KIER DISCUSSION PAPER SERIES

KYOTO INSTITUTE

OF

ECONOMIC RESEARCH

Discussion Paper No. 683

"Consumption Externalities and Wealth Distribution in a Neoclassical Growth Model"

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October 2009

KYOTO UNIVERSITY

KYOTO, JAPAN

Consumption Externalities and Wealth Distribution in a Neoclassical Growth Model *

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Abstract

This paper explores the distributional effect of consumption externalities in a neoclassical growth model with heterogeneous agents. The economy consists of two types of agents each of which perceives different degrees of intergroup as well as intragroup consumption external effects. It is shown that the stationary distribution and transitional dynamics are highly sensitive to the specification of preference structures of each type of agents.

Keywords: consumption externalities, heterogeneous agents, wealth distribution

JEL Classification Code: D31, E13, E21, O40

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*The first and second authors respectively acknowledge financial support by Grant-in-Aid for Scientific Research (No.17730139) and Grant-in-Aid for Young Scientists (Start-up) (No.10552200).

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1 Introduction

In recent years, there has been a renewed interest in the role of consumption externalities in macroeconomic dynamics. The basic assumption of this literature is that consumers' felicity depends not only on their private consumption but also on the average consumption in the economy at large. The presence of such a psychological external effect may alter saving behaviors of consumers and thus dynamic property of the model economy. The existing studies have inspected the effects of consumption externalities in the context of asset pricing (Abel 1990 and Galí 1994), optimal taxation (Ljungqvist and Uhlig 2000), equilibrium efficiency (Alonso-Carrera et al. 2003, Liu and Turnovsky 2005 and Nakamoto 2009), indeterminacy and sunspots (Weder 2000), and long-term economic growth (Carroll et al. 1997 and 2000, and Harbaugh 1996).[1]

One of the key features of this literature is that most of the foregoing studies employ representative agent models. In the representative-agent economies, the social average consumption coincides with the level of private consumption, so that the presence of consumption externalities affects aggregate dynamics in a quantitative manner rather than in a qualitative manner: the dynamic behavior of the model economy with consumption externalities is essentially the same as that of the economy without external effects. Furthermore, it has been well-known that, in the Ramsey model with inelastic labor supply, the steady-state levels of aggregate capital and aggregate consumption are not affected by the presence of consumption externalities.[2]

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[1]Some of the existing studies such as Ljunavust and Uhlig (2000) and Carroll et al. (1997 and 2000) assume the external habit formation in which the benchmark consumption is given by a weighted average of past levels of the average consumption in the economy. Unlike the internal habit formation, consumers consider that the benchmark consumption is not affected by their own consumption behavior under the external habit formation hypothesis. Thus this assumption represents consumption externalities with time delay rather than (internal) habit formation under which each agent takes its past consumption into account when deciding its optimal saving plans.

[2]See Fisher and Hof (2000).

Unlike the mainstream literature on dynamic macroeconomic analysis with consumption externalities, the present paper explores the effects of consumption externalities in a model of capital accumulation with heterogeneous agents. We construct a neoclassical growth model in which there are two groups of infinitely lived households. Each group of households differs in their initial wealth holding and in their preference structure as to consumption external effects. In this setting, external effects of consumption among the consumers are more complex than in the representative-agent counterpart, because the presence of consumption externalities may have distributional effects that is inevitably absent in the representative agent modelling.

More specifically, we distinguish intergroup externalities from intragroup externalities. Namely, we assume that a household may react differently in response to changes in the average consumption in her own group and that in the other group. For example, an agent would feel jealousy as to other members in her own group but would admire consumption behavior of other group's members. We assume the presence of such kind of asymmetric external effects in order to inspect how the different forms of external effects among the households affect wealth distribution and aggregate dynamics of the economy.

As it turns out, the steady state of aggregate economy is the same as that of the standard Ramsey model without consumption externalities. In addition, as well as in the standard model without externalities, the steady-state distribution of wealth is history dependent, that is, the long-run wealth distribution depends on the initial distribution of wealth among the agents. It is, however, demonstrated that dynamic behavior of the model economy towards the steady state is highly sensitive to the consumption external effects. In particular, in addition to the initial condition, the specification of consumers' preferences with respect to external effects plays a critical role in determining the stationary distribution of wealth. We carefully inspect how the strength and direction of intergroup as well as intragroup external effects determine the equilibrium trajectory of the economy leading to a specific stationary distribution of wealth.

It is to be noted that our study is closely related to García-Peñalosa and Turnovsky (2008). These authors also examine a heterogeneous-agent model of neoclassical growth with consumption externalities. On the one hand, the model used by García-Peñalosa and Turnovsky (2008) is more general than ours because their model allows variable labor supply, while our model does not. On the other hand, they assume that the utility function is identical for all agents and satisfies quasi-homotheticity. Given this restriction, the aggregate dynamics of the model economy is independent of wealth distribution so that García-Peñalosa and Turnovsky (2008) can focus on the distribution dynamics under a given pattern of macroeconomic dynamics. As emphasized above, since our model assumes that the external effects are asymmetric between the two groups of agents, the aggregate dynamics of macroeconomy depends on the wealth distribution. Hence, the contribution of our investigation is to examine distributional effect of consumption externalities in a general situation under which dynamic behavior of the macroeconomy cannot be separated from personal wealth distribution.

The reminder of this paper is organized as follows. Section 2 sets up the theoretical framework and Section 3 characterizes the steady-state equilibrium. Section 4 presents a detailed discussion on the relation between consumption externalities and the stationary distribution of wealth. Section 5 gives some discussions for our findings. Section 6 concludes.

2 The Model

2.1 Households

Suppose that there are two groups of infinitely-lived agents. Each group consists of a continuum of identical households who have the same form of instantaneous utility function and an identical rate of time preference. The felicity function and the initial holding of wealth of the representative household in each group are different from each other. For simplicity, we assume that population in the economy is constant over time and, therefore, the mass of each group will not change. We also assume that the economy is closed and the stock of capital is the only net asset held by agents.

The representative agent in group i ($i = 1, 2$) supplies one unit of labor in each moment and maximizes a discounted sum of utilities over an infinite time horizon. The objective functional of the representative agent in group i is given by

$$U_i = \int_0^{+\infty} e^{-\rho t} u^i(c_i, C_i, C_j) dt, \quad \rho > 0, \quad i, j = 1, 2, \quad i \neq j. \tag{1}$$

In the above, ρ denotes a given rate of time discount, c_i private consumption of group i agent, and C_i and C_j respectively represent the average levels of consumption in groups i and j. The instantaneous utility function, $u^i(\cdot)$, is assumed to be monotonically increasing and strictly concave in private consumption, c_i. It is also assumed that in the symmetric equilibrium where $C \equiv c_i = C_1 = C_2$, the utility function holds the Inada conditions: $\lim_{C \to 0} u_1^i(C, C, C) = \infty$ and $\lim_{C \to \infty} u_1^i(C, C, C) = 0$, where $u_m^i(\cdot)$ ($m = 1, 2, 3$) denotes the partial derivative of the utility function with respective to the m-th variable in $u^i(\cdot)$.

The key assumption about the instantaneous felicity function in (1) is that we distinguish intragroup externalities from intergroup externalities. That is, an agent's concern with the consumption levels of members in her own group may be different from the concern with consumption of agents in the other group. Following the taxonomy given by Dupor and Liu (2003), the external effect of consumption on an individual utility may be either negative (jealousy) or positive (admiration). In addition, each consumer is a conformist who likes being similar to others (keeping up with the Joneses) or an anti-conformist who wants to be different from others (running away from the Joneses). We allow, for example, an agent in a particular group feels jealousy as to consumption of others in her group but admires consumption of agents who belong to the other group. Such a situation may emerge, the agents in the rich group admire an increase in the benchmark level of consumption in the poor group, whereas they have jealousy as to the consumption level of other members in her own group. In addition, the agent would be a conformist as to consumption behavior of her group's members, but they like running away from consumption behavior of the other group's agents. As a result, even though there are only two types of agents, the external effects among the consumers cover a richer class of situations than that treated in the representative-agent economy where external effects are symmetric for all agents.

As usual, the negative externality (jealousy) is expressed by $u_j^i\left(\cdot\right)\left(=\partial u^i/\partial C_{j-1}\right) < 0 \quad (i=1,2,\ j=2,3)$, while positive externality (admiration) means that $u_j^i\left(\cdot\right)$ has a positive value. Similarly, the consumers' conformism is expressed by $u_{1j}^i\left(\cdot\right)\left(=\partial^2 u^i/\partial C_{j-1}\partial c_i\right) > 0$, and anti-conformism holds if $u_{1j}^i\left(\cdot\right)\left(=\partial^2 u^i/\partial C_{j-1}\partial c_i\right) < 0$. In what follows, we assume that, regardless of the forms of external effects, the effect of a change in the private consumption dominates the impact on her utility caused by external effect. More specifically, the utility function is assumed to satisfy the following properties:

$$u_1^i(\cdot) + u_2^i(\cdot) > 0, \tag{2a}$$

$$u_1^i(\cdot) + u_3^i(\cdot) > 0, \tag{2b}$$

$$u_{11}^i(\cdot) + u_{12}^i(\cdot) < 0, \tag{2c}$$

$$u_{11}^i(\cdot) + u_{13}^i(\cdot) < 0, \tag{2d}$$

$$u_1^i(\cdot) + u_2^i(\cdot) + u_3^i(\cdot) > 0, \tag{2e}$$

$$u_{11}^i(\cdot) + u_{12}^i(\cdot) + u_{13}^i(\cdot) < 0, \tag{2f}$$

where $i = 1$ and 2. Conditions $(2a)$ and $(2b)$ mean that the marginal utility of own consumption dominates impacts produced by consumption externalities. Conditions $(2c)$ and $(2d)$ show that the marginal utility of own consumption diminishes even in the presence of external effects. Conditions $(2e)$ and $(2f)$ ensure that, in a social symmetric equilibrium $C_1 = C_2$, the marginal utility of consumption in a group is positive and it monotonically decreases with private consumption.

The flow budget constraint for each agent is

$$\dot{k}_i = rk_i + w - c_i, \quad i = 1, 2, \tag{3}$$

where, k_i is capital stock owned by an agent in group i, c_i consumption, r_i the rate of return to asset and w_i the real wage rate. The initial holding of capital, $k_i(0)$, is exogenously given.

Each household maximizes U_i subject to (3) and the initial holding of capital, $k_i(0)$. Note that when selecting her optimal consumption plan, she takes the sequences of external effects, $\{C_1(t), C_2(t)\}_{t=0}^{\infty}$, as given. Letting the implicit price of capital k_i be q_i, the optimization conditions include

$$U_1^i(c_i, C_i, C_j) = q_i, \quad i, j = 1, 2, \ i \neq j, \tag{4}$$

$$\dot{q}_i = q_i(\rho - r), \ i = 1, 2 \tag{5}$$

along with the transversality condition, $\lim_{t\to\infty} e^{-\rho t} q_i k_i = 0$.

Remember that households in each group are identical. Thus in equilibrium it holds that $C_i = c_i$ $(i = 1, 2)$ for all $t \geq 0$. Keeping this in mind, from (4) and (5) we derive a set of Euler equations in such a way that

$$\underbrace{\begin{bmatrix} \Omega_1^1/C_1 & \Omega_2^1/C_2 \\ \Omega_2^2/C_1 & \Omega_1^2/C_2 \end{bmatrix}}_{J} \begin{bmatrix} \dot{C}_1 \\ \dot{C}_2 \end{bmatrix} = \begin{bmatrix} r - \rho \\ r - \rho \end{bmatrix}, \tag{6}$$

where

$$\begin{gathered} \Omega_1^i \equiv -\frac{(u_{11}^i(C_i, C_i, C_j) + u_{12}^i(C_i, C_i, C_j))C_i}{u_1^i(C_i, C_i, C_j)} > 0, \\ \Omega_2^i \equiv -\frac{u_{13}^i(C_i, C_i, C_j)C_j}{u_1^i(C_i, C_i, C_j)}, \quad i = 1, 2. \end{gathered} \tag{7}$$

Here, Ω_1^i denotes the elasticity of marginal utility of consumption within the agent's own group, which equals the inverse of an elasticity of intertemporal substitution in private consumption plus social consumption in its own group. This elasticity has a positive value due to condition $(2c)$. Additionally, Ω_2^i is the elasticity of marginal utility with respect to the other group's consumption. The sign of this term depends on how group i agents respond to consumption of group j agents. If agents are conformist to keep up with consumption of the other group's members (so that $u_{13}^i > 0$), then Ω_2^i has a negative sign. On the other hand, if they do not like being similar to consumption behaviors of the other group ($u^i_{13} < 0$), then Ω^i_2 is strictly positive. Note that, in view of $(2f)$, the following conditions hold:

$$\Omega^i_1 + \Omega^i_2 > 0, \qquad i = 1, 2. \tag{8}$$

2.2 Production

The representative firm produces a single good according to a constant-returns-to-scale technology expressed by

$$\bar{Y} = F\left(\bar{K}, N\right).$$

Here, $\bar{Y}$, $\bar{K}$ and N denote the total output, capital and labor, respectively. Using the homogeneity assumption, we write the production function as follows:

$$Y = f\left(K\right),$$

where $Y \equiv \bar{Y}/N$ and $K \equiv \bar{K}/N$. The production function, $f\left(K\right)$, is assumed to be monotonically increasing and strictly concave in capital-labor ratio, K, and fulfills the Inada conditions. The commodity market is assumed to be competitive so that the before-tax rate of return to capital and real wage are respectively determined by

$$r = f'(K), \quad w = f(K) - Kf'(K). \tag{9}$$

For simplicity, we assume that capital does not depreciate.

If we denote the number of agents in group i by N_i $\left(i = 1, 2\right)$, then the full-employment condition for labor and capital are:

$$N_1 + N_2 = N,$$

$$N_1 k_1 + N_2 k_2 = \bar{K}.$$

Letting $\theta_i = N_i/N$, the full-employment conditions are summarized as follows:

$$K = \theta_1 k_1 + \theta_2 k_2, \ , \quad 0 \leq \theta_i \leq 1, \quad \theta_1 + \theta_2 = 1. \tag{10}$$

For notational simplicity, in the following we normalize the total population, N, to one. Thus θ_i represents the mass of agents of type i as well as the population share of that type.

2.3 Dynamic System

First, we rewrite (6) as

$$\begin{bmatrix} \dot{C}_1 \\ \dot{C}_2 \end{bmatrix} = \frac{C_1 C_2}{\Omega_1^1 \Omega_1^2 - \Omega_2^1 \Omega_2^2} \begin{bmatrix} \Omega_1^2 / C_2 & -\Omega_2^1 / C_2 \\ -\Omega_2^2 / C_1 & \Omega_1^1 / C_1 \end{bmatrix} \begin{bmatrix} r - \rho \\ r - \rho \end{bmatrix}. \tag{11}$$

Second, (3) and (9) yield

$$\dot{k}_i = f(K) + (k_i - K) f'(K) - C_i, \quad i = 1, 2. \tag{12}$$

A complete dynamic system of our economy consists of (11) and (12) that describe dynamic motions of $(C_1, C_2,\ k_1, k_2)$.

It is worth noting that summing up the flow budget constraints (12) over all of the households and dividing the both sides by N, we obtain

$$\theta_1 \dot{k}_1 + \theta_2 \dot{k}_2 = f(K) - \theta_1 C_1 - \theta_2 C_2.$$

Thus, in view of (10), we obtain the final-good market equilibrium condition for the entire economy:

$$\dot{K} = f(K) - C,$$

where $C = \theta_1 C_1 + \theta_2 C_2$.

3 Steady-State Equilibrium

3.1 Steady-State Characterization

From (9) and (11) the steady-state level of aggregate capital, K^*, is determined by the modified Golden-Rule condition such that

$$f'(K^*) = \rho, \tag{13}$$

where

$$K^* = \theta_1 k_1^* + \theta_2 k_2^*. \tag{14}$$

Thus, taking account of (12), we see that the steady-state levels of consumption in each group are given by

$$C_i^* = f(K^*) + (k_i^* - K^*) f'(K^*), \quad i = 1, 2. \tag{15}$$

Notice that although the aggregate level of capital and consumption are uniquely determined, the steady-state restriction does not pins down the steady-state levels of individual capital stock, k_i^*. Obviously, the determination of k_i^* needs to specify trajectory starting from a specific set of initial capital stocks $k_1(0)$ and $k_2(0)$.

3.2 Local Determinacy

As shown above, the steady-state levels of k_1 and k_2 are path dependent. Therefore, if equilibrium determinacy holds in our setting, as to every set of k_1^* and k_2^*, we can find a unique converging path to that particular point. Considering this fact, we first specify the steady-state levels of capital holding of each type of agent and then inspect the presence of feasible set of initial distribution of capital that realizes the selected capital holdings in the steady state.

We now select a particular set of steady-state levels of k_1^* and k_2^* that fulfill (14). Linearizing dynamic equations (11) and (12) at the steady state, we obtain the following approximated system:

$$\begin{bmatrix} \dot{C}_1 \\ \dot{C}_2 \\ \dot{k}_1 \\ \dot{k}_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & \frac{C_1^*(\Omega_1^2 - \Omega_2^1)\theta_1 f''(K^*)}{\Omega_1^1\Omega_1^2 - \Omega_2^1\Omega_2^2} & \frac{C_1^*(\Omega_1^2 - \Omega_2^1)\theta_2 f''(K^*)}{\Omega_1^1\Omega_1^2 - \Omega_2^1\Omega_2^2} \\ 0 & 0 & \frac{C_2^*(\Omega_1^1 - \Omega_2^2)\theta_1 f''(K^*)}{\Omega_1^1\Omega_1^2 - \Omega_2^1\Omega_2^2} & \frac{C_2^*(\Omega_1^1 - \Omega_2^2)\theta_2 f''(K^*)}{\Omega_1^1\Omega_1^2 - \Omega_2^1\Omega_2^2} \\ -1 & 0 & f'(K^*) + (k_1^* - K^*) f''(K^*)\theta_1 & (k_1^* - K^*) f''(K^*)\theta_2 \\ 0 & -1 & (k_2^* - K^*) f''(K^*)\theta_1 & f'(K^*) + (k_2^* - K^*) f''(K^*)\theta_2 \end{bmatrix} \begin{bmatrix} C_1 - C_1^* \\ C_2 - C_2^* \\ k_1 - k_1^* \\ k_2 - k_2^* \end{bmatrix}. \tag{16}$$

Letting the eigenvalues of the coefficient matrix in (16) be λ_j $(j = 1, 2, 3, 4)$, we find the following:

Lemma 1. *The eigenvalues of the coefficient matrix of* (16) *are given by*

$$\lambda_j = \frac{f'(K^*) \pm \sqrt{f'(K^*)^2 - \frac{4f''(K^*)(C_1^*(\Omega_1^2 - \Omega_2^1)\theta_1 + C_2^*(\Omega_1^1 - \Omega_2^2)\theta_2)}{\Omega_1^1\Omega_1^2 - \Omega_2^1\Omega_2^2}}}{2}, \ 0, \ f'(K^*). \tag{17}$$

Proof. See Appendix A. □

Since there are two unpredetermined variables, C_1 and C_2, if the number of unstable roots (i.e. roots with positive real parts) are two, then there may exist a unique converging path towards the selected steady state. Lemma 1 shows that one of the eigenvalues is zero, so that we should have one root with a negative real part to establish local determinacy of equilibrium. Note that the presence of zero root implies that the steady-state equilibrium is path dependent: it depends on the initial position from which equilibrium path starts.[3] Consequently, it is sufficient to focus on the following eigen value:

$$\lambda_2 \equiv \frac{f'(K^*) - \sqrt{f'(K^*)^2 - \frac{4f''(K^*)(C_1^*(\Omega_1^2-\Omega_2^1)\theta_1+C_2^*(\Omega_1^1-\Omega_2^2)\theta_2)}{\Omega_1^1\Omega_1^2-\Omega_2^1\Omega_2^2}}}{2}.$$

If this has a negative value, we may establish local determinacy. For notational simplicity, in the following we denote $\lambda_2 = \lambda$.

To guarantee that λ has a negative value, we impose the following assumptions.[4]

Assumption 1. *Assume that* $\Omega_1^1\Omega_1^2 - \Omega_2^1\Omega_2^2 > 0$ *and* $C_1^*(\Omega_1^2 - \Omega_2^1)\theta_1 + C_2^*(\Omega_1^1 - \Omega_2^2)\theta_2 > 0$.

Given Assumption 1, the solution of the linearized system along the stable path can be written as

$$k_1(t) = k_1^* + A_1 e^{\lambda t} + A_2, \tag{18a}$$

$$k_2(t) = k_2^* + A_1\gamma_{21} e^{\lambda t} - \frac{\theta_1}{\theta_2}A_2, \tag{18b}$$

$$C_1(t) = C_1^* + A_1\gamma_{31} e^{\lambda t} + \frac{A_2}{f'(K^*)}, \tag{18c}$$

$$C_2(t) = C_2^* + A_1\gamma_{41} e^{\lambda t} - \frac{\theta_1}{\theta_2 f'(K^*)}A_2, \tag{18d}$$

where A_1 and A_2 are undetermined constants and γ_{21}, γ_{31} and γ_{41} are given by the following:[5]

$$\gamma_{21} = \frac{C_2^*(\Omega_1^1 - \Omega_2^2)}{C_1^*(\Omega_1^2 - \Omega_2^1)}, \tag{19a}$$

[3] See Li et al (2003).

[4] If $\Omega_1^1 < \Omega_2^2$ and $\Omega_1^2 < \Omega_2^1$, then the sign of λ is negative. We will mention this case in section 5.

[5] See Appendix B for the derivation.

$$\gamma_{31} = \frac{\lambda - f'(K^*) - (k_2^* - K^*)f''(K^*)\theta_2 + (k_1^* - K^*)f''(K^*)\theta_2 \frac{C_2^*(\Omega_1^1 - \Omega_2^2)}{C_1^*(\Omega_1^2 - \Omega_2^1)}}{-(f'(K^*) - \lambda)^2}, \tag{19b}$$

$$\gamma_{41} = \frac{(k_2^* - K^*)f''(K^*)\theta_1 + \{\lambda - f'(K^*) - (k_1^* - K^*)f''(K^*)\theta_1\} \frac{C_2^*(\Omega_1^1 - \Omega_2^2)}{C_1^*(\Omega_1^2 - \Omega_2^1)}}{-(f'(K^*) - \lambda)^2}. \tag{19c}$$

Furthermore, in view of (18a) and (18b), we can specify A_1 and A_2 as

$$A_1 = \frac{\frac{\theta_1}{\theta_2}(k_1(0) - k_1^*) + (k_2(0) - k_2^*)}{\frac{C_2^*(\Omega_1^1 - \Omega_2^2)}{C_1^*(\Omega_1^2 - \Omega_2^1)} + \frac{\theta_1}{\theta_2}}, \tag{20a}$$

$$A_2 = \frac{\frac{C_2^*(\Omega_1^1 - \Omega_2^2)}{C_1^*(\Omega_1^2 - \Omega_2^1)}(k_1(0) - k_1^*) - (k_2(0) - k_2^*)}{\frac{C_2^*(\Omega_1^1 - \Omega_2^2)}{C_1^*(\Omega_1^2 - \Omega_2^1)} + \frac{\theta_1}{\theta_2}}, \tag{20b}$$

where $k_1(0)$ and $k_2(0)$ are the initial levels of capital stock.

As a consequence, we can show the following result:

Proposition 1. *The economy converges to the specified steady-state equilibrium if and only if the initial capital holdings $k_1(0)$ and $k_2(0)$ satisfy $A_2 = 0$.*

Proof. When $A_2 = 0$, conditions from (18a) to (18d) shows that $\lim_{t\to\infty} k_i(t) = k_i^*$ and $\lim_{t\to\infty} C_i(t) = C_i^*$. Conversely, $k_i(t)$ converges k_i^* only if $A_2 = 0$. □

4 Wealth Distribution

This section is devoted to explore the relation between consumption externalities and the long-run distribution of wealth. In order to examine distributional dynamics of our economy in an analytically tractable manner, we assume that Ω_i^j $(i, j = 1, 2)$ in (7) are constant parameters.

This condition holds if, for example, the instantaneous utility function is given by the following:

$$u(c_i, C_i, C_j) = \frac{\left(c_i C_i^{\phi_i} C_j^{\eta_i}\right)^{1-\gamma_i} - 1}{1 - \gamma_i}, \quad i, j = 1, 2, \quad i \neq j. \tag{21}$$

where γ_i denotes the inverse of elasticity of intertemporal substitution in felicity, the parameter ϕ_i represents the extent of the intragroup consumption externalities, whereas η_i shows the intensity of intergroup externalities. In this specification, if

$\phi_i(1-\gamma_i) > (<)0$, then individuals' preference shows conformism (anti-conformism) for the average level of consumption in the same group, whereas if $\eta_i(1-\gamma_i) > (<)0$, it indicates conformism (anti-conformism) for the average level of consumption in the different group.[6]

4.1 Consumption Externalities and Capital Accumulation

The preference parameters $(\Omega_1^1 - \Omega_2^2)$ and $(\Omega_1^2 - \Omega_2^1)$ are critical to determine the direction of capital stock in respective groups towards the steady-state equilibrium. Under the specified utility function (21), $(\Omega_1^1 - \Omega_2^2)$ represents the divergence between the elasticity of intragroup marginal utility of type 1 agent and the elasticity of intergroup marginal utility of type 2 agent. Similarly, $(\Omega_1^2 - \Omega_2^1)$ denotes the divergence between the elasticity of intragroup marginal utility of type 2 agent and the elasticity of intergroup marginal utility of type 1 agent.[7] Let us differentiate $(18a)$ and $(18b)$ with respect to time:

$$\frac{dk_1(t)}{dt} = A_1 \lambda e^{\lambda t}, \tag{22a}$$

$$\frac{dk_2(t)}{dt} = A_1 \lambda \gamma_{21} e^{\lambda t}. \tag{22b}$$

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[6] From conditions $(2c)$ and $(2f)$, the following inequalities must be satisfied:

$$\Omega_1^i = \gamma_i - \phi_i(1-\gamma_i) > 0, \quad i = 1,2,$$

$$\Omega_i^i + \Omega_j^i = \gamma_1 - (\phi_1 + \eta_1)(1-\gamma_1) > 0, \quad i,j = 1,2, \quad i \neq j.$$

In addition, in the symmetric steady state where $C_1 = C_2$, condition $(2d)$ requires the following:

$$-\frac{u_{11}^i C}{u_1^i} - \frac{u_{13}^i C}{u_1^i} = \gamma_i - \eta_i(1-\gamma_i) > 0, \quad i = 1,2.$$

[7] From (21) we derive the constant values of $\left(\Omega_1^1 - \Omega_2^2\right)$ and $\left(\Omega_1^2 - \Omega_2^1\right)$ as follows:

$$\Omega_1^1 - \Omega_2^2 = \gamma_1 - \phi_1(1-\gamma_1) + \eta_2(1-\gamma_2),$$

$$\Omega_1^2 - \Omega_2^1 = \gamma_2 - \phi_2(1-\gamma_2) + \eta_1(1-\gamma_1).$$

From $(22a)$ and $(22b)$, it can be easily confirmed that whether or not the direction of capital stock held by an agent in group 2 is the same with that in group 1 is determined by the sign of γ_{21} alone. If there do not exist the intergroup consumption externalities, both signs of Ω_1^2 and Ω_1^1 are positive, implying that the capital stocks in each group converging each steady-state equilibrium move in the same direction as time goes. More interestingly, when there exist the intergroup consumption externalities so that either sign of $(\Omega_1^2 - \Omega_2^1)$ or $(\Omega_1^1 - \Omega_2^2)$ is negative, the sign of γ_{21} is negative. Hence, from $(22a)$ and $(22b)$ it is confirmed that the capital stock held by group 1 moves in the opposite direction to that in group 2.

Notice that when both signs of $(\Omega_1^1 - \Omega_2^2)$ and $(\Omega_1^2 - \Omega_2^1)$ are negative, assumption 1 does not hold. We shall discuss this case again in the next section.

In sum, we have shown:

Proposition 2. *(i) If* $(\Omega_1^1 - \Omega_2^2)(\Omega_1^2 - \Omega_2^1) > 0$, *then* $(k_1(t) - k_1^*)(k_2(t) - k_2^*) > 0$ *for all* $t \geq 0$.
(ii) If $(\Omega_1^1 - \Omega_2^2)(\Omega_1^2 - \Omega_2^1) < 0$, *then* $(k_1(t) - k_1^*)(k_2(t) - k_2^*) < 0$ *for all* $t \geq 0$.

When we specify the initial level of aggregate capital, we may present a more detailed discussion on the behaviors of the average capital of each group.

Proposition 3. *Suppose that* $K(0) < K^*(= f'^{-1}(\rho))$. *Then we obtain:*
(i) If $(\Omega_1^1 - \Omega_2^2)(\Omega_1^2 - \Omega_2^1) > 0$, *then* $\frac{dk_1(t)}{dt} > 0$ *and* $\frac{dk_2(t)}{dt} > 0$.
(ii) If $\Omega_1^1 < \Omega_2^2$ *and* $\Omega_1^2 > \Omega_2^1$ *(resp.* $\Omega_1^1 > \Omega_2^2$ *and* $\Omega_1^2 < \Omega_2^1$*), then* $\frac{dk_1(t)}{dt} > 0$ *and* $\frac{dk_2(t)}{dt} < 0$ $\left(\textit{resp. } \frac{dk_1(t)}{dt} < 0 \textit{ and } \frac{dk_2(t)}{dt} > 0\right)$.
When $K(0) > K^*$, *we obtain the following:*
(i) If $(\Omega_1^1 - \Omega_2^2)(\Omega_1^2 - \Omega_2^1) > 0$, *then* $\frac{dk_1(t)}{dt} < 0$ *and* $\frac{dk_2(t)}{dt} < 0$.
(ii) If $\Omega_1^1 < \Omega_2^2$ *and* $\Omega_1^2 > \Omega_2^1$ *(resp.* $\Omega_1^1 > \Omega_2^2$ *and* $\Omega_1^2 < \Omega_2^1$*), then* $\frac{dk_1(t)}{dt} < 0$ *and* $\frac{dk_2(t)}{dt} > 0$ $\left(\textit{resp. } \frac{dk_1(t)}{dt} > 0 \textit{ and } \frac{dk_2(t)}{dt} < 0\right)$.

Proof. The parameter A_1 in $(20a)$ can be rewritten as

$$A_1 = \frac{C_1^*(\Omega_1^2 - \Omega_2^1)(K(0) - K^*)}{\theta_1 C_1^*(\Omega_1^2 - \Omega_2^1) + \theta_2 C_2^*(\Omega_1^1 - \Omega_2^2)}, \tag{23}$$

where the sign of the denominator is positive due to Assumption 1.

Assume that $K(0) < K^*$. In this case, if $\Omega_1^2 > \Omega_2^1$, then $A_1 < 0$ in (23). This means that from (22a), $\frac{dk_1(t)}{dt} > 0$ where we impose $A_2 = 0$. In addition, if $\Omega_1^1 > \Omega_2^2$ so that $\gamma_{21} > 0$, from (22b) $\frac{dk_2(t)}{dt} > 0$. That is, both groups' capital stocks move in the same direction and the levels of capital stock increase in the transition.

If $\Omega_1^2 > \Omega_2^1$ and $\Omega_1^1 < \Omega_2^2$, then $A_1 < 0$ and $\gamma_{21} < 0$. In this case, from (22a) and (22b) we can show that $\frac{dk_1(t)}{dt} > 0$ and $\frac{dk_2(t)}{dt} < 0$. This means that both groups' capital stocks move in the opposite directions.

Making use of the similar procedure, we can obtain the other results.□

4.2 Initial Distribution and the Steady State

As easily confirmed by (18a) − (18d), if the condition $A_2 = 0$ is not satisfied, this economy cannot have a stable path that converges to the steady-state equilibrium. From (20b), $A_2 = 0$ gives a set of the initial capital holding $(k_1(0), k_2(0))$ that achieves the steady-state equilibrium (k_1^*, k_2^*).

We first consider the relationship between the initial level of capital held by each type of household and the steady-state distribution of wealth. Notice that from (13) and (15) the steady-state level of consumption of a group i agents is shown by $C_i^* \equiv w^* + \rho k_i^*$ where $w^* \equiv f(K^*) - K^* f'(K^*) (> 0)$. Since K^* $(= f'^{-1}(\rho))$ is uniquely given, the full employment condition (10) gives a relation between k_1^* and k_2^* in such a way that

$$k_1^* = -\frac{\theta_2}{\theta_1} k_2^* + \frac{1}{\theta_1} f'^{-1}(\rho).$$

Therefore, (20b) shows that condition $A_2 = 0$ can be expressed

$$\Psi(k_2^*) = \Gamma(k_2^*), \tag{24}$$

where

$$\Psi(k_2^*) \equiv \frac{(\Omega_1^1 - \Omega_2^2)(w^* + \rho k_2^*)}{(\Omega_1^2 - \Omega_2^1)(w^* + \rho k_1^*(k_2^*))} \text{ and } \Gamma(k_2^*) \equiv \frac{k_2(0) - k_2^*}{k_1(0) - k_1^*(k_2^*)}.$$

The steady-state level of capital stock held by group 2 agents is determined by (24).

We note that if $k_1^* = k_2^*$ in the steady-state equilibrium, then these levels are given by K^* $(= k_1^* = k_2^*)$. Let us consider the shape of $\Psi(k_2^*)$ for $k_2^* \in \left[0, \frac{K^*}{\theta_2}\right]$ in (24).

Lemma 2. *If* $\frac{\Omega_1^1-\Omega_2^2}{\Omega_1^2-\Omega_2^1}>0$ $\left(\frac{\Omega_1^1-\Omega_2^2}{\Omega_1^2-\Omega_2^1}<0\right)$, *then* $\Psi'(k_2^*)>0$ (<0) *for all* $0<k_2^*<\frac{K^*}{\theta_2}$.

Proof. First, substituting $k_2^*=K^*$, $k_2^*=0$ and $k_2^*=\frac{K^*}{\theta_2}$ into $\Psi(k_2^*)$, notice that $\Psi(K^*)=\frac{\Omega_1^1-\Omega_2^2}{\Omega_1^2-\Omega_2^1}$, $\Psi(0)=\frac{\left(\Omega_1^1-\Omega_2^2\right)w^*}{\left(\Omega_1^2-\Omega_2^1\right)\left(w^*+\rho\frac{K^*}{\theta_1}\right)}$ and $\Psi(\frac{K^*}{\theta_2})=\frac{\left(\Omega_1^1-\Omega_2^2\right)\left(w^*+\rho\frac{K^*}{\theta_2}\right)}{\left(\Omega_1^2-\Omega_2^1\right)w^*}$. Differentiating $\Psi(k_2^*)$ with respect to k_2^*, we can show that

$$\Psi'(k_2^*)=\frac{\Omega_1^1-\Omega_2^2}{\Omega_1^2-\Omega_2^1}\frac{\rho f(K^*)}{\theta_1(w^*+\rho k_1^*)^2}. \tag{25}$$

Thus, we can confirm that $\Psi'(k_2^*)>(<)0$ if $\frac{\Omega_1^1-\Omega_2^2}{\Omega_1^2-\Omega_2^1}>(<)0$. □

4.3 Consumption Externalities and Stationary Distribution

To make our argument clear, this subsection assumes that the initial levels of capital stock in both groups are the same: $k_1(0)=k_2(0)$.[8] If this is the case, (10) shows that $K(0)=k_1(0)=k_2(0)$ where $K(0)\equiv\theta_1k_1(0)+\theta_2k_2(0)$. Hence, our central concern in this subsection is to explore how the presence of consumption externalities affects the long-run wealth distribution between the households who hold the same amount of wealth at the outset. Under this assumption, we focus on the two cases.

Case (i) $\Omega_1^1>\Omega_2^2$ **and** $\Omega_1^2>\Omega_2^1$

In this case, Lemma 2 means that $\Psi(k_2^*)>0$ and $\Psi'(k_2^*)>0$ for all $k_2^*\in\left[0,\frac{K^*}{\theta_2}\right]$. We now consider the shape of $\Gamma(k_2^*)$. Since $\Psi(k_2^*)>0$ for all $k_2^*\in\left[0,\frac{K^*}{\theta_2}\right]$, it needs to hold that $\Gamma(k_2^*)>0$ in the steady-state equilibrium to satisfy equation (24). Considering that $\Gamma(k_2^*)=\frac{K(0)-k_2^*}{K(0)-k_1^*(k_2^*)}>0$, we see that the following two sub-cases hold:

$$\text{Case (i-a)}:\quad k_i^*>K(0)=k_i(0)\ \text{ for } k_2^*\in\left[K(0),\frac{K^*-\theta_1K(0)}{\theta_2}\right],\quad i=1,2$$

$$\text{Case (i-b)}:\quad k_i^*<K(0)=k_i(0)\ \text{ for } k_2^*\in\left[\frac{K^*-\theta_1K(0)}{\theta_2},K(0)\right]\quad i=1,2.$$

It is to be noted that the slope of $\Gamma(k_2^*)$ is given by

$$\Gamma'(k_2^*)=\frac{K^*-K(0)}{\theta_1(k_1(0)-k_1^*)^2}. \tag{26}$$

[8] In the next section, we shall mention the case that $k_1(0)\neq k_2(0)$.

Thus, the slope of $\Gamma(k_2^*)$ is positive in case (i-a), while it is negative in case (i-b).

Figures 1 and 2 depict the graphs of two functions $\Psi(k_2^*)$ and $\Gamma(k_2^*)$. These graphs use the facts that $\Gamma(K^*) = 1(>0)$ and $\Gamma(K(0)) = 0$. Figure 1 assumes that $K^* > K(0)$ so that from (26) $\Gamma(k_2^*)$ has a positive slope, whereas Figure 2 imposes the inequality $K^* < K(0)$ and thus $\Gamma(k_2^*)$ has a negative slope. As depicted in Figures 1 and 2, the graph of $\Gamma(k_2^*)$ always intersects line $\Psi(k_2^*) = 0$ only once, confirming that there always exists one crosspoint in the case that $\Omega_1^1 > \Omega_2^2$ and $\Omega_1^2 > \Omega_2^1$.[9] For instance, Figure 1 shows that point B_1 is above point A_1 where point A_1 is the intersection between $\Gamma(k_2^*)$ and $k_2^* = K^*$, and point B_1 is the intersection between $\Psi(k_2^*)$ and $k_2^* = K^*$. Those imply that the crosspoint E_1 is in the region $k_2^* > K^*$. As a result, from (14) we can conclude that $k_2^* > K^* > k_1^*$. Conversely, if point B_1 is below point A_1, the crosspoint E_1 would be in the region $k_2^* < K^*$ so that $k_2^* < K^* < k_1^*$. In other words, whether point A_1 is above or below point B_1 determines patterns of wealth distribution in the steady state.

Consequently, we obtain the following proposition.

Proposition 4. *When* $K^* > K(0)$ $(i = 1, 2)$*, it holds that*

$k_2^* > K^* > k_1^* > k_1(0) = k_2(0)$ *if* $(\Omega_1^1 - \Omega_2^2) > (\Omega_1^2 - \Omega_2^1)\,(>0)$,

$k_1^* > K^* > k_2^* > k_1(0) = k_2(0)$ *if* $(\Omega_1^2 - \Omega_2^1) > (\Omega_1^1 - \Omega_2^2)\,(>0)$.

When $K^* < K(0)$ $(i = 1, 2)$*, it holds that*

$k_1(0) = k_2(0) > k_1^* > K^* > k_2^*$ *if* $(\Omega_1^1 - \Omega_2^2) > (\Omega_1^2 - \Omega_2^1)\,(>0)$,

$k_1(0) = k_2(0) > k_2^* > K^* > k_1^*$ *if* $(\Omega_1^2 - \Omega_2^1) > (\Omega_1^1 - \Omega_2^2)\,(>0)$.

Proof. To prove the case where $K^* > K(0)$, let us make use of Figure 1.[10] When $\Psi(K^*) > \Gamma(K^*)$ (i.e., $\Omega_1^1 - \Omega_2^2 > \Omega_1^2 - \Omega_2^1$) as shown in Figure 1, the level of group 2's capital stock in the equilibrium E_1 is greater than the level of K^*, implying that $K(0) < k_1^* < K^* < k_2^*$. In contrast, assuming that $\Psi(K^*) < \Gamma(K^*)$ (i.e., $\Omega_1^1 - \Omega_2^2 < \Omega_1^2 - \Omega_2^1$), we can show that $K(0) < k_2^* < K^* < k_1^*$. On the other hand,

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[9]This also applies to the following case that sign $\left(\Omega_1^2 - \Omega_2^1\right) = -$sign $\left(\Omega_1^1 - \Omega_2^2\right)$.

[10]It holds that $\frac{K^* - \theta_1 K(0)}{\theta_2} > K^*$ in case (i-a).

as in Figure 2 we assume that $K^* < K(0)$.[11] This figure shows that $\Psi(K^*) > \Gamma(K^*)$ (i.e., $\Omega_1^1 - \Omega_2^2 > \Omega_1^2 - \Omega_2^1$) so that the level of group 2's capital stock in the equilibrium E_2 is lower than that of K^*. That is, it holds that $k_2^* < K^* < k_1^* < K(0)$. If $\Psi(K^*) < \Gamma(K^*)$ (i.e., $\Omega_1^1 - \Omega_2^2 < \Omega_1^2 - \Omega_2^1$), then $k_1^* < K^* < k_2^* < K(0)$. $\square$

Case (ii): sign $(\Omega_1^2 - \Omega_2^1) = -$sign $(\Omega_1^1 - \Omega_2^2)$

Now suppose that $(\Omega_1^2 - \Omega_2^1)$ and $(\Omega_1^1 - \Omega_2^2)$ have opposite signs, that is, $\frac{\Omega_1^1 - \Omega_2^2}{\Omega_1^2 - \Omega_2^1} < 0$.[12] In this case, from Lemma 2 $\Psi(k_2^*) < 0$ and $\Psi'(k_2^*) < 0$ for all $k_2^* \in [0, \frac{K^*}{\theta_2}]$.

When considering the shape of $\Gamma(k_2^*)$, it is useful to notice the following:

$$\Gamma(0) = \frac{K(0)}{K(0) - \frac{K^*}{\theta_1}}, \tag{27a}$$

$$\Gamma\left(\frac{K^*}{\theta_2}\right) = \frac{K(0) - \frac{K^*}{\theta_2}}{K(0)}, \tag{27b}$$

$$\Psi(0) = \frac{\Omega_1^1 - \Omega_2^2}{\Omega_1^2 - \Omega_2^1} \frac{w^*}{w^* + \rho\frac{K^*}{\theta_1}} (< 0), \tag{27c}$$

$$\Psi\left(\frac{K^*}{\theta_2}\right) = \frac{\Omega_1^1 - \Omega_2^2}{\Omega_1^2 - \Omega_2^1} \frac{w^* + \rho\frac{K^*}{\theta_2}}{w^*} (< 0). \tag{27d}$$

Comparing these values, we can obtain the following four sub-cases: case (ii-a): $\Psi(0) > \Gamma(0)$ and $\Psi\left(\frac{K^*}{\theta_2}\right) < \Gamma\left(\frac{K^*}{\theta_2}\right)$; case (ii-b): $\Psi(0) < \Gamma(0)$ and $\Psi\left(\frac{K^*}{\theta_2}\right) < \Gamma\left(\frac{K^*}{\theta_2}\right)$; case (ii-c): $\Psi(0) > \Gamma(0)$ and $\Psi\left(\frac{K^*}{\theta_2}\right) > \Gamma\left(\frac{K^*}{\theta_2}\right)$; case (ii-d): $\Psi(0) < \Gamma(0)$ and $\Psi\left(\frac{K^*}{\theta_2}\right) > \Gamma\left(\frac{K^*}{\theta_2}\right)$. Furthermore, using $(27a) - (27d)$, we may express these sub-cases as follows:

$$\text{Case (ii-a)}: \frac{\Omega_1^1 - \Omega_2^2}{\Omega_1^2 - \Omega_2^1} \in \left[\frac{w^* + \rho\frac{K^*}{\theta_1}}{w^*}\frac{K(0)}{K(0) - \frac{K^*}{\theta_1}}, \frac{w^*}{w^* + \rho\frac{K^*}{\theta_2}}\frac{K(0) - \frac{K^*}{\theta_2}}{K(0)}\right],$$

$$\text{Case (ii-b)}: \frac{\Omega_1^1 - \Omega_2^2}{\Omega_1^2 - \Omega_2^1} < \min\left[\frac{w^* + \rho\frac{K^*}{\theta_1}}{w^*}\frac{K(0)}{K(0) - \frac{K^*}{\theta_1}}, \frac{w^*}{w^* + \rho\frac{K^*}{\theta_2}}\frac{K(0) - \frac{K^*}{\theta_2}}{K(0)}\right],$$

$$\text{Case (ii-c)}: \frac{\Omega_1^1 - \Omega_2^2}{\Omega_1^2 - \Omega_2^1} > \max\left[\frac{w^* + \rho\frac{K^*}{\theta_1}}{w^*}\frac{K(0)}{K(0) - \frac{K^*}{\theta_1}}, \frac{w^*}{w^* + \rho\frac{K^*}{\theta_2}}\frac{K(0) - \frac{K^*}{\theta_2}}{K(0)}\right],$$

[11] It holds that $\frac{K^* - \theta_1 K(0)}{\theta_2} < K^*$ in case (i-b).

[12] If $C_1^*(\Omega_1^2 - \Omega_2^1)\theta_1 + C_2^*(\Omega_1^1 - \Omega_2^2)\theta_2 < 0$, there does not exist the stable root in this case. Hence, we note that the assumption 1 is imposed.

$$\text{Case (ii-d)}: \frac{\Omega_1^1 - \Omega_2^2}{\Omega_1^2 - \Omega_2^1} \in \left[\frac{w^*}{w^* + \rho \frac{K^*}{\theta_2}} \frac{K(0) - \frac{K^*}{\theta_2}}{K(0)}, \frac{w^* + \rho \frac{K^*}{\theta_1}}{w^*} \frac{K(0)}{K(0) - \frac{K^*}{\theta_1}} \right].$$

Assume that $K^* > K(0)$. This means that from (26) the slope of $\Gamma(k_2^*)$ is positive. Figure 3 depicts case (ii-a). As confirmed in Figure 3, taking account of the positive slope of $\Gamma(k_2^*)$, the number of intersection that satisfies $\Gamma(k_2^*) = \Psi(k_2^*)$ is two in case (ii-a). It is also seen that the number of the intersection in cases (ii-b) and (ii-c) is one. In addition, there is no intersection in case (ii-d). Alternatively, if $K^* < K(0)$ so that the slope of $\Gamma(k_2^*)$ is negative, then the relationship is reversed. That is, we find: there is no intersection in case (ii-a); there are two intersections in case (ii-d), and; there is one intersection in cases (ii-b) and (ii-c).

Taking account of these facts, the relationship between the value of $\frac{\Omega_1^1 - \Omega_2^2}{\Omega_1^2 - \Omega_2^1}$ and wealth distribution can be summarized as follows.

Lemma 3. *When $K^* > K(0)$, it holds that*

Case (ii-a): $k_1^* > K^* > K(0) > k_2^*$ or $k_2^* > K^* > K(0) > k_1^*$;

Case (ii-b): $k_2^* > K^* > K(0) > k_1^*$;

Case (ii-c): $k_1^* > K^* > K(0) > k_2^*$.

On the other hand, when $K^* < K(0)$, it holds that

Case(ii-b): $k_1^* > K(0) > K^* > k_2^*$;

Case (ii-c): $k_2^* > K(0) > K^* > k_1^*$;

Case (ii-d): $k_1^* > K(0) > K^* > k_2^*$ or $k_2^* > K(0) > K^* > k_1^*$.

In all cases in Lemma 3, $\frac{\Omega_1^1 - \Omega_2^2}{\Omega_1^2 - \Omega_2^1}$ is negative; however, it is not manifest whether either of $(\Omega_1^1 - \Omega_2^2)$ or $(\Omega_1^2 - \Omega_2^1)$ is negative. Considering that the signs of $(\Omega_1^1 - \Omega_2^2)$ and $(\Omega_1^2 - \Omega_2^1)$ determine the dynamic behavior of capital stock held by each group, we obtain the following proposition.

Proposition 5. *When $K^* > K(0)$, the following relations are established:*

Case (ii-a): if $\Omega_1^1 < \Omega_2^2$ and $\Omega_1^2 > \Omega_2^1$, then $k_1^* > K^* > K(0) > k_2^*$,

whereas if $\Omega_1^1 > \Omega_2^2$ and $\Omega_1^2 < \Omega_2^1$, then $k_2^* > K^* > K(0) > k_1^*$,

Case (ii-b): if $\Omega_1^1 > \Omega_2^2$ and $\Omega_1^2 < \Omega_2^1$, then $k_2^* > K^* > K(0) > k_1^*$,

whereas if $\Omega_1^1 < \Omega_2^2$ and $\Omega_1^2 > \Omega_2^1$, then there is no converging path towards the *steady state,*

Case (ii-c): if $\Omega_1^1 < \Omega_2^2$ *and* $\Omega_1^2 > \Omega_2^1$, $k_1^* > K^* > K(0) > k_2^*$.

whereas if $\Omega_1^1 > \Omega_2^2$ *and* $\Omega_1^2 < \Omega_2^1$, *then there is no converging path towards the steady state,*

Case (ii-d): there is no converging path towards the steady state,

When $K^* < K(0)$, *the following relations are established:*

Case (ii-a): there is no converging path towards the steady state,

Case (ii-b): if $\Omega_1^1 < \Omega_2^2$ *and* $\Omega_1^2 > \Omega_2^1$, *then* $k_1^* > K(0) > K^* > k_2^*$,

whereas if $\Omega_1^1 > \Omega_2^2$ *and* $\Omega_1^2 < \Omega_2^1$, *then there is no converging path towards the steady state,*

Case (ii-c): if $\Omega_1^1 > \Omega_2^2$ *and* $\Omega_1^2 < \Omega_2^1$, *then* $k_2^* > K(0) > K^* > k_1^*$,

whereas if $\Omega_1^1 < \Omega_2^2$ *and* $\Omega_1^2 > \Omega_2^1$, *then there is no converging path towards the steady state,*

Case (ii-d): if $\Omega_1^1 < \Omega_2^2$ *and* $\Omega_1^2 > \Omega_2^1$, *then* $k_1^* > K(0) > K^* > k_2^*$,

whereas if $\Omega_1^1 > \Omega_2^2$ *and* $\Omega_1^2 < \Omega_2^1$, *then* $k_2^* > K(0) > K^* > k_1^*$.

Proof. First, consider the case that $K^* > K(0)$. For instance, assume that $\Omega_1^1 < \Omega_2^2$ and $\Omega_1^2 > \Omega_2^1$ so that $\frac{\Omega_1^1 - \Omega_2^2}{\Omega_1^2 - \Omega_2^1} < 0$ where we assume that the value of $\frac{\Omega_1^1 - \Omega_2^2}{\Omega_1^2 - \Omega_2^1}$ is in the range in case (ii-a) or case (ii-c). Considering that $\gamma_{21} < 0$ in (19a) and $A_1 < 0$ in (23), from (22a) and (22b) the level of capital stock of group 1 monotonically increases and the level of group 2's capital stock decreases during the transition, implying that there exists a steady-state equilibrium that satisfies $k_1^* > K^* > K(0) > k_2^*$.

Instead, we consider the case that $\Omega_1^1 > \Omega_2^2$ and $\Omega_1^2 < \Omega_2^1$ where we assume that $\frac{\Omega_1^1 - \Omega_2^2}{\Omega_1^2 - \Omega_2^1} < 0$ in case (ii-a) or in case (ii-b). In this case, since the level of capital stock of group 1 decreases and that of capital stock of group 2 increases, there is the steady-state equilibrium with $k_2^* > K^* > K(0) > k_1^*$. Making use of the same procedure, we can show the other results in the case that $K^* < K(0)$.$\square$

From Propositions 4 and 5 we obtain intuitive explanations about the relationship between wealth distribution and consumption externalities. First, we consider the effect of the intragroup consumption externalities on wealth distribution. The intragroup consumption externalities only produce quantitative effects: although the intragroup consumption externalities cause the difference of wealth distribution in the long run, the capital stock held by each group converges to the steady state from the same direction. Assume that there do not exist the intergroup consumption externalities (i.e., $\Omega_2^2 = \Omega_2^1 = 0$) so that there exist $\Omega_1^1(> 0)$ and $\Omega_1^2(> 0)$ alone. From Proposition 3, this case shows that if $K^* > K(0)$ (resp. $K^* < K(0)$), the capital stocks held by both groups monotonically increase (resp. decrease). In addition, for simplicity, we assume that the pure elasticities of the marginal utility of own consumption are the same (i.e., $-\frac{u_{11}^1 C_1}{u_1^1} = -\frac{u_{11}^2 C_2}{u_1^2}$) in both groups, so that the inequality $\Omega_1^1 > (<)\Omega_1^2$ means $\frac{u_{12}^1 C_1^*}{u_1^1} < (>)\frac{u_{12}^2 C_2^*}{u_1^2}$. We now consider the case that $K^* > K(0)$, meaning that $f'(K(t)) > \rho$ for all t. For instance, if individuals' preference of group 1 exhibits strong conformism relative to that in group 2 so that $\frac{u_{12}^1 C_1^*}{u_1^1} > \frac{u_{12}^2 C_2^*}{u_1^2}$, we can show that $\frac{\dot{C}_1(t)}{C_1(t)} > \frac{\dot{C}_2(t)}{C_2(t)}(> 0)$. That is, because the agents in group 1 save more than the agents in group 2, the long-run level of capital stock in group 1 is larger than that in group 2, that is, $k_1^* > K^* > k_2^* > K(0) = k_1(0) = k_2(0)$. This result corresponds to the conclusion in Proposition 4. In contrast, if individuals' preference in group 1 shows strong anti-conformism relative to that in group 2 so that $\frac{u_{12}^1 C_1^*}{u_1^1} < \frac{u_{12}^2 C_2^*}{u_1^2}$, we obtain $k_2^* > K^* > k_1^* > K(0) = k_1(0) = k_2(0)$.

Next, suppose that there are intergroup as well as intragroup consumption externalities. We consider the case where $K^* > K(0)$. In this case, the sign of $(\Omega_1^1 - \Omega_2^2)$ or $(\Omega_1^2 - \Omega_2^1)$ can be negative.[13] Then, the presence of the intergroup consumption externalities may yield not only quantitative but also qualitative differences in dynamic behaviors of capital stock held by each group. More specifically, the qualitative impact of consumption externalities is demonstrated by the fact that the dynamic behavior of capital stock held by one group is opposite to that of the other group's capital. For example, assume that $\Omega_1^1 < \Omega_2^2$ and $\Omega_1^2 > \Omega_2^1$. Noting that both Ω_1^1 and Ω_1^2 have positive signs, this case may hold if there is intergroup conformism among agents in group 1 (i.e., $\Omega_2^1 = -\frac{u_{13}^1 C_2}{u_1^1}(< 0)$) so that $\Omega_1^2 > \Omega_2^1$, while if group 2's agents are intergroup anti-conformists (i.e., $\Omega_2^2 = -\frac{u_{13}^2 C_1}{u_1^2}(> 0)$) so that $\Omega_1^1 < \Omega_2^2$. As shown

[13]We shall deal with the case that both signs of $\left(\Omega_1^2 - \Omega_2^1\right)$ and $\left(\Omega_1^1 - \Omega_2^2\right)$ are negative in the next section.

in Proposition 3, when $K^* > K(0)$, the inequalities $\Omega_1^1 < \Omega_2^2$ and $\Omega_1^2 > \Omega_2^1$ show that the level of capital stock held by group 1 monotonically increases and group 2's capital stock monotonically decreases. Furthermore, taking account of the dynamic equations of consumption, we can show that $\frac{\dot{C}_1(t)}{C_1(t)} > 0 > \frac{\dot{C}_2(t)}{C_2(t)}$. In addition, when the condition $A_2 = 0$ is imposed, we can confirm whether there is a stable steady-state equilibrium that depends on the value of $\frac{\Omega_1^1 - \Omega_2^2}{\Omega_1^2 - \Omega_2^1}$. If cases (ii-a) and (ii-c) hold, the steady-state wealth distribution is characterized by $k_1^* > K^* > K(0) > k_2^*$. Furthermore, if the population size is the same in both groups $\theta_1 = \theta_2$, the difference between k_1^* and $k_1(0)(= K(0))$ is larger than that between k_2^* and $k_2(0)(= K(0))$. Namely, wealth inequality will be enhanced during the transition.

Finally, because of the presence of the intergroup consumption externalities, we notice that the steady state equilibrium may not exist if $\frac{\Omega_1^2 - \Omega_2^1}{\Omega_1^1 - \Omega_2^2} < 0$. For example, if $K^* > K(0)$, $\Omega_1^1 < \Omega_2^2$ and $\Omega_1^2 > \Omega_2^1$, then there is no feasible steady state equilibrium in cases (ii-b) and (ii-d). This fact is also a qualitative effect generated by the introduction of heterogeneous consumers.

5 Discussion

5.1 The Euler Equations and Consumption Externalities

We have examined the relationship between the wealth distribution and consumption externalities. In the last section, we found that the levels and the signs of the preference parameters $(\Omega_1^1 - \Omega_2^2)$ and $(\Omega_1^2 - \Omega_2^1)$ are critical to characterize the relationship. In this subsection, making use of the Euler equations, we present intuitive implication of our findings displayed above.

First, note that if intergroup externalities do not exist, the Euler equations for optimal consumption of each group are

$$\frac{\dot{C}_i}{C_i} = \frac{1}{\Omega_1^i}(r - \rho), \quad i = 1, 2$$

where $\frac{1}{\Omega_1^i} = -\frac{1}{\frac{u_{11}^i C_i}{u_1^i} + \frac{u_{12}^i C_i}{u_1^i}}$, which indicates the partial elasticity of intertemporal substitution in consumption from the intragroup perspective. In this case, a higher degree of intragroup conformism, i.e. $u_{12}^i > 0$, increases $1/\Omega_1^i$, which accelerates consumption growth of each group. Therefore, if there is no intergroup consumption externalities, the consumption conformism (resp. anti-conformism) enhances (diminishes) current savings and promotes capital accumulation. Hence, in the absence of intergroup externalities, we may conclude that although wealth distribution affects aggregate dynamics, transitional behavior of our economy is close to that of the representative-agent economy with consumption externalities.

Once we consider the intergroup consumption externalities, we find a much richer set of dynamic behaviors of consumption and capital. The Euler equations are now given by

$$\frac{\dot{C}_i}{C_i} = \frac{\alpha_i}{\hat{\Omega}}\left(r - \rho\right), \quad i = 1, 2$$

where

$$\hat{\Omega} = \Omega_1^1\Omega_1^2 - \Omega_2^1\Omega_2^2,$$

$$\begin{aligned} \alpha_1 &= \Omega_1^2 - \Omega_2^1 = -\left(\frac{u_{11}^2 C_2}{u_1^2} + \frac{u_{12}^2 C_2}{u_1^2}\right) - \left(-\frac{u_{13}^1 C_1}{u_1^1}\right) \\ \alpha_2 &= \Omega_1^1 - \Omega_2^2 = -\left(\frac{u_{11}^1 C_1}{u_1^1} + \frac{u_{12}^1 C_1}{u_1^1}\right) - \left(-\frac{u_{13}^2 C_2}{u_1^2}\right). \end{aligned}$$

Notice that the social marginal utility of own consumption including intragroup external effect is decreasing regardless of the sign of u_{12}^i (i.e., $\Omega_1^1 > 0$ and $\Omega_1^2 > 0$). The preference parameters Ω_2^1 and Ω_2^2 express the partial elasticity of intertemporal substitution in consumption from the intergroup perspective. Note that when the elasticities of substitution from the intragroup perspective dominate those from the intergroup perspective, $\hat{\Omega}$ has a positive value. Conversely, if agents in each group are more sensitive to the consumption of the other group, $\hat{\Omega}$ could be negative.

The elasticity of intertemporal substitution in consumption including the intergroup consumption externalities is given by $\alpha_i/\hat{\Omega}$. Unlike the intragroup consumption externalities alone, when the intergroup consumption externalities are incorporated, we need to consider the effect of the intergroup consumption externalities on the elasticity of intertemporal substitution in consumption through $\hat{\Omega}$ and α_i.

We first consider the effect on $\hat{\Omega}$. For simplicity, we assume that the sign of $\hat{\Omega}$ is positive. In this case, if Ω_2^1 and Ω_2^2 have the same signs and both become larger, then $\hat{\Omega}$ decreases, which is likely to enhance consumption growth in respective groups. In contrast, if agents in one group have conformism with respect to the other group's consumption but the other group's members have intergroup anti-conformism, then $\Omega_2^1 \Omega_2^2 < 0$. Thus when these intergroup external effects are large enough, $\hat{\Omega}$ has a larger value, which will depress consumption growth. As a result, we see that, as long as $\hat{\Omega}$ takes a positive value, the rate of consumption growth of each group is higher when both groups have the same type of sentiment about the other group's consumption behavior. In other words, other things being equal, the homogeneity of preference in the society at large tends to accelerate capital accumulation.

The actual value of consumption adjustment speed, however, depends on the magnitude of α_i as well. Assume that α_i $(i = 1, 2)$ take positive values. Then, we find that the positive sign of u_{13}^i $(i = 1, 2)$ leads to an increase in the growth rate of consumption, while the negative sign of u_{13}^i lowers it. As a result, we can conclude that when the agents in each group are conformists as for group's consumption behavior, consumption growth is enhanced.

The above discussion deals with the case that $\hat{\Omega} > 0$ and $\alpha_i > 0$ so that there is no qualitative effect generated by the intergroup consumption externalities. Namely, when $\alpha_i > 0$ $(i = 1, 2)$ so that $\alpha_i / \hat{\Omega} > 0$, the behaviors of optimal consumption of each group are similar to the case where there is no intergroup externalities; however, if $\alpha_i < 0$ $(i = 1, 2)$ and $\hat{\Omega} > 0$, dynamics of consumption and capital may show explicit qualitative differences. In this case, even if we restrict our attention to the normal case where $\hat{\Omega} > 0$, the consumption (so savings) behavior of each group takes the opposite directions. As an example, suppose that $\alpha_1 < 0$ and $\alpha_2 > 0$. This situation holds, if agents in group 1 has a strong degree of conformism towards the group 2 such that the sign of α_1 is negative. In this case a rise in the real rate of return to capital, r, depresses consumption growth of agents in group 1 and thus their capital formation is lowered. If such an impact is large enough, the steady-state equilibrium could be unstable.[14]

5.2 Alternative Initial Conditions

In Section 3 we restrict our attention to the case where the initial capital holding of each type of agent is identical. This restriction means that $\Gamma(K^*) = 1$ in (24) under which the long-run wealth distribution can be characterized by the signs of $(\Omega_1^2 - \Omega_2^1)$ and $(\Omega_1^1 - \Omega_2^2)$ alone: see Propositions 4 and 5. Here, we briefly discuss the case where $k_1(0) \neq k_2(0)$. Since $\Gamma(K^*) = \frac{k_2(0)-K^*}{k_1(0)-K^*}$, how the wealth is distributed in the long run is determined by the initial holdings of capital stock in respective groups as well as by the values and the signs of $(\Omega_1^1 - \Omega_2^2)$ and $(\Omega_1^2 - \Omega_2^1)$. Alternatively, even if assuming $k_1(0) \neq k_2(0)$, the behavior of capital stock from the initial period towards the steady-state equilibrium can be characterized by the sign of $\frac{\Omega_1^1-\Omega_2^2}{\Omega_1^2-\Omega_2^1}$ alone as confirmed in Propositions 2 and 3. That is, if $\frac{\Omega_1^1-\Omega_2^2}{\Omega_1^2-\Omega_2^1} > 0$, capital stocks of both groups change in the same direction, while if $\frac{\Omega_1^1-\Omega_2^2}{\Omega_1^2-\Omega_2^1} < 0$, the behavior of capital stock in a group is opposite to that of the other groups.

Note that the wealth distribution depends on the initial holdings of capital stock as well as on the preference structures. Let us consider the case that $\Omega_1^1 > \Omega_2^2$ and $\Omega_1^2 > \Omega_2^1$. In addition, assume that the steady-state levels of capital stock in both groups are greater than those of capital stock at initial period $k_i^* > k_i(0)$ and that the steady-state level of capital stock in group 2 is greater than that of group 1. In this case, the condition that determines the wealth distribution can be rewritten as $(\Omega_1^1 - \Omega_2^2) > \frac{\left(\Omega_1^2-\Omega_2^1\right)\left(k_2(0)-k_2^*\right)}{k_1(0)-k_1^*}$. Comparing it with the condition in Proposition 4, we see that even if the preference on consumption in group 2 does not show strong KUJ so that $(\Omega_1^1 - \Omega_2^2) < (\Omega_1^2 - \Omega_2^1)$, the large difference between $k_1(0) - k_1^*$ and the small difference between $k_2(0) - k_2^*$ would produce the long-run wealth distribution in such a way that $k_2^* > K^* > k_1^*$. Furthermore, if the preference shows the strong KUJ in

[14] One may consider the case that if $\hat{\Omega} < 0$ and $\alpha_i < 0$ $(i = 1, 2)$, the economy moves towards the steady-state equilibrium. In this case, the stability of the economy may be indeterminate as shown in Mino and Nakamoto (2008) where the paper incorporates the progressive taxation to guarantee the unique steady state.

group 2 so that the difference of the initial levels of capital stock is not critical for the determination of wealth distribution, we may confirm that individuals in group 2 become rich in the long run relative to the other group although they are poor in the initial period. This means that $k_2(0) < k_1(0) < k_1^* < k_2^*$.

In the case that either of $(\Omega_1^1 - \Omega_2^2)$ and $(\Omega_1^2 - \Omega_2^1)$ is negative, the behavior of capital stock in a group is opposite to that of the other group. Therefore, it may hold that $k_1^* < k_2(0) < k_1(0) < k_2^*$. Individuals in group 1 initially become rich, but their capital stock decreases over time and they finally become poor. In contrast, individuals in group 2 are initially poor, but continuing increase in their capital makes them relatively rich in the steady-state equilibrium.

6 Conclusion

In this paper we have studied how the presence of consumption externalities affects macroeconomic stability and stationary wealth distribution in a neoclassical growth model with heterogeneous households. We have distinguished intragroup externalities form intergroup external effects in order to examine the role of asymmetric external effects among the agents. Our findings reveal that the long-run distribution of wealth is highly sensitive to the strength of conformism (or anti-conformism) as to intragroup as well as to intragroup comparison of consumption levels.

Our central concern is to show that consumption externalities would play a more prominent role in the economy with heterogeneous agents than in the representative-agent counterpart. We thus have focused on the distributional effect of consumption externalities and have not discussed policy implications in detail. Our next task is to study the policy impacts, in particular, the effect of redistribution policies, in our setting.

Appendices

Throughout these appendices, we assume that $\Delta \equiv \Omega_1^1\Omega_1^2 - \Omega_2^1\Omega_2^2$, $\Pi_1 \equiv C_1^*(\Omega_1^2 - \Omega_2^1)$ and $\Pi_2 \equiv C_2^*(\Omega_1^1 - \Omega_2^2)$.

Appendix A.

We derive the eigenvalues in this economy. Given (16), the characteristic equation is

$$\lambda\left\{\lambda^3 - 2f'(K^*)\lambda^2 + \left(f'(K^*)^2 + \frac{f''(K^*)(\Pi_1\theta_1 + \Pi_2\theta_2)}{\Delta}\right)\lambda - \frac{f''(K^*)f'(K^*)(\Pi_1\theta_1 + \Pi_2\theta_2)}{\Delta}\right\} = 0. \tag{A.1}$$

Thus, one of the eigenvalues is zero. Furthermore, the equation $(A.1)$ can be rewritten as

$$\lambda\left(\lambda - f'(K^*)\right)\left(\lambda^2 - f'(K^*) + \frac{f''(K^*)(\Pi_1\theta_1 + \Pi_2\theta_2)}{\Delta}\right) = 0. \tag{A.2}$$

Hence, we can obtain the eigenvalues given by (17).

Appendix B.

The eigen-vectors corresponding to the stable root λ are given by:

$$\begin{bmatrix} -\lambda & 0 & \frac{\Pi_1\theta_1 f''(K^*)}{\Delta} & \frac{\Pi_1\theta_2 f''(K^*)}{\Delta} \\ 0 & -\lambda & \frac{\Pi_2\theta_1 f''(K^*)}{\Delta} & \frac{\Pi_2\theta_2 f''(K^*)}{\Delta} \\ -1 & 0 & f'(K^*) + (k_1^* - K^*)f''(K^*)\theta_1 - \lambda & (k_1^* - K^*)f''(K^*)\theta_2 \\ 0 & -1 & (k_2^* - K^*)f''(K^*)\theta_1 & f'(K^*) + (k_2^* - K^*)f''(K^*)\theta_2 - \lambda \end{bmatrix} \begin{bmatrix} 1 \\ \gamma_{21} \\ \gamma_{31} \\ \gamma_{41} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}. \tag{B.1}$$

Here, we can obtain the following equations:

$$-\lambda + \frac{\Pi_1\theta_1 f''(K^*)}{\Delta}\gamma_{31} + \frac{\Pi_1\theta_2 f''(K^*)}{\Delta}\gamma_{41} = 0, \tag{B.2a}$$

$$-\lambda\gamma_{21} + \frac{\Pi_2\theta_1 f''(K^*)}{\Delta}\gamma_{31} + \frac{\Pi_2\theta_2 f''(K^*)}{\Delta}\gamma_{41} = 0, \tag{B.2b}$$

$$-1 + \left\{f'(K^*) + (k_1^* - K^*)f''(K^*)\theta_1 - \lambda\right\}\gamma_{31} + (k_1^* - K^*)f''(K^*)\theta_2\gamma_{41} = 0, \tag{B.2c}$$

$$-\gamma_{21} + (k_2^* - K^*)f''(K^*)\theta_1\gamma_{31} + \left\{f'(K^*) + (k_2^* - K^*)f''(K^*)\theta_2 - \lambda\right\}\gamma_{41} = 0. \tag{B.2d}$$

First, using $\{(B.2a) \times \Pi_2 - (B.2b) \times \Pi_1\}$, we can show that

$$\lambda\left(\Pi_2 - \gamma_{21}\Pi_1\right) = 0. \tag{B.3}$$

Noting that $\lambda \neq 0$, we obtain:

$$\gamma_{21} = \frac{\Pi_2}{\Pi_1}. \tag{B.4}$$

Furthermore, using $(B.2c)$ and $(B.2d)$, we can derive γ_{31} and γ_{41} where $\gamma_{21} = \frac{\Pi_2}{\Pi_1}$ in $(19b)$ and $(19c)$.

Next, in the case of zero–root, we only substitute $\lambda = 0$ into equations $(B.2a) - (B.2d)$, leading to the following relations:

$$\frac{\Pi_1 \theta_1 f''(K^*)}{\Delta}\gamma_{32} + \frac{\Pi_1 \theta_2 f''(K^*)}{\Delta}\gamma_{42} = 0, \tag{B.5a}$$

$$\frac{\Pi_2 \theta_1 f''(K^*)}{\Delta}\gamma_{32} + \frac{\Pi_2 \theta_2 f''(K^*)}{\Delta}\gamma_{42} = 0, \tag{B.5b}$$

$$-1 + \{f'(K^*) + (k_1^* - K^*) f''(K^*)\theta_1\}\, \gamma_{32} + (k_1^* - K^*) f''(K^*)\theta_2 \gamma_{42} = 0, \tag{B.5c}$$

$$-\gamma_{22} + (k_2^* - K^*) f''(K^*)\theta_1 \gamma_{32} + \{f'(K^*) + (k_2^* - K^*) f''(K^*)\theta_2\}\, \gamma_{42} = 0. \tag{B.5d}$$

From $(B.5a)$ and $(B.5c)$, we can obtain $\gamma_{32} = \frac{1}{f'(K^*)}$ and $\gamma_{42} = -\frac{\theta_1}{\theta_2 f'(K^*)}$, respectively. Substituting these into $(B.5d)$, we derive $\gamma_{22} = -\frac{\theta_1}{\theta_2}$.

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Figure 1: The case (i-a)

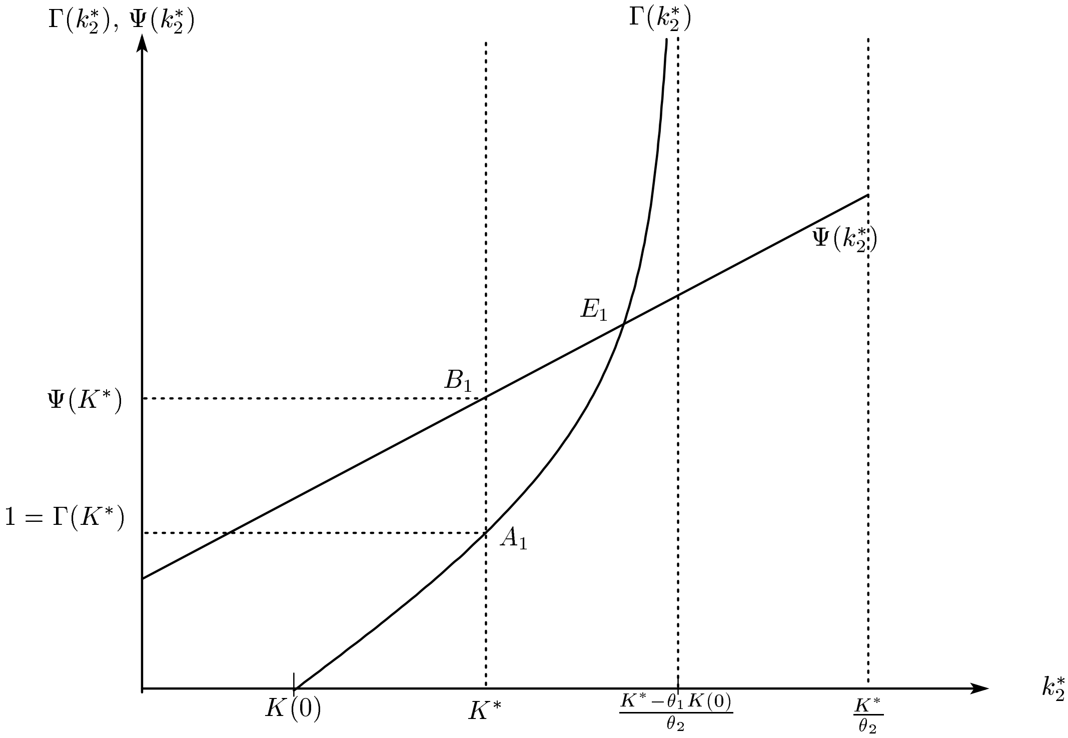

Figure 2: The case (i-b)

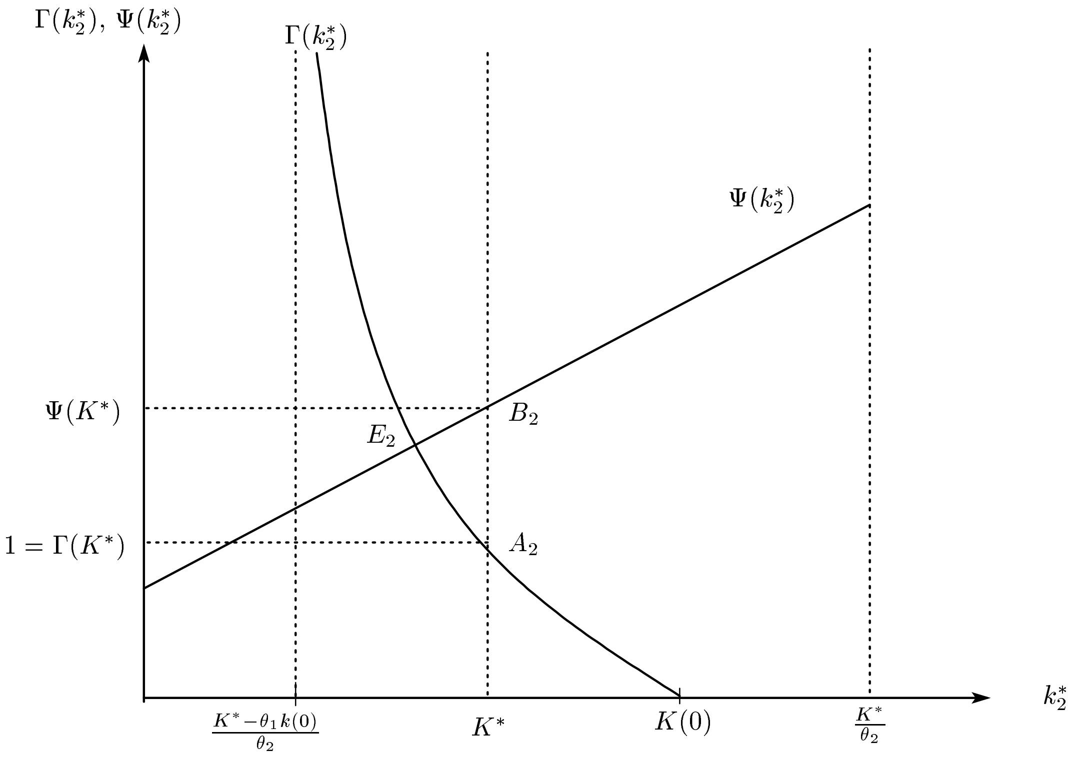

Figure 3: The case (ii-a)