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"Baby Boom and Baby Bust in Gender-Gap Model:
A Quantitative Analysis"

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# Baby Boom and Baby Bust in Gender-Gap Model:
# A Quantitative Analysis[*]

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**Abstract**

This paper explores what factor is important to replicate U.S. fertility transition in the last two centuries. We solve a multiperiod version of the model of Kimura and Yasui (J Econ Growth 15(4):323-351, 2010) numerically, conducting several experiments based on it. We find that the main trend of fertility transition in the last two centuries is attributed to changes in gender division of labor associated with capital accumulation and technological progress, the plunge during 1920-1940 to negative shocks on male labor supply by the World War II, and the upswing during 1940-1965 to an atypical burst of technological progress in household sector.

# 1   Introduction

Over the nineteenth and twentieth centuries, many Western developed countries experienced similar patterns of fertility transition. After the fall in fertility during the demographic transition, there were some periods of unusual deviations in fertility around an otherwise declining trend. There was first a plunge of fertility rates during 1920-1940, followed by a sharp upswing during 1940-1965 (the baby boom). During the period 1965-1985, the fertility rate returned to even lower levels than before the boom.

Recently, several researchers have attempted to identify the driving force behind the baby bust of 1920-1940 and the baby boom of 1940-1965 in secularly declining trend. Greenwood et al. (2005) attribute the secular decline in fertility and the temporary rise to increases in market-sector productivity and household-sector productivity respectively. They argue that the rise in household-sector productivity was spawned by the introduction of electricity and the development of associated household products such as appliances and frozen foods. Tamura and Simon (2010) identify declining young adult mortality and falling price for space as the causes of the secular decline and the baby boom respectively. Doepke et al. (2007) attribute the baby boom to the demand shock for female labor caused by World War II. They argue that women of the post-war generation were crowded out of the labor market and chose to have more children, leading to the baby boom.[1]

These studies share the common feature that unusual deviations in fertility around a secular declining trend were caused by exogenous shocks, e.g., exogenous technological changes, the WWII. On the contrary, the model of Kimura and Yasui (2010) generates such a fertility transition as a by-product of economic development associated with capital accumulation: the gender time lag in participation in paid work resulting from capital accumulation leads to a non-monotonic fertility dynamics.

Both the approach focusing on exogenous factors and the approach of Kimura and Yasui

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[1] Various explanations on the baby have been proposed by many researchers. See, for instance, Kimura and Yasui (2010), Doepke et al. (2007), and Greenwood et al. (2005), and the references therein.

(2010) focusing on the endogenous driving force succeed in capturing the main features of U.S. fertility transition: there was first a plunge of fertility rates during 1920-1940, followed by a sharp upswing during 1940-1965, and then the fertility rate returned to even lower levels than before the boom. However, both of them fail to quantitatively replicate large swings in the mid-twentieth century. It might be the case because the former is based on the model without structural change accompanying economic growth and the latter does not consider exogenous factors other than TFP growth.

The aim of this chapter is to introduce exogenous shocks into the model of Kimura and Yasui (2010) and quantitatively assess the capability of the model to generate U.S. fertility transition in the last two-centuries. The original model of Kimura and Yasui (2010) is unsuitable for considering exogenous shocks because it is a three-period overlapping-generations model and the length of a period in the quantitative analysis is 20 years, which is too long to consider various shocks, e.g., although we introduce the mobilization of male labor during the war as a war shock, the period when massive wartime mobilization was observed in U.S. was not more than 10 years.[2] We extend the three-period overlapping-generations model of Kimura and Yasui (2010) to the multiperiod overlapping-generations model and simulate the model with exogenous shocks.

The remainder of this paper is organized as follows. In the next section, we present a multiperiod version of the model of Kimura and Yasui (2010). Section 3 discusses our calibration strategy and presents the results of quantitative analysis. Section 4 concludes.

## 2 Model

Consider an overlapping generations model in which agents live for $T_C + T$ periods: each agent lives for $T_C$ periods as a child, for $T_A$ periods as an adult, and for $T_O$ periods as an elderly person, that is, $T = T_A + T_O$. We assume that $T_C \leq T_A$. In childhood, they do not make any decisions and consume a fixed quantity of time from their parents. In adulthood, they raise children,

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[2] See Doepke et al. (2007) for details.

supply labor to the market, engage in non-market work, and consume goods. In old age, they only consume goods. All decisions are made at the beginning of adulthood (adult age 0). Then agents decide the number of children, the amount of time spent on market and non-market work in each period, and the consumption plan over their lifetime.

The economy is populated by two kinds of agents: men and women. Men and women differ only in terms of their ability to earn wages in the labor market. It is assumed that there is no difference between men and women in the abilities to do non-market work and raise children. This assumption is employed for simplification, not crucial for our main results. What matters is that men have a comparative advantage in market work. The mechanism generating the gender wage gap is identical to that of Galor and Weil (1996); men and women have equal endowments of mental input, but men have more physical strength than women, and thus a gender wage gap reflecting this difference in physical strength exists. A man and a woman form a family and jointly decide the allocation of their time; they are assumed to have joint consumption and joint utility. Our basic unit of analysis is the couple and they are assumed to be together from birth so that we need not consider issues related to the formation of families.

## 2.1 Production

There is a single final good, the numeraire, which can either be consumed or invested. The final goods can be produced in two sectors: the non-market sector, where the only input is labor, and the market sector, which is relatively capital-intensive.

The production technology is the same as the one used in Kimura and Yasui (2010). The aggregate production function in market sector is given by

$$Y_t = A_t \left[ K_t^\alpha \left( L_t^m \right)^{1-\alpha} + b L_t^p \right],$$

where $K_t$, $L_t^m$, and $L_t^p$ are physical capital, mental labor, and physical labor respectively, and $b > 0$ and $\alpha \in (0,1)$ are parameters. It is important to note that physical capital and mental labor

are more complementary than physical capital and physical labor.

Assuming perfectly competitive factor markets, the return on a unit of physical labor at time $t$, $w_t^p$, the return on mental labor at time $t$, $w_t^m$, and the return on physical capital at time $t$, $r_t$, are respectively,

$$w_t^p = A_t b, \tag{1}$$

$$w_t^m = A_t (1 - \alpha) K_t^{\alpha} (L_t^m)^{-\alpha}, \tag{2}$$

$$r_t + \delta = A_t \alpha K_t^{\alpha-1} (L_t^m)^{1-\alpha}, \tag{3}$$

where $\delta \in [0, 1]$ is the depreciation rate of physical capital. If all the time available were devoted to market work, men could supply one unit of physical labor and one unit of mental labor and earn $w_t^p + w_t^m$, while women could supply only one unit of mental labor and earn $w_t^m$.

It follows from (2) and (3) that we can write the period-$t$ interest rate as a function of the period-$t$ mental wage rate:

$$r_t = A_t \alpha \left( \frac{w_t^m}{A_t (1 - \alpha)} \right)^{\frac{\alpha-1}{\alpha}} - \delta. \tag{4}$$

The production technology in non-market sector is given by

$$f(h_t) = \eta_t h_t^{\xi}, \tag{5}$$

where $h_t$ is the couple's time input to non-market work.

## 2.2 Couples' decision problem

Couples receive utility from the number of children that they have and from consumption stream over their lifetime. The utility function of a couple turning adult in period $t$ is

$$U_t = \sum_{j=0}^{T-1} \beta^j \ln c_{t,j} + \gamma \ln n_t \tag{6}$$

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where $c_{t,j}$ is consumption at adult age $j$ and $n_t$ is the number of children..

Each adult is endowed with a unit of time that can be devoted to market work, non-market work, and child rearing in each period. The time constraint of person $i$ of adult age $j$ in period $t$ is given by

$$l_{t,j}^i + h_{t,j}^i + q_{t,j}^i = 1, \ i \in \{H, W\},$$

where $l_{t,j}^i$, $h_{t,j}^i$, and $q_{t,j}^i$ denote the time spent on market work, non-market work, and child rearing, respectively. The person indexed by the superscript $H$ (resp. $W$) is the husband (resp. wife). Raising a pair of children takes fraction $z \in (0,1)$ of the time endowment of one person in each period. The time constraint for raising children can be written as

$$q_{t,j}^H + q_{t,j}^W = z n_t \text{ for } j \leq T_C - 1.$$

Offsprings consume their parents' time only in childhood.

The flow budget constraint that the couple faces in period $t + j$ is

$$c_{t,j} + a_{t,j+1} = (1 + r_{t+j}) a_{t,j} + m_{t,j} \text{ for } j = 0, ..., T - 1, \tag{7}$$

$$\text{with } a_{t,0} = a_{t,T} = 0,$$

where $a_{t,j}$, $r_{t+j}$ and $m_{t,j}$ are their assets, the interest rate, and their earnings in period $t + j$, respectively. The couple's earnings in period $t$ are

$$m_{t,j} = (w_{t+j}^m + w_{t+j}^p) l_{t,j}^H + w_{t+j}^m l_{t,j}^W + \eta (h_{t,j}^H + h_{t,j}^W)^{\xi} \tag{8}$$

The couple chooses how to allocate its time among different activities. The couple's division of labor can be classified into four cases. *Case 1*: The couple spends no time on market work. *Case2*: The wife spends no time on market work and the husband spends part of his time on market work. *Case 3*: The husband specializes in market work and the wife spends no time on

market work. *Case 4*: The husband specializes in market work and the wife also engages in market work.

Given the case the couple chooses, we can rewrite the couple's earnings equation (8) as the following.

$$
m_{t,j} = \begin{cases}
\eta \left(2 - zn_t\right)^{\xi} & \text{if Case 1,} \\
\left[2 - zn_t - \underline{h}(w_{t+j}^m)\right]\left(w_{t+j}^m + Ab\right) + \eta \left[\underline{h}(w_{t+j}^m)\right]^{\xi} & \text{if Case 2,} \\
w_{t+j}^m + Ab + \eta \left(1 - zn_t\right)^{\xi} & \text{if Case 3,} \\
w_{t+j}^m \left[2 - zn_t - \bar{h}(w_{t+j}^m)\right] + Ab + \eta \left[\bar{h}(w_{t+j}^m)\right]^{\xi} & \text{if Case 4,}
\end{cases} \quad \text{for } j \leq T_A - 1, \quad (9)
$$

$$
\text{where } \underline{h}(w_t^m) = \left(\frac{\eta \xi}{w_t^m + Ab}\right)^{\frac{1}{1-\xi}} \quad \text{and} \quad \bar{h}(w_t^m) = \left(\frac{\eta \xi}{w_t^m}\right)^{\frac{1}{1-\xi}}.
$$

Note that $\underline{h}$ and $\bar{h}$ denote the time spent on non-market work such that the marginal product of non-market work is equal to the men's wage rate, $w_t^m + w_t^p$, and the women's, $w_t^m$, respectively. In old age, the couple does not work:

$$
m_{t,j} = 0 \text{ for } j > T_A - 1.
$$

From equations (7) and (8), the lifetime budget constraint can be expressed as

$$
\sum_{j=0}^{T-1} q_{t,j} c_{t,j} = \sum_{j=0}^{T_A-1} q_t m_{t,j}, \quad (10)
$$

where $q_{t,j}$ is the present-value price, which is defined by $q_{t,j} = q_{t,j-1}/\left(1 + r_{t+j}\right)$ with $q_{t,0} = 1$.

A couple turning adult in period $t$ maximizes the utility function (6) subject to (10), given factor prices, $\left\{r_t, r_{t+1}, ..., r_{t+T}, w_t^m, w_{t+1}^m ..., w_{t+T}^m\right\}$. The solution to the optimization problem can

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be expressed as

$$c_{t,j} = c\left(r_t, r_{t+1}, \ldots, r_{t+T}, w_t^m, w_{t+1}^m \ldots, w_{t+T}^m\right) \text{ for } j = 0, \ldots, T-1, \tag{11}$$

$$l_{t,j} = l\left(r_t, r_{t+1}, \ldots, r_{t+T}, w_t^m, w_{t+1}^m \ldots, w_{t+T}^m\right) \text{ for } j = 0, \ldots, T_A - 1, \tag{12}$$

$$n_t = n\left(r_t, r_{t+1}, \ldots, r_{t+T}, w_t^m, w_{t+1}^m \ldots, w_{t+T}^m\right), \tag{13}$$

$$a_{t,j} = a\left(r_t, r_{t+1}, \ldots, r_{t+T}, w_t^m, w_{t+1}^m \ldots, w_{t+T}^m\right) \text{ for } j = 1, \ldots, T-1. \tag{14}$$

## 3 Equilibrium

Let $N_t$ denote the size of the cohort turning adult in period $t$. The market-clearing condition for capital is given by

$$K_t = \sum_{s=1}^{T-1} N_{t-s} a_{t-s,s}.$$

The market-clearing condition for mental labor is given by

$$L_t^m = \sum_{s=0}^{T_a - 1} N_{t-s} l_{t-s,s}, \tag{15}$$

$$\text{where } l_{t,j} = \begin{cases} 0 & \text{if } \text{Case 1,} \\ 2 - zn\left(w_{t+j}^m\right) - \underline{h}(w_{t+j}^m) & \text{if } \text{Case 2,} \\ 1 & \text{if } \text{Case 3,} \\ 2 - zn\left(w_{t+j}^m\right) - \bar{h}(w_{t+j}^m) & \text{if } \text{Case 4.} \end{cases} \tag{16}$$

The cohort size $N_t$ evolves according to the law of motion:

$$N_{t+T_C} = N_t n_t.$$

Some initial conditions must be given for specifying the dynamics of this model. In period 0, there are $T$ generations of adults and elderly people. The initial demographic structure of adults and elderly people is expressed by the $T$-dimensional vector $(N_{-T+1}, \ldots, N_{-1}, N_0)$. Denote their

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initial assets by $(a_{-T+1,T-1}, ..., a_{-2,2}, a_{-1,1})$. Given these initial conditions, the initial aggregate capital stock, $K_0$, is represented as

$$K_0 = \sum_{s=1}^{T-1} N_{-s} a_{-s,s}.$$

Furthermore, there are $T_C - 1$ generations of children, $(n_0 N_0, n_{-1} N_{-1}, ..., n_{-T} N)$.

Given the initial conditions mentioned just above, a competitive equilibrium is a time path for factor prices $\{r_t, w_t^m\}_{t=1}^{\infty}$ such that (i) the allocations $\{c_{t,j}, n_t\}_{t=1}^{\infty}$ solve the household's problem, given $\{r_t, w_t^m\}_{t=1}^{\infty}$, (ii) the allocations $\{K_t, L_t^m\}_{t=1}^{\infty}$ solve the firm's problem, given $\{r_t, w_t^m\}_{t=1}^{\infty}$, and (iii) the market-clearing conditions hold.

# 4 The Difference Equation System

To simulate the model, take the length of a period in the model to be 10 years. For that purpose, we consider the case of $T_C = 2$, $T_A = 2$, $T_O = 2$.

Define the Lagrangian associated with the couple's maximization:

$$\mathscr{L} = \sum_{j=0}^{3} \beta^j \ln c_{t,j} + \gamma \ln n_t + \lambda \left( \sum_{j=0}^{3} q_t m_{t,j} - \sum_{j=0}^{3} q_{t,j} c_{t,j} \right),$$

where $\lambda$ is the Lagrange multiplier associated with this problem. The first-order conditions imply

$$c_{t,j+1} = \beta (1 + r_{t+j+1}) c_{t,j} \text{ for } j = 0, ..., 2,$$

$$\frac{\gamma}{n_t} = -\lambda \sum_{j=0}^{1} q_t \frac{\partial m_{t,j}}{\partial n_t},$$

and

$$m_{t,0} + \frac{m_{t,1}}{(1+r_{t+1})} = c_{t,0} + \frac{c_{t,1}}{(1+r_{t+1})} + \frac{c_{t,2}}{(1+r_{t+1})(1+r_{t+2})} + \frac{c_{t,3}}{(1+r_{t+1})(1+r_{t+2})(1+r_{t+3})}.$$

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As is apparent from (9), $m_{t,j}$ takes different forms in different cases. That the couple can freely choose one of four cases at adult ages 0 and 1 makes the problem complex. There are 9 patterns in their case choice: (Case 2, Case 2), (Case 2, Case 3), ... , (Case 4, Case 3), and (Case 4, Case 4). Note that Case 1 dose not arise in equilibrium if capital stock is positive because the aggregate production function meets the Inada condition.

For deriving the solution, we first calculate the within-pattern maximized utility for 9 patterns. Then we compare them, thereby obtaining $\left( \left\{ c_{t,j}^{*} \right\}_{j=0}^{T}, n_{t}^{*} \right)$, which provides the couple with maximal utility.

Using (4), (11), (13) and (14), the solution to the optimization problem can be expressed as

$$c_{t,j} = c\left(w_{t}^{m}, w_{t+1}^{m}..., w_{t+4}^{m}\right) \text{ for } j = 0, 1, 2, 3, \tag{17}$$

$$n_{t} = n\left(w_{t}^{m}, w_{t+1}^{m}..., w_{t+4}^{m}\right), \tag{18}$$

$$l_{t,j} = l\left(w_{t}^{m}, w_{t+1}^{m}..., w_{t+4}^{m}\right) \text{ for } j = 0, 1, \tag{19}$$

$$a_{t,j} = a\left(w_{t}^{m}, w_{t+1}^{m}..., w_{t+4}^{m}\right) \text{ for } j = 1, 2, 3. \tag{20}$$

The transformed period-$t$ labor market clearing condition is

$$L_{t}^{m} = N_{t} l_{t,0} + N_{t-1} l_{t-1,1}. \tag{21}$$

The transformed period-$t$ asset market clearing condition is

$$k_{t} = \frac{1}{\theta_{t}} a_{t-1,1} + \frac{1}{n_{t-2}} a_{t-2,2} + \frac{1}{n_{t-3}\theta_{t}} a_{t-3,3}, \tag{22}$$

where $k_{t} \equiv K_{t}/N_{t}$ and $\theta_{t} \equiv N_{t+1}/N_{t}$. Using (18) and (20), we see that period-$t$ savings depend on $w_{t-3}, w_{t-2}, w_{t-1}, w_{t}, w_{t+1}, w_{t+2}$, and $\theta_{t}$.

Using (2) and (15), we get

$$w_t^m = A_t \left(1 - \alpha\right) \left(\frac{k_t}{l_{t,0} + \frac{1}{\theta_t} l_{t-1,1}}\right)^\alpha. \tag{23}$$

From (19), we see that (23) depends on $w_{t-1}$, $w_t$, $w_{t+1}$, $w_{t+2}$, $w_{t+3}$, and $\theta_t$.

It follows from (22) and (23) that

$$w_t = A_t \left(1 - \alpha\right) \left[\frac{\frac{1}{\theta_t} a_{t-1,1} + \frac{1}{n_{t-2}} a_{t-2,2} + \frac{1}{n_{t-3}\theta_t} a_{t-3,3}}{l_{t,0} + \frac{1}{\theta_t} l_{t-1,1}}\right]^a. \tag{24}$$

From (20), we find that RHS of this equation depends on $w_{t-3}$, $w_{t-2}$, $w_{t-1}$, $w_t$, $w_{t+1}$, $w_{t+2}$, $w_{t+3}$, and $\theta_t$.

The cohort size $N_t$ evolves according to the law of motion:

$$N_{t+2} = N_t n_t. \tag{25}$$

Using (25) and the definition of $\theta_t$, we obtain

$$\theta_{t+1} \equiv \frac{N_{t+1}}{N_t} = \frac{N_{t+1}}{\theta_t N_{t-1}} = \frac{n_{t-1}}{\theta_t}. \tag{26}$$

Equations (24) and (26) together implicitly define a difference equation system:

$$D\left(w_{t-3}, w_{t-2}, ..., w_{t+3}, \theta_t, \theta_{t+1}\right) = 0. \tag{27}$$

This nonlinear difference equation system is used to compute the time path for fertility.

# 5   Simulation

Take the length of a period in the model to be 10 years so that an individual lives for 20 years as a child, for 20 years as an adult, and for 20 years as an elderly person. There will be 17 model

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periods between 1830 and 2000.

Thus far we have not taken infant mortality into consideration, but it is not negligible for simulating the model over long periods; infant mortality is much lower today than 200 years ago. Although we do not explicitly model infant mortality, we assume that $n_t$ represents the number of pairs of "surviving" children following the convention of literature. We use the total fertility rate net of infant mortality as data compared with simulated fertility.[3]

## 5.1 Calibration

The model has two types of parameters. One is parameters constant over time: $\alpha$ (physical capital share), $\delta$ (depreciation rate of physical capital), $\beta$ (subjective discount factor), $b$ (Marginal product of physical labor per TFP), and $\xi$ (Curvature of non-market production function). The other is time-varying parameters: $z_t$ (time cost of having a child), $A_t$ (technological level in market sector), and $\eta_t$ (technological level in non-market sector).

We set $\alpha = 0.3$ because it is well known that capital share of income is roughly 30%. Following Doepke et al. (2007) and Greenwood et al. (2005), we set the value of $\delta$ so that the annual depreciation rate of physical capital is 4.7%, i.e., $\delta = 1 - (1 - 0.047)^{10}$. We set the yearly discount rate to 3%, which implies $\beta = 0.97^{10}$.

The remaining parameters are estimated based on historical evidence or chosen to match the data.

Time paths for TFP in the market sector similar to those found in the United States between 1830 and 2000 are given to the model. The initial level of TFP is normalized to unity. The estimates of the growth rates of TFP for 1830-1890 are taken from Gallman (2000, p.15, Table 1.4), those for 1890-1950 are from Carter et al., eds (2006, Series Cg270 and Cg278), and those for 1950-2000 are from Bureau of Labor Statistics.[4]

It is difficult to specify technological progress in the non-market sector $\{\eta_t\}$, because various

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[3]Total fertility rates and infant mortality rates are taken from Haines (2008, Table 1).
[4]Source: ftp://ftp.bls.gov/pub/special.requests/opt/mp/prod3.mfptablehis.zip

factors have contributed to it. We use the growth rates of agricultural productivity as a proxy of technological progress in the non-market sector. According to Atack et al. (2000, Table 6.I.), the annual growth rate of agricultural productivity from 1800 and 1900 is 0.49%. Based on this value. we set the time path of technological level in the non-market sector so that $\eta_{t+1}/\eta_t = (1+0.0049)^{10}$.

Next, consider the sequence of the time cost of having a child, $\{z_t\}$. Since we take the length of a period in the model to be 10 years, age-0 children and age-1 children in the model respectively correspond to children aged 0-9 and children aged 10-19 in the real world. In reality, the cost parents with age-0 children bear differ from the cost parents with age-1 children bear. For parents with age-0 children, spending on health, nutrition, and sanitation might account for a substantial fraction of total cost of child-rearing. For parents with age-1 children, on the contrary, education costs might take up a large portion in total spending on children. We assume the time cost function incorporating such ideas. The time cost function for parents with age-0 children, i.e., age-0 adults, in period $t$ is given by $COST_{t,0} = 0.5 \cdot 1/(1 - IMR_t) + 0.5SER_t$, where $IMR_t$ is the infant mortality rate and $SER_t$ is the school enrollment rate.[5] The formulation of this cost function reflects the idea that parents raising age-0 children principally bear health-related costs in the first half of the period and education-related costs in the second half. The time cost function for parents with age-1 children, i.e., age-1 adults, in period $t$ is given by $COST_{t,1} = SER_t$. Note that we compute $COST_{t,i}$ for $i = 0, 1$ by a linear approximation because there are some cohorts whose data are not available. Following the empirical results of Haveman and Wolfe (1995) and Knowles (1999), we set $z_{1990} = 0.4$, that is, $z_{t,i} = z_{1990} \cdot COST_{t,i}/COST_{1990}$ for $i = 0, 1$.

Other parameters, $\gamma$, $b$, $\xi$, and $\eta_{1800}$, are chosen to minimize the sum of squares of gap between predicted fertility and actual fertility for appropriate parameter sets.

Following the procedures above, we obtain the parameter values listed in Table 1.

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[5]The school enrollment rate of whites aged 5-19 for 1850-1990 are taken from Goldin (1999, CG.A.15).

Table 1: Baseline parameter values

| Parameter | Interpretation | Value |
|:---:|:---|:---:|
| $\alpha$ | Physical capital share | 0.3 |
| $b$ | Marginal product of physical labor per TFP | 0.7 |
| $\delta$ | Depreciation rate of physical capital | $1 - (1 - 0.047)^{10}$ |
| $\gamma$ | Weight of children in utility function | 0.5 |
| $\eta_{1800}$ | Efficiency of non-market production in 1800 | 4.9 |
| $\xi$ | Curvature of non-market production function | 0.39 |

## 5.2 Transitional Dynamics

Imagine starting the economy off in 1830, and suppose that it is a initial state belonging to Case 2. Figure 1 presents the simulations against the actual data.

We find that the model does well at explaining the main trend in fertility transition (a long secular decline, interrupted by a temporary rise), but fails to replicate a baby boom around 1960 and significant declines before and after the boom. Furthermore, the model cannot explain sharp rises of married-female participation in paid work in the latter half of the twentieth century, which would be one of the factors preventing the model from replicating the baby bust after the baby boom.

## 5.3 Experiments

We have simulated a multiperiod version of the model of Kimura and Yasui (2010). The simulation not only captures the main trend in fertility transition (a long secular decline, interrupted by a temporary rise) but also does better at replicating fertility transition than the simulation of Kimura and Yasui (2010). As Kimura and Yasui (2010), however, there remains a considerable discrepancy between fertility transition in the data and that in the model around the baby boom: the amplitude of variations in simulated fertility is much smaller than that in observed fertility.

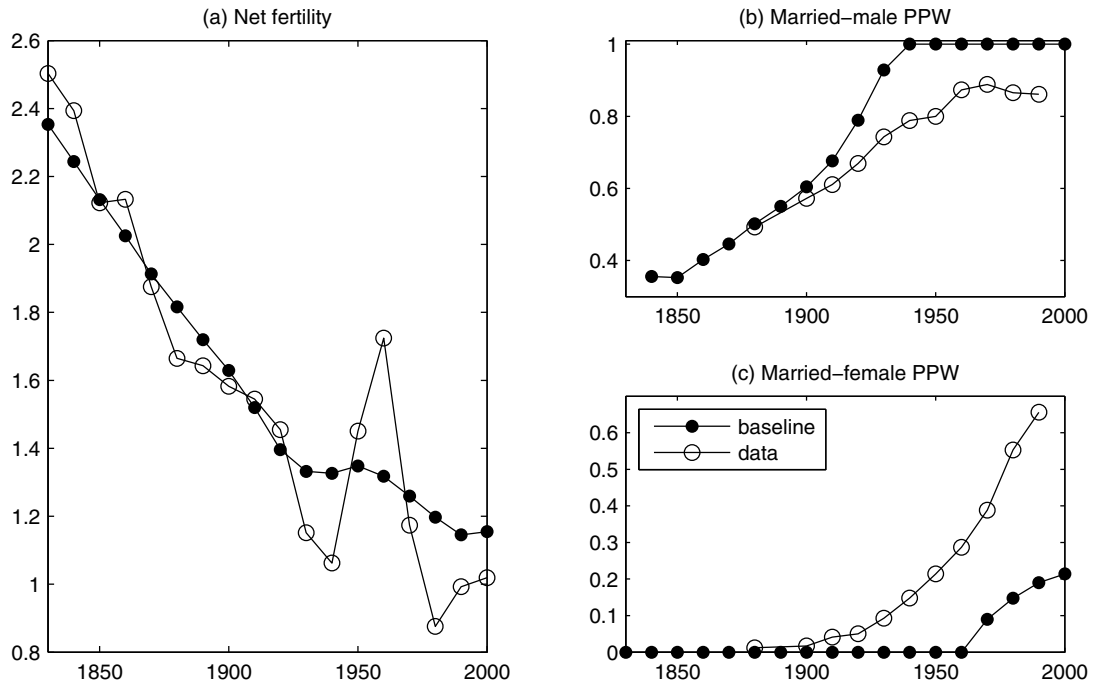Now consider what caused such sharp rises and declines of fertility in the twentieth century.

Figure 1: Baseline case

### 5.3.1 Modeling the War Shock

We now want to demonstrate how a shock caused by the World War II affects fertility in our model. We model the war shock as a one-time decline in the availability of male labor. Consider a sudden drop in the availability of male labor from 1 to $\tau \in (0,1)$ in period $t_W$. It is assumed that this drop is unpredictable until the beginning of period $t_W$. Basically, we concentrate on the rational-expectation equilibrium where couples' expectations about the future are realized, but the unpredictability interrupts the rational expectation. Forward-looking couples make their decisions anticipating the future sequences of factor prices, but the war shock forces couples to rearrange their plan in period $t_W$. Couples of age elderly 1 in period $t_W$ are affected by this shock only through the change of the interest rate in that period: their age-elderly-1 consumption level, $c_{t_W-3,3}$, differs from the initially planned level. However, nothing but $c_{t_W-3,3}$ changes for them because their fertility, labor-supply, and saving behaviors have been already finished by then.
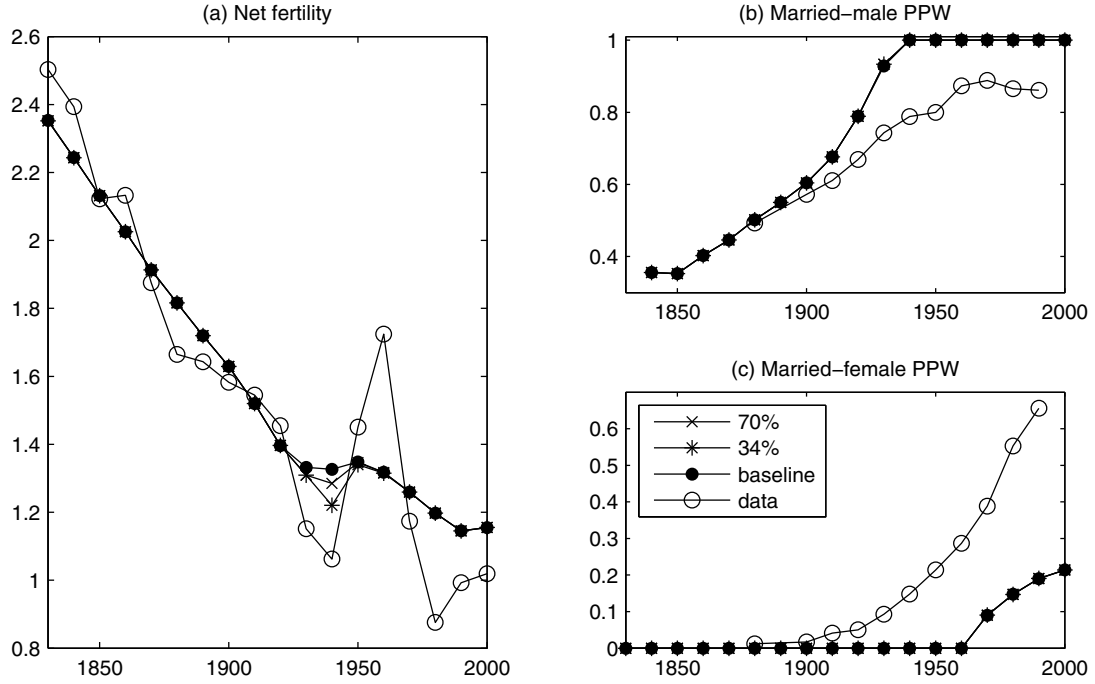
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Figure 2: Effect of war shock

Couples of age elderly 0 in period $t_W$ change their saving-consumption plan in response to the changes of factor prices induced by the war shock: $c_{t_W-2,2}$ and $c_{t_W-2,3}$ differ from the initially planned levels. Couples of age adult 1 in period $t_W$ change their time allocation between market work and non-market work in period $t_W$, $\{l^H_{t_W-1,1}, l^W_{t_W-1,1}\}$, and their consumption plan after the shock, $\{c_{t_W-1,1}, c_{t_W-1,2}, c_{t_W-1,3}\}$. For couples of age adult 0, on the contrary, the initially planned fertility, time allocation, and consumption are consistent with the realized ones because they make their decisions after observing the war shock.

Figure 2 depicts the results. If, following Doepke et al. (2007), we assume that $\tau = 0.7$, i.e., the amount of male labor available in period $t_W$ declines by 30%, then $|n_{1940} - n_{1930}| = 0.0245$, which explains 27.4% of observed drop of net fertility from period 1930 to period 1940. To replicate the observed drop, we need to set $\tau = 0.34$. We can find that the introduction of the war shock improves the simulation result, but only slightly.

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### 5.3.2 The time cost of having a child

For the baseline, we considered the case where the time cost of having a child increased because of exogenous increases in education costs. However, there are many other factors reducing the time cost, for instance, the prevalence of appliances and frozen foods and the higher availability of child-care services. Greenwood et al. (2005) argue that an atypical burst of technological progress in household sector occurred in the middle of the last century, which lowered the cost of having children. In our model, such technological progress means declines in the cost of having child-0 children.

As an experiment, here, we incorporate changes in the time cost of having a child similar to those considered in Greenwood et al. (2005). We set the time cost of having a child in period 1950 and 1960 so that $z_{1950,0}/z_{1940,0}$ and $z_{1956,0}/z_{1940,0}$ are equal to the corresponding ratios in the model of Greenwood et al. (2005). Fig. 3 shows that reducing the cost of having a child, $z_{t,0}$, in those periods generates a larger scale of baby boom and makes the simulated fertility rates closer to the observed ones. During the baby-bust period, however, the gap between predicted fertility and actual fertility becomes larger than that in the baseline case.

### 5.3.3 The technology of non-market production

Thus far, we have incorporated a war shock and changes in child-rearing costs into the model for generating large variations in fertility around the baby boom. From the experiments, we can infer what is behind large swings of fertility in the middle of the twentieth century. In our model, the size and the timing of the baby boom crucially depend on the sequence of the technology level of non-market production.

In the baseline case, we set $\eta_{t+1}/\eta_t = (1+0.0049)^{10}$. Suppose that the value is changed to $(1+0.0059)^{10}$ and $(1+0.0039)^{10}$. Figure 4 depicts the results. The timing of the baby boom depends on the growth rate of the efficiency of non-market production. The rapid technological progress in the non-market sector inhibits the shift of labor from the non-market sector to the market sector and retards the advent of baby boom.
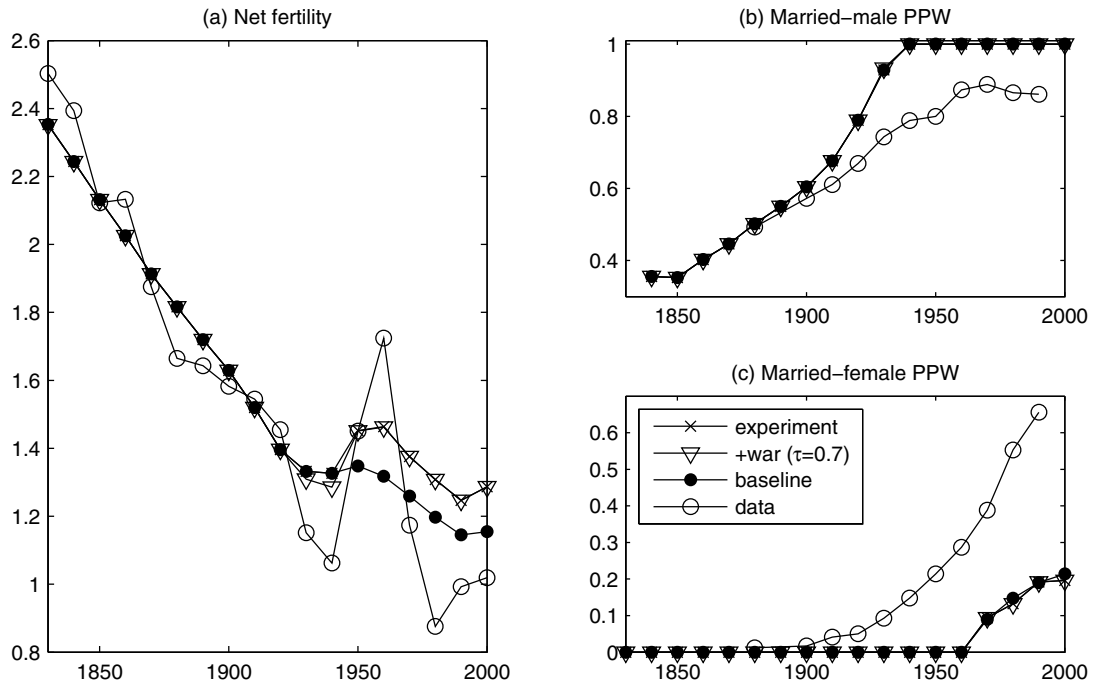
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Figure 3: Effect of change of child-rearing cost.

# 6 Conclusion

Our objective in this chapter was to explore what factor is important to replicate U.S. fertility transition in the last two centuries. We solved a multiperiod version of the model of Kimura and Yasui (2010) numerically, conducting several experiments based on it. We found that the main trend of fertility transition in the last two centuries is attributed to changes in gender division of labor associated with capital accumulation and technological progress, the plunge during 1920-1940 to negative shocks on male labor supply by the World War II, and the upswing during 1940-1965 to an atypical burst of technological progress in household sector. It remains, however, difficult to replicate large variations in fertility around the baby boom.

We conclude by suggesting some potential directions for future research. First, introducing human capital investments in children could produce larger swings in fertility. In this chapter, we assume that males and females differ only in their innate ability to earn wages in the mar-
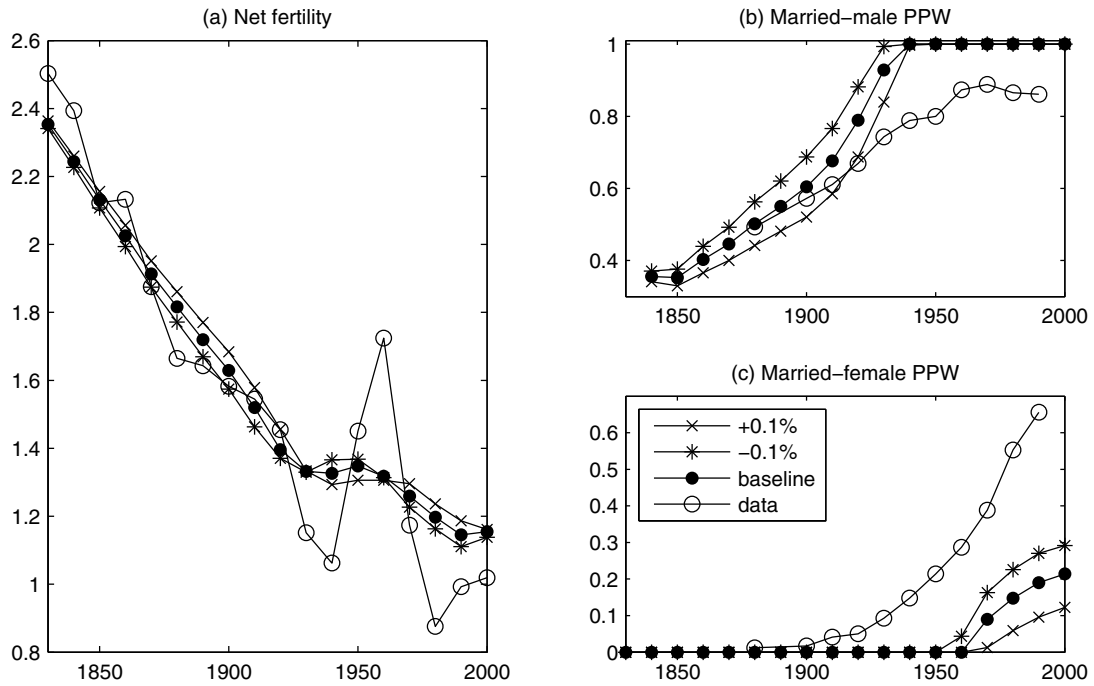
Figure 4: Effect of change of technological progress in the non-market production.

ket sector. Educational investments by parents after birth might magnify or diminish the innate difference, and thus could amplify time-series variation in fertility. Second, our model employs some assumptions for simplification, such as log-linear utility function and perfect substitutability between the goods produced in market sector and non-market sector. The relaxation of these assumptions might improve our simulation results.

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