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"Pricing and investment of cross-border transport infrastructure"

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Pricing and investment of cross-border transport infrastructure[*]

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Abstract

We develop a simple two-country model of international trade in which the transportation cost between countries is endogenously determined by decisions concerning capacity and price (road toll, rail fare) of infrastructure. We evaluate alternative regimes of pricing and investment, i.e., free access (e.g., public road), pricing and investment by two governments, and private operation. Comparisons between free-access and other regimes reveal that pricing plays a positive role of encouraging investment. However, pricing by governments results in lower welfare since excessively high prices are charged. We also show that higher welfare could be attained by elaborating the design of bidding systems for the right to build and operate the infrastructure.

JEL Classification: H54, R42, R48

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1 Introduction

Cross-border transport infrastructure plays the important role of supporting trade with neighboring countries, which accounts for a significant share of total trade (Bougheas et al., 1999; Limao and Venables, 2001). The recent wave of regional integration is leading to greater needs for investment in the transport infrastructure serving the region, since a well-developed transport infrastructure is essential for successful regional integration (Fujimura, 2004). Examples of initiatives to facilitate regional integration include the Trans-European Transport Network and the Asian Land Transport Infrastructure Development (ALTID) project.

Mun and Nakagawa (2008) discuss the problem of resource allocation concerning the provision of cross-border transport infrastructure connecting two neighboring countries. An investment in the infrastructure in one country decreases the transportation costs of both import and export goods, which benefits not only the home country but also the neighboring country. In this case, independent decision making leads to under-investment in infrastructure, since the investment decision of each country does not take into account the benefit to the other country. Mun and Nakagawa focus on the role of foreign aid to improve the efficiency, and show that the aid may make not only the recipient but also the donor better off. The limitation of this paper is that it focuses only on investment decisions assuming that the price for using the infrastructure is free.

Although free access to transport infrastructure (such as public roads) is widely applied, there exist types of transportation modes that impose user charges for infrastructure (railways, ports, airports). At the same time, there has been an increasing tendency of tolling on public roads, for several reasons such as congestion management, obtaining funds for road investments, etc. (Glaister and Graham, 2004; Lindsey, 2007). For example, a truck tolling system was recently introduced in Germany in 2005.

The present paper extends our earlier work by including pricing policies. We develop a simple two-country model of international trade where the transportation cost between two countries depends on the capacity and price (e.g., road toll, rail fare) of infrastructure. The government of each country chooses the capacity and price of infrastructure use within its territory so as to maximize the welfare of its citizens. The government faces the following trade-off: investment in capacity would lower the transport cost and thereby increase the gains from trade, but increase the fiscal burden; raising the price would increase the revenue but decrease the gains from trade.

The earlier result of under-investment in the case of free access may be modified if we incorporate pricing; the decision rule of choosing the capacity becomes efficient. Investment induces increase in the volume of traded goods, which raises not only the gains from trade but also the revenue from pricing. The latter effect plays the role of giving the governments incentives for greater investment. However, the pricing rule is inefficient in that each government charges an excessively high price for infrastructure use. Thus, despite the efficient investment rule, the resulting capacity does not attain the optimal level.

We evaluate two regimes, with and without pricing, in terms of the efficiency in provision of the infrastructure, welfare of the two countries, and the global welfare of the two-country economy as a whole. Furthermore, considering the fact that a growing number of toll roads have been constructed and operated privately or by various forms of

public-private partnerships (Roth, 1996), we examine alternative systems of providing cross-border transport infrastructure, such as private operation, control of investment decisions of private firms by design of bidding for franchise.[1]

Recently, economists have become interested in pricing and the investment decisions of the transport infrastructure by multiple governments (Bond, 2006; De Borger, Dunkerley, Proost, 2007; Fukuyama, 2006; Levinson, 2000).

Bond (2000) investigates the consequences of independent decision making by governments concerning infrastructure investment, and examined the effects of trade liberalization on the incentive to invest. Fukuyama (2005) also discusses a similar problem by means of numerical simulations. These papers focus only on investment decisions. Levinson (2000) looks at strategic interactions between governments on a serial network. Levinson keeps the capacity fixed, and focuses on the choice of revenue raising mechanisms (tax versus toll). He finds that larger regions are more likely to tax than smaller regions.

Our paper is closely related to the recent work by De Borger, Dunkerley and Proost (2007) that deals with pricing and investment in the setting of two transport links in series, each of which is controlled by a different government. Transport links are congestible and traffic on each link consists of local trips and transit. Transit trips are neither originated from nor destined to one of two regions, while trips between neighboring regions are assumed to be zero. De Borger, Dunkerley and Proost (2007) evaluate numerically the effects of toll discrimination between local and transit trips. Our paper considers trips between neighboring regions that would be quantitatively more important than transit traffic in many cases. We evaluate a larger set of alternative regimes such as private operation, etc., and obtain the ranking of different regimes analytically in the case that two countries are symmetric.

The rest of the paper is organized as follows. Section 2 describes the model of a two-country economy. Section 3 presents alternative regimes concerning pricing and investment of cross-border transport infrastructure: free-access, pricing and investment by national government, private operation, and user cost minimization. These alternative regimes are evaluated analytically and numerically in Section 4. Section 5 concludes the paper.

2   The Model

2.1   The setting

Consider an economy with two countries, indexed by i ($i = 1, 2$). There are l_i households in Country i. All households in the same country have identical preferences and labor skills. The two countries may be different in country size, income, and preferences.

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[1]De Palma and Lindsey (2000) analyze road pricing in the case that roads are operated privately. Yang and Meng (2000) investigate the effect of a new road project by BOT in the setting of network with route choice. Verhoef (2007) evaluates alternative highway franchise regimes for different network structure, such as parallel and serial. The primary interests of these works are the role of pricing to control traffic congestion, and they do not deal with the situation that multiple governments are involved in pricing and investment of the transport infrastructure.

This economy produces three goods, which are indexed by 1, 2, and z. The productions of Goods 1 and 2 are completely specialized, i.e., Country i produces Good i. Unlike Goods 1 and 2, Good z is produced in both countries, which is set as the numeraire. Labor is the only input for the production of the three goods. Each household in the economy consumes all three goods. Thus, Country i exports Good i and imports Good j $(j \neq i)$. Goods and factor markets are perfectly competitive.

When Goods 1 and 2 are traded, transportation costs are incurred. On the other hand, Good z is transported without cost. The transportation costs of Goods 1 and 2 depend on the capacity and price (e.g., road toll, rail fare) of infrastructure. We assume that the transport infrastructure is produced from Good z with constant returns to scale technology.

2.2 Consumption

The preferences of a household in Country i are represented by the following utility function

$$u_i\left(x_i^i, x_i^j, z_i\right),$$

where x_i^i, x_i^j, and z_i are respectively the consumption of Goods i, j, and z. We assume that u_i is strictly increasing, quasi-concave, and twice continuously differentiable. Each household is endowed with one unit of labor and levied a poll tax. The household's disposable income, y_i, is defined as

$$y_i = w_i - \tau_i,$$

where w_i is the wage rate in the country and τ_i is the poll tax. The budget constraint is given by

$$y_i = z_i + p_i^i x_i^i + p_i^j x_i^j,$$

where p_i^i and p_i^j represent the prices of Goods i and j in Country i. We suppose that Goods 1 and 2 are non-inferior goods.

Solving the utility maximization problem, we get the household's demand functions as

$$x_i^i\left(p_i^i, p_i^j, y_i\right), x_i^j\left(p_i^i, p_i^j, y_i\right), z_i\left(p_i^i, p_i^j, y_i\right),$$

and the indirect utility function

$$v_i\left(p_i^i, p_i^j, y_i\right).$$

2.3 Production

Each of the three goods is produced with a linear production technology. The production of Good i in Country i requires a_i^i amounts of labor. It follows that

$$p_i^i = w_i a_i^i, \text{ for } i = 1, 2.$$

Since Good z is set as the numeraire, the following relation should hold

$$1 = w_i a_i^z, \text{ for } i = 1, 2,$$

where a_i^z is the amount of labor required to produce one unit of Good z in Country i.

2.4 Transportation and trade

We suppose that there is a single location in each country at which goods are produced and consumed. We call this location as a market. Traded goods are transported between two markets. Labor is only input for the production of the transportation service. The transport cost from the market in Country i to the border, c_i, is defined as

$$c_i = f_i + w_i t_i,$$

where f_i is the price of infrastructure use within Country i and t_i is the amount of labor required for transportation. We interpret t_i as the transport time from the market in Country i to the border of the two countries. t_i depends on the capacity of the transport infrastructure in Country i, namely,

$$t_i = t_i\left(k_i\right),$$

where k_i is the capacity of the transport infrastructure in Country i. We assume that an investment in transport infrastructure increases capacity, thereby saves the labor (or time) required for transportation and that the investment is decreasing return to scale: $t_i' \equiv dt_i/dk_i < 0, t_i'' \equiv d^2 t_i/dk_i^2 > 0$.

The transport cost between two countries is given by $c_1 + c_2$. We assume perfect competition among trading firms, which eliminates positive profits. The prices of the traded goods satisfy

$$p_i^j = p_j^j + c_1 + c_2, \text{ for } i,j = 1,2, j \neq i. \tag{1}$$

3 Capacity and price of transport infrastructure

In this section, we consider the following five regimes: the first-best optimum, free access regime, pricing regime, private operation regime, and user cost minimization regime.

3.1 First-best optimum (Regime O)

In this paper, the first-best optimum is characterized as the solution to a global welfare maximization problem, which is called Regime O. The social planner chooses the price of infrastructure use, the capacity of infrastructure, and the poll tax. The problem to be solved is stated as follows

$$\max_{f_1,f_2,k_1,k_2,\tau_1,\tau_2,u_1,u_2} W\left(u_1,u_2\right)$$

subject to

$$u_i = v_i\left(p_i^i, p_j^j + f_1 + f_2 + w_1 t_1 + w_2 t_2, w_i - \tau_i\right), j \neq i$$

$$p_1^k k_1 + p_2^k k_2 = l_1 \tau_1 + l_2 \tau_2 + (f_1 + f_2)\left(l_1 x_1^2 + l_2 x_2^1\right),$$

where $W\left(.\right)$ is strictly increasing and continuously differentiable in u_i and quasi-concave with respect to the policy variables, and p_i^k is the amount of Good z required to produce

one unit of the transport infrastructure in Country i (or, unit cost of infrastructure). The second constraint is the budget constraint of the planner.

The first order conditions with respect to the level of infrastructure and the price of infrastructure use are

$$-w_i t'_i \left(l_1 x_1^2 + l_2 x_2^1 \right) = p_i^k \tag{2}$$
$$f_i = 0 \tag{3}$$

The LHS of (2) is the product of the quantity of Goods 1 and 2 transported between the two countries and the marginal change in the transport cost by an investment in transport infrastructure in Country i. The LHS is the marginal benefit of the investment, while the RHS is the marginal cost.

Condition (3) means that the price of infrastructure use should be zero. This is natural since marginal cost of usage, such as operation cost or congestion, does not exist in our model.

3.2 Free access, investment by national government (Regime F)

This regime has been discussed by Mun and Nakagawa (2008). In this regime, the government chooses the level of infrastructure and collects a poll tax to finance the expenditure while its price of infrastructure use is fixed to zero, i.e. the infrastructure is free access. We call this regime the free access regime or Regime F. The objective of the government is to maximize the welfare of its citizens

$$\max_{k_i} v_i \left(p_i^i, p_j^j + w_i t_i + w_j t_j, w_i - \tau_i \right),$$

where

$$\tau_i = \frac{p_i^k k_i}{l_i}. \tag{4}$$

The first order condition of the above problem yields the following investment rule

$$-w_i t'_i l_i x_i^j = p_i^k. \tag{5}$$

The LHS of equation (5) is marginal change in transport cost multiplied by the quantity of Good j that Country i imports from Country j. Namely, the LHS is the marginal benefit to Country i of an investment in the transport infrastructure. The RHS is the marginal cost of the investment. Comparison between conditions (2) and (5) reveals that the investment rule under free access is inefficient since Country i does not take account of the marginal benefit to Country j.

Country i's optimum level of investment is given as the solution to equation (5). Since the quantity of import goods depends on the capacity of the transport infrastructure in Country j, the solution to equations (5) is written as

$$k_i = K_i^F (k_j) \text{ for } i, j = 1, 2, j \neq i, \tag{6}$$

which is considered as Country i's reaction function. As shown in Mun and Nakagawa (2008), the investment on transport infrastructure is a strategic complement, namely,

$$\frac{dK_i^F}{dk_j} > 0 \text{ for } i, j = 1, 2, j \neq i.$$

3.3 Pricing and investment by national government (Regime P)

In this regime, the government in Country i chooses not only the level of the transport infrastructure but also the price of infrastructure use. We call this regime the pricing regime or Regime P. The government maximizes the welfare of its citizens subject to the budget constraint. The problem of the government is given by

$$\max_{k_i, f_i} v_i \left(p_i^i, p_j^j + c_j + f_{,i} + w_i t_i, w_i - \tau_i \right)$$

where τ_i is obtained from the budget constraint, as follows

$$\tau_i = \frac{p_i^k k_i - f_i \left(l_i x_i^j + l_j x_j^i \right)}{l_i}. \tag{7}$$

The first order condition with respect to the price f_i yields

$$l_i x_i^j = -l_i \frac{\partial \tau_i}{\partial f_i}. \tag{8}$$

The LHS of (8) is the quantity of Good j imported by Country i that is equal to the loss in consumer surplus. A rise in the price of infrastructure use increases the price of the import good in Country i, which harms the welfare of the consumers. The RHS of (8) is the reduction in the tax burden caused by increase in revenue from infrastructure. Differentiating the budget constraint, and substituting it to (8), we have the pricing rule as follows [2].

$$l_i x_i^j = \frac{l_i x_i^j + l_j x_j^i + f_i \left(l_i x_{ic}^j + l_j x_{jc}^i \right)}{1 - f_i x_{iy}^j}. \tag{9}$$

where $x_{ic}^j = \partial x_i^j / \partial (c_i + c_j)$, $x_{jc}^i = \partial x_j^i / \partial (c_i + c_j)$, $x_{iy}^j = \partial x_i^j / \partial y_i$. The numerator of the RHS of (9) is the marginal revenue for the government. When there are no income effects on the demands for Goods 1 and 2, the pricing rule is reduced to the following form

$$-l_j x_j^i = f_i \left(l_i x_{ic}^j + l_j x_{jc}^i \right). \tag{10}$$

In this case, the reduction in the poll tax is equal to the per-capita increase in the government revenue.

We now turn to the investment rule. From the first order condition, we get

$$-w_i t_i' l_i x_i^j = l_i \frac{\partial \tau_i}{\partial k_i}. \tag{11}$$

The LHS of (11) is the increase in Country i's consumer surplus from one unit increase in the capacity of the transport infrastructure, which is the marginal benefit of the investment.

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[2]Differentiating equation (7) with respect to τ_i and f_i and rearranging the resulting equation yields the effect of a rise in the price of infrastructure use on the poll tax as

$$\frac{\partial \tau_i}{\partial f_i} = -\frac{l_i x_i^j + l_j x_j^i + f_i \left(l_i x_{ic}^j + l_j x_{jc}^i \right)}{l_i \left(1 - f_i x_{iy}^j \right)},$$

Substituting this equation into (8), we have the pricing rule.

The RHS is the increase in the tax burden to finance the investment, which is perceived as the marginal cost of investment for the country. Condition (11) is the cost-benefit rule for the government that is concerned only with the welfare of its citizens.

Differentiating equation (7) with respect to τ_i and k_i yields

$$\frac{\partial \tau_i}{\partial k_i} = \frac{p_i^k - f_i \left(l_i x_{ic}^j + l_j x_{jc}^i \right) w_i t_i'}{l_i \left(1 - f_i x_{iy}^j \right)}. \tag{12}$$

The numerator of the RHS of (12) is the net effect on the government expenditure of a one-unit investment on transport infrastructure. Note that the investment increases the trade volume and the revenue from the infrastructure. The second term of the numerator is the rise in revenue from infrastructure, which reduces the tax burden. Substituting (9) and (12) into (11) and rearranging the resulting equation, we have the investment rule as follows.

$$-w_i t_i' \left(l_i x_i^j + l_j x_j^i \right) = p_i^k. \tag{13}$$

This equation is identical to the first-best investment rule. The government that is concerned only about the welfare of its citizens takes into account the benefit for the other country's citizens in the end. This is because the investment generates additional benefit as described earlier: increase the revenue induced by the expansion of trade volume. This revenue effect turns out to be equal to the benefit for the other country's citizens.

Country i's optimal level of investment and its optimal price of infrastructure use are the solutions to equations (9) and (13). These equations include the quantity of traded goods that depend on the capacity and price of the transport infrastructure not only in the home country but also in the foreign country. Thus, the solutions are given by

$$k_i^P = K_i^P \left(k_j, f_j \right), \tag{14}$$

$$f_i^P = F_i^P \left(k_j, f_j \right) \text{ for } i, j = 1, 2, j \neq i, \tag{15}$$

where the super script P represents this regime.

The response of these functions to a change in the transport cost in Country j is not straightforward. When the demand for an import good does not depend on the disposable income ($x_{iy}^j = 0$ for $i, j = 1, 2, j \neq i$), the response is given in the following lemma. The proof of this lemma is shown in Appendix A.

Lemma 1 *Suppose that in Country i ($i = 1, 2$), the demand of Good j ($j = 1, 2, j \neq i$) is independent of disposable income. Then, Country i's investment decisions in response to other country's policies are*

$$\frac{\partial K_i^P}{\partial k_j} > 0 \text{ and } \frac{\partial K_i^P}{\partial f_j} < 0$$

Country i's pricing decision depends on the shape of the demand function for import goods.

$$\frac{\partial F_i^P}{\partial k_j} \lesseqgtr 0 \text{ and } \frac{\partial F_i^P}{\partial f_j} \gtreqless 0$$

$$\iff l_j x_{jc}^i + f_i \left(l_i x_{icc}^j + l_j x_{jcc}^i \right) \gtreqless 0, \tag{16}$$

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where $x^j_{icc} = \partial^2 x^j_i / \partial \left(c_i + c_j\right)^2$ *and* $x^i_{jcc} = \partial^2 x^i_j / \partial \left(c_i + c_j\right)^2$.

This lemma implies that when the demand function for an import good is linear, the pricing decisions are strategic substitutes[3]: Country i increases its price of infrastructure use in response to an increase in Country j's capacity and to a decrease in Country j's price. When the demand is nonlinear and sufficiently convex, the pricing decisions may be strategic complements.

3.4   Private operation of transport infrastructure (Regime M)

Suppose an auction in which the right to build and operate the transport infrastructure is awarded to the firm offering the highest bid. We assume that this auction is competitive in that there are sufficient number of equally productive firms, that no firm has a market power in the bidding process, and that there is no collusion between firms. Thus, the profit of the winner will be zero. We call this regime the private operation regime or Regime M. In this case, the winning firm, the provider of the infrastructure service should solve the following problem:

$$\max_{k_1, k_2, f} f\left(l_1 x_1^2 + l_2 x_2^1\right) - p_1^k k_1 - p_2^k k_2$$

The first order conditions are given by

$$f\left(l_i x^j_{ic} + l_j x^i_{jc}\right) w_i t'_i - p_i^k = 0, \tag{17}$$
$$l_1 x_1^2 + l_2 x_2^1 + f\left(l_1 x^2_{1c} + l_2 x^1_{2c}\right) = 0. \tag{18}$$

From equations (17) and (18), we have the investment rule as

$$-w_i t'_i \left(l_i x^j_i + l_j x^i_j\right) = p_i^k. \tag{19}$$

This investment rule is identical to the first-best rule as in the case of pricing and investment by the national government.

The pricing rule is given by (18). The LHS is the marginal revenue for the provider. Since there is no operation cost, any change in traffic does not affect the cost of the provider. Thus, (18) is the condition of revenue maximization. It should be noted that when demand for import goods does not depend on income, the pricing rule in this regime is identical to that in the pricing regime (Regime P).[4]

Thus, we have the following lemma:

Lemma 2 *Suppose that in Country i $(i = 1, 2)$, the demand of Good j $(j = 1, 2, j \neq i)$ is independent of disposable income. Then, private operation (Regime M) and pricing and investment by the national government (Regime P) yield the same outcome.*

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[3]If the demand is linear, $x^j_{icc} = x^i_{jcc} = 0$. Applying these relations to (16), we have $\frac{\partial F_i^P}{\partial k_j} > 0$ and $\frac{\partial F_i^P}{\partial f_j} < 0$.

[4]Summing up the equations (10) for two countries, we have

$$l_1 x_1^2 + l_2 x_2^1 + \left(f_1 + f_2\right)\left(l_1 x^2_{1c} + l_2 x^1_{1c}\right) = 0$$

Since $f = f_1 + f_2$, the above equation coincides with (18).

Private operation by a single provider ignores the effect on consumers' welfare in the pricing decision, but avoids double margins in the independent pricing by governments. Lemma 2 shows that these positive and negative effects of the private operation are exactly offset in the absence of income effect.

3.5 User cost minimization (Regime U)

In this regime, competitive bidding is designed so that the right to construct and operate the transport infrastructure is awarded to the firm proposing the plan that minimizes the user cost.[5] We call this regime the user cost minimization regime or Regime U. The bidder's problem is

$$\min_{f, k_1, k_2} f + w_1 t_1 + w_2 t_2$$

subject to

$$f \left(l_1 x_1^2 + l_2 x_2^1 \right) - p_1^k k_1 - p_2^k k_2 = 0$$

The investment rule is again the same as in the first-best optimum.

$$-w_i t_i' \left(l_i x_i^j + l_j x_j^i \right) = p_i^k. \tag{20}$$

To interpret this result, (20) is rewritten as follows

$$\frac{-w_i t_i'}{p_i^k} = \frac{1}{l_i x_i^j + l_j x_j^i}. \tag{21}$$

The above condition states that marginal benefit-cost ratios should be equalized across policy instruments to minimize the user cost: investment in infrastructure and price reduction. The LHS of the above equation is the marginal benefit-cost ratio of investment: the benefit and cost are appeared respectively in the numerator and denominator. When the provider invests one unit on the infrastructure, the user cost decreases by $-w_i t_i'$. This investment incurs the cost of infrastructure, p_i^k. The RHS is the marginal benefit-cost ratio of price reduction. The benefit of price reduction by one unit is the reduction in the user cost by one unit. The cost of price reduction is the reduction in the provider's revenue by the quantity of the infrastructure use.

Substituting this investment rule in the zero profit condition yields the pricing rule:

$$f = -w_1 t_1' k_1 - w_2 t_2' k_2.$$

4 Evaluation of alternative regimes

To obtain the explicit result, we specify the form of the utility function as

$$u_i \left(x_i^i, x_i^j, z_i \right) = z_i - \frac{x_i^i}{\alpha_{ea}} \left[\ln \left(\frac{x_i^i}{\alpha_{eb}} \right) - 1 \right] - \frac{x_i^j}{\alpha_{ma}} \left[\ln \left(\frac{x_i^j}{\alpha_{mb}} \right) - 1 \right]$$

$$\text{for } i, j = 1, 2, j \neq i, \tag{22}$$

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[5]Note that trade volume must be maximized by user cost minimization. Thus this regime is equivalent to the patronage maximization discussed in Verhoef (2007).

where α_{ea}, α_{eb}, α_{ma}, and α_{mb} are parameters representing the preferences for consumptions of the export and import goods. Then, the demand of a household in Country i for Good j is given by

$$x_i^j = \alpha_{mb} \exp\left(-\alpha_{ma} p_i^j\right).$$

We also specify the function describing the transport technology as

$$t_i\left(k_i\right) = -\beta \ln \frac{k_i}{\bar{k}}, \tag{23}$$

$\bar{k}$ is the upper limit of the capacity of the transport infrastructure ($\bar{k} = 1$).

4.1 Analytical results

In this subsection, we present some analytical results derived for the specific forms of utility and transport technology functions.

4.1.1 A symmetric economy

Suppose that the sizes and technologies of countries are identical. We present the ranking of the regimes for several criteria such as the capacity of infrastructure, transport cost, and welfare. Since our specific utility function does not have an income effect, the private operation regime attains the same allocation as the pricing regime as shown by Lemma 2. Thus, we omit the results for the private operation regime. In the following, super scripts indicate the regimes: O represents the first best optimum, F the free access regime, P the pricing regime, and U the user cost minimization regime. The proofs of the following lemmas and proposition are given in Appendix A.

First, the capacities of infrastructure under alternative regimes are ranked as follows.

Lemma 3 *In a symmetric economy, the capacity in the regimes satisfy the following relations*

$$k_i^O > k_i^U > k_i^F > k_i^P \text{ if } \alpha_{ma}\beta w_i < \frac{1}{2}\log 2 \cong 0.346574,$$

$$k_i^O > k_i^U = k_i^F > k_i^P \text{ if } \alpha_{ma}\beta w_i = \frac{1}{2}\log 2, \qquad \text{for } i = 1, 2,$$

$$k_i^O > k_i^F > k_i^U > k_i^P \text{ if } \alpha_{ma}\beta w_i > \frac{1}{2}\log 2.$$

The investment on transport infrastructure is lowest in the pricing regime. Although the investment rule in the pricing regime is identical with the first best rule, the investment level in the pricing regime is lower than that in the free access regime in which the investment decision is not efficient. Note that the marginal benefit of investment (LHS of the expression for the investment rule) is proportional to the volume of trade. In the pricing regime, the volume of trade is smaller due to the excessively high price of infrastructure use. This effect reduces the marginal benefit, and leads to a smaller level of investment.

The ranking of the free access regime and the user cost minimization regime depends on the wage rate and the slope of demand function for import good and transport technology. If the demand is more elastic (smaller α_{ma}), transport technology is backward

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(smaller β), or wage rate is lower (smaller w_i), the level of investment in the user cost minimization is higher than that in the free access regime.

The transport cost depends not only on the level of investment but also on the price of infrastructure use. Thus, the ranking of transport cost is different from that of the capacity:

Lemma 4 *In a symmetric economy, the transport costs in the regimes satisfy the following relations*

$$c^P > c^U > c^F > c^O,$$

where $c^r = c_1^r + c_2^r$ for $r = O, F, P,$and U.

Unlike the levels of investment, the ranking is not contingent. The transport cost is highest under the pricing regime. User cost in Regime U is higher than that in Regime F. This result holds, even in the case that the level of investment is higher in Regime U. The price effect dominates the effect of larger capacity.

The utility function specified as (22) is a quasi-linear form, thereby the utility is measured in monetary terms. So we define global welfare as the sum of utilities of all households in the economy, that is, $W = l_1 v_1 + l_2 v_2$. We have the ordering of the global welfare as follows:

Proposition 1 *In a symmetric economy, the levels of the global welfare in the regimes are ranked as follows*

$$W^O > W^U > W^F > W^P \ \ \text{if} \ \ \alpha_{ma}\beta w_i < \theta \cong 0.229966,$$

$$W^O > W^U = W^F > W^P \ \ \text{if} \ \ \alpha_{ma}\beta w_i = \theta, \qquad \text{for } i = 1, 2,$$

$$W^O > W^F > W^U > W^P \ \ \text{if} \ \ \alpha_{ma}\beta w_i > \theta,$$

where θ is the solution of the following equation

$$\left(\frac{2}{e}\right)^{\frac{2\theta}{1-2\theta}} - 1 + \theta = 0.$$

Proposition 1 shows that the second-best regime may be different depending on the parameters. The second-best regime in a symmetric economy is the user cost minimization (Regime U) when the value of $(\alpha_{ma}\beta w_i)$ is relatively small, in other words, the demand is more elastic, transport technology is backward, or the wage rate is lower. Otherwise, the second-best regime is the free access (Regime F). However, the calibration for numerical analysis suggests that the value of $(\alpha_{ma}\beta w_i)$ is very small (much smaller than 0.1). Therefore, it is reasonable to say that the user cost minimization is more efficient than the free access.

Additionally, the pricing by national governments (Regime P) is less efficient than the free-access (Regime F). This result is easily anticipated from Lemmas 3 and 4. Ohsawa (2000) obtains the contrasting result that the pricing attains higher welfare than the free-access. His model is different from ours in that the pricing and investment decisions are determined by voting.

4.1.2 An asymmetric economy

This section examines the asymmetry between two countries in population size and the wage rates. We have some analytical results concerning the effect of difference in population size. Let us focus on the population distribution holding the sum of the population of the two countries constant. Let s_i be Country i's share of population such that $s_i = l_i / (l_1 + l_2)$.

Lemma 5 *Under the free-access regime, the effect of the population share of Country 1 on investment levels of the two countries are*

$$\frac{\partial k_1^F}{\partial s_1} \gtreqqless 0 \iff s_1 \lesseqqgtr 1 - \alpha_{ma}\beta w_2,$$

$$\frac{\partial k_2^F}{\partial s_1} \gtreqqless 0 \iff s_1 \lesseqqgtr \alpha_{ma}\beta w_1,$$

The effect on the user cost is

$$\frac{\partial c^F}{\partial s_1} \lesseqqgtr 0 \iff s_1 \lesseqqgtr \frac{w_1}{w_1 + w_2}.$$

In addition, suppose that the production technology in the two countries are identical, then, $w_1 = w_2$ and $p_1^1 = p_2^2$. In this case, the global welfare in the free access regime satisfies

$$\frac{\partial W^F}{\partial s_1} \gtreqqless 0 \iff s_1 \lesseqqgtr \frac{1}{2}.$$

The effect of population distribution on the level of investment may be either positive or negative. Since the value of $\alpha_{ma}\beta w_i$ is small as already mentioned, $\alpha_{ma}\beta w_1 < s_1 < 1 - \alpha_{ma}\beta w_2$ unless population distribution is extremely uneven. It is likely that $\partial k_1^F/\partial s_1 > 0$ and $\partial k_2^F/\partial s_1 < 0$. This is to say, the level of investment is increasing with the population share of home country.

When the wage rates in two countries are identical, the transport cost, c^F, is increasing with asymmetry in population size. This is because investment in infrastructure is decreasing returns: when the population size in the larger country increases and the size of the smaller country decreases by the same units, the larger country increases its investment and the smaller country decreases. However, the transport cost reduction in the larger country is smaller than the transport cost increase in the smaller country. Consequently, increasing the asymmetry increases the transport cost and decreases the global welfare.

We turn to the other regimes.

Lemma 6 *The effects of population distribution in Regimes O, P, and U are*

$$\frac{\partial f_1^P}{\partial s_1} < 0, \frac{\partial f_2^P}{\partial s_1} > 0, \frac{\partial \left(f_1^P + f_2^P\right)}{\partial s_1} = 0,$$

$$\frac{\partial f^U}{\partial s_1} = 0,$$

$$\frac{\partial k_i^r}{\partial s_1} \gtreqqless 0 \text{ and } \frac{\partial c^r}{\partial s_1} \lesseqqgtr 0 \iff p_1^1 \gtreqqless p_2^2, \ i = 1, 2, r = O, P, U.$$

Table 1: Parameter values

a_1^z, a_2^z	1	α_{ma}	0.833333
a_1^1, a_2^2	1	α_{mb}	0.453047
$l_1 + l_2$	1	α_{ea}	0.833333
p_1^k	0.026472	α_{eb}	0.67957
p_2^k	0.026472	β	0.06

In addition, when $w_1 = w_2$ and $p_1^1 = p_2^2$, we have

$$\frac{\partial W^O}{\partial s_1} = \frac{\partial W^P}{\partial s_1} = \frac{\partial W^U}{\partial s_1} = 0.$$

Lemma 6 shows that the effects of population distribution on the investment levels and transport cost depend on the difference in the FOB prices. This is explained as follows. In this model, the transport costs of Goods 1 and 2 are identical. The ratio of the transported quantity of Good 1 to that of Good 2 is determined by the ratio of the FOB price. Suppose that the FOB price of Good 2 is lower than that of Good 1. Then, a household in Country 1 consumes more Good 2 than a household in country 2 consumes Good 1. An increase in the size of Country 1 and corresponding decrease in the size of Country 2 raises the quantity transported between the two countries. This increase in the transportation raises the investment and reduces the transport cost.

In addition, if asymmetry between the two countries is only in population size, then the investment levels, transport cost between the two countries, and the global welfare levels are independent from the population distribution. This implies that the results of Lemmas 3, 4, and Proposition 1 concerning the ranking between Regimes O, P, U hold if the population distribution is asymmetric.

4.2 Numerical results

In this subsection, we evaluate the regimes in a more general setting by means of numerical simulations. In the simulation, the parameters are set so as to describe the real world pictures. We choose several observable indicators that are endogenous variables of the model, and set the parameter values such that the output of the model fits the prespecified (and reasonable) values of the indicators. Details are described in Appendix B. Table 1 represents the values of the parameters and indicators.

4.2.1 Asymmetry in country size

Figure 1 shows the results of simulations for various population distributions. Note that $\alpha_{ma}\beta w_i < \frac{1}{2}\log 2 < \theta$ under the parameter values in Table 1. The horizontal axis measures Country 1's share of the global population, s_1. A vertical dotted line is drawn so that it crosses the horizontal axis at $s_1 = 0.5$ in order to verify the results in lemmas and the proposition for the symmetric case. Curve O represents the first best optimum,

Curve F the free access regime, Curve P the pricing regime, and Curve U the user cost minimization regime.

Figure 1(a) shows the capacity of the transport infrastructure in Country 1. In the free access regime, the investment of the country increases with its size, but the sum of investments of the two countries is almost the same as that in the symmetric economy. When Country 1's population share is small, its investment in the free access regime is smaller than that in the pricing regime: Lemma 3 does not hold when the population distribution is quite uneven.

Figure 1(b) illustrates the transport time. When the global population distributes uniformly, the transport time in the free access regime is lower than that in the pricing regime. As the distribution becomes more uneven, this relation is reversed.

Figure 1(c) depicts the transport cost. In most situations, the transport cost in the pricing regime is significantly higher than those in the other regimes due to the high price for infrastructure use. For example, in the symmetric case, the price in the pricing regime is 1.2, while the price in the user cost minimization regime is 0.12. In an asymmetric economy, the transport time and cost in the free access regime are higher than those in the symmetric economy. The reason is that the transport technology is decreasing return to scale: the more a country invests, the less the effect of investment is. When the population distribution is extremely uneven, the transport cost in the free access regime exceeds that in the pricing regime.

Figure 1(d) shows the efficiency ratios of the four regimes. The efficiency ratio measured on the vertical axis is calculated by the following formula:

$$\text{Efficiency ratio} = \frac{W^r - W^P}{W^O - W^P} \text{ for } r = F, U.$$

This indicator is utilized for evaluating the relative efficiency of regimes. Note that the global welfare in the pricing regime turns out to be lowest in almost all cases. The efficiency ratio indicates welfare gain of Regime r from the lowest level (Regime P) relative to the maximum gain. This figure suggests that welfare loss in the pricing regime is very large. This is due to the distortion caused by pricing behavior, as explained for Figure 1(c).

In this setting, the user cost minimization regime is the second best. Overall, numerical results suggest that the ranking of regimes is not affected by introducing asymmetry in population size. However the countries might not have the incentive to adopt this regime. Figure 1(e) shows the welfare of the representative household in Country 1. When the population in Country 1 is sufficiently small, the household in Country 1 most prefers the pricing regime.

4.2.2 Asymmetry in productivity

Let us examine the effect of the asymmetry in productivity, which is represented by the difference in wage rates between countries. Consider an economy that differs from the symmetric economy only in the wage rate in Country 2. Other parameters are identical with those in Table 1. Figure 2 illustrates how the relative wage affects the investment, transport, and welfare. The horizontal axis of this figure is the relative wage rate of Country 2. The wage rate of Country 1 is fixed at one. The notation of the curves is the same as those in Figure 1.

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Figure 2(a) depicts the investment in Country 1. In each regime, Country 1 reduces its investment as the wage rate in Country 2 rises. Figure 2(b) shows the investment in Country 2. In each regime, Country 2's investment increases with its wage rate. Namely, a country increases its investment in response to an increase in the home country's wage, and decreases in response to an increase in the other country's wage. In addition, Figure 2(b) indicates that Country 2 invests more in the pricing regime than in the free access regime when its wage rate is low. In other cases, the level of investment in the pricing regime is lower. Figure 2(c) shows that ranking of transport time among regimes is not affected by the difference in wage rates. It is seen that the transport time in each regime is decreasing with the wage rate in Country 2, but its level is relatively stable against changes in wage differential. This is because increase in investment of one country is offset by decrease in the other country, as shown in Figure 2 (a) and (b). Figure 2(d) depicts the transport cost. In contrast to the transport time, the transport cost increases as Country 2's wage rate raises. Figure 2(e) illustrates the efficiency ratio. For any wage rate of Country 2, the ratio in the user cost minimization regime is higher than that in the free access regime. As the wage rate of Country 2 falls, the values of efficiency ratios for both regimes become close to one.

5 Conclusion

If the use of infrastructure is free of charge, the national government does not take into account the benefit of investment in a neighboring country, which leads to under-investment in terms of resource allocation for the two-country economy as a whole. On the other hand, it is shown that the investment rule becomes efficient if the government levies a charge for infrastructure use. The problem with charging is that the price would be inefficiently high, which negatively affects the level of investment despite the efficient rule. Comparison of these two regimes, free access vs. pricing by government, shows that the distortion of pricing in the latter exceeds the loss owing to lack of incentive in investment decision in the former regime. However, it is also shown that the user cost minimization regime attains higher welfare than free access. This result suggests that a more efficient mechanism with pricing could be designed. We need to make further efforts in the future to explore the possibility of finding better mechanisms.

Appendix A

In this appendix, we give the proof of Lemmas and Propositions.

Proof of Lemma 1

For conciseness, define the quantity of Good j imported by Country i as $N_i = l_i x_i^j$. We also let the sum of N_1 and N_2 denoted by N. We assume that the demands for an import good do not depend on the disposable income.

The first order conditions are written as follows

$$\frac{\partial v_i}{\partial f_i} = \frac{v_{iy}}{l_i}\left(N_j + f_i N_c\right) = 0, \tag{24}$$

$$\frac{\partial v_i}{\partial k_i} = \frac{v_{iy}}{l_i}\left(-N_i w_i t'_i - p_i^k + f_i N_c w_i t'_i\right) = 0, \tag{25}$$

where $v_{iy} = \partial v_i/\partial y_i$, $N_c = N_{ic} + N_{jc}$, $N_{ic} = \partial N_i/\partial\left(c_1 + c_2\right)$, $N_{jc} = \partial N_i/\partial\left(c_1 + c_2\right)$. Differentiating (24) and (25) with respect to f_i, k_i, and c_j yields

$$\frac{v_{iy}}{l_i}\left\{\left(N_{ic} + 2N_{jc} + f_i N_{cc}\right)df_i + \left(N_{jc} + f_i N_{cc}\right)w_i t'_i dk_i + \left(N_{jc} + f_i N_{cc}\right)dc_j\right\} = 0, \tag{26}$$

and

$$\frac{v_{iy}}{l_i}\left\{\left(N_{jc} + f_i N_{cc}\right)w_i t'_i df_i + \left[-\left(N_{ic} + f_i N_{cc}\right)\left(w_i t'_i\right)^2 - N w_i t''_i\right]dk_i\right.$$
$$\left. + \left(-N_{ic} + f_i N_{cc}\right)w_i t'_i dc_j\right\} = 0 \tag{27}$$

where $N_{cc} = N_{icc} + N_{jcc}$, $N_{icc} = \partial^2 N_i/\partial\left(c_1 + c_2\right)^2$, $N_{jcc} = \partial^2 N_j/\partial\left(c_1 + c_2\right)^2$. Solving equations (26) and (27) with respect to df_i and dk_i, we get

$$\frac{\partial F_i^P}{\partial c_j} = \frac{\left(N_{jc} + f_i N_{cc}\right)N w_i t''_i}{\Delta},$$

$$\frac{\partial K_i^P}{\partial c_j} = \frac{\left(N_c\right)^2 w_i t'_i}{\Delta},$$

where

$$\Delta = \begin{vmatrix} N_{ic} + 2N_{jc} + f_i N_{cc} & \left(N_{jc} + f_i N_{cc}\right)w_i t'_i \\ \left(N_{jc} + f_i N_{cc}\right)w_i t'_i & -\left(N_{ic} + f_i N_{cc}\right)\left(w_i t'_i\right)^2 - N w_i t''_i \end{vmatrix} > 0.$$

The fact that Δ is positive follows from the second order condition of the national welfare maximization problem. Thus, we get

$$\frac{\partial F_i^P}{\partial c_j} \gtreqless 0 \iff N_{jc} + f_i N_{cc} = l_j x_{jc}^i + f_i\left(l_i x_{icc}^j + l_j x_{jcc}^i\right) \gtreqless 0,$$

$$\frac{\partial K_i^P}{\partial c_j} < 0.$$

Since $w_i t'_i$ is negative, the lemma follows from the above equations.

Proof of Lemma 3

This lemma gives the ordering of the levels of investment on transport infrastructure in a symmetric economy. In this economy, we have

$$w_1 = w_2, l_1 = l_2, a_{11} = a_{22}.$$

From the stability condition of the equilibrium in the free access and the pricing regime, we have

$$0 < \alpha_{ma}\beta w_1 < \frac{1}{2}. \tag{28}$$

The levels of investment are given by

$$k_i^O = \left(\frac{2l_1 e^{-\alpha_{ma} p_2^2} \alpha_{mb} \beta w_1}{p_1^k} \right)^{\frac{1}{1-2\alpha_{ma}\beta w_1}},$$

$$k_i^F = \left(\frac{l_1 e^{-\alpha_{ma} p_2^2} \alpha_{mb} \beta w_1}{p_1^k} \right)^{\frac{1}{1-2\alpha_{ma}\beta w_1}},$$

$$k_i^P = \left(\frac{2l_1 e^{-\alpha_{ma} p_2^2} \alpha_{mb} \beta w_1}{e\, p_1^k} \right)^{\frac{1}{1-2\alpha_{ma}\beta w_1}},$$

$$k_i^U = \left(\frac{2l_1 e^{-\alpha_{ma} p_2^2} \alpha_{mb} \beta w_1}{e^{2\alpha_{ma}\beta w_1} p_1^k} \right)^{\frac{1}{1-2\alpha_{ma}\beta w_1}}.$$

Then, we have

$$\frac{k_i^O}{k_i^U} = \exp\left(\frac{2\alpha_{ma}\beta w_1}{1 - 2\alpha_{ma}\beta w_1} \right) > 1,$$

$$\frac{k_i^U}{k_i^P} = e > 1,$$

$$\frac{k_i^O}{k_i^F} = 2^{\frac{1}{1-2\alpha_{ma}\beta w_1}} > 1,$$

$$\frac{k_i^F}{k_i^P} = \left(\frac{e}{2} \right)^{\frac{1}{1-2\alpha_{ma}\beta w_1}} > 1.$$

Thus, we have

$$k_i^O > k_i^U > k_i^P \text{ and } k_i^O > k_i^U > k_i^P.$$

The ordering of the levels of the pricing regime and of the user cost minimization regime depends on $a_{ma}\beta w_1$, namely,

$$\frac{k_i^F}{k_i^U} = \left(\frac{e^{2\alpha_{ma}\beta w_1}}{2} \right)^{\frac{1}{1-2\alpha_{ma}\beta w_1}} > 1 \text{ if and only if } \frac{e^{2\alpha_{ma}\beta w_1}}{2} > 1.$$

Thus, we have the lemma.

Proof of Lemma 4

This lemma gives the ordering of the user costs in the regimes. The user costs are given by

$$c^O = \frac{-2\beta w_1}{1 - 2\alpha_{ma}\beta w_1} \log\left(\frac{2l_1 \exp\left(-\alpha_{ma} p_2^2\right) \alpha_{mb} \beta w_1}{p_1^k} \right),$$

$$c^F = \frac{-2\beta w_1}{1 - 2\alpha_{ma}\beta w_1} \log\left(\frac{l_1 \exp\left(-\alpha_{ma} p_2^2\right) \alpha_{mb} \beta w_1}{p_1^k} \right),$$

$$c^P = \frac{2\beta w_1}{1 - 2\alpha_{ma}\beta w_1} \left[\frac{1}{2\alpha_{ma}\beta w_1} - \log\left(\frac{2l_1 \exp\left(-\alpha_{ma} p_2^2\right) \alpha_{mb} \beta w_1}{p_1^k} \right) \right],$$

$$c^U = \frac{2\beta w_1}{1 - 2\alpha_{ma}\beta w_1} \left[1 - \log\left(\frac{2l_1 \exp\left(-\alpha_{ma} p_2^2\right) \alpha_{mb} \beta w_1}{p_1^k} \right) \right].$$

Then, we have

$$c^O - c^F = \frac{-2\beta w_1}{1 - 2\alpha_{ma}\beta w_1} \log 2 < 0,$$

$$c^F - c^U = \frac{-2\beta w_1}{1 - 2\alpha_{ma}\beta w_1} (1 - \log 2) < 0,$$

$$c^U - c^P = \frac{-1}{\alpha_{ma}} < 0.$$

Thus, we have

$$c^O < c^F < c^U < c^P.$$

Proof of Proposition 1

Finally, we give the welfare ordering of the regimes. In a symmetric economy, the levels of global welfare in the regimes are given by

$$W^O = \overline{W} + 2l_1 e^{-\alpha_{ma}p_2^2} \frac{\alpha_{mb}}{\alpha_{ma}} (1 - 2\alpha_{ma}\beta w_1) \left(\frac{2l_1 e^{-\alpha_{ma}p_2^2}\alpha_{mb}\beta w_1}{p_1^k} \right)^{\frac{2\alpha_{ma}\beta w_1}{1-2\alpha_{ma}\beta w_1}},$$

$$W^F = \overline{W} + 2l_1 e^{-\alpha_{ma}p_2^2} \frac{\alpha_{mb}}{\alpha_{ma}} (1 - \alpha_{ma}\beta w_1) \left(\frac{l_1 e^{-\alpha_{ma}p_2^2}\alpha_{mb}\beta w_1}{p_1^k} \right)^{\frac{2\alpha_{ma}\beta w_1}{1-2\alpha_{ma}\beta w_1}},$$

$$W^P = \overline{W} + \frac{4l_1 e^{-\alpha_{ma}p_2^2}}{e} \frac{\alpha_{mb}}{\alpha_{ma}} (1 - \alpha_{ma}\beta w_1) \left(\frac{2l_1 e^{-\alpha_{ma}p_2^2}\alpha_{mb}\beta w_1}{e\, p_1^k} \right)^{\frac{2\alpha_{ma}\beta w_1}{1-2\alpha_{ma}\beta w_1}},$$

$$W^U = \overline{W} + 2l_1 e^{-\alpha_{ma}p_2^2} \frac{\alpha_{mb}}{\alpha_{ma}} \left(\frac{2l_1 e^{-\alpha_{ma}p_2^2}\alpha_{mb}\beta w_1}{e\, p_1^k} \right)^{\frac{2\alpha_{ma}\beta w_1}{1-2\alpha_{ma}\beta w_1}},$$

where

$$\overline{W} = 2 \left(l_1 w_1 + l_1 \frac{\alpha_{eb}}{\alpha_{ea}} \exp\left(-\alpha_{ea}p_1^1\right) \right).$$

Then, we have

$$\frac{W^U - \overline{W}}{W^P - \overline{W}} = \frac{e}{2(1 - \alpha_{ma}\beta w_1)} > 1,$$

$$\frac{W^F - \overline{W}}{W^P - \overline{W}} = \left(\frac{e}{2}\right)^{\frac{1}{1-2\alpha_{ma}\beta w_1}} > 1.$$

Thus, we get $W^U > W^P$ and $W^F > W^P$.

Comparing the first best global welfare with the global welfare in the user cost minimization regime yields

$$\frac{W^O - \overline{W}}{W^U - \overline{W}} = (1 - 2\alpha_{ma}\beta w_1) e^{\frac{2\alpha_{ma}\beta w_1}{2\alpha_{ma}\beta w_1 - 1}}.$$

Consider the logarithm of the RHS. The RHS is greater than one if the logarithm is positive. The logarithm is given by

$$\log\left(1 - 2\alpha_{ma}\beta w_1\right) - f\left(1 - 2\alpha_{ma}\beta w_1\right),$$

where

$$f(x) = \frac{x-1}{x}.$$

Investigating the slope and the intercept of $f(x)$, we obtain $\log x > f(x)$ for $0 < x < 1$.[6] This yields $W^O > W^U$.

Comparing the first best global welfare with the global welfare in the free access regime yields

$$\frac{W^F - \overline{W}}{W^O - \overline{W}} = \left(\frac{1 - \alpha_{ma}\beta w_1}{1 - 2\alpha_{ma}\beta w_1}\right) 2^{\frac{2\alpha_{ma}\beta w_1}{2\alpha_{ma}\beta w_1 - 1}}$$

$$= g\left(\alpha_{ma}\beta w_1\right),$$

where

$$g(x) = \left(\frac{1-x}{1-2x}\right) 2^{\frac{2x}{2x-1}}.$$

Then, we have

$$g(0) = 1 \text{ and } g'(x) < 0 \text{ for } 0 < x < 1.$$

This yields

$$g(x) < 1 \text{ for } 0 < x < 1.$$

Thus, we obtain $W^O > W^F$. To summarize the results, we get

$$W^O > W^F > W^P \text{ and } W^O > W^U > W^P.$$

Finally, let us consider the welfare ordering of the free access regime and the user cost minimization regime. We have

$$W^U - W^F$$

$$= \frac{2\alpha_{mb}l_1 e^{-\alpha_{ma}p_2^2}}{\alpha_{ma}} \left(\frac{l_1 e^{-\alpha_{ma}p_2^2}\alpha_{mb}\beta w_1}{p_1^k}\right)^{\frac{2\alpha_{ma}\beta w_1}{1-2\alpha_{ma}\beta w_1}} \left[\left(\frac{2}{e}\right)^{\frac{2\alpha_{ma}\beta w_1}{1-2\alpha_{ma}\beta w_1}} - (1 - \alpha_{ma}\beta w_1)\right].$$

The bracket of the RHS is rewritten as

$$\left(\frac{2}{e}\right)^{\frac{2\alpha_{ma}\beta w_1}{1-2\alpha_{ma}\beta w_1}} - (1 - \alpha_{ma}\beta w_1) = h\left(\alpha_{ma}\beta w_1\right) - (1 - \alpha_{ma}\beta w_1),$$

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[6]We have $\log 1 = f(1) = 0$ and

$$\frac{d}{dx}\log x - f'(x) = \frac{x-1}{x^2} < 0 \text{ for } 0 < x < 1.$$

Thus, we have

$$\log x < f(x) \text{ for } 0 < x < 1.$$

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where

$$h\left(x\right) = \left(\frac{2}{e}\right)^{\frac{2x}{1-2x}}.$$

The function $h\left(x\right)$ satisfies

$$h\left(0\right) = 1, \lim_{x \to \frac{1}{2}} h\left(x\right) = 0,$$

$$h'\left(0\right) > -1,$$

$$h'\left(x\right) < 0 \text{ for } 0 < x < \frac{1}{2}.$$

Thus there is a unique point $\theta \in \left(0, \frac{1}{2}\right)$ such that

$$h\left(\theta\right) - \left(1 - \theta\right) = 0.$$

We also have

$$h\left(x\right) - \left(1 - x\right) > 0 \text{ for } x < \theta,$$
$$h\left(x\right) - \left(1 - x\right) < 0 \text{ for } x > \theta.$$

Thus, we have

$$W^U > W^F \text{ for } \alpha_{ma}\beta w_1 < \theta,$$
$$W^U = W^F \text{ for } \alpha_{ma}\beta w_1 = \theta,$$
$$W^U < W^F \text{ for } \alpha_{ma}\beta w_1 > \theta.$$

Proof of Lemma 5

In the free access regime, the levels of investment in Countries 1 and 2 and the transport cost satisfy

$$k_1^F = \left[\frac{s_1\left(l_1 + l_2\right)\alpha_{mb}\beta w_1}{p_1^k e^{\alpha_{ma}p_2^2}}\right]^{\frac{1-\alpha_{ma}\beta w_2}{1-\alpha_{ma}\beta(w_1+w_2)}} \left[\frac{\left(1 - s_1\right)\left(l_1 + l_2\right)\alpha_{mb}\beta w_2}{p_2^k e^{\alpha_{ma}p_1^1}}\right]^{\frac{\alpha_{ma}\beta w_2}{1-\alpha_{ma}\beta(w_1+w_2)}},$$

$$k_2^F = \left[\frac{s_1\left(l_1 + l_2\right)\alpha_{mb}\beta w_1}{p_1^k e^{\alpha_{ma}p_2^2}}\right]^{\frac{\alpha_{ma}\beta w_1}{1-\alpha_{ma}\beta(w_1+w_2)}} \left[\frac{\left(1 - s_1\right)\left(l_1 + l_2\right)\alpha_{mb}\beta w_2}{p_2^k e^{\alpha_{ma}p_1^1}}\right]^{\frac{1-\alpha_{ma}\beta w_1}{1-\alpha_{ma}\beta(w_1+w_2)}},$$

$$c^F = -\beta w_1 \ln k_1^F - \beta w_2 \ln k_2^F.$$

Differentiating the above equations with respect to s_1 yields

$$\frac{1}{k_1^F}\frac{\partial k_1^F}{\partial s_1} = \frac{1 - \alpha_{ma}\beta w_2 - s_1}{\left[1 - \alpha_{ma}\beta\left(w_1 + w_2\right)\right]s_1\left(1 - s_1\right)},$$

$$\frac{1}{k_2^F}\frac{\partial k_2^F}{\partial s_1} = \frac{\alpha_{ma}\beta w_1 - s_1}{\left[1 - \alpha_{ma}\beta\left(w_1 + w_2\right)\right]s_1\left(1 - s_1\right)},$$

$$\frac{\partial c^F}{\partial s_1} = \frac{\beta\left(w_1 + w_2\right)}{\left[1 - \alpha_{ma}\beta\left(w_1 + w_2\right)\right]s_1\left(1 - s_1\right)}\left(s_1 - \frac{w_1}{w_1 + w_2}\right).$$

Thus, we have

$$\frac{\partial k_1^F}{\partial s_1} \gtreqqless 0 \iff s_1 \lesseqqgtr 1 - \alpha_{ma}\beta w_2,$$

$$\frac{\partial k_2^F}{\partial s_1} \gtreqqless 0 \iff s_1 \lesseqqgtr \alpha_{ma}\beta w_1,$$

$$\frac{\partial c^F}{\partial s_1} \lesseqqgtr 0 \iff s_1 \lesseqqgtr \frac{w_1}{w_1 + w_2},$$

which constitute the first half of Lemma 5. The global welfare is given by

$$W^F = \overline{W}$$
$$+ \left(\frac{\alpha_{mb}}{\alpha_{ma}} \right) (l_1 + l_2) \left[(1 - \alpha_{ma}\beta w_1) e^{-\alpha_{ma}p_2^2} s_1 + (1 - \alpha_{ma}\beta w_2) e^{-\alpha_{ma}p_1^1} (1 - s_1) \right]$$
$$\times \left[\frac{s_1 (l_1 + l_2) \alpha_{mb}\beta w_1}{p_1^k e^{\alpha_{ma}p_2^2}} \right]^{\frac{\alpha_{ma}\beta w_1}{1 - \alpha_{ma}\beta(w_1 + w_2)}} \left[\frac{(1 - s_1) (l_1 + l_2) \alpha_{mb}\beta w_2}{p_2^k e^{\alpha_{ma}p_1^1}} \right]^{\frac{\alpha_{ma}\beta w_2}{1 - \alpha_{ma}\beta(w_1 + w_2)}}.$$

If $w_1 = w_2$ and $p_1^1 = p_2^2$, differentiation of the above equation with respect to s_1 yields

$$\frac{\partial W^F}{\partial s_1} = \left(\frac{\alpha_{mb}}{\alpha_{ma}} \right) \left[\frac{s_1 (l_1 + l_2) \alpha_{mb}\beta w_1}{p_1^k e^{\alpha_{ma}p_1^1}} \right]^{\frac{\alpha_{ma}\beta w_1}{1 - 2\alpha_{ma}\beta w_1}} \left[\frac{(1 - s_1) (l_1 + l_2) \alpha_{mb}\beta w_1}{p_2^k e^{\alpha_{ma}p_1^1}} \right]^{\frac{\alpha_{ma}\beta w_1}{1 - 2\alpha_{ma}\beta w_1}}$$
$$\times \frac{2 (1 - \alpha_{ma}\beta w_1) e^{-\alpha_{ma}p_1^1} (l_1 + l_2) \alpha_{ma}\beta w_1}{(1 - 2\alpha_{ma}\beta w_1) s_1 (1 - s_1)} \left(\frac{1}{2} - s_1 \right),$$

which is the latter half of Lemma 5.

Proof of Lemma 6

The prices of infrastructure use satisfy

$$f^O = 0,$$
$$f_i^P = \frac{l_j e^{-\alpha_{ma}p_i^i}}{l_1 e^{-\alpha_{ma}p_2^2} + l_2 e^{-\alpha_{ma}p_1^1}}, \quad f_1^P + f_2^P = \frac{1}{\alpha_{ma}}, \quad \text{for } i = 1, 2,$$
$$f^U = \beta (w_1 + w_2).$$

The levels of investment in the optimum, pricing, and user cost minimization regimes

are given by

$$k_1^O = \left[\frac{(l_1 + l_2)\left(s_1 e^{-\alpha_{ma} p_2^2} + (1 - s_1) e^{-\alpha_{ma} p_1^1}\right)\alpha_{mb}\beta w_1}{p_1^k}\right]^{\frac{1 - \alpha_{ma}\beta w_2}{1 - \alpha_{ma}\beta(w_1 + w_2)}}$$

$$\times \left[\frac{(l_1 + l_2)\left(s_1 e^{-\alpha_{ma} p_2^2} + (1 - s_1) e^{-\alpha_{ma} p_1^1}\right)\alpha_{mb}\beta w_2}{p_2^k}\right]^{\frac{\alpha_{ma}\beta w_2}{1 - \alpha_{ma}\beta(w_1 + w_2)}},$$

$$k_2^O = \left[\frac{(l_1 + l_2)\left(s_1 e^{-\alpha_{ma} p_2^2} + (1 - s_1) e^{-\alpha_{ma} p_1^1}\right)\alpha_{mb}\beta w_1}{p_1^k}\right]^{\frac{\alpha_{ma}\beta w_1}{1 - \alpha_{ma}\beta(w_1 + w_2)}}$$

$$\times \left[\frac{(l_1 + l_2)\left(s_1 e^{-\alpha_{ma} p_2^2} + (1 - s_1) e^{-\alpha_{ma} p_1^1}\right)\alpha_{mb}\beta w_2}{p_2^k}\right]^{\frac{1 - \alpha_{ma}\beta w_1}{1 - \alpha_{ma}\beta(w_1 + w_2)}},$$

$$k_i^P = \exp\left(\frac{-1}{1 - \alpha_{ma}\beta(w_1 + w_2)}\right) k_i^O,$$

$$k_i^U = \exp\left(\frac{-\alpha_{ma}\beta(w_1 + w_2)}{1 - \alpha_{ma}\beta(w_1 + w_2)}\right) k_i^O, \text{ for } i = 1, 2.$$

Differentiating the above equations with respect to s_1 yields

$$\frac{1}{k_i^r}\frac{\partial k_i^r}{\partial s_1} = \frac{(l_1 + l_2) e^{-\alpha_{ma} p_1^1}\left[e^{\alpha_{ma}\left(p_1^1 - p_2^2\right)} - 1\right]}{[1 - \alpha_{ma}\beta(w_1 + w_2)]\left(l_1 e^{-\alpha_{ma} p_2^2} + l_2 e^{-\alpha_{ma} p_1^1}\right)} \text{ for } i = 1, 2; r = O, P, U.$$

(29)

Similarly, differentiating the transport cost with respect to s_1 yields

$$\frac{\partial c^r}{\partial s_1} = \frac{-\beta(w_1 + w_2)(l_1 + l_2) e^{-\alpha_{ma} p_1^1}\left[e^{\alpha_{ma}\left(p_1^1 - p_2^2\right)} - 1\right]}{[1 - \alpha_{ma}\beta(w_1 + w_2)]\left(l_1 e^{-\alpha_{ma} p_2^2} + l_2 e^{-\alpha_{ma} p_1^1}\right)}, \text{ for } r = O, P, U. \quad (30)$$

Since the denominators of the RHSs of the equations (29) and (26) are positive, the signs of $\frac{1}{k_i^r}\frac{\partial k_i^r}{\partial s_1}$ and $\frac{\partial c^r}{\partial s_1}$ depend on the signs of the brackets of the numerator, $\left[e^{\alpha_{ma}\left(p_1^1 - p_2^2\right)} - 1\right]$. Namely, we have

$$\frac{\partial k_i^r}{\partial s_1} \gtreqless 0 \text{ and } \frac{\partial c^r}{\partial s_1} \lesseqgtr 0 \iff e^{\alpha_{ma}\left(p_1^1 - p_2^2\right)} \gtreqless 1 \text{ for } i = 1, 2, r = O, P, U.$$

When $w_1 = w_2$ and $p_1^1 = p_2^2$, the global welfare satisfies

$$\overline{W} = (l_1 + l_2) \left(w_1 + \frac{\alpha_{eb} e^{-\alpha_{ea} p_1^1}}{\alpha_{ea}} \right),$$

$$W^O = \overline{W}$$
$$+ \left(\frac{\alpha_{mb}}{\alpha_{ma}} \right) (1 - 2\alpha_{ma}\beta w_1) \left[(l_1 + l_2) e^{-\alpha_{ma} p_1^1} \right]^{\frac{1}{1 - 2\alpha_{ma}\beta w_1}} \left[\frac{(\alpha_{mb}\beta w_1)^2}{p_1^k p_2^k} \right]^{\frac{\alpha_{ma}\beta w_1}{1 - \alpha_{ma}\beta w_1}}$$

$$W^P = \overline{W}$$
$$+ \left(\frac{2\alpha_{mb}}{e\alpha_{ma}} \right) (1 - \alpha_{ma}\beta w_1) \left[(l_1 + l_2) e^{-\alpha_{ma} p_1^1} \right]^{\frac{1}{1 - 2\alpha_{ma}\beta w_1}} \left[\frac{(\alpha_{mb}\beta w_1)^2}{e^2 p_1^k p_2^k} \right]^{\frac{\alpha_{ma}\beta w_1}{1 - \alpha_{ma}\beta w_1}}$$

$$W^U = \overline{W}$$
$$+ \left(\frac{\alpha_{mb}}{\alpha_{ma}} \right) \left[(l_1 + l_2) e^{-\alpha_{ma} p_1^1} \right]^{\frac{1}{1 - 2\alpha_{ma}\beta w_1}} \left[\frac{(\alpha_{mb}\beta w_1)^2}{e^2 p_1^k p_2^k} \right]^{\frac{\alpha_{ma}\beta w_1}{1 - \alpha_{ma}\beta w_1}}$$

Thus, we have
$$\frac{\partial W^r}{\partial s_1} = 0, \text{ for } r = O, P, U.$$

Appendix B

In this appendix, we describe the details of the setting of the parameter values. We select several key indicators that are constructed by endogenous variables of the model, and set reasonable values for these indicators based on the available data. Then, we calculate the parameter values such that the values of the indicators computed by the model reproduce the predetermined values. Note that we suppose a symmetric economy and free access for the use of infrastructure (Regime F). The indicators we selected are: Openness; Transport cost ratio; Price elasticity of demand for import goods; Expenditure ratio for transport infrastructure. Their definitions and supposed values are given as follows

- Openness: $p_i^j l_i x_i^j / l_i w_i = 0.2$

- Transport cost ratio: $(c_1 + c_2) / p_i^i = 0.2$

- Price elasticity of import goods: $- \left(p_i^j / x_i^j \right) \left(\partial x_i^j / \partial p_i^j \right) = 1$

- Expenditure ratio: $p_i^k k_i / l_i w_i = 0.01$

The openness is defined as the share of import good consumption. According to IMF(2007), the openness of developed countries in Western Europe are around 0.3 to 0.4, while that of Japan is about 0.1 and that of the United States is about 0.2. As a representative value among the above countries, we set 0.2. The transport cost ratio is based on the estimates by Anderson and van Wincoop (2004). They reported that the full transport cost, which consists of time cost and direct transport cost, is 21 percent of the FOB price.

We round the value to one decimal place, and get 0.2. The price elasticity of import goods are chosen so as to be consistent with a standard Cob-Douglas preference. The expenditure ratio is the ratio of the government expenditure on transport infrastructure to the gross domestic product. In Japan, the expenditure of the special account for road construction and improvement in Fiscal 2006 is 3,916 billion yen, which is 0.8 percent of the Gross Domestic Product of Japan.[7] Thus, we set the ratio at 0.01.

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Figure 1 Effects of the population distribution

O:Optimum, F:Free Access, P: Pricng and investment by national government,
U: User cost minimization

(a) Investment in country 1
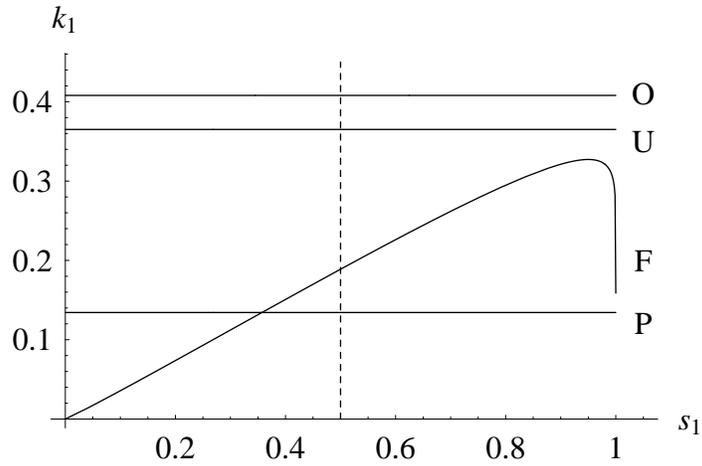
(b) Transport time
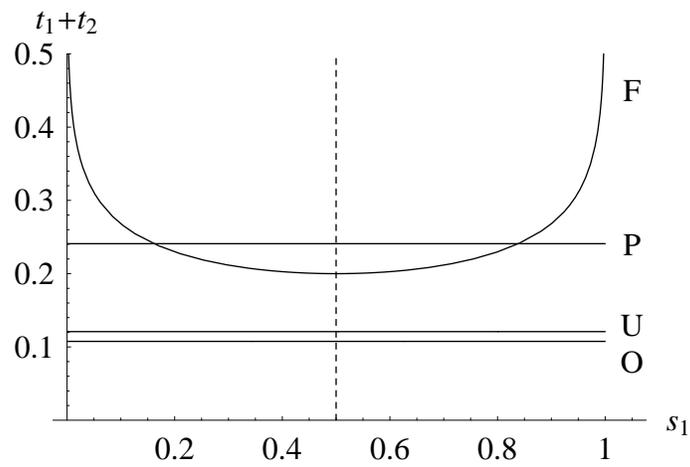
(c) Transport cost
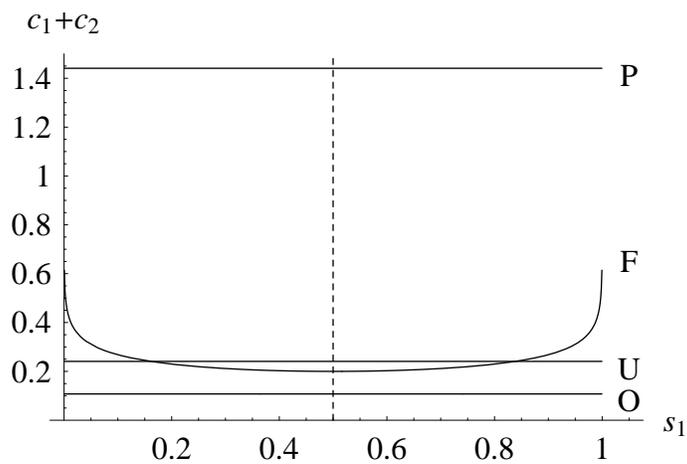
(d) Efficiency ratio

Efficiency ratio
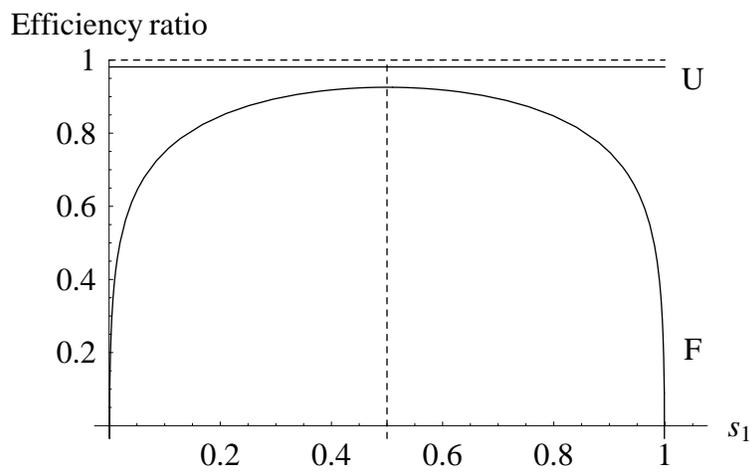
(e) National welfare of country 1
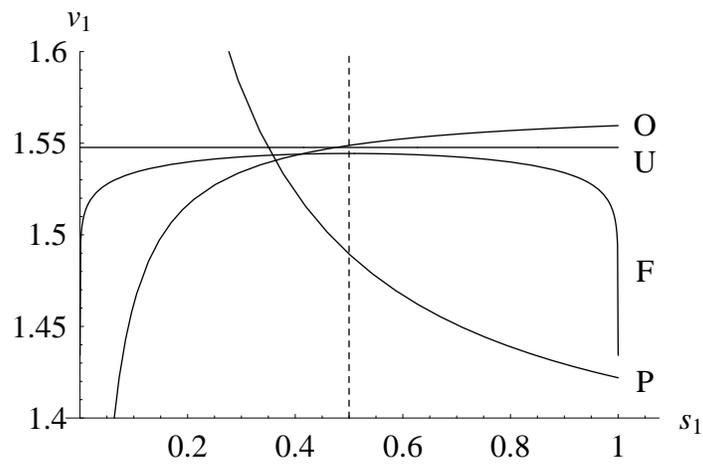
Figure 2 Effects of the difference in the wage rate

O:Optimum, F:Free Access, P: Pricng and investment by national government,
U: User cost minimization

(a) Investment in country 1
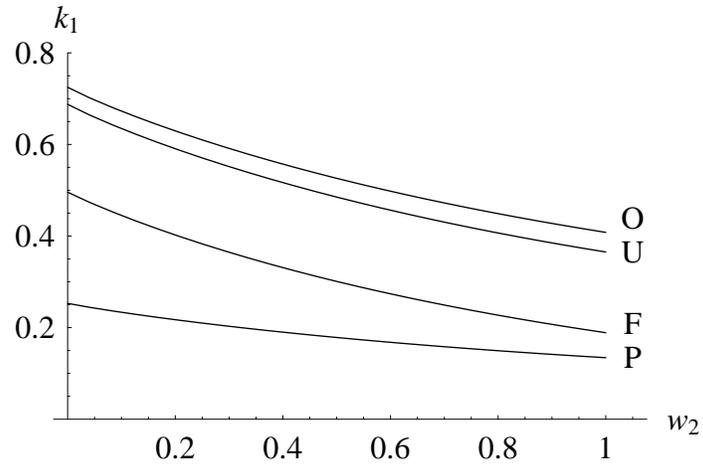
(b) Investment in country 2
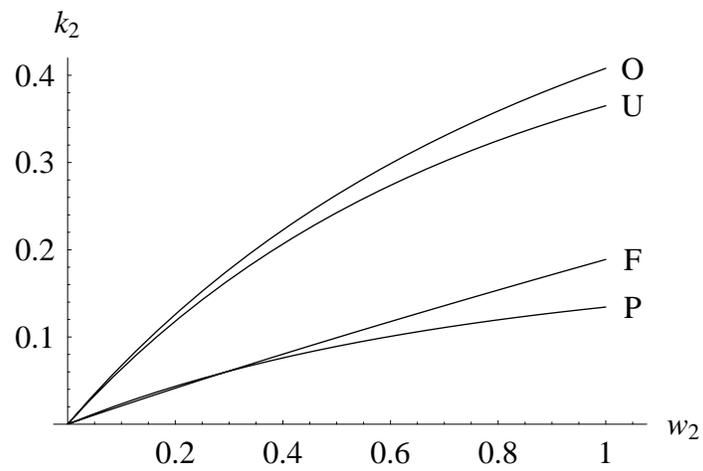
(c) Transport time
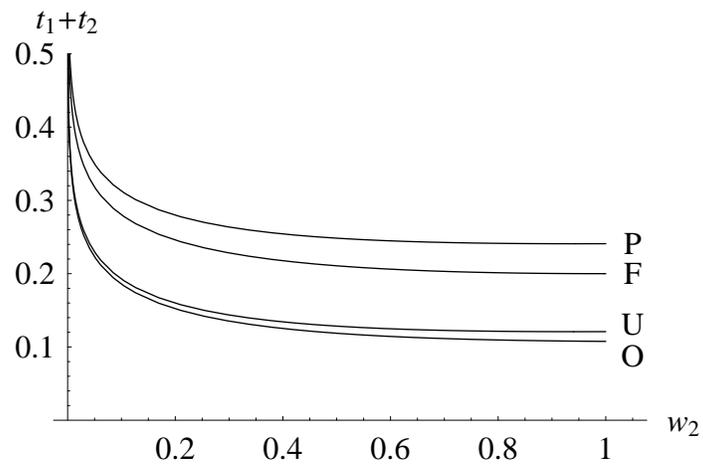
(d) Transport cost

(e) Efficiency Ratio