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Trade with Heterogeneous Multiple Priors

by

Atsushi Kajii

and

Takashi Ui

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KYOTO UNIVERSITY

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Atsushi Kajii
Institute of Economic Research
Kyoto University
kajii@kier.kyoto-u.ac.jp

Takashi Ui
Faculty of Economics
Yokohama National University
oui@ynu.ac.jp

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Abstract

This paper presents a general framework to understand the possibility of a purely speculative trade under asymmetric information, where the decision making rule of each trader conforms to the multiple priors model (Gibloa and Schmeidler, 1989): the agents are interested in the minimum of the conditional expected value of trade where the minimum is taken over the set of posteriors. In this framework, we derive a necessary and sufficient condition on the sets of posteriors, thus implicitly on the updating rules adopted by the agents, for non-existence of trade such that it is always common knowledge that every agent expects a positive gain.
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Key words: multiple priors; no trade; dynamic consistency; interim efficiency; rectangularity.

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1 Introduction

This paper presents a general framework to understand the possibility of a purely speculative trade under asymmetric information, where the decision making rule of each trader conforms to the multiple priors model (Gibloa and Schmeidler, 1989): the agents are interested in the minimum of the conditional expected value of trade where the minimum is taken over the set of posteriors. In this framework, we derive a necessary and sufficient condition on the sets of posteriors, thus implicitly on the updating rules adopted by the agents, for non-existence of interim agreeable trade, i.e., trade such that it is always common knowledge that every agent expects a positive gain.

The condition is closely related to the existence of a common prior in the single prior model, and thus our results give an insight about the role of a "common set of priors" in the multiple prior model. Since the model in this paper is an extension of the standard Bayesian trading model initiated by Milgrom and Stokey (1982), our results can be seen as a generalization of results obtained by Morris (1995) and Feinberg (2000), further elaborated by Samet (1998) and Ng (2003).

A simple analogy to the standard Bayesian model might suggest that the existence of a common prior, i.e., the non-emptiness of the intersection of agents' sets of priors, would imply non-existence of interim agreeable trade. Indeed, as is characterized by Billot *et al.* (2000), this is basically the case for *ex ante* agreeable trade. But for the *interim* trade agreement, the problem is more subtle. Even in the standard Bayesian model with a single prior, as is shown in the above literature, the necessary and sufficient condition for no interim agreeable trade is weaker than the common prior assumption; the condition is that the posteriors of the traders are consistent with some (fictitious) common prior. In our model of multiple priors, however, the issue is more complicated, since there are many possible ways to update multiple priors upon arrival of private information. We consider a collection of all (fictitious) priors consistent with the posteriors and call it a maximal rectangular prior set. The necessary and sufficient condition for non-existence of interim agreeable trade is that the intersection of these sets is non-empty.

Non-existence of purely speculative trade in general non-expected utility frameworks, including multiple prior models, with asymmetric information has been studied by Dow *et al.* (1990), Ma (2001), and Halevy (2004), so let us clarify our contribution with respect to these works. These papers were intended to characterize the condition on the ex ante preference relation which ensures the non-existence of interim agreeable trade for *any* information structure. The condition Ma (2001) and Halevy (2004) found for this purpose is essentially the weak decomposability axiom studied by Grant *et al.* (2000), which leads to a dynamically consistent decision making rule.

In contrast to these papers, we concentrate on multiple priors models. Since the weak decomposability is typically not satisfied for this class of models, one might think that our results are inconsistent with the aforementioned results. The key difference is that we assume a *fixed* information structure, and hence our condition on the set of possible posteriors is relative to the fixed information structure.

As a matter of fact, the reader will see that the dynamic consistency is relevant in our condition as well, and the relevant concept is the rectangularity introduced by Epstein and Schneider (2003) and Wakai (2002).[1] Wakai (2002) showed that the rectangularity is sufficient for no trade. Thus our main contribution is the necessity part of the argument, especially the necessity of a common prior. But we also contend that our formulation, inspired by a beautiful paper of Samet (1998), is much simpler but captures the essence of the problem. In particular, our approach clarifies the precise role of dynamic consistency in the analysis.

The organization of the paper is as follows. Section 2 describes the model and summarizes basic known results. Section 3 reports the main results. Section 4 discusses the implications of our results in comparison with those in the standard Bayesian model. Section 5 contains examples.

2 Setup

We consider the following information structure for a finite number of agents, who are allowed to have multiple posteriors.

- $\Omega = \{1, \dots, n\}$: a finite set of states.
- $\Delta(\Omega) = \{p \in \mathbb{R}^{\Omega} : p(\omega) \geq 0 \text{ for all } \omega \in \Omega, \sum_{\omega\in\Omega} p(\omega) = 1\}$: the set of probability distributions over Ω. A generic element of $\Delta(\Omega)$ is denoted by $p = (p(1), \dots, p(n))$. For $E \subseteq \Omega$, we write $p(E)$ for $\sum_{\omega\in E} p(\omega)$.

[1] We adopt the term "rectangularity" from Epstein and Schneider (2003).

- $p(\cdot|E)$: the conditional probability given E over Ω when $p(E) > 0$ for $p \in \Delta(\Omega)$. That is, $p(A|E) = p(A \cap E)/p(E)$ for $A, E \subseteq \Omega$. If $p(E) = 0$, let $p(\cdot|E)$ be an arbitrary element of $\Delta(\Omega)$.
- $\mathcal{P} = 2^{\Delta(\Omega)} \backslash \emptyset$: a collection of all non-empty subsets of $\Delta(\Omega)$.
- $\mathcal{I} = \{1, \ldots, I\}$: a finite set of agents. For each $i \in \mathcal{I}$,
  - Π_i is an information partition of Ω for agent i, and $\Pi_i(\omega)$ is the partition element containing $\omega \in \Omega$. A generic element of Π_i is denoted by π_i.
  - $\Phi_i : \Pi_i \to \mathcal{P}$ is a function such that:
    - $p(\pi_i) = 1$ for all $p \in \Phi_i(\pi_i)$ and $\pi_i \in \Pi_i$,
    - $\Phi_i(\pi_i)$ is closed and non-empty for all $\pi_i \in \Pi_i$.

  We refer to the function Φ_i as a posterior function of agent $i \in \mathcal{I}$.

Definition 1 We call $\langle \Omega, \mathcal{I}, \{\Pi_i\}_{i \in \mathcal{I}}, \{\Phi_i\}_{i \in \mathcal{I}} \rangle$ an information structure with multiple posteriors.

In our setup, we take posteriors rather than priors as primitives. The posterior function is intended to describe the beliefs of agent $i \in \mathcal{I}$ after a partition element $\pi_i \in \Pi_i$ is observed. A natural case of course will be where the posterior function is derived from an updating rule operated on a set of priors (cf. Gibloa and Schmeidler, 1993). We shall list below a couple of standard updating rules. Let $P_i \in \mathcal{P}$ be a non-empty, closed set such that $\{p \in P_i : p(\pi_i) > 0\} \neq \emptyset$ for all $\pi_i \in \Pi_i$ and $i \in \mathcal{I}$.

1. The full Bayesian updating (FB-updating) on P_i:

$$\Phi_i(\pi_i) := \{p(\cdot|\pi_i) : p(\pi_i) > 0,\ p \in P_i\}.$$

2. The maximum likelihood updating (ML-updating) on P_i:

$$\Phi_i(\pi_i) := \{p(\cdot|\pi_i) : p(\pi_i) = \max_{p' \in P_i} p'(\pi_i)\}.$$

The standard Bayesian model corresponds to the case where P_i is a singleton for every agent. When P_i is a singleton, the derived posterior functions in the above examples coincide trivially, and they are convex valued. It is easy to check that $\Phi_i(\pi_i)$ is a closed set for the ML-updating rule. For the FB-updating rule, if $\min_{p \in P_i} p(\pi_i) > 0$, $\Phi_i(\pi_i)$ is closed.

Alternatively, one can start with a given posterior function and ask what set of priors will be compatible with it, and this is the path we shall pursue in this paper. First of all, a minimal Bayesian consistency requirement is that any point in $\Phi_i(\pi_i)$ should be a conditional probability of some priors in the prior set.

Definition 2 A prior set $P_i \in \mathcal{P}$ is said to be *compatible* with Φ_i if, for any $\pi_i \in \Pi_i$ and $p \in \Phi_i(\pi_i)$, there exists $p' \in P_i$ with $p'(\cdot|\pi_i) = p$. A prior set $P_i \in \mathcal{P}$ is said to be *fully compatible* with Φ_i if P_i is compatible with Φ_i and $p \in P_i$ with $p(\pi_i) > 0$ implies $p(\cdot|\pi_i) \in \Phi_i(\pi_i)$ for any $\pi_i \in \Pi_i$.

To put if differently, $P_i \in \mathcal{P}$ is compatible with Φ_i if and only if $\{p(\cdot|\pi_i) : p(\pi_i) > 0,\ p \in P_i\} \supseteq \Phi_i(\pi_i)$ for all $\pi_i \in \Pi_i$; and $P_i \in \mathcal{P}$ is fully compatible with Φ_i if and only if $\{p(\cdot|\pi_i) : p(\pi_i) > 0,\ p \in P_i\} = \Phi_i(\pi_i)$ for all $\pi_i \in \Pi_i$.

If P_i is fully compatible with Φ_i, then,

$$P_i \subseteq \bigcup_{p \in P_i} \sum_{\pi_i \in \Pi_i} p(\pi_i)\Phi_i(\pi_i) \tag{1}$$

where the summation is the Minkowski sum. The inclusion above may be strict in general. A stronger consistency requirement below, introduced by Epstein and Schneider (2003) and Wakai (2002), asks for the equality:

Definition 3 A prior set $P_i \in \mathcal{P}$ is *rectangular* (with Φ_i) if P_i is fully compatible with Φ_i and

$$P_i = \bigcup_{p \in P_i} \sum_{\pi_i \in \Pi_i} p(\pi_i)\Phi_i(\pi_i).$$

To sum up, we have introduced three progressively stronger Bayesian consistency requirements for prior sets: compatibly, full compatibility, and rectangularity. See Example 1 in Section 5 which illustrates the differences of the three requirements.

We summarize below the relationship between the three Bayesian consistency requirements and the FB-updating or the ML-updating.

Lemma 1 *The following claims are true:*

- *If Φ_i is the ML-updating on P_i, then P_i is compatible with Φ_i.*
- *Φ_i is the FB-updating on P_i if and only if P_i is fully compatible with Φ_i.*
- *If P_i is rectangular, then the FB-updating and the ML-updating coincide.*

Proof. The first and second claims are apparent. We prove the last claim. Pick any $\hat{\pi}_i \in \Pi_i$ and fix any $p^* \in P_i$ with $p^*(\hat{\pi}_i) = \max_{p' \in P_i} p'(\hat{\pi}_i)$. For any $p \in P_i$ such that $p(\pi_i) > 0$ for $\pi_i \in \Pi_i$ with $p^*(\pi_i) > 0$, by the definition of rectangularity, $\bar{p} := \sum_{\pi_i \in \Pi_i} p^*(\pi_i) p(\cdot|\pi_i) \in P_i$. So $\bar{p}(\cdot|\hat{\pi}_i) = p(\cdot|\hat{\pi}_i)$ is obtained by the ML updating on P_i. ∎

For any Φ_i, there is a fully compatible, rectangular prior set; fix any $Q \in \mathcal{P}$ and define

$$P_i := \left\{ p \in \sum_{\pi_i \in \Pi_i} q(\pi_i)\Phi_i(\pi_i) : q \in Q \right\} = \bigcup_{q \in Q} \sum_{\pi_i \in \Pi_i} q(\pi_i)\Phi_i(\pi_i). \tag{2}$$

Then, P_i is rectangular with Φ_i. Conversely, any rectangular prior set P_i can be written of the form (2); let $Q = P_i$ in the construction (2). Then by the definition of rectangularity, $\bigcup_{p \in P_i} \sum_{\pi_i \in \Pi_i} p(\pi_i)\Phi_i(\pi_i) = P_i$.

For given Φ_i, define a rectangular prior set $P_i^* \in \mathcal{P}$ by the rule:

$$P_i^* := \left\{ p \in \sum_{\pi_i \in \Pi_i} q(\pi_i)\Phi_i(\pi_i) : \ q \in \Delta(\Omega) \right\} = \bigcup_{q \in \Delta(\Omega)} \sum_{\pi_i \in \Pi_i} q(\pi_i)\Phi_i(\pi_i). \tag{3}$$

We shall refer to P_i^* as the *maximal rectangular prior set* because P_i^* is maximal in the collection of all rectangular prior set ordered by the set inclusion relation. The set P_i^* will play an important role in the main result. We have the following properties of P_i^*:

- $\Phi_i(\pi_i) \subseteq P_i^*$ for each $\pi_i \in \Pi_i$; consider $q \in \Delta(\Omega)$ with $q(\pi_i) = 1$ in (3).
- $p \in P_i^*$ if and only if $p(\pi_i) > 0$ implies $p(\cdot|\pi_i) \in \Phi_i(\pi_i)$ by (3).
- From (1) and (3), P_i^* is the collection of all probability distributions over Ω contained in some prior set fully compatible with Φ_i. In particular, if P_i is fully compatible with Φ_i, then $P_i \subseteq P_i^*$.

- If Φ_i is singleton-valued, P_i^* is the set of all priors which generates the posteriors given by Φ_i in the standard sense. In other words, if $\bigcap_{i \in \mathcal{I}} P_i^* \neq \emptyset$, then the posteriors are consistent in the sense of Harsanyi (1967–1968).
- P_i^* is closed (and hence compact) since $\Phi_i(\pi_i)$ is closed for all $\pi_i \in \Pi_i$.

For $f \in \mathbb{R}^\Omega$ and $i \in \mathcal{I}$, let

$$E_i f(\omega) = \min_{p \in \Phi_i(\Pi_i(\omega))} p \cdot f \tag{4}$$

where $p \cdot f = \sum_{\omega \in \Omega} p(\omega) f(\omega)$. That is, E_i assigns the smallest conditional expected value of f at ω. Similarly, we write

$$E_i^0 f = \min_{p \in P_i} p \cdot f \tag{5}$$

when the reference to the prior set P_i is clear from the context: $E_i^0 f$ is the smallest expected value.

We consider a collection of functions $\{f_i \in \mathbb{R}^\Omega\}_{i \in \mathcal{I}}$ with $\sum_{i \in \mathcal{I}} f_i = 0$. We interpret a function f_i as a financial asset; when $\omega \in \Omega$ is realized, agent $i \in \mathcal{I}$ who owns f_i receives the value of $f_i(\omega)$. Assume that the initial position of each agent is neutral: he receives 0 regardless of ω. Since we require $\sum_{i \in \mathcal{I}} f_i = 0$, the collection $\{f_i\}_{i \in \mathcal{I}}$ can be understood as a trade arrangement. Now if agent i with a posterior function Φ_i adopts the very pessimistic decision rule of maximizing the minimum expected value (Gibloa and Schmeidler, 1989), then the agent is willing to accept f_i at ω if $E_i f_i(\omega) > 0$. Thus, if $E_i f_i(\omega) > 0$ for every $i \in \mathcal{I}$, we shall deem that a trade arrangement $\{f_i\}_{i \in \mathcal{I}}$ where agent $i \in \mathcal{I}$ receives f_i is *interim* agreeable to all agents. Similarly, an *ex ante* agreement can be defined. We discuss the relationship between them in Section 4.3.

For the case of a single prior, Samet (1998) showed the following result.

Proposition 1 *Suppose that $\Phi_i(\pi_i)$ is a singleton for all $\pi_i \in \Pi_i$ and $i \in \mathcal{I}$. Let P_i^* be the maximal rectangular prior set for $i \in \mathcal{I}$. Then $\bigcap_{i \in \mathcal{I}} P_i^* \neq \emptyset$ if and only if there exists no $\{f_i \in \mathbb{R}^\Omega\}_{i \in \mathcal{I}}$ with $\sum_{i \in \mathcal{I}} f_i = 0$ such that $E_i f_i(\omega) > 0$ for all $\omega \in \Omega$, for $i \in \mathcal{I}$.*

Note that $\bigcap_{i \in \mathcal{I}} P_i^* \neq \emptyset$ implies that there exists a common prior $p \in \bigcap_{i \in \mathcal{I}} P_i^*$ such that $\Phi_i(\pi_i) = \{p(\cdot|\pi_i)\}$ for all $\pi_i \in \Pi_i$ and $i \in \mathcal{I}$ if $p(\pi_i) > 0$, or $\{\Phi_i\}_{i \in \mathcal{I}}$ is consistent in the sense of Harsanyi (1967–1968). Thus, the proposition says that there exists a common prior if and only if there exists no interim agreeable trade arrangement.

Let $\delta_E \in \mathbb{R}^\Omega$ be an indicator function of $E \subseteq \Omega$ such that $\delta_E(\omega) = 1$ if $\omega \in E$ and $\delta_E(\omega) = 0$ otherwise. Note that $E_i\delta_E(\omega)$ is a posterior probability of $E \subseteq \Omega$ held by agent $i \in \mathcal{I}$ at $\omega \in \Omega$. As a corollary of Proposition 1, we have the agreement theorem of Aumann (1976).[2]

Corollary 1 *Suppose that $\Phi_i(\pi_i)$ is a singleton for all $\pi_i \in \Pi_i$ and $i \in \mathcal{I}$. If $\bigcap_{i\in\mathcal{I}} P_i^* \neq \emptyset$ and $E_i\delta_E(\omega)$ is constant over all $\omega \in \Omega$ for all $i \in \mathcal{I}$, then $E_i\delta_E(\omega) = E_j\delta_E(\omega)$ for all $i, j \in \mathcal{I}$.*

Samet (1998) showed the following separation theorem of many convex sets in a simplex in order to show the above results.

Lemma 2 *Let $K_1, \ldots, K_I$ be convex, closed, subsets of $\Delta(\Omega)$. Then, $\bigcap_{i=1}^I K_i = \emptyset$ if an only if there are $f_1, \ldots, f_I \in \mathbb{R}^\Omega$ such that $\sum_{i=1}^I f_i = 0$ and $x_i \cdot f_i > 0$ for each $x_i \in K_i$, for $i = 1, \ldots, I$.*

3 Results

Let us first provide a result which relates the conditional expectation and the unconditional expectation with the Bayesian consistency requirements.

Lemma 3 *Let $c \in \mathbb{R}$.* (i) *Let P_i be fully compatible with Φ_i for $i \in \mathcal{I}$. Then, $p \cdot f > c$ for all $p \in P_i$ if $E_i f(\omega) > c$ for all $\omega \in \Omega$.* (ii) *Let P_i^* be the maximal rectangular prior set for $i \in \mathcal{I}$. Then, $p \cdot f > c$ for all $p \in P_i^*$ if and only if $E_i f(\omega) > c$ for all $\omega \in \Omega$.*

Proof. Let P_i be fully compatible with Φ_i. Suppose that $E_i f(\omega) = \min_{p\in\Phi_i(\Pi_i(\omega))} p \cdot f > c$ for all $\omega \in \Omega$. Pick any $q \in P_i$. Note that $q = \sum_{\pi_i\in\Pi_i} q(\pi_i) q(\cdot|\pi_i)$. For each $\pi_i \in \Pi_i$ with $q(\pi_i) > 0$, full compatibility implies that $q(\cdot|\pi_i) \in \Phi_i(\pi_i)$ and thus $q(\cdot|\pi_i) \cdot f \geq \min_{p\in\Phi_i(\pi_i)} p \cdot f > c$. Therefore, $q \cdot f = \sum_{\pi_i\in\Pi_i} q(\pi_i)(q(\cdot|\pi_i) \cdot f) > c$, which establishes (i).

For (ii), if $p \cdot f > c$ for all $p \in P_i^*$, we have $E_i f(\omega) = \min_{p\in\Phi_i(\Pi_i(\omega))} p \cdot f > c$ for all $\omega \in \Omega$ since $\Phi_i(\Pi_i(\omega)) \subseteq P_i^*$ for all $\omega \in \Omega$. The converse also holds because of full compatibility of P_i^*. ∎

[2]We can replace Ω with a common knowledge event in Proposition 1. The condition that $E_i f_i(\omega) > 0$ for all $i \in \mathcal{I}$ and $\omega \in \Omega$ can be re-stated that "a positive gain from trade is common knowledge" at $\omega \in \Omega$. Corollary 1 can be modified analogously.

The converse of (i) does not hold even if P_i is rectangular. See Example 1 in Section 5.

We report our main results which extend Proposition 1 and Corollary 1 to the case where $\Phi_i(\pi_i)$ is not a singleton.

Proposition 2 *Let P_i^* be the maximal rectangular prior set for $i \in \mathcal{I}$ and let $\mathrm{co}\,(P_i^*)$ be its convex hull. Then, $\bigcap_{i\in\mathcal{I}} \mathrm{co}\,(P_i^*) \neq \emptyset$ if and only if there exists no $\{f_i \in \mathbb{R}^\Omega\}_{i\in\mathcal{I}}$ with $\sum_{i\in\mathcal{I}} f_i = 0$ such that $E_i f_i(\omega) > 0$ for all $\omega \in \Omega$, for $i \in \mathcal{I}$.*

Proof. Since $\mathrm{co}\,(P_i^*)$ is convex and closed for each $i \in \mathcal{I}$, by Lemma 2, $\bigcap_{i\in I} \mathrm{co}\,(P_i^*) \neq \emptyset$ if and only if there are no $f_1, \ldots, f_I \in \mathbb{R}^\Omega$ such that $\sum_{i\in\mathcal{I}} f_i = 0$ and $p_i \cdot f_i > 0$ for each $p_i \in \mathrm{co}\,(P_i^*)$, for $i \in \mathcal{I}$. Note that $p_i \cdot f_i > 0$ for each $p_i \in \mathrm{co}\,(P_i^*)$ if and only if $p_i \cdot f_i > 0$ for each $p_i \in P_i^*$. Thus, Lemma 3 completes the proof. ∎

Corollary 2 *If $\bigcap_{i\in\mathcal{I}} \mathrm{co}\,(P_i^*) \neq \emptyset$, and if, for each $i \in \mathcal{I}$,*

$$\min_{p\in\Phi_i(\Pi_i(\omega))} p(E), \quad \max_{p\in\Phi_i(\Pi_i(\omega))} p(E)$$

are constant over all $\omega \in \Omega$, then, for $i, j \in \mathcal{I}$,

$$\min_{p\in\Phi_i(\Pi_i(\omega))} p(E) \leq \max_{p\in\Phi_j(\Pi_j(\omega))} p(E).$$

Proof. Let

$$\underline{c}_i = \min_{p\in\Phi_i(\Pi_i(\omega))} p(E) = E_i\delta_E(\omega), \ \overline{c}_i = \max_{p\in\Phi_i(\Pi_i(\omega))} p(E) = -E_i(-\delta_E)(\omega)$$

for $i \in \mathcal{I}$. We show that $\underline{c}_i \leq \overline{c}_j$.

Suppose that $\mathcal{I} = \{1, 2\}$. If $\overline{c}_j < \underline{c}_i$ with $i \neq j$, let $f_i = \delta_E - c$ and $f_j = c - \delta_E$ where $\overline{c}_j < c < \underline{c}_i$. Then, $E_i f_i(\omega) > 0$ and $E_j f_j(\omega) > 0$ for all $\omega \in \Omega$, which contradicts to $\bigcap_{i\in\mathcal{I}} \mathrm{co}\,(P_i^*) \neq \emptyset$. Thus, we must have $\underline{c}_i \leq \overline{c}_j$.

Suppose that $\mathcal{I} = \{1, \ldots, I\}$ with $I \geq 3$. If $\overline{c}_j < \underline{c}_i$ with $i \neq j$, let $f_i = \delta_E - c_i$, $f_j = c_j - \delta_E$, $f_k = (c_i - c_j)/(I - 2)$ for $k \neq i, j$ where $\overline{c}_j < c_j < c_i < \underline{c}_i$. Then, $E_i f_i(\omega) > 0$ for all $i \in \mathcal{I}$, which contradicts to $\bigcap_{i\in\mathcal{I}} \mathrm{co}\,(P_i^*) \neq \emptyset$. Thus, we must have $\underline{c}_i \leq \overline{c}_j$, completing the proof. ∎

4 Discussions

We discuss three important differences between Proposition 1 with singleton-valued posterior functions and Proposition 2 with general posterior functions.

4.1 On the existence of common priors sets

If Φ_i is singleton-valued, the implication of $\bigcap_{i\in\mathcal{I}} P_i^* \neq \emptyset$ is clear, as we have already discussed: the agents look as if they share a common prior from the observer's point of view. The common prior assumption is necessary and sufficient for the interim no trade, i.e., non-existence of interim agreeable trade, by Proposition 1.

This leads us to the following question: if Φ_i is not singleton-valued, what is the implication of $\bigcap_{i\in\mathcal{I}} \mathrm{co}(P_i^*) \neq \emptyset$? To consider this question, we discuss a sufficient condition for $\bigcap_{i\in\mathcal{I}} \mathrm{co}(P_i^*) \neq \emptyset$.

Lemma 4 *Let $P_i \in \mathcal{P}$ for $i \in \mathcal{I}$ and let $\mathrm{co}\,(P_i)$ be its convex hull. If $\bigcap_{i\in\mathcal{I}} \mathrm{co}(P_i) \neq \emptyset$ and P_i is fully compatible with Φ_i for all $i \in \mathcal{I}$, then $\bigcap_{i\in\mathcal{I}} \mathrm{co}(P_i^*) \neq \emptyset$.*

Proof. Since P_i is fully compatible with Φ_i, we have $P_i \subseteq P_i^*$ and thus $\mathrm{co}(P_i) \subseteq \mathrm{co}(P_i^*)$. This implies that $\bigcap_{i\in\mathcal{I}} \mathrm{co}(P_i^*) \supseteq \bigcap_{i\in\mathcal{I}} \mathrm{co}(P_i) \neq \emptyset$. ∎

For example, suppose that Φ_i is derived from the FB-updating on P_i and $P_i = P_j$ for all $i, j \in \mathcal{I}$. That is, there exists a "common prior set" $P \in \mathcal{P}$ fully compatible with Φ_i for all $i \in \mathcal{I}$.[3] Then, trivially, $\bigcap_{i\in\mathcal{I}} \mathrm{co}(P_i) \neq \emptyset$. Thus, by Lemma 4, we have $\bigcap_{i\in\mathcal{I}} \mathrm{co}(P_i^*) \neq \emptyset$, and by Proposition 2, there are no interim agreeable trade arrangements if $\min_{p\in P_i} p(\pi_i) > 0$ and thus $\Phi_i(\pi_i)$ is closed.[4]

If "full compatibility" is replaced by "compatibility" in the lemma, the consequence of the lemma may not be true in general. Remember that if Φ_i is the FB-updating, then P_i is fully compatible with Φ_i, while if Φ_i is the ML-updating, then P_i is compatible with Φ_i, but not necessarily fully compatible with Φ_i. Thus, the ML-updating may result in

[3] As we have pointed out in Lemma 1, the ML-updating and the FB-updating coincide if P_i is rectangular. So the discussion here is readily extended to the ML-updating if the rectangularity is satisfied.

[4] A similar result can be obtained even if we replace the assumption of $\min_{p\in P_i} p(\pi_i) > 0$ with $\max_{p\in P_i} p(\pi_i) > 0$ by defining the FB-updating rule in terms of the closure of that in the original definition: $\Phi_i(\pi_i) := \mathrm{Cl}\,\{p(\cdot|\pi_i) : p(\pi_i) > 0,\ p \in P\}$.

$\bigcap_{i\in\mathcal{I}} \mathrm{co}(P_i^*) = \emptyset$ even if every agent has a common prior set. For such an example, see Example 2 in Section 5.

On the other hand, as far as full compatibility is retained, $\bigcap_{i\in\mathcal{I}} \mathrm{co}(P_i^*) \neq \emptyset$ may hold even if $\bigcap_{i\in\mathcal{I}} P_i = \emptyset$, but $\bigcap_{i\in\mathcal{I}} \mathrm{co}(P_i) \neq \emptyset$. Such an example is found in Example 3 in Section 5.

To sum up, if Φ_i is not singleton-valued, the "common prior set" assumption with full compatibility of the prior set is sufficient for the interim no trade, but not necessary. Even the empty intersection of agents' prior sets may result in non-existence of interim agreeable trade. Thus, it is *not* the case that, for the non-existence of interim agreeable trade, the agents must behave as if they have a common set of multiple priors.

4.2 Uncertainty aversion: the role of zero endowment assumption

We have assumed that the initial position of each agent is neutral, i.e., the endowments of financial assets are zero. When Φ_i is singleton-valued, the zero endowment assumption is not restrictive. If $\Phi_i(\pi_i)$ is a singleton for all $\pi_i \in \Pi_i$, then E_i is a linear operator: for all $f, g \in \mathbb{R}^\Omega$, $E_i f(\omega) - E_i g(\omega) = E_i(f-g)(\omega)$. Thus, we have the following claim as an immediate consequence of Proposition 1:

Corollary 3 *Suppose that $\Phi_i(\pi_i)$ is a singleton for all $\pi_i \in \Pi_i$ and $i \in \mathcal{I}$. Fix any $\{g_i \in \mathbb{R}^\Omega\}_{i\in\mathcal{I}}$ with $\sum_{i\in\mathcal{I}} g_i = 0$. Then the following two conditions are equivalent:* (i) *$\bigcap_{i\in\mathcal{I}} P_i^* \neq \emptyset$;* (ii) *There exists no $\{f_i \in \mathbb{R}^\Omega\}_{i\in\mathcal{I}}$ with $\sum_{i\in\mathcal{I}} f_i = 0$ such that $E_i f_i(\omega) > E_i g_i(\omega)$ for all $\omega \in \Omega$, for $i \in \mathcal{I}$.*

To interpret, think of $\{g_i\}_{i\in\mathcal{I}}$ as an initial arrangement of trade: $g_i(\omega)$ is the amount agent i receives if state ω is realized. Imagine that an alternative trading arrangement $\{f_i\}_{i\in\mathcal{I}}$ is proposed to the agents. Then, the condition for $\{f_i\}_{i\in\mathcal{I}}$ in (ii) means that $\{f_i\}_{i\in\mathcal{I}}$ is deemed desirable to *all* the agents at *any* state $\omega \in \Omega$. Roughly speaking, there cannot be a mutually beneficial trade when the agents know that the environment is zero-sum if and only if there exists a common prior, for any initial arrangement of trade.

However, the same logic does not extend to the case of multiple priors, i.e., the zero endowment assumption is restrictive when Φ_i is not singleton-valued. That is, even if $\bigcap_{i\in\mathcal{I}} \mathrm{co}(P_i^*) \neq \emptyset$, there may exist two trade arrangements $\{f_i\}_{i\in\mathcal{I}}$ and $\{g_i\}_{i\in\mathcal{I}}$ with $\sum_{i\in\mathcal{I}} f_i = 0$ and $\sum_{i\in\mathcal{I}} g_i = 0$ such that $E_i f_i(\omega) > E_i g_i(\omega)$ for all $\omega \in \Omega$, for $i \in \mathcal{I}$.

We shall give an example in Example 4 of Section 5, but let us first discuss why this is the case. Mathematically, this occurs because of concavity of the operator E_i which is the minimum of linear operators given by priors. In terms of economics, because of "uncertainty aversion" of traders, there can be an agreeable trade arrangement $\{f_i\}_{i\in\mathcal{I}}$ over $\{g_i\}_{i\in\mathcal{I}}$ which may reduce "uncertainty" of trade $\{g_i\}_{i\in\mathcal{I}}$ for *all* the traders. Thus even if there is a common understanding on the priors set, the environment starting with $\{g_i\}_{i\in\mathcal{I}}$ is not necessarily zero-sum. Thus in such a case, trading on private information should not be deemed purely speculative to begin with.

As one can expect, a no trade result similar to Corollary 3 is obtained as a straightforward corollary to Proposition 2 if $\{g_i\}_{i\in\mathcal{I}}$ are assumed to be constant; by construction, there is no "uncertainty" in trade $\{g_i\}_{i\in\mathcal{I}}$ if it is constant and thus $E_i f(\omega) - E_i g(\omega) = E_i(f-g)(\omega)$ holds.

Corollary 4 *Fix any $c_i \in \mathbb{R}$ for $i \in \mathcal{I}$ with $\sum_{i\in\mathcal{I}} c_i = 0$. Then the following two conditions are equivalent:* (i) $\bigcap_{i\in\mathcal{I}} \mathrm{co}(P_i^*) \neq \emptyset$; (ii) *There exists no $\{f_i \in \mathbb{R}^\Omega\}_{i\in\mathcal{I}}$ with $\sum_{i\in\mathcal{I}} f_i = 0$ such that $E_i f_i(\omega) > c_i$ for all $\omega \in \Omega$, for $i \in \mathcal{I}$.*

4.3 Efficiency and dynamic consistency

We have considered non-existence of interim agreeable trade. In this subsection, we study its relation with non-existence of ex ante agreeable trade. More precisely, we consider whether ex ante efficiency implies interim efficiency (cf. Milgrom and Stokey, 1982), which is the issue of dynamic consistency.

Let a prior set $P_i \in \mathcal{P}$ be given and Φ_i be a posterior function derived from some updating rule on P_i. To study ax ante efficiency and interim efficiency of trade arrangements, we restrict our attention to trade arrangements with bounded volumes:

$$\mathbf{T} = \{\{f_i \in \mathbb{R}^\Omega\}_{i\in\mathcal{I}} : \sum_{i\in\mathcal{I}} f_i = 0,\ \underline{b}_i \leq f_i \leq \overline{b}_i \text{ for all } i \in \mathcal{I}\},$$

where $\underline{b}_i < 0 < \overline{b}_i$ for every $i \in \mathcal{I}$. As before, we write

$$E_i^0 f = \min_{p\in P_i} p \cdot f$$

for $i \in \mathcal{I}$. We say that:

- $\{f_i\}_{i\in\mathcal{I}} \in \mathbf{T}$ is ex ante efficient in $\mathbf{T}$ if there exists no $\{g_i\}_{i\in\mathcal{I}} \in \mathbf{T}$ such that $E_i^0 g_i > E_i^0 f_i$ for all $i \in \mathcal{I}$.
- $\{f_i\}_{i\in\mathcal{I}} \in \mathbf{T}$ is interim efficient in $\mathbf{T}$ if there exists no $\{g_i\}_{i\in\mathcal{I}} \in \mathbf{T}$ such that $E_i g_i(\omega) > E_i f_i(\omega)$ for all $\omega \in \Omega$ and $i \in \mathcal{I}$.

Since $\mathbf{T}$ is compact, both an ex ante efficient trade arrangement and an interim efficient trade arrangement exist.

As a bench mark, let us first recall the case of a single prior, where the expected value can be written as:

$$E_i^0 f = \sum_{\omega\in\Omega} E_i f(\omega) \times p_i(\omega) \,. \tag{6}$$

where $p_i \in \Delta(\Omega)$ is a prior for $i \in \mathcal{I}$. As is well known, ex ante efficiency implies interim efficiency in this case.

For the case of multiple priors, the relation between ex ante efficiency and interim efficiency is more delicate. As Dow *et al.* (1990) demonstrated, it is known that ex ante efficiency does not necessarily imply interim efficiency. We shall clarify this issue by investigating the implications of each of the three progressively stronger Bayesian consistency requirements, compatibility, full compatibility, and rectangularity, we introduced in Section 2. We begin by characterizing ex ante efficiency of zero endowments. Consider the trivial partition $\Pi_i = \{\Omega\}$, and let Φ_i be the FB-updating on $P_i \in \mathcal{P}$. Then, as an immediate corollary of Proposition 2, we have the following characterization of ex ante efficiency of zero endowments, which can be regarded as a special case of Billot *et al.* (2000):[5]

Corollary 5 *The following conditions are equivalent:* (i) $\bigcap_{i\in\mathcal{I}} \mathrm{co}(P_i) \neq \emptyset$*;* (ii) *there exists no* $\{f_i \in \mathbb{R}^\Omega\}_{i\in\mathcal{I}}$ *with* $\sum_{i\in\mathcal{I}} f_i = 0$ *such that* $E_i^0 f_i > 0$ *for all* $i \in \mathcal{I}$.

For the case of a single prior, this result is reduced to the trivial fact that the agents agree to trade ex ante if and only if they disagree about the likelihood of the states; that is, no trade is ex ante efficient if and only if the players have the same prior.

First, suppose that P_i is compatible but not fully compatible with Φ_i. Then, we can easily construct an example of ex ante efficient trade which is not interim efficient, even

[5] Different from our result, Billot *et al.* (2000) considered strictly risk averse traders and assumed convex sets of priors. For the issue of ex ante agreeable trade, see also Kajii and Ui (2004a).

if every agent has a common prior set. If every agent has a common prior set, then zero trade is ex ante efficient by Corollary 5. But if P_i is not fully compatible, we may have $\bigcap_{i\in\mathcal{I}} \mathrm{co}(P_i^*) = \emptyset$, as we discussed in Section 4.1, and thus zero trade may not be interim efficient. See Example 2 in Section 5.

Second, suppose that P_i is fully compatible with Φ_i. If a constant trade is ex ante efficient, then it is interim efficient. That is, dynamic consistency holds for constant trade arrangements. To see this, look at (i) of Lemma 3, which says that if an interim improvement over constant is possible for all agents, then an ex ante improvement is possible. Note that the role of constant trade is similar to that discussed in Section 4.2, and that the efficiency of constant trade implies $\bigcap_{i\in\mathcal{I}} \mathrm{co}(P_i^*) \neq \emptyset$ by Corollary 4. Thus, ex ante efficiency implies interim efficiency only when $\bigcap_{i\in\mathcal{I}} \mathrm{co}(P_i^*) \neq \emptyset$.

Finally, suppose that P_i is rectangular with Φ_i. Then, as is pointed out by Epstein and Schneider (2003) and Wakai (2002), the ex ante efficiency does imply the interim efficiency for any trade arrangements. That is, dynamic consistency holds for any trade arrangements. We state it formally for completeness:

Lemma 5 *Suppose that P_i is rectangular for all $i \in \mathcal{I}$. If $\{f_i\}_{i\in\mathcal{I}} \in \mathbf{T}$ is ex ante efficient in $\mathbf{T}$, then it is interim efficient in $\mathbf{T}$.*

Proof. Fix any $\{f_i\}_{i\in\mathcal{I}} \in \mathbf{T}$. For each $i \in \mathcal{I}$, let

$$q_i^* \in \arg\min_{q\in P_i} \sum_{\pi_i\in\Pi_i} \left(\min_{p\in\Phi_i(\pi_i)} p\cdot f_i \right) q(\pi_i)\,,\ \ r_i(\cdot|\pi_i) \in \arg\min_{p\in\Phi_i(\pi_i)} p\cdot f_i$$

for $\pi_i \in \Pi_i$. By the definition of rectangularity, we must have

$$p_i' := \sum_{\pi_i\in\Pi_i} q_i^*(\pi) r_i(\cdot|\pi_i) \in P_i.$$

This implies that $p_i' \in \arg\min_{p\in P_i} p\cdot f_i$, and hence $E_i^0 f_i = \sum_{\omega\in\Omega} E_i f_i(\omega) \times q_i^*(\omega)$. So if $\{f_i\}_{i\in\mathcal{I}} \in \mathbf{T}$ is ex ante efficient, then it is interim efficient. ∎

Based upon the above observation, Proposition 2 can be re-interpreted as follows. Recall that P_i^* is the maximal rectangular prior set. If P_i^* is taken as though it is the set of priors for agent i, the condition $\bigcap_{i\in\mathcal{I}} \mathrm{co}(P_i^*) \neq \emptyset$ holds if and only if zero endowment is *ex ante* efficient by Corollary 5. By Lemma 5, if zero endowment is ex ante efficient, then it is interim efficient because P_i^* is fully compatible with Φ_i.

5 Examples

In this section, we provide examples which clarify the meaning of concepts and results in the paper.

Example 1

This example is intended to illustrate the differences among the consistency requirements for prior sets, compatibility, full compatibility, and rectangularity as well as the maximal rectangular prior set in Section 2. Let $\Omega = \{1,2,3\}$, $\Pi_i = \{\{1,2\},\{3\}\}$, $\Phi_i(\{1,2\}) = \{(\frac{1}{2},\frac{1}{2},0),(0,1,0)\}$, $\Phi_i(\{3\}) = \{(0,0,1)\}$.

- $P_i = \{(\frac{1}{2},\frac{1}{2},0),(0,\frac{1}{2},\frac{1}{2}),(1,0,0)\}$ is compatible but is not fully compatible with Φ_i since the posterior of $(1,0,0)$ given $\{1,2\}$ is not included in $\Phi_i(\{1,2\})$.
- A smaller set $P_i' = \{(\frac{1}{2},\frac{1}{2},0),(0,\frac{1}{2},\frac{1}{2})\}$ is fully compatible. The set P_i', however, is not rectangular, since $\frac{1}{2}(\frac{1}{2},\frac{1}{2},0)+\frac{1}{2}(0,0,1)$ is not in P_i' although $p = (0,\frac{1}{2},\frac{1}{2}) \in P_i'$ gives $p(\{1,2\}) = p(\{3\}) = \frac{1}{2}$ and $(\frac{1}{2},\frac{1}{2},0) \in \Phi_i(\{1,2\})$, $(0,0,1) \in \Phi_i(\{3\})$.
- On the other hand, $P_i'' = \{(\frac{1}{4},\frac{1}{4},\frac{1}{2}),(0,\frac{1}{2},\frac{1}{2})\}$ is fully compatible and rectangular.
- The maximal rectangular prior set is

$$P_i^* = \mathrm{co}\left(\left\{(\frac{1}{2},\frac{1}{2},0),(0,0,1)\right\}\right) \cup \mathrm{co}\left(\{(0,1,0),(0,0,1)\}\right).$$

- Let $f \in \mathbb{R}^\Omega$ be such that $f = (-1,2,1)$. Then, $E_i f(1) = E_i f(2) = \frac{1}{2}$ and $E_i f(3) = 1$. For P_i, $E_i^0 f = -1$, but for P_i', $E_i^0 f = \frac{1}{2}$, confirming (i) of Lemma 3.
- Let $f \in \mathbb{R}^\Omega$ be such that $f = (1,2,-1)$. For P_i'', $E_i^0 f = \frac{1}{4} > 0$, but $E_i f(3) = -1$. On the other hand, for P_i^*, $E_i^0 f = -1 < 0$, confirming (ii) of Lemma 3.

Example 2

As we discussed in Section 4.1, if there exists a common prior set and posterior functions are derived from the FB-updating, then the condition of Proposition 2 is satisfied. This may not be the case if posterior functions are derived from the ML-updating. The following example illustrates this observation.

Let $\Omega = \{1,2,3,4\}$, $\mathcal{I} = \{1,2\}$, $\Pi_1 = \{\{1,2\},\{3,4\}\}$, $\Pi_2 = \{\{1,3\},\{2,4\}\}$. Consider

$$P_1 = P_2 = \left\{ p^t \in \Delta(\Omega) : p^t = \left(\frac{1+t}{5}, \frac{2t}{5}, \frac{2-2t}{5}, \frac{2-t}{5} \right),\ 0 \leq t \leq 1 \right\}.$$

Let Φ_i be the ML-updating on P_i for $i = 1,2$. Thus, P_i is compatible with Φ_i for $i = 1,2$. Then, we can show that $\mathrm{co}(P_1^*) \cap \mathrm{co}(P_2^*) = \emptyset$. To see this, calculate $\{1\} = \arg\max_{t\in[0,1]} p^t(\{1,2\})$, $\{0\} = \arg\max_{t\in[0,1]} p^t(\{3,4\})$, $\{0\} = \arg\max_{t\in[0,1]} p^t(\{1,3\})$, $\{1\} = \arg\max_{t\in[0,1]} p^t(\{2,4\})$. Let $E = \{1,4\}$. Then,

$$\min_{p\in\Phi_1(\Pi_1(\omega))} p(E) = \max_{p\in\Phi_1(\Pi_1(\omega))} p(E) = \frac{1}{2},$$
$$\min_{p\in\Phi_2(\Pi_2(\omega))} p(E) = \max_{p\in\Phi_2(\Pi_2(\omega))} p(E) = \frac{1}{3}$$

for all $\omega \in \Omega$. Thus, by Corollary 2, we must have $\mathrm{co}(P_1^*) \cap \mathrm{co}(P_2^*) = \emptyset$.

Example 3

Let $\Omega = \{1,2,3\}$, $\mathcal{I} = \{1,2\}$, and $\Pi_1 = \Pi_2 = \{\Omega\}$. Let $P_1 = \{(1,0,0),(0,\frac{1}{2},\frac{1}{2})\}$ and $P_2 = \{(0,1,0),(\frac{1}{2},0,\frac{1}{2})\}$. Let Φ_i be the FB-updating on P_i for $i = 1,2$. Clearly, $\bigcap_{i\in\mathcal{I}} P_i = \emptyset$, but $(\frac{1}{3},\frac{1}{3},\frac{1}{3}) \in \bigcap_{i\in\mathcal{I}} \mathrm{co}(P_i) \neq \emptyset$. Thus, by Lemma 4, we must have $\bigcap_{i\in\mathcal{I}} \mathrm{co}(P_i^*) \neq \emptyset$.

Example 4

The following example, discussed by Kajii and Ui (2004b), illustrates that $\bigcap_{i\in\mathcal{I}} \mathrm{co}(P_i^*) \neq \emptyset$ but there are trade arrangements $\{f_i\}_{\in\mathcal{I}}$ and $\{g_i\}_{\in\mathcal{I}}$ such that $E_i f_i(\omega) > E_i g_i(\omega)$ for all $\omega \in \Omega$, $i \in \mathcal{I}$.

Let $\mathcal{I} = \{1,2\}$ and consider a state space $\Omega = \{1,2,3a,3b,4a,4b\}$ where the players have assigned probability 0.2 to the events $\{1\}$ and $\{2\}$ and probability 0.3 to the events $\{3a,3b\}$ and $\{4a,4b\}$, respectively. The difference between state $3a$ and state $3b$ and that between $4a$ and $4b$ are ambiguous in the sense that the players do not know how the probabilities assigned to $\{3a,3b\}$ and $\{4a,4b\}$ should be allocated to these states. Thus the players have a common set of priors, which is:

$$P_1 = P_2 = \{p \in \Delta(\Omega) : p(\{1\}) = p(\{2\}) = 0.2,\ p(\{3a,3b\}) = p(\{4a,4b\}) = 0.3\}.$$

The private information of players are given by the following partition.

$$\Pi_1 = \{\{1,3a,3b\},\{2,4a,4b\}\},\ \Pi_2 = \{\{1,3a,4a\},\{2,3b,4b\}\}.$$

Let Φ_i be the FB-updating on P_i for $i = 1, 2$. We have:

$$
\begin{aligned}
\Phi_1(\{1, 3a, 3b\}) =& \{p \in \Delta(\Omega) : p(\{1\}) = 0.4,\ p(\{3a, 3b\}) = 0.6\},\\
\Phi_1(\{2, 4a, 4b\}) =& \{p \in \Delta(\Omega) : p(\{2\}) = 0.4,\ p(\{4a, 4b\}) = 0.6\},\\
\Phi_2(\{1, 3a, 4a\}) =& \Big\{p \in \Delta(\Omega) : p(\{1\}) = \frac{0.2}{0.2 + x + y},\ p(\{3a\}) = \frac{x}{0.2 + x + y},\\
& p(\{4a\}) = \frac{y}{0.2 + x + y} \text{ where } x \in [0, 0.3],\ y \in [0, 0.3]\Big\},\\
\Phi_2(\{2, 3b, 4b\}) =& \Big\{p \in \Delta(\Omega) : p(\{2\}) = \frac{0.2}{0.2 + x + y},\ p(\{3b\}) = \frac{x}{0.2 + x + y},\\
& p(\{4b\}) = \frac{y}{0.2 + x + y} \text{ where } x \in [0, 0.3],\ y \in [0, 0.3]\Big\}.
\end{aligned}
$$

Let $E = \{1, 2\}$. Note that the updated probabilities of E are:

$$\{p(E) \,|\, p \in \Phi_1(\Pi_1(\omega))\} = \{0.4\},\ \{p(E) \,|\, p \in \Phi_2(\Pi_2(\omega))\} = [0.25, 1]$$

for all $\omega \in \Omega$. Consider trade arrangements $\{f_i\}_{i \in \mathcal{I}}$ and $\{g_i\}_{i \in \mathcal{I}}$ such that $f_1(\omega) = -f_2(\omega) = -1$ if $\omega \in E$ and $f_1(\omega) = -f_2(\omega) = 1$ otherwise, and $g_i(\omega) = -f_i(\omega)$ for all $\omega \in \Omega$ and $i = 1, 2$. Then, $E_1 f_1(\omega) = \min_{p \in \{0.4\}} p \cdot (-1) + (1 - p) \cdot 1 = 0.2$, $E_1 g_1(\omega) = \min_{p \in \{0.4\}} p \cdot 1 + (1 - p) \cdot (-1) = -0.2$, $E_2 f_2(\omega) = \min_{p \in [0.25, 1]} p \cdot 1 + (1 - p) \cdot (-1) = -0.5$, and $E_2 g_2(\omega) = \min_{p \in [0.25, 1]} p \cdot (-1) + (1 - p) \cdot 1 = -1$. Thus, $E_1 f_1(\omega) > E_1 g_1(\omega)$ and $E_2 f_2(\omega) > E_2 g_2(\omega)$ for all $\omega \in \Omega$ even if $P_1 = P_2 \subseteq \mathrm{co}(P_1^*) \cap \mathrm{co}(P_2^*) \neq \emptyset$.

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