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"Consumption Externalities and Equilibrium Dynamics with Heterogenous Agents"

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Consumption Externalities and Equilibrium Dynamics with Heterogenous Agents*

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Abstract

This paper explores the effect of consumption externalities on equilibrium dynamics of a standard neoclassical growth model in which there are two types of agents. To emphasize the presence of heterogenous agents, we distinguish intergroup consumption externalities from intragroup consumption externalities. We show that if there are intragroup consumption externalities alone, then the steady state equilibrium satisfies saddle-point stability and the equilibrium path of the economy is uniquely determined. In contrast, even if the intragroup consumption externalities do not exist, the intergroup external effects of consumption may yield either unstability or local indeterminacy of the steady-state equilibrium. In addition to analytical considerations, we show the relationship between the stability and the consumption externalities in numerical examples.

Keywords: consumption externalities, heterogeneous agents, progressive taxation, equilibrium determinacy

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1 Introduction

Recently, there is a renewed interest in consumption external effects in dynamic macroeconomic analyses. While the earlier contributions such as Abel (1990) and Gali (1991) focus on the role of consumption externalities in the asset-pricing models, the recent studies treat a wider class of issues. For example, the recent investigations consider external effects of consumption on optimal taxation (Linquvist and Sargent 1997), on the relation between savings and long-term economic growth (Carroll et al. 1997 and 2000) as well as on interactions of consumption and production externalities (Weder 2000 and Liu and Turnovsky 2005). A common feature of this literature is that most studies employ the representative agent frameworks. In this literature the consumption external effect is formulated in such a way that an individual consumer's felicity depends on the average level of consumption in the economy as well as on her own consumption. In the equilibrium of representative-agent economies the individual and the average levels of consumption coincide each other and, therefore, the presence of consumption externalities generally produce quantitative effect rather than qualitative effect: the equilibrium dynamics and the steady state characterization are usually the same as the models without consumption externalities.

Unlike the mainstream literature mentioned above, this paper examines the role of consumption externalities in the presence of heterogenous agents. Since the external interactions among the consumers tend to be much more complex in an economy with heterogenous agents than in the representative-agent counterpart, the presence of consumption external effects would yield fundamental impacts on the dynamic behavior of the economy if we consider heterogeneity of consumers. Using a simple model of the neoclassical growth model with two types of agents, we confirm our prediction. We show that even in the symmetric steady state where every agent has the same levels of income and wealth, the dynamic behavior of the economy may not exhibit a regular saddle point stability. The equilibrium path of the economy may be either unstable or indeterminate. Thus consumption externalities, together with heterogeneity of agents, would yield a variety of dynamic behaviors even in the absence of production externalities or complex preference structure associated with variable labor supply.

The analytical framework of this paper is the standard neoclassical growth model with infinitely-lived agents. In this setting it has been well known that there exists a continuum of steady states if all the agents have an identical time discount rate, while the agent with the lowest time discount rate ultimately owns the entire capital stock if the time discount rate of each agent is not identical. To avoid those extreme outcomes, we introduce progressive income taxation into the base model. As pointed out by Sarte (1997), the presence of progressive income tax scheme may yield a unique interior steady state in which every agent holds a positive amount of capital, even though the agents have heterogenous rates of time preferences. Owing to progressive income taxation, the steady state equilibrium of our economy with heterogenous agents is essentially the same as the stationary equilibrium of the representative agent economy. Hence, we may elucidate how the introduction of heterogeneity of agents affect the role of consumption externalities in the transition process of an economy.

Our investigation presents two findings. First, either if there are only intragroup consumption externalities or if the magnitude of intergroup consumption externalities is small enough, then a uniquely given steady state exhibits a regular saddle point property. In this case, the equilibrium path is determinate and it converges to the steady state equilibrium. Our second finding is that if the intergroup external effects have large impacts on the individual consumption decision, then the steady state equilibrium is either totally unstable or locally indeterminate. In the latter case, there exists a continuum of converging paths around the steady state, so that expectations-driven economic fluctuations may emerge. If this is the case, the presence of heterogenous agents plays a pivotal role for characterizing the standard neoclassical growth model with consumption externalities.

It is to be noted that Alonso-Carrera et al. (2008) and Chen and Hasu (2007) reveal that equilibrium indeterminacy may hold in the representative agent models with consumption externalities. Alonso-Carrera et al. (2008) show that if labor-leisure choice is allowed and if the utility function is not homothetic with respect to private and average consumption levels, then the one-sector growth model with consumption externalities may generate indeterminacy of equilibrium.[1] Chen and Hasu (2007) examines a two-sector growth model and shows that the presence of consumption externalities affects resource allocation between two production sectors, which may cause multiple equilibria. Indeterminacy shown in these studies is, therefore, partially depends on the complex preference structure or on the production side of the model economy. In contrast, our study uses a one-sector neoclassical growth model with fixed labor supply, so that the presence of heterogenous agents is the main source of multiple equilibria.

The next section sets up the analytical framework. Section 3 examines the dynamic behavior of our model economy and presents intuitive implication of the stability conditions. Section 4 presents numerical examples. Concluding remarks are given in Section 5.

2 The Model

Suppose that there are two groups of infinitely-lived agents. Each group consists of a continuum of identical households. The preference and the initial holding of wealth of the representative household in each group are different from each other. For simplicity, we assume that population in the economy is constant over time, so that the mass of each group will not change. We also assume that the economy is closed and the government does not issue interest bearing bonds. Thus the stock of capital is the only net asset held by agents.

[1]More precisely, the presence of indeterminacy requires that the marginal substitution between private and average consumption is not constant along the equilibrium path where the average consumption of the economy at large coincides with the level of private consumption.

2.1 Households

The representative agent in group i ($i = 1, 2$) supplies one unit of labor in each moment and maximizes a discounted sum of utilities over an infinite time horizon. The objective functional of the representative agent in group i is given by

$$U_i = \int_0^{+\infty} e^{-\rho_i t} u^i (c_i, C_i, C_j) dt, \quad \rho_i > 0, \quad i, j = 1, 2, \quad i \neq j. \tag{1}$$

In the above, ρ_i denotes a given rate of time discount of group i agent, c_i her private consumption, and C_i and C_j respectively represent the average levels of consumption in groups i and j. The instantaneous utility function, $u^i(\cdot)$, is assumed to be monotonically increasing and strictly concave in private consumption, c_i. It is also assumed that in the symmetric equilibrium where $c_i = C_1 = C_2$, the utility function holds the Inada conditions: $\lim_{C \to 0} u_1^i(C, C, C) = \infty$ and $\lim_{C \to \infty} u_1^i(C, C, C) = 0$, where $u_m^i(\cdot)$ ($m = 1, 2, 3$) denotes the partial derivative of the utility function with respective to the m-th variable in $u^i(\cdot)$.

The key assumption about the instantaneous felicity function in (1) is that we distinguish intragroup externalities from intergroup externalities. Namely, an agent's concern with the consumption levels of members in her own group may be different from the concern with consumption of agents in the other group. The presence of intergroup external effects produces the outcomes specific to models with heterogenous agents.

Following the taxonomy given by Dupor and Liu (2003), the external effect of consumption on an individual consumption may be either negative (jealousy) of positive (admiration). In addition, each consumer would be a conformist who likes being similar to others (keeping up with the Joneses) or a anti-conformist who wants to be different from others (running away from the Joneses). We allow, for example, an agent in a particular group feels jealousy as to consumption of others in her group but admires consumption of agents belongs to the other group. Such a situation may emerge, the agents in the rich group admire an increase in the benchmark level of consumption in the poor group, whereas they have jealousy as to the consumption level of other members in her group. In addition, the agent is a conformist as to consumption behavior of her group's members, but keeps away from consumption behavior of the other group's agents. As a result, even though there are only two types of agents, the external effects among the consumers cover a richer class of situations than that treated in the representative-agent economy.

As usual, the negative externality (jealousy) is expressed by $u^i_j(\cdot)\,(=\partial u^i/\partial C_j) < 0 \quad (i=1,2,\ j=2,3)$, while positive externality (admiration) means that $u^i_j(\cdot)$ has a positive value. Similarly, the consumers' conformism is shown by $u^i_{1j}(\cdot)\,(=\partial^2 u^i/\partial C_j \partial c_i) > 0$, and anti-conformist holds if $u^i_{1j}(\cdot)\,(=\partial^2 u^i/\partial C_j \partial c_i) < 0$. In what follows, we assume that, regardless of the forms of external effects, the effects of a change in the private consumption dominate the impact on her utility caused by external effect. More specifically, the utility function is assumed to satisfy the following properties:

$$u^i_1(\cdot) + u^i_j(\cdot) > 0, \quad i=1,2, \quad j=2,3, \tag{2a}$$

$$u^i_{11}(\cdot) + u^i_{1j}(\cdot) < 0, \quad i=1,2, \quad j=2,3, \tag{2b}$$

$$\sum_{j=1}^{3} u^i_j(\cdot) > 0, \quad \text{and} \quad \sum_{j=1}^{3} u^i_{1j}(\cdot) < 0, \quad i=1,2. \tag{2c}$$

Conditions $(2a)$ mean that the marginal utility of own consumption dominates impacts produced by consumption externalities. Conditions $(2b)$ show that the marginal utility of own consumption diminishes even considering external effects. Conditions $(2c)$ ensure that, in a social symmetric equilibrium $C_1 = C_2$, the marginal utility of consumption in a group is positive and it decreases with private consumption.

The flow budget constraint for each agent is

$$\dot{k}_i = \hat{r}_i k_i + \hat{w}_i - c_i + T_i, \quad i=1,2, \tag{3}$$

where, k_i is capital stock owned by an agent in group i, c_i consumption, $\hat{r}_i$ after-tax rate of return to asset, $\hat{w}_i$ the after-tax real wage rate and T_i expresses a transfer from the government. The initial holding of capital, $k_i(0)$, is exogenously given.

2.2 Production

The representative firm produces a single good according to a constant-returns-to-scale technology expressed by $\bar{Y} = F\left(\bar{K}, N\right)$ where $\bar{Y}$, $\bar{K}$ and N denote the total output, capital and labor, respectively. Using the homogeneity assumption, we write the production function $Y = f(K)$ where $Y \equiv \bar{Y}/N$ and $K \equiv \bar{K}/N$. The productivity function, $f(K)$, is assumed to be monotonically increasing and strictly concave in the capital-labor ratio, K, and fulfills the Inada conditions. The commodity market is competitive so that the before-tax rate of return to capital and real wage are respectively determined by

$$r = f'(K), \quad w = f(K) - Kf'(K). \tag{4}$$

For simplicity, we assume that capital does not depreciate.

If we denote the number of agents in group i by N_i $(i = 1, 2)$, then the full-employment condition for labor and capital are $N_1 + N_2 = N$ and $N_1 k_1 + N_2 k_2 = \bar{K}$. Letting $\theta_i = N_i/N$, we can the full-employment conditions as follows:

$$K = \theta_1 k_1 + \theta_2 k_2, \ , \quad 0 < \theta_i < 1, \quad \theta_1 + \theta_2 = 1. \tag{5}$$

For notational simplicity, in the following we normalize the total population, N, to one. Thus θ_i represents the mass of agents of type i as well as the population share of that type.

2.3 Fiscal Rules

The government levies distortionary income tax and distributes back its tax revenue as a transfer to each agent. In the main part of the paper, we assume that the same rate of tax applies to both capital and labor incomes. The rate of tax applies to income of an agent in group i is $\tau_i = \tau(y_i)$, $(i = 1, 2)$ where τ_i is the rate of tax and y_i $(= rk_i + w_i)$ denotes the total income of an agent in group i. The tax function $\tau(y_i)$: $\Re_+ \to \Re_+$ is continuous, monotonically increasing, a twice differentiable function and satisfies $0 < \tau(y_i) < 1$.

The after-tax rate of return and real wage received by type i agents are respectively written as

$$\hat{r}_i = (1 - \tau(y_i))\, r, \qquad \hat{w}_i = (1 - \tau(y_i))\, w, \quad i = 1, 2. \tag{6}$$

As a result, the flow budget constraint for the household (3) is rewritten as

$$\dot{k}_i = (1 - \tau(y_i))\, y_i - c_i + T_i, \qquad i = 1, 2.$$

We assume that the government follows the balanced-budget rule and, therefore, its flow budget constraint (in per-capita term) is

$$\theta_1 T_1 + \theta_2 T_2 = \theta_1 \tau(y_1)\, y_1 + \theta_2 \tau(y_2)\, y_2.$$

In addition, if we assume that the government pays back an identical amount of transfer to each agent, the lump-sum transfers of the group 1 and the group 2 are given by

$$T_1 = T_2 = \theta_1 \tau(y_1)\, y_1 + \theta_2 \tau(y_2)\, y_2. \tag{9}$$

2.4 Consumption and Capital Formation

Under the fiscal rules given above, the type i agent's flow budget constraint is expressed as

$$\dot{k}_i = (1 - \tau(y_i))\,(rk_i + w) - c_i + T_i, \quad i = 1, 2, \tag{10}$$

where T_i is determined by (9). Following Guo and Lansing (1998), we assume that the households perceive the rule of progressive taxation on private income, but she takes the transfer payment, T_i, as given. Therefore, the household of type i maximizes (1) subject to (10), the initial holding of capital, $k_i(0)$ as well as to the anticipated, given of $\{C_i(t), C_j(t), r(t), w(t), Y(t), T_i(t)\}_{t=0}^{\infty}$.

Let the elasticity of marginal utility denote the following.

$$\Omega_s^i \equiv -\frac{(u_{11}^i(C_i, C_i, C_j) + u_{12}^i(C_i, C_i, C_j))C_i}{u_1^i(C_i, C_i, C_j)} > 0, \tag{11}$$

$$\Omega_o^i \equiv -\frac{u_{13}^i(C_i, C_i, C_j)C_j}{u_1^i(C_i, C_i, C_j)}, \qquad i, j = 1, 2. \tag{12}$$

Here, Ω_s^i denotes the elasticity of marginal utility of consumption within the agent's own group, which equals the inverse of an elasticity of intertemporal substitution in private consumption plus social consumption in its own group. This elasticity has a positive value due to condition $(2b)$. Additionally, Ω_o^i is the elasticity of marginal utility with respect to the other group's consumption. The sign of this term depends on how group i agents respond to consumption of group j agents. If agents are conformist to keep up with consumption of the other group's members (so that $u^i_{13} > 0$), then Ω^i_o has a negative sign. On the other hand, if they do not like being similar to the other group's agents ($u^i_{13} < 0$), then Ω^i_o is strictly positive. Note that, in view of (2c), the following is satisfied:

$$\Omega^i_s + \Omega^i_o > 0, \qquad i = 1, 2. \tag{13}$$

The Euler equations are given by

$$\begin{bmatrix} \Omega^1_s/C_1 & \Omega^1_o/C_2 \\ \Omega^2_o/C_1 & \Omega^2_s/C_2 \end{bmatrix} \begin{bmatrix} \dot{C}_1 \\ \dot{C}_2 \end{bmatrix} = \begin{bmatrix} (1-\tau(y_1) - y_1\tau'(y_1))r - \rho_1 \\ (1-\tau(y_2) - y_2\tau'(y_2))r - \rho_2 \end{bmatrix}.$$

Solving this set of equations with respect to $\dot{C}_1$ and $\dot{C}_2$, we obtain

$$\begin{bmatrix} \dot{C}_1 \\ \dot{C}_2 \end{bmatrix} = \frac{C_1C_2}{\Omega^1_s\Omega^2_s - \Omega^1_o\Omega^2_o} \begin{bmatrix} \Omega^2_s/C_2 & -\Omega^1_o/C_2 \\ -\Omega^2_o/C_1 & \Omega^1_s/C_1 \end{bmatrix} \begin{bmatrix} (1-\tau(y_1) - y_1\tau'(y_1))r - \rho_1 \\ (1-\tau(y_2) - y_2\tau'(y_2))r - \rho_2 \end{bmatrix}. \tag{14}$$

Equations (9) and (10) yield

$$\dot{k}_i = (1 - \tau(y_i))\, y_i - C_i + \theta_1\tau(y_1)\, y_1 + \theta_2\tau(y_2)\, y_2, \qquad i = 1, 2. \tag{15}$$

Summing up the flow budget constraint (10) over all of the households and dividing the both sides by N, we obtain

$$\theta_1\dot{k}_1 + \theta_2\dot{k}_2 = \theta_1 y_1 + \theta_2 y_2 - \theta_1 C_1 - \theta_2 C_2.$$

Thus, in view of $y_i = rk_i + w$ and (5), we obtain the final-good market equilibrium condition for the entire economy: $\dot{K} = f(K) - C$ where $C = \theta_1 C_1 + \theta_2 C_2$.

3 Macroeconomic Stability

3.1 Dynamic System

Equations (4) and (5) give

$$y_i = rk_i + w = f(K) + (k_i - K)f'(K), \quad i = 1, 2, \tag{16}$$

where $K = \theta_1 k_1 + (1-\theta_1) k_2$. Plugging (16) into (14) and (15) , we obtain a complete dynamic system that depicts the dynamic behaviors of k_1, k_2, C_1 and C_2.

The solution of this dynamic system that fulfills the initial conditions on $k_1(0)$ and $k_2(0)$ as well as the transversality conditions for the households' optimization problem, $\lim_{t\to\infty} u_1^i (C_i(t), C_i(t), C_j(t)) e^{-\rho t} k_i(t) = 0$, where $i = 1$, or 2 presents the perfect-foresight competitive equilibrium of our model economy.

3.2 Steady-State Equilibrium

In the steady-state equilibrium, k_i and C_i $(i = 1, 2)$ stay constant over time. In view of (14) and (15), the steady-state conditions are given by

$$C_i^* = y_i^* + \theta_j \left(\tau\left(y_j^*\right) y_j^* - \tau(y_i^*) y_i^*\right), \quad i, j = 1, 2, \quad i \neq j, \tag{17}$$

$$\rho_i = f'(K^*)(1 - \tau(y_i^*) - y_i^* \tau'(y_i^*)), \quad i = 1, 2, \tag{18}$$

where C_i^* and k_i^* denote steady-state levels of k_i and C_i.

To simplify analytical argument, we make the following assumption:

Assumption 1. $\tau(y_i) + y_i \tau'(y_i)$ $(i = 1, 2)$ *is a monotonic increasing function of the income* y_i.

Given Assumption 1, it is easy to confirm the following fact:

Proposition 1. *Given Assumption 1, there is a unique steady state equilibrium. If* $\rho_1 = \rho_2$*, then the unique steady state is symmetric in the sense that* $k_1^* = k_2^* = K$ *and* $C_1^* = C_2^*$ *is uniquely determined. In addition, if* $\rho_1 < (>) \rho_2$*, then* $k_1^* > (<) k_2^*$ for $i = 1$ and 2.

Proof. See Appendix A. ■

The symmetry of steady state comes from the rates of time preference in both groups. In other words, if the rates of time preference are the same, there is the symmetric steady-state equilibrium. If these rates are not the same, there exists the unique steady state that the level of capital stock in patient group is greater than the other. Obviously, the transition dynamics out of the steady state of both groups are not necessarily symmetric because the initial holdings of capital stock as well as the utility functions in both groups are not necessarily the same.

3.3 Stability

Let us examine the local stability condition of the steady-state equilibrium defined above. Linear approximation of dynamic system, (14) and (15) , around the steady state equilibrium yields the following:

$$\begin{bmatrix} \dot{C}_1 \\ \dot{C}_2 \\ \dot{k}_1 \\ \dot{k}_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & \partial \dot{C}_1/\partial k_1 & \partial \dot{C}_1/\partial k_2 \\ 0 & 0 & \partial \dot{C}_2/\partial k_1 & \partial \dot{C}_2/\partial k_2 \\ -1 & 0 & \partial \dot{k}_1/\partial k_1 & \partial \dot{k}_1/\partial k_2 \\ 0 & -1 & \partial \dot{k}_2/\partial k_1 & \partial \dot{k}_2/\partial k_2 \end{bmatrix} \begin{bmatrix} C_1(t) - C_1^* \\ C_2(t) - C_2^* \\ k_1(t) - k_1^* \\ k_2(t) - k_2^* \end{bmatrix}$$

Hence, the characteristic equation of this system is given by

$$\lambda^4 - \mathrm{Tr}J\lambda^3 + \mathrm{W}J\lambda^2 - \mathrm{Z}J\lambda + \mathrm{Det}J = 0, \tag{19}$$

where

$$\mathrm{Tr}J = f'(k^*)\left\{1 - \theta_2(\tau'(y_1^*)y_1^* + \tau(y_1^*))\right\} + f'(k^*)\left\{1 - \theta_1(\tau'(y_2^*)y_2^* + \tau(y_2^*))\right\} > 0, \tag{20}$$

$$\mathrm{W}J = f'(k^*)^2\left\{1 - \theta_2(\tau'(y_1^*)y_1^* + \tau(y_1^*)) - \theta_1(\tau'(y_2^*)y_2^* + \tau(y_2^*))\right\} + \frac{\partial \dot{C}_1}{\partial k_1} + \frac{\partial \dot{C}_2}{\partial k_2}, \tag{21}$$

$$\mathrm{Z}J = \frac{\partial \dot{C}_1}{\partial k_1}\frac{\partial \dot{k}_2}{\partial k_2} - \frac{\partial \dot{C}_1}{\partial k_2}\frac{\partial \dot{k}_2}{\partial k_1} + \frac{\partial \dot{C}_2}{\partial k_2}\frac{\partial \dot{k}_1}{\partial k_1} - \frac{\partial \dot{C}_2}{\partial k_1}\frac{\partial \dot{k}_1}{\partial k_2}, \tag{22}$$

$$\mathrm{Det}J = \frac{C_1^* C_2^* f'(k^*)^2 A}{\Omega_s^1\Omega_s^2 - \Omega_o^1\Omega_o^2}$$

$$A \equiv f'(k^*)^2(2\tau'(y_1^*) + y_1^*\tau''(y_1^*))(2\tau'(y_2^*) + y_2^*\tau''(y_2^*)) - f''(k^*)(\theta_1(1 - \tau(y_1) - y_1\tau'(y_1))$$
$$(2\tau'(y_2) + y_2\tau''(y_2)) + \theta_2(1 - \tau(y_2) - y_2\tau'(y_2))(2\tau'(y_1) + y_1\tau''(y_1)))(> 0). \tag{23}$$

The precise expression of matrix's coefficients and the term $\mathrm{Z}J$ in (22) is displayed in Appendix B of the paper.

Note that this model involves two jumpable variables, C_1 and C_2. Thus the necessary and sufficient conditions for local determinacy is that the characteristic equation

(19) has two roots with negative real parts. Considering the form of (23), we see that the sign of the determinant depends on only the households' preferences shown by $(\Omega_s^1\Omega_s^2 - \Omega_o^1\Omega_o^2)$ under the assumption 1.

As for the stability of the steady state equilibrium our main finding is as follows:

Proposition 2. *Given Assumptions 1, if* $\Omega_s^1\Omega_s^2 - \Omega_o^1\Omega_o^2 < 0$, *then the unique steady-state equilibrium is either locally unstable or indeterminate.*

Proof. Equation (23) shows that the determinant of J is strictly negative when $\Omega_s^1\Omega_s^2 - \Omega_o^1\Omega_o^2 < 0$. In this case the number of characteristic roots with negative sign is either one or three. The former case means that the stable manifold is one dimensional around the steady state and thus there no converging path can be selected for arbitrarily given levels of initial capital stocks, $k_1(0)$ and $k_2(0)$. If the number of stable roots is three, then there may exist a continuum of converging path starting from the given initial distribution of capital stocks.■

First, this is not the case if Ω_o^1 and Ω_o^2 have different signs. That is, it holds that $\Omega_s^1\Omega_s^2 - \Omega_o^1\omega_o^2 > 0$. Concerning the different signs of Ω_o^1 and Ω_o^2, we may make the intuitive explanation as follows. Assume that agents in group 1 live in the urban area, while agents in group 2 live in the rural area. Then, it could be plausible to assume that agents in group 2 like being similar to the average consumption in people living in cities, whereas agents in group 1 have anti-conformism as to the average consumption in the rural area. If this is the case, it holds that $\Omega_o^1 > 0$ and $\Omega_o^2 < 0$. In addition, as proposition 1 shows, if $\rho_2 > \rho_1$, then group 1 agents are richer than group 2 agents at least in the steady state. Therefore, if we restrict our discussion to the steady state equilibrium, the above implication may be replaced with the assumption that poor people have conformism for the consumption behavior of rich people, but there is no other way around.

Second, even if the intragroup consumption externalities do not exist (i.e., $\phi_i = 0$ in (24)), the steady state is still locally unstable or indeterminate when the condition, $\Omega_s^1\Omega_s^2 - \Omega_o^1\Omega_o^2 < 0$ is satisfied.[2]

[2]In numerical examples of Section 4, we show that the indeterminacy arises when the intergroup

Proposition 2 means that the necessary condition under which the steady state is a regular saddlepioint so that local determinacy holds is that $\Omega_s^1\Omega_s^2 - \Omega_o^1\Omega_o^2 > 0$. We find that a set of sufficient conditions for holding determinacy is the following:

Proposition 3. *Suppose that $\rho \equiv \rho_1 = \rho_2$. Given Assumptions 1, if* (i) $\Omega_s^1\Omega_s^2 > \Omega_o^1\Omega_o^2$, (ii) $\theta_1\Omega_s^2 + \theta_2\Omega_s^1 > \theta_1\Omega_o^1 + \theta_2\Omega_o^2$ and (iii) $\Omega_s^1 + \Omega_s^2 > \left(1 - \frac{\rho}{f'(K^*)}\right)(\theta_1\Omega_s^2 + \theta_2\Omega_s^1 - \theta_1\Omega_o^1 - \theta_2\Omega_o^2)$, *then the unique steady state has the saddle-path stability.*

Proof. Let us denote roots of the characteristic equation by $\lambda_s\,(s = 1, 2, 3, 4)$. As (20) shows, the sign of the trace of J, which equals $\sum_{s=1}^4 \lambda_s$, is strictly positive. Hence, at least one of the characteristic roots has positive real part. The assumption (i) holds that the determinant has a positive sign, which implies that the determinant $J\ (= \Pi_{s=1}^4 \lambda_s)$ is strictly positive: see (23). This means that the number of characteristic roots with positive real parts is either two or four. The assumptions (ii) and (iii) hold that ZJ, which equals $\lambda_1\lambda_2\lambda_3 + \lambda_2\lambda_4\lambda_1 + \lambda_3\lambda_4\lambda_1 + \lambda_2\lambda_3\lambda_4$, has a negative sign, so that at least the number of stable root is above one. Consequently, there are two characteristic roots with positive real part. This demonstrates that there is a two–dimensional stable manifold around the steady state, implying that the competitive equilibrium path converging to the steady state is uniquely determined. See the detail proof in Appendix B. ■

Furthermore, we see that if there are no intergroup external effects, then the equilibrium path is determinate. The following result indicates that intergroup external effects play a pivotal role to prevent the dynamic system of holding a regular saddlepoint property:

Corollary 1. *When $\Omega_o^1 = \Omega_o^2 = 0$, the unique steady state satisfies local determinacy.*

Proof. At first, we note that the trace of J always has a positive sign. Assuming that $\Omega_o^1 = \Omega_o^2 = 0$, the conditions (i)–(iii) in Proposition 3 are satisfied. Hence, the

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consumption externalities exist but the intragroup ones do not exist in which the existence of intragroup consumption externalities expands to the region of indeterminacy.

steady state satisfies saddle-point stability. ■

To understand the results shown above more clearly, suppose that the utility function is given by the CRRA type such that

$$u^i(c_i(t), C_i(t), C_j(t)) = \frac{1}{1-\gamma_i}\left(c_i C_i^{\phi_i} C_j^{\eta_i}\right)^{1-\gamma_i}, \quad i, j = 1, 2, \quad i \neq j. \tag{24}$$

Here, γ_i denotes the inverse of elasticity of intertemporal substitution in felicity. The parameter ϕ_i represents the extent of the intragroup consumption externalities, whereas η_i shows the degree of inter-group externalities. From (24) we find that Ω_s^i and Ω_o^i are given by constant parameters as follows:

$$\Omega_s^i = \gamma_i - \phi_i(1-\gamma_i)(>0), \tag{25a}$$

$$\Omega_o^i = -\eta_i(1-\gamma_i). \tag{25b}$$

Using $(25a)$ and $(25b)$, we can show that

$$\begin{aligned} &\Omega_s^1\Omega_s^2 - \Omega_o^1\Omega_o^2 \\ &= \{\gamma_1 - \phi_1(1-\gamma_1)\}\{\gamma_2 - \phi_2(1-\gamma_2)\} - \eta_1\eta_2(1-\gamma_1)(1-\gamma_2). \end{aligned} \tag{26}$$

Corollary 1 says that when $\Omega_o^1 = \Omega_o^2 = 0$ (i.e., $\eta_1 = \eta_2 = 0$ in (24)), then the economy has a unique converging path towards the unique steady-state equilibrium, regardless of the different preferences of each type of agents. Concerning $(25a)$, this is due to the positive sign of determinant from $\Omega_s^1\Omega_s^2 = \{\gamma_1 - \phi_1(1-\gamma_1)\}\{\gamma_2 - \phi_2(1-\gamma_2)\} > 0$. Intuitively, assuming that households in a group have neither jealousy nor admiration about the consumption level of the other group's members, the different rates of time preference as well as the different form of utility function do not affect the stability of the steady state so that the economy satisfies saddle-path stability and the competitive equilibrium path is uniquely determined.

Next, suppose that $\Omega_o^1 \neq 0$ and $\Omega_o^2 \neq 0$. That is, we consider the intergroup consumption externalities. In this case, we can show the following sufficient condition to satisfy the saddlepoint stability.

First, suppose that there is no intragroup consumption externality so that Ω_s^1 and Ω_s^2 are equal to the pure rates of risk aversion without the intragroup consumption externalities (i.e, $\Omega_s^i = \gamma_i, \;\; i = 1, 2$). In this case, Proposition 3 shows that the unique steady state is saddlepoint stable even if the intergroup consumption externalities exist. For example, if $\Omega_s^i = \gamma_i > \Omega_o^i > 0$ and $\frac{\theta_j}{\theta_i} > \frac{\Omega_o^i}{\Omega_s^i} = \frac{\Omega_o^i}{\gamma_i}$ in both groups, the steady state satisfies local determinacy where $\left(1 - \frac{\rho}{f'(K^*)}\right)$ always has a positive sign as confirmed in (18). It means that if individuals' preferences exhibit unit-conformism as to the other group's consumption behaviors, the economy satisfies the saddlepoint stability when the degree of intergroup anti-conformism is small enough or the degree of risk aversion is large enough.

Second, suppose that there is intragroup consumption externality. Unlike the above, $\Omega_s^i = \gamma_i - \phi_i(1 - \gamma_i)$ $(i = 1, 2)$, meaning that Ω_s^i is not equal to the rate of risk aversion. Thus, we need to consider the rate of risk aversion including the degree of intragroup consumption externalities. Because the conditions given by Proposition 3 are not changed, the steady state satisfies local determinacy if the value of Ω_o^i is small enough or Ω_s^i is large enough; however, as confirmed in numerical examples later, the presence of intragroup consumption externalities yields a richer region of indeterminacy.

The above proposition fails to specify when indeterminacy emerges. Since it is hard to present the analytical conditions for local indeterminacy (the sufficient conditions under which that the characteristic equation has three stable roots), we inspect numerical examples in Section 4.

3.4 Intuition

As shown by Proposition 2, the key to determine dynamic behavior of our model economy is the sign of $\Omega_s^1 \Omega_s^2 - \Omega_o^1 \Omega_o^2$. To obtain an intuitive implication why this sign plays a pivotal role, it is useful to inspect the first order conditions for consumers' optimization. Letting q_i be the shadow value of capital held by Group i's households, the first-order condition for each type of household is given by

$$u_1^1 \left(c_1, C_1, C_2\right) = q_1,$$

$$u_1^2 \left(c_2, C_2, C_1\right) = q_2,$$

where q_i changes according to

$$\dot{q}_i = q_i \left[(1 - \tau(y_i) - y_1 \tau'(y_i)) r - \rho_i \right], \quad i = 1, 2.$$

Using the consistency conditions, $c_i = C_i \ (i = 1, 2)$, the optimal consumption levels can be written as

$$C_1 = D^1 (q_1, q_2),$$
$$C_2 = D^2 (q_1, q_2).$$

The partial derivatives of consumption demand functions are given by

$$\frac{\partial C_1}{\partial q_1} = D_1^1 = -\frac{\Omega_s^2}{\Omega_s^1 \Omega_s^2 - \Omega_o^1 \Omega_o^2}, \qquad \frac{\partial C_1}{\partial q_2} = D_2^1 = \frac{\Omega_o^1}{\Omega_s^1 \Omega_s^2 - \Omega_o^1 \Omega_o^2}$$
$$\frac{\partial C_2}{\partial q_1} = D_1^2 = \frac{\Omega_o^2}{\Omega_s^1 \Omega_s^2 - \Omega_o^1 \Omega_o^2}, \qquad \frac{\partial C_2}{\partial q_2} = D_2^2 = -\frac{\Omega_s^1}{\Omega_s^1 \Omega_s^2 - \Omega_o^1 \Omega_o^2}$$

Note that if the own effects of a change in consumption on the marginal utility dominates the cross effects,we obtain $\Omega_s^1 \Omega_s^2 - \Omega_o^1 \Omega_o^2 > 0$. In this case a higher q_i depresses C_i. In addition, if the households are conformists with respect to the other group's consumption behavior, Ω_o^i has a negative value, so that a rise in q_j lowers C_i. In contrast, if the cross effects dominates the own effects of consumption change, $\Omega_s^1 \Omega_s^2 - \Omega_o^1 \Omega_o^2$ has a negative sign. Namely, if households are highly sensitive to the consumption level of the other group's agents, a higher marginal value of capital may increase the current consumption. Again when Ω_o is positive, a higher q_j also raises C_i.

Now suppose that a sunspot shock hits the economy and the households in both groups anticipate that the rate of return to capital will rise. This raises the their marginal utility value of capital, q_i, so that households plans to increase their savings. If $\Omega_s^1 \Omega_s^2 - \Omega_o^1 \Omega_o^2 > 0$ and conformism prevails, then rises in q_1 and q_2 depresses current consumption demand of both types of households, so that capital accumulation of the entire economy is accelerated. As a result, the aggregate capital increases and the rate of return to capital decreases. This contradicts the initial anticipated rise in the rate of return to capital and, hence, the households' expectations are not self-fulfilled. The equilibrium path of the economy is, therefore, uniquely determined.

When $\Omega_s^1\Omega_s^2 - \Omega_o^1\Omega_o^2 < 0$, the expected rises in q_1 and q_2 increase the current consumption of both types of households. Thus, savings increase and the resulting lower aggregate capital raises the rate of return, r. In this case, the initial expectations are self-fulfilled, which means that there may exist multiple (a infinite number) of equilibrium paths.

Note that if the households do not concern with the other group's consumption, then $\Omega_o^i = 0$. In this case, the price effects on the current consumption demand are given by

$$\frac{\partial C_1}{\partial q_1} = -\frac{1}{\Omega_s^1} < 0, \qquad \frac{\partial C_2}{\partial q_2} = -\frac{1}{\Omega_s^2} < 0, \qquad \frac{\partial C_1}{\partial q_2} = \frac{\partial C_2}{\partial q_1} = 0.$$

Therefore, a higher q_i always depresses C_i.As a result, sunspot driven expected rise in the value of capital lowers consumption of both groups,which decreases the rate of return to capital. Therefore, multiple equilibrium cannot exist.

Finally it is to be remember that the sign of $\Omega_s^1\Omega_s^2 - \Omega_o^1\Omega_o^2$ is a part of necessary conditions for determinacy/indeterminacy. In fact, as Proposition 3 shows, a set of sufficient conditions for determinacy involve other conditions in addition to $\Omega_s^1\Omega_s^2 - \Omega_o^1\Omega_o^2 > 0$. If $\Omega_s^1\Omega_s^2 - \Omega_o^1\Omega < 0$, the steady state could be unstable rather than holding indeterminacy. Therefore, the intuitive discussion shown above is a not a precise description of the stability conditions but a rough sketch of the dynamics.

4 Numerical Analysis

In the previous section we have confirmed that if the sign of $\Omega_s^1\Omega_s^2 - \Omega_o^1\Omega_o^2$ is negative, then the steady-state equilibrium is either locally indeterminate or unstable. For the purpose of distinguishing the conditions for indeterminacy from these for instability, this section conducts numerical experiments by specifying the utility, production and tax functions.

At first, we make use of the utility function in (24) whose Ω_s^i and Ω_o^i are constant as confirmed in (25a) and (25b). Next, note that $\Omega_s^i > 0$ in (25a) because the marginal utility of private consumption is decreasing regardless of the intragroup consumption externalities. Furthermore, the following conditions are restricted:

$$-\frac{u_{11}^i C_i}{u_1^i} - \frac{u_{13}^i C_j}{u_1^i} = \gamma_i - \eta_i(1-\gamma_i) > 0, \quad i = 1,2. \tag{27a}$$

Condition $(2c)$, requires the following:

$$\Omega_s^i + \Omega_o^i = \gamma_i - (\phi_i + \eta_i)(1-\gamma_i) > 0, \quad i = 1,2. \tag{27b}$$

As for the production function, it is given by Cobb-Douglas one:

$$f(K) = AK^\alpha, \ \ 0 < \alpha < 1, \ \ A > 0, \tag{28}$$

where $K = \theta_1 k_1 + \theta_2 k_t$.

The tax function is specified as

$$\tau(y_i) = y_i^\xi, \tag{29}$$

where $\xi \geq 1$ so that the assumption 1 is always satisfied.

Our object is to clarify the effects of intergroup consumption externalities on the stability in numerical examples. Hence, we make use of the magnitudes of parameters as simply as possible. First, we assume that the rates of time preference are the same in both groups (i.e., $\rho_1 = \rho_2$) so that the steady-state levels of capital stock and consumption are the same within groups. That is, the steady-state equilibrium given by numerical examples is symmetric. Second, we assume that the population sizes in two groups are the same (i.e., $\theta_1 = \theta_2$). Third, we make use of the linear tax function not to consider the curvature of tax function (i.e., $\xi = 1$) so that the tax function is specified as $\tau(y_i) = y_i$.

The parameter set of our numerical examples is given by:

$$\theta_1 = \theta_2 = 0.5, \ \ A = 0.6, \quad \alpha = 0.25, \quad \xi = 1, \ \ \rho_1 = \rho_2 = 0.005.$$

To achieve the plausible rate of interest, we set that the rates of time preference are 0.005 and the production parameter A is 0.6. As a result, from (18) the steady-state level of aggregate capital stock, $K^* = k_1^* = k_2^*$ is given by 0.45 so that the after-tax rate of return, $\hat{r}$ in (6) is 0.14 and the before-tax rate of return, r is 0.27.

As for the parameter values concerning the preference structure, we consider the following three sets:[3]

$$\text{(i) } \gamma_1 = \gamma_2 = 0.2, \quad \phi_1 = 0, \quad \eta_1 = -0.9,$$
$$\text{(ii) } \gamma_1 = 5, \quad \gamma_2 = 0.2, \quad \phi_1 = -0.9, \quad \eta_1 = -0.9,$$
$$\text{(iii) } \gamma_1 = \gamma_2 = 5, \quad \phi_1 = -0.9, \quad \eta_1 = 0.9.$$

Let us confirm the properties of respective cases. In case (i), we consider that the intragroup consumption externalities in group 1 do not exist. That is, $\Omega_s^1 = \gamma_1 = 0.2$. Alternatively, cases (ii) and (iii) suppose that the agents in group 1 have the high degree of intragroup consumption externalities. This is because the positive value of Ω_s^1 is as small as possible to confirm the negative sign of determinant. That is, Ω_s^1 in these cases (ii) and (iii) are given by 1.4. Because Ω_s^1 has a positive sign in all cases (i)-(iii), the condition $(25a)$ is satisfied.

Next, we take account of the agents' preferences in group 1. In case (i), group 1's agents do not have any interests about the consumption behavior of their own group's members (i.e., $\phi_i = 0$), while they have jealousy and anti-conformism about the average level of consumption in group 2. In case (ii), the agents in group 1 have jealousy and conformism about the consumption behavior of members in both groups. In case (iii), group 1's agents have jealousy and conformism for the average level of consumption in the same group like in case (ii), but have admiration and anti-conformism about the average level of consumption in group 2.

Given those parameter magnitudes, we change ϕ_2 and η_2 with appropriate intervals. Figures 1, 2 and 3 respectively depict the case with preference parameters (i), (ii) and (iii) displayed above. In these figures, we divide (ϕ_2, η_2) space according to the stability conditions. For instance, the areas with a green cross show the combination of ϕ_2 and η_2 that yields local determinacy, while those with a red triangle show its combination that yields the unstability. The marker with a blue circle indicates the steady-state equilibrium with the local indeterminacy. In addition, the

[3] As the pure values of risk aversion γ_i approach to the unity, it would be difficult to confirm the indeterminacy because the value of Ω_s^i is large and thus $\Omega_s^1\Omega_s^2 > \Omega_o^1\Omega_o^2$ as confirmed in Result 2.

parameter sets with the black square do not satisfy the standard conditions of utility functions in the sense that the marginal utility of private consumption is positive and decreasing given by $(2a) - (2c)$.

Let us look at Figure 1 which confirms whether or not the indeterminacy could arise even when the intragroup consumption externality in group 1 does not exist. From the figure, we can see that the blue circles exist in the regions that $\phi_2 = 0$ and η_2 is around between -0.2 and -0.4. It means that even if the intragroup consumption externalities in both groups do not exist, the indeterminacy arises. Next, when seeing that ϕ_2 has a positive sign as well as a negative sign in the region of blue circle, we can show that whether agents in group 2 have conformism or anti-conformism does not discharge the critical role for producing the indeterminacy. Alternatively, the sign of η_2 needs to be negative in the region of blue circle, meaning that the agents in group 2 are anti-conformist for the average level of consumption in group 1. Moreover, taking account of the preferences of agents in group 1, the preferences of all agents in both groups respectively indicate the anti-conformism for consumption behavior of members in the other group in the area of blue circle.

While the pure rates of risk aversion, γ_1 and γ_2 in both groups are below the unity in Figure 1, Figure 2 is the case that the pure rate of risk aversion in group 1 is above the unity and that in group 2 is below the unity. Hence, as seen in $(25a)$ and $(25b)$, the preferences show the opposite characteristic if the signs of ϕ_1 and ϕ_2 (or η_1 and η_2) are the same. For example, when ϕ_1 and ϕ_2 have negative signs, the agents in group 1 are conformist about the average level of consumption in group 1; however, those in group 2 are anti-conformist about that in group 2. In this case, figure 2 shows that all of blue circle are enclosed in black square. This means that the number of stable roots is three, but the utility function in group 2 does not satisfy the standard conditions that the marginal utility of private consumption is positive and decreasing.

Finally, in Figure 3, we deal with the case that the pure rates of risk aversion are above the unity in both groups. In this case, when the agents in both groups are anti-conformist for the consumption behavior of members in the other group, the blue circle which shows indeterminacy is observed.

5 Concluding Remarks

We have shown that if there are heterogenous agents and consumption external effects perceived by consumers are not uniform, then the equilibrium path of the standard Ramsey economy may not display a regular saddle point property. The equilibrium dynamics could be unstable or indeterminate if the intergroup consumption externalities have distinctive effects on the consumers' behaviors. In numerical examples, we suppose the symmetric groups in the sense that the population size and the rates of time preference are the same in both groups. In this time, even if the intragroup consumption externalities do not exist, we confirm that the intergroup consumption externalities may produce the indeterminacy.

We are going to extend this model in two points. First, in this manuscript we make use of the CRRA types of utility function whose elasticities are given by constant parameters. Instead, for instance, when making use of the stone-geary utility functions, which may indicate $u(c_i, C_i, C_j) = \frac{\left((c_i - \bar{c}_i)C_i^{\phi_i}C_j^{\eta_i}\right)^{1-\gamma_i}}{1-\gamma_i}$ where $\bar{c}_i$ is a parameter shown by a subsistence level of consumption, these elasticities depend on the level of private consumption as well as the preference parameters. In this case, it would be useful to observe a change in the region of indeterminacy related with the subsistence level. Second, in this paper we have employed a simple Ramsey model with fixed labor supply and a constant returns to scale technology. It would be interesting to reconsider our discussion in models with increasing returns and/or endogenous labor supply.

Appendix

Through the appendices, we make use of $\Delta(y_i) \equiv \tau(y_i) + y_i\tau'(y_i) (> 0)$ $(i = 1, 2)$. From (18) and Assumption 1, we can show that

$$1 - \Delta(y_i^*) > 0, \quad \Delta'(y_i) = 2\tau'(y_i) + y_i\tau''(y_i) > 0.$$

Furthermore, we use the following.

$$\frac{\partial y_i}{\partial k_i} = f'(K) + f''(K)\theta_i(k_i - K), \text{ and } \frac{\partial y_i}{\partial k_j} = f''(K)\theta_j(k_i - K), \quad i, j = 1, 2, \quad i \neq j.$$

Concerning these signs, for instance, if $\rho_1 \geq \rho_2$ so that $k_2^* \geq K^* \geq k_1^*$, it holds that $\frac{\partial y_1}{\partial k_1} > 0$, $\frac{\partial y_2}{\partial k_2} \lesseqgtr 0$, $\frac{\partial y_1}{\partial k_2} > 0$ and $\frac{\partial y_2}{\partial k_1} < 0$

Appendix A

We show that the steady state is uniquely determined under assumption 1 and furthermore the steady-state level of capital stock with patient group is greater than that with impatient group. For simplicity, we assume that the rate of time preference in group 1 is more than that in group 2, $\rho_1 \geq \rho_2$ so that households in group 1 are impatient and those in group 2 are patient.

Totally differentiating (18) in group 1, we obtain $k_1 = k_1(k_2)$ where

$$\frac{\partial k_1}{\partial k_2} = \frac{-f''(K^*)(1 - \Delta(y_1^*))\theta_2 + f'(K^*)\Delta'(y_1^*)\frac{\partial y_1}{\partial k_2}}{f''(K^*)(1 - \Delta(y_1^*))\theta_1 - f'(K^*)\Delta'(y_1^*)\frac{\partial y_1}{\partial k_1}}. \tag{A.1}$$

Now, we substitute $(A.1)$ into (18) in group 2 as follows:

$$\Gamma(k_2^*) \equiv f'(K^*)(1 - \Delta(y_2^*)) \, (= \rho_2) \, . \tag{A.2}$$

Differentiating $(A.2)$ with respect to k_2 yields

$$\Gamma'(k_2^*) = \frac{f'(K^*)^2 \left\{-\theta_2 f''(K^*)(1 - \Delta(y_2^*))\Delta'(y_1^*) - \theta_1 f''(K^*)(1 - \Delta(y_1^*))\Delta'(y_2^*) + f'(K^*)\Delta'(y_1^*)\Delta'(y_2^*)\right\}}{f''(K^*)(1 - \Delta(y_1^*))\theta_1 - f'(K^*)\Delta'(y_1^*)\frac{\partial y_1}{\partial k_1}}. \tag{A.3}$$

Note that the sign of brace is positive.

Concerning the denominator in $(A.3)$, $\frac{\partial y_1}{\partial k_1} (= f'(K) + f''(K)\theta_1(k_1 - K))$ has a positive sign because the assumption $\rho_1 \geq \rho_2$ causes $k_2^* \geq K^* \geq k_1^*$. Therefore, the sign of denominator is negative.

Because the sign of $\Gamma'(k_2^*)$ is negative, we can from $(A.2)$ confirm that there exists the unique level of group 2's capital stock in the steady state, which uniquely determines the steady-state level of capital stock in group 1.

Let us relate the steady-state levels of capital stock in both groups and time preference rate. Using (18) in two groups, we can show the following.

$$\frac{\rho_1}{1-\Delta(y_1^*)} = \frac{\rho_2}{1-\Delta(y_2^*)}. \tag{A.4}$$

$(A.4)$ and (17) mean that when $\rho_1 = \rho_2$, it holds that $k_1^* = k_2^*$ and $C_1^* = C_2^*$. When $\rho_1 \geq \rho_2$, we can see that $\Delta(y_2^*) \geq \Delta(y_1^*)$. Taking account of the assumption 1, we obtain that $y_2^* \geq y_1^*$ so that $k_2^* \geq k_1^*$.

Appendix B

The coefficients of the matrix J are given by

$$\frac{\partial \dot{C}_1}{\partial k_1} = \frac{\Omega_s^1\Omega_s^2 - \Omega_o^1\Omega_o^2}{C_2^*}\Bigg(\Omega_s^2\left(f''(K^*)\theta_1(1-\Delta(y_1^*)) + f'(K^*)\Delta'(y_1^*)\frac{\partial y_1}{\partial k_1}\right)$$
$$-\,\Omega_o^1\left(f''(K^*)\theta_1(1-\Delta(y_2^*)) + f'(K^*)\Delta'(y_2^*)\frac{\partial y_2}{\partial k_1}\right)\Bigg),$$

$$\frac{\partial \dot{C}_1}{\partial k_2} = \frac{\Omega_s^1\Omega_s^2 - \Omega_o^1\Omega_o^2}{C_2^*}\Bigg(\Omega_s^2\left(f''(K^*)\theta_2(1-\Delta(y_1^*)) + f'(K^*)\Delta'(y_1^*)\frac{\partial y_1}{\partial k_2}\right)$$
$$-\,\Omega_o^1\left(f''(K^*)\theta_2(1-\Delta(y_2^*)) + f'(K^*)\Delta'(y_2^*)\frac{\partial y_2}{\partial k_2}\right)\Bigg),$$

$$\frac{\partial \dot{C}_2}{\partial k_1} = \frac{\Omega_s^1\Omega_s^2 - \Omega_o^1\Omega_o^2}{C_1^*}\Bigg(\Omega_s^1\left(f''(K^*)\theta_1(1-\Delta(y_2^*)) + f'(K^*)\Delta'(y_2^*)\frac{\partial y_2}{\partial k_1}\right)$$
$$-\,\Omega_o^2\left(f''(K^*)\theta_1(1-\Delta(y_1^*)) + f'(K^*)\Delta'(y_1^*)\frac{\partial y_1}{\partial k_1}\right)\Bigg),$$

$$\frac{\partial \dot{C}_2}{\partial k_2} = \frac{\Omega_s^1\Omega_s^2 - \Omega_o^1\Omega_o^2}{C_1^*}\Bigg(\Omega_s^1\left(f''(K^*)\theta_2(1-\Delta(y_2^*)) + f'(K^*)\Delta'(y_2^*)\frac{\partial y_2}{\partial k_2}\right)$$
$$-\,\Omega_o^2\left(f''(K^*)\theta_2(1-\Delta(y_1^*)) + f'(K^*)\Delta'(y_1^*)\frac{\partial y_1}{\partial k_2}\right)\Bigg),$$

$$\frac{\partial \dot{k}_1}{\partial k_1} = \frac{\partial y_1}{\partial k_1} + \theta_2\left(\Delta_2\frac{\partial y_2}{\partial k_1} - \Delta_1\frac{\partial y_1}{\partial k_1}\right),$$

$$\frac{\partial \dot{k}_1}{\partial k_2} = \frac{\partial y_1}{\partial k_2} + \theta_2\left(\Delta_2\frac{\partial y_2}{\partial k_2} - \Delta_1\frac{\partial y_1}{\partial k_2}\right),$$

$$\frac{\partial \dot{k}_2}{\partial k_1} = \frac{\partial y_2}{\partial k_1} + \theta_1\left(\Delta_1\frac{\partial y_1}{\partial k_1} - \Delta_2\frac{\partial y_2}{\partial k_1}\right),$$

$$\frac{\partial \dot{k}_2}{\partial k_2} = \frac{\partial y_2}{\partial k_2} + \theta_1\left(\Delta_1\frac{\partial y_1}{\partial k_2} - \Delta_2\frac{\partial y_2}{\partial k_2}\right).$$

Thus, the detail expression of the term ZJ in (22) is

$$\begin{aligned}
\mathrm{Z}J &= (1-\theta_2\Delta(y_1^*)-\theta_1\Delta(y_2^*))f''(K^*)\left(\frac{1-\Delta(y_1^*)}{C_2^*}(\Omega_s^2\theta_1-\Omega_o^2\theta_2)+\frac{1-\Delta(y_2^*)}{C_1^*}(\Omega_s^1\theta_2-\Omega_o^1\theta_1)\right)\\
&+ f'(K^*)^2\left(\Delta'(y_1^*)\left(\frac{\Omega_s^2}{C_2^*}(-1+\theta_1\Delta(y_2^*))-\frac{\Omega_o^2}{C_1^*}\theta_2\Delta(y_2^*)\right)+\Delta'(y_2^*)\left(\frac{\Omega_s^1}{C_1^*}(-1+\theta_2\Delta(y_1^*))-\frac{\Omega_o^1}{C_2^*}\theta_1\Delta(y_1^*)\right)\right).
\end{aligned} \tag{B.1}$$

Result 2: the case that $\Omega_o^1=\Omega_o^2=0$:

When $\Omega_o^1=\Omega_o^2=0$, ZJ in (22) can be rewritten as

$$\begin{aligned}
\mathrm{Z}J &= (1-\theta_2\Delta(y_1^*)-\theta_1\Delta(y_2^*))f''(K^*)\left(\frac{1-\Delta(y_1^*)}{C_2^*}\Omega_s^2\theta_1+\frac{1-\Delta(y_2^*)}{C_1^*}\Omega_s^1\theta_2\right)\\
&+ f'(K^*)^2\left(\Delta'(y_1^*)\frac{\Omega_s^2}{C_2^*}(-1+\theta_1\Delta(y_2^*))+\Delta'(y_2^*)\frac{\Omega_s^1}{C_1^*}(-1+\theta_2\Delta(y_1^*))\right)(<0).
\end{aligned} \tag{B.2}$$

Result 3: the case that $\rho_1=\rho_2$:

When $\rho_1=\rho_2$, we notice that $y\equiv y_1^*=y_2^*$ so that it holds that $C^*\equiv C_1^*=C_2^*$ and $\Delta(y^*)\equiv\Delta(y_1^*)=\Delta(y_2^*)$. Then, $(B.1)$ can be represented as

$$\begin{aligned}
\mathrm{Z}J &= \frac{(1-\Delta(y^*))^2f''(K^*)}{C^*}\left(\theta_1\Omega_s^2+\theta_2\Omega_s^1-\theta_1\Omega_o^1-\theta_2\Omega_o^2\right)\\
&- \frac{f'(K^*)^2\Delta'(y^*)}{C^*}\left(\Omega_s^1+\Omega_s^2-\Delta(y^*)(\theta_1\Omega_s^2+\theta_2\Omega_s^1-\theta_1\Omega_o^1-\theta_2\Omega_o^2)\right).
\end{aligned} \tag{B.3}$$

Note that $\Delta(y^*)=1-\frac{\rho}{f'(K^*)}$ in (18). Assuming that $\theta_1\Omega_s^2+\theta_2\Omega_s^1>\theta_1\Omega_o^1+\theta_2\Omega_o^2$ and $\Omega_s^1+\Omega_s^2>\left(1-\frac{\rho}{f'(K^*)}\right)(\theta_1\Omega_s^2+\theta_2\Omega_s^1-\theta_1\Omega_o^1-\theta_2\Omega_o^2)$, the sign of Z$J$ in $(B.3)$ is negative.

Because the assumption $\Omega_s^1\Omega_s^2>\Omega_o^1\Omega_o^2$ makes the sign of determinant negative, we can confirm the saddlepoint stability.

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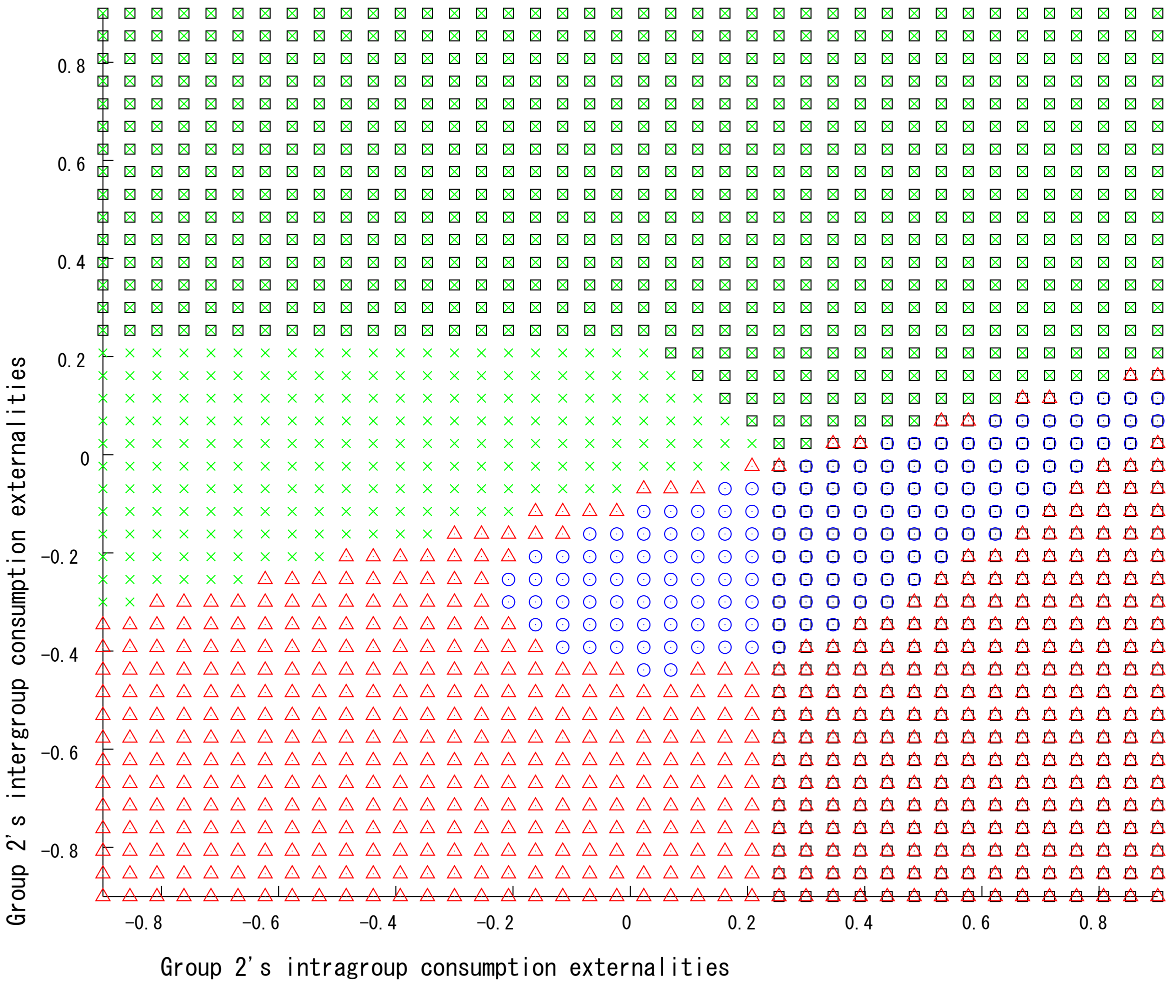

Figure 1: The case 1 that $\phi_1 = 0$ and $\eta_1 = -0.9$

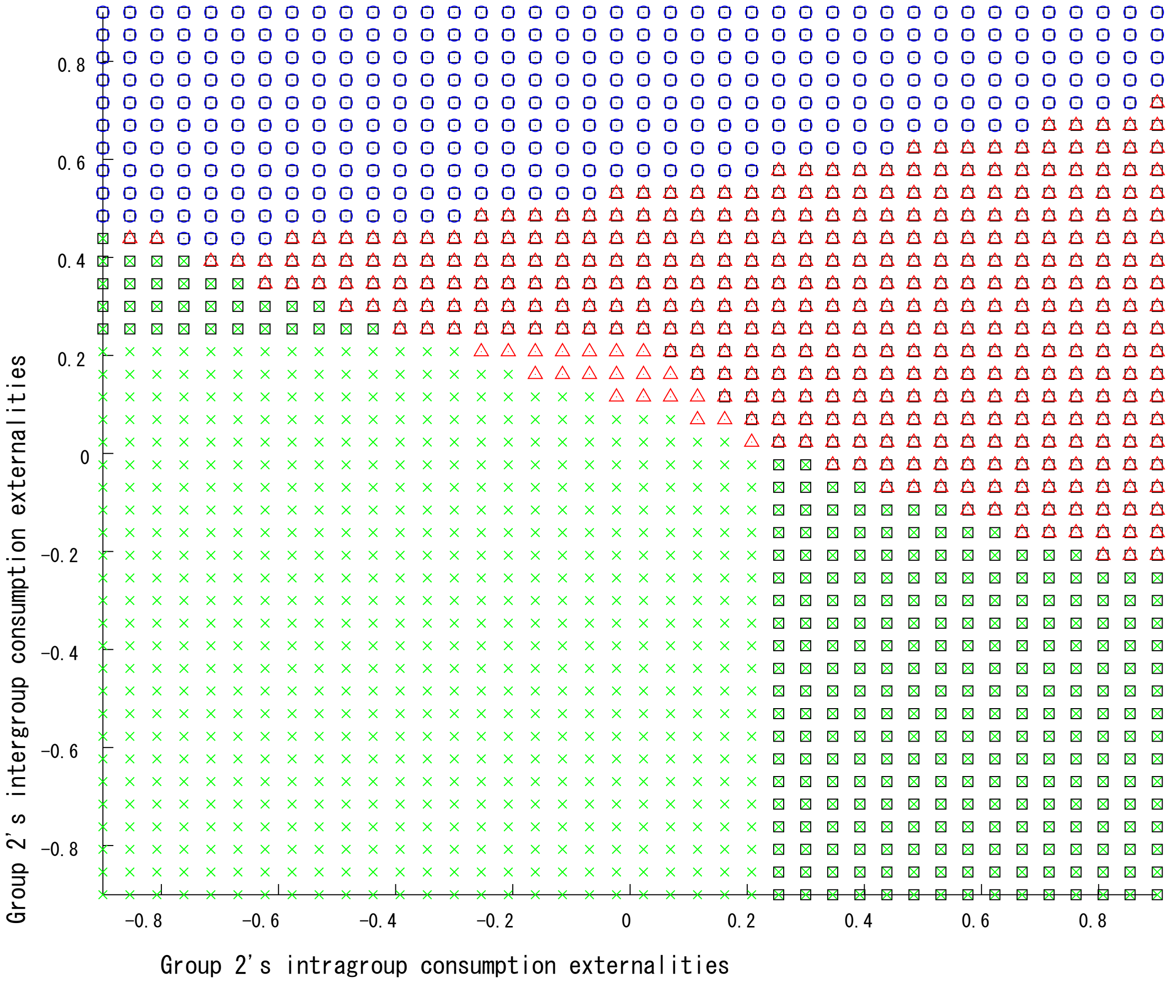

Figure 2: The case 2 that $\phi_1 = 0$ and $\eta_1 = 0.9$

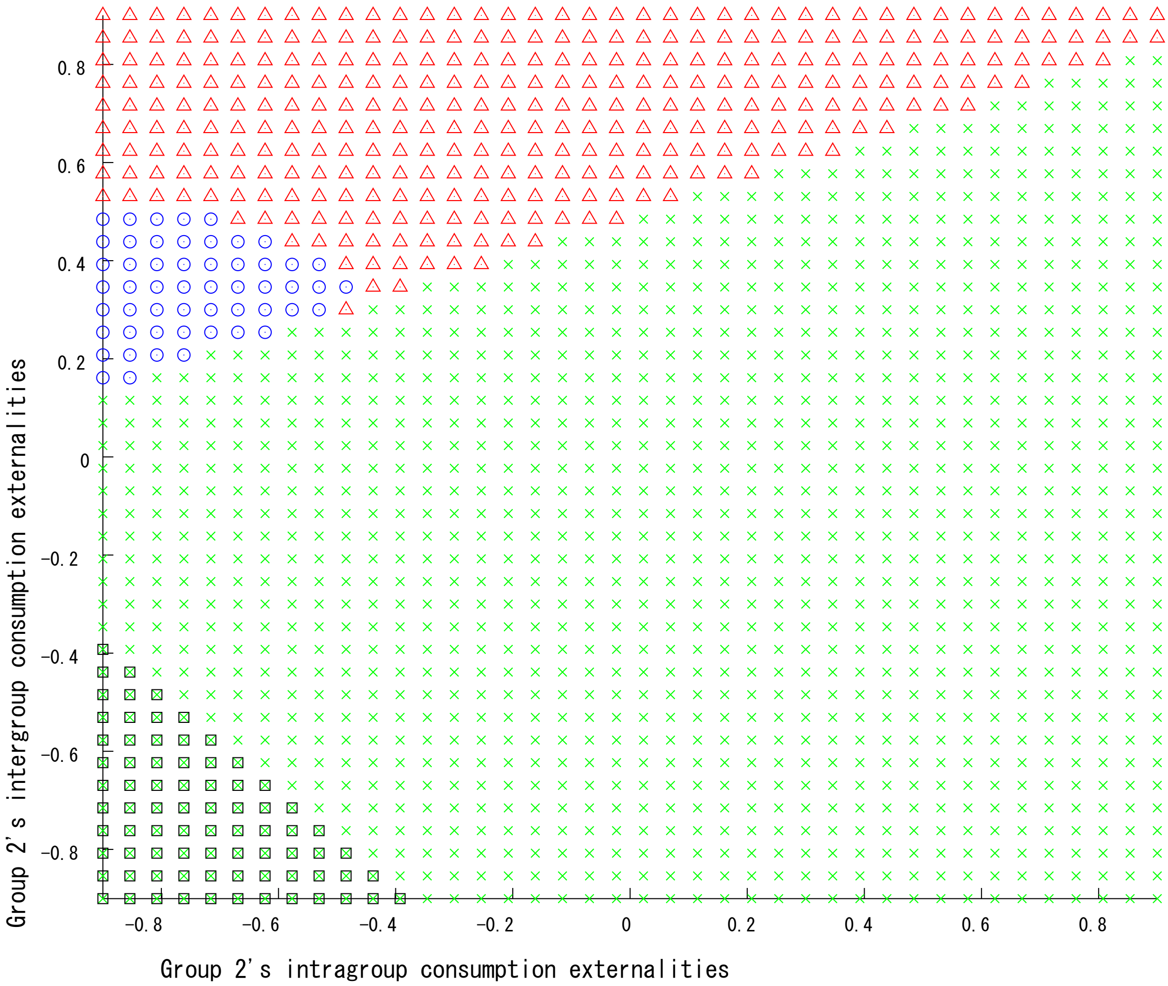

Figure 3: The case 2 that $\phi_1 = 0$ and $\eta_1 = 0.9$