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Internal vs. External Habit Formation in a Growing Economy with Overlapping Generations*

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Abstract

This paper explores the roles of internal and external habit formation in a simple model of endogenous growth with overlapping generations. Unlike the representative agent settings in which the distinction between internal and external habits may not yield significant qualitative differences in working of the model economy, we show that the internal and external habit persistence in overlapping generations economies may have qualitatively different effects on the steady-state characterization as well as on the dynamic behavior of the economy. We also confirm that in an overlapping generations framework, whether the habits in the utility function takes subtractive or multiplicative forms may be critical both for long-run growth and for transitional dynamics.

keywords: internal habits formation, external habit formation, overlapping generations economy, long-run growth

JEL classification: E32, J24, O40

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1 Introduction

When discussing habit persistence of consumption, the literature has distinguished internal (inward-looking) habits from external (outward-looking) habits. The internal habit formation means that the current level of utility is affected by the accumulated stock of past private consumption. Since in this situation the future felicity depends on the choice of current consumption, a rational consumer takes this fact into account in selecting her optimal consumption plan. In contrast, in the presence of external habit formation, the accumulated level average past consumption in the economy at large affects the current utility of an individual consumer. The external habit formation is a form of intertemporal consumption externalities and thus the consumer does not internalize external habits when making her optimal decision.

Using the representative agent models, several authors have examined implications of the distinction between internal and external habit persistence. A sample of this literature includes Carroll, Overland and Weil (1997, 2000) and Harbaugh (1996) on the relation between savings and long-run growth, Gomez (2008) on the speed of convergence, Grishchenko (2007) on asset pricing, and Uribe (2002) on exchange rate determination. Although these studies demonstrate that whether habit formation is internal or external to the consumers may yield significant quantitative differences, it may not change working of a model economy in fundamental ways: the dynamic behavior and steady-state characterization of a model with internal habit persistence are generally the same as those of a model with external habit formation. The main reason for this result is that in representative-agent models the average consumption habit stock of the entire economy equals the private habit stock in equilibrium. Therefore, the internal habit stock always takes the same level as the external habit stock. Such a restriction may contribute to reducing qualitative differences between outcomes generated by external and internal habit formation.

Unlike most of the foregoing studies, in this paper we examine the effects of internal and external habits in the context of an overlapping generations (OLG) economy. In an OLG economy there are heterogeneous cohorts in each period and, hence, the habit stock of consumers in each cohort is no more equal to the average habit stock of the economy. In addition, the presence of heterogeneous agents makes intertemporal consumption externalities more complex than those in the representative-agent economies. Those facts suggest that there may exist significant differences between the roles of internal and external habit formation in an OLG economy. In order to confirm this prediction, we construct a simple growth model with overlapping generations where both internal and external consumption habits are present. As for internal habits, we employ the standard formulation in which the level of consumption in young age affects the consumer's felicity when old. In formulating external habit formation, we follow de la Croix (1996) and de la Croix and Michel (1999) who assume that young agents' consumption behavior is affected by their parents' consumption levels. Such an 'inherited taste' represents intergenerational external effects in consumption, which is a natural expression of external habit formation in an OLG economy.

In this paper we utilize an endogenous growth version of Diamond's (1965) model and introduce two types of habits mentioned above into the base model. Inspecting the model behavior, we obtain two main findings. First, as was predicted, the steady-state characterization and dynamic stability of the economy are highly sensitive to whether habit persistence is internal or external to the agents. In particular, in the presence of external habits persistence, the internal habit formation in consumption may yield more complex effects than in the representative agent economies. Furthermore, stability of the balanced-growth path depends heavily upon the strength of habit persistence and the growth effects of internal habit formation depends on the strength of external habits. Second, the dynamic behavior and balanced-growth equilibrium are also affected by the way how consumption habit stocks are introduced into the felicity function. To see this, we use two of the most popular formulations used in the literature: subtractive and multiplicative forms. If habit formation has a subtractive form in the utility, we generally obtain straightforward conclusions. In contrast, the formulation of multiplicative habits yields rather complex results and many of the outcomes depend on whether the agents are conformists or anti-conformists. As well as the distinction between internal and external habits, in the representative agent economies, the analytical outcomes of subtractive and multiplicative habit formation yield quantitative effects alone.[1] Our study thus demonstrates that the way how consumption habit formation

[1] Using models with internal habit persistence, Wendner (2002 and 2003) and Bossi and Gomis-Porqueras (2009) compare the economic consequences of subtractive and multiplicative forms of consumption habits. Wendner (2002) uses multiperiod model of consumer's optimization problem, while Bossi and Gomis-Porqueras

is formulated plays a more prominent role in an OLG economy than in the representative agent counterpart.

The present paper is closely related to Carroll et al. (1997 and 2000) and Alonso-Carrera et al. (2007). Carroll et al. (1997 and 2000) examine representative agent models of endogenous growth with an Ak technology. They compare the model with internal habits with one with external habits. As mentioned above, they conclude that while the distinction between internal and external habit formation may produce relevant quantitative differences, the equilibrium dynamics and steady state of the model economy are not affected by that distinction. Our discussion clarifies that their findings do not necessarily hold in overlapping generations settings. Alonso-Carrera et al. (2007) construct an overlapping generations model with internal as well as external habit formation. They use an exogenous (neoclassical) growth model and assume the presence of bequest motives. Their central concern is to analyze the effects of habit formation on integenerational wealth transmission.[2] Although the basic model structure of this paper is similar to those examined by Carroll et al. (1997 and 2000) and Alonso-Carrera et al. (2007), our research concern does not overlap these existing studies. Therefore, the present paper is a compliment rather than a substitute to the foregoing contributions.[3]

The rest of the paper is organized as follows. The next section sets up the analytical framework. Section 3 examines a model with subtractive habit formation, while Section 4 reconsiders the base model in the case of multiplicative habits. In both sections, we focus on the properties of balanced-growth equilibrium and equilibrium dynamics out of the balanced-growth path. Section 5 presents some numerical examples and Section 6 concludes.

(2009) uses an exchange economy model with overlapping generations.

[2] Alonso-Carrera et al. (2008) introduce intragenerational wealth heterogeneity into a small country version of their base model and examine the effect of habit formation on the stationary wealth distribution.

[3] As for introducing consumption habit persistence into OLG models, Lahiri and Puhakka (1998) is probably the first study that examines the role of internal consumption habits in an OLG economy. They introduce internal habit formation into the Gale-Samuelson type model of an exchange economy. Bossi and Gomis-Porqueras (2009) extend the discussion of Lahiri and Puhakka (1998). In addition, Bunzel (2006) considers the monetary OLG model with internal habit persistence.

2 Analytical Framework

2.1 Preferences

Consider an overlapping generations economy where there are three types of agents, young, adult and old. When young, the agents neither consume nor work. They only observe their parents' (i.e. adult agents') consumption behavior and 'inherit' their tastes. During adult period, the agents work, consume and save. At the same time, they again observe consumption behaviors of their parents (i.e. old agents) and inherit them.[4] When old, the agents only consume. Population is constant and each cohort consists of a continuum of identical agents with a unit mass. In the following, the t-th generation means the cohort born at the beginning of period $t-1$.

The life-time utility function of an agent in the t-th generation is given by

$$U_t = u\left(c_t, h_t\right) + \beta u\left(x_{t+1}, h_{t+1}\right), \quad 0 < \beta < 1, \tag{1}$$

where c_t denotes consumption when adult, x_{t+1} is consumption when old, and β is a given discount factor. Here, h_t and h_{t+1} represent stocks of consumption habits in period t and $t+1$, respectively. For analytical clarity, the following discussion employs specific forms of utility functions. More specifically, we consider alternative forms of popular specifications: subtractive habits and multiplicative habits. When assuming the preference structure with a subtractive form of habit formation, we use the following function:

$$U_t = \frac{\left(c_t - \theta_0 h_t\right)^{1-\sigma}}{1-\sigma} + \beta \frac{\left(x_{t+1} - \theta_1 h_{t+1}\right)^{1-\sigma}}{1-\sigma}, \quad \theta_0,\ \theta_1,\ \sigma > 0. \tag{2}$$

Ever since Ryder and Heal (1973), this form of utility function including habit persistence has been frequently employed in the literature. Similarly, the felicity function involving multiplicative habits is formulated as

$$U_t = \frac{\left(c_t h_t^{-\theta_0}\right)^{1-\sigma}}{1-\sigma} + \beta \frac{\left(x_{t+1} h_{t+1}^{-\theta_1}\right)^{1-\sigma}}{1-\sigma}, \quad 0 < \theta_0,\ \theta_1 < 1, \quad \sigma > 1. \tag{3}$$

[4] The assumption of taste inheritance in the adult age follows de la Croix (1996) and de la Croix and Michel (1999). In this paper we also assume that the old agents inherit consumption behavior of their parents when they are old.

We compare the effects of internal and external habit formations by use of alternative functional forms given by (2) and (3) .[5]

In general, h_t depends c_{t-1} alone and h_{t+1} is determined by c_t and x_t. Since we have assumed that young agents do not consume, the stock of consumption habits held by the adult agents does not involve their past consumption. Thus the habit stock held by the adult agent is inherited from their parents' consumption in period $t-1$:

$$h_t = c_{t-1}. \tag{4}$$

We assume that the habit stock held when old, h_{t+1}, increases with c_t and x_t :

$$h_{t+1} = h\left(c_t, x_t\right). \tag{5}$$

Namely, the habit stock held in the old period consists of the old-age consumption of their parents as well as their own consumption in the previous period.[6] In what follows, when we deal with the subtractive form of habit persistence, we specify $h\left(c_t, x_t\right)$ in such a way that

$$h\left(c_t, x_t\right) = x_t + \frac{\eta}{\theta_1} c_t, \quad \eta > 0. \tag{6}$$

When we discuss the model with multiplicative habits, we use

$$h\left(c_t, x_t\right) = x_t^{\varepsilon_1} c_t^{\varepsilon_2}, \quad 0 \leq \varepsilon_1, \quad \varepsilon_2 \leq 1. \tag{7}$$

It is to be noted that, as we will see in Footnote 11 in Section 4, if σ is less than one, then the felicity function with (7) cannot yield an interior solution for the household's selection of optimal consumption. To avoid obtaining this result, we use a linear function similar to (6) when we consider the case of multiplicative habits when $0 < \sigma < 1$.[7]

[5] In the recent literature, Alonso-Carrera et al. (2005), Alverz-Cuadrado et al. (2007), Bunzel (2006) and Carroll et al. (1997, 2000) use the multiplicative forms of habit persistence, whereas de la Croix (1996, 2001), Lahiri and Puhakka (1998), Wendner (2002, 2003) employ the subtractive form of habit formation.

[6] More generally, we may assume that h_{t+1} also depends on the average consumption in the economy at large when agents are young, that is, $h_{t+1} = h\left(c_t, \bar{c}_t, x_t\right)$, where $\bar{c}_t$ denotes the average consumption when young. In this formulation, $\bar{c}_t$ expresses the intertemporal as well as intragenerational consumption externalities. Since introduction of $\bar{c}_t$ effect adds a purely quantitative effects on the model behavior, the following discussion does not consider $\bar{c}_t$.

[7] See Section 4.3.

To sum up, external habits are expressed by c_{t-1} in (4) and x_t in (5), which also represent intertemporal consumption externalities. In contrast, c_t in (5) means the internal habit formation perceived by the adult agents when deciding their optimal consumption-saving plan.[8]

2.2 Consumption and Saving

An adult agent supplies one unit of labor inelastically and spends its wage income for consumption and saving. In the old age, the agent consumes all of her savings. Thus the flow budget constraints for adult and old agents are respectively given by

$$c_t + s_t = w_t, \tag{8}$$

$$x_{t+1} = R_{t+1} s_t, \tag{9}$$

where w_t, s_t and R_{t+1} denote real wage in period t, saving of the adult and the gross rate of return to capital in period $t+1$, respectively. Combining these flow constraints, we obtain the life-time budget constraint:

$$c_t + \frac{x_{t+1}}{R_{t+1}} = w_t. \tag{10}$$

In the beginning of period t, an agent in cohort born in period $t-1$ maximizes U_t given by (1) subject to (10), (4) and (5). Considering that c_{t-1} and x_t are external to the t-th generation, we manipulate the first-order conditions for an optimum to obtain the following:

$$\frac{u_1(c_t, h_t)}{u_1(x_{t+1}, h_{t+1})} + \beta \frac{u_2(x_{t+1}, h_{t+1})(\partial h_{t+1}/\partial c_t)}{u_1(x_{t+1}, h_{t+1})} = \beta R_{t+1}. \tag{11}$$

[8] In this paper we focus on intertemporal consumption externalities from parents to children. If we consider intratemporal external effects as well, the utility function may be set as

$$U_t = u(c_t, h_t, e_t) + \beta u(x_{t+1}, h_t, e_{t+1})$$

where e_t and e_{t+1} represent intratemporal consumption external effects determined by

$$e_t = e(\bar{c}_t, \bar{x}_t), \quad e_{t+1} = e(\bar{x}_{t+1}, \bar{c}_{t+1}).$$

Here, a variable with an upper bar means the average level of that variable. This formulation states that the felicity of an agent is also affected by the current level of average consumption in the economy at large. Note that in this formulation the felicity of an old agent depends on the average consumption of the next generation, $\bar{c}_{t+1}$, which may change the dynamic properties of the model economy: see Mino (2007 and 2008) for dynamic analyses of OLG models with intratemporal consumption externalities.

In the presence of internal habit formation, a substitution between consumption when adult, c_t, and consumption when old, x_{t+1}, has two effects. One is the direct intertemporal substitution between current and future consumption, which is represented by the first term in the left-hand side of (11). Additionally, a change in the current consumption, c_t, alters the level of habit stock in the old age, which affects the felicity when old. The second term in the left-hand side of (11) expresses such an indirect substitution effect caused by the presence of inward-looking consumption habits. The optimal choice between c_t and x_{t+1} thus requires that the sum of the direct and indirect rates of substitution equals the discounted rate of interest in period $t+1$. Alternatively, we may rewrite (11) as

$$\frac{u_1(c_t, h_t)}{u_1(x_{t+1}, h_{t+1})} = \beta \left[R_{t+1} - \frac{u_2(x_{t+1}, h_{t+1})(\partial h_{t+1}/\partial c_t)}{u_1(x_{t+1}, h_{t+1})} \right]. \tag{12}$$

As formulated in (2) and (3), the standard formulation of internal habits assumes that $u_2(x, h) < 0$. One unit decrease in the current consumption, c_t, increases one unit of saving, which yields the real interest, R_{t+1}, plus the return to a reduction in the internal consumption habit stock held in the next period represented by $-[u_2(.)(\partial h_{t+1}/\partial c_t)]/u_1(.)(> 0)$. Thus (12) states that the marginal rate of intertemporal substitution in consumption equals the discounted subjective rate of return to investment that includes a change in the stock of internal consumption habits.

Finally, since only young agents work, capital accumulation is determined by the following condition:

$$k_{t+1} = s_t = w_t - c_t. \tag{13}$$

2.3 Production

To avoid unnecessary complexity, we use one of the simplest models of endogenous growth. The production function is given by

$$y_t = A k_t^{\alpha} \left(\bar{k}_t N_t \right)^{1-\alpha}, \quad A > 0, \quad \alpha < 1,$$

where y_t is aggregate output, k_t is capital and N_t denotes the total labor supply. In addition, $\bar{k}_t$ expresses production externalities generated by capital stock in the sense of Romer (1986). Because of normalization of the number of young agents, we set $N_t = 1$ for all t. In addition

$\bar{k}_t = k_t$ in equilibrium. Therefore, the social production function that internalizes production externalities is written as

$$y_t = Ak_t. \tag{14}$$

If capital depreciates at the rate of $\delta \in [0,1]$, the gross rate of return, R_t, equals $1 + \partial Y_t/\partial k_t - \delta$. We assume that capital completely depreciates within a period $(\delta = 1)$, so that R_t is given by

$$R_t = \alpha A, \tag{15}$$

implying that the gross rate of return to capital stays constant over time.[9] Similarly, the real wage rate equals the private marginal productivity of labor, and hence

$$w_t = (1-\alpha)\, y_t/N_t = (1-\alpha)\, Ak_t. \tag{16}$$

Because of an Ak structure of production, the economy can sustain persistent growth.[10]

3 Subtractive Habits

We first consider the model with a subtractive form of habit formation. Substituting (4) and (6) into (2), we obtain the following utility function:

$$U_t = \frac{(c_t - \theta_0 c_{t-1})^{1-\sigma}}{1-\sigma} + \beta \frac{(x_{t+1} - \theta_1 x_t - \eta c_t)^{1-\sigma}}{1-\sigma}, \quad \theta_0, \ \theta_1, \ \eta > 0. \tag{17}$$

[9] Since one period may be 25 or 30 years in a two-period lived OLG economy, the assumption of complete depreciation of capital would be plausible.

[10] An alternative implication of the Ak modelling is that the production inputs consist of physical and human capitals:

$$y_t = Ak_t^{\alpha} h_t^{1-\alpha},$$

where h_t denotes stock of human capital. Suppose that both types of capital stocks are produced by use of final goods. Then the non-arbitrage condition between holding physical and human capital is given by $\partial y_t/\partial k_t = \partial y_t/\partial h_t$, which yields $\alpha y_t/k_t = (1-\alpha)\, y_t/h_t$ so that $h_t = \frac{1-\alpha}{\alpha} k_t$. Substituting this into the production function, we obtain

$$y_t = A \left(\frac{1-\alpha}{\alpha} \right)^{1-\alpha} k_t.$$

Since this formulation does not assume the presence of external increasing returns, unlike the Romer-type modelling, there is no intratemporal inefficiency in production. Otherwise, both formulations give essentially the same results. See, for example, Chapter 8 in Acemoglu (2009) for the detailed discussion on the Ak growth model with two capital stocks.

Here, θ_0 and θ_1 represent the strength of taste inheritance, while η expresses the degree of internal habit persistence in the old age.[11]

3.1 Dynamic System

Given (17) , the optimal condition (11) is expressed as

$$(c_t - \theta_0 c_{t-1})^{-\sigma} = \beta(R_{t+1} + \eta)(x_{t+1} - \theta_1 x_t - \eta c_t)^{-\sigma}. \tag{18}$$

Using (9) and (13) , we obtain $x_{t+1} = R_{t+1}k_{t+1}$. Thus from (15), it holds that

$$x_{t+1} = \alpha A k_{t+1}. \tag{19}$$

In view of (15) and (19) , the optimization condition (18) can be written as

$$\alpha A k_{t+1} - (\eta + \phi)c_t + \phi\theta_0 c_{t-1} - \theta_1 \alpha A k_t = 0, \tag{20}$$

where

$$\phi \equiv [\beta(\eta + \alpha A)]^{\frac{1}{\sigma}}.$$

In addition, equations (13) and (16) yield:

$$k_{t+1} = (1 - \alpha)Ak_t - c_t. \tag{21}$$

Now define

$$z_{t+1} \equiv \frac{k_{t+1}}{c_t}.$$

Then (21) yields

$$\frac{c_t}{k_t} = \frac{(1-\alpha)A}{1 + z_{t+1}}. \tag{22}$$

By use of the above equation, we find that (20) gives the following difference equation of z_t:

$$z_t = \frac{\phi\theta_0(1 + z_{t+1})}{A\left[(\eta + \phi)(1 - \alpha) + \theta_1\alpha - \alpha\left(A(1-\alpha) - \theta_1\right) z_{t+1}\right]}. \tag{23}$$

Since this equation contains all the equilibrium and optimization conditions, it gives a complete dynamic system of our economy.

[11] As mentioned in Section 1, Alonso-Carrera et al. (2007) study an overlapping generations economy model with internal as well as external habit formation. In our notation, their utility function is

$$U_t = \alpha \log (c_t - \theta c_{t-1}) + \beta \log (x_{t+1} - \eta c_t) + \gamma \log b_t,$$

where b_t denotes bequests. Unlike our formulation, the above setting ignores the taste inheritance in the old age.

3.2 Balanced-Growth Equilibrium

In the steady-state equilibrium, x_t, c_t and k_t grow at a common rate. As a result, z_t $(= k_t/c_{t-1})$ stays constant over time. Letting z^* be the steady-state value of z_t, from (23) we see that z^* satisfies

$$z^* = \frac{\phi\theta_0(1+z^*)}{A[(\eta+\phi)(1-\alpha)+\theta_1\alpha-\alpha\left(A(1-\alpha)-\theta_1\right)z^*]},$$

which gives

$$z^* = \frac{\xi \pm \sqrt{\xi^2 - 4\kappa\phi\theta_0}}{2\kappa}, \tag{24}$$

where

$$\xi \equiv A\left[(\eta+\phi)(1-\alpha)+\theta_1\alpha\right]-\phi\theta_0,$$
$$\kappa \equiv \alpha A\left[A(1-\alpha)-\theta_1\right],$$
$$\phi \equiv \left[\beta(\alpha A+\eta)\right]^{\frac{1}{\sigma}}.$$

To ensure the existence of z^*, we assume that

$$\xi^2 - 4\kappa\phi\theta_0 > 0. \tag{25}$$

Inspecting (23) and the steady-state condition shown above, we may summarize the steady-state characterization in the case of subtractive form of habit formation in the following manner:

Proposition 1 *In the model with a subtractive form of habit formation, (i) if $A(1-\alpha) > \theta_1$, then there exist dual balanced-growth paths: one with a higher growth rate is locally stable and the other with a lower growth rate is locally unstable, and; (ii) if $A(1-\alpha) < \theta_1$, then the balanced-growth path is uniquely given and it is globally unstable.*

Proof. Let us write (23) as

$$z_t = G\left(z_{t+1}\right),$$

where $G\left(z_{t+1}\right)$ is the right-hand side of (23). Note that

$$G'\left(z_{t+1}\right) = \frac{\phi\theta_0(\eta+\phi+\alpha A)(1-\alpha)}{A\left[(\eta+\phi)(1-\alpha)+\theta_1\alpha-\alpha\left(A(1-\alpha)-\theta_1\right)z_{t+1}\right]^2} > 0,$$

and

$$G\left(0\right) = \frac{\phi\theta_0}{A[(\eta+\phi)(1-\alpha)+\theta_1\alpha]} > 0.$$

In addition, it is easy to see that if $A(1-\alpha) > \theta_1$, then $G''(z_{t+1}) > 0$ and $\lim_{z_{t+1}\to\bar{z}} G(z_{t+1}) = +\infty$, where

$$\bar{z} = \frac{(\eta+\phi)(1-\alpha)+\theta_1\alpha}{\alpha[A(1-\alpha)-\theta_1]}.$$

Hence, $G(z_{t+1})$ is monotonically increasing and strictly convex in z_{t+1} so that there are two values of z^* satisfying $z^* = G(z^*)$. Since $G(z_{t+1})$ is a monotonic function, the dynamic equation can be expressed as

$$z_{t+1} = G^{-1}(z_t).$$

Moreover, as Figure 1 shows, we see that

$$\left.\frac{dz_{t+1}}{dz_t}\right|_{z_t=z^*} = \frac{1}{G'(z^*)}$$

is strictly larger (resp. smaller) than 1, when z^* has a relatively small (resp. large) value. Since the initial value of $z_t = k_t/c_{t-1}$ is historically given in the beginning of period t, the above shows that the steady state with a higher z^* is locally stable, while the steady state with a lower z^* is locally unstable. In the case that $A(1-\alpha) < \theta_1$, it is satisfied that $G''(z_{t+1}) < 0$ so that $G(z_{t+1})$ is strictly concave, implying that the steady state with a positive z^* is uniquely given. Since $\left.\frac{dz_{t+1}}{dz_t}\right|_{z_t=z^*} = 1/G'(z^*) < 1$ holds in this case, the unique balanced-growth path is globally unstable: see Figure 2. Finally, notice that (14) and (21) give $y_{t+1}/y_t = k_{t+1}/k_t = (1-\alpha)A - c_t/k_t$. Therefore, the gross growth rate of income on the balanced-growth path is given by

$$g^* = \frac{(1-\alpha)Az^*}{1+z^*}. \tag{26}$$

This equation shows that the balanced-growth rate, g^*, increases with the steady-state level of z^*. Hence, the balanced-growth path with a higher value of z^* sustains higher growth of income and capital. ■

In what follows, we focus on the stable balanced-growth path obtained under the condition of $A(1-\alpha) > \theta_1$. More specifically, we examine how the strength of consumption habits affects the long-term growth rate of the economy. Since the balanced-growth rate of income monotonically increases with the steady-state value of z_t, we may consider the relation between external and internal habit formation and long-term growth by exploring the effects of changes in parameters, θ_0, θ_1 and η on z^*. Remember that when $A(1-\alpha) > \theta_1$, the steady state satisfying local stability is one with a higher level of z^*. Hence, from (24) we focus on

$$z^* = \frac{\xi + \sqrt{\xi^2 - 4\kappa\phi\theta_0}}{2\kappa}, \tag{27}$$

where $\kappa = \alpha A[A(1-\alpha) - \theta_1] > 0$. In view of definitions of ξ, κ and ϕ given in (24), it is easy to see that

$$\frac{\partial z^*}{\partial \theta_0} < 0, \quad \frac{\partial z^*}{\partial \theta_1} > 0.$$

Since the balanced-growth rate is positively related to z^*, the above results show that a higher degree of taste inheritance in the adult age decelerates growth, while a stronger degree of aspiration for their parents' consumption in the old age accelerates capital formation. Note that a higher aspiration for parents' consumption in adult age raises the marginal utility in that period, so that the adult agents increase their consumption level relative to that in their old age. As a result, savings are reduced and thus capital accumulation is decelerated. In contrast, a rise in the degree of external effect in old age increases the marginal utility of consumption when old. Thus the agents substitute consumption in their adult age with that in their old age and, therefore, savings will increase to yield a higher balanced-growth rate of income.

The effects of a change in the intensity of internal habit formation, η, are more complex than those of θ_0 and θ_1. Since both ξ and ϕ increase with η, the effect of a change on z^* is ambiguous if $A(1-\alpha) - \theta_1 > 0$. If there is no taste inheritance in the adult age (i.e. $\theta_0 = 0$), then z^* monotonically increases with η. In fact, when $\theta_0 = 0$, the steady-state levels of z are $z^* = 0$ and ξ/κ. The interior steady-state value of z is

$$z^* = \frac{\xi}{\kappa} = \frac{\left[\eta + \beta^{\frac{1}{\sigma}}(\alpha A + \eta)^{\frac{1}{\sigma}}\right](1-\alpha) + \theta_1\alpha}{\alpha[A(1-\alpha) - \theta_1]}, \tag{28}$$

which is an increasing function of η. Hence, the possible presence of a negative relation between z^* (so the balanced-growth rate) and the degree of internal habit persistence is attributed to the presence of external habit formation in the adult period.

In sum, we have shown:

Proposition 2 *On the balanced-growth path satisfying local stability, a higher degree of subtractive habit persistence in the adult age lowers the balanced-growth rate, while a stronger* *external habit persistence when old enhances long-term growth. A rise in the intensity of internal habit persistence increases the balanced-growth rate if there is no taste inheritance in the adult age. Otherwise, the growth effect of internal habit persistence depends on the parameter values involved in the model.*

We may obtain more precise interpretation of the comparative statics results in the above proposition. To do so, let us write (11) as

$$\frac{\hat{x}_{t+1}}{\hat{c}_t} = [\beta(\alpha A + \eta)]^{\frac{1}{\sigma}}, \tag{29}$$

where $\hat{c}_t = c_t - \theta_0 c_{t-1}$ and $\hat{x}_{t+1} = x_{t+1} - \theta_1 x_t - \eta c_t$. This equation states that the optimal choice between the habit-adjusted consumption in adult and old ages depends upon the discounted rate of return to capital plus the degree of internal habit persistence. Note that changes in the strength of external habits, θ_0 and θ_1, do not alter the ratio of the habit-adjusted consumption in adult and old ages. Hence, if θ_0 increases under given levels of c_{t-1} and x_t, both c_t and x_{t+1} increase to keep $\hat{x}_{t+1}/\hat{c}_t$ constant. As a result, consumption in both adult and old age rise, so that capital accumulation will decline. When θ_1 becomes higher under a given level of x_t, consumption in the adult age, c_t will decline to keep $\hat{x}_{t+1}/\hat{c}_t$ constant, implying that savings of adult agents increase and capital accumulation is accelerated.

As for the effects of a change in the degree of internal habit persistence, η, it is to be noted that if there is no external habit in the adult age, i.e. $\theta_0 = 0$, (29) becomes

$$\frac{\hat{x}_{t+1}}{\hat{c}_t} = \frac{x_{t+1} - \theta_1 x_t - \eta c_t}{c_t} = [\beta(\alpha A + \eta)]^{\frac{1}{\sigma}}$$

In this case, a rise in η increases the habit-adjusted discounted rate of return, $\beta\,(\alpha A + \eta)$, and the habit adjusted-consumption during the old age, $\hat{x}_{t+1}$, under a given level of c_t. To keep the above condition, c_t must decline so that capital accumulation will be enhanced. In particular, on the balanced-growth path we may rewrite (29)

$$\frac{\hat{x}_{t+1}}{\hat{c}_t} = \left(1 - \frac{\theta_1}{g^*}\right)\alpha A z^* - \eta = [\beta(\alpha A + \eta)]^{\frac{1}{\sigma}}.$$

In deriving the above, we use $x_{t+1}/x_t = g^*$ and $x_{t+1}/c_t = \alpha A k_{t+1}/c_t = \alpha A z^*$. Remember that from (26) g^* is positively related to z^*. Thus a rise in the habit-adjusted rate of return $\beta\,(\alpha A + \eta)$ is associated with a higher rate of balanced-growth.

In contrast, when $\theta_0 > 0$, (29) is given by

$$\frac{\hat{x}_{t+1}}{\hat{c}_t} = \frac{x_{t+1} - \theta_1 x_t - \eta c_t}{c_t - \theta_0 c_{t-1}} = [\beta(\alpha A + \eta)]^{\frac{1}{\sigma}}.$$

On the balanced-growth path this above is written as

$$\frac{\hat{x}_{t+1}}{\hat{c}_t} = \frac{g^*}{g^* - \eta}\left[\left(1 - \frac{\theta_1}{g^*}\right)\alpha A z^* - \eta\right] = [\beta(\alpha A + \eta)]^{\frac{1}{\sigma}},$$

This equation indicates that both g^* and z^* could decrease, when a rise in η increases the rate of marginal substitution between the habit-adjusted consumption $\hat{c}_t$ and $\hat{x}_t$.

If there are internal habits alone $(\theta_0 = \theta_1 = 0)$, then dynamic equation (20) is replaced with $\alpha A k_{t+1} - (\eta + \phi)c_t = 0$, which is rewritten as

$$g_t = \frac{\eta + \phi}{\alpha A}\frac{c_t}{k_t},$$

where $g_t = k_{t+1}/k_t$. Thus, using (21) and (28), we see that if $\theta_0 = \theta_1 = 0$, the gross growth rate of capital is given by

$$g_t = \frac{A(1-\alpha)\left[\eta + (\beta(\eta + \alpha A))^{\frac{1}{\sigma}}\right]}{\alpha A + \eta + (\beta(\eta + \alpha A))^{\frac{1}{\sigma}}}. \tag{30}$$

In this case, the predetermined variable at the beginning of period t is k_t alone, the initial value of $g_t\,(= k_{t+1}/k_t)$ is not fixed. Therefore, the economy without external habits always stays on the balanced-growth path. As (30) shows, a rise η increases g_t, which means that a stronger internal habit persistence produces a higher income growth if there is no external habit formation.

Consequently, we have found:

Proposition 3 *In the case of subtractive habit formation, there exists a unique and stable balanced-growth path if there is external habit persistence alone. If there is only internal habit formation, then the economy always stays on a unique balanced-growth path and a stronger habit persistence increases the balanced-growth rate of the economy.*

4 Multiplicative Habits

In this section we assume that habit stocks are involved in the felicity function as multiplicative form of (3). We first assume that the utility function is given by

$$U_t = \frac{\left(c_t c_{t-1}^{-\theta}\right)^{1-\sigma}}{1-\sigma} + \beta\frac{\left(x_{t+1} c_t^{-\gamma} x_t^{\gamma-\theta}\right)^{1-\sigma}}{1-\sigma}, \quad \sigma > 1, \quad 0 < \gamma < \theta < 1. \tag{31}$$

In the above, θ and $\gamma - \theta$ respectively denote the degrees of external habits in adult and old ages, while γ represents the intensity of internal habit formation in the old age. Since we will focus on the balanced-growth equilibrium in which c_t and x_t grow at a common, constant rate, the felicity in each period should have the same degree of homogeneity. To satisfy this requirement, we assume that habit persistence in the old age, $c_t^{-\gamma} x_t^{\gamma-\theta}$, is homogeneous of degree $-\theta$ in c_t and x_t to keep the homogeneity of U_t function. In addition, we assume that $\sigma > 1$ in order to obtain an interior solution for the household's optimization problem.[12] When we consider the case of $0 < \sigma < 1$ in Section 4.3, we will use an alternative functional form.

4.1 Dynamic System

The representative agent in cohort born in period $t-1$ chooses c_t,and x_{t+1} to maximize the utility (31) subject to (10). The first-order condition for an optimum is given by:

$$\left(\frac{x_{t+1}}{c_t}\right)^{\sigma}\left(\frac{c_t}{c_{t-1}}\right)^{\gamma(1-\sigma)} - \beta\gamma\left(\frac{x_{t+1}}{c_t}\right) z_t{}^{(\gamma-\theta)(1-\sigma)} = \beta\alpha A z_t{}^{(\gamma-\theta)(1-\sigma)}. \tag{32}$$

Note that $c_t/c_{t-1} = (c_t/k_t)(k_t/c_{t+1})$. Thus by use of (19) and (22), we transform (32) into

$$(\alpha A z_{t+1})^{\sigma}\left[\frac{(1-\alpha)A}{1+z_{t+1}}\right]^{\gamma(1-\sigma)} = \beta\alpha A(\gamma z_{t+1}+1) z_t^{-\theta(1-\sigma)},$$

which leads to

$$z_t = \left[\frac{\Omega(\gamma z_{t+1}+1)(z_{t+1}+1)^{\gamma(1-\sigma)}}{z_{t+1}^{\sigma}}\right]^{\frac{1}{\theta(1-\sigma)}}, \tag{33}$$

where

$$\Omega \equiv \frac{\beta\alpha A}{(\alpha A)^{\sigma}\left[(1-\alpha)A\right]^{\gamma(1-\sigma)}}.$$

Dynamic equation (33) presents a complete dynamic system that summarizes the equilibrium dynamics of the model with the multiplicative form of habit formation.

[12] Plugging $c_t = w_t - s_t$ into (31), we obtain

$$U_t = \frac{(w_t - s_t)^{1-\sigma} c_{t-1}^{-\theta(1-\sigma)}}{1-\sigma} + \beta\frac{(w_t - s_t)^{-\gamma(1-\sigma)}\left(x_{t+1}x_t^{\gamma-\theta}\right)^{1-\sigma}}{1-\sigma}$$

Under given levels of c_{t-1} and x_t, the above expression means that $\lim_{s_t \to w_t} U_t = +\infty$, if $0 < \sigma < 1$. Therefore, when σ is less than one, the optimal choice of c_t is a corner solution: $c_t = 0$. See Alonso-Carrera et al. (2005) and Hiraguchi (2008) for the detail.

4.2 Balanced-Growth Equilibrium

Inspecting dynamic equation (33) , we first find the following result:

Proposition 4 *If the utility function is given by (31) in the case of multiplicative habits, then there is a unique balanced-growth equilibrium that satisfies global stability.*

Proof. Let us rewrite (33) as

$$\begin{aligned} z_t &= \left[\frac{\Omega(\gamma z_{t+1}+1)(z_{t+1}+1)^{\gamma(1-\sigma)}}{z_{t+1}^{\sigma}}\right]^{\frac{1}{\theta(1-\sigma)}}, \\ &= \Omega^{\frac{1}{\theta(1-\sigma)}}\left(\gamma+\frac{1}{z_{t+1}}\right)^{\frac{1}{\theta(1-\sigma)}}(z_{t+1}+1)^{\frac{\gamma}{\theta}} z_{t+1}^{\frac{1}{\theta}} \equiv F\left(z_{t+1}\right). \end{aligned} \tag{34}$$

The steady-state value of z_t fulfilling $z^* = F(z^*)$ is given by

$$\frac{(z^*)^{\theta-1}}{(z^*+1)^{\gamma}} = \left[\frac{z^*}{\Omega(\gamma z^*+1)}\right]^{\frac{1}{\sigma-1}}. \tag{35}$$

Considering our assumptions that $0 < \theta < 1$ and $\gamma > 0$, we see that the left-hand side of (35) monotonically decreases with z^* and it takes $+\infty$ when $z^* = 0$, while it is zero when $z^* = +\infty$. Similarly, the right-hand side of (35) is strictly increasing in z^* and it is zero when $z^* = 0$. Therefore, there is a unique, positive value of z that fulfills (35) . To check stability of the balanced- growth path, notice that $F(0) = 0$ and that

$$F'(z_{t+1}) = F(z_{t+1})\left[\frac{1}{\theta(\sigma-1)\left(\gamma z_{t+1}^2+z_{t+1}\right)}+\frac{\gamma}{\theta(z_{t+1}+1)}+\frac{1}{\theta z_{t+1}}\right] > 0.$$

In addition, we see that $\lim_{z_{t+1}\to\infty} F(z_{t+1}) = +\infty$ for $\sigma > 1$. Furthermore, in the steady state we obtain

$$F'(z^*) = \frac{1}{\theta(\sigma-1)(\gamma z^*+1)}+\frac{\gamma z^*}{\theta(z^*+1)}+\frac{1}{\theta} > 1,$$

because $0 < \theta < 1$ and $\sigma > 1$. As a consequence, it holds that

$$0 < \left.\frac{dz_{t+1}}{dz_t}\right|_{z_t=z^*} = \frac{1}{F'(z^*)} < 1,$$

which demonstrates that the unique interior steady state is globally stable. ■

We also confirm that in the case of $\sigma > 1$, if θ increases, the steady-state value of z^* satisfying (35) decreases. Additionally, it is also seen that the effect of a change in γ on z^* is ambiguous. In Section 6 we numerically examine the growth effect of a change in η.

We summarize these comparative statics results as follows:

Proposition 5 *If the utility function is given by* (31) , *a higher degree of external habit formation depresses long-run growth, while the growth effect of a rise in the strength of internal habit persistence depends on the parameter values involved in the model economy.*

It is to be noted that unlike the subtractive form of habit persistence discussed in the previous section, we have imposed restrictions on the parameter specification to sustain the balanced-growth equilibrium. Therefore, a rise in θ increases the habit persistence effect of consumption in the adult age but it decreases the degree of habit persistence in the old age: see the utility function (31) . This means that a rise in γ stimulates consumption when the agents are adult so that capital accumulation is lowered. In contrast, a change in γ affects the degrees of strength of internal and external habit persistence in the old age at the same time. In section 4.3 we use an alternative functional form of utility function that can treat those effects separately.

In order to emphasize the difference between the subtractive and multiplicative forms of habit persistence, it is useful to write condition (12) as

$$\frac{\hat{x}_{t+1}}{\hat{c}_t} = \left[\beta\left(\alpha A + \gamma \frac{x_{t+1}}{c_t}\right)\right]^{\frac{1}{\sigma}} \left(\frac{h_t}{h_{t+1}}\right)^{\frac{\theta}{\sigma}}, \tag{36}$$

where $\hat{c}_t = c_t c_{t-1}^{-\theta}$ and $\hat{x}_{t+1} = x_{t+1} c_t^{-\gamma} x_t^{\gamma-\theta}$ respectively denote the habit-adjusted consumption in the adult and old ages. A key difference between (29) and (36) is that in the case of multiplicative habits, the marginal rate of substitution between habit-adjusted consumption in the adult and old ages is not constant over time. Furthermore, the level of relative habit stocks, h_t/h_{t+1}, also affects the optimal selection between $\hat{c}_t$ and $\hat{x}_{t+1}$. The qualitative as well as quantitative divergence in outcomes between the subtractive and multiplicative formations of consumption habits mainly come from these differences in the first-order condition.

Two special cases deserve to be mentioned. First, suppose that there is no external habit formation in the adult age $(\theta = 0)$. Then (34) becomes

$$\frac{\Omega(\gamma z_{t+1} + 1)(z_{t+1} + 1)^{\gamma(1-\sigma)}}{z_{t+1}^{\sigma}} = 1.$$

This system contains only $z_{t+1} = k_{t+1}/c_t$. Since the value of c_t is not specified at the beginning of t, the economy, again, always stays on the balanced-growth path. In view of the definition of Ω, we find that the steady-state level of z is determined by

$$\frac{1}{z^*(z^*+1)^{\gamma}} = \left[\frac{(\alpha A)^{\sigma}\left[(1-\alpha)A\right]^{\gamma(1-\sigma)} z^*}{\beta\alpha A(\gamma z^*+1)}\right]^{\frac{1}{\sigma-1}}.$$

It is easy to see that, given our assumption that $\sigma > 1$, a rise in γ raises z^* satisfying the above equation. Therefore, as well as in the case of subtractive habits, a larger internal habit persistence enhances long-term growth if there is no external habit formation.

Next, assume that there is no internal habit formation, so that $\gamma = 0$. In this case we can consider the case of $0 < \sigma < 1$, because the household's optimization problem involves an interior solution for all $\sigma > 0$ if there is no internal habit persistence. When $\gamma = 0$, the steady-state value of z is explicitly given by

$$z^* = \left[\beta(\alpha A)^{1-\sigma}\right]^{\frac{1}{\theta(1-\sigma)+\sigma}}, \tag{37}$$

so that the balanced-growth path is uniquely determine. The dynamic equation for this case is reduced to

$$z_t = \left(\beta(\alpha A)^{1-\sigma}\right)^{\frac{1}{\theta(1-\sigma)}} z_{t+1}^{-\frac{\sigma}{\theta(1-\sigma)}} \equiv N(z_{t+1}). \tag{38}$$

Inspecting (38), we find the following. When $\sigma > 1$, it is easy to see that function $N(z_{t+1})$ defined in (38) is monotonically increasing and strictly convex function of z_t: see Figure 3-a. Therefore, the unique balanced-growth equilibrium satisfies global stability. An increase in the degree of external effect, θ, shifts down the graph of $N(z_{t+1})$, so that the steady-state value of z_t increases. Since there is positive relation between the steady-state levels of z_t and the balanced-growth rate of income, a higher θ raises the balanced-growth rate. In contrast, if $\sigma < 1$, as depicted by Figure 3-b, the graph of $N(z_{t+1})$ is monotonically decreasing with $N(0) = +\infty$ and $N(+\infty) = 0$. Thus the steady state is uniquely determined. However, a rise in θ produces a downward shift of the graph of $N(z_{t+1})$ and thus the steady-state level of z decreases, implying that a higher degree of external habit persistence depresses long-term growth if the intertemporal elasticity of felicity is higher than one.

In the steady-state equilibrium of (38) it holds that $\left(\beta(\alpha A)^{1-\sigma}\right)^{\frac{1}{\sigma}}(z^*)^{\theta\left(1-\frac{1}{\sigma}\right)-1} = 1$. we obtain

$$\begin{aligned}\left.\frac{dz_{t+1}}{dz_t}\right|_{z_t=z_*} &= \theta\left(1-\frac{1}{\sigma}\right)\left(\beta(\alpha A)^{1-\sigma}\right)^{\frac{1}{\sigma}} z^{*\theta\left(1-\frac{1}{\sigma}\right)-1} \\ &= \theta\left(1-\frac{1}{\sigma}\right).\end{aligned}$$

Consequently, the dynamics system exhibits local stability around the balanced-growth equilibrium if $-1 < \theta \left(1 - \frac{1}{\sigma}\right)$ so the

$$\frac{\theta}{1+\theta} < \sigma. \tag{39}$$

Otherwise, the dynamic system is locally unstable. As shown above, the balanced-growth path is uniquely determined, the economy satisfies global stability if (39) is met. As a result, if $\theta / (1+\theta) < \sigma < 1$, then the economy cyclically converges to the balanced-growth path.

We may summarize the above argument as the following proposition:

Proposition 6 *If consumption habit formation takes a multiplicative form and if there is no internal habit persistence, then the balanced-growth path is uniquely given and it is globally stable if and only if*

$$\frac{\theta}{1+\theta} < \sigma.$$

In addition, $\sigma > 1$, *a higher degree of external habit formation,* θ, *raises the balanced-growth rate, while it lowers the balanced-growth rate if* $\theta / (1+\theta) < \sigma < 1$.

To interpret these results, notice the following facts:

$$\text{sign}\, \frac{\partial^2 U_t}{\partial c_t \partial c_{t-1}} = \text{sign}\, \frac{\partial^2 U_t}{\partial x_{t+1} \partial x_t} = \text{sign}\, \theta \left(\sigma - 1\right).$$

Thus if $\sigma > 1$, the marginal utility of private consumption increases with her parent's consumption levels, that is, the agents are conformists in the sense that they like being similar to their parents consumption behaviors (keeping up with their parents). In contrast, if $\sigma < 1$, then the marginal utility of private consumption decreases with c_{t-1} and x_t so that the agents are anti-conformist (running away from their parents). The former case promotes consumption (and capital) growth on the balanced-growth path, but in the latter case, the presence of external habits depresses long-term growth. These results are similar to the comparative statics results on the balanced-growth path of a representative agent economy with external consumption habits and an Ak technology: see Carroll et al. (1997 and 2000).

4.3 An Alternative Specification

So far, we have assumed that if there is internal habit persistence, the elasticity of intertemporal substitution in felicity, $1/\sigma$, should be less than unity to yield an interior solution for the agents' optimization problem. To consider the case of $1/\sigma > 1$, we now use the following utility function:

$$U_t = \frac{\left(c_t c_{t-1}^{-\theta}\right)^{1-\sigma}}{1-\sigma} + \beta \frac{\left(x_{t+1} h_{t+1}^{-\theta}\right)^{1-\sigma}}{1-\sigma}, \quad \sigma > 0, \ \sigma \neq 1, \quad 0 < \theta < 1, \tag{40}$$

where

$$h_{t+1} = \varepsilon x_t + \eta c_t, \qquad \varepsilon > 0, \quad \eta > 0.$$

It is easy to see that as long as $\varepsilon > 0$, this function may yield an interior optimum solution for the household's consumption plan even for the case that $0 < \sigma < 1$.

In this case the first-order condition for an optimum is given by

$$\left(\frac{\hat{c}_t}{\hat{x}_{t+1}}\right)^{-\sigma} = \beta \left(\frac{h_t}{h_{t+1}}\right)^{\theta} \left(\alpha A + \eta \frac{\theta x_{t+1}}{\varepsilon x_t + \eta c_t}\right),$$

which yields

$$\left(\frac{c_t}{x_{t+1}}\right)^{-\sigma} \left(\frac{c_{t-1}}{\varepsilon x_t + \eta c_t}\right)^{-\theta(1-\sigma)} - \frac{\beta \theta \eta x_{t+1}}{\varepsilon x_t + \eta c_t} = \beta R. \tag{41}$$

Using $z_t = k_t / c_{t-1}$, $x_t = \alpha A k_t$ and $c_t / k_t = (1-\alpha) A / (1 + z_{t+1})$, we can rewrite (41) in the following way:

$$(\alpha A z_{t+1})^{\sigma} \left(\alpha \varepsilon A z_t + \eta \frac{c_t}{c_{t-1}}\right)^{\theta(1-\sigma)} = \beta R + \frac{\alpha (1-\alpha) A \beta \theta \eta z_{t+1}}{\alpha \varepsilon (1 + z_{t+1}) + \eta (1-\alpha)}.$$

Noting that $c_t / c_{t-1} = z_t (c_t / k_t)$, we find that the above equation gives

$$\begin{aligned} z_t = {} & (\alpha A z_{t+1})^{-\frac{\sigma}{\theta(1-\sigma)}} \left[\beta R + \frac{\alpha (1-\alpha) A \beta \theta \eta z_{t+1}}{\alpha \varepsilon (1 + z_{t+1}) + \eta (1-\alpha)}\right]^{\frac{1}{\theta(1-\sigma)}} \\ & \times \left[\frac{1 + z_{t+1}}{\alpha \varepsilon A (1 + z_{t+1}) + \eta (1-\alpha) A}\right] \equiv H(z_{t+1}). \end{aligned} \tag{42}$$

The steady-state level of z_t satisfies $z = H(z)$. Since $H(.)$ function is rather complex, it is more convenient to express the steady-state condition, $z = H(z)$, for the case of $\sigma > 1$ in the following manner:

$$I(z) = J(z), \tag{43}$$

where

$$I(z) = z^{\frac{\sigma + \theta(1-\sigma)}{\theta(1-\sigma)}} \left[\frac{\alpha \varepsilon A (1 + z) + \eta (1-\alpha) A}{1 + z}\right],$$

$$J(z) = (\alpha A)^{-\frac{\sigma}{\theta(1-\sigma)}} \left[\beta R + \frac{\alpha (1-\alpha) A \beta \theta \eta z}{\alpha \varepsilon (1 + z) + \eta (1-\alpha)}\right]^{\frac{1}{\theta(1-\sigma)}}.$$

When $\sigma > 1$, we see that $I(0) = +\infty$ and $I(+\infty) = 0$ as well as

$$J(0) = (\alpha A)^{-\frac{\sigma}{\theta(1-\sigma)}} (\beta R)^{\frac{1}{\theta(1-\sigma)}} > 0,$$

$$J(+\infty) = (\alpha A)^{-\frac{\sigma}{\theta(1-\sigma)}} \left[\beta R + \frac{1-\alpha}{\varepsilon} A\beta\theta\eta\right]^{\frac{1}{\theta(1-\sigma)}} \quad (< J(0)).$$

Furthermore, both $I(z)$ and $J(z)$ monotonically decreases with z. As a consequence, the relation between these two functions are depicted in Figures 4-a which shows that there is at least one steady-state value of z. It is also seen that a rise in the degree of internal habits, γ shifts the graphs of $I(z)$ up and that of $J(z)$ down. Therefore, a higher degree of internal habits raises the steady-state value of z and, hence, the balanced-growth rate will increase as long as the balanced-growth path is stable.

As for the case where $0 < \sigma < 1$, it is helpful to express the steady-state condition as

$$\begin{aligned} z^{\frac{\sigma+\theta(1-\sigma)}{\theta(1-\sigma)}} &= (\alpha A)^{-\frac{\sigma}{\theta(1-\sigma)}} \left[\beta R + \frac{\alpha(1-\alpha) A\beta\theta\eta z}{\alpha\varepsilon(1+z) + \eta(1-\alpha)}\right]^{\frac{1}{\theta(1-\sigma)}} \\ &\times \left[\frac{1+z}{\alpha\varepsilon(1+z) + \eta(1-\alpha)}\right] \equiv L(z). \end{aligned} \tag{44}$$

The both sides of the above monotonically increases with z and it holds that

$$L(0) = (\alpha A)^{-\frac{\sigma}{\theta(1-\sigma)}} [\beta R]^{\frac{1}{\theta(1-\sigma)}} \left[\frac{1}{\alpha\varepsilon + \eta(1-\alpha)}\right] > 0,$$

$$L(+\infty) = (\alpha A)^{-\frac{\sigma}{\theta(1-\sigma)}} \left[\beta R + \frac{1-\alpha}{\varepsilon} A\beta\theta\eta\right]^{\frac{1}{\theta(1-\sigma)}} \left[\frac{1}{\alpha\varepsilon}\right] \quad (> L(0)).$$

Again, as Figure 4-b demonstrates, there is at least one steady-state level of z. Since in this case a rise in γ may shifts down or up the graph of $L(z)$, its effect on the long-term growth depends on the parameter values.

This example indicates that in the presence of both internal and external habit persistence, not only the growth effect of outward-looking habit formation but also the growth effect of inward-looking habits may critically depend on whether the agents are conformists $(\theta(1-\sigma) < 0)$ or anti-conformists $(\theta(1-\sigma) > 1)$.

5 Numerical Examples

In this section we examine some numerical examples of the basic models. All the examples shown below set the following baseline parameter values:

$$\alpha = 0.3, \quad \beta = 0.5, \quad A = 5.0$$

Following the standard specification, the income share of capital, α, is assumed to be 0.3. Since in our three-period lived OLG model, one period may be considered 25 years, so that the discount factor β is set at 0.5, which means that the annual discount factor is about 0.9726 $\left(\simeq (0.5)^{1/25}\right)$. The magnitude of the total factor productivity, A, is set at 5.0 to make the balanced-growth rates plausible ones.[13]

5.1 Subtractive Habits

First, consider the model with the subtractive form of habit formation. As shown in Proposition 1, the balanced-growth path is unstable if $A(1-\alpha) < \theta_1$. Therefore, we focus on the stable balanced-equilibrium for the case of $A(1-\alpha) > \theta_1$.

Case (i): A change in θ_0

When we consider the growth effect of a change in the strength of taste inheritance in the adult age, we assume:

$$\theta_1 = 0.8, \quad \eta = 0.6, \quad \sigma = 2.0$$

Then we specify the magnitude of θ_0 and calibrate the steady-state value of z^* given in (27). Substitute these values into (26), we obtain the following rates of balanced-growth:

$$\begin{aligned} g^* &= 3.00213 \text{ for } \theta_0 = 0.5, \\ g^* &= 2.98875 \text{ for } \theta_0 = 0.6, \\ g^* &= 2.97412 \text{ for } \theta_0 = 0.7. \end{aligned}$$

Note that we have assumed that one period is about 25 years, the annual growth rates of income are respectively given by 0.04495 (for $\theta_0 = 0.5$), 0.04476 (for $\theta_0 = 0.6$), and 0.04456 (for $\theta_0 = 0.7$). As shown in Proposition 1, a higher degree of taste inheritance in the old age depresses the balanced-growth rate.

If we replace $\sigma = 2.0$ with $\sigma = 0.8$, then the balanced-growth rate respectively becomes

$$\begin{aligned} g^* &= 3.00967 \text{ for } \theta_0 = 0.5, \\ g^* &= 2.99627 \text{ for } \theta_0 = 0.6, \\ g^* &= 2.98165 \text{ for } \theta_0 = 0.7. \end{aligned}$$

[13] It is to be noted that the gross rate of return to capital is $R = \alpha A = 1.5$ in this example. Thus the annual net rate of return is $r = (1.5)^{1/25} - 1 = 0.01635$.

A higher intetemporal elasticity of substitution in felicity yields a higher balanced-growth rate. The growth effect of a change in θ_0 is almost the same as that obtained when $\sigma = 0.2$.

Case (ii): A change in θ_1

As well as in case (i), we conduct calculation of the balanced-growth rate for alternative values of θ_1 and σ. In this example, we set $\theta_0 = 0.5$. The results are summarized in the table below.

	$\sigma = 0.2$	$\sigma = 0.8$
$\theta_1 = 0.5$	2.96158	2.97070
$\theta_1 = 0.6$	2.99629	2.98499
$\theta_1 = 0.7$	$2,98953$	2.99745

These figures indicate that a larger impact of taste inheritance in the old age promotes long-term growth of income.

Case (iii) A change in η

We have seen that when there are external habit persistence, the growth effect of a change in the degree of internal habit persistence, η, is analytically ambiguous. If we assume that $\theta_0 = 0.5$ and $\theta_1 = 0.8$, then the balanced-growth rates are given by the following table:

	$\sigma = 0.2$	$\sigma = 0.8$
$\eta = 0.5$	2.96870	2.97896
$\eta = 0.6$	3.00313	3.00967
$\eta = 0.7$	3.10057	3.12544

Hence, in this example, a rise in the degree of internal habit persistence accelerates growth in the long-run equilibrium.

5.2 Multiplicative Habits

In the case of multiplicative form of habit formation, we still assume that $\alpha = 0.3$, $\beta = 0.5$ and $A = 5.0$. We use (35) to calculate z^* and derive the balanced-growth rate g^* by use of (26). The following examples fix the level of σ at 1.2.

Since the model with multiplicative form of habits contains two parameters concerning the degree of internal and external habit persistence, we calculate the balanced-growth rate,

g^*, for alternative magnitudes of θ and γ

	$\theta = 0.2$	$\theta = 0.5$
$\gamma = 0.2$	1.2841	1.2621
$\gamma = 0.3$	1.3371	1.3126
$\gamma = 0.4$	1.3936	1.3706
$\gamma = 0.5$	1.4518	1.4320

In this table θ takes two values, 0.2 and 0.5, while we change γ from 0.2 to 0.5. As Proposition 5 states, a higher degree of external habit persistence (a higher value of θ) depresses the long-run growth rate of income under a given level of γ. Although analytical inspection cannot specify whether or not a rise in the degree of internal habit, γ, enhances long-run growth, our example gives a positive relation between γ and g^* under a given level of θ.

Comparing with the case of subtractive habits, we see that the balanced-growth rate of the model with multiplicative habits is relatively low. For example, when $\theta = 0.2$, the net annual growth rates of income are: 0.01005 (for $\gamma = 0.2$), 0.0137 (for $\gamma = 0.3$), 0.01451 (for $\gamma = 0.4$) and 0.0150 (for $\gamma = 0.5$). This result simply comes from the model specification and we may obtain higher balanced-growth rates by setting $A = 7.0$ or 8.0.

Finally, consider the special cases. If there is no internal habit formation $(\gamma = 0)$, then the steady-state value of z is given by (37). Using this relation to calculate the balanced-growth rate, we find the following:

	$\sigma = 1.2$	$\sigma = 0.8$
$\theta = 0.2$	1.1867	1.1392
$\theta = 0.3$	1.1776	1.1523
$\theta = 0.4$	1.1681	1.1648
$\theta = 0.5$	1.1583	1.1769

Hence, we see that a rise in the strength of external habit persistence lowers growth if $\sigma > 1$, but it raises the long-run growth rate if $0 < \sigma < 1$.

Next, consider the case where there is no habit inheritance in the adult age, i.e. $\theta = 0$ (but there are external habits in the old age). Since we should assume that $\sigma > 1$. we obtain the following balanced-growth rates under $\sigma = 1.2$:

$$\begin{aligned} g^* &= 1.3002 \quad \text{for } \gamma = 0.2 \\ g^* &= 1.3521 \quad \text{for } \gamma = 0.3, \\ g^* &= 1.4066 \quad \text{for } \gamma = 0.4, \\ g^* &= 1.4638 \quad \text{for } \gamma = 0.5. \end{aligned}$$

In this example, both internal and external habit persistences in the old age increase. As shown above, a higher degree of habits persistence in the old age enhances growth.

6 Conclusion

This paper has examined how the presence of internal as well as external habit formation affects long-run equilibrium and dynamic behavior of an overlapping generations economy with endogenous growth. We have shown that if there is only external habit formation, which represents taste inheritance from parents to children, then it plays the role similar to that in the representative agent economies. Namely, in the presence of envy (aspiration) and conformism, external habit formation may promote long-run growth, while anti-conformism would reduce the long-term growth rate of income. In contrast, internal habit formation in an overlapping generations model with endogenous growth presents more complex effects on the dynamic behavior as well as the steady-state characteristic of the economy. Therefore, when the consumers have both types habits persistence, the distinction between external and internal habit persistence would be more relevant in OLG economies than in the representative agent counterpart where both types of habits generally yield the same type of impact on the economy. Additionally, we have confirmed that the dynamic behavior and steady-state conditions of an OLG economy is highly sensitive to whether habit stocks are introduced into the felicity function in subtractive or multiplicative form. Since functional forms of habit formation generally yields quantitative effects alone in the representative-agent models, our findings have demonstrated that the how the preference structure is formulated would be critical in examining overlapping generations economies with habit persistence in consumption.

In this paper we have focused on the issue how the alternative formulations of habit persistence of consumption may yield different outcomes in an endogenously growing economy. Our next task is to investigate policy implications in growing OLG economies in the presence of alternative forms of habit formation.

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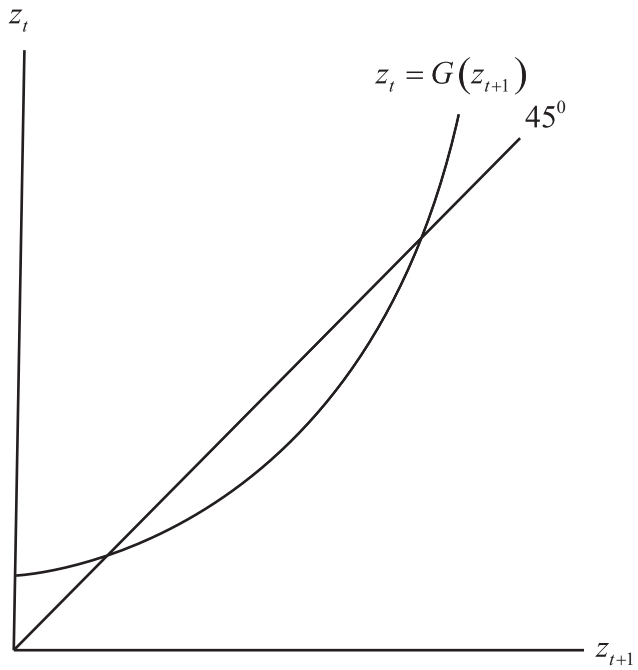

Figure 1-a: $A(1-\alpha) > \theta_1$

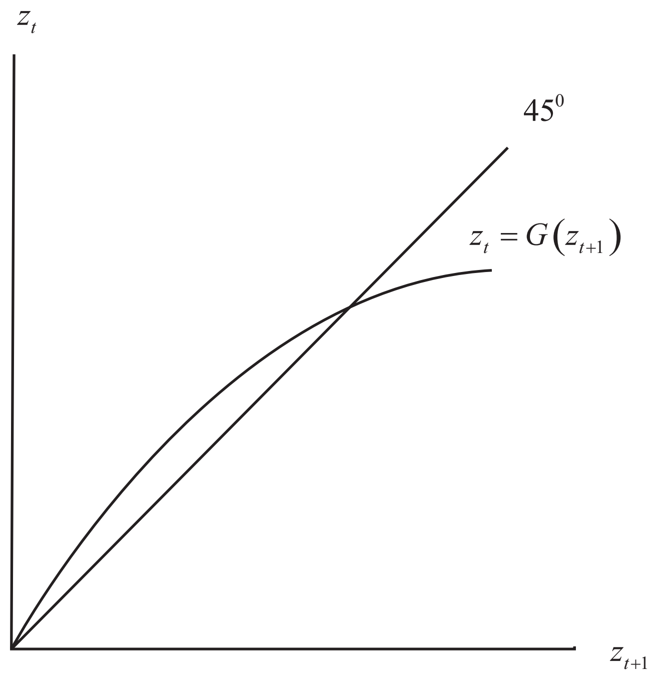

Figure 1-b: $A(1-\alpha) < \theta_1$

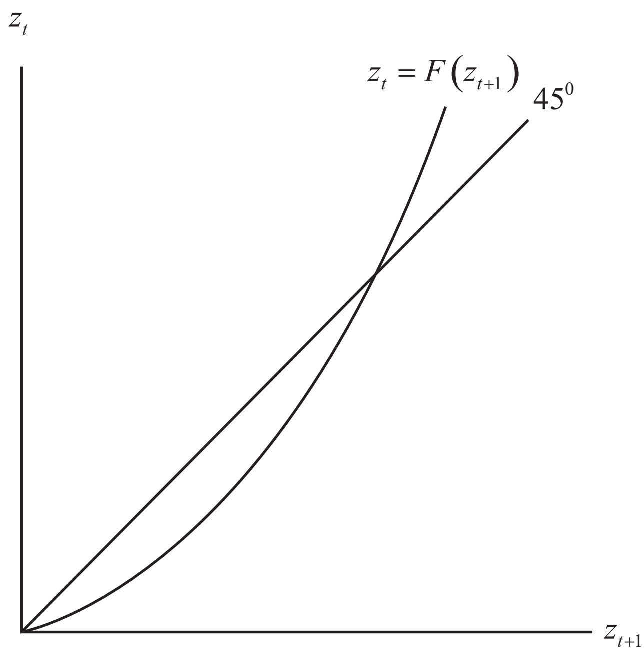

Figure 2

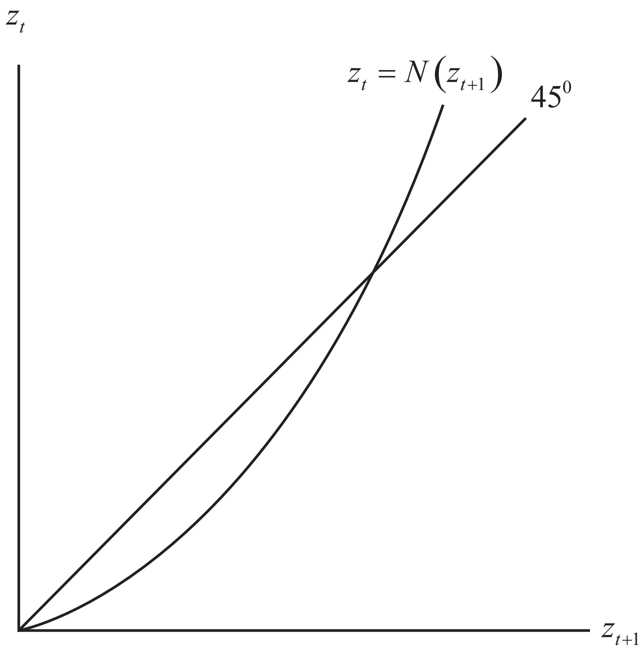

Figure 3-a: $\sigma > 1$

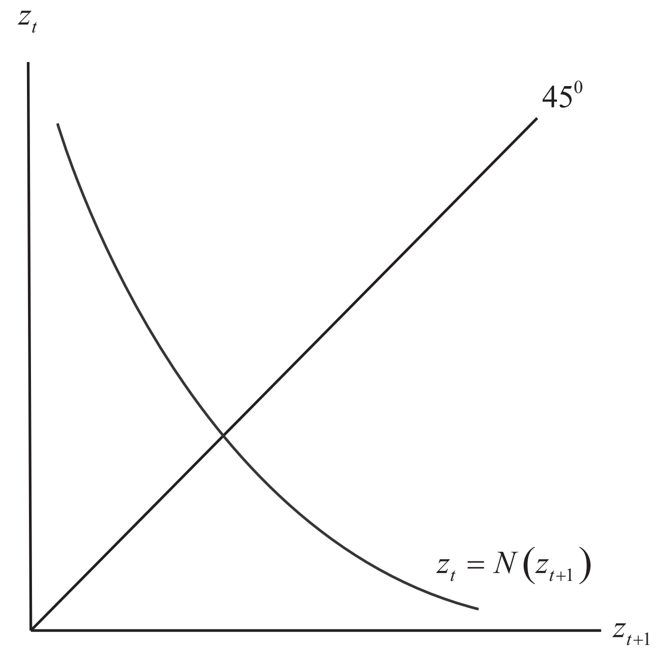

Figure 3-b : $0 < \sigma < 1$

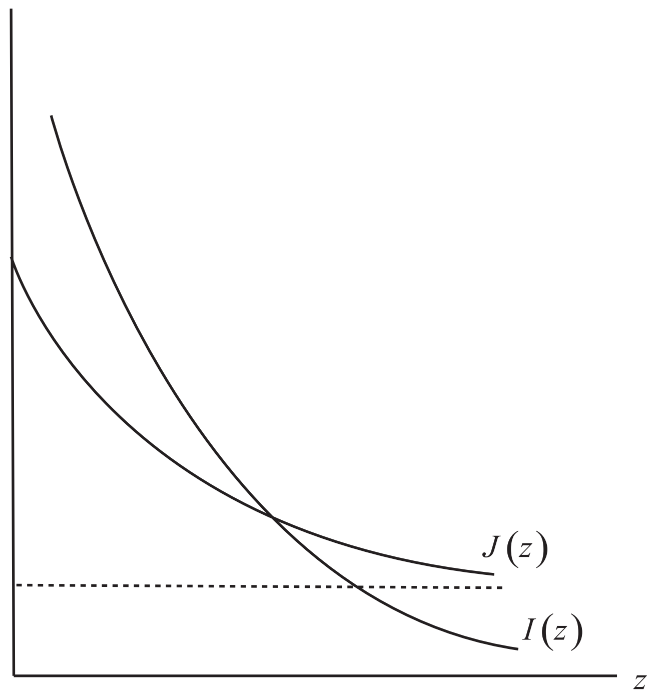

Figure 4-a: $\sigma > 1$

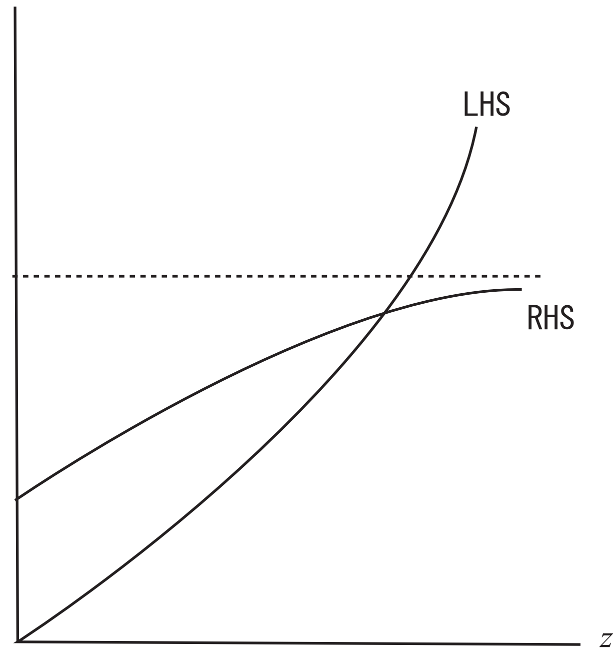

Figure 4-b: $0 < \sigma < 1$