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“Evolutionary Sequential Trading”

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Evolutionary Sequential Trading*

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1 Introduction

Easley and O'Hara (1992) show how trading volume affects the speed of price adjustment in a market with asymmetric information. They have the following setting. Group of informed traders are fixed and they have identical private information about the value of the asset. Other traders do not have such private information. At each time, one randomly chosen trader can place one unit of sell/buy order. A market maker determines the price without knowing whether the trader has the private information or not. The market maker learns from the past trading history, so that the bid-ask spread (resp. the price) converges to zero (the strong-form efficient level) exponentially over time.

They assume that the ratio of informed trader is fixed as μ over time. However this assumption may be taken as quintessential assurance of information revelation in their result. In a practical sense, we should analyze the decision making problem that includes endogenous choices. Once we consider the cost, it is not apparent whether the price converges to the strong-form efficient level or not. When a chosen trader knows that the price enough reflects the information, he may not try to spend his money to get the private information. Then the market maker may expect that there are not so many traders who have the private information in the past trade, and set the wide bid/ask spread. This may not result in the exponential convergence shown in their paper.

Unfortunately it is difficult to extend the model in Easley and O'Hara (1992) to the one endogenizing information choices directly. The fact that the market maker remembers all the past trade might be the cause of the difficulty to analyze. Therefore we change the setting by assuming the traders' limited memories, and analyze their evolutionary decision making. We obtain the result in which the ratio of the informed traders is proportional to the width of the bid ask spread, and the price converges to the strong-form efficient level exponentially as is shown in Easley and O'Hara (1992).

2 Model

We consider the trading in a day. The eventual value of the asset is represented by a random variable V. Before the start of the trading day, an information event that relates to the asset value occurs with the probability α, where $0 < \alpha < 1$. If the information event occurs, the fraction μ of the traders can know the occurrence of the information event, and observe the identical signal. We require that $0 < \mu < 1$. The other $1 - \mu$ traders and the market maker do not observe it. Formally, we define an information event as the occurrence of a signal Ψ about V. The signal can take one of two values, L and H, with probabilities $\delta > 0$ and $1 - \delta > 0$ respectively. We let the expected value of the asset conditional on the signal be $E[V|\Psi = L] = \underline{V}$ or $E[V|\Psi = H] = \overline{V}$. If no information event has occurred, we denote this as $\Psi = 0$ and the expected value of the asset simply remains at its unconditional level $V^* = \delta\underline{V} + (1-\delta)\overline{V}$.

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Then a randomly chosen trader comes to the market and places one unit of sell/buy order one by one. The market maker sets the bid/ask prices before each order.

Informed traders are risk-neutral and take prices as given. She will buy if she has seen a high signal and an ask price below $\overline{V}$; she will sell if she has seen a low signal and a bid price above $\underline{V}$.

An trade for liquidity reasons such as the timing of consumption or portfolio considerations may arise. She will buy (resp. sell) with the probability $\epsilon^B > 0$ $\left(\epsilon^S > 0\right)$. If it is not the case, she does not want to trade, because the quotes should be unprofitable for her.

We assume that the market maker is risk-neutral and acts competitively. That is, his price quotes yield zero expected profit conditional on a trade at the quotes.

This trading structure can be understood most easily by the tree diagram given in Figure 1.

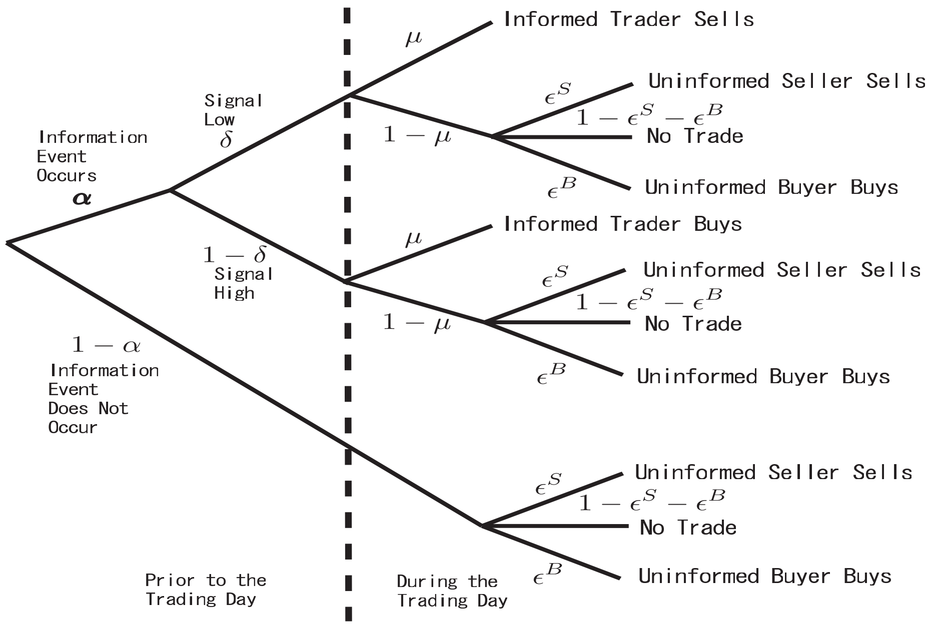

Figure 1

At the first node nature selects whether the information event occurs. If there is an information event, then the type of signal (either L or H) is determined at the second node. These two nodes are reached only at the beginning of the day. Traders are selected at each time t to trade based on the probabilities described above. Thus, if an information event has occurred, an informed trader is selected with probability μ, and she then chooses either to buy or sell. Similarly, with probability $1-\mu$ an uninformed trader is selected and she may choose to buy, sell or not trade with the indicated probabilities. For trade in the next time interval, only the trader selection process is repeated, so that the game proceeds from the right dotted line on the tree diagram. This continues throughout the day.

The market maker is Bayesian who has limited memory. He does not know whether the information event has occurred, whether it is good or bad news given that it has occurred, or whether any particular trader is informed. Each market participant can watch the market. Over time, this allows the market maker to learn whether the information event has occurred and whether it is good or bad news given that it has occurred and revise his belief. But he cannot remember all past trades. He remembers only m outcomes of the past trading. That is, he remembers the number of the buy orders β, sell orders s, and no trade n $(\beta + s + n \leq m)$ in the past trades. Note that the quotes submitted by the market maker depend not on the entire history but only on the ratios of the number of each order (See Proposition 3 in Easley and O'Hara (1992)). He can remember all the past trades before the time m. At t $(t > m)$, he randomly forgets one of m outcomes in his memory at $t-1$ with equal probabilities, and remembers the new outcome at t.[1]

[1] He does not believe that he will forget one outcome. When he set the quotes, he considers the $m+1$ trade (m past outcomes and a potential sell/buy/no-trade order at the present time) as is noted below.

With a calculation similar to the one in Easley and O'Hara (1992), the market maker's beliefs given his memory are given by:

$$\Pr\{\Psi = 0 | (n, s, \beta)\} =$$
$$\frac{(1-\alpha)\left(\epsilon^S\right)^s\left(\epsilon^B\right)^\beta}{(1-\alpha)\left(\epsilon^S\right)^s\left(\epsilon^B\right)^\beta + (1-\mu)^n\left[\alpha\delta\left(\mu+(1-\mu)\epsilon^S\right)^s\left((1-\mu)\epsilon^B\right)^\beta + \alpha(1-\delta)\left((1-\mu)\epsilon^S\right)^s\left(\mu+(1-\mu)\epsilon^B\right)^\beta\right]}.$$

The probabilities of low and high signals are calculated similarly.

Since beliefs depend on (n, s, β), quotes will also depend on these variables. The bid b and the ask a can be written as:

$$\begin{aligned}
b &= \Pr\{\Psi = L | (n, s+1, \beta)\}\underline{V} \\
&\quad + \Pr\{\Psi = H | (n, s+1, \beta)\}\overline{V} + \Pr\{\Psi = 0 | (n, s+1, \beta)\}V^*, \text{ and} \\
a &= \Pr\{\Psi = L | (n, s, \beta+1)\}\underline{V} \\
&\quad + \Pr\{\Psi = H | (n, s, \beta+1)\}\overline{V} + \Pr\{\Psi = 0 | (n, s, \beta+1)\}V^*.
\end{aligned}$$

We line up Ψ, n, s, and β and define the collection as a *state*. We can calculate the transition probabilities between all pairs of the states, and regard the process as Markov process. This process is not irreducible. For example, $(L, \cdot, \cdot, \cdot)$ is not accessible from $(H, \cdot, \cdot, \cdot)$. We can find three absorbing sets there.

$$\begin{aligned}
&\{(L, n, s, \beta)\}_{n+s+\beta=m}, \\
&\{(H, n, s, \beta)\}_{n+s+\beta=m}, \text{ and} \\
&\{(0, n, s, \beta)\}_{n+s+\beta=m}.
\end{aligned}$$

When we fix a outcome of $\Psi = L$, H, or 0 arbitrary, "partial processes" in which the set of states consists of $n + s + \beta = m$ are finite, irreducible, and recurrent. Therefore each partial process has a unique stationary distribution. Multiplying these stationary distributions given Ψ by the probabilities of Ψ occurring, the distribution of the union of them is also stationary, which we call *stationary state.*

3 Stationary State

In this section, we solve the stationary state analytically.

Proposition 1 *The stationary state* $\left(x_{(\Psi,n,s,\beta)}\right)$ *is as follows:*

$$x_{(\Psi,n,s,\beta)} = \begin{cases} \alpha\delta\frac{m!}{n!s!\beta!}p_n(L)^n p_s(L)^s p_\beta(L)^\beta & \textit{if} \quad \Psi = L \\ \alpha(1-\delta)\frac{m!}{n!s!\beta!}p_n(H)^n p_s(H)^s p_\beta(H)^\beta & \textit{if} \quad \Psi = H, \textit{ and} \\ (1-\alpha)\frac{m!}{n!s!\beta!}p_n(0)^n p_s(0)^s p_\beta(0)^\beta & \textit{if} \quad \Psi = 0 \end{cases},$$

$$\begin{aligned}
&\textit{where} \\
&p_n(\Psi) = \Pr\{\textit{No Trade}|\Psi\}, \\
&p_s(\Psi) = \Pr\{\textit{Sell}|\Psi\}, \textit{ and} \\
&p_\beta(\Psi) = \Pr\{\textit{Buy}|\Psi\}.
\end{aligned}$$

Proof. $\left(x_{(\Psi,n,s,\beta)}\right)$ *is probability distribution because* $x_{(\Psi,n,s,\beta)} \geq 0$ *holds for every* Ψ, n, s, β, *and*

$$\begin{aligned}
\sum_{\Psi,n,s,\beta} x_{(\Psi,n,s,\beta)} &= \alpha\delta\left(p_n(L) + p_s(L) + p_\beta(L)\right)^3 \\
&\quad + \alpha(1-\delta)\left(p_n(H) + p_s(H) + p_\beta(H)\right)^3 \\
&\quad + (1-\alpha)\left(p_n(0) + p_s(0) + p_\beta(0)\right)^3 \\
&= 1.
\end{aligned}$$

The transition probabilities in both the entire process and the partial processes are

$$
\begin{aligned}
\Pr\{(\Psi,n,s,\beta)\to(\Psi,n-1,s,\beta+1)\} &= \frac{n}{m}p_\beta(\Psi),\\
\Pr\{(\Psi,n,s,\beta)\to(\Psi,n-1,s+1,\beta)\} &= \frac{n}{m}p_s(\Psi),\\
\Pr\{(\Psi,n,s,\beta)\to(\Psi,n,s-1,\beta+1)\} &= \frac{s}{m}p_\beta(\Psi),\\
\Pr\{(\Psi,n,s,\beta)\to(\Psi,n,s,\beta)\} &= \frac{n}{m}p_n(\Psi)+\frac{s}{m}p_s(\Psi)+\frac{\beta}{m}p_\beta(\Psi),\\
\Pr\{(\Psi,n,s,\beta)\to(\Psi,n,s+1,\beta-1)\} &= \frac{\beta}{m}p_s(\Psi),\\
\Pr\{(\Psi,n,s,\beta)\to(\Psi,n+1,s-1,\beta)\} &= \frac{s}{m}p_n(\Psi),\\
\Pr\{(\Psi,n,s,\beta)\to(\Psi,n+1,s,\beta-1)\} &= \frac{\beta}{m}p_n(\Psi),\ \textit{and}\\
\Pr\{(\Psi,n,s,\beta)\to \textit{others}\} &= 0.
\end{aligned}
$$

Therefore, for all $n,s,\beta\in N\ (n+s+\beta=m)$,

$$
\begin{aligned}
&\sum_{(\Psi,n',s',\beta')}\Pr\{(\Psi,n',s',\beta')\to(\Psi,n,s,\beta)\}\frac{m!}{n!s!\beta!}p_n(\Psi)^n p_s(\Psi)^s p_\beta(\Psi)^\beta\\
&=\frac{m!}{(n+1)!s!(\beta-1)!}p_n(\Psi)^{n+1}p_s(\Psi)^s p_\beta(\Psi)^{\beta-1}\cdot\frac{n+1}{m}p_\beta(\Psi)\\
&+\frac{m!}{(n+1)!(s-1)!\beta!}p_n(\Psi)^{n+1}p_s(\Psi)^{s-1}p_\beta(\Psi)^{\beta}\cdot\frac{n+1}{m}p_s(\Psi)\\
&+\frac{m!}{n!(s+1)!(\beta-1)!}p_n(\Psi)^{n}p_s(\Psi)^{s+1}p_\beta(\Psi)^{\beta-1}\cdot\frac{s+1}{m}p_\beta(\Psi)\\
&+\frac{m!}{n!s!\beta!}p_n(\Psi)^n p_s(\Psi)^s p_\beta(\Psi)^\beta\cdot\left(\frac{n}{m}p_n(\Psi)+\frac{s}{m}p_s(\Psi)+\frac{\beta}{m}p_\beta(\Psi)\right)\\
&+\frac{m!}{n!(s-1)!(\beta+1)!}p_n(\Psi)^{n}p_s(\Psi)^{s-1}p_\beta(\Psi)^{\beta+1}\cdot\frac{\beta+1}{m}p_s(\Psi)\\
&+\frac{m!}{(n-1)!(s+1)!\beta!}p_n(\Psi)^{n-1}p_s(\Psi)^{s+1}p_\beta(\Psi)^{\beta}\cdot\frac{s+1}{m}p_n(\Psi)\\
&+\frac{m!}{(n-1)!s!(\beta+1)!}p_n(\Psi)^{n-1}p_s(\Psi)^{s}p_\beta(\Psi)^{\beta+1}\cdot\frac{\beta+1}{m}p_n(\Psi)\\
&=\frac{m!}{n!s!\beta!}p_n^n p_s^s p_\beta^\beta,
\end{aligned}
$$

where

$$0!=1,\ \textit{and}\ (-1)!=0.$$

■

4 Evolutionary Decision Making on Free Entry

We slightly change the setting of the model in this section. We assume that the traders can choose either to get the information by bearing the expenses, or not. The ratio of traders who can get the information is μ at the initial state. One trader is randomly chosen before the trading, and she can decide either to get the information with the cost $c>0$, or not. She expects that the present state is the stationary state on that occasion. If she chooses to get information (resp. does not get information), the value of μ becomes greater (less) than now in a small range. If she chooses to get the information, and if the information event occurs, then she can get the information. Otherwise, her behavior does not depend on her choice, and she behave as an uninformed trader. We can calculate the stationary state under the new μ. Another trader is randomly chosen in the new stationary state, and she makes the same decision... We call the state where both choices between acquiring and not acquiring are indifferent each other as a *stable state.*

The following scenario may facilitate the understandings. The traders often get up late. The market has already opened, and there are too many trade outcomes for the market maker to remember because of his limited memory m. The state in the market seems to have converged to the stationary state. The proportion μ of the traders subscribe the newspapers, and can read them before they trade. Since they get up now, they do not know if important events occurred today until they read the newspaper. The market maker is busy and has no time to read the newspaper. The fee of subscription is c. After reading the newspaper, she can place her order once in order to maximize her today's payoff. Even if she can not get the information and if she does not want to trade, she may have to buy or sell with the probabilities ϵ^B or ϵ^S. A newspaper gentry goes door-to-door and solicits subscriptions at a pace of a house a day (probably after the trading hours). When the newspaper gentry comes, a trader can choose whether she enters her subscription, renews, or cancels. The market maker knows how many people read the newspapers.

What is a stable state in such a dynamic? When the number of traders who read the newspapers is too small (resp. large), it is difficult (easy) for the market maker to learn through the outcomes, the price reflects the information less (more), and the value of reading the newspaper is large (small), which results in the increase (decrease) of the willingness for the traders to read the newspapers. Reading and not reading must be indifferent in the stable state.

Now we can prove that the average value of the bid-ask spread is proportional to μ in the stable state.

Proposition 2 *Suppose that $\epsilon^S = \epsilon^B$. Then, in the stable state the following holds:*

$$\sum x_{(\Psi,n,s,\beta)} (a-b) = \frac{\mu}{\epsilon^S} c,$$

where $a-b$ is the bid-ask spread when the memories are (n,s,β).

Proof. *Denote the expected payoff of the trader paying c as X, and not paying as Y. Since the expected payoff of the market maker is zero,*

$$\mu X + (1-\mu) Y = 0.$$

On the other hand, there is no difference between paying and not paying for the trader,

$$X - c = Y.$$

Therefore

$$\mu c = -Y.$$

■

Since the bid-ask spread depends on μ, it is difficult to express it as the function of μ explicitly. In order to understand the behavior of μ, we see simple numerical examples in what follows. The figure below shows the relationship between m and μ in the stable states in the case where $\overline{V} = 100$, $\underline{V} = 0$, $\alpha = 0.9$, $\epsilon^S = \epsilon^B = 0.05$, and $c = 1$. μ and the average value of the bid-ask spread decrease exponentially with the capacity of memory m. Thus, we show that the prices converge to the strong form efficient level of the price exponentially over time (as is shown in Easley and O'Hara (1992)) in the model in which the traders can choose whether they get the private information or not.

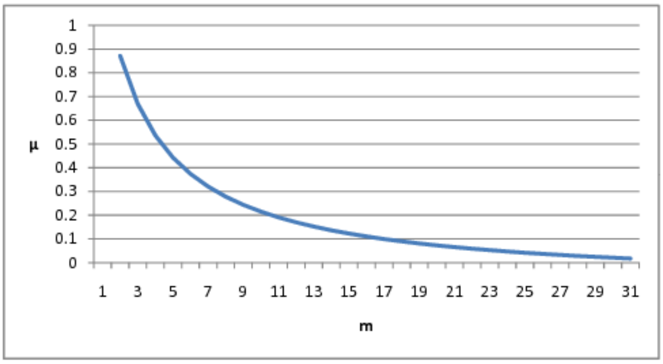

Figure 2

Intuitively, as m becomes greater, the market maker can learn more from the past trade flow, and the asymmetric information is eased. He can put small bid-ask spread since the loss by the trade with informed traders is less. Since paying information cost is fruitless, the traders become less willing to get the information. Even if they may have to trade for liquidity reasons, the loss is not large so much because the price is almost strong form efficient. The above result holds with various parameters.

5 Conclusion

We analyze the model in which the traders can choose either to get private information or not when the capacity of memory of the market maker is limited. We obtain the result that the prices exponentially converge to the strong-form efficient level.

References

[1] Easley, David, and Maureen O'Hara, 1992, Time and the Process of Security Price Adjustment, *The Journal of Finance*, 47, 2, 577-605.