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“Context dependence and consistency in dynamic choice under uncertainty: the case of anticipated regret”

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Context dependence and consistency in dynamic choice under uncertainty: the case of anticipated regret*

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Abstract

We examine if and to what extent choice dispositions can allow dependence on contexts *and* maintain consistency over time, in a dynamic environment under uncertainty. We focus on a 'minimal' case of context dependence, opportunity dependence due to being affected by anticipated regret.

There are two sources of potential inconsistency, one is arrival of information and the other is changing opportunities. First, we go over the general method of resolution of potential inconsistency, by taking any kinds of inconsistency as given constraints. Second, we characterize a class of choice dispositions that are consistent to information arrival but may be inconsistent to changing opportunities. Finally, we consider the full requirement of dynamic consistency and show that it necessarily implies independence of choice opportunities.

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1 Introduction

1.1 Dynamic consistency

The analysis of dynamic choice normally assumes that the decision maker has a 'unity' of personality. The decision maker's life path is typically described as a solution to certain *planning* problem, in which the candidate life plans are evaluated according to the viewpoint of his 'initial self.' The unity is required, since otherwise the life plan prescribed by the initial self may not be followed by 'himselves' in the future, and the analysis based on such planning solution is misleading.

Thus an analyst needs to go through checking so-called dynamic consistency condition: consider that the successive selves associated with different date-events are different *potentially*, and check that nevertheless they have no disagreement about a desirable life plan.

Dynamic consistency imposes that the sequence of choice dispositions of the decision maker's successive selves has to be connected across date-events in a recursive manner. In choice under subjective uncertainty, it is known to require that beliefs at different date-events should be connected by means of Bayesian updating or its generalized version (Epstein and Le Breton [7], Ghirardato [9], Epstein and Schneider [8]).

1.2 Dependence on contexts

On the other hand, in the static choice literature, various empirical studies find that choice not only depends on payoff-relevant information but also significantly on *contexts*. Among many others, one can raise: dependence of risk attitudes on a status quo point and many other kinds of framing effects found by Kahneman and Tversky [19]; endowment effect found by Thaler [43]; extremeness aversion found by Simonson and Tversky [37], that consumers tend to choose alternatives that are placed physically in the the middle, apart from preferential values; naive diversification found by Benartzi and Thaler [2], that investors tend to split portfolios equally between available assets, apart from their return distributions; violation of transitivity of choice found by Loomes, Starmer and Sugden [25] in the setting of choices between bets, which they explain by fear of ex-post regret.

These findings motivate various descriptive theories of context dependent choice such as prospect theory (Kahneman and Tversky [19]), the theories of reference dependent choices (Masatlioglu and Ok [28], Rubinstein and Salant [32]), the theory of choice from lists (Rubinstein and Salant [31]) and the theories of anticipated regret (Loomes and Sugden

[26], Sugden [42], Hayashi [17]).

Also, we see a normative thought behind the use of context dependent choice rules in practice that choice rather *should* depend on contexts in order to utilize richer information they possess. In the statistical decision making literature for example, minimax regret (Savage [33, 34]) is frequently used. As explained later, minimax regret, as well as other models of anticipated regret in general, is sensitive to the presence of salient unchosen alternatives, in particular which are optimal at some states if one could choose at the hindsight of that. The basis of using such rule will be attributed to the view that one should worry about being 'wrong,' and be 'pessimistic' about the wrongness. Here what is 'correct' and what is 'wrong' are determined by ex-post optimal choices, that is, an action is correct at some state if it is optimal there, and wrong at some state if it is not optimal there. Such notions depend on what are available to choose — the decision maker cannot worry about being wrong when he does not choose what he cannot choose.

To understand, imagine a choice problem in which the set of state contingent payoffs is a segment spanned by $(0, 0)$ and $(a, -b)$, where the first coordinate is about state 1 and the second is about state 2, and $a, b > 0$. The maximin principle prescribes to choose $(0, 0)$ even when b is very small, which is safe but severely wrong at state 1 when a is very large. In practical problems, this often leads to the prescription that one should not do anything. The Bayesian method prescribes either of the two endpoints in almost all cases, but it can be severely wrong at either state. An intermediate model of uncertainty aversion (such as multiple-prior model or second-order Bayesian model) can pick an intermediate point, but it still does not take the notion of correctness or wrongness into account, which is seen when one extends the right endpoint $(a, -b)$ to, say, $(2a, -2b)$: if the rule prescribes an intermediate point in the current situation, it does not move after extending the endpoint, but now in the new situation the current choice is more severely wrong at state 1. One can make a similar argument for the case that the choice opportunity shrinks. The minimax regret principle takes the notion of correctness and wrongness into account, and prescribes $\left(\frac{a^2}{a+b}, -\frac{ab}{a+b}\right)$: the correct choice ex-post is $(a, -b)$ at state 1 and $(0, 0)$ at state 2; a candidate choice $\left(x, -\frac{b}{a}x\right)$ yields regret anticipated with regard to each state, which is the measure of wrongness; the anticipated regret is $a - x$ at state 1 and $\frac{b}{a}x$ at state 2; be pessimistic in the sense that one should worry about the maximal anticipated regret; minimize the maximal regret, which is in the current case obtained by equating $a - x$ and $\frac{b}{a}x$.

The idea of allowing choice to depend on opportunities is often adopted in the social choice literature as well, particularly in the literature of cooperative bargaining (see for example Kalai and Smorodinsky [20], Chun [4]).[1]

These arguments, descriptive and normative, bring up a problem to the dynamic analysis side. As date-event information evolves, that is, as time proceeds and uncertainty resolves over time, contexts evolve and change as well accordingly, and that might affect future choice as well. The problem is even more serious, because the change of contexts in the future may be caused endogenously by the current choice, whereas the evolution of time and uncertainty is exogenous. Can dependence on contexts be compatible with dynamic consistency? It is not difficult to imagine that context dependence has a potential conflict with dynamic consistency (since dependence in general involves more variables), but is that a necessity?

To be more to the point, the current paper focuses on a case of 'minimal' context dependence, the opportunity dependence due to being driven by anticipated regret — the decision maker worries about being wrong, that is, being inferior to ex-post optimum.

As choice opportunity varies, ex-post optimal actions change. Hence choice depends on the presence of salient alternatives which may not be chosen but are ex-post optimal at some states. This suggests that eliminating alternatives at some stage may change how to choose from remaining ones.

It is a 'minimal' departure in the following senses.

1. No cognitive/epistemic bias or irrationality: The decision maker is fully aware of all the payoff-relevant factors, able to calculate the consequence of every action perfectly in the form of a Savage act. He has perfect recall, and correctly perceives and understands arriving information.

2. No extrinsic framing factors: We are not attributing dynamic inconsistencies to extrinsic framing factors. Certain kinds of extrinsic framing factors that change over time may cause inconsistency, and it may be even necessary, but let us leave it aside here.

[1]The view that one should allow such dependence on opportunities is criticized by Chernoff [3]. He argues that the dependence leads to inconsistency, in the sense that the final choice is not robust to the order of how one eliminates alternatives in the meantime. He proposes a *static* consistency axiom based on this argument, while the dynamic nature of the underlying elimination processes is taken to be implicit. The current paper relates to Chernoff's concern in the way that we consider an extended setting that explicitly treats the processes of eliminating alternatives together with the evolution of time and uncertainty.

3. Payoff-based: No naive diversification or naive extremeness aversion, in which the decision maker tends to choose something that is placed physically in the middle of the given set of alternatives.

4. No changing tastes, no present bias, no habit formation, no preference for intertemporal variety.

5. No dynamic inconsistency issue due to non-expected utility preferences: We maintain that choice over objective (and reduced) lotteries follows the standard expected utility theory a la von-Neumann and Morgenstern.

6. Consequentialism maintained: We still maintain the assumption that the decision maker looks only at future, and does not care about what might have occurred at unrealized events, what he did or could have done before. We allow dependence on contexts about what *are* available, but we exclude dependence on what *were* available.[2]

Although it is minimal, it has a necessary conflict with dynamic consistency, as we show later.

1.3 What does dynamic consistency mean, by the way?

We are going to consider changing contexts/opportunities as the additional source of inconsistency, as well as the well-known one, arrival of information. This requires us to be much clearer about what logical process we are going through when we are checking the dynamic consistency condition. The point below has to be made clear.

> Dynamic consistency is a requirement that the decision maker's selves associated with different decision nodes (including both date-events and contexts/opportunities) should agree over the prescription of a *commitment life plan*, where those selves'

[2] We are aware that potential dynamic inconsistency might be rather saved by incorporating a non-consequentialist viewpoint. Allowing non-consequentialism can be seen as making the updating procedure depend on 'dynamic contexts.' In the present paper, we are treating a process of choice dispositions which allows dependence on *static* contexts at each date-event. Taking dynamic contexts into account broadens the possibility of dynamic consistency since the variety generated by contexts is given to the side of consistency definition, whereas having a sequential dependence on static contexts may in general conflict with dynamic consistency. See for example Hanany and Klibanoff [15, 16], who take the non-consequentialist approach for the inconsistency problem due to information arrival. Let us leave the above point aside as an orthogonal issue, though.

prescriptions are collected *hypothetically.* To check consistency, each self has to be asked hypothetically to prescribe a commitment plan that starts from his node, *as if* he is given a full discretion to govern the future course of actions. That is, we have to ask each self, "if the plan starts from you and if you can and have to make perfect commitment, and if this is the set of available plans, which plan would you pick?"

Such collection of prescriptions is possible or conceivable *only* in a hypothetical manner, because, if a self at some point really makes a choice with perfect commitment, there is no choice problem left to the subsequent selves any longer.

This hypothetical exercise is necessary, however. If we are given just a real path of actions, it is vacuously consistent.[3]

To understand, forget about uncertainty for a moment and consider a deterministic two-period setting in which only the final consumption at the end of period 2 matters. At period 1, the decision maker is asked, "which one out of x and y would you pick?" To check dynamic consistency, we have to ask him the same question at period 2, "which one out of x and y would you pick?" If they disagree, this decision maker is dynamically inconsistent. To see the satisfaction of dynamic consistency, we have to see that these two selves agree for all such sequences of questions.

We must be more precise. The questions above should be more precisely stated as

Question to the period-1 self: "Which one out of x and y would you pick, *if the choice is final*?"
Question to the period-2 self: "Which one out of x and y would you pick, *if the choice is final*?"

Now we can see that we cannot have such a sequence of questions when real choice is involved, because, if this decision maker really takes an action according to the answer to the first question, there is no choice problem left about which we can ask the second question. Thus, the concept of dynamic consistency presumes 'perfect commitment at every possible decision node,' which is an oxymoron and makes sense only in a hypothetical manner.

[3]This point might apply to the static notion of choice consistency as well, though in the static argument certain kind of 'repeated statics' or repeated static experiment is accepted to work.

1.4 The conflict between opportunity dependence and dynamic consistency

Choice driven by anticipated regret is opportunity dependent, in the sense that choice is affected by the presence of unchosen alternatives. In other words, what is chosen from a larger set may not be chosen from a smaller set containing it. This has a potential conflict with dynamic consistency, because eliminating some alternatives at some stage may change how the decision maker chooses from the remaining ones in the subsequent stages.

Is such conflict a *necessity*? Our answer is Yes. In this subsection we explain its intuition by means of examples, in which the decision maker is assumed to follow Savage's minimax regret at every decision node. The use of minimax regret might sound too extreme, but here we use it simply because it is the most familiar model of anticipated regret, and facilitates to convey what kind of consistency we are talking about. We are not basing the argument on an extreme case.

In the later part of the paper, we show that the inconsistency as explained in the examples below entails in *any* model that allows dependence on opportunities, not only in particular models of anticipated regret. In our Lemma 2 provided in Section 6, it is shown that the satisfaction of full dynamic consistency indeed requires that we cannot have any minor dependence on opportunity. Thus the conflict is a necessity.

Now we proceed to the examples.

Example 1 To focus on the source of inconsistency, assume that no information is provided at period 0 and 1, and the whole uncertainty resolves at period 2, and also assume that the decision maker cares only about the consumption at the final period. Thus, inconsistency problems related to discounting and updating are absent, and inconsistency if any can arise only from changing opportunities.

For the simplicity purpose explained above, assume that the decision maker follows Savage's minimax regret at each period.

There are three acts (state-contingent payoffs), f, g, h given by

	s_1	s_2
f	2	2
g	4	1
h	-1	5

If the decision maker chooses from $\{f, g, h\}$ at period 0 with perfect commitment, she follows the table below,

	Income		Regret		Max Regret
	s_1	s_2	s_1	s_2	
f	2	2	2	3	3
g	4	1	0	4	4
h	-1	5	5	0	5
Best	4	5			

which leads to the choice f.

On the other hand, if the decision maker at period 1 is to choose from $\{f, g\}$ with perfect commitment (since there is no choice problem at period 2, it is naturally a commitment problem), she follows

	Income		Regret		Max Regret
	s_1	s_2	s_1	s_2	
f	2	2	2	0	2
g	4	1	0	1	1
Best	4	2			

which leads to the choice g. Thus, period-0 self and period-1 self disagree on the commitment life path, and this disagreement is due to the difference of choice opportunities.

Why is it a problem? What is the real choice problem in which the above disagreement bites? Consider that the real choice problem at period 0 is between two choice opportunities to carry over, $\{f, g\}$ and $\{h\}$, which lacks perfect commitment. If the period-0 self takes that he can govern his future behavior, he chooses $\{f, g\}$ because he wants f. However, this is not fulfilled because the period-1 self picks g when $\{f, g\}$ is given.

Thus a dynamic inconsistency problem pops up, in the sense that under the lack of commitment an ex-ante plan chosen by the current decision maker may not be followed by himself in the future. This is due to the opportunity dependence, such that discarding h at period 0 changes how one chooses from $\{f, g\}$ at period 1.

Next example explains how opportunity dependence conflicts with dynamic consistency when both arrival of information and changing opportunity proceed together over time.

Example 2 An investor is holding a stock. No new information is provided at period 0. At period 1, either H_1 or L_1 realizes. At period 2, either H_2 or L_2 realizes.

At period 1, the stock price goes up by 5 if H_1 realizes and falls by 7 if L_1 realizes. At period 2, if the shock at period 1 is H_1 , the price goes up by 14 if H_2 realizes and falls by

4 if L_2 realizes; if the period-1 shock is L_1, the price goes up by 11 if H_2 realizes and falls by 9 if L_2 realizes.

The problem is when to sell the stock, where he has to sell it at period 2 anyway. Once he sells, payoff is finalized. Normalize the net gain of selling it at period 0 to 0.[4]

Let for example 'Hold, (Hold if H_1, Hold if L_1)' refer to the plan that he holds the stock at period 0 and continues to hold it if H_1 realizes at period 1 and continues to hold also if L_1 realizes, and similarly for other plans. Then we can write down the corresponding Savage acts as below.

	(H_1, H_2)	(H_1, L_2)	(L_1, H_2)	(L_1, L_2)
Hold, (Hold if H_1, Hold if L_1)	19	1	4	-16
Hold, (Hold if H_1, Sell if L_1)	19	1	-7	-7
Hold, (Sell if H_1, Hold if L_1)	5	5	4	-16
Hold, (Sell if H_1, Sell if L_1)	5	5	-7	-7
Sell	0	0	0	0

Again for simplicity assume that the investor follows Savage's minimax regret at each decision node. If he chooses a plan presuming that he can exercise perfect commitment, he picks 'Hold, (Hold if H_1, Sell if L_1).' However, if he holds the stock and L_1 realizes at period 1, the choice problem at period 1 there is now

	(L_1, H_2)	(L_1, L_2)
Hold	4	-16
Sell	-7	-7

There are two changes. One is the realization of L_1. Since we are maintaining consequentialism, the left two columns in the first table, the payoffs contingent on H_1, are excluded from consideration. The other is that the choice option of selling at period 0 is gone. Since we are maintaining consequentialism also in the sense that past choice opportunities do not matter, we exclude the bottom row in the first table from consideration. In the updated problem at period 1, the investor following Savage's minimax regret chooses Hold, which is against the plan prescribed by the initial self.

[4]Since the model is about *anticipated* regret, it does not question whether the investor indeed monitors the price movement after making choice. However, it is easier to understand the example if one takes that the price movement is publicly observable and the investor monitors it even after selling.

1.5 Consistency vs. resolution

Here we need to clearly state the distinction between the requirement of dynamic consistency and the resolution of possible inconsistencies.[5]

As we already discussed, for the argument we need to consider a sequence of preferences or choice functions associated with different decision nodes, each of which prescribes which *commitment life plan* is desirable to follow from the viewpoint of the corresponding node. Dynamic consistency is a requirement that there should be no disagreement between these prescriptions, or that one should make such sequence so that there is no disagreement *in the beginning.*

On the other hand, resolution is about how to determine the behavior, taking any possible disagreement as a *given constraint.*

They are in the relation of mutual trade-offs. When the sequence of prescriptions is dynamically consistent, there is no role for resolution. Also, at least logically, having more sources of inconsistency requires more tasks about resolution.[6] In this paper, we consider three scenarios.

1. Resolution without any consistency (Section 4)

2. Some consistency, leaving some necessity of resolution (Section 5)

3. Full satisfaction of consistency, leaving no necessity of resolution (Section 6)

In Section 4, we go over a general description of resolution, in which we allow any kind of inconsistency and do not question whether the inconsistency is due to arrival of information or due to changing opportunities. There we impose a well-known requirement, sophistication, which says that the decision maker at the current node is fully self-aware of potential inconsistencies and correctly foresees how his future selves will behave. The sophistication axiom characterizes the backward induction behavior.

Resolution itself does not require or use any unity of personality. Future selves here can be, as it were, totally different persons from the current self. They can be simply somebody else, whom the current self knows very well about. We do think the decision maker has

[5]Some authors refer to resolution as 'consistent planning,' but we take the current terminology so as to make the distinction clearer.

[6]This does not exclude the possibility that having *several* kinds of inconsistencies together turns out to make the life rather easier *as a coincidence.* In fact, this can happen in particular kinds of problems. In the companion paper, Hayashi [18] shows in a stopping problem that the resolution becomes rather simpler when the decision maker follows minimax regret with multiple-priors, where the belief process follows so called ε-contamination which is *not* consistent.

some level of unity, though it may not be perfect. This motivates the material in Section 5.

What's the merit of considering 'some' consistency? There are several dimensions about consistency and inconsistency. For example, one may be consistent in the dimension of knowledge but may be inconsistent in the dimension of will or self-governance.[7] Another may be consistent to changing contexts/opportunities but may be inconsistent to information arrival. As we will see in Section 6, the full satisfaction of dynamic consistency requires consistency in essentially all the dimensions. Should we then say that 'some' consistency is meaningless if it is not perfect?

Our view is that some (or maybe each) dimension of consistency has an independent merit to look at, even though it alone does not fulfill perfect consistency, and also that there may be another dimension in which the decision maker is significantly inconsistent despite of consistency in the first dimension. In particular, we view that we can and we should be able to talk about consistency to information arrival, even when it alone does not achieve the full requirement of dynamic consistency. In fact, it is what the existing arguments are pursuing and imposing when there is no inconsistency due to opportunity dependence.

In Section 5, we formulate this idea in the form of the Consistency to Information Arrival (CIA) axiom. It coincides with the standard definition of dynamic consistency when information arrival is the only source of inconsistency, but it is only a part of the full requirement of dynamic consistency when opportunity dependence has a role. The CIA axiom limits attention to the cases that information arrives but choice opportunity remains the same (after conditioning). Since choice opportunity remains the same, opportunity dependence has no role to play there and the only possible inconsistency there is about arrival of information. The CIA axiom states that there should be no choice reversal in such situations.

The CIA axiom thus involves hypothetical comparison of choices, where the choice problems compared cannot be connected by means of real actions, since real action necessarily changes future choice opportunities. However, as we discussed above and will do in Section 5 as well, this hypothetical exercise is necessary, and it is exactly what *all* the existing notions of dynamic consistency are presuming. We are just making it clear on a larger canvas.

The existing notions of dynamic consistency, most of which limit attention to informa-

[7] This is typically the case in the literature of self-control problem (see Strotz [41], Phelps and Pollak [30] and Laibson [24]).

tion arrival as the only source of inconsistency, ignore the potential inconsistency due to opportunity dependence. There, consistency to information arrival alone guarantees the full satisfaction of dynamic consistency. Here it does not, though as we claim above it has a merit to look at by itself. Therefore the full requirement of dynamic consistency here has to consider endogenous change of future choice opportunity as well. It is what we do in Section 6. Not surprisingly, we obtain a 'folk theorem,' that the full satisfaction of dynamic consistency necessarily implies independence of choice opportunities.

1.6 Related literature

Hammond [14] considers dynamic decision making under certainty, and shows the folk theorem there.[8] To see how, consider an opportunity-dependent decision maker who chooses a from $\{a, b, c\}$ but chooses b from $\{a, b\}$, where such dependence is for simplicity taken to be the same across periods. Notice that they are the responses in which perfect commitment is presumed. Now consider a real problem that at period 0 he chooses between two choice opportunities to carry over, $\{a, b\}$ and $\{c\}$, and at period 1 he makes final choice. If the period-0 self takes that he can govern the future behavior, he chooses $\{a, b\}$ because he wants a out of $\{a, b\} \cup \{c\} = \{a, b, c\}$. However, this is not fulfilled because the period-1 self picks b when $\{a, b\}$ is given. In section 6, we confirm that the folk theorem holds even when arrival of information and changing opportunities interact over time.

In the setting of objective risk, Machina [27] considers a sequence of preferences indexed by time, that are defined over *commitment* lotteries. He argues that there is a necessary conflict between dynamic consistency and weakening the independence axiom a la von-Neumann/Morgenstern, as far as the consequentialist view and the assumption of reduction of compound lotteries are maintained. One way to achieve both maintaining dynamic consistency and allowing non-expected utility preferences is to drop the reduction assumption, which in other words says that the decision maker may care about the timing of resolution of risk (see for example Kreps and Porteus [23]). Another way is to incorporate a non-consequentialist viewpoint, while maintaining the reduction assumption. Segal [35] allows anchoring the updated preference on the lottery that was 'chosen' to be optimal in the previous stage, and provides a sequence of dynamically consistent preferences that do not boil down to the expected utility model. As already mentioned, we assume that

[8] More precisely, opportunity independence here refers to the contraction property by Chernoff [3]. Also, the folk theorem argument already appears in Chernoff's paper as a motivation for the contraction property.

choice over objective outcomes follows the standard expected utility theory, and leave this problem aside as an orthogonal issue.

In the setting of subjective uncertainty, Epstein and Le Breton [7] consider a process of preferences over *commitment* random variables, in which preference at each date-event is complete and transitive (see also Ghirardato [9]). Thus, the only potential source of dynamic inconsistency there is arrival of information. They impose the consistency condition that preference reversal cannot occur after arrival of information. When the process of preferences falls in the model of subjective expected utility (Savage [34]), the condition implies that the corresponding process of beliefs follows Bayesian updating. Also, when the condition is applied to variable events, it even implies the sure thing principle.

In the subjective uncertainty setting with a fixed information structure, Epstein and Schneider [8] consider a process of preferences that accommodate ambiguity aversion and maintain dynamic consistency. They assume that the preference process falls in the model of multiple-prior expected utility by Gilboa-Schmeidler [11], and show that the corresponding process of multiple-prior beliefs has to satisfy so-called rectangularity, a generalization of Bayesianity, so as to maintain dynamic consistency. Again, notice that arrival of information is the only source of potential inconsistency there, and rectangularity is sufficient for dynamic consistency as well.

Hanany and Klibanoff [15, 16] generalize the definition of conditional preference so that it depends not only on a realized event but also on the choice opportunity one 'had' in the previous stage and what was 'chosen' to be optimal there. This weakens the standard consequentialist viewpoint, that conditional preference should depend only on a given realized event and should ignore everything outside of it. For this general class of conditional preferences, they characterize a broader class of updating rules, that may or may not be Bayesian or rectangular but satisfy dynamic consistency.

Siniscalchi [38] considers preference over decision trees, instead of commitment random variables. Here a decision tree can be viewed as a random variable over choice problems consisting of random variables over choice problems and so on, and it is an element of a larger domain than that of commitment random variables. He allows an almost arbitrary class of updating rules and types of uncertainty aversion, which allows potential dynamic inconsistency due to arrival of information. In his setting, belief updating and determination of behavior are mutually orthogonal issues, which is the case in our setting as well. It is imposed there that the decision maker is sophisticated, in the same spirit of our resolution notion, that he has a correct foresight about how his future selves will behave and takes it as given. Arrival of information is still the only source of dynamic inconsistency there, but it makes choice opportunity matter in the sense that the decision maker has a significant preference for commitment: if the current self foresees that himself after knowing some information rather makes a bad choice from his perspective, he rather prefers to make a commitment rather than to leave a choice opportunity to his future self.

Finally, we mention a recent paper by Krähmer and Stone [22], that considers a model of dynamic choice under uncertainty in which the regret-driven decision maker plays backward induction. Their model falls in our general model, and is close to its special case, the smooth model of regret aversion. The difference is that we are aiming to characterize how regret is anticipated in the dynamic setting, how dynamic inconsistency is resolved, and how beliefs are updated, while Krähmer-Stone start with the backward induction model with the notion of ex-post regret and Bayesian updating, and aim to see its behavioral implications.

2 Setting

2.1 Information structure

Time is discrete and finite. It varies from 0 to T. Let Ω be a finite set of states of the world. Information structure is fixed, and it is given in the form of a sequence of partitions $\{\mathcal{F}_t\}_{t=0}^T$ as follows:

1. For each $t = 0, \cdots, T$, $\mathcal{F}_t$ is a partition of Ω. For every $t = 0, \cdots, T-1$ and $E_t \in \mathcal{F}_t$,

$$E_t = \bigcup_{E_{t+1} \in \mathcal{F}_{t+1}, E_{t+1} \subset E_t} E_{t+1}$$

   holds;

2. $\mathcal{F}_0 = \{\Omega\}$ and $\mathcal{F}_T = \{\{\omega\} : \omega \in \Omega\}$.

Given $t = 0, \cdots, T-1$ and $E_t \in \mathcal{F}_t$, let

$$\mathcal{F}_{t+1}|E_t = \{E_{t+1} \in \mathcal{F}_{t+1} : E_{t+1} \subset E_t\}.$$

2.2 Hierarchical domain of choice problems

For simplicity, we assume that utility (or payoff) of each possible final outcome is given as a real number and its range covers the entire real line $\mathbb{R}$. It is justified by making certain ex-post randomization argument following Anscombe and Aumann [1] and a suitable extension of that.

We consider a hierarchical domain of choice problems. An action taken in a given choice problem results in a subsequent choice problem again, after *one-step* realization of uncertainty. This is indeed how we formulate a dynamic choice problem in many applications. Formally, the hierarchical domain of choice problems $((\mathcal{B}_{E_t})_{E_t \in \mathcal{F}_t})_{t=0}^{T-1}$ is defined by

1. for each $E_{T-1} \in \mathcal{F}_{T-1}$,
$$\mathcal{B}_{E_{T-1}} = \mathcal{K}(\mathbb{R}^{E_{T-1}})$$

2. for each $t = 0, \cdots T-2$ and $E_t \in \mathcal{F}_t$,
$$\mathcal{B}_{E_t} = \mathcal{K}\left(\prod_{E_{t+1} \in \mathcal{F}_{t+1} | E_t} \mathcal{B}_{E_{t+1}}\right),$$

where $\mathbb{R}^S$ denotes the set of functions from a given set S to $\mathbb{R}$, and $\mathcal{K}(X)$ denotes the set of nonempty compacts subsets of a given metric space X.

2.3 Subdomain of choice problems over commitment plans

Also we consider a subdomain of 'static' choice problems. In this paper, we take it to be the subdomain of choice problems where the decision maker has to make perfect commitment.

The subdomain of commitment choice problems at period $t = 0, \cdots, T-1$ with event $E_t \in \mathcal{F}_t$, denoted $\mathcal{C}_{E_t}$, is given by

$$\mathcal{C}_{E_t} = \mathcal{K}\left(\mathbb{R}^{E_t}\right).$$

When such a choice problem $C_{E_t} \in \mathcal{C}_{E_t}$ is given and the decision maker chooses its element $f_{E_t} \in C_{E_t}$, she has to *commit* to this random variable (since $f_{E_t} \in \mathbb{R}^{E_t}$) and there is no choice afterward, hence the choice is essentially static.[9] Thus an element of $\mathcal{C}_{E_t}$ is called a *commitment choice problem*. With the slight abuse of notation, for each $t = 0, \cdots, T-1$ and $E_t \in \mathcal{F}_t$, we have '$\mathcal{C}_{E_t} \subset \mathcal{B}_{E_t}$.' Since choice at period $T-1$ is necessarily final, we have $\mathcal{C}_{E_{T-1}} = \mathcal{B}_{E_{T-1}}$ for every $E_{T-1} \in \mathcal{F}_{T-1}$.

[9] To be notationally rigorous, one has to write it with a layer of brackets like $\{\cdots\{\{f_{E_t}\}\}\}$ and similarly for the description of C_{E_t}. But we omit this when perfect commitment is imposed and when no confusion arises.

2.4 Policy functions

In standard dynamic choice models, a sequence of preferences over commitment random variables is taken to be the primitive, and a processes of policy functions is derived as a result of optimization. In contrary, here we take a process of policy functions as the primitive of the model. To allow possible multiplicity of choices, we consider a set of processes of policy functions.

A process of policy functions $\varphi = ((\varphi_{E_t})_{E_t \in \mathcal{F}_t})_{t=0}^{T-1}$ is a sequence of functions, where

$$\varphi_{E_t} : \mathcal{B}_{E_t} \to \prod_{E_{t+1} \in \mathcal{F}_{t+1} | E_t} \mathcal{B}_{E_{t+1}}$$

satisfies $\varphi_{E_t}(B_{E_t}) \in B_{E_t}$ for every $t = 0, \cdots, T-1$, $E_t \in \mathcal{F}_t$ and $B_{E_t} \in \mathcal{B}_{E_t}$. We call such sequence a *choice process*. Let Φ denote a *set* of such choice processes. We call it a *choice process set*.

In commitment choice problems at a given date-event, there is no distinction between a choice process set and the family of sets of choices induced by that (choice correspondence in other words). Thus, the correspondence of commitment choices that is induced by Φ is given as follows: for each $t = 0, \cdots, T-1$ and $E_t \in \mathcal{F}_t$, given $C_{E_t} \in \mathcal{C}_{E_t}$, $\Phi_{E_t}(C_{E_t}) \subset \mathbb{R}^{E_t}$ is defined by

$$\Phi_{E_t}(C_{E_t}) = \{\varphi_{E_t}(C_{E_t}) : \varphi \in \Phi\}.$$

3 Choice over commitment plans

We assume that the decision maker's 'self' at each decision node gives choice prescription about commitment plans, following a model of regret-based choice. Choice over commitment plans is essentially static, hence we don't go beyond borrowing a static choice model from the literature and adapting it to the current setting. In the current paper we adopt the static model by Hayashi [17], since (i) it includes Savage's minimax regret as a special case and includes a larger class of regret-driven choices; (ii) it includes subjective expected utility maximization as a special case; (iii) it covers the intransitive preference model by Loomes and Sugden [26] when applied to binary choices; (iv) it enables clearer treatment of opportunity dependence since it is build on choice functions rather than preference relations; (v) it is axiomatic and enables a clearer connection between dynamic consistency and opportunity dependence.[10]

[10]See the related models and axiomatizations by by Milnor [29], Stoye [39, 40]

Though, of course we would not insist that it is the only model that captures dependence on opportunity due to being affected by anticipated regret. We pick the current model just because it is the most efficient way to tackle the problem within our current scope.

Except for one thing, the adaptation is more or less straightforward, hence we avoid repeating the whole axiomatization of it. The non-obvious thing is about the very definition of anticipated regret. To see why, go back to Example 2. Consider that the investor at period 0 is wondering about the case that he holds the asset and sees L_1 at period 1. Does he anticipate regretting to hold the asset at that point? It's not clear, because if he continues to hold further and sees H_2 at period 2 then it is not regrettable. This suggests that in order to describe one anticipated regret we may need to imagine and track one whole path of uncertainty resolution and future actions.

Since there is no extra information presumed about how the decision maker limits the scope about the imagination of 'ex-post' optimum, in the current adaptation we take the widest scope as our default. That is, we require that the notion of ex-post optimum is perceived with regard to terminal states. Formally, it is embodied by the following dynamic adaptation of the definition of ex-post dominance. Given $t = 0, \cdots, T-1$ and $E_t \in \mathcal{F}_t$, consider two sets of commitment plans, $C_{E_t}, D_{E_t} \in \mathcal{C}_{E_t}$. Say that C_{E_t} *ex-post dominates* D_{E_t} *from the viewpoint of* E_t if for all $\omega \in E_t$ there exists $f_{E_t} \in C_{E_t}$ such that $f_{E_t}(\omega) \geq g_{E_t}(\omega)$ for all $g_{E_t} \in D_{E_t}$. This says that whatever states in E_t realizes C_{E_t} guarantees a better ex-post choice than D_{E_t} does. In other words, if one could choose at the hindsight of *terminal* states, C_{E_t} is unambiguously better than D_{E_t}. Write the relationship by $C_{E_t} \geq^{EP}_{E_t} D_{E_t}$.

The main axiom for the static model is that adding ex-post dominated acts to the existing set of alternatives does not cause choice reversal, because it does not change what is good ex-post. Below is its adaptation to the current setting.[11]

Irrelevance of Ex-post Dominated Acts: For every $t = 0, \cdots, T-1$ and $E_t \in \mathcal{F}_t$, for every $C_{E_t}, D_{E_t} \in \mathcal{C}_{E_t}$ with $C_{E_t} \geq^{EP}_{E_t} D_{E_t}$,

$$\Phi_{E_t}(C_{E_t} \cup D_{E_t}) \cap C_{E_t} \neq \emptyset \implies \Phi_{E_t}(C_{E_t} \cup D_{E_t}) \cap C_{E_t} = \Phi_{E_t}(C_{E_t})$$

In contrary to the purely static model, there is a non-obvious choice of the dominance relation that is used in the weakened independence axiom. For example, the current adaptation may not allow a notion of *interim* regret, in which the decision maker summarizes

[11] The idea originates from Milnor [29]. See also Stoye [39, 40] for a more sophisticated treatment of it.

information about future uncertainty and actions into a 'continuation value' in the form of a one-step-ahead measurable function and anticipates regret with regard to the realization of one-step-ahead uncertainty.

The Irrelevance of Ex-post Dominated Acts axiom plus additional axioms characterize the general model of regret-based choice below. The additional axioms are: (i) an admissibility axiom, that an alternative should not be chosen if there is another available alternative that is ex-post better at every terminal state; (ii) an independence axiom with regard to ex-post mixture (randomization a la Anscombe-Aumann) of outcomes between sets and singleton sets; and (iii) upper hemi-continuity, a mild technical axiom.

Given $t = 0, \cdots, T-1$ and $E_t \in \mathcal{F}_t$, a function $\Psi_{E_t} : \mathbb{R}_+^{E_t} \to \mathbb{R}_+$ is said to be *weakly monotone* if for every $x_{E_t}, y_{E_t} \in \mathbb{R}_+^{E_t}$, $\Psi_{E_t}(x_{E_t}) \geq \Psi_{E_t}(y_{E_t})$ if $x_{E_t}(\omega) \geq y_{E_t}(\omega)$ for all $\omega \in E_t$, and $\Psi_{E_t}(x_{E_t}) > \Psi_{E_t}(y_{E_t})$ additionally if $x_{E_t}(\omega) > y_{E_t}(\omega)$ for all $\omega \in E_t$ with $x_{E_t}(\omega) > 0$.

General model of regret-based choice: Given a list of weakly monotone and homothetic functions $((\Psi_{E_t})_{E_t \in \mathcal{F}_t})_{t=0}^{T-1}$, for each $t = 0, \cdots, T-1$ and $E_t \in \mathcal{F}_t$, it holds that

$$\Phi_{E_t}(C_{E_t}) = \arg\min_{f_{E_t} \in C_{E_t}} \Psi_{E_t}\left[\left(\max_{g_{E_t} \in C_{E_t}} g_{E_t}(\omega) - f_{E_t}(\omega)\right)_{\omega \in E_t}\right]$$

for every $C_{E_t} \in \mathcal{C}_{E_t}$.

Here the function Ψ_{E_t} given E_t explains how the decision maker there aggregates regret anticipated with regard to terminal states. Let us call it regret-aggregating function. There are two aspects explained by the function, one is belief about states and the other is how one is pessimistic about anticipated inferiority to ex-post optimum.

The general model has two notable special cases. One is minimax regret with multiple-priors, which is a generalization of Savage's minimax regret. Here the set of priors is to explain two different roles, belief itself and attitude toward anticipated regret, which is typically the case in other different types of use of multiple-priors as well (see for example Ghirardato and Marinacci [10]).

Given $t = 0, \cdots, T-1$ and $E_t \in \mathcal{F}_t$, let $\Delta(E_t)$ denote the set of probability measures over E_t.

Subclass 1 (**Minimax regret with multiple-priors**): Given a list of closed convex sets $((P_{E_t})_{E_t \in \mathcal{F}_t})_{t=0}^{T-1}$, for each $t = 0, \cdots, T-1$ and $E_t \in \mathcal{F}_t$, it holds that $P_{E_t} \cap int\Delta(E_t) \neq$

$\emptyset$ and

$$\Phi_{E_t}(C_{E_t}) = \arg \min_{f_{E_t} \in C_{E_t}} \max_{p_{E_t} \in P_{E_t}} \sum_{\omega \in E_t} \left(\max_{g_{E_t} \in C_{E_t}} g_{E_t}(\omega) - f_{E_t}(\omega) \right) p_{E_t}(\omega)$$

for every $C_{E_t} \in \mathcal{C}_{E_t}$.

This includes Savage's minimax regret as a special case that $P_{E_t} = \Delta(E_t)$ for all $t = 0, \cdots, T-1$ and $E_t \in \mathcal{F}_t$. Also, it includes subjective expected maximization as a special case that P_{E_t} is a singleton for all $t = 0, \cdots, T-1$ and $E_t \in \mathcal{F}_t$.

The second notable special case is the smooth model of anticipated regret, in which the decision maker holds a probabilistic belief but distorts anticipated regret before taking expectation. Here the distortion parameter explains how one is averse to the anticipated inferiority to ex-post optimum, which we call regret aversion parameter.

Subclass 2 (**Smooth model of anticipated regret**): Given a list of probability measures $((p_{E_t})_{E_t \in \mathcal{F}_t})_{t=0}^{T-1}$ and a list of positive numbers $((\alpha_{E_t})_{E_t \in \mathcal{F}_t})_{t=0}^{T-1}$, for each $t = 0, \cdots, T-1$ and $E_t \in \mathcal{F}_t$, it holds that $p_{E_t} \in int\Delta(E_t)$ and

$$\Phi_{E_t}(C_{E_t}) = \arg \min_{f_{E_t} \in C_{E_t}} \sum_{\omega \in E_t} \left(\max_{g_{E_t} \in C_{E_t}} g_{E_t}(\omega) - f_{E_t}(\omega) \right)^{\alpha_{E_t}} p_{E_t}(\omega)$$

for every $C_{E_t} \in \mathcal{C}_{E_t}$.

This includes subjective expected utility maximization as a special case that $\alpha_{E_t} = 1$ for all $t = 0, \cdots, T-1$ and $E_t \in \mathcal{F}_t$. Also, it covers Savage's minimax regret as a limit case that $\alpha_{E_t} \to \infty$ for all $t = 0, \cdots, T-1$ and $E_t \in \mathcal{F}_t$.

4 Resolution without consistency

First, we go over the general problem of how the decision maker behaves in non-commitment situations, without assuming any 'unity' of personality, by taking *any* possible dynamic inconsistencies as a given constraint.

The model of choice over commitment plans described in the previous section allows two kinds of potential inconsistency, which may pop up in the non-commitment situations. One is due to arrival of new information. Since we have not specified any belief updating rule yet, arrival of information has a potential to cause inconsistency. The other is due to opportunity dependence, which is discussed in the introduction. Here we consider how to make a sophisticated choice, in the presence of any of these two kinds of inconsistency, without making any distinction between them. The argument of sophistication here is quite well-known, since it appears in many models of dynamic inconsistency. Hence we obviously do not claim novelty about it, but we contain it for completeness.

Given $\varphi \in \Phi$, define the continuation path resulting from 'one time deviation' $a_{E_t} \in B_{E_t} \in \mathcal{B}_{E_t}$ at period t with event E_t as follows : define a commitment random variable $h_{E_t}(\varphi, a_{E_t}) \in \mathbb{R}^{E_t}$ by

$$h_{E_t}(\varphi, a_{E_t})(\omega) = \varphi_{E_{T-1}}(\cdots \varphi_{E_{t+1}}(a_{E_t}(E_{t+1}))(E_{T-1}))(\omega)$$

for each $\omega \in \Omega$, where $E_{t+1}, \cdots, E_{T-1}$ is the sequence of events which follow E_t and include ω.

Given $B_{E_t} \in \mathcal{B}_{E_t}$ and φ, define the reduced problem with commitment, denoted $B_{E_t}(\varphi) \in \mathcal{C}_{E_t}$, by

$$B_{E_t}(\varphi) = \{h_{E_t}(\varphi, a_{E_t}) : a_{E_t} \in B_{E_t}\},$$

which consists of the commitment variables generated as above.

We impose an axiom that any chosen action must be chosen also in the reduced problem, and vice versa.

Axiom S (**Sophistication**): $\varphi \in \Phi$ if and only if for every $B_{E_t} \in \mathcal{B}_{E_t}$,

$$h_{E_t}(\varphi, \varphi_{E_t}(B_{E_t})) \in \Phi_{E_t}(B_{E_t}(\varphi)).$$

Under the lack of commitment, the decision maker at period t takes it into account how his future selves will behave, which is described by $((\varphi_{E_\tau})_{E_\tau \in \mathcal{F}_\tau})_{\tau=t+1}^{T-1}$. Given this foresight, he computes the consequence of his current action a_{E_t}, which is described by means of the commitment random variable $h_{E_t}(\varphi, a_{E_t})$. The current decision maker takes each possible continuation path to be a commitment random variable. Given that, he makes choice as if he is facing a commitment problem, in which once he takes an action the successive selves start acting like automatic machines that he cannot control at all. The axiom says that such way of making choice has to coincide with how he behaves indeed.

The substantive assumption behind it is that the current decision maker has no power to control or govern his future choices, and has to take future selves' behaviors as given. That is, the future selves can be treated as if they are totally different persons. The decision maker modeled like this looks somewhat schizophrenic, but we adopt it as a step toward explicitly treating how intrapersonal conflicts and tensions are resolved. One might consider cases that the decision maker has incomplete but some amount of power to govern his future behaviors, or that he mistakenly believes he can do so, but it is beyond the current scope.[12]

Here we state the consequence of the sophistication axiom.

Theorem 1 Assume that the choice process set Φ allows the representation as in the basic model. Then, the following statements are equivalent:
(a) The choice process set Φ satisfies axiom S.
(b) The choice process set Φ satisfies that $\varphi \in \Phi$ if and only if

$$\varphi_{E_t}(B_{E_t}) \in \arg\min_{a_{E_t} \in B_{E_t}} \Psi_{E_t} \left[\left(\max_{b_{E_t} \in B_{E_t}} h_{E_t}(\varphi, b_{E_t})(\omega) - h_{E_t}(\varphi, a_{E_t})(\omega) \right)_{\omega \in E_t} \right]$$

for every $t = 0, \cdots, T-1$, $E_t \in \mathcal{F}_t$ and $B_{E_t} \in \mathcal{B}_{E_t}$.

Proof. Since (b) $\Longrightarrow$ (a) is routine, we prove (a) $\Longrightarrow$ (b).
'Only if' part: Suppose $\varphi \in \Phi$ and take any $B_{E_t} \in \mathcal{B}_{E_t}$. By Sophistication, $h_{E_t}(\varphi, \varphi_{E_t}(B_{E_t})) \in \Phi_{E_t}(B_{E_t}(\varphi))$. By definition of $B_{E_t}(\varphi)$,

$$\Phi_{E_t}(B_{E_t}(\varphi)) = \arg\min_{h_{E_t} \in B_{E_t}(\varphi)} \Psi_{E_t} \left[\left(\max_{g_{E_t} \in B_{E_t}(\varphi)} g_{E_t}(\omega) - h_{E_t}(\omega) \right)_{\omega \in E_t} \right].$$

Therefore

$$h_{E_t}(\varphi, \varphi_{E_t}(B_{E_t})) \in \arg\min_{h_{E_t} \in B_{E_t}(\varphi)} \Psi_{E_t} \left[\left(\max_{g_{E_t} \in B_{E_t}(\varphi)} g_{E_t}(\omega) - h_{E_t}(\omega) \right)_{\omega \in E_t} \right],$$

which is equivalent to

$$\varphi_{E_t}(B_{E_t}) \in \arg\min_{a_{E_t} \in B_{E_t}} \Psi_{E_t} \left[\left(\max_{b_{E_t} \in B_{E_t}} h_{E_t}(\varphi, b_{E_t})(\omega) - h_{E_t}(\varphi, a_{E_t})(\omega) \right)_{\omega \in E_t} \right].$$

'If' part: analogous. ■

[12] There is another approach, that treats resolution of intrapersonal conflicts in an 'implicit' manner (see for example Gul and Pesendorfer [12, 13], Epstein [6]). It is done by looking at preference *over* menus, while choice *from* a given menu is left unspecified or assumed to follow preference over singleton menus eventually. It attempts to describe how one deals with his intrapersonal conflicts through analyzing nontrivial preference for flexibility or commitment. Such description is obtained as a part of the representation of preference, where the ranking between menus is explained *as if* the decision maker is assigning certain weights between conflicting dispositions within him.

The implicit approach allows the use of dynamic programming when extended to a hierarchical domain of menus, in which dynamic consistency does not seem to be a problem. However, this is because looking at preference over menus (value function or indirect utility, in other words) in the beginning presumes that intrapersonal conflict if any has been already resolved in some way, and it limits analysis to what kind of conflict can explain the observed non-standard features of the menu preference.

Corollary 1 Assume that the choice process set Φ allows the representation Sublass 1. Then, the following statements are equivalent:
(a) The choice process set Φ satisfies axiom S.
(b) The choice process set Φ satisfies that $\varphi \in \Phi$ if and only if

$$\varphi_{E_t}(B_{E_t}) \in \arg \min_{a_{E_t} \in B_{E_t}} \max_{p_{E_t} \in P_{E_t}} \sum_{\omega \in E_t} \left(\max_{b_{E_t} \in B_{E_t}} h_{E_t}(\varphi, b_{E_t})(\omega) - h_{E_t}(\varphi, a_{E_t})(\omega) \right) p_{E_t}(\omega)$$

for every $t = 0, \cdots, T-1$, $E_t \in \mathcal{F}_t$ and $B_{E_t} \in \mathcal{B}_{E_t}$.

Corollary 2 Assume that the choice process set Φ allows the representation Sublass 2. Then, the following statements are equivalent:
(a) The choice process set Φ satisfies axiom S.
(b) The choice process set Φ satisfies that $\varphi \in \Phi$ if and only if

$$\varphi_{E_t}(B_{E_t}) \in \arg \min_{a_{E_t} \in B_{E_t}} \sum_{\omega \in E_t} \left(\max_{b_{E_t} \in B_{E_t}} h_{E_t}(\varphi, b_{E_t})(\omega) - h_{E_t}(\varphi, a_{E_t})(\omega) \right)^{\alpha_{E_t}} p_{E_t}(\omega)$$

for every $t = 0, \cdots, T-1$, $E_t \in \mathcal{F}_t$ and $B_{E_t} \in \mathcal{B}_{E_t}$.

5 Some unity: consistency to information arrival

The resolution of dynamic inconsistency discussed in the previous section allows that the decision maker's future self can be a totally different personality than the current self. Also, it does not question whether inconsistency comes from arrival of information or from changing choice opportunities.

In this section, we investigate under which condition the decision maker still has certain level of unity as an individual, in terms of consistency to information arrival. That is we rule out one of the two types of inconsistency, the inconsistency due to arrival of information, which delivers consistent connections of beliefs and uncertainty (regret) attitudes across date-events. Notice that such decision maker still may have the inconsistency due to opportunity dependence.

To see in what sense one is consistent to information arrival, compare two choice problems: (a) a commitment choice problem at a given node; (b) a commitment choice problem at any given node in the next period, which consists of conditional revision of *all* the acts in (a) upon the realization of one-step-ahead uncertainty. There is no change of opportunity between (a) and (b). Hence the inconsistency due to opportunity dependence is absent, and the only possible inconsistency is about the arrival of information. Our consistency to information arrival condition states that there is no choice reversal between (a) and (b).

In the existing studies of choice under uncertainty in which independence of choice opportunity is assumed and arrival of information is the only source of possible inconsistency, the above condition alone is necessary and sufficient for the full satisfaction of dynamic consistency. However, in the current case in which choice is opportunity dependent, the above condition is necessary but not sufficient for the full satisfaction, and it has an implication only about how beliefs and uncertainty attitudes evolve over time.

The consistency to information arrival requirement is stated as

Axiom CIA (**Consistency to Information Arrival**): For every $t = 0, \cdots, T-2$, $E_t \in \mathcal{F}_t$ and $(C_{E_{t+1}})_{E_{t+1} \in \mathcal{F}_t | E_t}$,

$$\prod_{E_{t+1} \in \mathcal{F}_{t+1} | E_t} \Phi_{E_{t+1}}(C_{E_{t+1}}) = \Phi_{E_t}\left(\prod_{E_{t+1} \in \mathcal{F}_{t+1} | E_t} C_{E_{t+1}} \right).$$

Notice that axiom CIA is *not* about a *real* course of actions. In reality, making some action necessarily changes choice opportunity in the future. Therefore, the consistency to information arrival requirement has to be about *hypothetical* connection between commitment choice problems at different periods that may not be connected by means of real actions.

Hypothetical exercise is necessary

The hypothetical connection stated above might sound absurd as it says. However, it is exactly what *all* the existing arguments about dynamic consistency are presuming. All the existing models of dynamic choice either under certainty or uncertainty[13] presume that the analyst can see the comparison across decision nodes about the rankings between all the pairs of alternatives (consumption streams, random variables, random consumption streams, etc.), by seeing *as if* all the alternatives are available throughout the lifetime (after conditioning on date-event) and the decision maker can exercise 'perfect commitment at each decision node,' which cannot be true along the real course of actions.[14]

To see this, consider the standard dynamic consistency requirement in the model of preference process, with regard to arrival of information: for all t, $E_t \in \mathcal{F}_t$ and $E_{t+1} \in \mathcal{F}_{t+1} | E_t$, for all $f_{E_{t+1}}, g_{E_{t+1}} \in \mathbb{R}^{E_{t+1}}$ and $h_{E_t \setminus E_{t+1}} \in \mathbb{R}^{E_t \setminus E_{t+1}}$,

$$f_{E_{t+1}} \succsim_{E_{t+1}} g_{E_{t+1}} \text{ if and only if } (f_{E_{t+1}}, h_{E_t \setminus E_{t+1}}) \succsim_{E_t} (g_{E_{t+1}}, h_{E_t \setminus E_{t+1}})$$

[13] such as Koopmans [21], Epstein [5], Machina [27], Kreps and Porteus [23], Epstein and Le Breton [7], Ghirardato [9], Epstein and Schneider [8]

[14] An explicit dynamic consistency axiom does not appear in Koopmans [21] and Epstein [5], but they essentially assume that preference over future consumption paths is identical over time, and impose the stationarity axiom as the dynamic consistency requirement.

where $\succsim_{E_t}$ refers to the period-t/event-E_t preference over commitment random variables that are conditional on E_t, and $\succsim_{E_{t+1}}$ refers to the period-$t+1$/event-E_{t+1} preference over commitment random variables that are conditional on E_{t+1}. As formal objects, the acts appeared above are commitment random variables. The relation $(f_{E_{t+1}}, h_{E_t \setminus E_{t+1}}) \succ_{E_t} (g_{E_{t+1}}, h_{E_t \setminus E_{t+1}})$ says 'if the decision maker at period-t/event-E_t chooses between $(f_{E_{t+1}}, h_{E_t \setminus E_{t+1}})$ and $(g_{E_{t+1}}, h_{E_t \setminus E_{t+1}})$ when perfect commitment is imposed, he chooses $(f_{E_{t+1}}, h_{E_t \setminus E_{t+1}})$.' Hence, if he really makes choice in such a manner, there is no choice problem left at any subsequent date/event: if he really chooses $(f_{E_{t+1}}, h_{E_t \setminus E_{t+1}})$ over $(g_{E_{t+1}}, h_{E_t \setminus E_{t+1}})$ at period-t/event-E_t with perfect commitment, he cannot have the choice problem between $f_{E_{t+1}}$ and $g_{E_{t+1}}$ at period-$t+1$/event-E_{t+1}.

In the choice function framework, the above form of dynamic consistency condition is written as

$$\Phi_{E_{t+1}}(\{f_{E_{t+1}}, g_{E_{t+1}}\}) \times \{h_{E_t \setminus E_{t+1}}\} = \Phi_{E_t}\left(\{f_{E_{t+1}}, g_{E_{t+1}}\} \times \{h_{E_t \setminus E_{t+1}}\}\right),$$

which is exactly our CIA axiom applied to the above binary choices.

When arrival of information is the only source of potential inconsistency, Consistency to Information Arrival alone guarantees the full satisfaction of dynamic consistency (the full requirement that should be stated in our extended setting comes in the next section). It is exactly what the existing arguments are imposing.

The axiom CIA is supposed to operate in a hypothetical universe of choice problems that are connected at different periods hypothetically, not by means of real actions. But this point applies to all the existing definitions of dynamic consistency.

Consistency to Information Arrival imposes that the regret-aggregating functions are connected in a recursive manner.

Theorem 2 Assume that the choice process set Φ allows the representation as in the basic model. Then, the following statements are equivalent:
(a) The choice process set Φ satisfies axiom CIA.
(b) The choice process set Φ has the additional property that for every $t = 0, \cdots, T-2$, $E_t \in \mathcal{F}_t$ and every $x_{E_t}, y_{E_t} \in \mathbb{R}_+^{E_t}$,

$$\Psi_{E_{t+1}}(x_{E_{t+1}}) \geqq \Psi_{E_{t+1}}(y_{E_{t+1}}) \text{ for every } E_{t+1} \in \mathcal{F}_{t+1}|E_t$$

implies

$$\Psi_{E_t}(x_{E_t}) \geqq \Psi_{E_t}(y_{E_t})$$

and the conclusion holds with strict inequality additionally if $\Psi_{E_{t+1}}(x_{E_{t+1}}) > \Psi_{E_{t+1}}(y_{E_{t+1}})$ for some $E_{t+1} \in \mathcal{F}_{t+1}|E_t$, where $x_{E_{t+1}}$ denotes the restriction of x_{E_t} to E_{t+1}.

Proof. (a) $\Longrightarrow$ (b): For simplicity of the argument, we directly treat regret vectors instead of payoff vectors. Take any $x_{E_t}, y_{E_t} \in \mathbb{R}_+^{E_t}$ such that $\Psi_{E_{t+1}}(x_{E_{t+1}}) \geqq \Psi_{E_{t+1}}(y_{E_{t+1}})$ for every $E_{t+1} \in \mathcal{F}_{t+1}|E_t$.

For each $E_{t+1} \in \mathcal{F}_{t+1}|E_t$, let $C_{E_{t+1}} \in \mathbb{R}_+^{E_{t+1}}$ be any compact set with $x_{E_{t+1}}, y_{E_{t+1}} \in C_{E_{t+1}}$ such that $\min_{z_{E_{t+1}} \in C_{E_{t+1}}} z_{E_{t+1}}(\omega) = 0$ for all $\omega \in E_{t+1}$ and $\Psi_{E_{t+1}}(y_{E_{t+1}}) = \min_{z_{E_{t+1}} \in C_{E_{t+1}}} \Psi_{E_{t+1}}(z_{E_{t+}}$ By construction, $y_{E_{t+1}}$ minimizes $\Psi_{E_{t+1}}$ in $C_{E_{t+1}}$ for each $E_{t+1} \in \mathcal{F}_{t+1}|E_t$.

By CIA ($\subset$ direction), y_{E_t} is chosen from $\prod_{E_{t+1} \in \mathcal{F}_{t+1}|E_t} C_{E_{t+1}}$ at period t with event E_t, which implies that y_{E_t} is minimizing Ψ_{E_t} in $\prod_{E_{t+1} \in \mathcal{F}_{t+1}|E_t} C_{E_t}$. Therefore, $\Psi_{E_t}(x_{E_t}) \geqq \Psi_{E_t}(y_{E_t})$.

Suppose additionally that $\Psi_{E_{t+1}}(x_{E_{t+1}}) > \Psi_{E_{t+1}}(y_{E_{t+1}})$ for some $E_{t+1} \in \mathcal{F}_{t+1}|E_t$. Then, $x_{E_{t+1}}$ cannot be chosen from $C_{E_{t+1}}$.

By CIA ($\supset$ direction), x_{E_t} cannot be chosen from $\prod_{E_{t+1} \in \mathcal{F}_{t+1}|E_t} C_{E_{t+1}}$ at period t with event E_t, which implies that x_{E_t} is not minimizing Ψ_{E_t} in $\prod_{E_{t+1} \in \mathcal{F}_{t+1}|E_t} C_{E_t}$. Therefore, $\Psi_{E_t}(x_{E_t}) > \Psi_{E_t}(y_{E_t})$.

(b) $\Longrightarrow$ (a): Obvious. ■

On the model of minimax regret with multiple-priors, Theorem 2 implies that the process of multiple-priors follows a recursive relationship.

Definition 1 The list of sets $((P_{E_t})_{E_t \in \mathcal{F}_t})_{t=0}^{T-1}$ is *rectangular* if for all $t = 0, \cdots, T-2$, $E_t \in \mathcal{F}_t$:
(i) $p_{E_t}(E_{t+1}) > 0$ for all $p_{E_t} \in P_{E_t}$ and $E_{t+1} \in \mathcal{F}_{t+1}|E_t$;
(ii)

$$P_{E_t} = \left\{ (p_{E_t}(E_{t+1}) p_{E_{t+1}})_{E_{t+1} \in \mathcal{F}_{t+1}|E_t} : p_{E_t} \in P_{E_t},\ p_{E_{t+1}} \in P_{E_{t+1}},\ E_{t+1} \in \mathcal{F}_{t+1}|E_t \right\}.$$

Corollary 3 Assume that the choice process set Φ allows the representation Sublass 1. Then, the following statements are equivalent:
(a) The choice process set Φ satisfies axiom CIA.
(b) The choice process set Φ has the additional property that $((P_{E_t})_{E_t \in \mathcal{F}_t})_{t=0}^{T-1}$ is rectangular.

Proof. (a) $\Longrightarrow$ (b): Part (i) follows from Theorem 2. We prove (ii). Let

$$Q_{E_t} = \left\{ (p_{E_t}(E_{t+1}) p_{E_{t+1}})_{E_{t+1} \in \mathcal{F}_{t+1}|E_t} : p_{E_t} \in P_{E_t},\ p_{E_{t+1}} \in P_{E_{t+1}},\ E_{t+1} \in \mathcal{F}_{t+1}|E_t \right\}$$

[15] It suffices to consider the case that such C_{E_t} is finite, though its cardinality needs to be at least $|E_t| + 2$ in general.

It suffices to establish

$$\max_{p_{E_t}\in P_{E_t}}\sum_{\omega\in E_t} x(\omega)p_{E_t}(\omega)=\max_{q_{E_t}\in Q_{E_t}}\sum_{\omega\in E_t} x(\omega)q_{E_t}(\omega)$$

for every $x_{E_t}\in\mathbb{R}_+^{E_t}$.

Given $x_{E_t}\in\mathbb{R}_+^{E_t}$, define $x^*_{E_t}\in\mathbb{R}_+^{E_t}$ by

$$x^*_{E_t}(\omega)=\max_{p_{E_{t+1}}\in P_{E_{t+1}}}\sum_{\omega'\in E_{t+1}} x_{E_t}(\omega')p_{E_{t+1}}(\omega')$$

where $E_{t+1}\in\mathcal{F}_{t+1}|E_t$ is taken so that $\omega\in E_{t+1}$.

By Theorem 2, we have

$$\max_{p_{E_t}\in P_{E_t}}\sum_{\omega\in E_t} x_{E_t}(\omega)p_{E_t}(\omega)=\max_{p_{E_t}\in P_{E_t}}\sum_{\omega\in E_t} x^*_{E_t}(\omega)p_{E_t}(\omega).$$

By definition of $x^*_{E_t}$,

$$\max_{p_{E_t}\in P_{E_t}}\sum_{\omega\in E_t} x^*_{E_t}(\omega)p_{E_t}(\omega)=\max_{p_{E_t}\in P_{E_t}}\sum_{E_{t+1}\in\mathcal{F}_{t+1}|E_t}\max_{p_{E_{t+1}}\in P_{E_{t+1}}}\sum_{\omega\in E_{t+1}} x_{E_t}(\omega)p_{E_t}(E_{E_{t+1}})p_{E_{t+1}}(\omega),$$

where the right-hand-side is equal to $\max_{q_{E_t}\in Q_{E_t}}\sum_{\omega\in E_t} x(\omega)q_{E_t}(\omega)$.

(b) $\Longrightarrow$ (a): Obvious. ■

On the smooth model of anticipated regret, Theorem 2 implies that the process of beliefs follows Bayesian updating and the regret aversion parameters are constant across date-events.

Corollary 4 Assume that the choice process set Φ allows the representation Sublass 2. Then, the following statements are equivalent:

(a) The choice process set Φ satisfies axiom CIA.

(b) The choice process set Φ has the additional property that

$$p_{E_t}=(p_{E_t}(E_{t+1})p_{E_{t+1}})_{E_{t+1}\in\mathcal{F}_{t+1}|E_t}$$

for all $t=0,\cdots,T-2$, $E_t\in\mathcal{F}_t$, and

$$\alpha_{E_t}=\alpha>0$$

for all $t=0,\cdots,T-1$, $E_t\in\mathcal{F}_t$.

Proof. (a) $\Longrightarrow$ (b): Fix any $t = 0, \cdots, T-2$, $E_t \in \mathcal{F}_t$ and $E_{t+1} \in \mathcal{F}_t | E_t$. Consider $x_{E_t}, y_{E_t} \in \mathbb{R}_+^{E_t}$, such that

$$x_{E_t}(\omega) = y_{E_t}(\omega) = 1 \text{ for all } \omega \in E_t \setminus E_{t+1}.$$

By Theorem 2, we have

$$\sum_{\omega \in E_{t+1}} x_{E_t}^{\alpha_{E_{t+1}}}(\omega) p_{E_{t+1}}(\omega) = \sum_{\omega \in E_{t+1}} y_{E_t}^{\alpha_{E_{t+1}}}(\omega) p_{E_{t+1}}(\omega) \implies \sum_{\omega \in E_t} x_{E_t}^{\alpha_{E_t}}(\omega) p_{E_t}(\omega) = \sum_{\omega \in E_t} y_{E_t}^{\alpha_{E_t}}(\omega) p_{E_t}(\omega),$$

where the latter is saying that

$$\sum_{\omega \in E_{t+1}} x_{E_t}^{\alpha_{E_t}}(\omega) p_{E_t}(\omega | E_{t+1}) = \sum_{\omega \in E_{t+1}} y_{E_t}^{\alpha_{E_t}}(\omega) p_{E_t}(\omega | E_{t+1}).$$

Thus, two functions $\sum_{\omega \in E_{t+1}} z_{E_t}^{\alpha_{E_{t+1}}}(\omega) p_{E_{t+1}}(\omega)$ and $\sum_{\omega \in E_{t+1}} z_{E_t}^{\alpha_{E_t}}(\omega) p_{E_t}(\omega | E_{t+1})$ agree on the ranking over $\mathbb{R}_+^{E_{t+1}}$. By the uniqueness of representation, $\alpha_{E_t} = \alpha_{E_{t+1}}$ and $p_{E_t}(\cdot | E_{t+1}) = p_{E_{t+1}}(\cdot)$.

Since this is true for all $E_{t+1} \in \mathcal{F}_t | E_t$, $E_t \in \mathcal{F}_t$, $t = 0, \cdots, T-1$, we obtain the desired result.

(b) $\Longrightarrow$ (a): Obvious. ■

6 Full dynamic consistency

The previous section discusses that belief consistency has an independent merit to look at, but it alone does not fulfill dynamic consistency perfectly, since there is inconsistency due to opportunity dependence. In this section we consider the implication of imposing full dynamic consistency, which says there is no kind of disagreement between successive selves, and leaves no necessity of resolution. Here it has to involve both arrival of information and changing opportunity, while we talked only about the first one in the previous section.

Given a *general* choice problem at period t in which commitment is not necessarily presumed, consider two lists of *commitment* choice problems derived from it *hypothetically*: (a) a commitment choice problem at period t, which is derived by giving the period-t self a perfect power to control choices at all the subsequent periods; (b) a list of commitment choice problems at period $t+1$, that are derived from the original problem by postponing choice at period t to each decision node at period $t+1$, in which the corresponding period-$t+1$ self is given the power. The dynamic consistency condition basically states that there is no reversal between (a) and (b).

Again, the consistency requirement is about hypothetical connections between *commitment choice problems at different decision nodes* (which is an oxymoron), and and it is a necessary argument to go through.

Given $t = 0, \cdots, T-1$, $E_t \in \mathcal{F}_t$ and a choice problem $B_{E_t} \in \mathcal{B}_{E_t}$, let

$$C_{E_t}(B_{E_t}) = \bigcup_{a_{E_t} \in B_{E_t}} \prod_{E_{t+1} \in \mathcal{F}_{t+1}|E_t} \cdots \bigcup_{a_{E_{T-2}} \in a_{E_{T-3}}(E_{T-2})} \prod_{E_{T-1} \in \mathcal{F}_{T-1}|E_{T-2}} a_{E_{T-2}}(E_{T-1})$$

be the set of all the commitment random variables that are attained if the decision maker at E_t can exercise perfect commitment so as to control all future selves' choices. Notice that it satisfies the recursive formula

$$C_{E_t}(B_{E_t}) = \bigcup_{a_{E_t} \in B_{E_t}} \prod_{E_{t+1} \in \mathcal{F}_{t+1}|E_t} C_{E_{t+1}}(a_{E_t}(E_{t+1})).$$

Now the full dynamic consistency condition is stated as

Axiom DC (**Dynamic Consistency**): For every $t = 0, \cdots, T-2$, $E_t \in \mathcal{F}_t$, and $B_{E_t} \in \mathcal{B}_{E_t}$,

(i)

$$\prod_{E_{t+1} \in \mathcal{F}_{t+1}|E_t} \Phi_{E_{t+1}} \left(\bigcup_{a_{E_t} \in B_{E_t}} C_{E_{t+1}}(a_{E_t}(E_{t+1})) \right) \subset \Phi_{E_t} \left(C_{E_t}(B_{E_t}) \right);$$

(ii)

$$\Phi_{E_t} \left(C_{E_t}(B_{E_t}) \right) \subset \bigcup_{a_{E_t} \in B_{E_t}} \prod_{E_{t+1} \in \mathcal{F}_{t+1}|E_t} \Phi_{E_{t+1}} \left(C_{E_{t+1}}(a_{E_t}(E_{t+1})) \right).$$

To understand DC-(i), consider hypothetically that the decision maker at time-t/event-E_t postpones choice and delegates it to himself at the next date-event. Upon each possible realization of one-step-ahead uncertainty, the next-period self makes choice with perfect commitment. Collect the list of such (anticipated) responses contingent on the realization of one-step-ahead uncertainty. It is the left-hand-side of DC-(i). The right-hand-side of DC-(i) is the commitment life plan which the current self picks if he can exercise perfect commitment. Now DC-(i) says that if a commitment life plan is supported by all the selves at all the events at the next period, it should be chosen by the current self as well when he can exercise perfect commitment.

To understand DC-(ii), consider that the current self can make choice with perfect commitment, assuming that he can perfectly govern his future actions. Consider a commitment life plan chosen there. It is the left-hand-side of DC-(ii). Then, DC-(ii) says that there must be a corresponding *real* action at the current period that induces every self in the next period to agree to the prescribed life plan, given each possible realization of one-step-ahead uncertainty and delivery of new choice opportunity.

Lemma 1 DC implies CIA.

Proof. By applying DC to the elements of $\mathcal{B}_{E_t}$ that take the form $\left\{\prod_{E_{t+1}\in\mathcal{F}_{t+1}|E_t} C_{E_{t+1}}\right\}$ ■

Next lemma is a kind of folk theorem. We confirm that the full dynamic consistency requirement implies opportunity independence in the sense that the sequence of choice correspondences satisfies the contraction property except at the initial node.[16] Notice that this folk theorem itself is *model-free* — it holds in any model of dynamic choice under uncertainty in which arrival of information and changing opportunities are the sources of potential inconsistency, as far as the above-noted consequentialism is maintained.

Lemma 2 (**Folk Theorem**): DC implies that the sequence $((\Phi_{E_t})_{E_t\in\mathcal{F}_t})_{t=1}^{T-1}$ satisfies the contraction property (Chernoff [3]) at each node: for all $t=1,\cdots,T-1$ and $E_t\in\mathcal{F}_t$, for all finite sets $C_{E_t}, D_{E_t}\in\mathcal{C}_{E_t}$ with $C_{E_t}\subset D_{E_t}$,

$$\Phi_{E_t}(D_{E_t})\cap C_{E_t}\subset\Phi_{E_t}(C_{E_t}).$$

The claim extends to general compact sets when $((\Phi_{E_t})_{E_t\in\mathcal{F}_t})_{t=1}^{T-1}$ satisfies upper hemi-continuity.

Proof. Fix any $t=0,\cdots,T-1$ and $E_t\in\mathcal{F}_t$. Pick any $f_{E_t}\in\Phi_{E_t}(D_{E_t})\cap C_{E_t}$. Suppose $f_{E_t}\notin\Phi_{E_t}(C_{E_t})$.
Let $E_{t-1}\in\mathcal{F}_{t-1}$ be such that $E_{t-1}\supset E_t$. Let $\{h_{\widetilde{E}_t}\}_{\widetilde{E}_t\in\mathcal{F}_t|E_{t-1},\widetilde{E}_t\neq E_t}$ be any list, where $h_{\widetilde{E}_t}\in\mathbb{R}^{\widetilde{E}_t}$ for each $\widetilde{E}_t\in\mathcal{F}_t|E_{t-1},\widetilde{E}_t\neq E_t$.
Let

$$B_{E_{t-1}}=\left\{D_{E_t}\times\prod_{\widetilde{E}_t\in\mathcal{F}_t|E_{t-1},\widetilde{E}_t\neq E_t}\{h_{\widetilde{E}_t}\}\right\}.$$

By assumption, we have

$$(f_{E_t},\{h_{\widetilde{E}_t}\}_{\widetilde{E}_t\in\mathcal{F}_t|E_{t-1},\widetilde{E}_t\neq E_t})\in\Phi_{E_t}(D_{E_t})\times\prod_{\widetilde{E}_t\in\mathcal{F}_t|E_{t-1},\widetilde{E}_t\neq E_t}\Phi_{\widetilde{E}_t}(\{h_{\widetilde{E}_t}\}).$$

[16]Dynamic consistency does not have an implication with regard to the expansion property (Sen [36]), because changing opportunity over time proceeds only in the direction of narrowing down.

By DC-(i) applied to $B_{E_{t-1}}$, we obtain

$$(f_{E_t}, \{h_{\widetilde{E}_t}\}_{\widetilde{E}_t \in \mathcal{F}_t | E_{t-1}, \widetilde{E}_t \neq E_t}) \in \Phi_{E_{t-1}} \left(D_{E_t} \times \prod_{\widetilde{E}_t \in \mathcal{F}_t | E_{t-1}, \widetilde{E}_t \neq E_t} \{h_{\widetilde{E}_t}\} \right).$$

Let

$$B'_{E_{t-1}} = \left\{ C_{E_t} \times \prod_{\widetilde{E}_t \in \mathcal{F}_t | E_{t-1}, \widetilde{E}_t \neq E_t} \{h_{\widetilde{E}_t}\},\ (D_{E_t} \setminus C_{E_t}) \times \prod_{\widetilde{E}_t \in \mathcal{F}_t | E_{t-1}, \widetilde{E}_t \neq E_t} \{h_{\widetilde{E}_t}\} \right\}$$

on the other hand. By assumption, we have

$$\begin{aligned}(f_{E_t}, \{h_{\widetilde{E}_t}\}_{\widetilde{E}_t \in \mathcal{F}_t | E_{t-1}, \widetilde{E}_t \neq E_t}) \quad \notin \quad & \left(\Phi_{E_t}(C_{E_t}) \times \prod_{\widetilde{E}_t \in \mathcal{F}_t | E_{t-1}, \widetilde{E}_t \neq E_t} \Phi_{\widetilde{E}_{t+1}}(\{h_{\widetilde{E}_t}\}) \right) \\ & \cup \left(\Phi_{E_t}(D_{E_t} \setminus C_{E_t}) \times \prod_{\widetilde{E}_t \in \mathcal{F}_t | E_{t-1}, \widetilde{E}_t \neq E_t} \Phi_{\widetilde{E}_{t+1}}(\{h_{\widetilde{E}_t}\}) \right).\end{aligned}$$

By DC-(ii) applied to $B'_{E_{t-1}}$, we obtain

$$(f_{E_t}, \{h_{\widetilde{E}_t}\}_{\widetilde{E}_t \in \mathcal{F}_t | E_{t-1}, \widetilde{E}_t \neq E_t}) \notin \Phi_{E_{t-1}} \left(D_{E_t} \times \prod_{\widetilde{E}_t \in \mathcal{F}_t | E_{t-1}, \widetilde{E}_t \neq E_t} \{h_{\widetilde{E}_t}\} \right),$$

a contradiction. ■

The basic model satisfies the contraction property over finite sets if and only if it obeys subjective expected utility maximization.[17] Combining this fact and Lemma 1, Lemma 2 and Theorem 2, we obtain

Theorem 3 Assume that the choice process set Φ allows the representation as in the basic model. Then, the following statements are equivalent:
(a) The choice process set Φ satisfies axiom DC.
(b) There exist a homothetic and strictly increasing function $\widetilde{\Psi}_0 : \mathbb{R}_+^{\mathcal{F}_1} \to \mathbb{R}_+$ and a process of full-support probability measures $((p_{E_t})_{E_t \in \mathcal{F}_t})_{t=1}^{T-1}$ such that

$$\Phi_{E_t}(C_{E_t}) = \arg \max_{f_{E_t} \in C_{E_t}} \sum_{\omega \in E_t} f_{E_t}(\omega) p_{E_t}(\omega)$$

[17]In the static setting, Hayashi [17] shows that the general regret-based model satisfies the Nash-Arrow type independence irrelevant alternatives condition if and only if it obeys subjective expected utility maximization. The same argument goes through with the contraction property applied just to finite sets, which is a milder condition.

for every $C_{E_t} \in \mathcal{C}_{E_t}$, $E_t \in \mathcal{F}_t$, $t = 1, \cdots, T-1$, and

$$\Phi_0(C_0) = \arg\min_{f_0 \in C_0} \widetilde{\Psi}_0 \left[\left(\sum_{\omega \in E_1} \max_{g_0 \in C_0} g_0(\omega) p_{E_1}(\omega) - \sum_{\omega \in E_1} f_0(\omega) p_{E_1}(\omega) \right)_{E_1 \in \mathcal{F}_1} \right]$$

for every $C_0 \in \mathcal{C}_0$. Moreover, $((p_{E_t})_{E_t \in \mathcal{F}_t})_{t=1}^{T-1}$ satisfies the property that

$$p_{E_t} = (p_{E_t}(E_{t+1}) p_{E_{t+1}})_{E_{t+1} \in \mathcal{F}_{t+1} | E_t}$$

for all $t = 1, \cdots, T-2$ and $E_t \in \mathcal{F}_t$.

Remark 1 When $\mathcal{F}_1 = \mathcal{F}_0$, that is, when no information is revealed at period 1, the period-0 choice reduces to the expected utility maximization as well, and there is no room for opportunity dependence at all. This essentially says that full satisfaction of dynamic consistency requires opportunity independence, since we can easily add an extra stage, say, period 0.5, with no information revelation.

Remark 2 Epstein-Schneider [8] provides a class of intertemporal preferences that allows uncertainty aversion but satisfies dynamic consistency. Our result does not contradict to theirs. The regret-based model coincides with the model of uncertainty aversion such as maximin expected utility (Gilboa-Schmeidler [11]) only when opportunity sets are symmetric in the sense that ex-post maximum values are equal across terminal states. On the whole domain of choice problems, the only intersection between the two is subjective expected utility maximization. In the regret-based model, subjective uncertainty plays a role through causing a dependence on choice opportunities. It makes the decision maker 'pessimistic' about how to evaluate the inferiority to ex-post optimum, but here the notion of pessimism is dependent on choice opportunity since the point of ex-post optimum is so. This is a different channel which is orthogonal to how subjective uncertainty plays a role in the model of uncertainty aversion.

Thus, since dynamic consistency implies there is essentially no role for opportunity dependence, it further implies there is no role for subjective uncertainty beyond subjective expected utility maximization.

Proof. (a) $\Longrightarrow$ (b): It follows from the combination of Lemma 1, Lemma 2 and Theorem 2.

(b) $\Longrightarrow$ (a): It is easy to see that the choice correspondences at period 1 and after satisfy DC since they obey subjective expected utility maximization with Bayesian updating. Thus

we focus on the relation between period 0 and 1.
To check DC-(i), consider $f_0 \in C_0(B_0)$ such that for each $E_1 \in \mathcal{F}_1$,

$$f_{E_1} \in \arg \max_{h_{E_1} \in \bigcup_{a_0 \in B_0} C_{E_1}(a_0(E_1))} \sum_{\omega \in E_1} h_{E_1}(\omega) p_{E_1}(\omega),$$

where f_{E_1} denotes the restriction of f_0 to E_1.

Then it achieves

$$f_0 \in \arg \min_{h_0 \in C_0(B_0)} \widetilde{\Psi}_0 \left[\left(\sum_{\omega \in E_1} \max_{g_0 \in C_0(B_0)} g_0(\omega) p_{E_1}(\omega) - \sum_{\omega \in E_1} h_0(\omega) p_{E_1}(\omega) \right)_{E_1 \in \mathcal{F}_1} \right],$$

because f_0 restricted to E_1 minimizes $\sum_{\omega \in E_1} \max_{g_0 \in C_0(B_0)} g_0(\omega) p_{E_1}(\omega) - \sum_{\omega \in E_1} f_0(\omega) p_{E_1}(\omega)$ for each $E_1 \in \mathcal{F}_1$.
To check DC-(ii), let $f_0 \in \Phi_0(C_0(B_0))$, that is,

$$f_0 \in \arg \min_{h_0 \in C_0(B_0)} \widetilde{\Psi}_0 \left[\left(\sum_{\omega \in E_1} \max_{g_0 \in C_0(B_0)} g_0(\omega) p_{E_1}(\omega) - \sum_{\omega \in E_1} h_0(\omega) p_{E_1}(\omega) \right)_{E_1 \in \mathcal{F}_1} \right].$$

Then, there is $a_0 \in B_0$ such that $f_0 \in \prod_{E_1 \in \mathcal{F}_1} C_{E_1}(a_0(E_1))$ achieves the minimum. Then, it has to be the case that for each $E_1 \in \mathcal{F}_1$, f_{E_1} maximizes $\sum_{\omega \in E_1} f_{E_1}(\omega) p_{E_1}(\omega)$ in $C_{E_1}(a_0(E_1))$, which results in $f_0 \in \prod_{E_1 \in \mathcal{F}_1} \Phi_{E_1}(C_{E_1}(a_0(E_1)))$.
— Otherwise, we can take $h_{E_1} \in C_{E_1}(a_0(E_1))$ for some $E_1 \in \mathcal{F}_1$, where $\sum_{\omega \in E_1} h_{E_1}(\omega) p_{E_1}(\omega) > \sum_{\omega \in E_1} f_{E_1}(\omega) p_{E_1}(\omega)$. Then, $(h_{E_1}, (f_{\widetilde{E}_1})_{\widetilde{E}_1 \in \mathcal{F}_1 \setminus \{E_1\}}) \in C_0(B_0)$ and it obtains a smaller value of $\widetilde{\Psi}_0$, which is a contradiction to the assumption. ■

On the model of minimax regret with multiple-priors, the above result implies that the multiplicity of beliefs is allowed only for the one-step-ahead uncertainty between period 0 and 1.

Corollary 5 Assume that the choice process set Φ allows the representation Subclass 1. Then, the following statements are equivalent:
(a) The choice process set Φ satisfies axiom DC.
(b) There exist a set of probability measures $P_0 \subset \Delta(\Omega)$ and a process of full-support probability measures $((p_{E_t})_{E_t \in \mathcal{F}_t})_{t=1}^{T-1}$ such that

$$\Phi_{E_t}(C_{E_t}) = \arg \max_{f_{E_t} \in C_{E_t}} \sum_{\omega \in E_t} f_{E_t}(\omega) p_{E_t}(\omega)$$

for every $C_{E_t} \in \mathcal{C}_{E_t}$, $E_t \in \mathcal{F}_t$, $t = 1, \cdots, T-1$, and

$$\Phi_0(C_0) = \arg \min_{f_0 \in C_0} \max_{p_0 \in P_0} \sum_{E_1 \in \mathcal{F}_1} \left(\sum_{\omega \in E_1} \max_{g_0 \in C_0} g_0(\omega) p_{E_1}(\omega) - \sum_{\omega \in E_1} f_0(\omega) p_{E_1}(\omega) \right) p_0(E_1)$$

for every $C_0 \in \mathcal{C}_0$. Moreover, P_0 and $((p_{E_t})_{E_t \in \mathcal{F}_t})_{t=1}^{T-1}$ satisfy the property that $p_0(E_1) > 0$ for all $E_1 \in \mathcal{F}_1$ and $p_0 \in P_0$, and

$$P_0 = \{(p_0(E_1)p_{E_1})_{E_1 \in \mathcal{F}_1} : p_0 \in P_0\},$$

and

$$p_{E_t} = (p_{E_t}(E_{t+1})p_{E_{t+1}})_{E_{t+1} \in \mathcal{F}_{t+1}|E_t}$$

for all $t = 1, \cdots, T-2$ and $E_t \in \mathcal{F}_t$.

On the smooth model of anticipated regret, since Corollary 4 already delivers Bayesian updating and constancy of regret attitudes across date-events, the folk theorem implies expected utility maximization at every decision node.

Corollary 6 Assume that the choice process set Φ allows the representation Subclass 2. Then, the following statements are equivalent:
(a) The choice process set Φ satisfies axiom DC.
(b) There exists a process of full-support probability measures $((p_{E_t})_{E_t \in \mathcal{F}_t})_{t=0}^{T-1}$ such that

$$\Phi_{E_t}(C_{E_t}) = \arg \max_{f_{E_t} \in C_{E_t}} \sum_{\omega \in E_t} f_{E_t}(\omega) p_{E_t}(\omega)$$

for every $C_{E_t} \in \mathcal{C}_{E_t}$, $E_t \in \mathcal{F}_t$, $t = 0, \cdots, T-1$. Moreover, $((p_{E_t})_{E_t \in \mathcal{F}_t})_{t=0}^{T-1}$ satisfy the property that

$$p_{E_t} = (p_{E_t}(E_{t+1})p_{E_{t+1}})_{E_{t+1} \in \mathcal{F}_{t+1}|E_t}$$

for all $t = 0, \cdots, T-2$ and $E_t \in \mathcal{F}_t$.

7 Concluding comments

We have examined if and to what extent choice dispositions can allow dependence on contexts and at the same time maintain dynamic consistency, on the case of opportunity dependence due to being affected by anticipated regret. First, we went over the general method of resolution of potential inconsistency, by taking any kinds of inconsistency as given constraints. Second, we characterized a class of choice dispositions that are consistent to arrival of information but may be inconsistent to changing opportunities. Finally, we considered the full requirement of dynamic consistency and showed that it necessarily implies independence of choice opportunities. The last result states that opportunity dependence and full dynamic consistency cannot coexist.

Our result does not yet exclude a possibility that there exists some type of context dependence which rather reinforces dynamic consistency. However, it should be noted that the pursuit of (full) dynamic consistency severely limits the varieties of choice dispositions that are admitted in the static life.

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