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“Public Provision of Private Child Goods”

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Public Provision of Private Child Goods*

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Abstract

This paper analyzes the public provision of private goods for children in a politico-economic model with endogenous fertility. The government provides every child with goods that can also be purchased by parents in private markets, and the level of provision is determined by majority rule. Households with many children benefit from the public provision more than those with fewer children; thus, a political conflict arises between them. The distribution of the number of children across households, which is a crucial factor for determining which group is politically dominant, is endogenously determined by households' fertility decision. The sequential interaction between fertility and political decisions might lead to multiple equilibria: equilibrium with high-fertility and low-private/public-spending-ratio and equilibrium with low-fertility and high-private/public-spending-ratio. Our model could explain the large differences in fertility and structure of child-related spending across countries.

JEL classification: I28; J13; H42

Keywords: Education; Fertility; Publicly Provided Private Goods

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1 Introduction

In most countries, the government provides goods that are available in private markets; these include education, health care, transportation, postal service, and police protection. Issues related to such dual-provision schemes increasingly come to the forefront in policy debates and are investigated by academic economists. This paper concentrates on an important aspect of such schemes that has been neglected by past studies in political economics: some of dual-provision schemes, such as education and childcare services, are targeted at children, and these affect not only the private purchase but also the fertility decision in each household.

The current debate on public policies for children often focuses on the effects of these policies on fertility rates against the backdrop of heightened concerns over declining fertility rates in some developed countries. The difference in the generosity of public services for children is considered to be a source of the fertility differential across countries; in fact, positive correlation between generosity and the fertility rate is often found in data (e.g., d'Addio and d'Ercole, 2005; Feyrer et al., 2008). The argument that generous public policies for children should reduce the out-of-pocket child-related costs borne by households and thus make them fertile can be deduced from standard textbook models of family economics (e.g., Cigno, 1991; Ermisch, 2003). However, there remains an unanswered question: why does the public policy for children vary greatly across countries? This paper demonstrates that an explanation to this question follows from standard economic models in a less extreme form.

Using a simple functional form for preferences and standard assumptions about the quantity-quality tradeoff for children, we develop an extension of the public-provision-of-private-goods model of Epple and Romano (1996b) and Gouveia (1997) . The basic framework is the standard "demand" model of fertility behavior (see Birdsall (1988) for a summary), in which each household receives utility from the quantity and quality of children and chooses both of them in the face of a constraint on the total amount of time that can be devoted to child rearing and labor supply. The government provides every child with goods that improve his or her quality—for instance, education—these goods can also be purchased by parents in private markets. The funding level for public provision is collectively determined by majority rule. Households with many children benefit from the public provision more than those with fewer children; thus, a political conflict arises between them. What is important in our model is that the number of children, which is crucial for determining whether or not a household belongs to the group supportive of public provision, is an endogenous variable derived from the household's fertility decision.

We show that under a realistically plausible assumption on the timing of fertility and voting decisions, the single-peakedness of the preferences over tax rates is satisfied in the voting process and the median-voter theorem holds (see Black, 1948). There exists at least one rational-expectation political equilibrium such that the voting outcome expected by households in their fertility decision is consistent with the voting outcome actually chosen in the economy populated by those households. Furthermore, multiple equilibria arise for reasonable parameters: both the situations—where a majority of the population prefers a relatively high funding level or a relatively low funding level—are consistent with the behaviors of fully-rational households. Since the public provision for children induces parents to substitute quantity for quality by affecting the relative price of quantity to quality, the equilibrium with high (low) public funding corresponds to the one with low (high) private funding and high (low) fertility.

Since the types of issues we consider are prominent in the education debate, we refer to the good on which we focus in our analysis as education.[1] In contrast to our model, many past studies in this literature, including the influential work of Joseph Stiglitz (Stiglitz, 1974), assume that public education and private one are mutually exclusive: agents are not allowed to further supplement public education, but are allowed to opt out of it in order to attend private education. This assumption is based on the fact that an individual cannot be a full-time student at both public and private schools concurrently. However, there are several reasons why we believe that agents can privately supplement the public education. First, some forms of private supplements, such as private tutoring and summer

[1] The results of this paper are applicable, more generally, to dual-provision settings wherein each household can privately supplement the public provision, public provision is targeted at children, and parents place value on the child's consumption of that good. Other striking examples would be childcare and healthcare services.

schools, are feasible and actually observed. The purchase of materials contributing to the intellectual development of children, such as personal computers and encyclopedias, can also be considered as a private supplement. Second, in many developed countries, private schools are supported by the government in a variety of ways (e.g., subsidies, tax exemptions, and vouchers); in such cases, we can regard the private school expenses of households as private supplements in effect.[2] The assumption that agents can opt out but not top up is only one extreme, as is the one that agents can top up but not opt out: the reality must be somewhere between these two assumptions.[3]

Our model is especially applicable to the discussion about private tutoring—namely, education outside the formal schooling system: besides the formal school education that is usually provided free of charge by the government, parents send their children to private-tutoring facilities with the intention of improving their performance at school or increasing their chances of success at the entrance examination of universities.[4] In East Asian countries, typically, the ratio of public spending on education to GDP is low, whereas private education-related spending is high owing to the prevalence of private tutoring.[5] Furthermore, in East Asian countries, fertility is much lower than that in countries with similar GDP per capita. These distinctive features of East Asian countries have often been attributed to exogenous factors, such as the preferences and education system (e.g., the eagerness to educate children, existence of competitive entrance examinations to universities). The multiplicity derived in this paper implies that a seemingly exogenous education system might be collectively determined by rational self-interested agents.

Here, we characterize our research in the context of studies on the relationship between

[2]The extent of financial support for private schools at the elementary and secondary levels vary greatly across countries. For instance, private schools are fully supported by the government in Belgium, partially supported in France and New Zealand, and hardly supported in the United States (Toma, 1996).

[3]de la Croix and Doepke (2008) develop a politico-economic model with endogenous fertility under a polar assumption contrasting to ours: agents can opt out but not supplement public education. They employ probabilistic voting as the political mechanism because of the non-single-peakedness of preferences. One advantage of our model is tractability: we can rely on the single-peakedness of the topping-up model (Epple and Romano, 1996b; Gouveia, 1997)

[4]Private tutoring has long been a major phenomenon in parts of East Asia. Recently, it has grown rapidly in other areas, such as other parts of Asia, Europe, and North America, and is drawing more attention from educational scientists (Bray, 2006).

[5]See, for example, Kim and Lee (2001) and Bray and Kwok (2003) for details of the reality of education in East Asia.

fertility and investments in human capital. A large part of the literature is based on the growth theory and tries to explain the demographic transition in the process of economic development (e.g., Becker et al., 1990; Galor and Weil, 2000). In contrast, our model is static and regarded as a model explaining the fertility differential among post-demographic transition countries. Almost all developed countries experienced significant reductions in fertility in the past few centuries; however, the eventual fertility rates currently being observed vary greatly among countries: total fertility rates in 2004 are as low as 1.3 in Germany, Italy, Japan, and Spain and nearly 1.8 in Denmark, France, Norway, and Sweden (United Nations, 2007). Given the fact that countries with similar GDP per capita have varying fertility rates, it seems that the current fertility differential among developed countries cannot be explained only in terms of demographic transition. This paper improves on the literature by stressing that both the manner of investing (i.e., the ratio of public investment to private investment) and the quantity of investment play an important role in the determination of fertility.

In our model, self-fulfilling prophecies of voting can occur and lead to multiple equilibria. The intuition is simple; if pivotal households anticipate a high funding level for public education, they will choose to have a large number of children. Once the households decide to have many children, it is actually optimal for them to vote for a high funding level. Conversely, a low-funding-level equilibrium can also be sustained. The result that agents' expectations about future policy may be self-fulfilling when public policy is endogenized is not new; in particular, Saint-Paul and Verdier (1997) derive the similar multiplicity result in a voting model regarding the interaction between capital income tax and saving decisions.[6] However, we would like to emphasize that the mechanism generating multiplicity here is especially important for the issues discussed in this paper. What really matters for the mechanism to work well is that each household makes an economic decision prior to the determination of public policy and also that the decision is irreversible. Owing to the absence of a market for trading children, the fertility decision is one of the most

[6]The possibility that the feedback mechanism between economic and political decisions generates multiple equilibria has been pointed out in various contexts (e.g., Glomm and Ravikumar, 1996; Hassler et al., 2003).

irreversible decisions for any household.

The remainder of this paper is organized as follows. In the next section, we present our model and characterize the equilibrium while taking public policy as given. In Section 3, we endogenize the policy through majority voting and derive the political equilibrium. Section 4 concludes the paper.

2 The Model

2.1 The Economic Environment

We consider an economy with a large number of households with identical preferences over the numeraire commodity, c_i, the number of children, n_i, and the quantity of education for each child, e_i.[7] We normalize the population size to unity. Household i's preferences are represented by:[8]

$$\gamma \left(\phi \ln e_i + \ln n_i\right) + (1 - \gamma) \ln c_i,$$

where $\gamma \in (0, 1)$ and $\phi \in (0, 1)$ denote the relative weight given to children and the relative weight given to their education, respectively.[9] Households are differentiated on the basis of wage rate (i.e., the quantity of numeraire commodity that they can acquire per unit of time in the labor market), which is the only source of heterogeneity in our model. We assume that wages across households are distributed according to the cumulative distribution function $F(\cdot)$ with finite mean. It is assumed that the support of $F(\cdot)$ is $\mathbb{R}_+$ and that $F(\cdot)$ is strictly increasing and twice continuously differentiable.

To educate children, parents must utilize educational services, which can be obtained through private purchase and government provision. All children receive the same quantity of public education in this economy. On the other hand, households are allowed to supplement the publicly provided quantity, and the quantity of private purchase may differ

[7]We follow the standard quantity-quality fertility model in assuming that parents treat their children equally.

[8]The logarithmic utility function is chosen for the sake of brevity, as is often the case in this literature (e.g., de la Croix and Doepke, 2008); what matters for our results is the homotheticity of preferences over the bundle (e_i, n_i, c_i).

[9]The assumption $\phi < 1$ guarantees that the second-order condition for maximization is satisfied in the household's maximization problem; if $\phi \geq 1$, households obtain infinite utility by reducing the number of children close to zero and thereby increasing their education level close to infinity (see Equation (8)).

across households. It should be noted that the fact that the quantity of public education is common to all children does not mean that all households receive the same quantity of public services, because the number of children may differ across households. Household i's education level for each child, e_i, is given by

$$e_i = s_i + g, \tag{1}$$

where s_i and g denote the quantity of private purchase and quantity of public provision per child, respectively. The publicly provided quantity cannot be traded and thus, s_i must be nonnegative.[10]

Each household is endowed with a unit of time that can be devoted to child-rearing and labor-market activities. Raising one child takes fraction $z \in (0, 1)$ of a household's time. The government collects taxes from all households at a constant rate t. Each household allocates the after-tax income to consumption expenditures and private educational services. The budget constraint for household i whose wage is w_i is then given by

$$w_i (1 - zn_i) (1 - t) = c_i + s_i n_i. \tag{2}$$

It is assumed that one unit of educational services is produced with one unit of the numeraire in the private sector and that private educational services are provided by price-taking suppliers. The assumption that child-rearing requires parents' time input while child-education requires educational services traded in the market, which is often used in the quantity-quality model, is crucial for generating fertility differentials. It implies that the cost of having a child relative to the cost of education rises as the wage rises.

Public provision is financed by a proportional tax, t, on income. Assuming that the technology for converting expenditures into quantity of service is the same as that in the

[10] Utility is a function of the total amount of education e_i, but does not depend on the share of the quantity of private purchase, s_i, nor the share of the publicly provided quantity, g. This assumption follows the convention of the literature on the public provision of private goods.

private sector, the government's budget constraint is

$$t \int_0^\infty [1 - zn(w)]\, wdF(w) = g \int_0^\infty n(w)\, dF(w), \tag{3}$$

where $n(w)$ denotes the number of children of the household with wage w. To simplify notation, we introduce the following new variables: $\bar{n}$ and $\bar{m}$, which denote the average (as well as total) number of children, $\int_0^\infty n(w)\, dF(w)$, and the average (as well as total) labor income, $\int_0^\infty [1 - zn(w)]\, wdF(w)$, respectively. Equation (3) can be rewritten as

$$t\bar{m} = g\bar{n}. \tag{4}$$

Since public education is as efficient as private education and there is no externality in education, redistribution is the sole motive for public education in this model, as is the convention of the positive literature. It is noted that we cannot control t and g independently because they are linked through the government's budget constraint. If we choose one as a control variable, then the other is endogenously determined. Following the convention of the literature, we treat t as a policy variable that is determined by the government: it is exogenously fixed in this section, and chosen through voting in the next section.

2.2 The Household Decision Problem

Household i solves the following maximization problem:

$$\max_{s_i, n_i, c_i} \ \gamma(\phi \ln e_i + \ln n_i) + (1 - \gamma) \ln c_i, \tag{5}$$

$$\begin{aligned} \text{s.t.} \quad & \text{equations (1), (2), and (4),} \\ & s_i \geq 0,\ n_i \geq 0, \qquad (6) \\ & \text{given } t \in (0,1),\ \bar{m},\ \bar{n}. \end{aligned}$$

The solution to this problem can either be interior or at a corner; there is a threshold wage rate, $\hat{w}$, below which households choose a corner solution of not supplementing the publicly provided quantity. The threshold is

$$\hat{w} = \frac{t\bar{m}}{\phi z (1-t) \bar{n}}. \tag{7}$$

For a household with a sufficiently high wage such that $w_i > \hat{w}$ holds, there is an interior solution for the optimal private supplement level; the first-order conditions imply

$$s_i = \frac{\phi z (1-t) w_i \bar{n} - t\bar{m}}{(1-\phi)\bar{n}} \quad \text{and} \quad n_i = \frac{\gamma (1-\phi)(1-t) w_i \bar{n}}{z (1-t) w_i \bar{n} - t\bar{m}}. \tag{8}$$

It follows that $\partial s_i / \partial w_i > 0$ and $\partial n_i / \partial w_i < 0$. The higher the wage rate, the larger is the quantity of private education and the smaller the number of children. The above result is drawn from the following two assumptions: (i) the cost of having a child increases with wage, while the cost of education per child is independent of wage and (ii) due to the presence of public education, the education level of children remains positive even if the parents do not privately purchase any educational services.

It is noteworthy that the presence of public education, g, guarantees that the number of children is a nontrivial function of wage rate. Consider the impact of the rise in wage rate. On the one hand, it increases the cost of having children, ceteris paribus, because the time spent for child-rearing could have instead been used to work and purchase educational services. Consequently, parents are induced to substitute quality for quantity (the substitution effect). On the other hand, the rise in wage rate makes the household wealthier, and thus it desires to have more children and purchase more educational services (the income effect). If $g = 0$, then those two effects on fertility cancel out and there is no change in fertility. If $g > 0$, then the marginal utility of education decreases more slowly than the increase in wealth: the substitution effect dominates the income effect and fertility falls. The presence of g mitigates the decline of the marginal utility of education.[11] The lowest and highest possible fertility rates for households that choose a positive supplement are

[11] Using the mechanism mentioned here, de la Croix and Doepke (2003, 2004) and Moav (2005) derive first-order conditions similar to those of our model. They assume that the human capital of children is positive even if they are not educated at all: all agents are born with an elementary level of human capital by assumption. In our model, alternatively, the presence of public education guarantees that children receive a certain level of education.

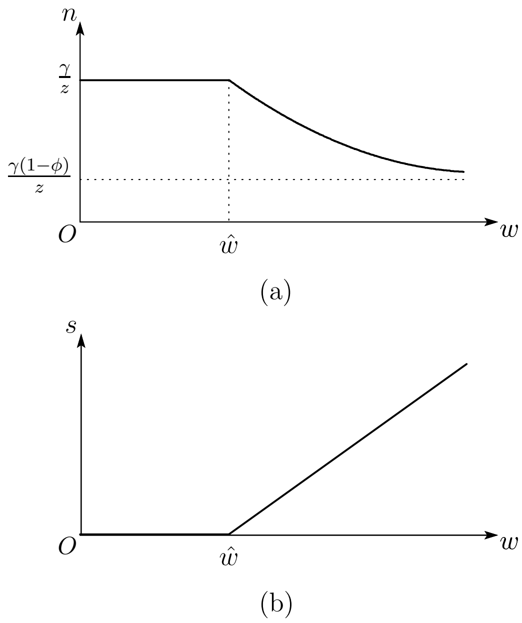

Figure 1: (a) the relationship between fertility and wage rate and (b) the relationship between private education and wage rate

given by $\lim_{w_i \to \infty} n_i = \gamma(1-\phi)/z$ and $\lim_{w_i \to \hat{w}} n_i = \gamma/z$, respectively.

For a household whose wage is sufficiently low such that $w_i \leq \hat{w}$ holds, the optimal private supplement level is zero; the first-order conditions imply

$$s_i = 0 \quad \text{and} \quad n_i = \frac{\gamma}{z}. \tag{9}$$

Once a household is at a corner solution and does not supplement the public provision, fertility no longer increases as wage decreases.

Using the results described above, we can depict the relationship between fertility and wage rate and the one between private education and wage rate (Figure 1). These relationships are consistent with the well-known evidence: high-income (low-income) households choose relatively low (high) fertility rates with relatively high (low) investment in their offspring's education.

We conclude this subsection by developing some comparative static results.

Proposition 1. *(i) For households that choose a positive supplement, a rise in t, a decrease in $\bar{n}$, and an increase in $\bar{m}$ decrease the education level of their children and increase* *fertility. (ii) A rise in t, a decrease in $\bar{n}$, and an increase in $\bar{m}$ decrease the proportion of households choosing a positive supplement.*

Proof. Using (4) and s_i in (8), the total expenditure on education per child is represented by

$$e_i = s_i + g = \frac{\phi}{1-\phi} \frac{z(1-t) w_i \bar{n} - t\bar{m}}{\bar{n}}. \tag{10}$$

The results are derived by differentiating (7), n_i in (8), and (10) with respect to t, $\bar{n}$, and $\bar{m}$, respectively. □

A rise in t affects a household's behavior through two channels. First, provided that $\bar{n}$ and $\bar{m}$ are fixed, it implies an increase in the quantity of public provision, which lowers the cost of having a child relative to the cost of education for a given education level. Second, it implies a reduction in the after-tax income per unit of time, which lowers the cost of having a child relative to the cost of education. As a result, it induces parents to substitute quantity for quality of children; the combined public and private expenditure on education per child falls, whereas the public expenditure rises. An increase in $\bar{n}$ implies a decline in the quantity of public provision (the congestion effect on public education), inducing parents to substitute quality for quantity. An increase in $\bar{m}$ implies an increase in the quantity of public provision because it broadens the tax base, inducing parents to substitute quantity for quality.

Proposition 1 states that the public provision of goods for children reduces each child's consumption level of those goods when private supplements are allowed and fertility is endogenous. This is a result that stands in sharp contrast to previous related studies. In the standard public-provision-of-private-goods model, owing to the "residual" nature of private demand, public provision affects the demand of the good only through the income effect; agents increase (decrease) the combined public and private demand if their effective income rises (falls) as a result of the public provision (see Epple and Romano, 1996b; Gouveia, 1997). It is the endogeneity of fertility that differentiates our result from theirs; public provision has a price effect as well as an income effect because it changes the relative price between the quantity and quality of children.

2.3 Equilibrium

Thus far, we have analyzed the individual decision problem given other households' actions, that is, given $\bar{n}$ and $\bar{m}$. In reality, however, both $\bar{n}$ and $\bar{m}$ are endogenously determined by aggregating each household's maximization behavior. In this subsection, we characterize the equilibrium of this economy. It follows from n_i in (8) and n_i in (9) that, in equilibrium, the fertility rate $\bar{n}$ must satisfy the following condition:

$$\bar{n} = F(\hat{w})\frac{\gamma}{z} + \int_{\hat{w}}^{\infty} \frac{\gamma(1-\phi)(1-t)w\bar{n}}{z(1-t)w\bar{n} - t\bar{m}} dF(w), \tag{11}$$

where the first term and the second term of the RHS represent the fertility rates of households with no supplement and those with positive supplement, respectively. We call this equation the "fertility curve;" it is the mapping from the average labor income to the fertility rate. Furthermore, in equilibrium, the average labor income $\bar{m}$ must satisfy the following condition:

$$\bar{m} = (1-\gamma)\int_0^{\hat{w}} w dF(w) + \int_{\hat{w}}^{\infty} w \frac{(1-\gamma+\gamma\phi)z(1-t)w\bar{n} - t\bar{m}}{z(1-t)w\bar{n} - t\bar{m}} dF(w), \tag{12}$$

where the first term and the second term of the RHS represent the labor incomes of households with no supplement and those with positive supplement, respectively. We call this equation the "labor-income curve;" it is the mapping from the fertility rate to the average labor income. The equilibrium of this economy is given by a tuple $(\bar{n}, \bar{m})$ that satisfies (11) and (12) simultaneously. Using the equilibrium conditions, (11) and (12), we obtain the following proposition.

Proposition 2. *Given $t \in (0,1)$, there exists a unique equilibrium.*

Proof. See Appendix A.

Figure 2 depicts the equilibrium. The fertility curve provides a unique fertility rate that is associated with a labor income. This uniqueness arises from the strategic substitutability among households in the fertility decision: if $\bar{n}$ rises, each household is induced to reduce fertility through the congestion effect on public education. Since a rise in $\bar{m}$ induces each

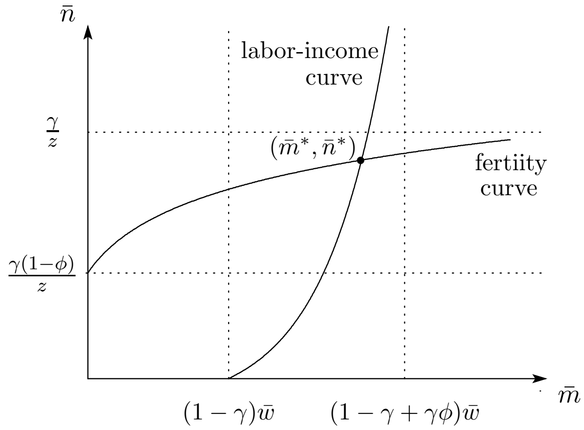

Figure 2: Equilibrium

household to increase the fertility by improving public education, the fertility curve is upward sloping. The result that the labor-income curve provides a unique labor income for a particular fertility rate and is upward sloping also comes from the analogous argument.

The result of Proposition 2 does not depend on the assumption of logarithmic utility; it holds for a larger class of preferences such that the demand for quantity of children is increasing in the government provision, thereby generating strategic substitutability among households. The above is verified as follows. Each household's fertility decision is affected by the change of others' fertility decisions only through the change of $g = t\bar{m}/\bar{n}$. Therefore, the change such that $\bar{m}$ and $\bar{n}$ change equiproportionately does not alter any household's fertility. Suppose that $(\bar{n}', \bar{m}')$ is an equilibrium. Since the equiproportional change of both $\bar{m}$ and $\bar{n}$ does not alter any household's fertility, any points on the straight line passing through the origin and $(\bar{n}', \bar{m}')$ never be an equilibrium. For $\bar{n}$ $(> \bar{n}')$ to be an equilibrium, g must rise, that is, $\bar{m}$ must rise proportionately more than $\bar{n}$. On the other hand, for $\bar{m}$ $(> \bar{m}')$ to be an equilibrium, g must fall, that is, $\bar{n}$ must rise proportionately more than $\bar{m}$. The above two possibilities cannot occur simultaneously. It follows from an analogous argument that change in any direction is excluded, and thus, points other than $(\bar{n}', \bar{m}')$ cannot be an equilibrium.

Given the equilibrium fertility rate $\bar{n}^*$ and the equilibrium average labor income $\bar{m}^*$, it follows from (4), (7), (8), and (9) that the equilibrium allocation of this economy,

$\{\{n_i^*\}, \{e_i^*\}, g^*\}$, is completely specified. Using the equilibrium conditions, (11) and (12), we can analyze the effect of tax rates on the equilibrium. The results are summarized in the following proposition.

Proposition 3. *(i) A rise in t raises the fertility rate. (ii) A rise in t decreases the average labor income. (iii) A rise in t increases the proportion of households choosing no supplement. (iv) A rise in t raises the ratio of public spending on education to the total spending.*

Proof. See Appendix A.

This proposition states that the enhancement of public education lowers private spending and raises the fertility rate in equilibrium. This is because the public provision of education induces parents to substitute quantity of children for quality by changing the relative price between the two, as has been shown in Proposition 1.

It is often argued that generous public policies for children should reduce the out-of-pocket child-related costs borne by households and make them fertile; this is supported by some cross-country data (e.g., d'Addio and d'Ercole, 2005; Feyrer et al., 2008). Proposition 3 formally demonstrates that such an argument holds in equilibrium where heterogeneous households interact with each other through the government's policy.[12]

3 Political Equilibrium

3.1 Timing of Events

In this section, we demonstrate that the tax rate, which is exogenously specified in Section 2, can be endogenized by introducing voting among the population, and that there exists a rational-expectation political equilibrium.

[12] Besides cross-country data, there are some time-series data suggesting that the mechanism highlighted here is not just a theoretical possibility but should also play an important role in policy debates. The experience of Japan since the 1970s is a prominent example. In Japan, the education curricula in elementary schools, junior high schools, and senior high schools are prescribed by a teaching guideline of the Education Ministry. Since the 1970s, the Japanese government amended the contents of the guideline several times and gradually reduced the amount of course hours at each education stage: the number of course hours in the compulsory education system decreased by more than 10%. As predicted by our model, the fall of the fertility rate and the increase of households' education-related spending for primary and lower-secondary education levels were simultaneously observed around the same time as such policy changes.

We consider a two-stage game; the timing of events is as follows.[13]

1. Each household decides the number of children it has (this decision is irreversible). In making this decision, each household treats the fertility choices of other households as given and has perfect foresight regarding the outcome of the future voting process over public funding for education.

2. All adults in the economy vote on tax rates and the tax rate is determined by majority rule. As a result, the quantity of public education per child is also determined because the fertility and attendant labor supply have already been determined in the first stage. After voting and understanding the outcome of the voting process, each household allocates its after-tax income to consumption expenditures and private educational services.

This timing is motivated by the fact that the public funding for education is more adjustable than fertility. Usually, in a democracy, the government budget must be considered and approved annually and there is an election every few years, which might bring about a major change in the political climate. On the other hand, in most countries, the completion of an elementary school education curriculum takes at least a decade from the time of birth while the completion of a secondary school education curriculum takes about fifteen years. Parents cannot have a child who is ready for school at birth or easily disown a child who has reached school age. At the time of making a fertility decision, each household must anticipate other households' fertility behaviors and the future political outcome in an economy populated by those households. Adopting such timing assumptions, we can formulate the political equilibrium of our model as follows.

Definition (Rational-expectation majority voting equilibrium). A political equilibrium is a set $\{t^*, \{n^*(w)\}\}$ such that

1. all households make the optimal fertility decision $n^*(w)$, taking as given the tax rate t^* and other households' fertility decisions $\{n^*(w)\}$;

[13] We will investigate the implications of alternative timing assumptions later.

2. given $\{n^*(w)\}$, there does not exist another t' such that t' is preferred over t^* by more than half of the population.

On the basis of the assumption that households must decide the number of children they have before voting and that this decision is irreversible, all households take as given their fertility decisions $\{n^*(w)\}$ at the time of voting. We can derive the equilibrium by the following procedure: (i) we derive the equilibrium condition for fertility set $\{n(w)\}$, given a tax rate t; (ii) we derive the equilibrium condition for tax rate t, given a fertility set $\{n(w)\}$; and then (iii) we search a set $\{t, \{n(w)\}\}$ that simultaneously satisfies both the conditions.

3.2 Voting Stage

We have already derived the equilibrium condition for fertility set $\{n(w)\}$, given a tax rate t in Section 2 and can now apply the same results. Here, we consider the equilibrium condition for tax rate t, given a fertility set $\{n(w)\}$. At the time of voting, the number of children and the attendant labor supply in each household have already been determined. Thus, focusing on the voting stage, we observe that this model has almost the same structure as the model of Epple and Romano (1996b) (hereafter referred to as ER). It follows from an analogous argument of ER that preferences over tax rates are single-peaked and a majority voting equilibrium always exists. It is noted that there is a difference between the ER model and our model. In the ER model, households with income below (above) the mean income prefer a positive (zero) tax rate. In our model, households whose number of children per unit of income is larger (smaller) than the mean number of children per unit of income prefer a positive (zero) tax rate. The tax price of education to households with a larger number of children per unit of income than the mean is below the market price; thus, they benefit from positive taxes. However, the incentive to redistribute via public provision of education is limited and a household's utility is maximized at a tax rate strictly less than 1. Since the public service cannot be sold, the costs arise in the form of inefficient allocation between education and consumption.[14] The above description is

[14] In this model, the labor supply is endogenously determined as a result of the fertility decision. However, the limitation on the incentive to redistribute in our model differs from that in Meltzer and Richard (1981),

formally presented by the following lemma.

Lemma 1. *(i) The most preferred tax rate for household i with $n_i/[w_i(1-zn_i)] < \bar{n}/\bar{m}$ is zero. The most preferred tax rate for household i with $n_i/[w_i(1-zn_i)] > \bar{n}/\bar{m}$ is $\gamma\phi/(1-\gamma+\gamma\phi)$. Household i with $n_i/[w_i(1-zn_i)] = \bar{n}/\bar{m}$ is indifferent among t in $[0, \gamma\phi/(1-\gamma+\gamma\phi)]$ but prefers any t in that interval to the remainder. (ii) If household i with $n_i/[w_i(1-zn_i)] < \bar{n}/\bar{m}$ is median in terms of the number of children per unit of income, $t = 0$ is chosen by voting. If household i with $n_i/[w_i(1-zn_i)] > \bar{n}/\bar{m}$ is the median, $t = \gamma\phi/(1-\gamma+\gamma\phi)$ is chosen. If household i with $n_i/[w_i(1-zn_i)] = \bar{n}/\bar{m}$ is the median, any t in $[0, \gamma\phi/(1-\gamma+\gamma\phi)]$ can be chosen.*

Proof. See Appendix A.

Figure 3 depicts the description of Lemma 1; the preferences over tax rates are single-peaked and the median-voter theorem holds.[15] The statement of Lemma 1 is consistent with some empirical results: childless voters are less likely to support public-school spending than voters with children (e.g., Rubinfeld, 1977).[16]

Since the decisive voter is the household with the median number of children per unit of income, we need to find out which household is the median in terms of this criterion. For this purpose, we can use the result of Section 2: for any given t, a household's number of children does not increase with wage, and the quantity of labor supply does not decrease with wage—that is, labor income is strictly increasing with wage. Therefore, a household's number of children per unit of income is strictly decreasing with wage, and the household with median wage also has the median number of children per unit of income. To obtain predictions about the voting outcome, we have only to focus on the voting behavior of the

where the presence of labor supply incentives limits the incentive to redistribute. Since fertility decisions are made prior to the voting stage, each household does not consider the effect of public policy on labor supply incentives at the time of making the political decision; the incentive here to redistribute is limited solely by diminishing marginal utility.

[15] The result that households where the number of children per unit of income is larger than the mean number of children per unit of income prefer the same tax rate comes from the assumption of logarithmic utility. However, the existence of majority voting equilibrium does not depend on such specification (see Epple and Romano, 1996b).

[16] Lemma 1 describes the political outcome under the assumption that the dual-provision regime is already established. Even if the regime itself can be determined by majority rule, the dual-provision regime is not defeated by the market-only regime or government-only regime and Lemma 1 holds. It can be proved by exactly the same way as the proof of Proposition 2 in Epple and Romano (1996b).

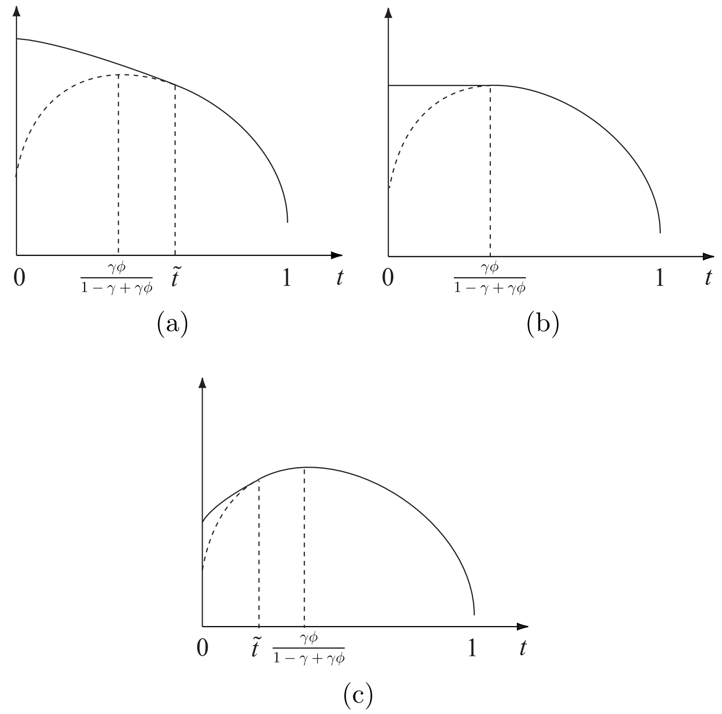

Figure 3: The indirect utility functions of (a) household i with $n_i / [w_i (1 - zn_i)] < \bar{n}/\bar{m}$, (b) household i with $n_i / [w_i (1 - zn_i)] = \bar{n}/\bar{m}$, and (c) household i with $n_i / [w_i (1 - zn_i)] > \bar{n}/\bar{m}$

households with median wage. It follows from Lemma 1 that the optimal voting rule of such households is given by

$$\begin{cases} \text{vote on } t = 0 & \text{if} \quad \frac{n_M}{w_M(1-zn_M)} < \frac{\bar{n}}{\bar{m}}, \\ \text{vote on any } t \in \left[0, \frac{\gamma\phi}{1-\gamma+\gamma\phi}\right] & \text{if} \quad \frac{n_M}{w_M(1-zn_M)} = \frac{\bar{n}}{\bar{m}}, \\ \text{vote on } t = \frac{\gamma\phi}{1-\gamma+\gamma\phi} & \text{if} \quad \frac{n_M}{w_M(1-zn_M)} > \frac{\bar{n}}{\bar{m}}, \end{cases}$$

where w_M is the median wage, which is defined by $F(w_M) = 1/2$, and n_M is the number of children of households with median wage.

3.3 Rational-expectation Political Equilibrium

Here, we define the following function:

$$\Upsilon(t) \equiv \frac{n_M}{w_M (1 - zn_M)} - \frac{\bar{n}}{\bar{m}},$$

which represents the difference between the median number of children per unit of income and the mean number of children per unit of income as a function of t. It follows from n_i in (8) and n_i in (9) that $\Upsilon(t)$ can be written as

$$\Upsilon(t) = \begin{cases} \frac{\gamma(1-\phi)(1-t)\bar{n}}{(1-\gamma+\gamma\phi)z(1-t)w_M\bar{n}-t\bar{m}} - \frac{\bar{n}}{\bar{m}} & \text{if} \quad t < \hat{t}, \\ \frac{\gamma}{(1-\gamma)zw_M} - \frac{\bar{n}}{\bar{m}} & \text{if} \quad t \geq \hat{t}, \end{cases} \tag{13}$$

where $\hat{t}$ is defined by $w_M = \hat{t}\bar{m}/\left[\phi z\left(1-\hat{t}\right)\bar{n}\right]$, that is, $\hat{t}$ is the threshold tax rate for the median households. If t is less (more) than $\hat{t}$, the median households choose a positive (zero) supplement. It should be noted that $\bar{n}$ and $\bar{m}$ are also functions of t (see (19) and (20)). The function $\Upsilon(t)$ captures the marginal preferences of the median households over tax rates in equilibrium. If $\Upsilon(t) > 0$, the median households prefer higher tax rates as long as t is lower than $\gamma\phi/(1-\gamma+\gamma\phi)$; if $\Upsilon(t) < 0$, they prefer lower tax rates as long as t is positive; and if $\Upsilon(t) = 0$, then they prefer the present tax rate. Using this function, we can formulate the equilibrium condition as follows.

Equilibrium condition. The equilibrium tax rate t^* must satisfy

$$\begin{cases} \Upsilon(0) \leq 0 & \text{if} \quad t^* = 0, \\ \Upsilon(t^*) = 0 & \text{if} \quad t^* \in \left(0, \frac{\gamma\phi}{1-\gamma+\gamma\phi}\right), \\ \Upsilon\left(\frac{\gamma\phi}{1-\gamma+\gamma\phi}\right) \geq 0 & \text{if} \quad t^* = \frac{\gamma\phi}{1-\gamma+\gamma\phi}. \end{cases} \tag{14}$$

It follows from this equilibrium condition that we obtain the existence result.

Proposition 4. *There exists at least one rational-expectation political equilibrium for any wage distribution $F(\cdot)$.*

Proof. It is easily verified that $\Upsilon(0) \leq 0$ if and only if the median wage is not below the mean wage, $w_M \geq \bar{w}$. If $w_M \geq \bar{w}$, then $\Upsilon(0) \leq 0$ and $t = 0$ is an equilibrium tax rate. If $w_M < \bar{w}$, then $\Upsilon(0) > 0$ and $t = 0$ is not an equilibrium tax rate. In this case, there are two possibilities at the other corner: $\Upsilon(\gamma\phi/(1-\gamma+\gamma\phi)) \geq 0$ or $\Upsilon(\gamma\phi/(1-\gamma+\gamma\phi)) < 0$. If $\Upsilon(\gamma\phi/(1-\gamma+\gamma\phi)) \geq 0$, then $t = \gamma\phi/(1-\gamma+\gamma\phi)$ is an equilibrium tax rate. If $\Upsilon(\gamma\phi/(1-\gamma+\gamma\phi)) < 0$, then $t = \gamma\phi/(1-\gamma+\gamma\phi)$ is not an equilibrium tax rate; however, as $\Upsilon(\cdot)$ is continuous, there is at least one value of t at which $\Upsilon(t) = 0$. Thus the result is obtained. □

Below are some of the results on the property of equilibrium.

Proposition 5. *(i) Government provision is positive if the median wage is less than the mean wage. (ii) There can exist multiple equilibria. Multiple equilibria arise if and only if there exists t' such that $\Upsilon(t') = 0$ and $\Upsilon(t)$ is increasing at t'. (iii) If multiple equilibria exist, then the private supplement of median households is positive in at least one equilibrium. (iv) Suppose that there exist multiple equilibria. Then, for any two equilibria, a and b, we have $\bar{n}^*(t^a) < \bar{n}^*\left(t^b\right) \leftrightarrow t^a < t^b$.*

Proof. See Appendix A.

The functional form of $\Upsilon(t)$ might be complex and it is difficult to obtain further analytical results without any specification. Figure 4 presents a numerical example of $\Upsilon(t)$. The example uses lognormal distribution $F\left(\mu, \sigma^2\right)$ as the wage distribution, where μ and σ^2 are the mean and variance of the underlying normal distribution, respectively. We assume that $z = 0.1$, $\gamma = 0.4$, $\phi = 0.8$, and $\sigma = 0.55$. These parameter values are chosen on the basis of two criteria: (i) the parameter values themselves should be reasonable, and (ii) the values of the endogenous variables that follow from those parameter values should also be reasonable (see Appendix B for details). Figure 4 exhibits multiple equilibria; there are three tax rates that satisfy the equilibrium condition (14).[17] It is noted that the middle-tax equilibrium is the knife-edge case, in which $\Upsilon(t)$ is increasing at $\Upsilon(t) = 0$: that is, there are two stable equilibria. We now explain the intuition of the condition for multiplicity.

Multiple equilibria arise if and only if there is more than one tax rate supported by the households with median wage. Given a tax rate t, there are two groups in the economy: one prefers higher tax rates and the other, lower. $\Upsilon(t) > (<)\ 0$ implies that the households

[17] The occurrence of multiplicity is not restricted to that parameter configuration. Multiplicity broadly occurs under various parameter sets that satisfy the two criteria mentioned above. This can be verified numerically by varying parameters z, γ, ϕ, and σ. The numerial results are available from the authors upon request.

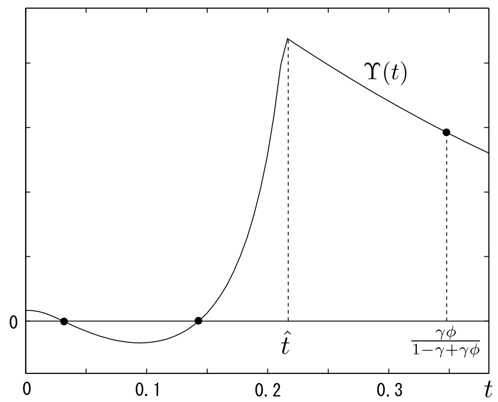

Figure 4: Political Equilibrium

with median wage belong to the group supporting higher (lower) tax rates at t. If $\Upsilon(t) >$ $(<)\ 0$ for any t, then a majority of the population is better off as t rises (falls): different tax rates cannot be supported by the majority. The existence of t such that $\Upsilon(t) = 0$ implies that there exists a tax rate such that the political group to which median households belong changes on reaching that rate; this is a necessary condition for different tax rates to be supported by the majority.

The slope of $\Upsilon(t)$ represents the relative responsiveness of the median households to the change of tax rate in their fertility decisions as compared to the average households.[18] A positive slope implies that the median households are more responsive than the average households: if t rises, then the median number of children per unit of income increases more than the average number of children per unit of income. When $\Upsilon(t)$ is increasing in t, decisive households can free-ride on the inelastic fertility behavior of average households; in the high-tax equilibrium, the decisive households benefit from the high tax rate because their fertility is high, their tax base is small, and thus, the tax price of education is low. Therefore, the decisive households will vote for a high tax rate and sustain the equilibrium. The opposite happens in the low-tax equilibrium.[19]

[18] It is noted that the "average households" do not mean the households with average wage but mean the virtual households whose behavior is equivalent to the average behavior of all households. In this model, generally, $\bar{n} \neq n(\bar{w})$.

[19] Saint-Paul and Verdier (1997) formulate a necessary condition for multiplicity in a politico-economic model where heterogeneous agents ex post vote on policy as the "elasticity principle:" the behavior of decisive agents, which determines their identity at the time of voting, is more responsive to the expected policy than that of average agents. In our model, that condition corresponds to the positive slope of $\Upsilon(t)$.

The multiplicity is characterized by the coexistence of a high-fertility, low-private/public-education-ratio equilibrium and a low-fertility, high-private/public-education-ratio equilibrium. This implies that countries that appear similar in their overall characteristics (e.g., income per capita and wage inequality) sometimes end up with widely different public policies for children, private/public spending ratios and fertility rates, and the same countries may jump from one equilibrium to another from time to time. The solution of the numerical example above is presented in Appendix B. As predicted by the analytical results, the fertility rate, $\bar{n}^*$, the proportion of households with no supplement, $F(\hat{w}^*)$, and the ratio of public spending, $g^*\bar{n}^*/E^*$, are higher in the high-tax equilibrium than in the low-tax equilibrium. The majority supplements the public provision in the low-tax equilibrium, whereas it does not do so in the high-tax equilibrium; each equilibrium is sustained by the majority households' behaviors.[20]

3.4 Alternative Timing Assumption

The timing of events is crucial for the property of political equilibrium. If we assume that each household can adjust fertility after the funding level for public education has been determined, the results are greatly altered: (i) preferences over tax rates are not single-peaked and (ii) the multiplicity of equilibria disappears. For multiplicity to arise, the fertility decision affecting the identity of decisive voters must be made prior to the political decision. If the political decision is made prior to the fertility decision, the decisive households vote on the generically unique tax rate that maximizes their utility. Thus, the disappearance of multiplicity is a rather natural result.

We now discuss the non-single-peakedness in detail. For deriving the indirect utility over tax rates in the adjustable case, we have only to solve the maximization problem given by (5) and (6), and substitute maximizers into the utility function. That is, the difference from the nonadjustable case (Lemma 1) is that the number of children, n_i, is not given but controllable in the voting process. Figure 5 illustrates the indirect utility

[20]In Japan and South Korea, more than half of the households send their children to private-tutoring facilities (see for instance Bray (2006)). Based on our theory, the distinctive feature of these countries, which is characterized by the prevalence of private tutoring and very low fertility, might be self-fulfillingly sustained by the majority households.

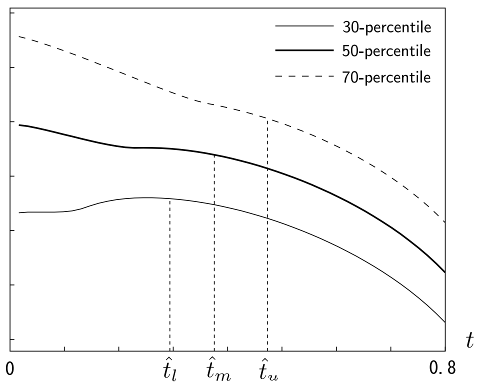

Figure 5: Preferences over tax rates in the adjustable case

over tax rates for households with different wages under the same parameter set as the numerical example above, where $\hat{t}_l$, $\hat{t}_m$, $\hat{t}_u$ are the threshold tax rates for 30-percentile, 50-percentile, and 70-percentile households in wage distribution, respectively. Preferences over tax rates are no longer single-peaked, but a majority voting equilibrium still exists because our choice of utility function satisfies a single-crossing condition (Gans and Smart, 1996). In the case of the parameter set that we are considering, as is clear from Figure 5, $t = 0$ is chosen by majority voting.

In a politico-economic model regarding public provision of private goods, it is known that, generally, preferences are not single-peaked if agents can opt out of but not supplement public provision (Stiglitz, 1974; Epple and Romano, 1996a; Glomm and Ravikumar, 1998), whereas preferences are single-peaked if agents can supplement, but not opt out (Epple and Romano, 1996b; Gouveia, 1997). The non-single-peakedness in our nonadjustable case can be explained by an analogous argument of the opting-out model.[21] In the standard opting-out model, the fact that agents can freely opt out of the public regime after policy determination simply leads to non-single-peakedness.[22] In our endogenous fer-

[21] Preferences over tax rates in the opting-out model are typically similar to those of Figure 5 (see Glomm and Ravikumar, 1998).

[22] Intuitively, in the standard opting-out model, non-single-peakedness occurs according to the following mechanism. At low levels of public-service quality, a household that prefers high-quality service might prefer private alternatives. A small increase in quality might be bad for the household because the tax rate rises whereas the increase in service quality is not sufficient to induce the household to consume the public service. A large increase in public service quality, however, might be good for the household. The household might be induced to use the public alternative, and the increased tax cost may be offset by the savings from forgoing private services.

tility model, households cannot opt out, but the fact that households can decide whether or not to have children implies that, in effect, they have a choice between opting out or not.

As stated above, it seems reasonable to assume that the fertility decision is made prior to the political decision. However, if a government can commit to a specified policy over a certain amount of time, say, at least a decade or so, then the order of decisions is reversed. The discussion of this subsection suggests that the availability of commitment might greatly alter the implication of the policy targeted at children. The above comes from the fact that the fertility decision is irreversible and that child-rearing takes time to complete.

4 Conclusion

Our objective in this paper was to analyze the public provision of children's goods in a model where agents are allowed to supplement the publicly provided quantity by private purchase and the fertility is endogenously determined. Using a simple functional form for preferences and standard assumptions about the quantity-quality tradeoff for children, we developed an extension of the model of Epple and Romano (1996b) and Gouveia (1997). Some key results are as follows. If we choose the realistically plausible assumption regarding the timing of events, then the single-peakedness of the preferences over tax rates is satisfied in the voting process and a rational-expectation political equilibrium always exists. The sequential interaction between fertility and political decisions might lead to multiple equilibria: a high-fertility, low-private/public-spending-ratio equilibrium and a low-fertility, high-private/public-spending-ratio equilibrium. Whether a government can commit to a specified policy over a certain amount of time crucially affects the implication of the public policy targeted at children.

This paper offers a theory explaining the large differences in the fertility and structure of child-related spending across countries. The public spending per child is high in Nordic countries and much lower in Southern European and East Asian countries. Based on our model, these countries can be thought of as in different equilibria. Such differences can of course be explained not only by multiplicity but also by differences in underlying economic conditions. However, the explanation obtained using multiplicity has obvious appeal: the characteristics of an economy can suddenly change owing to the shift from one equilibrium to another even if it is only the policy expectation that changes. While fertility rates have declined over the past decades in almost all OECD countries, the pace of decline and the level achieved differ across countries. The fertility decline was more drastic in some countries than others; one striking example is South Korea. Up to the early 1970s, the total fertility rate of South Korea had been over 4. In less than twenty years, that rate reduced to about 1.5. This drastic decline of fertility has coincided with sharp rises in both the education/consumption ratio in households and the ratio of private spending on education to public spending; the ratio of elementary pupils who participate in private tutoring was 12.9% in 1980, whereas that in 2003 was 83.1% (Kwak, 2004). Even if we take the rapid economic development of South Korea into consideration, it appears to be difficult to explain the drastic change in such a short period of time only in terms of the change in underlying economic conditions such as income level and wage distribution.

In the politico-economic literature, the political conflict over public education has often been discussed in terms of the intergenerational conflict (e.g. Levy, 2005): given the size of government, the young with school-age children support public education, whereas the old prefer direct income redistribution. Our model concentrates on the intragenerational conflict between the group with many children and the group with fewer children, where the number of children is endogenously determined by each household. This paper is more likely to complement rather than contradict the studies on the intergenerational conflict. According to previous studies, the smaller the size of the population of the young relative to that of the old, the less public education is likely to be politically supported. If the young do not support public education, the funds for education would shrink further. Furthermore, the low fertility rate of the young generation leads to the aging of population structures; therefore, the public education is less likely to be politically supported in the future. The shift from a high-fertility equilibrium to a low-fertility equilibrium, implied by our model, might trigger further declines in the public spending on education and the fertility rate.

Appendix A

For convenience, we rewrite equations (11) and (12) respectively as

$$\Omega\left(\bar{m},\bar{n};t\right)=0, \tag{15}$$

$$\text{where } \Omega\left(\bar{m},\bar{n};t\right)\equiv\bar{n}-F\left(\hat{w}\right)\frac{\gamma}{z}-\int_{\hat{w}}^{\infty}\frac{\gamma\left(1-\phi\right)\left(1-t\right)w\bar{n}}{z\left(1-t\right)w\bar{n}-t\bar{m}}dF\left(w\right),$$

and

$$\Psi\left(\bar{m},\bar{n};t\right)=0, \tag{16}$$

$$\begin{aligned}\text{where } \Psi\left(\bar{m},\bar{n};t\right)\equiv\bar{m}-\left(1-\gamma\right)&\int_{0}^{\hat{w}}wdF\left(w\right)\\ -&\int_{\hat{w}}^{\infty}w\frac{\left(1-\gamma+\gamma\phi\right)z\left(1-t\right)w\bar{n}-t\bar{m}}{z\left(1-t\right)w\bar{n}-t\bar{m}}dF\left(w\right).\end{aligned}$$

Proof of Proposition 2

Existence. By the implicit function theorem, it follows from (15) that

$$\frac{d\bar{n}}{d\bar{m}}=-\frac{\Omega_{\bar{m}}}{\Omega_{\bar{n}}}=\frac{\int_{\hat{w}}^{\infty}\frac{\gamma(1-\phi)(1-t)tw\bar{n}}{[z(1-t)w\bar{n}-t\bar{m}]^2}dF\left(w\right)}{1+\int_{\hat{w}}^{\infty}\frac{\gamma(1-\phi)(1-t)tw\bar{m}}{[z(1-t)w\bar{n}-t\bar{m}]^2}dF\left(w\right)}>0. \tag{17}$$

It follows from (16) that

$$\frac{d\bar{m}}{d\bar{n}}=-\frac{\Psi_{\bar{n}}}{\Psi_{\bar{m}}}=\frac{\int_{\hat{w}}^{\infty}\frac{\gamma(1-\phi)z(1-t)tw^2\bar{m}}{[z(1-t)w\bar{n}-t\bar{m}]^2}dF\left(w\right)}{1+\int_{\hat{w}}^{\infty}\frac{\gamma(1-\phi)(1-t)tzw^2\bar{n}}{[z(1-t)w\bar{n}-t\bar{m}]^2}dF\left(w\right)}>0. \tag{18}$$

Both the fertility curve and the labor-income curve are upward sloping in the $(\bar{m},\bar{n})$ space. Evaluating (11) at $\bar{m}=0$ and at $\bar{m}=\infty$, we obtain $\bar{n}|_{\bar{m}=0}=\gamma\left(1-\phi\right)/z$ and $\bar{n}|_{\bar{m}=\infty}=\gamma/z$, respectively. It follows that the fertility curve intersects the $\bar{n}$-axis at $\gamma\left(1-\phi\right)/z$, and it approaches γ/z as $\bar{m}\to\infty$. Evaluating (12) at $\bar{n}=0$ and at $\bar{n}=\infty$, we obtain $\bar{m}|_{\bar{n}=0}=\left(1-\gamma\right)\bar{w}$ and $\bar{m}|_{\bar{n}=\infty}=\left(1-\gamma+\gamma\phi\right)\bar{w}$, respectively, where $\bar{w}$ is the mean of wage distribution $\int_0^\infty wdF(w)$. It follows that the labor-income curve intersects the $\bar{m}$-axis at $(1-\gamma)\bar{w}$, and it approaches $(1-\gamma+\gamma\phi)\bar{w}$ as $\bar{n}\to\infty$. As the two curves are continuous, they must intersect at least once.

Uniqueness. Using (15) and (16), we obtain the following relationship:

$$
\begin{aligned}
&\text{(the slope of the fertility curve)} - \text{(the slope of the labor-income curve)}\\
&= -\frac{\Omega_{\bar{m}}}{\Omega_{\bar{n}}} - \left(-\frac{\Psi_{\bar{m}}}{\Psi_{\bar{n}}}\right)\\
&= -\frac{1+\int_{\hat{w}}^{\infty}\frac{\gamma(1-\phi)t(1-t)w(zw\bar{n}+\bar{m})}{[z(1-t)w\bar{n}-t\bar{m}]^2}dF(w)}{\left\{1+\int_{\hat{w}}^{\infty}\frac{\gamma(1-\phi)(1-t)tw\bar{m}}{[z(1-t)w\bar{n}-t\bar{m}]^2}dF(w)\right\}\int_{\hat{w}}^{\infty}\frac{\gamma(1-\phi)z(1-t)tw^2\bar{m}}{[z(1-t)w\bar{n}-t\bar{m}]^2}dF(w)} < 0.
\end{aligned}
$$

The slope of the labor-income curve is larger than that of the fertility curve for any $\bar{m}$ in the $(\bar{m},\bar{n})$ space. Therefore, the intersection point of the two curves is unique (if any). Further, the labor-income curve crosses the fertility curve from beneath. $\square$

Proof of Proposition 3

(i) By the implicit function theorem, it follows from (15) that we obtain the following:

$$
\frac{d\bar{n}}{dt} = -\frac{\Omega_t}{\Omega_{\bar{n}}} = \frac{\int_{\hat{w}}^{\infty}\frac{\gamma(1-\phi)w\bar{n}\bar{m}}{[z(1-t)w\bar{n}-t\bar{m}]^2}dF(w)}{1+\int_{\hat{w}}^{\infty}\frac{\gamma(1-\phi)(1-t)tw\bar{m}}{[z(1-t)w\bar{n}-t\bar{m}]^2}dF(w)} \geq 0.
$$

Equality holds if $\bar{m}=0$ and strict inequality holds if $\bar{m}\neq 0$. The following can be derived from (16):

$$
\frac{d\bar{m}}{dt} = -\frac{\Psi_t}{\Psi_{\bar{m}}} = -\frac{\int_{\hat{w}}^{\infty}\frac{\gamma(1-\phi)zw^2\bar{n}\bar{m}}{[z(1-t)w\bar{n}-t\bar{m}]^2}dF(w)}{1+\int_{\hat{w}}^{\infty}\frac{\gamma(1-\phi)z(1-t)tw^2\bar{n}}{[z(1-t)w\bar{n}-t\bar{m}]^2}dF(w)} \leq 0.
$$

Equality holds if $\bar{n}=0$ and inequality holds if $\bar{n}\neq 0$. A rise in t rotates the fertility curve upward around a point at $\bar{m}=0$ and the labor-income curve to the left around a point at $\bar{n}=0$ in the $(\bar{m},\bar{n})$ space. In the diagram, the total effect on the equilibrium fertility rate is ambiguous; however, the following calculation establishes that it is unambiguously determined and positive:

$$\begin{aligned}\frac{d\bar{n}^*}{dt} =& \frac{\Omega_{\bar{m}}\Psi_t - \Omega_t\Psi_{\bar{m}}}{\Omega_{\bar{n}}\Psi_{\bar{m}} - \Omega_{\bar{m}}\Psi_{\bar{n}}} \\ =& \frac{\int_{\hat{w}}^{\infty} \frac{\gamma(1-\phi)w\bar{n}\bar{m}}{[z(1-t)w\bar{n}-t\bar{m}]^2} dF(w)}{1+\gamma(1-\phi)(1-t)t\int_{\hat{w}}^{\infty} \frac{zw^2\bar{n}+w\bar{m}}{[z(1-t)w\bar{n}-t\bar{m}]^2} dF(w)} > 0. \end{aligned} \tag{19}$$

(ii) The total effect on the equilibrium average labor income is also ambiguous in the diagram, but the following calculation establishes that it is unambiguously determined and negative:

$$\begin{aligned}\frac{d\bar{m}^*}{dt} =& \frac{\Omega_{\bar{n}}\Psi_t - \Omega_t\Psi_{\bar{n}}}{\Omega_{\bar{m}}\Psi_{\bar{n}} - \Omega_{\bar{n}}\Psi_{\bar{m}}} \\ =& -\frac{\int_{\hat{w}}^{\infty} \frac{\gamma(1-\phi)zw^2\bar{n}\bar{m}}{[z(1-t)w\bar{n}-t\bar{m}]^2} dF(w)}{1+\gamma(1-\phi)(1-t)t\int_{\hat{w}}^{\infty} \frac{zw^2\bar{n}+w\bar{m}}{[z(1-t)w\bar{n}-t\bar{m}]^2} dF(w)} < 0. \end{aligned} \tag{20}$$

(iii) Differentiating (7) with respect to t by noting that the relationship between $\bar{n}^*$ and t is given by (19) and that the relationship between $\bar{m}^*$ and t is given by (20), we obtain the following:

$$\frac{d\hat{w}^*}{dt} = \frac{\bar{m}}{\phi z(1-t)^2\bar{n}\left\{1+\gamma(1-\phi)(1-t)t\int_{\hat{w}}^{\infty} \frac{zw^2\bar{n}+w\bar{m}}{[z(1-t)w\bar{n}-t\bar{m}]^2} dF(w)\right\}} > 0.$$

(iv) The ratio of public spending on education to total spending on education, $g\bar{n}/E$, is represented by

$$\frac{g\bar{n}}{E} = \frac{t\bar{m}}{F(\hat{w})\frac{\gamma}{z}\frac{t\bar{m}}{\bar{n}} + \gamma\phi(1-t)\int_{\hat{w}}^{\infty} w dF(w)},$$

where E denotes the total spending on education, and the numerator and denominator of this equation represent the public spending on education and total spending on education, respectively. Differentiating this equation with respect to t, we get

$$\frac{\partial(g\bar{n}/E)}{\partial t} = \frac{\bar{m}\gamma\phi\int_{\hat{w}}^{\infty} w dF(w) + tE\int_{\hat{w}}^{\infty} \frac{\gamma(1-\phi)w\bar{m}^2}{[z(1-t)w\bar{n}-t\bar{m}]^2} dF(w)}{E^2\left\{1+\gamma(1-\phi)(1-t)t\int_{\hat{w}}^{\infty} \frac{zw^2\bar{n}+w\bar{m}}{[z(1-t)w\bar{n}-t\bar{m}]^2} dF(w)\right\}} > 0.$$

Proof of Lemma 1

Assume that the average fertility rate is $\bar{n}$ and the average labor income is $\bar{m}$. The decision of household i with wage w_i and the number of children n_i is given by

$$\max_{s_i} \ \gamma\phi \ln\left(s_i + \frac{t\bar{m}}{\bar{n}}\right) + \gamma \ln n_i + (1-\gamma)\ln\left[w_i\left(1-zn_i\right)(1-t) - s_i n_i\right].$$

The first-order condition is

$$\begin{cases} s_i = \frac{\gamma\phi w_i(1-zn_i)(1-t)\bar{n}-(1-\gamma)t\bar{m}n_i}{(1-\gamma+\gamma\phi)n_i\bar{n}} & \text{if} \quad t < \tilde{t}\left(w_i, n_i\right), \\ s_i = 0 & \text{if} \quad t \geq \tilde{t}\left(w_i, n_i\right), \end{cases}$$

$$\text{where } \tilde{t}\left(w_i, n_i\right) \equiv \frac{\gamma\phi w_i\left(1-zn_i\right)\bar{n}}{(1-\gamma)\,\bar{m}n_i + \gamma\phi w_i\left(1-zn_i\right)\bar{n}}.$$

The indirect utility in the case of $t \leq \tilde{t}\left(w_i, n_i\right)$ is

$$A + (1-\gamma+\gamma\phi)\ln\left[w_i\left(1-zn_i\right)(1-t) + \frac{t\bar{m}}{\bar{n}}n_i\right],$$

where A is a constant term. This is strictly decreasing (increasing) in t for household i with $n_i/w_i\left(1-zn_i\right) < (>)\ \bar{n}/\bar{m}$. The indirect utility in the case of $t > \tilde{t}\left(w_i, n_i\right)$ is

$$\gamma\phi \ln\frac{t\bar{m}}{\bar{n}} + \gamma \ln n_i + (1-\gamma)\ln w_i\left(1-zn_i\right)(1-t).$$

This is strictly concave, and maximized at $t = \gamma\phi/\left(1-\gamma+\gamma\phi\right)$ if $\tilde{t}(w_i, n_i) < \gamma\phi/(1-\gamma+\gamma\phi)$ and maximized at $t = \tilde{t}(w_i, n_i)$ if $\tilde{t}(w_i, n_i) > \gamma\phi/(1-\gamma+\gamma\phi)$. It is verified that for household i with $n_i/w_i\left(1-zn_i\right) < (>)\ \bar{n}/\bar{m}$, $\tilde{t}\left(w_i, n_i\right)$ is larger (smaller) than $\gamma\phi/\left(1-\gamma+\gamma\phi\right)$. To sum up, for household i with $n_i/\left[w_i\left(1-zn_i\right)\right] < \bar{n}/\bar{m}$, the indirect utility is strictly decreasing in t and maximized at $t = 0$, whereas for household i with $n_i/w_i\left(1-zn_i\right) > \bar{n}/\bar{m}$, the indirect utility is strictly concave and maximized at $t = \gamma\phi/\left(1-\gamma+\gamma\phi\right)$ (see Figure 4). It follows that preferences over tax rates are clearly single-peaked and standard theorems guaranteeing the existence of a majority voting equilibrium hold. Thus, the result is obtained. $\square$

Proof of Proposition 5

(i) This follows immediately from the result that $\Upsilon(0) > 0$ if and only if the median wage is less than the mean wage. (ii) This follows immediately from the continuity of $\Upsilon(\cdot)$ and the equilibrium condition (14). (iii) It is easily verified from (13) and Proposition 3 that $\Upsilon(t)$ is decreasing in t for a sufficiently high t such that $t > \hat{t}$, when the median households choose zero supplement. Therefore, if there exists t' such that $\Upsilon(t') = 0$ and $\Upsilon(t)$ is increasing at t', then $t' < \hat{t}$, when the median households choose positive supplement. Thus, the result is obtained. (iv) This follows immediately from Proposition 3.

Appendix B

Choice of the Parameters in the Numerical Example

The choice of z. Parameter z represents the time cost for having a child. We follow the procedure of de la Croix and Doepke (2004) for the determination of this parameter. Evidence in Haveman and Wolfe (1995) and Knowles (1999) suggests that the opportunity cost of a child is equivalent to about 20% of the parents' time endowment. This cost only accrues as long as the child is living with the parents. If we assume that children live with parents for 15 years and that the adult period lasts 30 years, the overall time cost should be 50% of the time cost per year with the child present. Accordingly, we choose $z = 0.1$.

The choice of γ. Parameter γ, which is the weight of children in the utility function, directly influences the number of children in each household. In particular, for households with no private education, given a value of z, their fertility is determined exclusively by the value of γ. We set it such that the upper bound on the number of children (that is, the number of children in households with no private education) is four, because there is generally no income class where the average fertility rate reaches five. This leads to $\gamma = 0.4$ because we choose $z = 0.1$.

The choice of ϕ. Parameter ϕ represents the relative weight given to the quality of

	Low-tax equilibrium	High-tax equilibrium
t^*	0.031	0.348
$\bar{n}^*$	1.262	3.582
$F(\hat{w}^*)$	0.021	0.763
$g^*\bar{n}^*/E^*$	0.089	0.812

Table 1: Results of the Numerical Example

children. The upper bound on the fertility differential of this economy is given by

$$\frac{\lim_{w\to 0} n(w)}{\lim_{w\to\infty} n(w)} = \frac{1}{1-\phi}.$$

It follows that parameter ϕ determines the upper bound on the fertility differential. We choose $\phi = 0.8$ so that the fertility ratio between households with the lowest and highest wage is 5.

The choice of σ. We assume that the wage distribution follows a lognormal distribution $F(\mu, \sigma^2)$, where μ and σ^2 are the mean and variance of the underlying normal distribution, respectively. The Gini coefficient of this distribution is $G = 2N(\sigma/\sqrt{2}) - 1$, where N is the standard normal distribution function. We choose $\sigma = 0.55$, corresponding to a Gini coefficient of 0.3027.

Results of the Numerical Example

See Table 1.

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