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Tradeoff between Inflation Stabilization and
Growth Maximization

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Abstract

This paper analyzes monetary policy implication in an endogenous growth model in which the average growth rate is inefficiently low and in which the capital accumulation technology is concave. This paper does two exercises. First, we derive the utility-based welfare criterion of the model. The welfare measure suggests that even if the natural rate of growth moves parallel to its efficient rate, the increase of inflation volatility may improve welfare through the increase of average growth. Second, we test this hypothesis numerically and show that in our calibrated model the tradeoff between inflation stabilization and average growth maximization exists. In addition, the tradeoff is resolved by highly growth-stimulating (investment stabilization) policy. The reason is the existence of concavity in the capital accumulation technology, through which investment stabilization rises average growth.

Keywords: Endogenous Growth; Monetary Stabilization Policy

JEL classification: E3

1 Introduction

Should monetary policy concentrate price or inflation stabilization? To propose a new point of view, this paper analyzes monetary policy implication in an endogenous growth model in which the average growth rate is inefficiently low and in which the capital accumulation technology is concave.

It is known that in real economy, an exogenous increase of average growth is highly welfare-improving. In his seminal study, Lucas (1987) shows that the welfare gain of small increase of average growth is much more than that of perfectly elimination of business cycle fluctuation. More recently, Barlevy (2004) shows that, in real stochastic endogenous growth models with concave capital accumulation technology, stabilizing business cycle fluctuation stimulates the average growth, hence stabilization of business cycle has much more welfare gain than that calculated by Lucas (1987). Though Barlevy (2004) does not study any policy problem, his study suggests that the introduction of endogenous growth and the concavity of capital accumulation technology change the existing implications of the studies about economic stabilization policy. Our motivation is a test of the effect on monetary policy implication of Barlevy (2004)'s mechanism. That is, my question is as follows. When both of sticky price and growth (externality) distortions exist, does the two distortions have some tradeoff? If it exists, how should monetary policymaker resolve those distortions?

To answer these questions, this paper does two exercises. First, we derive the utility-based welfare criterion of the model. The welfare measure suggests that even if the natural rate of growth moves parallel to its efficient rate, the increase of inflation volatility may improve welfare through the increase of average growth. Second, we tests this hypothesis numerically and show that in our calibrated model the tradeoff between inflation stabilization and average growth maximization exists. In addition, the tradeoff is resolved by highly growth-stimulating (investment stabilization) policy. The reason is the existence of concavity in the capital accumulation technology, through which investment stabilization rises average growth.

This paper is summarized as follows. Section 2 presents an endogenous growth model with externality in good production, concavity in the capital accumulation technology, and Calvo (1983)-type nominal rigidities. Section 3 derives the natural and efficient rates of the model. Section 4 derives the utility-based welfare criterion. Section 5 does a numerical exercise. Section 6 concludes.

2 The Model

2.1 Households

The representative household has its preference represented by the following utility function:

$$E_0 \sum_{t=0}^{\infty} \beta^t \log C_t, \tag{1}$$

where C_t denotes the final good comsumption.

The intertemporal budget constraint of the household is given by

$$\frac{B_t}{P_t} + C_t + I_t = \frac{R_{t-1}D_{t-1}}{P_t} + r_t^K K_t + \Gamma_t + T_t \tag{2}$$

where B_t denotes the quantity of the riskless nominal bond, of which nominal interest rate is R_t, I_t denotes investment spending, K_t denotes capital stock, r_t^K denotes the rate of return on capital, Γ_t denotes his dividend income transfered from firms, and T_t denotes the transfer from the government.

There is i type of good in the economy. The final good is produced by the standard Dixit-Stiglitz aggregator:

$$Y_t = \left[\int_0^1 Y_t(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}, \tag{3}$$

where $Y_t(i)$ denotes the quantity of good i and Y_t denotes the demand for final good. Thus the demand for good i and the aggregate price index, P_t, can be derived as follows.

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\theta} Y_t, \tag{4}$$

$$P_t = \left(\int_0^1 P_t(i)^{1-\theta} di \right)^{\frac{1}{1-\theta}}, \tag{5}$$

where $P_t(i)$ denotes the nominal price of good i.

For simplicity, we assume the full depreciation of capital. In addition, following Barlevy (2004), we assume that the technology of capital production is concave. Thus, the dynamic process governing capital accumulation is summarized as:

$$K_{t+1} = \left(\frac{I_t}{K_t} \right)^{1-\phi} K_t, \quad 0 \le \phi < 1. \tag{6}$$

When $\phi = 0$, the technology of capital production is the standard linear technology, $K_{t+1} = I_t$. Otherwise, the growth rate of capital stock is concave in investment-capital ratio.

The household maximizes his lifetime utility (1), subject to his intertemporal budget constraint (2) and the capital accumulation process (6). The first order conditions are as follows:

$$\frac{1}{P_t} C_t = \beta E_t \frac{R_t}{P_{t+1} C_{t+s}}, \tag{7}$$

$$1 = q_t(1-\phi) \left(\frac{I_t}{K_t} \right)^{-\phi}, \tag{8}$$

$$\frac{q_t}{C_t} = \beta \frac{1}{C_{t+1}} \left[r_{t+1}^K + q_{t+1}\phi \left(\frac{I_t}{K_t} \right)^{1-\phi} \right], \tag{9}$$

where q_t denotes the Lagrange multiplier with respect to (6).

2.2 Firms

Good i is monopolistically supplied by firm i, which has the following technology:

$$Y_t(i) = A_t K_t(i)^{1-\epsilon} Z_t^\epsilon, \tag{10}$$

where $Y_t(i)$ denotes production of good i, A_t denotes the aggregate productivity, and $K_t(i)$ denotes the capital service employed by firm i, and Z_t denotes the knowledge level of the economy, that each individual firm takes as given. The production technology is decreasing return to scale for each individual firm, hence the real marginal cost each firm faces, $mc_t(i)$, is not identical across firms. Cost minimization derives the first-order condition:

$$r_t^K = (1 - \epsilon) A_t K_t(i)^{-\epsilon} Z_t^\epsilon mc_t(i). \tag{11}$$

We assume sticky prices following Calvo (1983), in which in any period a rumdomly selected fraction $1 - \xi$ of firms can reset their price. The profit maximization problem of a firm that can reset its price in period t can be described as:

$$\max_{P_t^*} \quad E_t \sum_{s=0}^\infty \xi^s Q_{t,t+s} \left[P_t^* Y_{t+s|t} - (1-\tau)\Psi_{t+s}(Y_{t+s|t}) \right] \tag{12}$$

$$\text{s.t.} \quad Y_{t+s|t} = \left(\frac{P_t^*}{P_{t+s}} \right)^{-\theta} Y_{t+s}, \tag{13}$$

where $Y_{t+s|t}$ denotes the demand in period-$t+s$ for good produced by the firm that sets its price at period t, $Q_{t,t+s}$ is the nominal stochastic discount factor that is defined as:

$$Q_{t,t+s} = \beta^s \frac{P_t C_t}{P_{t+s} C_{t+s}}, \tag{14}$$

$\tau = \frac{1}{\theta}$ (i.e., monopoly distortion is eliminated in steady state), and Ψ denotes the nominal total production cost function. The first order condition of this problem is:

$$E_t \sum_{s=0}^\infty (\beta\xi)^s Q_{t,t+s} Y_{t+s|t} \left[P_t^* - P_{t+s} mc_{t+s|t} \right] = 0, \tag{15}$$

where P_t^* is the optimal nominal price, and $mc_{t+s|t}$ denotes the real marginal cost that the firm faces. From (10) and (11),

$$mc_{t+s|t} = \frac{r_{t+s}^K Y_{t+s|t}^{\frac{\epsilon}{1-\epsilon}} A_{t+s}^{\frac{-1}{1-\epsilon}} Z_{t+s}^{\frac{-\epsilon}{1-\epsilon}}}{1 - \epsilon}. \tag{16}$$

2.3 Government

To focus the effect of monetary policy, balanced budget is assumed:

$$\int_0^1 \tau P_t Y_t(i) mc_t di = -P_t T_t. \tag{17}$$

The details of monetary policy will be described below.

2.4 Equilibrium

From (5) and the assumption of Calvo pricing,

$$P_t^{1-\theta} = (1-\xi)(P_t^*)^{1-\theta} + \xi P_{t-1}^{1-\theta}, \tag{18}$$

or:

$$1 = (1-\xi)(p_t^*)^{1-\theta} + \xi \pi_t^{\theta-1}, \tag{19}$$

where $\pi_t \equiv \frac{P_t}{P_{t-1}}$ denotes the (gross) inflation rate and $p_t^* \equiv P_t^*/P_t$.

The final good market clearing condition is:

$$Y_t = C_t + I_t. \tag{20}$$

The good i market clearing condition is;

$$A_t K_t(i)^{1-\epsilon} Z_t^\epsilon = \left(\frac{P_t(i)}{P_t}\right)^{-\theta} Y_t. \tag{21}$$

and the resource constraint of capital:

$$K_t = \int_0^1 K_t(i) di \tag{22}$$

must hold. From these equations,

$$K_t = \left(\frac{Y_t}{A_t Z_t^\epsilon}\right)^{\frac{1}{1-\epsilon}} \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{\frac{-\theta}{1-\epsilon}} di, \tag{23}$$

therefore,

$$K_t = \left(\frac{Y_t d_t}{A_t Z_t^\epsilon}\right)^{\frac{1}{1-\epsilon}}, \tag{24}$$

where d_t denotes the degree of relative price dispersion, which is defined by:

$$d_t \equiv \left[\int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{\frac{-\theta}{1-\epsilon}} di\right]^{1-\epsilon}, \tag{25}$$

or, using (19), we obtain a recursive representation,

$$d_t^{\frac{1}{1-\epsilon}} = (1-\xi)(p_t^*)^{\frac{-\theta}{1-\epsilon}} + \xi \pi_t^{\frac{\theta}{1-\epsilon}} d_{t-1}^{\frac{1}{1-\epsilon}}. \tag{26}$$

Here, similar as Gali (2008), we define the average real marginal cost mc_t as the factor price divided by the (private) aggregate marginal product of capital:

$$mc_t \equiv \frac{r_t^K}{(1-\epsilon)Y_t/K_t}. \tag{27}$$

(4), (16), (24), and (27) implies:

$$mc_{t+s|t} = \left(\frac{P_t^*}{P_{t+s}}\right)^{\frac{-\epsilon\theta}{1-\epsilon}} d_{t+s}^{\frac{1}{\epsilon-1}} mc_{t+s}. \tag{28}$$

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Thus, the optimal pricing condition (15) can be rewritten by using the average real marginal cost as:

$$E_t \sum_{s=0}^{\infty} (\beta\xi)^s Q_{t,t+s} \left(\frac{P_t^*}{P_{t+s}}\right)^{-\theta} Y_{t+s} \left[P_t^* - P_{t+s} \left(\frac{P_t^*}{P_{t+s}}\right)^{\frac{-\epsilon\theta}{1-\epsilon}} d_{t+s}^{\frac{1}{\epsilon-1}} mc_{t+s}\right] = 0, \tag{29}$$

Finally, we simply assume the knowledge level of the economy is represented by the average capital stock (learning-by-doing):

$$Z_t = \bar{K}_t = K_t. \tag{30}$$

In the economy, output, capital stock, consumption, and investment grow at same rate on the balanced-growth path. To write the equilibrium conditions by only stationary variables, we define:

$$y_t \equiv \frac{Y_t}{K_t}, \quad g_t \equiv \frac{K_t}{K_{t-1}},$$

$$c_t \equiv \frac{C_t}{K_t}, \quad i_t \equiv \frac{I_t}{K_t}.$$

and after some algebra, we find that the following proposition holds.

Proposition 1 Given the state variable d_{-1}, the exogenous stochastic process $\{A_t\}$, and monetary policy $\{R_t\}$, the competitive equilibrium $\{y_t,\ g_{t+1},\ i_t,\ c_t,\ d_t,\ \chi_t^1,\ \chi_t^2\},\ \{mc_t,\ \pi_t,\ p_t^*\}$ satisfies the following equations:

$$1 = (1-\xi)(p_t^*)^{1-\theta} + \xi\pi_t^{\theta-1} \tag{31}$$

$$\frac{1}{c_t} = E_t\beta \frac{R_t}{g_{t+1}c_{t+1}\pi_{t+1}} \tag{32}$$

$$\chi_t^1 = \frac{y_t}{c_t}(p_t^*)^{\frac{1-\epsilon+\epsilon\theta}{1-\epsilon}} + E_t\beta\xi\pi_{t+1}^{\theta-1}\left(\frac{p_t^*}{p_{t+1}^*}\right)^{\frac{1-\epsilon+\epsilon\theta}{1-\epsilon}}\chi_{t+1}^1 \tag{33}$$

$$\chi_t^2 = \frac{y_t}{c_t}d_t^{\frac{1}{\epsilon-1}}mc_t + E_t\beta\xi\pi_{t+1}^{\theta-2+\frac{\epsilon\theta}{1-\epsilon}}\chi_{t+1}^2 \tag{34}$$

$$\chi_t^1 = \chi_t^2 \tag{35}$$

$$d_t^{\frac{1}{\epsilon-1}} = (1-\xi)(p_t^*)^{\frac{-\theta}{1-\epsilon}} + \xi\pi_t^{\frac{\theta}{1-\epsilon}}d_{t-1}^{\frac{1}{\epsilon-1}} \tag{36}$$

$$y_t d_t = A_t \tag{37}$$

$$g_{t+1} = i_t^{1-\phi} \tag{38}$$

$$\frac{i_t}{(1-\phi)c_t} = E_t\beta\frac{1}{c_{t+1}}\left[(1-\epsilon)y_{t+1}mc_{t+1} + \frac{\phi}{1-\phi}i_{t+1}\right] \tag{39}$$

$$y_t = c_t + i_t \tag{40}$$

The equilibrium conditions consists on two blocks: the nominal and real side. On the one hand, the nominal side of equilibrium conditions, which consists on (31)-(36), determines the sequence $\{\pi_t, p_t^*, mc_t, d_t, \chi_t^1, \chi_t^2\}$, for given $\{R_t, y_t, c_t, g_{t+1}\}$. On the other hand, the real side of equilibrium conditions, which consists on (37)-(40), determines the sequence $\{y_t, c_t, i_t, g_{t+1}\}$, for given $\{d_t, mc_t, A_t\}$. If good prices are fully flexible, then $mc_t = 1$; hence the real allocation is determined by only the real side of equilibrium conditions, i.e., the monetary policy is neutral even in short-run.

2.5 Zero-Inflation Steady State

The variables without time index denotes the steady state values of the corresponding ones. At the zero-inflation steady state, the nominal side of equilibrium conditions can be easily derived.

$$\pi = 1, \quad p^* = 1, \quad mc = 1, \quad d = 1. \tag{41}$$

The real allocation is derived from the real side of equilibrium condition:

$$y = A, \quad i = \frac{(1-\phi)(1-\epsilon)\beta A}{1-\beta\phi},$$

$$c = \frac{1-\beta(1-\epsilon-\epsilon\phi)}{1-\beta\phi}A, \quad g = \left(\frac{(1-\phi)(1-\epsilon)\beta A}{1-\beta\phi}\right)^{1-\phi}. \tag{42}$$

2.6 Log-linearization

$\hat{x}_t$ denotes the log-deviation of an endogenous variable x_t from its steady state value.

The definition of price level (31) and the optimal pricing equation (33)-(35) are log-approximated up to first order to obtain the New Keynesian Phillips curve:

$$\pi_t = \beta E_t \pi_{t+1} + \lambda \hat{mc}_t + O(||\zeta||^2). \tag{43}$$

where:

$$\lambda \equiv \frac{(1-\xi\beta)(1-\xi)}{\xi}\Theta, \quad \Theta \equiv \frac{1-\epsilon}{1-\epsilon+\epsilon\theta}. \tag{44}$$

The Euler equation (32) is log-approximated up to first order to obtain the (endogenous growth version of) dynamic IS equation:

$$\hat{c}_t = \hat{g}_{t+1} + E_t\hat{c}_{t+1} - (\hat{R}_t - E_t\pi_{t+1}) + O(||\zeta||^2). \tag{45}$$

The aggregate production function (37) and the capital accumulation equation (38) are log-linearized to:

$$\hat{y}_t + \hat{d}_t = \hat{A}_t, \tag{46}$$

$$\hat{g}_{t+1} = (1-\phi)\hat{i}_t, \tag{47}$$

The capital market optimal condition (39), and the final good market clearing condition (40) is log-approximated up to first order to obtain:

$$\hat{i}_t - \hat{c}_t = -E_t\hat{c}_{t+1} + X^1 E_t(\hat{y}_{t+1} + \hat{mc}_{t+1}) + X^2 E_t\hat{i}_{t+1} + O(||\zeta||^2). \tag{48}$$

$$\hat{y}_t = \frac{c}{y}\hat{c}_t + \frac{i}{y}\hat{i}_t + O(||\zeta||^2) \tag{49}$$

where:

$$X^1 \equiv \frac{(1-\epsilon)y}{(1-\epsilon)y + \frac{\phi}{1-\phi}i} \tag{50}$$

and $X^2 = 1 - X^1$.

3 The Natural and Efficient Rates

3.1 The Natural Rate

If goods' prices are fully flexible, $mc_t = 1$ for all t. Therefore the real allocation $\{y_t, c_t, i_t, g_{t+1}\}$ satisfies (37) with $d_t = 1$, (38), (39), and (40). This flexible-price model can be solved analytically. The allocation in the flexible-price economy is as follows.

$$y_t^n = A_t \tag{51}$$

$$c_t^n = \frac{1 - \beta(1 - \epsilon + \epsilon\phi)}{1 - \beta\phi} A_t \tag{52}$$

$$i_t^n = \frac{(1 - \phi)(1 - \epsilon)\beta}{1 - \beta\phi} A_t \tag{53}$$

$$g_{t+1}^n = \left(\frac{(1 - \phi)(1 - \epsilon)\beta}{1 - \beta\phi} A_t \right)^{1-\phi} \tag{54}$$

where x_t^n denotes the realized value of an endogenous variable x_t. We define the natural rate as $\{y_t^n, c_t^n, i_t^n, g_{t+1}^n\}$. As discussed in Edge (2003), there are two competing definitions of the natural rate in the economies with endogenous state variables. The first concept is the one defined by Neiss and Nelson (2003). Their definition of the natural rate based on the endogenous state variables (in their model, the capital stock) which would have been in place had the economy always existed in a flexible-price world. The second concept is the one defined by Woodford (2003). He defines the natural rate based on the actual endogenous state variables with which the economy enters each period. In our model, however, we do not need to distinguish the two concepts of the natural rates. This is because, in our endogeneous growth model with only one type of capital, the model variables are defined in the form of the ratio to capital hence there is not any endogenous state variables.[1]

For our purposes, we rewrite (51)-(54) as the form of log-deviation.

$$\hat{y}_t^n = \hat{A}_t \tag{55}$$

$$\hat{c}_t^n = \hat{A}_t \tag{56}$$

$$\hat{i}_t^n = \hat{A}_t \tag{57}$$

$$\hat{g}_{t+1}^n = (1 - \phi)\hat{A}_t \tag{58}$$

3.2 The Efficient Rate

When prices are fully flexible ($\xi = 0$) and when there is no externality ($\epsilon = 0$), the all distortion in this economy is eliminated so that the real allocation in the competitive equilibrium coincides with the efficient one. Therefore, from (51)-(54), the efficient rate $\{y_t^e, c_t^e, i_t^e, y_t^e\}$ satisfies the following equations.

$$y_t^e = A_t \tag{59}$$

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[1]Of course, from the point of view of original level variables, it is important to distinguish the two concepts of the natural rate. For example, the Neiss and Nelson' natural level of capital is defined as $K_t^n \equiv K_0 \times g_1^n \cdots \times g_t^n$. By contrast, the Woodford's natural level of capital is $K_t^n \equiv K_t (= K_0 \times g_1 \times \cdots \times g_t)$.

$$c_t^e = \frac{1-\beta}{1-\beta\phi} A_t \tag{60}$$

$$i_t^e = \frac{(1-\phi)\beta}{1-\beta\phi} A_t \tag{61}$$

$$g_{t+1}^e = \left(\frac{(1-\phi)\beta}{1-\beta\phi} A_t \right)^{1-\phi} \tag{62}$$

or as the log-deviation form,

$$\hat{y}_t^e = \hat{A}_t \tag{63}$$

$$\hat{c}_t^e = \hat{A}_t \tag{64}$$

$$\hat{i}_t^e = \hat{A}_t \tag{65}$$

$$\hat{g}_{t+1}^e = (1-\phi)\hat{A}_t \tag{66}$$

(52)-(54) and (60)-(62) imply the following proposition.

Proposition 2 If there is externality in the good production($\epsilon > 0$), the consumption-capital ratio is inefficiently high and the investment-capital ratio and the growth rate of capital are inefficently low in the flexible-price economy, that is,

$$c_t^n > c_t^e, \quad i_t^n < i_t^e, \quad \text{and} \quad g_{t+1}^n < g_{t+1}^e. \tag{67}$$

Proof. From $0 < \beta < 1, 0 < \phi < 1, 0 < \epsilon < 1$,

$$\begin{aligned} c_t^n - c_t^e &= \frac{1-\beta(1-\epsilon+\epsilon\phi)}{1-\beta\phi} A_t - \frac{1-\beta}{1-\beta\phi} A_t \\ &= \frac{\epsilon\beta(1-\phi)}{1-\beta\phi} A_t > 0. \end{aligned} \tag{68}$$

$$\begin{aligned} i_t^n - i_t^e &= (y_t^n - c_t^n) - (y_t^e - c_t^e) \\ &= c_t^e - c_t^n < 0. \end{aligned} \tag{69}$$

$$g_{t+1}^n - g_{t+1}^e = (i_t^n)^{1-\phi} - (i_t^e)^{1-\phi} < 0. \tag{70}$$

The intuition is very simple. Due to externality in the good production, the equilibrium rate of return on capital is lower than the efficient rate; households invest less, hence growth rate is lower. However, the next proposition shows the externality does not affect the business cycle dynamics in the flexible-price economy. (the proof is so easy that we omit it.)

Proposition 3 The natural rates of output, consumption, investment and growth, measured by log-deviation from steady state, fluctuate parallel to the effcient rates of those, that is,

$$\hat{y}_t^n = \hat{y}_t^e, \quad \hat{c}_t^n = \hat{c}_t^e,$$
$$\hat{i}_t^n = \hat{i}_t^e, \quad \hat{g}_{t+1}^n = \hat{g}_{t+1}^e.$$

As discussed in Woodford (2003), The standard New Keynesian model does not face the inflation-output tradeoff, even when its steady state is inefficient, because the natural rate of output moves parallel to its efficient rate. Therefore the strict price stabilization policy is optimal in that economy. In our model, as in the standard New Keynesian model, the natural rates moves parallel to their efficient rates. However, as showed below, the strict price stabilization is not optimal. To see that, in the next section we will derive the utility-based welfare criterion for the welfare implication of monetary policy.

4 Deriving A Utility-Based Welfare Criterion

In this section, we derive the utility-based loss function by using the second-order approximation to the household's utility function, following Rotemberg and Woodford (1997), Woodford (2003), and Edge (2003).

4.1 The Second-Order Approximation to the Utility Function

The welfare level on the economy is defined as the households' expected discounted sum of utility:

$$W_0 = E_0 \sum_{t=0}^{\infty} \beta^t \log C_t. \tag{71}$$

First, we rewrite it by using only stationary variables.

$$
\begin{aligned}
W_0 &= E_0 \left(\log C_0 + \beta \log C_1 + \beta^2 \log C_2 + \cdots \right) \\
&= E_0 \left[(\log c_0 + \log K_0) + \beta(\log c_1 + \log K_1) + \beta^2(\log c_2 + \log K_2) + \cdots \right] \\
&= E_0 \sum_{t=0}^{\infty} \beta^t \log c_t + E_0 \left[\log K_0 + \beta(\log K_0 + \log g_1) + \beta^2(\log K_0 + \log g_1 + \log g_2) + \cdots \right] \\
&= \frac{1}{1-\beta} \log K_0 + E_0 \sum_{t=0}^{\infty} \beta^t \log c_t + E_0 \left[\frac{\beta}{1-\beta} \log g_1 + \frac{\beta^2}{1-\beta} \log g_2 + \cdots \right] \\
&= E_0 \sum_{t=0}^{\infty} \beta^t \left[\log c_t + \frac{\beta}{1-\beta} \log g_{t+1} \right] + t.i.p. \tag{72}
\end{aligned}
$$

where $t.i.p.$ denotes the terms independent of policy. We define the with-in period utility as:

$$U_t \equiv \log c_t + \frac{\beta}{1-\beta} \log g_{t+1}. \tag{73}$$

Next, c_t is eliminated from the with-in period utility U_t by using the final good market clearing condition (40) and it is approximated up to second-order.

$$
\begin{aligned}
U_t &= \log(y_t - i_t) + \frac{\beta}{1-\beta} \log g_{t+1} \\
&= U + \frac{y}{c}\hat{y}_t - \frac{i}{c}\hat{i}_t + \frac{\beta}{1-\beta}\hat{g}_{t+1} - \frac{yi}{2c^2}\hat{y}_t^2 + \frac{yi}{c^2}\hat{y}_t\hat{i}_t - \frac{yi}{2c^2}\hat{i}_t^2 + O(||\zeta||^3). \tag{74}
\end{aligned}
$$

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4.2 Simplifying the Approximated With-in Period Utility Function

4.2.1 Eliminating the Terms of $\hat{y}_t$

First, we focus on the term of $\hat{y}_t$. From (46), $\hat{y}_t = \hat{A}_t - \hat{d}_t$, hence here we approximate $\hat{d}_t$ up to second-order.

Proposition 4

$$\hat{d}_t = \frac{1}{2}\frac{\theta}{\Theta}var_i\{\log P_t(i)\} + O(||\zeta||^3). \tag{75}$$

Proof: See Appendix in Chapter 4 in Gali (2008).

Therefore, (74) can be rewritten as:

$$U_t - U = -\frac{i}{c}\hat{i}_t + \frac{\beta}{1-\beta}\hat{g}_{t+1}$$
$$+ \frac{yi}{c^2}\hat{A}_t\hat{i}_t - \frac{yi}{2c^2}\hat{i}_t^2 - \frac{1}{2}\frac{y}{c}\frac{\theta}{\Theta}var_i\{\log P_t(i)\} + O(||\zeta||^3) + t.i.p. \tag{76}$$

4.2.2 Eliminating the Terms of $\hat{i}_t$

Next, we focus on the term of $\hat{i}_t$. From (47), $\hat{i}_t = \hat{g}_{t+1}/(1-\phi)$, hence (76) can be rewritten as:

$$U_t - U = \Phi\frac{\beta}{1-\beta}\hat{g}_{t+1} + \frac{yi}{c^2(1-\phi)}\hat{A}_t\hat{g}_{t+1} - \frac{yi}{2c^2(1-\phi)^2}\hat{g}_{t+1}^2$$
$$- \frac{1}{2}\frac{y}{c}\frac{\theta}{\Theta}var_i\{\log P_t(i)\} + O(||\zeta||^3) + t.i.p., \tag{77}$$

where[2]

$$\Phi \equiv \frac{\epsilon(1+\beta\phi)}{1-\beta[1-\epsilon(1+\phi)]} \geq 0 \tag{78}$$

If the steady-state allocation is efficient ($\epsilon = 0$), then $\Phi = 0$. In addition, Φ is increasing in ϵ. Thus, Φ can be interpreted as the degree of the *steady-state distortion*, as discussed in Woodford (2003). For simplicity, we assume that the steady-state distortion is sufficiently small so that the product term between Φ and a second-order term can be ignored as negligible.[3]

4.2.3 Simplifying the Cross-Product Terms

In order to simplifying the second-order cross-product term of the growth rate and shock, $\frac{yi}{c^2(1-\phi)}\hat{A}_t\hat{g}_{t+1}$, we use the definition of the natural rate. From (58), $A_t = \hat{g}_{t+1}^n/(1-\phi)$; hence we can be rewritten (77) as:

$$U_t - U = \Phi\frac{\beta}{1-\beta}\hat{g}_{t+1} + \frac{yi}{c^2(1-\phi)^2}\hat{g}_{t+1}\hat{g}_{t+1}^n - \frac{yi}{2c^2(1-\phi)^2}\hat{g}_{t+1}^2$$
$$- \frac{1}{2}\frac{y}{c}\frac{\theta}{\Theta}var_i\{\log P_t(i)\} + O(||\zeta||^3) + t.i.p., \tag{79}$$

---

[2]Φ can be derived by using the steady-state allocation equation (42).

[3]The case of large steady-state distortion is discussed in Benigno and Woodford (2005).

Here, we find that the within-period utility can be described by the terms of growth rate and price dispersion.

4.2.4 Expressing the With-in Period Utility by the "Gap" Terms

In order to obtain the welfare implication below, we express the approximated with-in period utility (79) by the "gap" terms, i.e., the difference between the actual rate and the natural or efficient rate of the growth of capital stock. The way to express (79) by the gap terms is simple. Note that the natural rate of growth of capital stock, $\hat{g}_{t+1}^n$ is independent of policy. Therefore, we can simply add $\hat{g}_{t+1}^n$ or its square or its product with arbitary constant. In order to express (79) by the gap terms, we add:

$$-\Phi\frac{\beta}{1-\beta}\hat{g}_{t+1}^n - \frac{yi}{2c^2(1-\phi)^2}(\hat{g}_{t+1}^n)^2 \tag{80}$$

to (79). Then, (79) can be rewritten as:

$$U_t - U = \Phi\frac{\beta}{1-\beta}(\hat{g}_{t+1} - \hat{g}_{t+1}^n) - \frac{yi}{2c^2(1-\phi)^2}\left(\hat{g}_{t+1}^2 - 2\hat{g}_{t+1}\hat{g}_{t+1}^n + (\hat{g}_{t+1}^n)^2\right)$$
$$- \frac{1}{2}\frac{y}{c}\frac{\theta}{\Theta}var_i\{\log P_t(i)\} + O(||\zeta||^3) + t.i.p. \tag{81}$$

Here we define the *capital growth gap*, $\check{g}_{t+1}$, as the log-difference between the actual rate and the natural counterpart:

$$\check{g}_{t+1} \equiv \log g_{t+1} - \log g_{t+1}^n. \tag{82}$$

Using the fact that $g = g^n$,

$$\check{g}_{t+1} = \hat{g}_{t+1} - \hat{g}_{t+1}^n + (\log g - \log g^n)$$
$$= \hat{g}_{t+1} - \hat{g}_{t+1}^n. \tag{83}$$

Hence, (81) can be rewritten by the gap terms:

$$U_t - U = \Phi\frac{\beta}{1-\beta}\check{g}_{t+1} - \frac{yi}{2c^2(1-\phi)^2}\check{g}_{t+1}^2 - \frac{1}{2}\frac{y}{c}\frac{\theta}{\Theta}var_i\{\log P_t(i)\} + O(||\zeta||^3) + t.i.p. \tag{84}$$

From the welfare point of view, more important is the difference between the actual rate and the efficient counterpart. Then we define the *welfare-relevant growth gap*, $\boldsymbol{g}_{t+1}$ as:

$$\boldsymbol{g}_{t+1} \equiv \log g_{t+1} - \log g_{t+1}^e$$
$$= \hat{g}_{t+1} - \hat{g}_{t+1}^e + (\log g - \log g^e)$$
$$= \hat{g}_{t+1} - \hat{g}_{t+1}^n + (\log g - \log g^e)$$
$$= \check{g}_{t+1} + \boldsymbol{g}. \tag{85}$$

Hence,

$$U_t - U = \Phi\frac{\beta}{1-\beta}(\boldsymbol{g}_{t+1} - \boldsymbol{g}) - \frac{yi}{2c^2(1-\phi)^2}(\boldsymbol{g}_{t+1} - \boldsymbol{g})^2 - \frac{1}{2}\frac{y}{c}\frac{\theta}{\Theta}var_i\{\log P_t(i)\} + O(||\zeta||^3) + t.i.p. \tag{86}$$

11

4.3 The Welfare Criterion

In this subsection, we derive the approximated overall utility written by the gap terms. Summing up (86) across periods, we obtain:

$$
W_0 = E_0 \sum_{t=0}^{\infty} \beta^t (U_t - U) + t.i.p
$$

$$
= E_0 \sum_{t=0}^{\infty} \beta_t \Big[\Phi \frac{\beta}{1-\beta} (\boldsymbol{g}_{t+1} - \boldsymbol{g}) - \frac{yi}{2c^2(1-\phi)^2} (\boldsymbol{g}_{t+1} - \boldsymbol{g})^2
$$

$$
- \frac{1}{2} \frac{y}{c} \frac{\theta}{\Theta} var_i \{ \log P_t(i) \} \Big] + O(||\zeta||^3) + t.i.p. \tag{87}
$$

Here we use the following proposition.

Proposition 5

$$
\sum_{t=0}^{\infty} \beta^t var_i \{ \log P_t(i) \} = \frac{\xi}{(1-\beta\xi)(1-\xi)} \sum_{t=0}^{\infty} \beta^t \pi_t^2. \tag{88}
$$

Proof: See Chapter 6 in Woodford (2003).

From (87) and (88), we obtain the utility-based welfare criterion.

Proposition 6 The utility-based welfare criterion of our model can be described as follows:

$$
W_0 = E_0 \sum_{t=0}^{\infty} \beta_t \Big[\Phi \frac{\beta}{1-\beta} \boldsymbol{g}_{t+1} - \frac{yi}{2c^2(1-\phi)^2} (\boldsymbol{g}_{t+1} - \boldsymbol{g})^2 - \frac{1}{2} \frac{y}{c} \frac{\theta}{\lambda} \pi_t^2 \Big] \tag{89}
$$

Proposition 6 gives us an important point of view about our model. In our model, the relevant variables for welfare are the growth rate of capital and inflation. In addition, more importantly, the welfare depends on not only the fluctuation but also the average rate of capital growth whenever $\Phi > 0$, that is, there is the externality in good production ($\epsilon > 0$). The reason is quite simple. When there is the externality in good production, the average rate of growth is inefficiently low even in the flexible price economy; hence, given the other condition, a rise of the average growth weaken the inefficiency from the externality and hence the welfare improves. However, note that proposition 6 does not necessarily claim that there is a tradeoff between inflation and growth. Clearly there is not the tradeoff between the inflation stabilization and the growth stabilization even if $\epsilon > 0$. If central bank completely stabilizes inflation, the difference between the actual growth rate of capital and its efficient rate is always constant; hence the welfare loss from the second and third term of (89) are zero. Accordingly, a necessary condition that there is tradeoff between inflation and growth is that the volatile inflation stimulates the average growth.[4] To investigate whether the tradeoff exists, we do a numerical exercise in the next section.

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[4]It is not the sufficient condition because the volatile inflation necessarily causes the inefficient fluctuation of growth.

5 The Tradeoff between Inflation Stabilization and Growth Maximization

As discussed above, it is not clear that the model economy has the tradeoff between inflation and growth. To see that, a numerical experiment is done in this section.

5.1 Calibration

The parameter values are summarized in Table 1. The calibration strategy is as follows.

The (zero-inflation) steady-state growth rate is set to 1.0045. β is set so that the annual real rate of interest equal to 1.04. θ is set so that the steady-state average markup is 20%. ξ is set to 0.6. ϵ and ϕ are set to 0.05 and 0.9, respectively. The exogenous process governing the productivety shock is assumed that:

$$\hat{A}_{t+1} = \rho_A \hat{A}_t + \epsilon_{t+1}^A, \quad \epsilon_{t+1}^A \sim N(0, \sigma_A^2). \tag{90}$$

where ρ_A and σ_A is set to 0.9 and 0.005, respectively.

Monetary authority sets the nominal interest rate according to a log-linear interest rate feedback rule:

$$\hat{R}_t = \alpha_\pi \pi_t + \alpha_I \hat{i}_t. \tag{91}$$

5.2 Results

We simulate the model economy in order to see the effect of alternative monetary policy rules (changing the value of α_π and α_I, in the ranges of $1 < \alpha_\pi < 5$ and $0 < \alpha_I < 4$) on the inflation volatility, the capital growth volatility, the mean growth, and the welfare cost. Figure 1-2 show those results. First, from Figure 1, we can see that the inflation volatility decreases as α_π is larger, and increases as α_I is larger; hence, if there were not tradeoff between inflation and growth then welfare would higher as α_π is larger and α_I is smaller through the inflation stabilization. However, we can see the opposite results in Figure 2, in which we find that the strong anti-inflationary stance is detriment for welfare.[5] These results imply that the tradeoff between inflation and growth exists in this economy. Why do such results occur? The reason is the existence of concavity in the capital accumulation technology. Barlevy (2004) shows that in endogenous growth models in which the capital accumulation technology is concave, the reduction of investment volatility stimulates the average growth. Also in our economy, the capital accumulation technology is concave; hence monetary policy that stabilizes investment (implied by large α_I) stimulates average growth. In our model the growth rate is inefficiently low due to the externality in good production, so that the policy which stimulates growth tends to improve welfare. In addition, as pointed out by Lucas (1987) and Lucas (2003), the increase of growth rate has a strong welfare effect, compared to simply stabilization. Accordingly, the tradeoff between growth and inflation is resolved by highly growth-stimulating and inflation-volatilizing policy.

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[5]Note that the welfare "cost" is showed in Figure 2; hence the high value in this figure implies low welfare level.

6 Conclusion

This paper has been analyzed monetary policy implication in an endogenous growth model in which the average growth rate is inefficiently low and in which the capital accumulation technology is concave.

This paper shows two implication with respect to the interaction between growth and volatility. First, we derive the utility-based welfare criterion of the model. The welfare measure suggests that even if the natural rate of growth moves parallel to its efficient rate, the increase of inflation volatility may improve welfare through the increase of average growth. Second, we tests this hypothesis numerically and show that in our calibrated model the tradeoff between inflation stabilization and average growth maximization exists. In addition, the tradeoff is resolved by highly growth-stimulating (investment stabilization) policy. The reason is the existence of concavity in the capital accumulation technology, through which investment stabilization rises average growth.

This study has some future research directions. First, we do not derive the Ramsey optimal policy. The derivation of the approximated welfare function enables us to derive the Ramsey allocation analytically. The analytical solution would brings many fruitful results to study of monetary policy. Second, Our model has some extreme assumption, for example, log-utility, full depreciation of capital. The relaxation of these assumptions would make analytically solving model difficult or impossible. However, solving numerically such models which has more reality could test the robustness of our results.

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Table 1: Structural parameters

A	β	ϵ	ϕ	ρ_A	σ_A	θ	ξ
1.16	1.0045/1.01	0.05	0.9	0.9	0.005	6	0.6

Fig. 1: The Inflation Volatility

Fig. 2: Welfare Cost

welfarecost

ALPI

ALPPAI
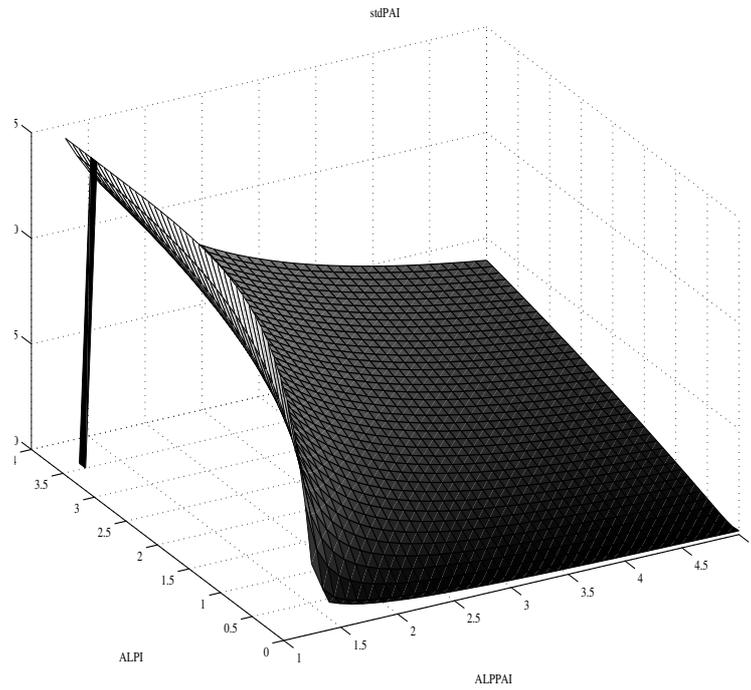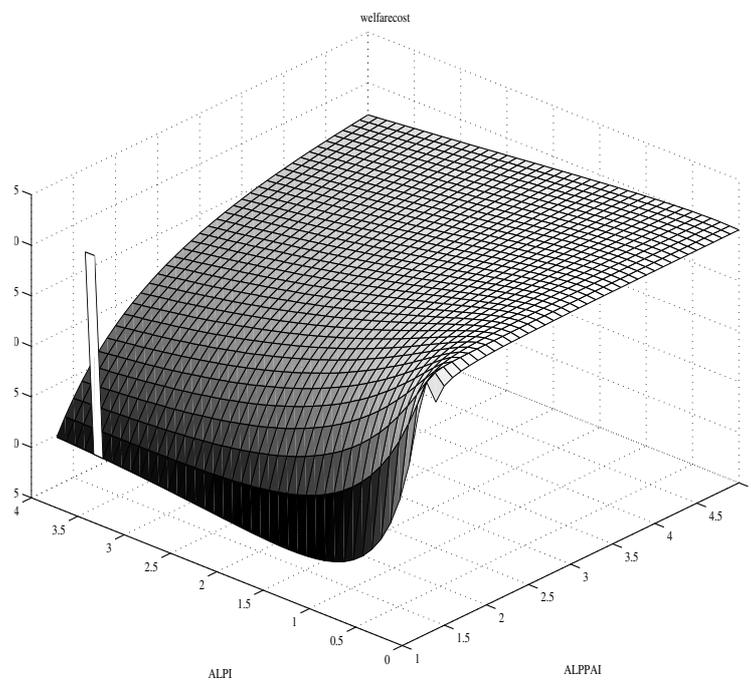
Fig. 3: The Growth Volatility

Fig. 4: The Average Growth Rate