t^*$. This indicates that although an improved teaching effort generates a higher student enrollment in the controlled tuition fee scheme, the degree is smaller than the fixed tuition fee scheme due to an increase in the tuition fee. Hence, small capacity ($\overline{t} < k\hat{n}$) hinders a university from exceeding the threshold of the teaching effort that can achieve a higher student enrollment ($\overline{n} > \hat{n}$). We also need to note that the condition for $\overline{n} > \hat{n}$ is stricter than that for $\bar{t} > \hat{t}$ from the abovementioned argument.

In view of Proposition 8(4), whether an overall student enrollment will increase or decrease in response to a change in the tuition fee scheme depends on parameters, such as the university capacity, research fund, initial tuition fee, and mobility cost. Let us focus exclusively on university capacity and research funding. If research funding, F, becomes large, the condition on research output ($\bar{t} > 2\sqrt{k(k-F)}$: decreasing in F) can be more easily satisfied while the condition on student enrollment ($\bar{t} > k\hat{n} = \frac{2ks(2F-k)}{k^2-4s^2}$: increasing in F) is not. This is why the condition on student enrollment may not be maintained despite the condition for research output being satisfied.

What kind of universities would observe an increase in both research output and student enrollment in response to a change in the tuition fee scheme from "fixed" to "controlled"? As we have already discussed, a small college with little capacity may opt for minimum research and teaching efforts (Proposition 6[1-ii]). When research funding is sufficiently large, even a small college can operate at full capacity (Proposition 6[2]). However, when a university is still relatively small, and research funding is not sufficient, an increase in student enrollment may not be guaranteed, although research output is likely to increase (Proposition 8[4]). In conclusion, the answer is that only a large university with sufficiently large capacity is expected to enroll more students and produce greater research output by a change from a fixed to a controlled tuition fee scheme.

5 Concluding Remarks

This paper examined how a university's mutually connected research and teaching activities of interact to generate research output and student enrollment, based on the setting in which a university obtains external research funding from a financing agency and earns tuition revenue from students by setting its tuition fee.

This paper theoretically argued that substitutability between research and teaching activities is of great importance, especially when considering how external research funding affects research output and student enrollment. Somewhat paradoxically, it was demonstrated that if substitutability is strong enough, both student enrollment and research output may decrease in response to an increase in research funding. Intuitively, since strong substitutability may drastically decrease teaching effort and student enrollment in response to increased research effort caused by an incremental increase in research funding, a smaller research budget may result in decreased research output in the end. Thus, policymakers and university officials should consider this possibility when they intend to have universities produce higher research output by enhancing research funding.

Simply assuming a zero degree of substitutability in an illustrative model, this paper found that the results significantly vary according to whether a tuition fee is fixed or controlled. In the case of a fixed tuition fee, while research funding can increase both research output and student enrollment when university capacity is not fully used (multiplier effect), student enrollment is crowded out when a university operates at full capacity (crowding-out effect). This simple result derives from the intrinsic nature of a university evaluated ultimately by research output and not teaching outcome.

By contrast, when a government controls a tuition fee to maximize tuition revenue, a marginal amount of research funding never positively affects student enrollment because of the emergence of a binary divide among universities (namely, multiple equilibria). This implies that while a university with large capacity (large university) operates at full capacity, a university with small capacity (small college) opts for marginal activities. In these two cases, increased research funding leads only to increased research output and not student enrollment. In particular, for a large university, the crowding-out effect operates to decrease student enrollment because it operates at full capacity. With these in mind, this paper revealed that to make a small college grow from engaging in marginal activities, providing a sufficiently large amount of research funding or enhancing the capacity of a university is required.

In conclusion, whether both research output and student enrollment increase with external research funding depends on certain conditions. In one case, student enrollment may be decreased while research output is increased. In the other extreme case, when substitutability is strong enough, even research output may be decreased in response to increased research funding.

The issues to be further scrutinized are briefly described in what follows. First, although a financing agency in this model only allocates constant research funding to a university, we can consider a dynamic model in which it depends on research productivity or research output so that the decision of a financing agency is also endogenized. Second, relative to the above, a multiple-universities model can be introduced to include competition for research funding as well as students. It is expected from this formulation that heterogeneous universities are divided into research-specific and teaching-specific universities, which is pointed out by Del Rey (2001) and De Fraja and Iossa (2002). This change in the model setting is likely to consequentially affect total research output and student enrollment.

6 Appendices

6.1 Proofs of Propositions and Lemmas

The mathematical demonstrations are gathered in this subsection. The proofs of Propositions and Lemmas follow.

Proposition 1 (1) $\frac{\partial r^*}{\partial F} = \frac{1}{|A_F|}[\frac{\partial^2 C}{\partial t^2} - r^* s(\frac{\partial^2 n}{\partial t^2})] > 0$ because $\frac{\partial^2 C}{\partial t^2} > 0$ and $\frac{\partial^2 n}{\partial t^2} \leq 0$ from the assumptions.
(2-i) When $\frac{\partial t^*}{\partial F} = \frac{1}{|A_F|}[s(\frac{\partial n}{\partial t}) - \frac{\partial^2 C}{\partial r \partial t}] > 0$ holds, it is obvious that $\frac{\partial^2 C}{\partial r \partial t} < s(\frac{\partial n}{\partial t})$. As for student enrollment, $\frac{\partial n^*}{\partial F} = (\frac{\partial n}{\partial t})(\frac{\partial t^*}{\partial F}) > 0$ because $\frac{\partial t^*}{\partial F} > 0$. Since $R^* = r^*(F + sn^*)$, we can derive $\frac{\partial R^*}{\partial F} = (\frac{\partial r^*}{\partial F})(F + sn^*) + r^*[1 + s(\frac{\partial n^*}{\partial F})] > 0$ in the above condition. (2-ii) By the same derivation of (2-i), if $\frac{\partial^2 C}{\partial r \partial t} > s(\frac{\partial n}{\partial t})$ holds, $\frac{\partial t^*}{\partial F} < 0$ and $\frac{\partial n^*}{\partial F} < 0$.
(3) Transforming the condition for $\frac{\partial R^*}{\partial F} < 0$, we obtain:

$$r^* + \frac{\partial r^*}{\partial F}(F + sn^*) + r^* s\left[\frac{\partial n}{\partial t}\frac{1}{|A_F|}\left[s\left(\frac{\partial n}{\partial t}\right) - \frac{\partial^2 C}{\partial r \partial t}\right]\right] < 0$$
$$\Longleftrightarrow \frac{\partial^2 C}{\partial r \partial t} > s\left(\frac{\partial n}{\partial t}\right) + \underbrace{\frac{|A_F|r^* + \left[\frac{\partial^2 C}{\partial t^2} - r^* s\left(\frac{\partial^2 n}{\partial t^2}\right)\right](F + sn^*)}{r^* s(\frac{\partial n}{\partial t})}}_{J} = s\left(\frac{\partial n}{\partial t}\right) + J, \tag{21}$$

where $J = \frac{|A_F|r^* + \left[\frac{\partial^2 C}{\partial t^2} - r^* s\left(\frac{\partial^2 n}{\partial t^2}\right)\right](F + sn^*)}{r^* s(\frac{\partial n}{\partial t})}$. By assuming the case of $\frac{\partial^2 C}{\partial r \partial t} > s(\frac{\partial n}{\partial t}) > 0$, we find that the determinant is positive: $|A_F| > 0$. Since $|A_F|$ in Equation (21) also includes $\frac{\partial^2 C}{\partial r \partial t}$, we need to check whether this inequality still holds. While the left-hand side of Equation (21) is increasing in $\frac{\partial^2 C}{\partial r \partial t}$, the right-hand side is decreasing in $\frac{\partial^2 C}{\partial r \partial t}$ because $|A_F| = (\frac{\partial^2 C}{\partial r^2})[\frac{\partial^2 C}{\partial t^2} - r^* s(\frac{\partial^2 n}{\partial t^2})] - [\frac{\partial^2 C}{\partial r \partial t} - s(\frac{\partial n}{\partial t})]^2$ is decreasing in $\frac{\partial^2 C}{\partial r \partial t}$ for $\frac{\partial^2 C}{\partial r \partial t} > s(\frac{\partial n}{\partial t})$. In addition, provided that $\frac{\partial^2 C}{\partial r \partial t} = s(\frac{\partial n}{\partial t})$, the right-hand side of Equation (21) is equivalent to:

$$s\left(\frac{\partial n}{\partial t}\right) + \underbrace{\frac{r^*[\frac{\partial^2 C}{\partial t^2} - r^* s(\frac{\partial^2 n}{\partial t})] + [\frac{\partial^2 C}{\partial t^2} - r^* s(\frac{\partial^2 n}{\partial t^2})](F + sn^*)}{r^* s(\frac{\partial n}{\partial t})}}_{\text{positive}} > s\left(\frac{\partial n}{\partial t}\right).$$

Hence, we can find a particular point, $\frac{\partial^2 C}{\partial r \partial t} = s(\frac{\partial n}{\partial t}) + \Omega$ with $\Omega > 0$, which leads to $\frac{\partial^2 C}{\partial r \partial t} = s(\frac{\partial n}{\partial t}) + \frac{|A_F|r^* + [\frac{\partial^2 C}{\partial t^2} - r^* s(\frac{\partial^2 n}{\partial t^2})](F + sn^*)}{r^* s(\frac{\partial n}{\partial t})}$. Accordingly, Ω exactly corresponds to $\Omega = J = \frac{|A_F|r^* + [\frac{\partial^2 C}{\partial t^2} - r^* s(\frac{\partial^2 n}{\partial t^2})](F + sn^*)}{r^* s(\frac{\partial n}{\partial t})} > 0$ in Equation (21). In conclusion, $\frac{\partial R^*}{\partial F} < 0$ for $\frac{\partial^2 C}{\partial r \partial t} > s(\frac{\partial n}{\partial t}) + \Omega$ is established. ■

Lemma 1 Equation (11) can be transformed into $t = (\frac{2s}{k} - \varepsilon)r$. By substituting this into in Equation (10), we obtain $r = (\frac{2s}{k} - \varepsilon)^2 r + \frac{kF-2s^2}{k}$. Solving this equation with respect to r provides $\hat{r} = \frac{k(kF-2s^2)}{(1-\varepsilon^2)k^2+4ks\varepsilon-4s^2}$. These two equations also derive $\hat{t} = \frac{(2s-k\varepsilon)(kF-2s^2)}{(1-\varepsilon^2)k^2+4ks\varepsilon-4s^2}$. $\hat{r}$ and $\hat{t}$ are strictly positive under the following assumptions, $(1-\varepsilon^2)k^2 + 4ks\varepsilon - 4s^2 > 0$, $kF - 2s^2 > 0$, and $2s - k\varepsilon > 0$. These conditions are summarized into $F > \frac{2s^2}{k}$ and $-1 + \frac{2s}{k} < \varepsilon < \frac{2s}{k}$. ■

Proposition 2 $\frac{\partial r^*}{\partial \varepsilon} = \frac{2k^2(kF-2s^2)(k\varepsilon-2s)}{[(1-\varepsilon^2)k^2+4ks\varepsilon-4s^2]^2} < 0$ because $F > \frac{2s^2}{k}$ and $\varepsilon \in (-1 + \frac{2s}{k}, \frac{2s}{k})$ are assumed in an interior equilibrium solution. Since Equation (11) implies $t^* = (\frac{2s}{k} - \varepsilon)r^*$, we can derive $\frac{\partial t^*}{\partial \varepsilon} = -r^* + (\frac{2s}{k} - \varepsilon)\frac{\partial r^*}{\partial \varepsilon} < 0$ because $\frac{2s}{k} - \varepsilon > 0$ and $\frac{\partial r^*}{\partial \varepsilon} < 0$. The student enrollment is represented as $n^* = \frac{2(t^*-s)}{k}$, and hence, $\frac{\partial n^*}{\partial \varepsilon} = (\frac{2}{k})\frac{\partial t^*}{\partial \varepsilon} < 0$. Finally, noting that $R = r^*(F + sn^*)$, we obtain $\frac{\partial R^*}{\partial \varepsilon} = (\frac{\partial r}{\partial \varepsilon})(F + sn^*) + r^* s(\frac{\partial n^*}{\partial \varepsilon}) < 0$. ■

Lemma 2 $\hat{n} = \frac{2(\hat{t}-s)}{k} = \frac{2}{k}[\frac{2s(kF-2s^2)}{k^2-4s^2} - s] = \frac{2s(2F-k)}{k^2-4s^2}$. Since Equation (10) indicates $\hat{r} = b$, we derive $\hat{R} = \hat{r}b = \hat{r}^2 = [\frac{k(kF-2s^2)}{k^2-4s^2}]^2$. The condition of student enrollment requires $0 \le \hat{n} \le 1 \Leftrightarrow 0 \le \frac{2(t-s)}{k} \le 1 \Leftrightarrow 0 \le \frac{2s(2F-k)}{k^2-4s^2} \le 1$. Solving these inequalities with respect to t and F, we obtain $s \le t \le s + \frac{k}{2}$ and $\frac{k}{2} \le F \le \frac{k}{2} + \frac{k^2-4s^2}{4s}$, respectively. Clearly, these two conditions coincide with each other. We will see hereafter the condition regarding F. In the first place, let us consider the case, $\frac{2s^2}{k} < F \le \frac{k}{2}$, where a university needs to decide whether to undertake significantly positive research and teaching efforts. More precisely, a university chooses either minimum teaching effort that assures infinitesimally small student enrollment (i.e., $\boldsymbol{e}^* = (F+\delta_r, s+\delta_t) \approx (F, s)$) or nil (i.e., $\boldsymbol{e}^* = (0, 0)$), considering the payoffs obtained from them. By approximate calculation, the payoffs become $U(F, s) = \frac{F^2-s^2}{2}$ and $U(0, 0) = 0$, respectively. It can be demonstrated that $U(F, s) < U(0, 0)$ if and only if $F < s$. Hence, if $\frac{2s^2}{k} < F < s$ holds, the equilibrium solution is $\boldsymbol{e}^* = (0, 0)$, $n^* = 0$, and $R^* = 0$ (statement [1]). Otherwise, if $s < F \le \frac{k}{2}$, we obtain $\boldsymbol{e}^* = (F + \delta_r, s + \delta_t) \approx (F, s)$, $n^* = \delta_n \approx 0$, and $R^* = (F + \delta_r)^2 \approx F^2$ considering Assumption 1 (statement [2]). Next, consider $F \ge \frac{k}{2} + \frac{k^2-4s^2}{4s}$, where a university enrolls all students in the jurisdiction ($n^* = 1$). In this case, the teaching effort, $t^* = s + \frac{k}{2}$, is chosen at the right corner, and thereby, a university budget amounts to $b = s + F$ (the tuition revenue is $n^*s = s$ for $n^* = 1$). Then, a university gains $U(s + F, s + \frac{k}{2}) = \frac{[2(s+F)]^2-(2s+k)^2}{8}$. Comparing the utility at $\boldsymbol{e} = (0, 0)$, we can derive $U(s + F, s + \frac{k}{2}) > U(0, 0) = 0$ for $F > \frac{k}{2}$. But $F > \frac{k}{2}$ is always satisfied for the setting, $F \ge \frac{k}{2} + \frac{k^2-4s^2}{4s}$. Because we can conclude $U(s + F, s + \frac{k}{2}) > U(0, 0)$, the equilibrium solution is $\mathbf{e}^* = (s + F, s + \frac{k}{2})$, $n^* = 1$, and $R^* = (s + F)^2$ for $F \ge \frac{k}{2} + \frac{k^2-4s^2}{4s}$ (statement [4]). Finally, when $\frac{k}{2} < F < \frac{k}{2} + \frac{k^2-4s^2}{4s}$, a university obtains the payoff, $U(\hat{r}, \hat{t}) = \hat{R} - (\frac{\hat{r}^2}{2} + \frac{\hat{t}^2}{2}) = \frac{1}{2}[\frac{(kF-2s^2)^2}{k^2-4s^2}] > 0$. Hence, the equilibrium solution is $\boldsymbol{e}^* = (\hat{r}, \hat{t})$, $n^* = \hat{n} \in (0, 1)$, and $R^* = \hat{R}$ for $\frac{k}{2} < F < \frac{k}{2} + \frac{k^2-4s^2}{4s}$ (statement [3]). ■

Proposition 3 (1) When $\frac{2s^2}{k} < F < s$ holds, F does not affect research effort and research output so that $\frac{\partial r^*}{\partial F} = 0$ and $\frac{\partial R^*}{\partial F} = 0$. Alternatively, when $F > s$ holds, we find r^* and R^* increasing in F, and hence, obtain $\frac{\partial r^*}{\partial F} > 0$ and $\frac{\partial R^*}{\partial F} > 0$.
(2) When $\frac{k}{2} < F < \frac{k}{2} + \frac{k^2-4s^2}{4s}$ holds, teaching effort ($t^* = \hat{t} = \frac{2s(kF-2s^2)}{k^2-4s^2}$) and student enrollment

($n^* = \hat{n} = \frac{2s(2F-k)}{k^2-4s^2}$) are positively related to F, namely, $\frac{\partial t^*}{\partial F} = \frac{2ks}{k^2-4s^2} > 0$ and $\frac{\partial n^*}{\partial F} = \frac{4s}{k^2-4s^2} > 0$. When $\frac{2s^2}{k} < F \leq \frac{k}{2}$ and $F \geq \frac{k}{2} + \frac{k^2-4s^2}{4s}$, it is clear that $\frac{\partial t^*}{\partial F} = \frac{\partial n^*}{\partial F} = 0$.
(3) Since $\frac{\partial r^*}{\partial F} = \frac{k^2}{k^2-4s^2}$ and $\frac{\partial t^*}{\partial F} = \frac{2ks}{k^2-4s^2} > 0$ for $\frac{k}{2} < F < \frac{k}{2}+\frac{k^2-4s^2}{4s}$, we can derive $\frac{dr^*}{dt^*} = \frac{\partial r^*}{\partial F} / \frac{\partial t^*}{\partial F} = \frac{k}{2s} > 1$ under the assumption of $s < \frac{k}{2}$. ■

Proposition 4 (1) We can demonstrate $\frac{\partial r^*}{\partial k} = \frac{2s^2(k^2-4kF+4s^2)}{(k^2-4s^2)^2} < \frac{2(\frac{k^2}{4})[k^2-4kF+4(\frac{k^2}{4})]}{(k^2-4s^2)^2} = \frac{k^3(k-2F)}{(k^2-4s^2)^2} < 0$ because $s < \frac{k}{2}$ and $F > \frac{k}{2}$ are in a positive interior equilibrium solution. Although $\frac{\partial t^*}{\partial k}$ can be directly calculated, we employ $t^* = (\frac{2s}{k})r^*$ suggested from the first-order condition of t. From this, we obtain $\frac{\partial t^*}{\partial k} = 2[\frac{ks(\frac{\partial r^*}{\partial k})-sr^*}{k^2}] < 0$. In addition, $n^* = \frac{2(t^*-s)}{k} > 0$ leads to $\frac{\partial n^*}{\partial k} = 2[\frac{k(\frac{\partial t^*}{\partial k})-(t^*-s)}{k^2}] < 0$ for $n^* > 0$ (namely, $t^* > s$). Finally, $\frac{\partial R^*}{\partial k} = 2r^*(\frac{\partial r^*}{\partial k}) < 0$ because $R^* = (r^*)^2$.
(2) In the first place, $\frac{\partial r^*}{\partial s} = \frac{4k^2 s(2F-k)}{(k^2-4s^2)^2} > 0$ can be demonstrated. From $t^* = (\frac{2s}{k})r^*$, we derive $\frac{\partial t^*}{\partial s} = \frac{2}{k}[r^* + s(\frac{\partial r^*}{\partial s})] > 0$. Next, note that the sign of $\frac{\partial n^*}{\partial s}$ depends on $\frac{\partial(t^*-s)}{\partial s}$. Thus, we obtain $\frac{\partial(t^*-s)}{\partial s} = (\frac{\partial t^*}{\partial s}) - 1 = \frac{2}{k}[r^* + s(\frac{\partial r^*}{\partial s})] - 1$. As we have already derived $\frac{\partial r^*}{\partial s} > 0$, it can be shown that $\frac{\partial(t^*-s)}{\partial s} > r^*(\frac{2}{k})-1 = \frac{2r^*-k}{k}$. Examining the sign of the numerator derives $2r^*-k = 2[\frac{k(kF-2s^2)}{k^2-4s^2}]-k = \frac{k^2(2F-k)}{k^2-4s^2} > 0$ for $F > \frac{k}{2}$. Hence, we can conclude $\frac{\partial n^*}{\partial s} > 0$. Lastly, $\frac{\partial R^*}{\partial s} = 2r^*(\frac{\partial r^*}{\partial s}) > 0$ is demonstrated. ■

Proposition 5 Because $r + t = \bar{a}$ applies at equilibrium when $\hat{r} + \hat{t} > \bar{a}$ holds, we can derive $t = \bar{a}-r$. By substituting it into the payoff function of a university, $U = r[F+\frac{2s(\bar{a}-r-s)}{k}]-\frac{r^2}{2}-\frac{(\bar{a}-r)^2}{2}$. The first-order condition $\frac{\partial U}{\partial r} = 0$ provides $r^* = \frac{kF-2s^2+\bar{a}(2s+k)}{2(2s+k)}$. By substituting r^* back into $t = \bar{a} - r$, we also obtain $t^* = \frac{-(kF-2s^2)+\bar{a}(2s+k)}{2(2s+k)}$. Hence, we can show $\frac{\partial r^*}{\partial F} = \frac{k}{2(2s+k)} > 0$ and $\frac{\partial t^*}{\partial F} = -\frac{k}{2(2s+k)} < 0$. Since $n^* = \frac{2(t^*-s)}{k}$, we obtain $\frac{\partial n^*}{\partial F} = (\frac{2}{k})\frac{\partial t^*}{\partial F} = -\frac{1}{2s+k} < 0$. Furthermore, we can denote $R^* = r^*(F+sn^*)$, and thus, $\frac{\partial R^*}{\partial F} = \frac{\partial r^*}{\partial F}(F+sn^*)+r^*[1+s(\frac{\partial n^*}{\partial F})]$. Since $1+s(\frac{\partial n^*}{\partial F}) = 1-\frac{s}{2s+k} = \frac{s+k}{2s+k} > 0$, we can see that a decrease in tuition revenue is smaller than an increase in research funding. In sum, we conclude $\frac{\partial R^*}{\partial F} > 0$. ■

Proposition 6 (1) When $F < k$ holds, the saddle point $\tilde{e} = (k, \sqrt{2k(k-F)})$ in the diagram of (r, t) appears in the north-east space from the point, $(F, 0)$. Hence, a university finds it optimal to select either maximum boundary efforts, $\overline{\boldsymbol{e}} = (\bar{r}, \bar{t})$ that satisfies both $r+t = \bar{a}$ and $r = F+\frac{t^2}{2k}$ (from Assumption 2), or minimum boundary efforts, $\boldsymbol{e}^0 = (F + \delta_r, \delta_t)$ where δ_r and δ_t are infinitesimal positive values. First, if research and teaching efforts are binding at $\overline{\boldsymbol{e}} = (\bar{r}, \bar{t}) = (F + \frac{\bar{t}^2}{2k}, \bar{t})$, the payoff of a university reaches $U(F + \frac{\bar{t}^2}{2k}, \bar{t}) = \frac{1}{2}(F + \frac{\bar{t}^2}{2k})^2 - \frac{\bar{t}^2}{2} = \frac{\bar{t}^4+4k(F-k)\bar{t}^2+4k^2F^2}{8k^2}$. We need to compare this payoff with $U(F+\delta_r, \delta_t) \approx U(F, 0) = \frac{F^2}{2}$ at the minimum boundary efforts. Solving the quadratic equation of $U(F + \frac{\bar{t}^2}{2k}, \bar{t}) = U(F, 0) \Leftrightarrow \frac{\bar{t}^4+4k(F-k)\bar{t}^2+4k^2F^2}{8k^2} = \frac{F^2}{2}$, we obtain $\bar{t}^2 = 4k(k-F) \Leftrightarrow \bar{t} = 2\sqrt{k(k-F)} > \sqrt{2k(k-F)}$ for $F < k$. From this relation, if $\bar{t} > 2\sqrt{k(k-F)}$ holds, the equilibrium solution is $\boldsymbol{e}^* = \overline{\boldsymbol{e}} = (F + \frac{\bar{t}^2}{2k}, \bar{t})$, $s^* = \bar{s} = \frac{\bar{t}}{2}$, $n^* = \bar{n} = \frac{2(t^*-s^*)}{k} = \frac{\bar{t}}{k}$, and $R^* = r^*(F + s^*n^*) = (F + \frac{\bar{t}^2}{2k})^2$. By contrast, if $\bar{t} < 2\sqrt{k(k-F)}$ holds, the equilibrium solution is

$\boldsymbol{e}^* = \boldsymbol{e}^0 = (F + \delta_r, \delta_t) \approx (F, 0)$, $s^* = \frac{\delta_t}{2} \approx 0$, $n^* = \frac{\delta_t}{k} \approx 0$, and $R^* = (F + \delta_r)(F + \frac{\delta_t^2}{2k}) \approx F^2$.
(2) With respect to $F > k$, while the saddle point $\tilde{e} = (k, \sqrt{2k(k-F)})$ does not appear, $\boldsymbol{e}^0 = (F, 0)$ becomes a new saddle point. Clearly, $U(F + \frac{\bar{t}^2}{2k}, \bar{t}) = \frac{\bar{t}^4 + 4k(F-k)\bar{t}^2 + 4k^2F^2}{8k^2} > \frac{\bar{t}^2 + 4k^2F^2}{8k^2} = \frac{\bar{t}^2}{8k^2} + \frac{F^2}{2} > \frac{F^2}{2} = U(F, 0)$. Hence, the equilibrium solution is $\boldsymbol{e}^* = \overline{\boldsymbol{e}} = (F + \frac{\bar{t}^2}{2k}, \bar{t})$ for any $\bar{t}$. ■

Proposition 7 (1) Since $r^* + t^* = \bar{r} + \bar{t} = \bar{a}$ is satisfied as an equilibrium solution, $\boldsymbol{e}^* = \overline{\boldsymbol{e}} = (F + \frac{\bar{t}^2}{2k}, \bar{t})$, we obtain $r^* = F + \frac{\bar{t}^2}{2k} = F + \frac{(\bar{a}-r^*)^2}{2k}$. Considering a derivative with respect to F on both sides of this equation, we have $\frac{\partial r^*}{\partial F} = 1 - (\frac{\bar{a}-r^*}{k})\frac{\partial r^*}{\partial F}$, which can be transformed into $(1 + \frac{\bar{a}-r^*}{k})\frac{\partial r^*}{\partial F} = 1$. As $1 + \frac{\bar{a}-r^*}{k}$ is obviously positive, we obtain $\frac{\partial r^*}{\partial F} > 0$. Moreover, since the capacity, $\bar{a}$, being constant indicates $\frac{\partial r^*}{\partial F} + \frac{\partial t^*}{\partial F} = 0$, we can conclude $\frac{\partial t^*}{\partial F} = -\frac{\partial r^*}{\partial F} < 0$. As for the other comparative statics, $\frac{\partial s^*}{\partial F} = (\frac{1}{2})\frac{\partial t^*}{\partial F} < 0$, $\frac{\partial n^*}{\partial F} = (\frac{1}{k})\frac{\partial t^*}{\partial F} < 0$, and $\frac{\partial R^*}{\partial F} = 2r^*(\frac{\partial r^*}{\partial F}) > 0$.
(2) A marginal increase in F moves the equilibrium solution only for research effort and research output, but not teaching effort and student enrollment, as suggested by the solution. Therefore, $\frac{\partial r^*}{\partial F} > 0$, $\frac{\partial t^*}{\partial F} = 0$, $\frac{\partial s^*}{\partial F} = 0$, $\frac{\partial n^*}{\partial F} = 0$, and $\frac{\partial R^*}{\partial F} > 0$ hold. ■

Proposition 8 (1) If we assume $\bar{t} > \hat{t} = \frac{2s(kF-2s^2)}{k^2-4s^2}$, $\bar{r} > \hat{r}$ is also expected to be satisfied because of $\hat{r} + \hat{t} < \bar{a}$ and $\bar{r} + \bar{t} = \bar{a}$ obtained from the construction. (1-i) As shown in Proposition 6, when the capacity, $\bar{a}$, is large enough that $\bar{t} > 2\sqrt{k(k-F)}$ and $\frac{k}{2} < F < k$ are satisfied ($F > \frac{k}{2}$ is required for $\hat{n} > 0$), a university prefers $\overline{\boldsymbol{e}} = (\bar{r}, \bar{t}) = (F + \frac{\bar{t}^2}{2k}, \bar{t})$ to $\boldsymbol{e}^0 = (F + \delta_r, \delta_t) \approx (F, 0)$. (1-ii) We have also proved that if $F > k$ is satisfied, a university always prefers $\overline{\boldsymbol{e}} = (\bar{r}, \bar{t}) = (F + \frac{\bar{t}^2}{2k}, \bar{t})$ for any $\bar{t}$. For such an F to exist within $F \in (\frac{k}{2}, \frac{k}{2} + \frac{k^2-4s^2}{4s})$ (the condition of which is that $\hat{e} = (\hat{r}, \hat{t})$ is a positive interior equilibrium solution under the fixed tuition fee scheme), it must be the case that $\frac{k}{2} + \frac{k^2-4s^2}{4s} > k$. Hence, solving this quadratic inequality, we need to consider $s < \frac{k(\sqrt{5}-1)}{4} \approx 0.309k$. This satisfies the condition of $s < \frac{k}{2} = 0.5k$ that is necessary for the second-order condition.
(2) To check whether $\bar{n}$ is larger than $\hat{n}$, we examine whether $\bar{n} - \hat{n} = \frac{\bar{t}}{k} - \frac{2s(2F-k)}{k^2-4s^2} = \frac{(k^2-4s^2)\bar{t} - 2ks(2F-k)}{k(k^2-4s^2)} > 0$ holds. By solving this inequality with respect to $\bar{t}$, we can show $\bar{n} > \hat{n}$ for $\bar{t} > \frac{2ks(2F-k)}{k^2-4s^2} = k\hat{n}$. Furthermore, $k\hat{n} - \hat{t} = \frac{2ks(2F-k)}{k^2-4s^2} - \frac{2s(kF-2s^2)}{k^2-4s^2} = \frac{2s(kF-2s^2)}{k^2-4s^2} > 0 \Leftrightarrow k\hat{n} > \hat{t}$ is satisfied. Hence, $\bar{t} > k\hat{n}$ is a stricter condition than $\bar{t} > \hat{t}$.
(3) As has been already shown, $R = r(F + sn) = r^2$ holds in this modeling from the first-order condition of r. When (1) is applied, we obtain $\overline{R} = \bar{r}^2 > \hat{r}^2 = \hat{R}$.
(4) Let us denote $f(F) = 2\sqrt{k(k-F)}$ and $g(F) = \frac{2ks(2F-k)}{k^2-4s^2}$. When $\bar{t} > f(F)$ and $\bar{t} < g(F)$, we derive $\overline{R} > \hat{R}$ and $\bar{n} < \hat{n}$. Obviously, $f(F)$ is decreasing and $g(F)$ is increasing in F monotonically. We have $f(\frac{k}{2}) = \sqrt{2}k > 0$, $g(\frac{k}{2}) = 0$, $f(k) = 0$, $g(k) = \frac{2k^2s}{k^2-4s^2} > 0$, and $g(\frac{k}{2} + \frac{k^2-4s^2}{4s}) = k > 0$. Suppose $s < \frac{k(\sqrt{5}-1)}{4} \approx 0.309k$ as before. As the diagram of Figure 6 illustrates, $f(F)$ and $g(F)$ must intersect only once at some point $F \in (\frac{k}{2}, k)$ from the intermediate-value theorem. Hence, we can find that there exist $\bar{t} > f(F)$ and $\bar{t} < g(F)$ in the area of (A). ■

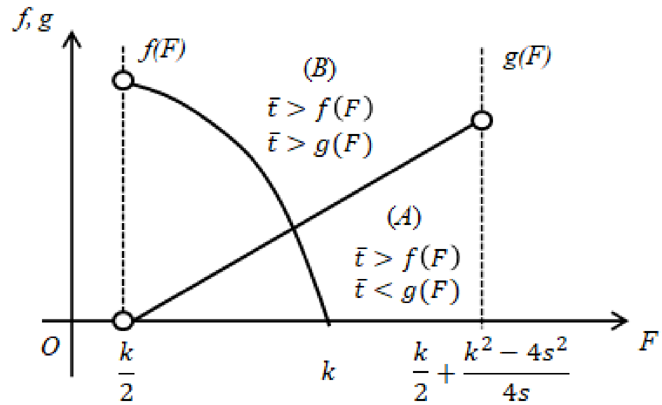

FIGURE 6. Diagram of $f(F)$ and $g(F)$.

6.2 Investigation of saddle points

In Subsection 4.2, the first-order conditions of maximizing $U(r,t) = Fr - \frac{r^2}{2} + (\frac{r-k}{2k})t^2$ with respect to r and t are given by Equations (19) and (20): $\frac{\partial U}{\partial r} = F - r + \frac{t^2}{2k} = 0$ and $\frac{\partial U}{\partial t} = (\frac{r-k}{k})t = 0$, respectively. By solving these two equations simultaneously, we derive the following two solutions: $(r,t) = (k, \sqrt{2k(k-F)})$ and $(F, 0)$.

We define the Hessian matrix of $U(r,t)$ as follows:

$$\tilde{U} = \begin{bmatrix} \frac{\partial^2 U}{\partial r^2} & \frac{\partial^2 U}{\partial r \partial t} \\ \frac{\partial^2 U}{\partial t \partial r} & \frac{\partial^2 U}{\partial t^2} \end{bmatrix} = \begin{bmatrix} -1 & \frac{t}{k} \\ \frac{t}{k} & \frac{r-k}{k} \end{bmatrix}.$$

Let us first consider $(r,t) = (k, \sqrt{2k(k-F)})$ with $F < k$. In this case, since $|\tilde{U}| = -\frac{t^2}{k} = -\frac{2k(k-F)}{k} < 0$, $(r,t) = (k, \sqrt{2k(k-F)})$ is a saddle point. Alternatively, evaluating at $(r,t) = (F,0)$, we obtain $|\tilde{U}| = -\frac{F-k}{k} > 0$ for $F < k$, which implies that $(r,t) = (F,0)$ is a local maximum. Next, suppose $F > k$; then we derive only $(r,t) = (F,0)$ as a solution to the simultaneous equations. Since the determinant at this point is $|\tilde{U}| = -\frac{F-k}{k} < 0$, $(r,t) = (F,0)$ is a saddle point.

6.3 Comparative statics of other parameters

Student mobility cost (k)

Considering the derivatives on both sides of Equations (3) and (4) by k, respectively, we obtain the following relations:

$$s\left[\left.\frac{\partial n}{\partial k}\right|_{t=t_c} + \left(\frac{\partial n}{\partial t}\right)\frac{\partial t^*}{\partial k}\right] - \left[\left(\frac{\partial^2 C}{\partial r^2}\right)\frac{\partial r^*}{\partial k} + \left(\frac{\partial^2 C}{\partial r \partial t}\right)\frac{\partial t^*}{\partial k}\right] = 0$$
$$\Longleftrightarrow \left(\frac{\partial^2 C}{\partial r^2}\right)\frac{\partial r^*}{\partial k} + \left[\frac{\partial^2 C}{\partial r \partial t} - s\left(\frac{\partial n}{\partial t}\right)\right]\frac{\partial t^*}{\partial k} = s\left(\left.\frac{\partial n}{\partial k}\right|_{t=t_c}\right), \tag{22}$$

$$s\left[\left(\frac{\partial n}{\partial t}\right)\frac{\partial r^*}{\partial k}+r^*\left[\frac{\partial^2 n}{\partial t\partial k}+\left(\frac{\partial^2 n}{\partial t^2}\right)\frac{\partial t^*}{\partial k}\right]\right]-\left[\left(\frac{\partial^2 C}{\partial r\partial t}\right)\frac{\partial r^*}{\partial k}+\left(\frac{\partial^2 C}{\partial t^2}\right)\frac{\partial t^*}{\partial k}\right]=0$$

$$\Longleftrightarrow\left[\frac{\partial^2 C}{\partial r\partial t}-s\left(\frac{\partial n}{\partial t}\right)\right]\frac{\partial r^*}{\partial k}+\left[\frac{\partial^2 C}{\partial t^2}-sr^*\left(\frac{\partial^2 n}{\partial t^2}\right)\right]\frac{\partial t^*}{\partial k}=sr^*\left(\frac{\partial^2 n}{\partial t\partial k}\right), \tag{23}$$

where $t = t_c$ means a derivative on the condition that t is constant. The matrix notation of Equations (22) and (23) is reduced to:

$$\begin{bmatrix}\frac{\partial^2 C}{\partial r^2} & \frac{\partial^2 C}{\partial r\partial t}-s(\frac{\partial n}{\partial t})\\ \frac{\partial^2 C}{\partial r\partial t}-s(\frac{\partial n}{\partial t}) & \frac{\partial^2 C}{\partial t^2}-sr^*(\frac{\partial^2 n}{\partial t^2})\end{bmatrix}\begin{bmatrix}\frac{\partial r^*}{\partial k}\\ \frac{\partial t^*}{\partial k}\end{bmatrix}=\begin{bmatrix}s(\frac{\partial n}{\partial k}\big|_{t=t_c})\\ sr^*(\frac{\partial^2 n}{\partial t\partial k})\end{bmatrix}. \tag{24}$$

Let us denote the first matrix as A_k. Suppose that $|A_k| = (\frac{\partial^2 C}{\partial r^2})[\frac{\partial^2 C}{\partial t^2}-sr^*(\frac{\partial^2 n}{\partial t^2})]-[\frac{\partial^2 C}{\partial r\partial t}-s(\frac{\partial n}{\partial t})]^2 > 0$ is satisfied for an analytical purpose. [17]

We solve Equation (24) with respect to $\frac{\partial r^*}{\partial k}$ and $\frac{\partial t^*}{\partial k}$:

$$\begin{aligned}\frac{\partial r^*}{\partial k}&=\frac{\begin{vmatrix}s(\frac{\partial n}{\partial k}\big|_{t=t_c}) & \frac{\partial^2 C}{\partial r\partial t}-s(\frac{\partial n}{\partial t})\\ sr^*(\frac{\partial^2 n}{\partial t\partial k}) & \frac{\partial^2 C}{\partial t^2}-sr^*(\frac{\partial^2 n}{\partial t^2})\end{vmatrix}}{|A_k|}\\ &=\frac{1}{|A_k|}\left[s\left(\frac{\partial n}{\partial k}\bigg|_{t=t_c}\right)\left[\left(\frac{\partial^2 C}{\partial t^2}\right)-sr^*\left(\frac{\partial^2 n}{\partial t^2}\right)\right]-sr^*\left(\frac{\partial^2 n}{\partial t\partial k}\right)\left[\frac{\partial^2 C}{\partial r\partial t}-s\left(\frac{\partial n}{\partial t}\right)\right]\right],\end{aligned} \tag{25}$$

$$\begin{aligned}\frac{\partial t^*}{\partial k}&=\frac{\begin{vmatrix}\frac{\partial^2 C}{\partial r^2} & s(\frac{\partial n}{\partial k}\big|_{t=t_c})\\ \frac{\partial^2 C}{\partial r\partial t}-s(\frac{\partial n}{\partial t}) & sr^*(\frac{\partial^2 n}{\partial t\partial k})\end{vmatrix}}{|A_k|}\\ &=\frac{1}{|A_k|}\left[sr^*\left(\frac{\partial^2 C}{\partial r^2}\right)\left(\frac{\partial^2 n}{\partial t\partial k}\right)-s\left(\frac{\partial n}{\partial k}\bigg|_{t=t_c}\right)\left[\frac{\partial^2 C}{\partial r\partial t}-s\left(\frac{\partial n}{\partial t}\right)\right]\right].\end{aligned} \tag{26}$$

In Equations (25) and (26), the signs of $\frac{\partial r^*}{\partial k}$ and $\frac{\partial t^*}{\partial k}$ are not decisive depending on $\frac{\partial^2 C}{\partial r\partial t}$. As usually expected, the conditions for the negative impact of a mobility cost on research and teaching efforts, $\frac{\partial r^*}{\partial k} < 0$ and $\frac{\partial r^*}{\partial k} < 0$, are as follows:

$$\frac{\partial r^*}{\partial k}<0\Longleftrightarrow\frac{\partial^2 C}{\partial r\partial t}<\underbrace{s\left(\frac{\partial n}{\partial t}\right)}_{\text{positive}}+\underbrace{\frac{(\frac{\partial n}{\partial k}\big|_{t=t_c})[(\frac{\partial^2 C}{\partial t^2})-sr^*(\frac{\partial^2 n}{\partial t^2})]}{r^*(\frac{\partial^2 n}{\partial t\partial k})}}_{\text{positive}}, \tag{27}$$

[17] As mentioned in Footnote 8, when $\frac{\partial^2 C}{\partial r\partial t} > 0$ is assumed, $|A_k|$ is always satisfied from the assumption made with $C(r,t)$ and $n(t,k,s)$.

$$\frac{\partial t^*}{\partial k} < 0 \Longleftrightarrow \frac{\partial^2 C}{\partial r \partial t} < \underbrace{s\left(\frac{\partial n}{\partial t}\right)}_{\text{positive}} + \underbrace{\frac{r^*(\frac{\partial^2 C}{\partial r^2})(\frac{\partial^2 n}{\partial t \partial k})}{\frac{\partial n}{\partial k}\Big|_{t=t_c}}}_{\text{positive}}. \tag{28}$$

Equations (27) and (28) suggest that if research and teaching activities are complementary or independent (i.e., $\frac{\partial^2 C}{\partial r \partial t} \leq 0$), a rise in a mobility cost decreases both research and teaching efforts. Accordingly, we can obtain $\frac{\partial n^*}{\partial k} = \frac{\partial n^*}{\partial k}\Big|_{t=t_c} + (\frac{\partial n}{\partial t})(\frac{\partial t^*}{\partial k}) < 0$ and $\frac{\partial R^*}{\partial k} = \frac{\partial r^*}{\partial k}(F + sn^*) + r^* s(\frac{\partial n^*}{\partial k}) < 0$, which implies that both student enrollment and research output will decrease.

On the contrary, if these conditions are not satisfied (the substitutability is sufficiently positive), we may obtain $\frac{\partial r^*}{\partial k} > 0$ and $\frac{\partial t^*}{\partial k} > 0$. In addition, the substitutability being very strong may generate $\frac{\partial n^*}{\partial k} = \frac{\partial n^*}{\partial k}\Big|_{t=t_c} + (\frac{\partial n^*}{\partial t})(\frac{\partial t^*}{\partial k}) > 0$ and $\frac{\partial R^*}{\partial k} = \frac{\partial r^*}{\partial k}(F + sn^*) + r^* s(\frac{\partial n^*}{\partial k}) > 0$. Although this argument seems surprising, the intuition is straightforward. That is, a rise in mobility cost reduces the budget of a university through a decrease in student enrollment so that a university relinquishes some degree of research effort. However, if substitutability is strong enough, teaching effort increases in response to the decreased research effort, which culminates in higher student enrollment in the end. When this latter positive effect on student enrollment is sufficiently large, research effort and research output may increase because of an enhanced research budget.

Tuition fee (s)

A tuition fee is assumed to be an exogenous variable. If we take the derivatives on both sides of Equations (3) and (4) by s, respectively, we obtain:

$$\begin{aligned}
& n^* + s\left[\frac{\partial n}{\partial s}\Big|_{t=t_c} + \left(\frac{\partial n}{\partial t}\right)\frac{\partial t^*}{\partial s}\right] - \left[\left(\frac{\partial^2 C}{\partial r^2}\right)\frac{\partial r^*}{\partial s} + \left(\frac{\partial^2 C}{\partial r \partial t}\right)\frac{\partial t^*}{\partial s}\right] = 0 \\
& \Longleftrightarrow \left(\frac{\partial^2 C}{\partial r^2}\right)\frac{\partial r^*}{\partial s} + \left[\frac{\partial^2 C}{\partial r \partial t} - s\left(\frac{\partial n}{\partial t}\right)\right]\frac{\partial t}{\partial s} = n^* + s\left(\frac{\partial n}{\partial s}\Big|_{t=t_c}\right),
\end{aligned} \tag{29}$$

$$\begin{aligned}
& \left[s\left(\frac{\partial n}{\partial t}\right)\frac{\partial r^*}{\partial s} + r^*\left(\frac{\partial n}{\partial t}\right)\right] + r^* s\left[\frac{\partial^2 n}{\partial t \partial s} + \left(\frac{\partial^2 n}{\partial t^2}\right)\frac{\partial t^*}{\partial s}\right] - \left[\left(\frac{\partial^2 C}{\partial r \partial t}\right)\frac{\partial r^*}{\partial s} + \left(\frac{\partial^2 C}{\partial t^2}\right)\frac{\partial t^*}{\partial s}\right] = 0 \\
& \Longleftrightarrow \left[\frac{\partial^2 C}{\partial r \partial t} - s\left(\frac{\partial n}{\partial t}\right)\right]\frac{\partial r^*}{\partial s} + \left[\left(\frac{\partial^2 C}{\partial t^2}\right) - r^* s\left(\frac{\partial^2 n}{\partial t^2}\right)\right]\frac{\partial t^*}{\partial s} = r^*\left[\frac{\partial n}{\partial t} + s\left(\frac{\partial^2 n}{\partial t \partial s}\right)\right].
\end{aligned} \tag{30}$$

From Equations (29) and (30), the following matrix notation is derived:

$$\begin{bmatrix} \frac{\partial^2 C}{\partial r^2} & \frac{\partial^2 C}{\partial r \partial t} - s(\frac{\partial n}{\partial t}) \\ \frac{\partial^2 C}{\partial r \partial t} - s(\frac{\partial n}{\partial t}) & \frac{\partial^2 C}{\partial t^2} - r^* s(\frac{\partial^2 n}{\partial t^2}) \end{bmatrix} \begin{bmatrix} \frac{\partial r}{\partial s} \\ \frac{\partial t}{\partial s} \end{bmatrix} = \begin{bmatrix} n^* + s(\frac{\partial n}{\partial s}\Big|_{t=t_c}) \\ r^*[\frac{\partial n}{\partial t} + s(\frac{\partial^2 n}{\partial t \partial s})] \end{bmatrix}.$$

Suppose again that the determinant of the first matrix, $|A_s| = (\frac{\partial^2 C}{\partial r^2})[\frac{\partial^2 C}{\partial t^2} - r^* s(\frac{\partial^2 n}{\partial t^2})] - [\frac{\partial^2 C}{\partial r \partial t} - s(\frac{\partial n}{\partial t})]^2 > 0$. We obtain $\frac{\partial r^*}{\partial s}$ and $\frac{\partial t^*}{\partial s}$ such that:

$$
\begin{aligned}
\frac{\partial r^*}{\partial s} &= \frac{\begin{vmatrix} n^* + s(\frac{\partial n}{\partial s}\Big|_{t=t_c}) & \frac{\partial^2 C}{\partial r \partial t} - s(\frac{\partial n}{\partial t}) \\ r^*[\frac{\partial n}{\partial t} + s(\frac{\partial^2 n}{\partial t \partial s})] & \frac{\partial^2 C}{\partial t^2} - r^* s(\frac{\partial^2 n}{\partial t^2}) \end{vmatrix}}{|A|} \\
&= \frac{1}{|A_s|}\left[\left[\frac{\partial^2 C}{\partial t^2} - r^* s\left(\frac{\partial^2 n}{\partial t^2}\right)\right]\left[n^* + s\left(\frac{\partial n}{\partial s}\Big|_{t=t_c}\right)\right] - r^*\left[\frac{\partial n}{\partial t} + s\left(\frac{\partial^2 n}{\partial t \partial s}\right)\right]\left[\frac{\partial^2 C}{\partial r \partial t} - s\left(\frac{\partial n}{\partial t}\right)\right]\right], \quad (31) \\
\frac{\partial t^*}{\partial s} &= \frac{\begin{vmatrix} \frac{\partial^2 C}{\partial r^2} & n^* + s(\frac{\partial n}{\partial s}\Big|_{t=t_c}) \\ \frac{\partial^2 C}{\partial r \partial t} - s(\frac{\partial n}{\partial t}) & r^*[\frac{\partial n}{\partial t} + s(\frac{\partial^2 n}{\partial t \partial s})] \end{vmatrix}}{|A|} \\
&= \frac{1}{|A_s|}\left[r^*\left(\frac{\partial^2 C}{\partial r^2}\right)\left[\frac{\partial n}{\partial t} + s\left(\frac{\partial^2 n}{\partial t \partial s}\right)\right] - \left[n^* + s\left(\frac{\partial n}{\partial s}\Big|_{t=t_c}\right)\right]\left[\frac{\partial^2 C}{\partial r \partial t} - s\left(\frac{\partial n}{\partial t}\right)\right]\right]. \quad (32)
\end{aligned}
$$

Equations (31) and (32) reveal that the signs of $\frac{\partial r^*}{\partial s}$ and $\frac{\partial t^*}{\partial s}$ depend on those of $\frac{\partial^2 C}{\partial r \partial t}$, $n^* + s(\frac{\partial n}{\partial s}\Big|_{t=t_c})$ (the degree of the tuition fee elasticity of student enrollment given teaching effort), and $\frac{\partial^2 n}{\partial t \partial s}$. For analytical simplicity, we posit $\frac{\partial^2 n}{\partial t \partial s} = 0$. We can rewrite Equations (31) and (32) as follows:

$$
\begin{aligned}
\frac{\partial r^*}{\partial s} &= \frac{1}{|A_s|}\left[\left[\frac{\partial^2 C}{\partial t^2} - r^* s\left(\frac{\partial^2 n}{\partial t^2}\right)\right]\left[n^* + s\left(\frac{\partial n}{\partial s}\Big|_{t=t_c}\right)\right] - r^*\left(\frac{\partial n}{\partial t}\right)\left[\frac{\partial^2 C}{\partial r \partial t} - s\left(\frac{\partial n}{\partial t}\right)\right]\right], \quad (33) \\
\frac{\partial t^*}{\partial s} &= \frac{1}{|A_s|}\left[r^*\left(\frac{\partial n}{\partial t}\right)\left(\frac{\partial^2 C}{\partial r^2}\right) - \left[n^* + s\left(\frac{\partial n}{\partial s}\Big|_{t=t_c}\right)\right]\left[\frac{\partial^2 C}{\partial r \partial t} - s\left(\frac{\partial n}{\partial t}\right)\right]\right]. \quad (34)
\end{aligned}
$$

Thus, the conditions for $\frac{\partial r^*}{\partial s} < 0$ and $\frac{\partial t^*}{\partial s} < 0$ are derived as follows:

$$
\frac{\partial r^*}{\partial s} < 0 \Longleftrightarrow \frac{\partial^2 C}{\partial r \partial t} > \underbrace{s\left(\frac{\partial n}{\partial t}\right)}_{\text{positive}} + \frac{[\frac{\partial^2 C}{\partial t^2} - r^* s(\frac{\partial^2 n}{\partial t^2})][n^* + s(\frac{\partial n}{\partial s}\Big|_{t=t_c})]}{r^*(\frac{\partial n}{\partial t})}, \quad (35)
$$

$$
\frac{\partial t^*}{\partial s} < 0 \Longleftrightarrow \frac{\partial^2 C}{\partial r \partial t} > \underbrace{s\left(\frac{\partial n}{\partial t}\right)}_{\text{positive}} + \underbrace{\frac{r^*(\frac{\partial^2 C}{\partial r^2})(\frac{\partial n}{\partial t})}{n^* + s(\frac{\partial n}{\partial s}\Big|_{t=t_c})}}_{\text{positive}} > 0 \;\text{ for } n^* + s\left(\frac{\partial n}{\partial s}\Big|_{t=t_c}\right) > 0, \quad (36)
$$

$$
\Longleftrightarrow \frac{\partial^2 C}{\partial r \partial t} < \underbrace{s\left(\frac{\partial n}{\partial t}\right)}_{\text{positive}} + \underbrace{\frac{r^*(\frac{\partial^2 C}{\partial r^2})(\frac{\partial n}{\partial t})}{n^* + s(\frac{\partial n}{\partial s}\Big|_{t=t_c})}}_{\text{negative}} \;\text{ for } n^* + s\left(\frac{\partial n}{\partial s}\Big|_{t=t_c}\right) < 0. \quad (37)
$$

The comparative statics of a tuition fee is much more complicated than a mobility cost.

Equation (34) indicates that when a certain degree of substitutability occurs, research effort is decreased, i.e., $\frac{\partial r^*}{\partial s} < 0$, by a rise in a tuition fee. On the other hand, there are two cases for $\frac{\partial t^*}{\partial s} < 0$ according to the sign of $n^* + s(\frac{\partial n}{\partial s}\Big|_{t=t_c})$. Let us first focus on Equation (36). When the tuition fee elasticity of student enrollment is inelastic given teaching effort, a rise in a tuition fee increases university's tuition revenue, and research effort increases accordingly. There is also more room for enhancing teaching activity due to the increased university budget. But if substitutability is strong, teaching effort is decreased in the equilibrium. With respect to Equation (37), since the tuition fee elasticity is elastic, a tuition fee increase reduces total tuition revenue, and the university is compelled to reduce research effort in response. Then, if substitutability is not strong, teaching effort is also ultimately decreased in equilibrium. In both Equation (36) and Equation (37), if teaching effort is reduced, student enrollment also decreases. Moreover, as for research output, $\frac{\partial R^*}{\partial s} = \frac{\partial r^*}{\partial s}(F + sn^*) + r^*[n^* + s[\frac{\partial n}{\partial s}\Big|_{t=t_c} + (\frac{\partial n}{\partial t})(\frac{\partial t^*}{\partial s})]] < 0$ may be derived when substitutability is strong, and the tuition fee elasticity is inelastic. Finally, we can obtain the conditions for $\frac{\partial r^*}{\partial s} > 0$ and $\frac{\partial t^*}{\partial s} > 0$ by reversing inequality signs of Equations (35)–(37), and the essence of the reasoning is the same as in the previous discussion.

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