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"Skill-Biased Technical Change and Wage Inequality:
The U.S. versus Europe"

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Skill-Biased Technical Change and Wage Inequality: The U.S. versus Europe[*]

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Abstract

This paper analyzes the effect of the recent technical change on the labor market and explains the observed differences in wage inequality among advanced countries. In particular, we focus on the difference between the wage inequality in the U.S. and continental Europe. By introducing human capital investment into Acemoglu (1999)'s model, we show that *ex ante* homogeneous economies would have distinct *ex post* wage inequality. In addition, we show that the differences in tax or education system can explain the difference in wage inequality between the U.S. and Europe.

Keywords: *skill-biased technical change, wage inequality, human capital investment, matching*
JEL Codes: E24, J24, J31, J64

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1   Introduction

There is a large body of literature that indicates a rise in wage inequality in the U.S. during the past three decades. For instance, Goldin and Katz (1999) show that the time series of wage inequality in the U.S. over the past 100 years forms a "U-shape" and that the present level of inequality is at a historical peak in the postwar period. According to Katz and Autor (1999), various indices such as the 90-10 weekly wage ratio, the standard deviation of log wage, and the Gini coefficient confirm that the rise in wage inequality in the U.S. started in the early 1970s. As indicated by Acemoglu (2002), the rise in wage inequality involved both the rise in the incomes of the workers at the top and the fall in the incomes of those at the bottom.

Blanchard and Wolfers (2000) show that in continental Europe, there is little change in wage inequality.[1] Table 1 shows the trends in earning dispersion among advanced countries, and it is clear that there is a contrast between Europe and the U.S.

The aim of this paper is to analyze the relationship between the recent technical change and the observed differences in income distribution among advanced countries. There is a consensus that technical changes such as the development of information technology increase the relative productivity of skilled workers to unskilled workers.[2] Motivated by these empirical studies, Acemoglu (1999) provides the model in which skill-biased technical change causes a drastic change in the employment structure, which increases the wages of skilled workers and collapses the employment of unskilled workers. The explanation of his model applies suitably to the observations in the U.S. However, the recent technical change has not occured only in the U.S. Why have the European countries not experienced the spread of inequality?

In order to access this question, we extend Acemoglu's model by introducing human capital investment. We assume that both firms and workers must decide their capacity before they enter the labor market.[3] Since search friction prevents them from knowing their partners in advance, there is strategic complementarity between the investment by firms and workers. That is, large investment by firms enhances their incentive to educate their workers and vice versa. Due to the strategic complementarity, there is a possibility that multiple equilibria exist, which explains the difference of

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[1]Machin (1996) only shows that the trend in the U.K. is similar to that in the U.S. The U.K. economy has exhibited a rise in wage inequality that was similar to that in the U.S. economy.

[2]For instance, see Krueger (1993), Autor et al. (1998), and Berman et al. (1998).

[3]In Acemoglu's model, the skill of workers is assumed to be exogenous.

income distribution between the U.S. and Europe.

The conclusions of this paper are as follows. First, like Acemoglu (1999), we find that the development of skill-biased technical change gradually increases the relative wage of skilled workers in the beginning. If the difference in productivity between skilled and unskilled workers becomes sufficiently large, a drastic change in the employment structure occurs and all unskilled workers lose their jobs. Second, contrary to Acemoglu, we find that the occurence of such a change is indeterminate if the difference in productivity is intermediate. Thus, we can explain the difference in income distribution without assuming fundamental heterogeneity. Third, taxation on labor prevents a drastic change. Fourth, the effect of one type education policy on income distribution may be opposite to that of another. While investment in primary education increases the productivity of unskilled workers and prevents a drastic change, scholarships for higher education promote such a change. These results are consistent with the observed features of the U.S. and European economies.

This paper investigates the interaction of technical change with institutional factors in the frictional labor market. Thus it is related to studies such as Ljungqvist and Sargent (1998, 2008), Mortensen and Pissarides (1999), and Hornstein et al. (2007). However, the focus of these studies is on the rise in unemployment in Europe relative to the U. S., whereas this paper focuses on the difference in the change in income distribution.

The remainder of this paper is organized as follows. Section 2 describes the structure of the model. Section 3 solves the model and finds Nash equilibria. We show that there exist two types of equilibria – pooling equilibrium and separating equilibrium. We also show that a skill-biased technical change causes a switch from a pooling equilibrium to a separating equilibrium in the economy. Section 4 considers the differences in labor market institutions. We can attribute the difference in income distribution between the U.S. and Europe to their institutional differences. Section 5 discusses the limitation of the model. Finally, Section 6 summarizes and comments on the conclusions of the paper.

2   The Basic Model

In this section, we describe the framework of the model. The basic structure of the model is the same as that of Acemoglu (1999) with the exception that human capital investment is endogenized in our model. In a labor market, there are continuums of risk-neutral workers and profit-maximizing firms.

Each measure is normalized to one. All firms are *ex ante* homogeneous. Production takes place in one firm-one worker pairs; however, agents do not know their partners in advance due to the existence of search friction. All pairs produce homogeneous consumption goods. The economy lasts for two periods and the timing of events is as follows. First, both workers and firms irreversibly determine their capacities i.e., the amount of "human capital" and "physical capital", respectively. Second, both workers and firms enter the labor market and meet their partners randomly. They are able to observe their partners' capacity when they meet and decide whether or not to form a pair and engage in production.

Although firms must determine the amount of physical capital, k, before they enter the labor market, they need not to incur the associated cost if they do not produce with the worker in the second period. However, if firms produce with the worker in the second period, they must incur a cost, ck.

Workers also must decide whether or not to acquire skill in the first period. We assume that the workers are *ex ante* heterogeneous with respect to the costs of acquiring skill. The continuous function $G(\cdot)$ represents the distribution of cost measured by consumption goods. Let $\tau(\cdot)$ be the inverse of this function, and suppose that it satisfies

$$\tau' > 0, \ \tau(0) = 0, \ \tau(1) = \infty. \tag{1}$$

We can interpret $\tau(\theta)$ as the cost of acquiring skill for the worker for whom the fraction of more capable workers is θ. (1) implies that the most capable worker can acquire skill without incuring a cost and that the least capable worker cannot acqurie skill. We normalize the human capital of unskilled workers to one and denote that of skilled workers as $\eta > 1$. The productivity of a pair comprising the worker with human capital h and the firm with physical capital k is given by

$$y(h,k) = k^{1-\alpha}h^{\alpha}. \tag{2}$$

The firm and the worker determine the wage through Nash bargaining after they agree that they form a pair. Therefore, capital cost ck has already sunk at the stage of wage determination. Let β denote the bargaining power of workers. Then, the wages of skilled workers, $w^H(k)$, and those of unskilled workers, $w^L(k)$, satisfy

$$w^H(k) = \beta k^{1-\alpha}\eta^{\alpha} \quad \text{and} \quad w^L(k) = \beta k^{1-\alpha}.$$

Note that wages depend on the amount of capital of the firm. To simplify

the algebra, we normalize $c = (1 - \beta)$.

The expected profit of the firm depends on the hiring strategy and the amount of physical capital. Let x^i be the probability that a firm produces with a worker of type $i = H, L$, conditional on matching with this worker. Then, the expected profit of the firm with physical capital k can be written as

$$V(k, x^H, x^L) = \phi x^H (1 - \beta)[k^{1-\alpha}\eta^\alpha - k] + (1 - \phi)x^L(1 - \beta)[k^{1-\alpha} - k], \quad (3)$$

where ϕ is the fraction of skilled workers, which is an endogenous variable in our model. Note that workers accept any pair since their payoffs are zero if they fail to form a pair.

Human capital investment depends on education cost and the wage difference between skilled and unskilled workers. The worker whose education cost is τ chooses to be skilled if the expected wage difference is higher than the cost, that is,

$$\int_{k\in\mathcal{K}} x^H(k)w^H(k)dF(k) - \int_{k\in\mathcal{K}} x^L(k)w^L(k)dF(k) \geq \tau.$$

Similarly, the worker chooses to be unskilled if the expected wage difference is smaller than the cost, that is,

$$\int_{k\in\mathcal{K}} x^H(k)w^H(k)dF(k) - \int_{k\in\mathcal{K}} x^L(k)w^L(k)dF(k) < \tau,$$

where $F(k)$ is the distribution of physical capital that firms choose and $\mathcal{K}$ is the support of this distribution. We denote the probability of adoption as a function of k since in an equilibrium, firms with the same amount of capital choose the same hiring strategies.

Since $\tau(\phi)$ is continuous and increasing, there is a worker who is indifferent between acquiring skill and not. In an equilibrium, the fraction of skilled workers, ϕ, is determined by the following condition:

$$\int_{k\in\mathcal{K}} x^H(k)w^H(k)dF(k) - \int_{k\in\mathcal{K}} x^L(k)w^L(k)dF(k) = \tau(\phi). \quad (4)$$

We define the equilibrium as the set $\{\phi, F(k), x^H(k), x^L(k)\}$ that satisfies the following conditions. First, given the distribution of capital and probability of adoption, thr educational choices of workers must be optimal, i.e., condition (4) must hold. Second, firms choose the amount of physical capital and the probability of adoption to maximize their expected profit

represented by (3). Therefore, the following condition must hold in an equilibrium.

$$\forall k' \in \mathcal{K} \qquad (k', x^H(k'), x^L(k')) \in \arg \max_{\{k, x^H, x^L\}} V(k, x^H, x^L). \qquad (5)$$

3   Nash Equilibrium

We consider the best responses of firms and workers in turn. Suppose that firms employ unskilled workers when they are indifferent between employing them and not. Then, we obatin the following lemma.

Lemma 1. *Given the fraction of skilled workers, ϕ, the best response strategies of firms are as follows:*

$$\eta \leq \Gamma(\phi) \quad \Rightarrow \quad \arg \max V(k, x^H, x^L) = (k^P, 1, 1), \qquad (6)$$
$$where \quad k^P = a[\phi\eta^\alpha + (1 - \phi)]^{\frac{1}{\alpha}}$$
$$\eta > \Gamma(\phi) \quad \Rightarrow \quad \arg \max V(k, x^H, x^L) = (k^S, 1, 0), \qquad (7)$$
$$where \quad k^S = a\eta,$$

where $\Gamma(\phi) \equiv \left(\frac{1-\phi}{\phi^\alpha - \phi}\right)^{\frac{1}{\alpha}}$ and $a \equiv (1 - \alpha)^{\frac{1}{\alpha}}$.

Proof. See Acemoglu (1999). $\qquad \square$

Since $\Gamma(\cdot)$ is a decreasing function, Lemma 1 implies that the higher fraction of skilled workers or the higher relative productivity of skilled workers discourages firms from employing unskilled workers. In these cases, production with an unskilled worker is not profitable since firms have chosen large physical capital in order to target only skilled workers.

Since firms are homogeneous, all firms choose the pooling strategy, $(k^P, 1, 1)$, if $\eta \leq \Gamma(\phi)$. In this case, the distribution of physical capital and the probability of adoption are given by

$$\mathcal{K} = \{k^P\}, x^H(k^P) = 1, x^L(k^P) = 1. \qquad (8)$$

Similarly, if $\eta > \Gamma(\phi)$, we have

$$\mathcal{K} = \{k^S\}, x^H(k^S) = 1, x^L(k^S) = 0. \qquad (9)$$

In both cases, the distribution of capital stock is singleton because all firms choose the same strategy in an equilibrium.

Next, we consider the best responses of workers. In an equilibrium, the distribution of capital is (8) or (9). Suppose that all firms choose the separating strategy, $(k^S, 1, 0)$. Then, workers decide whether or not to be skilled given that the distribution of capital satisfies (9). In this case, the optimal education condition, which is represented by (4), becomes

$$\tau(\phi) = \beta a^{1-\alpha} \eta \equiv \omega_S(\eta), \tag{10}$$

where $\omega_S(\eta)$ is the expected wage differential when all firms choose the separating strategy. Since $\tau(\cdot)$ is strictly increasing and satisfies $\tau(0) = 0$ and $\tau(1) = \infty$, equation (10) has a unique solution (see Figure 1). Let $\phi_S(\eta)$ denote the fraction of skilled workers that solves (10). Then, we have

$$\phi_S(\eta) = G \circ \omega_S(\eta). \tag{11}$$

Note that the solution of equation (10) depends on the relative productivity of skilled workers. Whether or not the pair of firms' separating strategies and workers' educational choices represented by (11) forms the equilibrium depends on the relative productivity of skilled workers. We define the threshold, η^*, which satisfies [4]

$$\eta^* = \Gamma(\phi_S(\eta^*)).$$

Since $\phi_S'(\cdot) > 0$ and $\Gamma'(\cdot) < 0$, we have $\eta > \Gamma(\phi_S(\eta))$ for any $\eta > \eta^*$. Therefore, whenever $\eta > \eta^*$, choosing the separating strategy is optimal for firms given that workers' strategies are represented by (11). That is, workers' and firms' strategies are mutually optimal. Hence, a strategy profile represented by $\{\phi_S, \mathcal{K} = \{k^S\}, x^H = 1, x^L = 0\}$ is a Nash equilibrium if $\eta > \eta^*$. We call this type of equilibrium a *separating equilibrium*. Next, we consider the case where all firms choose the pooling strategy $(k^P, 1, 1)$. Then, the optimal education condition becomes

$$
\begin{aligned}
&\beta(k^P)^{1-\alpha}\eta^\alpha - \tau(\phi) = \beta(k^P)^{1-\alpha} \\
&\Leftrightarrow \tau(\phi) = \beta a^{1-\alpha}[\phi\eta^\alpha + (1-\phi)]^{\frac{1-\alpha}{\alpha}}(\eta^\alpha - 1) \equiv \omega_P(\phi, \eta),
\end{aligned}
\tag{12}
$$

where $\omega_P(\phi, \eta)$ is the expected wage differential when all firms choose the pooling strategy. Since $\tau(0) < \omega(0, \eta)$ and $\tau(1) > \omega(1, \eta)$ for any $\eta > 1$, equation (12) has at least one solution. We assume that equation (12) has a

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[4]Since $\Gamma' < 0, G' > 0, \lim_{\phi \to 0} \Gamma(\phi) = \infty$ and $\lim_{\phi \to 1} \Gamma(\phi) = 1/a$, $\eta^* > 0$ exists uniquely. We suppose that $\eta^* > 1$.

unique solution for any η and denote it as $\phi_P(\eta)$.[5] Then, we can show that $\phi_P'(\eta) > 0$ and

$$\forall \eta > 1 \quad \phi_P(\eta) < \phi_S(\eta). \tag{13}$$

That is, more workers acquire skill when all firms choose the separating strategy than when they choose the pooling strategy. This is because the expected wage differential between skilled and unskilled workers is always higher in the former case, i.e.,

$$\forall \eta > 1 \ \forall \phi \in (0,1) \quad \omega_P(\phi, \eta) < \omega_S(\eta). \tag{14}$$

As in the case of a separating equilibrium, we define η^{**} by [6]

$$\eta^{**} = \Gamma(\phi_P(\eta^{**})).$$

Then, we have $\eta \leq \Gamma(\phi_P(\eta))$ for any $\eta \leq \eta^{**}$. Therefore, a strategy profile represented by $\{\phi_P, \mathcal{K} = k^P, x^H = 1, x^L = 1\}$ is a Nash equilibrium whenever $\eta \leq \eta^{**}$. We call this type of equilibrium a *pooling equilibrium*.

Figure 2 depicts the boundary that divides the firms' optimal strategies, $\Gamma(\phi)$, and the fractions of skilled workers, $\phi_S(\eta)$ and $\phi_P(\eta)$, which are results of workers' optimal responses to the separating strategy and the pooling strategy, respectively. Note that the $\phi_S(\eta)$ curve always lies on the right of the $\phi_P(\eta)$ curve. Thus, their intersections with the $\Gamma(\phi)$ curve, which correspond to η^{**} and η^*, respectively, satisfy $\eta^* < \eta^{**}$.

As we have seen, a set $\{\phi_S, \mathcal{K} = k^S, x^H = 1, x^L = 0\}$ satisfies the equilibrium condition if $\eta > \eta^*$ and a set $\{\phi_P, \mathcal{K} = k^P, x^H = 1, x^L = 1\}$ satisfies the equilibrium condition if $\eta \leq \eta^{**}$. Therefore, we have at least one equilibrium for any $\eta > 1$, and we have multiple equilibria if $\eta \in (\eta^*, \eta^{**}]$. The possibility that multiple equilibria exist is due to the strategic complementarity between firms' and workers' strategies. If all firms choose separating (pooling) strategies, more (less) workers acquire skill, which enhances the incentive for firms to choose separating (pooling) strategies. The following proposition summarizes these results.

Proposition 1. *A Nash equilibrium always exists. When the relative productivity of skilled workers is sufficiently high, $\eta > \eta^*$, there is a separating*

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[5]We consider the more general case that the equation (12) has multiple solutions in Appendix A. While we would have multiple pooling equilibria in the general case, the main results in this paper hold.

[6]By the same argument in the case of η^*, we can show that η^{**} exists uniquely. From (14), we have $\eta^* < \eta^{**}$ (see Figure 2).

equilibrium. When the relative productivity is sufficiently low, $\eta \leq \eta^{**}$, there is a pooling equilibrium. If the level of the relative productivity is intermediate, $\eta \in (\eta^*, \eta^{**}]$, both types of equilibria exist.

The features of each type of equilibrium are the same as those in Acemoglu (1999). In a pooling equilibrium, all firms employ both types of workers with probability one and choose the amount of capital stock that is suitable for both types of workers. Hence, there is no unemployment and the wage differential between skilled and unskilled workers is lower than that in the Walrasian equilibrium. [7]

In a separating equilibrium, all firms create high-quality jobs for skilled workers and refuse to employ unskilled workers. Hence, the earnings and employment of the unskilled workers collapse in a separating equilibrium. The wages of skilled workers, w_{sep}^H, are higher than those in a pooling equilibrium because we have

$$w_{sep}^H = \beta a \frac{\eta}{1-\alpha} > \beta a [\phi_P(\eta)\eta^\alpha + (1 - \phi_P(\eta))]^{\frac{1-\alpha}{\alpha}} \frac{\eta^\alpha}{1-\alpha} = w_{pool}^H.$$

In what follows, we analyze the effect of skill-biased technical change on income distribution. Proposition 1 shows that the distribution of skill, income, and employment depends on the level of relative productivity of skilled workers. We consider the case in which the relative productivity is sufficiently low and a pooling equilibrium is realized. In such a case, an increase in relative productivity caused by the skill-biased technical change always amplifies the wage inequality. However, the wages of unskilled workers also increase if the rise in relative productivity does not involve "the switch of equilibrium" from pooling equilibrium to separating equilibrium. This is because the rise in relative productivity increases the amount of capital stock that unskilled workers can use.[8] On the contrary, when the rise in relative productivity involves the switch of equilibrium, i.e., it takes the economy from a pooling equilibrium to a separating equilibrium, the wages of skilled workers jump up and the employment of unskilled workers

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[7]Let w_{pool}^H and w_{pool}^L denote the wages of skilled and unskilled workers, respectively, in a pooling equilibrium. Since $w_{pool}^H = \beta(k^P)^{1-\alpha}\eta^\alpha$ and $w_{pool}^L = \beta(k^P)^{1-\alpha}$, the relative wage of skilled workers, η^α, is lower than the relative wage in the Walrasian equilibrium, η, which is equal to the relative productivity.

[8]An increase in relative productivity raises the level of capital stock, k^P, because it increases the expected marginal productivity of physical capital. This is due to both the increase in productivity of skilled workers and the increase in the fraction of skilled workers. The latter effect is due to endogenous human capital investment; hence, Acemoglu's model does not have such an effect.

collapses.

Whether the switch occurs or not is indeterminate because the condition that η exceeds η^* is not sufficient for the switch to occur. If η is smaller than η^{**}, the switch would not happen since both types of equilibria can be present in such cases. In other words, it is indeterminate whether the switch of equilibrium occurs when the relative productivity of skilled workers is intermediate. Therefore, even if the skill-biased technical change causes a drastic rise in wage inequality in an economy, the same type of technical change does not necessarily bring about a similar change in another economy as long as the technical change is not so drastic. Hence, our discussion can provide an answer to the question as to why the technical shock yields the *ex post* heterogeneity in income distribution among *ex ante* homogeneous economies. Note that this result depends on the existence of multiple equilibria; hence, Acemoglu's model does not lead to this result.

4   Labor Market Institutions and Inequality

In the previous section, we analyze the mechanism that *ex ante* homogeneous economies can attain extremely different income distribution. However, there are various differences between the U.S. and Europe. Therefore, it seems natural to inquire how the *ex ante* heterogeneity affects income distribution. In this section, we consider the effect of tax and education system on income distribution in order to answer this question.

4.1   Taxes on Labor

We introduce taxes on labor into the model. We assume that there is a proportional tax on labor; hence the cost of hiring a worker for firms, w_f, does not coincide with the wage that the worker receives,

$$w = (1 - t)w_f, \tag{15}$$

where w is the disposable income of workers and t is the tax wedge. Table 2, which shows the tax wedges of the advanced countries, indicates that tax rates in Europe are higher than those in North America. Since a worker and a firm divide the total surplus that is realized by the matching, $y - w_f + w$, by Nash bargaining, we have

$$w = \beta(y - w_f + w). \tag{16}$$

From (15) and (16), we have

$$w = \frac{\beta(1-t)y}{(1-t)+\beta t} \quad \text{and} \quad w_f = \frac{\beta y}{(1-t)+\beta t}.$$

Then, we modify the expected profit of a firm as follows:

$$
\begin{aligned}
\tilde{V}(k, x^H, x^L) = {} & \phi x^H (1-\beta) \left[\frac{(1-t)k^{1-\alpha}\eta^\alpha}{(1-t)+\beta t} - k \right] \\
& + (1-\phi)x^L (1-\beta) \left[\frac{(1-t)k^{1-\alpha}}{(1-t)+\beta t} - k \right].
\end{aligned}
$$

By the same argument as that in the previous section, we arrive at the following lemma.

Lemma 2. *Given $\phi \in (0,1)$, the best response strategies of firms are as follows:*

$$
\begin{aligned}
\eta \le \Gamma(\phi) \quad &\Rightarrow \quad \arg\max \tilde{V}(k, x^H, x^L) = (\tilde{k}^P, 1, 1), \\
& \qquad where \quad \tilde{k}^P = a \left[\frac{(1-t)}{(1-t)+\beta t} \right]^{\frac{1}{\alpha}} [\phi \eta^\alpha + (1-\phi)]^{\frac{1}{\alpha}} \\
\eta > \Gamma(\phi) \quad &\Rightarrow \quad \arg\max \tilde{V}(k, x^H, x^L) = (\tilde{k}^S, 1, 0), \\
& \qquad where \quad \tilde{k}^S = a\eta \left[\frac{(1-t)}{(1-t)+\beta t} \right]^{\frac{1}{\alpha}}.
\end{aligned}
$$

Proof. See Appendix B. □

Lemma 2 implies that the boundary that divides firms' optimal strategies is the same as that in the previous section. This is because the ratio of the expected profit of firms when they choose the separating strategy to that when they choose the pooling strategy does not depend on tax rate. The optimal amount of capital is decreasing in tax rate; hence a higher tax rate reduces the investment by firms.

We define $\tilde{\phi}_S$ and $\tilde{\phi}_P$ as the fraction of skilled workers in the case when all firms choose the separating strategy and the pooling strategy, respectively. Hence, $\tilde{\phi}_S$ solves

$$\tau(\phi) = \beta a^{1-\alpha} \eta \{(1-t)/[(1-t)+\beta t]\}^{\frac{1}{\alpha}}, \tag{17}$$

and $\tilde{\phi}_P$ solves[9]

$$\tau(\phi) = \beta a^{1-\alpha}\{(1-t)/[(1-t)+\beta t]\}^{\frac{1}{\alpha}}[\phi\eta^\alpha + (1-\phi)]^{\frac{1-\alpha}{\alpha}}(\eta^\alpha - 1). \quad (18)$$

Since $(1-t)/(1-t+\beta t)$ is decreasing in t, $\tilde{\phi}_S$ and $\tilde{\phi}_P$ are decreasing in t. Therefore, we have the following proposition.

Proposition 2. *A Nash equilibrium always exists. If $\eta > \tilde{\eta}^*$, there is a separating equilibrium. If $\eta \leq \tilde{\eta}^{**}$, there is a pooling equilibrium. If $\eta \in (\tilde{\eta}^*, \tilde{\eta}^{**}]$, both types of equilibria exist. Here, $\tilde{\eta}^*$ and $\tilde{\eta}^{**}$ are defined by*

$$\tilde{\eta}^* = \Gamma(\tilde{\phi}_S(\tilde{\eta}^*)), \quad \tilde{\eta}^{**} = \Gamma(\tilde{\phi}_P(\tilde{\eta}^{**})).$$

A higher tax rate makes the condition under which a separating equilibrium exists more restrictive and makes the condition under which a pooling equilibrium exists less restrictive.

4.2 Policy on Education

Next, we consider the effects of educational policies. Here, we analyze two types of policies, investment in primary education and scholarships.

Since investment in primary education increases the productivity of unskilled workers, it has the same effect as the decrease in the relative productivity of skilled workers.[10] Therefore, the model predicts that a pooling equilibrium is realized in the economy where the quality of primary education is high. Table 3, which shows the average literacy test scores by education level, indicates that the less-educated workers in Germany, Sweden, and the Netherlands are more capable than those in the U.S. Freeman and Schettkat (2001) also show that the variance of income and skill in Germany is smaller than that in the U. S. These observations are consistent with the prediction of the model.

Pecuniary assistance for higher education, such as scholarships, reduces the cost of skill acquisition. Hence, it increases the fraction of skilled workers given firms' strategies. Therefore, the effect of this policy on income distribution is the opposite of that of investment in primary education. That is, a separating equilibrium is realized in the economy where workers can receive sufficient scholarships.

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[9]We assume that equation (18) has a unique solution.

[10]While this policy also increases the productivity of skilled workers, it seems natural to assume that it reduces the difference of the productivity between skilled and unskilled workers.

5   Discussion

Thus far, we have focused on the difference in income distribution between the U.S. and Europe and have provided an explanation for the inequality. However, the result that the unemployment rate in a pooling equilibrium is lower than that in a separating equilibrium is not consistent with the facts. Table 4, which presents the transitions in unemployment rates from 1973 to 2002 in advanced countries, shows that unemployment rates in the countries that have experienced a rise in inequality are lower than those in the countries that have had greater equality during the past two decades.

We think that a clue to solving this problem is the fact that the expected profit in a separating equilibrium is higher than that in a pooling equilibrium if both types of equilibria exist. This result can be explained by the following steps. First, the expected profit in a separating equilibrium must be larger than that in the case where the firm chooses the pooling strategy and the fraction of skilled workers is ϕ_S. This is obvious by the definition of Nash equilibrium. Second, the fraction of skilled workers in separating equilibrium is larger than that in a pooling equilibrium. Third, the expected profit in the case where the firm chooses the pooling strategy is increasing in the fraction of skilled workers. By these steps, we obtain the above mentioned result.

The difference in the expected profit could make the number of firms in a separating equilibrium larger than that in a pooling equilibrium if we endogenize the number of firms and the probability that each agent meets their partner as in the standard search model (e.g., Pissarides (2000)).[11] If the increase in the number participants in the labor market leads to an increase in the number of meeting, the entry of new firms decreases the unemployment rate given the hiring policies of firms. Therefore, the large number of firms in a separating equilibrium could make the unemployment rate lower than that in a pooling equilibrium.

However, endogenizing the number of firms makes the model more complicated because it yields another strategic complementarity between the educational choice of workers and the entry decisions of firms. Since the aim of this paper is to study the difference in income distribution, we leave the analysis in the difference of unemployment rates for further research.

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[11]Note that in our model each agent can meet their partner with probability one even though they cannot always match with their partner.

6   Concluding Remarks

We now summarize the result of this paper and mention the possibility of further research.

There are two types of equilibria in the model. In a pooling equilibrium, the wage differential is relatively low and all workers are employed. In a separating equilibrium, unskilled workers are unemployed, and skilled workers gain higher wages than that is realized in pooling equilibrium. Proposition 1 states that the type of equilibrium that is realized depends on the relative productivity of skilled workers. A pooling equilibrium is realized when the relative productivity is sufficiently small and a separating equilibrium is realized when the relative productivity is sufficiently large. If the relative productivity is intermediate, there are multiple equilibria. Since the skill-biased technical change, which is regarded as one of the causes of the recent wage inequality, increases the relative productivity of skilled workers, it can cause a shift from a pooling equilibrium to a separating equilibrium in the economy. However, whether or not a drastic change occurs is indeterminate when the rise in the relative productivity is not very large. As a result, the same type of tehcnical shock can result in different income distribution and employment structures *ex post* in *ex ante* homogeneous economies. High tax rates or high productivity of unskilled workers, which are observed in European economies, prevent the switch of equilibrium. Therefore we can regard these factors as the cause of the difference in the wage inequality between the U.S. and Europe.

While this paper provides an explanation for the observed difference in income distribution, the analysis of the difference in unemployment is a topic for further research.

Appendix

A. Multiple Pooling Equilibria

In Section 3, although we assume that equation (12) has a unique solution, this assumption is not crucial to the main result. Here, we consider the more general case in which equation (12) may have multiple solutions.

Let $\Phi_p(\eta)$ be the set of solutions of (12) when the relative productivity is η. We define $\underline{\phi}(\eta)$ and $\overline{\phi}(\eta)$ by

$$\tau(\underline{\phi}(\eta)) = \omega_P(0, \eta), \quad \tau(\overline{\phi}(\eta)) = \omega_P(1, \eta).$$

Since $\omega_P(\phi, \eta)$ is increasing in ϕ and $\omega_P(1, \eta) < \omega_S(\eta)$, we have

$$\forall \eta > 1 \; \forall \phi \in \Phi_P(\eta) \quad \underline{\phi}(\eta) < \phi < \overline{\phi}(\eta) < \phi_S(\eta).$$

As in Section 3, we define $\underline{\eta}$ and $\overline{\eta}$ by $\underline{\eta} = \Gamma(\underline{\phi}(\underline{\eta}))$ and $\overline{\eta} = \Gamma(\overline{\phi}(\overline{\eta}))$. Note that $\underline{\eta} > \overline{\eta} > \eta^*$ because $\underline{\phi}(\eta) < \overline{\phi}(\eta) < \phi_S(\eta)$ for any η. By the same argument as that in Section 3, we have

$$\forall \phi \in \Phi_P(\eta) \quad \eta \leq \overline{\eta} = \Gamma(\overline{\phi}(\overline{\eta})) \leq \Gamma(\overline{\phi}(\eta)) < \Gamma(\phi)$$

if $\eta \leq \overline{\eta}$. Therefore, the pair of any solution of equation (12) and the pooling strategy is a Nash equilibrium if $\eta \leq \overline{\eta}$. On the other hand, we have

$$\forall \phi \in \Phi_P(\eta) \quad \eta > \underline{\eta} = \Gamma(\underline{\phi}(\underline{\eta})) > \Gamma(\underline{\phi}(\eta)) > \Gamma(\phi)$$

if $\eta > \underline{\eta}$. Thus, we have no pooling equilibrium when η exceeds $\underline{\eta}$. By summarizing these results, we have the following proposition.

Proposition 3. *A Nash equilibrium always exists even if we allow the possibility that more than one pooling equilibrum exists. When the relative productivity of skilled workers is sufficiently high, $\eta > \eta^*$, there is a separating equilibrium. When the relative productivity is sufficiently low, $\eta \leq \overline{\eta}$, there is at least one pooling equilibrium. If the level of relative productivity is intermediate, $\eta \in (\eta^*, \overline{\eta}]$, both types of equilibria exist. There is no pooling equilibrium if $\eta > \underline{\eta}$.*

B. Proof of Lemma 2

We consider the optimal investment problem of the firm given the probability of adoption. When a firm hires both types of workers, i.e., $x^H = x^L = 1$, the amount of physical capital that maximizes $\tilde{V}(k, 1, 1)$ is given by

$$\tilde{k}^P = a \left[\frac{(1-t)}{(1-t) + \beta t} \right]^{\frac{1}{\alpha}} [\phi \eta^\alpha + (1-\phi)]^{\frac{1}{\alpha}}.$$

Then, the expected profit is

$$\tilde{V}^P \equiv V(\tilde{k}^P, 1, 1) = \frac{\alpha}{1 - \alpha}(1 - \beta)a[\phi \eta^\alpha + (1 - \phi)]^{\frac{1}{\alpha}} \left[\frac{(1-t)}{(1-t) + \beta t} \right]^{\frac{1}{\alpha}}.$$

When the firm hires only skilled workers, i.e., $x^H = 1$ and $x^L = 0$, the amount of physical capital that maximizes $\tilde{V}(k, 1, 0)$ is given by

$$\tilde{k}^S = a\eta \left[\frac{(1-t)}{(1-t) + \beta t} \right]^{\frac{1}{\alpha}}.$$

Then, the expected profit is

$$\tilde{V}^S \equiv V(\tilde{k}^S, 1, 0) = \frac{\alpha}{1-\alpha}(1-\beta)\phi a\eta \left[\frac{(1-t)}{(1-t) + \beta t} \right]^{\frac{1}{\alpha}}.$$

Since we can show that the other hiring strategies are not optimal, as shown in Lemma 1 in Acemoglu (1999), we can determine the optimal strategy of the firm by comparing $\tilde{V}^P$ and $\tilde{V}^S$. Since $\tilde{V}^P \geq \tilde{V}^H$ if and only if $\eta \leq \Gamma(\phi)$, the result is proved.

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Table 1: Trends in earning dispersion (Males)(1980-1995): OECD (1996)

		1980	1985	1990	1995
Canada	D9/D5	1.67 (81)	1.68 (86)	1.75	1.73 (94)
	D5/D1	2.07 (81)	2.40 (86)	2.28	2.18 (94)
Finland	D9/D5	1.67	1.73 (86)	1.72	1.73 (94)
	D5/D1	1.46	1.50 (86)	1.49	1.46 (94)
France	D9/D5	2.03	2.08	2.13	2.13 (94)
	D5/D1	1.66	1.61	1.62	1.61 (94)
Germany	D9/D5	1.63 (83)	1.66	1.65	1.64 (93)
	D5/D1	1.46 (83)	1.42	1.40	1.37 (93)
Italy	D9/D5	1.43	1.53 (86)	1.55 (91)	1.65 (93)
	D5/D1	1.63	1.44 (86)	1.43 (91)	1.60 (93)
Japan	D9/D5	1.63	1.68	1.73	1.73 (94)
	D5/D1	1.60	1.65	1.64	1.60 (94)
Sweden	D9/D5	1.61	1.58	1.56	1.62 (93)
	D5/D1	1.31	1.35	1.33	1.36 (93)
U.K.	D9/D5	1.62	1.71	1.81	1.86
	D5/D1	1.55	1.64	1.72	1.78
U.S.	D9/D5	1.76	1.84	1.96	2.04
	D5/D1	1.85	2.03	2.02	2.13

Table 1 is quoted from OECD (1996). D1 and D9 refer to the upper earnings limits of the first and ninth declies of employees ranked in order of their earnings from the lowest to highest. D5 is defined similarly and thus corresponds to median earnings.

Table 2: Tax rates on labor: Nickell et al. (2005)

	1980-87	1988-1995	1996-2000
Canada	42	50	53
Finland	58	64	62
France	65	67	68
Germany (W)	50	52	50
Italy	56	67	64
Japan	33	33	37
Sweden	77	78	77
U.K.	51	47	44
U.S.	44	45	45

Table 2 is quoted from Table 4 in Nickell et al. (2005). The tax wedge is defined as the sum of the rates of consumption tax, income tax, and payroll tax.

Table 3: Average literacy test scores by education level (1994): Nickell and Layard (1999)

	ISCED2	ISCED3	ISCED5	ISCED6/7	Total
Germany	2.42	2.97	3.11	3.39	2.84
Netherlands	2.52	2.96	n.a.	3.27	2.74
Sweden	2.96	3.07	3.31	3.56	3.04
Switzerland	2.20	2.82	3.09	3.16	2.67
Canada	2.20	2.67	2.97	3.55	2.62
U.S.	1.92	2.44	2.86	3.31	2.56

ISCED2, 3, 5, and 6/7 represent lower secondary, upper secondary, first stage of tertiary education, and second stage of tertiary education, respectively.

Table 4: Unemployment (Standardized Rate): Nickell et al. (2005)

	1973-9	1980-7	1988-95	1996-9	2000-1	2002
Canada	6.9	9.7	9.5	8.7	7.0	7.7
Finland	4.1	5.1	9.9	12.2	9.4	9.1
France	4.3	8.9	10.5	11.9	8.9	8.7
Germany (W)	2.9	6.1	5.6	7.1	6.4	6.8
Italy	4.5	6.7	8.1	9.9	8.4	7.4
Japan	1.8	2.5	2.5	3.9	4.9	5.4
Sweden	1.6	2.3	5.1	8.7	5.3	4.9
U.K.	4.8	10.5	8.8	6.9	5.2	5.1
U.S.	6.4	7.6	6.1	4.8	4.4	5.8
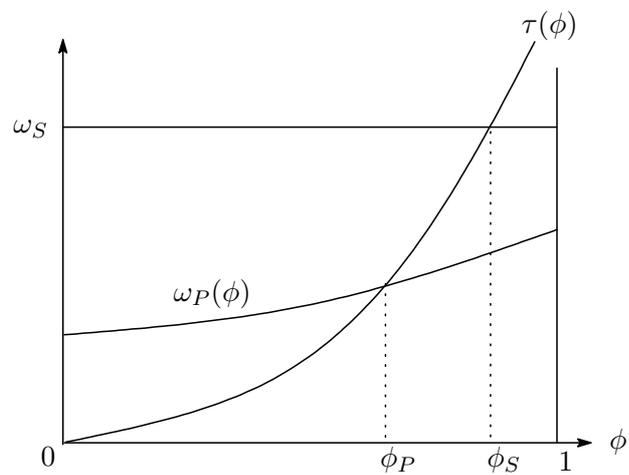
Figure 1: Human Capital Investment
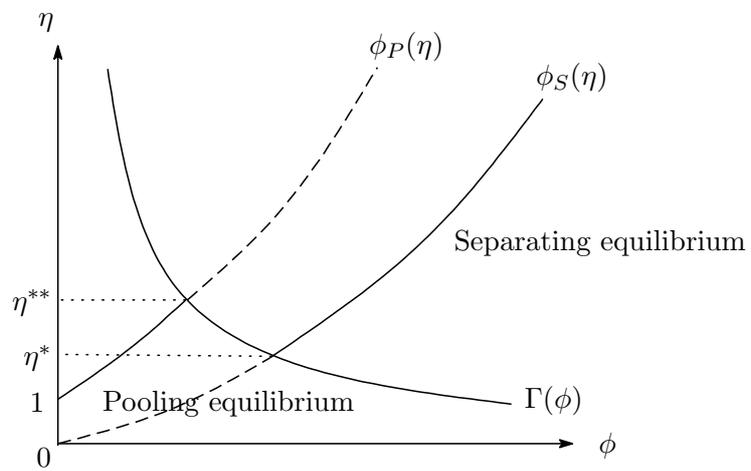
Figure 2: Nash Equilibria