Skills, Agglomeration and Segmentation

Tomoya Mori*and Alessandro Turrini†

June 2001

Abstract

We investigate the role of skill heterogeneity in explaining location patterns induced by pecuniary externalities (Krugman [30]). In our setting, sellers with higher skills better perform in the marketplace, and their sales are larger. Selling to distant locations leads to lower sales because of both (pecuniary) transport costs and communication costs that reduce the perceived quality of goods. A symmetry-breaking result is obtained: symmetric configurations cannot be stable, and regional inequality is inevitable. The relatively more skilled choose to stay in the location with higher aggregate income and skill, while the relatively less skilled in the other. The model allows us to analyze the links between the extent of interregional inequality and the extent of interpersonal skill inequality.

Keywords: Skill heterogeneity; Agglomeration; Core-periphery model; Regional inequality; Interpersonal inequality; Transport and communication costs.

JEL Classification: R12, R13, F12, F16.

Acknowledgments: We thank Richard Arnott, Duncan Black, Pierre-Philippe Combes, Gilles Duranton, Masa Fujita, Tatsuo Hatta, Gianmarco Ottaviano, Diego Puga, Akihisa Shibata, Tony Smith, Takaaki Takahashi and participants of the seminars/conferences for their constructive comments and encouragement. A part of the research was conducted while we were visiting CORE, Université catholique de Louvain, Belgium, where we benefited from the stimulating discussions with Jacques Thisse. This research has been supported by the Fonds National de la Recherche Scientifique of Belgium, the Nomura Science Foundation, the Grant in Aid for Research (Nos. 09CE2002 and 09730009) of the Ministry of Education, Science and Culture in Japan.

*Institute of Economic Research, Kyoto University, Yoshida-honmachi, Sakyo, Kyoto, 606-8501 Japan. Phone: +81-75-753-7121; Fax: +81-75-753-7198; E-mail: mori@kier.kyoto-u.ac.jp.

†Department of Economics, University of Bergamo, Piazza Rosate 2, 24100 Bergamo, Italy. Phone: +39-35-277506; Fax: +39-35-249975; E-mail: alessandro.turrini@uni-bocconi.it.

1 Introduction

Economic activities are not evenly distributed in space. Some places are crowded with firms and workers, while others, poor of people and human capital, lag behind. The economic landscape is full of humps and bumps, that result from an unequal distribution of labor across space, both in terms of its quantity and quality. We generally observe the most educated and talented workers clustering together in wealthy and economically important areas, while low skilled workers are more often located in low income areas. Which factors account for the observed agglomeration of workers, and for their spatial segmentation across skill and income levels? This paper studies the role of skill heterogeneity in "new economic geography" models of location.

The spatial distribution of human capital is far from being uniform. Workers tend to cluster together, the higher their level of human capital and skill. Figure 1 helps to illustrate the point. There, we report the Lorenz curves of the inter-city distribution of workers in Japan, year 1990.[1] Lorenz curves are depicted separately for the total population and for the workers belonging to particular education categories.[2] The figure clearly indicates that workers with higher education levels are more geographically concentrated.[3] The data also show that workers with higher education are more easily found in big cities: the rank correlation between city size and the share of college graduates in a city is 0.73, while that between city size and the share of workers with at most a junior high school degree is -0.67. Tokyo alone accounts for more than 40% of college graduates among all metropolitan areas. These patterns are not typical of Japan: similar evidence is obtained for the case of the US.[4]

Figure 1

In this paper we present a model for the location decisions of workers endowed with different skill levels that produces results consistent with available evidence.

The lumpy interregional distribution of human capital results from the location decisions of workers, which may differ according to their skill levels (based on their education, experience and innate ability). The existing distribution of skill affects in turn the outcome of the migration choices of workers.[5] Two

[1]Data source: Japan Statistics Bureau [29, Vol.3, Part 2]. A city here is a Standard Metropolitan Employment Area (SMEA) which is an aggregation of counties based on the commuting pattern. For the precise definition of SMEA, see Yamada and Tokuoka [42].

[2]The horizontal axis ranks cities in terms of their shares of workers with a given education level (the rank is normalized by the total number of cities, 124). The vertical axis reports the cumulated share of workers of a given education level in the cities up to a given position in the rank.

[3]The value of the Gini index for the distribution of the total population is 0.69, while the corresponding value for the cases of workers with education up to junior high school, high school, community/technical college, and college, is 0.60, 0.69, 0.77, and 0.82 respectively.

[4]In 1990, the Gini index value for the inter-city distribution of workers is 0.61, while it is 0.66 for the case of college graduates (aged above 25). The rank correlation between city size and college graduates share in the city workforce is 0.40. These figures are calculated from Black [5]'s data set which is based on the Population Census of the US.

[5]Borjas, Bronars, and Trejo [8] and Rauch [37] give empirical support to the view that these patterns may be due to the

approaches to study location decisions and the interregional distribution of workers have received the greatest favor. The first puts at the centre of agglomeration forces the existence of knowledge spillovers, the second deals with the existence of pecuniary externalities arising from the interaction of increasing returns to scale with transport costs. The analysis of economic agglomeration as an outcome of knowledge spillovers goes back to Marshall [33].[6] Recent theoretical explanations for interregional sorting of workers according to their skill levels are found in Black [5] and Black and Henderson [7].[7]

The interaction between scale economies and transport costs has a long-standing tradition in spatial economics, but the study of agglomerations on the basis of pecuniary externalities became part of mainstream analysis only after the "core-periphery" model by Krugman [30].[8] This model offers a comprehensive and analytically convenient representation of how the interaction among scale economies, transport costs and an immobile source of demand generates centripetal and centrifugal forces. However, the existing versions of the core-periphery model do not account for the location patterns of economic agents of different skill, leaving unexplained the observed tendency towards spatial segmentation according to skill levels.

As for empirical evidence, though there is a consensus about the pervasiveness of positive externalities that generate agglomeration forces in reality, it is quite difficult to disentangle empirically between the different sources of externalities that explain the observed agglomeration patterns.[9]

In this paper, we analyze the spatial distribution of human capital on the ground of pecuniary externalities without resorting to knowledge spillovers, offering an alternative explanation for the interregional sorting of workers according to skill levels.[10] We reinterpret the familiar core-periphery model, considering mobile worker-sellers whose earnings consist of rents associated with their skills. The economy we have in mind is made up of many, heterogenous sellers who offer goods that are differentiated both horizontally (variety) and vertically (quality), where quality requires skills. Selling to distant markets entails a "physical" transport cost: a fraction of the shipped good melts away. This is a basic feature of the core-periphery model, and a common one for almost all new economic geography models. In our model, however, selling to faraway consumers leads to an additional communication cost, that shows up

workers' tendency to self-select according to their skill levels across locations. The difference between one's own skill level and the average skill level of one's initial home town appears to be indeed an important explanatory factor for individual migration decisions.

[6] See also Jacobs [28] and Lucas [31].

[7] See also Bénabou [3][4] for models of intra-urban spatial segregation based on human capital externalities.

[8] Surveys can be found in Fujita [20] and Fujita, Krugman and Venables [21].

[9] Davis and Weinstein[13] find evidence of a significant "home market effect," namely, a more than proportional relation between local production and local demand implied by the existence of scale economies and factor mobility. Rauch [37] presents results in favor of the existence of knowledge spillovers, showing that the US workers are more productive in cities where average levels of educational attainment are higher. Results similar to Rauch [37] are found in Glaeser, Sheinkman, and Shleifer [26], Ciccone and Hall [12], and Dobkins and Ioannides [16], while Ciccone[9], Ciccone and Peri[10] and Ciccone, Peri and Almond [11] reject the hypothesis of significant human capital externalities. Finally, Dumais, Ellison and Glaeser [17] find empirical confirmation of both pecuniary externalities and knowledge spillovers, but claim that labor market pooling have the major role in explaining agglomeration patterns.

[10] See also Fernandez[18] for a review of models that explain within-cities spatial sorting according to skill levels on the ground of heterogeneity in preferences for local public goods.

as a reduction in goods' quality. One can think of many instances. Buyers may know the quality and characteristics of goods more precisely through direct contact rather than through mailing or advertising. The use of complex goods may require strict interaction between buyers and sellers, and interaction is hampered by distance. In general, we observe empirically a strong "home bias" in consumption: transport and trade costs are not enough to explain the preference of consumers for homemade goods (see, e.g., Helliwell [27]).[11]

Why does the presence of communication costs matter in our version of the core-periphery model? The reason is that they lead to a different impact of distance on the behavior of workers depending on their skill level. Because sellers of higher skill perform better in the marketplace and obtain larger demand for their goods, they will normally be hit relatively less by distance from the market. Real-life instances seem supportive of this view. In general, we see that only the most successful professionals and entrepreneurs start selling their goods and services in faraway markets. When only transport costs of the iceberg type are present, however, selling to a distant location corresponds to a constant share of sales (and profits) lost in transit. This means that more skilled and less skilled sellers would be affected in the same proportion by distance.[12] Adding communication costs to the core-periphery structure turns out to be a very convenient way to avoid costs of distance that are perfectly proportional to the value of the shipped goods, as it is the case when only transport costs of the iceberg type are present.[13] This way, the more skilled are somehow "closer to the global market," and, as in Rosen's [38] "Superstar economy," that an increase in the economy-wide demand raises disproportionately the rents associated with a higher level of skills. In this context, since the earnings of agents with higher skills are less dependent upon local markets, they are *relatively* more footloose than the less skilled. This is the feature that explains self-sorting according to skill levels in our model. The more skilled are found in locations where local competition is tougher, but where the goods are of higher quality and a wider range of varieties is available, while the less skilled are in locations where competition is low, but where goods are of lower quality and less variety is offered. Transport costs and regional characteristics (local market size and cost of living) have a different impact on the location decisions of workers depending on their skill level. The spatial distribution of population and skill, in turn, affects the market size and the cost of living in different locations.

One of the major results from the equilibrium analysis is that, in contrast with the predictions of the core-periphery model and all of its existing variants, the symmetric equilibrium where all locations are

[11]This observed home bias in consumption justifies the rather "ad-hoc" Armington [1] assumption in applied trade analysis.

[12]Of course, such a feature of iceberg transport costs is immaterial when all agents are homogenous as in the original version of the core-periphery model.

[13]An alternative approach would be to assume pecuniary transport costs that are, for instance, linear in the quantity of the good shipped. However, adopting such an approach in our analysis while keeping tractability of the model would necessarily require sacrificing other parts of the model structure (see, e.g., Ottaviano [36, Ch.3]).

identical cannot be a stable one. Starting from a symmetric configuration, perturbations to the agents' distribution always lead to a self-reinforcing sorting of mobile workers by their skill level. The reason is that regional variables in this setting are affected not only by the distribution of agents across locations (as in the standard core-periphery framework), but also by the spatial distribution of skill. In our model, however, the concentrated configuration (all mobile agents in one location) is not the only candidate stable equilibrium. In fact, spatial segmentation according to skill levels is the typical stable outcome: the more skilled choose to stay in the location where aggregate skill and income is higher, while the less skilled stay in the other. Our model can also serve as a tool to analyze the impact of interpersonal inequality on the spatial distribution of population and skill. We show that an increase in interpersonal inequality has different effects on the extent of spatial agglomeration depending on the level of transport and communication costs and on the mass of human capital in the economy.

The basic message of our analysis is that agents' heterogeneity in the presence of pecuniary externalities may work as a source of regional inequality. The possibility of international inequality has been already explained on the pure ground of pecuniary externalities.[14] Our contribution, we believe, is to show that *amending the standard core-periphery model with heterogenous agents is sufficient to predict the inevitability of regional inequality.* Workers' segmentation across skill levels can thus be thought of as a pervasive tendency that does not necessarily require the presence of human capital externalities. Market interactions among unequal agents can be sufficient to generate spatial sorting according to skill levels. This naturally leads to consider distributional issues as a key to understanding agglomeration patterns, and may help in explaining why regional inequalities appear to be associated with interpersonal income inequality in the real world.[15]

The remainder of the paper is organized as follows. The next section presents the structure of the model. Section 3 introduces the notion of temporary equilibrium and adjustment process. Section 4 defines the equilibrium. Section 5 classifies possible equilibrium configurations, and is devoted to the analysis of their existence and stability. Section 6 studies the global dynamics of the economy. Section 7 discusses some implications of our model. The concluding comments end the paper.

[14] Matsuyama [34] proposes an argument based on increasing returns arising from the trade in inputs and the international division of labor.

[15] It is interesting to compare the US and Japan in this respect, the former being one of the most unequal among the advanced countries (the richest 5% of the population accounts for more than 56% of the national wealth in 1983), and the latter a relatively equal country (the richest 5% of the population accounts for 25% in 1984, see Wolff [41] for these figures.). In the US, the ratio of the real wage in the richest city and that in the poorest city is above 7.0 (Dobkins and Ioannides [15]), while in Japan, this ratio is almost five times smaller (Nakamura and Tabuchi [35]).

2 The model

2.1 The economy

The economy consists of two regions, denoted by a and b, which are symmetric except for the mass of skilled workers that are located in each. Variables in the model bear a subscript $r = a, b$ to indicate the region. Firms produce differentiated goods under imperfect competition and free-entry. Consumers (skilled and unskilled workers) like variety, according to the Dixit-Stiglitz [14] formulation. There are two types of primary production factors: skilled and unskilled labor, where each worker embodies a unit of corresponding labor. The production requires two inputs: skill ("talent") provided by the former and an intermediate input, produced out of the latter. Skilled workers are perfectly mobile between the regions, while the unskilled are assumed to be immobile. The markets for the production factors and the intermediate good are competitive.

Unskilled workers are homogeneous. A unit mass of them is located in each of the two regions. Skilled workers are heterogeneous, in that they are distinguished by different skill levels. There is a unit mass of skilled workers in the whole economy. Each skilled worker is characterized by her skill level s; a more talented worker is associated with a larger value of s. The lowest and the highest skill levels among all skilled workers are denoted, respectively, by $\underline{s}$ and $\overline{s}$, where $0 < \underline{s} < \overline{s} < \infty$. Skilled workers are distributed over the interval, $S \equiv [\underline{s}, \overline{s}]$, according to the density function $f(s)$. The aggregate (and average) skill in the economy is denoted by $\hat{s}$, which equals $\int_{s \in S} sf(s)ds$. No additional assumptions are made on the distribution of skills. To express the regional distribution of workers of each skill level, let $f_r(s)$ $(r = a, b)$ represent the density of workers with skill level s in region r; denote by n_r *the share of skilled population*, and by s_r *the share of aggregate skill* in region r. Then we can write:

$$n_r = \int_{s \in S} f_r(s)ds, \quad s_r = \frac{1}{\hat{s}} \int_{s \in S} sf_r(s)ds. \tag{1}$$

where

$$s_a + s_b = 1, \quad n_a + n_b = 1, \tag{2}$$

$$f_a(s), f_b(s) \geq 0 \quad \text{and} \quad f_a(s) + f_b(s) = f(s) \quad \text{for each } s. \tag{3}$$

The intermediate input is produced through a linear technology using unskilled labor as sole input. Final production requires the services of skilled labor and a given amount of the intermediate input proportional to output. While the intermediate inputs provide standardized production services, skilled workers' talent adds "value," or "quality," to each unit of the final good. Goods of higher quality are more appreciated by consumers. Consequently, we consider products that are differentiated along the horizontal dimension (variety) and the vertical one (quality).[16]

The intermediate inputs are costlessly mobile across regions. Shipping final goods instead incurs iceberg transport cost: a fraction of the good is lost in transit. We further assume that consuming final

[16] The same formulation is found in Manasse and Turrini [32].

goods outside the region in which they are produced incurs an additional communication cost. Namely, the perceived quality of products is lower if the goods are shipped to the other region.

2.2 Distribution of skills and feasible spatial configurations

In our economy, the interpersonal distribution of skill and the spatial distribution of skilled workers are necessarily related. A graphical representation of the economy as in Figure 2 helps the intuition.[17] In the graph, the horizontal distance from O [resp., O'] measures the (skilled) population share in region a [resp., b]. The vertical distance from O [resp., O'] measures the share of aggregate skill in region a [resp., b]. The shaded area in each diagram, denoted by M, represents the feasible domain of n_a and s_a. At each point on the upper [resp., lower] boundary of M the spatial configuration is such that all workers in region a are at least [resp., at most] as skilled as those in region b. The construction of these boundaries is as follows. Take any aggregate skill share s_a in region a. At the corresponding point on the upper boundary, region a has the smallest population share consistent with s_a. In other words, at each point on the upper boundary, there exists a *threshold skill level* z, such that all workers with skill level higher than z are in region a; those with skill level lower than z are in region b; and those with skill level z may locate in either of the two regions. The lower is s_a, the higher the threshold skill level z: only a few of the most skilled workers are needed in region a to attain s_a. The lower boundary is obtained in a symmetrical way. In this case, for a given value of s_a, we must find the largest population share of region a consistent with s_a.

Figure 2

The boundaries of area M have a straightforward interpretation: each of them is *the Lorenz curve of the interpersonal distribution of skill in the economy.* The size of area M corresponds to the value of the Gini index for the skill distribution. The larger is M, the greater the extent of inequality in the interpersonal distribution of skills. The standard core-periphery model is a particular case where M has size zero, and the feasible allocations are restricted on the 45° line which passes through the symmetric configuration at E ($n_a = s_a = 1/2$). Since n_a and s_a must necessarily be in area M, it follows also that the feasible regional allocations of population and skill are necessarily shaped by the extent of interpersonal inequality in the economy.

2.3 Technology and preferences

The consumption good can be differentiated along a continuum of varieties $i \in R$. Each variety i is produced out of intermediate inputs and skill. The size of firms is normalized in such a way that one firm

[17] For the ease of exposition, in Figure 2, a continuous distribution of the population over the skill range S is assumed.

employs one skilled worker only.[18] We further assume that each skilled worker can employ her skill in the production of at most one variety of the consumption good. The intermediate input requirements for each variety are proportional to output. Let $w_r(s)$ and υ_r denote, respectively, the return to the skill of a worker endowed with "talent" s and the marginal cost (consisting of expenses for intermediate inputs) of final production in region r. Then, the cost, $C_r(Q,s)$, of producing Q units of any variety in region r by employing a worker with skill level s is given by

$$C_r(Q,s) = w_r(s) + \upsilon_r Q. \tag{4}$$

Firms are atomistic profit-maximizers. They produce goods which are imperfect substitutes and set their price taking as given other firms' choices (the "large group" Chamberlinian hypothesis holds). Consumer utility increases with the extent of variety in consumption. As it is standard in monopolistic competition models, love for variety plus increasing returns in production insure that no firm is willing to supply the same variant offered by a rival. Since each firm requires the skill of one worker, we necessarily have the total variety size equal to 1. In turn, the condition of free-entry ensures that *skilled workers receive all the operating profits realized by firms.* In analogy with Rosen [38], skilled workers' income thus consists of skill rents associated with firms' operating profits.

As for the production of intermediate inputs, we simply assume that one unit of unskilled labor produces one unit of the intermediate input. It follows from the assumption of perfect competition in the market of intermediates that wages for unskilled workers in region r equal υ_r.

We now turn to the problem of consumers. Recall that in our formulation more talented workers produce a good of better quality, and better quality is appreciated by consumers. For simplicity, and without affecting our qualitative results, we assume that the skill level of the worker employed for the production of the good exactly conveys the quality of the good. So, hereafter we use the terms, skill level and quality, interchangeably. Consumers derive utility from a combination of the quantity of each variety i, $x(i)$, and the perceived quality of the good of variety i. Because of love for variety, individuals will smooth their consumption across all available varieties, both domestic and imported. However, in the case of shipped varieties, the perceived quality level is lower. While for the case of domestic goods the perceived quality is equal to the skill level $s(i)$ associated with the good, when the good is imported the perceived quality equals $s(i) - c$, where $c > 0$ represents the communication cost. So, the geographic provenience of goods has a direct consequence on consumer welfare.[19] As will be clear in the following analysis, the presence of communication costs plays a crucial role. The reason is that *it introduces a non-convexity in the technology for selling goods to distant locations.* This non-convexity shows up across

[18] An alternative interpretation is that each worker is running a firm. In the remainder of the paper we refer to our mobile agents as workers or worker-sellers.

[19] Notice the analogy of this specification of communication costs with that used in many models aimed at explaining location patterns on the ground of agents' need for direct contact and face-to-face interaction. See Fujita and Smith [22] for a comprehensive survey.

sellers, with more skilled agents selling larger amounts of their goods, being thus proportionally less affected by distance compared with the less skilled. This, in turn, explains a different location behavior for workers endowed with different skill levels.

A Cobb-Douglas specification is chosen to nest the quantity $x(i)$ and the quality $s(i)$ in consumers' utility function, while a standard CES specification is used to nest different varieties.[20] Denoting the mass of varieties produced within and outside region r by N_r and N_{-r}, respectively, the utility level of a consumer located in region r is given by:

$$u_r = \left[\int_{i \in N_r} s(i)^{1-\rho} x(i)^{\rho} di + \int_{i \in N_{-r}} (s(i) - c)^{1-\rho} x(i)^{\rho} di \right]^{1/\rho}. \tag{5}$$

As usual, $\rho \in (0,1)$, and $\sigma = 1/(1-\rho) > 1$ is the elasticity of substitution across different varieties. Since firms are atomistic, σ is also the price elasticity of the demand for each variety.

Unlike the models based on the transport cost of only the iceberg type, in our model, the goods are not traded if the communication costs are too high ($c > s$). Since the analysis in this case is unnecessarily involved, we will focus on the case in which all varieties are traded at equilibrium. Notice that our model has qualitatively the same structure as that of the original core-periphery model, if all mobile agents are homogeneous, and communication costs are zero. For our purpose, it suffices to slightly perturb the structure of the core-periphery model while preserving its tractability. Namely, we maintain the following assumption:[21]

Assumption 1 *$0 < c < \underline{s}$.*

Indeed, we show that when interpersonal skill inequality is taken into account in the context of the core-periphery model, the mere presence of communication costs is sufficient to explain the spatial sorting of population by skill levels.

3 Temporary equilibrium

In this section, we derive the equilibrium conditions under a given regional distribution of workers. These are obviously the necessary conditions for a long-run equilibrium in which workers have no incentive to relocate even if they can. In equilibrium, all workers must be choosing to work at the best available conditions. To derive such conditions, we first define a *temporary equilibrium:* a state of the economy in which the opportunities of workers are restricted within the region where they are "temporarily" located, i.e., *a temporary equilibrium is realized when, given the regional distribution of workers, (i) consumers maximize utility; (ii) firms maximize profits; (iii) profits are zero; (iv) all workers of the same skill attain*

[20]This specification is chosen for analytical convenience only. Adopting a different formulation which still implies a positive relation between quality and associated sales would lead to the same results as ours.

[21]A detailed analysis under $c \geq \underline{s}$ is available from the authors upon request.

the same utility level in their region; (v) all markets clear. The economy is assumed to attain a temporary equilibrium instantaneously once the regional distribution of workers is given.

Note first that since intermediates are costlessly transportable, it must be that $\upsilon_a = \upsilon_b = \upsilon$. By choice of numeraire, we fix $\upsilon = 1$. Recall also that the final good, instead, is subject to transport costs of the iceberg type: for one unit to be shipped to the other region, $1 - \tau$ units are lost in transit, and a fraction τ arrives to distant consumers, where $\tau \in (0, 1)$.

Keeping this in mind, consider the profit maximization problem of a firm in region r. Since all firms share the same production technology and the price elasticity of demand is constant and equal to σ, all firms set a common mill price given by[22]

$$p = \sigma/(\sigma - 1). \tag{6}$$

Thus, we omit henceforth the variety index. Moreover, we see that if product quality matters for firms' performance, it matters only in terms of quantities sold.

Expressions for total demand for a good with quality s in region r differ depending on whether the good is produced locally or in the alternative region. Denoting, respectively, by $X_r(s)$ and $X_r^*(s)$ the demand in region r for a local good and that for a product provided in the alternative region $-r$, from the consumer utility maximization we obtain

$$X_r(s) = sp^{-\sigma} I_r P_r^{\sigma-1}, \quad X_r^*(s) = (s - c)\tau^{\sigma-1} p^{-\sigma} I_r P_r^{\sigma-1}, \tag{7}$$

where I_r is the aggregate income in region r, given by[23]

$$I_r = 1 + \int_{s \in S} f_r(s) w_r(s) ds, \tag{8}$$

and P_r is the price index in region r, given by

$$P_r \equiv p\hat{s}^{\frac{1}{1-\sigma}} \left[s_r + (s_{-r} - cn_{-r}/\hat{s})\tau^{\sigma-1} \right]^{\frac{1}{1-\sigma}}. \tag{9}$$

Given the fact that each unit of differentiated good requires one unit of intermediate good, the total amount of differentiated goods produced in the economy at equilibrium equals the mass of unskilled labor, that is, two units. However, the market share of each seller depends on her skill level. The more skilled sellers are able to sell more as (7) indicates. Note also that transport and communication costs affect firms' sales to distant regions differently. We observe in (7) that transport costs affect the sales of worker-sellers of different skills in the same proportion, while communication costs fall proportionally more on the less skilled.

[22] As it is always the case with monopolistic competition, iceberg transport costs, and a CES-Dixit-Stiglitz representation of preferences, firms set the same markup over marginal costs in all locations (see, e.g., Fujita, Krugman, and Venables [21, Ch.4]). It is important to bear in mind that when we speak about "tougher competition" in such a framework we refer to a downward shift in the individual demand curve at given price-cost margins.

[23] The first [resp., second] term in the RHS of (8) is the aggregate income of immobile [resp., mobile] workers in region r.

The equilibrium (operating) profit, $\pi_r(s)$, of a quality-s firm in region r can be easily obtained as a function of $X_r(s)$ and $X^*_{-r}(s)$. Recalling that the earnings of each skilled worker coincides with the firms's operating profit we obtain

$$\pi_r(s) = w_r(s) = \frac{1}{\sigma - 1}[X_r(s) + X^*_{-r}(s)]. \tag{10}$$

The wage rate is thus proportional to the quantity sold. Since by (7) the sales of the more skilled are less dependent on their local demand, their wage rate is also less dependent on their location. By solving (7)-(10) for $I_r P_r^{\sigma-1}$ –which if multiplied by $p^{-\sigma}$ is the local demand for a firm offering a "standard" good (i.e., of quality 1)– we obtain

$$I_r P_r^{\sigma-1} = \frac{A_{-r} + B_r}{A_a A_b - B_a B_b}, \tag{11}$$

where $A_r \equiv P_r^{1-\sigma} - \frac{1}{\sigma}p^{1-\sigma}s_r\bar{s}$ and $B_r \equiv \frac{1}{\sigma}(p/\tau)^{1-\sigma}(s_r\bar{s} - cn_r)$. It can be shown that $A_r > B_r > 0$, which in turn implies $A_a A_b - B_a B_b > 0$. Thus, two types of local firm demand, $X_r(s)$ and $X^*_r(s)$, are determined uniquely and are strictly positive. It follows from (10) that the temporary equilibrium wage rate $w_r(s)$ is also determined uniquely and is strictly positive. The temporary equilibrium utility level of workers endowed with skill s and located in region r corresponds to their real wage, namely, their wage deflated by the locally prevailing cost of living index:

$$u_r(s) = w_r(s)/P_r. \tag{12}$$

4 Equilibrium

A temporary equilibrium is not necessarily an equilibrium of the economy in the "long-run." Since skilled workers are freely mobile, they will search for the best opportunities available in the whole economy, moving across regions if profitable. Formally, workers are assumed to migrate toward the region that gives them a higher utility level. The adjustment process is assumed to be myopic and given by:

$$\dot{f}_a(s) = \Phi(s|u, n) \quad \text{and} \quad \dot{f}_b(s) = -\Phi(s|u, n) \tag{13}$$

subject to (3), where $\dot{f}_r(s)$ is the time derivative of $f_r(s)$, $u \equiv \{u_a(s), u_b(s)\}_{s\in S}$ the set of temporary utility levels of each worker in each region, and $n \equiv \{f_a(s), f_b(s)\}_{s\in S}$ is the worker distribution between the two regions. We assume that the function $\Phi(\cdot|u, n)$ is a "smooth" functional of u and n, and satisfies the following condition:[24]

$$\Phi(s|u, n) \geq 0 \quad \text{if and only if} \quad (u_a(s) - u_b(s))f_a(s)f_b(s) \geq 0. \tag{14}$$

[24] In our setting, it can be verified that the temporary utility level of each skilled worker smoothly changes given a small change in the distribution of workers between the two regions. Here, we further assume that this smooth response of the utility levels in turn translates into that of migration rates. The concept of the smoothness of a functional is based on the so-called Volterra [40] derivative.

The implication of condition (14) for the adjustment process (13) is obvious: the region where workers can attain higher utility is more attractive, under the constraint concerning the population distribution given by (3). The migration continues until nobody has an incentive to relocate.

The above considerations lead to the following definition of the (long-run) equilibrium: *an equilibrium is a state of the economy where (i) all the conditions for the temporary equilibrium are satisfied; (ii) all workers of the same skill attain the same utility level irrespective of their location.* So, an equilibrium is a temporary equilibrium that satisfies also the condition that $\dot{f}_r(s) = 0$ for all s and r.

Now, how will the equilibrium distribution of workers across regions look like? In searching for equilibria we perform the following thought experiment. Starting from an arbitrary temporary equilibrium we ask whether a worker-seller of any skill level s located in region a has an incentive to relocate to region b.[25] She is willing to do so provided the utility level in region b is at least as high as what she is enjoying in region a. Denote by $u(s)$ *the relative standard of living* in region b (i.e., $u(s) \equiv u_b(s)/u_a(s)$) of a worker with skill level s. By applying (10) to (12), and using (7) and (11), we obtain the following expression:

$$u(s) = \frac{1}{P} \frac{sX + (s-c)\tau^{\sigma-1}}{s + (s-c)\tau^{\sigma-1}X}. \tag{15}$$

where P is *the relative cost of living index* in region b given by

$$P \equiv P_b/P_a = \left(\frac{s_a + (s_b - cn_b/s)\tau^{\sigma-1}}{s_b + (s_a - cn_a/s)\tau^{\sigma-1}} \right)^{1/(\sigma-1)}, \tag{16}$$

and X represents *the relative local firm demand,* given by the ratio between the local demand for a firm selling a good of an arbitrary quality level s in region b and that in region a:

$$X \equiv \frac{X_b(s)}{X_a(s)} = \frac{A_a + B_b}{A_b + B_a}. \tag{17}$$

A couple of remarks are in order. First, the relative local firm demand X is independent of s.[26] Thus, if the local demand for some variety s is larger in one region, then it is larger also for all the other varieties. Second, it is important to avoid misinterpretations concerning the meaning of X. If, say, $X > 1$, then an arbitrary firm has larger local sales if located in region b. This, however, does not mean that, on aggregate, sales, expenditure, and nominal income are greater in region b. Thus, it is important to bear in mind that X refers to the relative size of local markets from the viewpoint of the representative firm.

A region's attractiveness depends on the individual-specific skill level s, region-specific variables (X and P) and the level of transport and communication costs, τ and c. For workers of a given skill level, what matters are regional variables. Regional differences in market size and the cost of living index are shaped, in turn, by the allocation of population and skill across locations.

[25] In the following, we use the terms, relative standard of living, relative utility level, and relative profitability, interchangeably.

[26] Note that this result does not depend upon the Cobb-Douglas specification chosen to nest quality and quantity in preferences.

A worker in region a of skill level s has a strict incentive to relocate if and only if $u(s) > 1$. The problem is perfectly symmetric when considering a worker with skill s who is instead temporarily located in region b. In equilibrium, we must have $f_a(s) > 0$ [resp., $f_b(s) > 0$] if and only if $u(s) \leq 1$ [resp., $u(s) \geq 1$].

What matters for the location of worker-sellers in this economy? Recall by (10) that the sales and earnings of more skilled sellers depends less upon locations. From a consumer's point of view, all agents equally care about the overall price level P_r, which may differ greatly between locations when economic activities are not equally distributed in space. It is now not difficult to envisage that a highly-skilled worker-seller will be attracted to the region with a lower overall price level even if the local market for her product is smaller. This guess will be confirmed in the next section.

5 Configurations, existence, and stability of equilibria

What equilibrium configurations are possible in our economy? Are there different ones? If so, under what conditions will each one be realized? Are these equilibrium configurations mutually exclusive or not? Are they stable? In this section we address these issues. First, we claim that in our setting, we have at most the following three equilibrium configurations:

Definition 1 (*Equilibrium configurations*) *(i) Dispersed equilibrium: the most skilled worker in each region is more skilled than the least skilled worker in the other region; (ii) Concentrated equilibrium: all mobile workers locate in one region;(iii) Segmented equilibrium: the most skilled worker in one region is equally skilled or less skilled than the least skilled worker in the other region.*

Dispersed equilibria include all the possible equilibria in which both regions have workers of each skill level. Graphically, they are located in the interior of the area delimited by the Lorenz curves (area M in Figure 2). A special case of the dispersed equilibrium is the *symmetric equilibrium* (at point E in Figure 2), where the aggregate skill and population are equal (though the skill distribution may differ) across regions. Segmented equilibria are located along the Lorenz curves. The concentrated equilibrium is at one of the two corners, O and O', in the figure, and is an extreme case of the segmented equilibrium, where the threshold skill level is out of range $[\underline{s}, \overline{s}]$. Our model shares the possibility of the symmetric and concentrated equilibria with the standard core-periphery model. However, as we will see below, segmentation is the most typical stable equilibrium in our model.

In this section, we derive conditions for each equilibrium configuration defined in Definition 1. We also analyze the stability of the dispersed and concentrated equilibria, while that of the segmented equilibria is left for the global stability analyses presented in Section 6.

5.1 Segmented equilibrium

We start from the segmented equilibrium, since it is easy to ascertain that in our setup this configuration is the rule. In these equilibria, workers with similar skill endowments tend to cluster in the same location. By (15), it can be shown that the relative local firm demand directly affects the relative profit. Taking any $s \in (\underline{s}, \overline{s})$ we have

$$u(s) \geq u(s') \quad \text{if and only if} \quad X \leq 1 \quad \text{for} \quad s' < s. \tag{18}$$

How do we interpret these relations? Consider $X < 1$, so that region a offers a larger local market to each firm. Recall that a smaller value of $u(\cdot)$ makes region a more attractive. From (18) we see that region a is less attractive for high-skilled workers (or, if we prefer, for firms selling high-quality goods). This means that being located in a region offering a smaller local market for their goods is comparatively less disadvantageous (or more advantageous) for the more skilled. The explanation is simple. Communication costs fall more heavily on low-quality suppliers. Hence, those sellers that supply goods of higher quality are relatively footloose, since their sales can penetrate distant markets easily. We can reverse the argument for the case of $X > 1$. When $X = 1$, the relative profitability coincides for all firms. We can therefore state the following lemma:

Lemma 1 *Whenever local sales of any given firm differ across locations (i.e., $X \neq 1$) the equilibrium can only be either concentrated or segmented. Moreover, in a segmented equilibrium, all the workers with sufficiently high skills are located where their local sales are smaller.*

Lemma 1 says that in a segmented equilibrium, less skilled workers tend to cluster in a region where their local sales are abundant but where locally supplied goods are of relatively low quality. Conversely, workers with sufficiently high skills find it convenient to locate where the local market for their good is smaller, but where locally provided goods are of relatively high quality. This result is shaped by the interaction between transport and communication costs and skill heterogeneity. Since for the less skilled it is more difficult to penetrate distant markets due to the loss of quality in transit, they are more tied to the local market. The more skilled are instead relatively footloose and are at ease in locating themselves where locally supplied goods are of high quality and the "true" cost of living index is lower. We will see in the subsequent sections that the low price level in the high-skill region will indeed more than compensate for the disadvantage of a small local market.

Due to the possible multiplicity of equilibria, the stability of the segmented equilibria is not apparent. This is investigated in Section 6.

5.2 Dispersed equilibrium

By Lemma 1, in any dispersed equilibrium we must have $X = 1$ (equal sales for all firms in both regions). Moreover, we see from (18) that when $X = 1$, the relative standard of living must be the same for all agents. A dispersed configuration thus requires $u(s) = 1$ for all s. Finally, we note from (15), that $X = 1$ and $u(s) = 1$ also imply that $P = 1$ at any dispersed equilibrium. As will be clear, the only configuration consistent with $X = P = 1$ is the symmetric equilibrium, where the mass of population and that of skill are equal in the two locations. Consider a worker endowed with an arbitrary skill level s. From (15), we can show that the impact of the local market size and cost of living on the relative standard of living is described by the following condition:

$$u(s) < 1 \quad \text{if and only if} \quad \begin{cases} X < 1 \text{ and } P \geq 1, \text{ or} \\ X \leq 1 \text{ and } P > 1. \end{cases} \tag{19}$$

That is, given the same price level [resp., local firm market size], the region yielding a larger local market [resp., lower price level] is more attractive to any worker. We know that, at any dispersed equilibrium, all workers must be indifferent between the two locations. It follows that *the regional allocation of workers is necessarily indeterminate in this case.* However, the values of X and P depend upon how the aggregate skill and population are shared between the two regions. So, to characterize the dispersed equilibrium we must find out the particular class of worker distributions which are consistent with $X = P = 1$.

How does the regional shares of population and skill (n_a, s_a) affect the value of X and P ? It can be shown from (2) and (17) that its impact on X is described as follows:

$$X \geq 1 \quad \text{if and only if} \quad n_a \geq \left(1 - \frac{\sigma - 1}{\sigma + 1}\tau^{1-\sigma}\right)\frac{b}{c}(s_a - 1/2) + 1/2. \tag{20}$$

This relation indicates first that given an aggregate skill in a region (s_a), a larger number of competitors (n_a) will make the local firm demand less than that of the other region (X_a is smaller than X_b). Next, consider the $(X = 1)$-line in the (n_a, s_a)-space. Equation (20) suggests that the line passes through (1/2,1/2), and rotates anti-clockwise as transport costs increase (τ decreases). It is convenient to have a glance at Figure 3, in which Diagram (a) [resp., (b)] depicts the $(X = 1)$-line on the feasible (n_a, s_a)-domain shown in Figure 2 for the case of low [resp., high] transport costs. The mechanism for this can be understood easily if we consider the location incentive of worker-sellers when all of them are concentrated in one region, say region a (which is at point O' in the diagrams). For each worker-seller, the benefit of leaving region a is that she can enjoy a larger market share in region b where there are no local competitors, while the cost is that she is moving away from the large market in region a. When transport costs are low enough (sufficiently large τ), the competitors in region a can easily reach the market in region b, and hence, the cost dominates, i.e., $X < 1$ (the slope of the $(X = 1)$-line is between 0 and 1 as in Diagram (a)). If instead transport costs are high enough (sufficiently small τ), so that the market is spatially segmented, the benefit dominates, hence $X > 1$ (the slope of the $(X = 1)$-line is either negative or greater than 1 as in Diagram (b)). It is to be noted that the presence of communication costs softens the impact of the spatial distribution of skill on the relative local firm demand. When c is very high, what really matters for the local sales of each worker-seller is the number of local competitors, not their skills. Next, by (2) and (16), we have

$$P \geq 1 \quad \text{if and only if} \quad n_a \geq (1-\tau^{1-\sigma})\frac{b}{c}(s_a - 1/2) + 1/2. \tag{21}$$

The interpretation of (21) is straightforward. Given the regional allocation of aggregate skill (s_a), a larger number of sellers in a (n_a) lowers the cost of living there (P_a decreases relatively to P_b) due to consumers' love for variety. A larger number of local sellers is associated indeed with more varieties available locally. On the other hand, given the regional allocation of population, a larger aggregate skill in a region implies a greater average quality of goods, and hence, a smaller cost of living. It follows that the slope of the ($P = 1$)-line is always negative in the (n_a, s_a)-space (refer to Figure 3). The presence of communication costs again softens the impact of the spatial distribution of skill on the relative cost of living: given a larger value of c, the relative price level more crucially depends on the size of locally available product variety rather than their quality.

From (20) and (21), the relation between the ($X = 1$)- and the ($P = 1$)-lines can be described as follows. Both lines rotate anti-clockwise in the (n_a, s_a)-space as τ decreases, and converge to $s_a = 1/2$ as τ approaches 0. It is apparent that the two lines never coincide except at the symmetric equilibrium (1/2,1/2). This proves that the symmetric equilibrium is the only dispersed equilibrium.

Is the symmetric equilibrium stable? The answer can be a positive one only if we can show that the symmetric equilibrium is restored after any possible perturbation to the distribution of workers. It is not difficult to show that this can never be the case. Consider a perturbation such that $s_a > 1/2$. Then, along the ($X = 1$)-line we must have $P > 1$, so that $u(s) < 1$ for all s. This means that in any arbitrarily small neighborhood of the symmetric equilibrium there always exists a perturbation to the regional distribution of workers (along the ($X = 1$)-line) which leads to a self-reinforcing agglomeration process towards region a. Symmetrically, we can always find a perturbation that induces $s_a < 1/2$ and that leads to a self-reinforcing agglomeration towards region b. Thus, we have the following proposition:

Proposition 1 *The symmetric equilibrium always exists, and is unstable. Moreover, it is the only dispersed equilibrium.*

In spite of the fact that the perfectly symmetric regional distribution of workers always induces an equilibrium, this equilibrium cannot be a stable one. This is a major point of departure of our analysis from the standard core-periphery model (and its existing variants), since there the symmetric equilibrium can instead be stable. The instability arising in our model is generated by the interaction of skill heterogeneity with communication costs. It is to be noted that *the instability result persists* *even if the skill heterogeneity of workers is marginal, and as long as communication costs are positive, even if very close to zero.* This indicates that the simultaneous presence of worker heterogeneity and communication cost is a necessary and sufficient condition for the instability result.

This observation leads to some remarks of possible interest. The first concerns the robustness of the results arising under the conventional framework of the core-periphery model. Agent heterogeneity reveals a major source of instability, that has so far received insufficient attention. Second, the presence of external economies related to human capital may not be necessary to predict the inevitability of regional differences in endowments and income. Pecuniary externalities are enough to do the job. Finally, we remark on the role of communication costs. It is becoming a popular view that falling communication costs due to the advancements in telecommunication technologies will in the end eliminate the relevance of physical space for economic activities. Production will become footloose, and big, crowded cities will be replaced by "telematic villages" (see Glaeser [24] for a discussion). Our model predicts that this view is correct only if taken as a hyperbolic statement. To recover the stability of dispersed equilibria, and hence, the regional convergence, communication costs must totally disappear: lowering them will not be sufficient.

5.3 Concentrated equilibrium

This is another polar case that emerges in the standard core-periphery models, where all mobile workers concentrate in one region. In our model, however, the concentrated equilibrium can also be interpreted as a special case of the segmented equilibrium. In this particular case, although workers with different skills tend to have different location incentives, all worker-sellers prefer to be located in the same region.

In analyzing existence and stability of the concentrated equilibrium, it is convenient to answer first the following question: "Who are the workers that first leave the concentration when it ceases to be an equilibrium?" An easy argument shows that those workers that have the highest incentive to move away from a concentrated configuration are the least skilled. Suppose all mobile agents are located in one region, say a. Then, the cost of living is higher in region b, where all goods must be transported from region a (i.e., $P > 1$). By (15) this means that region b must offer larger earnings to attract workers (i.e., $X > 1$). We know by (18) that in this case we must have $u(s) < u(s')$ for $s' < s$, namely, that relocating to region b is more advantageous for less skilled workers. It follows that if a worker has an incentive to move to region b, then all the less skilled workers want to do the same. Hence, we conclude that whenever the concentration breaks, the least skilled worker is the first one to relocate.

Next, under what condition is the concentration an equilibrium? To simplify the analysis, we consider the situation in which communication costs alone (i.e., when $\tau = 1$) are not enough to break the concentration. From the discussion above, when all workers are located in region a, a sufficient condition for the concentrated equilibrium is $X < 1$, i.e., point O' is at the left of the $(X = 1)$-line in the (n_a, s_a)-space as in Figure 3(a). By (20), this is guaranteed if $\tau \geq \hat{\tau}$, where

$$\hat{\tau} \equiv \left(\frac{\sigma - 1}{1 + \sigma} \frac{1}{1 - c/\hat{s}} \right)^{\frac{1}{\sigma - 1}}. \tag{22}$$

So, the desired situation is realized under $\hat{\tau} \leq 1$. Equation (22) implies that this condition is equivalent to $\hat{s} \geq \frac{\sigma+1}{2} c$. Under Assumption 1, it can be written as

Assumption 2 $\frac{\sigma+1}{2} \underline{s} \leq \hat{s}$.

In the rest of the paper, we maintain this assumption. In this context, when transport costs are low, the concentration is an equilibrium. As transport costs increase, workers become less footloose, and seek the region with lower competition. As a result, the concentration tends to break. However, if the goods are more differentiated (σ is smaller), transport costs matter less, which in turn implies lower location dependence of income for all workers. In fact, if the goods are too differentiated ($\sigma < 2$), once all workers agglomerate in one region, no one would move away given any level of transport costs.[27]

The following proposition summarizes the results obtained [see Appendix A.1 for a proof of statement (ii)]:[28]

Proposition 2 *(i) When all skilled workers are concentrated in one location, if a worker with skill s' has an incentive to move to the other region, then all workers with $s \leq s'$ want to move as well. (ii) For $\sigma \leq 2$, the concentration is always an equilibrium, while for $\sigma > 2$, the concentration is an equilibrium if and only if $\tau^c \leq \tau \leq 1$, where $\tau^c \in (0, \hat{\tau})$ is the unique solution to the equation: $u(s = \underline{s}; \tau) = 1$.*

Notice that if the concentration is an equilibrium, it is by definition stable. Unlike the symmetric equilibrium, our model shares with the standard core-periphery models the basic ingredients of the equilibrium condition for the concentration. Provided that the degree of scale economies is not too high, the concentration is viable when the transport costs are sufficiently low. In our setting, however, we need another condition for this result, namely, that the level of communication costs is not too high.

6 Global dynamics and inevitable regional inequality

In this section, we derive the global behavior of the economy. From the results in the previous section, we know that stable equilibrium configurations are either concentrated or segmented, i.e., on the Lorenz curves. Given the smoothness of the migration process, any worker distribution (except those yielding

[27] This corresponds to the so-called "black-hole" condition in the core-periphery model (see Fujita, Krugman, and Venables [21, Sec. 4.6]).

[28] Statement (i) of the proposition holds without Assumption 2.

a symmetric equilibrium) will inevitably converge to an equilibrium exhibiting regional inequality. In particular, we can show that a cycle is impossible in our economy.

Below, we illustrate the global dynamics of the economy under the assumption that $\sigma > 2$, so that the concentration is not always an equilibrium (Proposition 2 (ii)). The results concerning the possible equilibrium configurations and their stability are summarized in the following proposition [see Appendix A.2 for the proof].

Proposition 3 *(i) If* $\hat{\tau} < \tau < 1$, *then the concentrated equilibria exist, and there is no segmented equilibrium. (ii) If* $\tau^c < \tau < \hat{\tau}$, *then the concentrated equilibria exist, and there may also exist one pair of segmented equilibria (one stable equilibrium and one unstable) with* $X < 1$ *and one pair with* $X > 1$. *(iii) If* $0 < \tau < \tau^c$, *then the concentrated configuration is not an equilibrium, and there exist at least one stable segmented equilibrium with* $X < 1$ *and one with* $X > 1$. *(iv) There is no cycle.*

Figure 3 conveys the basic message of the proposition. Diagram (a) depicts the case of low transport cost ($\hat{\tau} < \tau$), while Diagram (b) depicts the case of high transport cost ($\tau < \tau^c$), where the gray area is the same feasible domain M shown in Figure 2.[29] The arrows in the diagrams indicate the typical direction of adjustments.

Figure 3

In Diagram (a), it can be seen by (19) that $u(s) < 1$ for all s in area $A'EB$, so that all dynamic trajectories starting in the area will eventually reach the concentrated equilibrium at O'. Similarly, those starting in area AEB' will reach O. The reason is straightforward. In these areas, both skill and population are relatively concentrated in one region. In this region, the cost of living is lower, since goods of higher quality and a wider variety are available in the more populated region. Moreover, since the competition is global when transport costs are low, locating in the less populated region (i.e., with fewer local competitors) will not improve sellers' market size much. Thus, the cost of living is the dominant location determinant in this case. In areas AEB and $A'EB'$, the adjustment direction at each point is instead ambiguous. Since we have $X < 1$ and $P < 1$ in area AEB, despite region a having a larger market (which is good for firms), it has a higher cost of living (which is bad for consumers). The ambiguity of the migration direction in area $A'EB'$ can be explained similarly. It can be shown, however, that in Diagram (a) any dynamic trajectory starting in these areas will eventually either reach the symmetric equilibrium E, or enter area AEB' or $A'EB$ (refer to Lemmas 2 and 3 in Appendix A.2). Note in particular that there are no dispersed equilibria in the interior of areas AEB and $A'EB'$ (Proposition 1). Also, by (20), we have $X < 1$ at any point along OB. Then, by Lemma 1, any segmented configuration along OB

[29] We omit the diagram for the case of intermediate transport costs ($\tau^c \leq \tau \leq \hat{\tau}$), since it adds no significant results. For this case, refer to Figure 5(c,c') in Appendix A.2.

cannot be an equilibrium, since in this configuration the less skilled are located in region b, where the local firm demand is smaller. By the same argument, there is no equilibrium along $O'B'$. Hence, any initial worker distribution in M will eventually end up with a complete concentration in one region.[30]

In Diagram (b), areas $A'EB$ and AEB' are characterized by the same migration pattern as those in Diagram (a). Thus, dynamic trajectories starting in these areas eventually enter areas $A'EB'$ and AEB, respectively. Under high transport costs ($\tau < \tau^c$), unlike the case of Diagram (a), the intensity of competition differs significantly across regions when skill and population are relatively concentrated in one region. In particular, when all sellers are located in one region (O or O'), the least skilled would always want to deviate from the concentration to escape from tough competition (Proposition 2 (i)). Also, as explained above, a segmented configuration is not an equilibrium along OB and $O'B'$. It follows that the only possible stable equilibria are the segmented ones along OA and $O'A'$. The arrows in Diagram (b) are for simplicity drawn for the case in which there is a unique segmented equilibrium on OA and on $O'A'$.

In the case of intermediate transport costs ($\tau^c \leq \tau \leq \hat{\tau}$), the possible equilibrium configurations are either the same as those in Diagram (a), or a situation in between Diagrams (a) and (b) may happen, where both the concentrated and segmented equilibria coexist, as statement (ii) in the proposition indicates.

7 Discussion

We saw that regional inequality is inevitable in our setting. This raises a number of questions. Where should we expect the most skilled agents to be located? What is the role of transport and communication costs in shaping the spatial distribution of economic activities? What are the links between skill inequality and agglomeration? The subsections below discuss these issues.

7.1 Skills and location patterns

Where do the most talented go? Our analysis above indicates that in any stable equilibrium, the aggregate skill in the region populated by the highly-skilled workers must be at least a half of the total, i.e., $\hat{s}/2$. This can be seen from the fact that the share of aggregate skill in region a at point A' – the intersection of the $(X = 1)$-line and the upper Lorenz curve– can never be smaller than 1/2 by (20). As for the population share in this high-skill region, things are less clear-cut. Figure 3(b) shows that in all segmented equilibria the region inhabited by the high skilled must have a minimum population size (i.e., when the skilled go to region a, n_a is necessarily bigger than the value found at the intersection of the $(X = 1)$-line and the upper Lorenz curve). This minimum population size, however, could be below 1/2, provided the $(X = 1)$-line is negatively sloped (i.e., transport costs are high).[31]

[30] Except the measure zero area which is contained in a stable manifold leading to E.

[31] Recall that all the segmented equilibria on the upper and lower Lorenz curves are respectively along the $O'A'$ and along OA segments in Figure 3(b).

We can say the following. The most skilled are always found in wealthier locations, where aggregate human capital, and aggregate income (and welfare) is higher.[32] Wealthier locations may end up being less populated, because these are inhabited by the "privileged few" who are skilled and rich.

How can we explain that the highly-skilled are to be found in wealthier locations? The reason is tied to the different impact of communication costs on sellers of different skills. Since the more skilled are always closer to the global market (less dependent upon local markets), they are not as sensitive as the less skilled to being located where their local individual demand is low. In other words, the local firm demand is small in rich thriving areas. Fierce competition is the price to pay for being located where the best are: the local total demand must be shared among the top sellers, and the share accruing to each is necessarily small. However, note that the cost of living index is lower in the high-skill region than in the low-skill region, since a wider range of high-quality goods are available there. This low price index compensates for the disadvantages of a limited local market, yielding a high standard of living in the high-skill region.[33] We summarize these remarks in the next corollary of Proposition 3.

Corollary 1 *(i) In any stable equilibrium, one region has higher aggregate skill and income, as well as a higher average utility level than the other. (ii) Workers at the top of the skill ladder are always located where aggregate skill is higher.*

7.2 The role of transport and communication costs

Distance matters in our model because of both transport and communications costs. As for transport costs, we see from Proposition 3 that whenever transport costs are sufficiently low (i.e., τ approaches 1) the equilibrium configuration is necessarily concentrated. The maximum extent of agglomeration occurs. This is a basic result of the standard core-periphery model. As transport costs rise, however, we cannot hope for a symmetric configuration in our model. The configuration will be segmented, with one high-skill region and one with low skills. We can say more. If transport costs are not too high, the high-skill region will be the one with the larger population. This is easily understood recalling that, by (20), the $(X = 1)$-line is positively sloped if $\tau > (\frac{\sigma-1}{\sigma+1})^{1/(\sigma-1)}$ (refer to Figure 3(a)). By the previous arguments, any segmented equilibrium must exhibit larger population in the high-skill region in this case. Recall also that the $(X = 1)$-line rotates clockwise as transport costs fall. So, we see that a reduction in transport costs still tends to raise the extent of agglomeration as in the standard core-periphery model. As τ becomes close to 1, most of the mobile agents must be located in the high-skill region.

[32] Denote by $I = I_{\mathsf{b}}/I_{\mathsf{a}}$ relative nominal incomes. After some algebra it is shown that that $I > 1 \Leftrightarrow s_{\mathsf{a}} < \frac{1-c/\mathsf{b}}{2-c/\mathsf{b}} + \frac{c/\mathsf{b}}{2-c/\mathsf{b}} n_{\mathsf{a}}$. It is easily checked that this condition is satisfied if and only if the most skilled are located in b.

[33] A straightforward corollary of this result is that immobile workers are necessarily better-off in the high-skill location. Their nominal income is the same in both locations, but the real income is higher in the high-skill region because of a lower cost-of-living index.

What about communication costs? It is possible to show that a fall of communication costs may have very different effects on the equilibrium configuration depending on the level of transport costs.

Consider the case where transport costs are low ($\tau > (\frac{\sigma-1}{\sigma+1})^{1/(\sigma-1)}$). Then, communication costs are dominating in the overall cost for transit. Then given a decrease in communication costs, even low-skilled worker-sellers become footloose. We see that in this context the agglomeration of human capital is necessarily associated with that of population. This can be seen from the fact that the range of possible segmented equilibria (OA and $O'A'$ on the Lorenz curves) shrinks after a fall in communication costs (refer to (20)). On the other hand, if transport costs are high ($\tau < (\frac{\sigma-1}{\sigma+1})^{1/(\sigma-1)}$), through a symmetric mechanism, even highly-skilled workers are tied to the local market, and seek the region with less competition. As a result, the agglomeration of human capital is not likely to be associated with that of population as communication costs fall. These findings are summarized in the next corollary to Proposition 3:

Corollary 2 *(i) When transport costs are low ($\tau > (\frac{\sigma-1}{\sigma+1})^{1/(\sigma-1)}$) the region with higher aggregate skill has also a larger population. (ii) When $\tau > (\frac{\sigma-1}{\sigma+1})^{1/(\sigma-1)}$ [resp., $\tau < (\frac{\sigma-1}{\sigma+1})^{1/(\sigma-1)}$], the minimum share of population in the high-skill region rises [resp. falls] as communication costs fall.*

The above results may shed light on some real-world phenomena. It may help to explain for instance the empirical fact that high concentrations of human capital have been associated with those of population in recent decades characterized by substantially decreasing communication costs and a fairly low level of transport costs (Glaeser et. al. [25]; Black and Henderson [6]). The model also helps in qualifying predictions concerning the impact of reduced communication costs on spatial agglomeration. In general, the analysis gives a further argument in favor of those that are skeptical about an inevitable dispersion of economic activities as a result of lower costs for communications: segmentation or concentration will remain the only stable outcome even with very small communication costs.[34] However, communication costs may matter for the extent of agglomeration because they affect the relation between the equilibrium location of skills and population.

7.3 Agglomeration and interpersonal inequality

How interpersonal (skill and then income) inequality relates to regional inequality? Due to the pervasiveness of equilibrium multiplicity a general assessment concerning the links between inequality and agglomeration is beyond the scope of our analysis. Formally, we cannot pin down the location and number of the segmented equilibria along OA and $O'A'$ on the Lorenz curves in Figure 3(b). Given the relatively simple model structure, however, once the functional form of the skill distribution among workers is specified, we can numerically explore the behavior of the model fairly exhaustively over the

[34] See, for instance, Gaspar and Glaeser [23], for an argument in favor of greater need of face-to-face contact, and agent proximity in the presence of lower communication costs.

possible parameter range. For simplicity, we assume a uniform distribution of population over the skill range $[\underline{s}, \overline{s}]$.[35] The key parameters are the aggregate skill in the economy, $\mathbf{s}$, the degree of interpersonal inequality (as measured by the Gini index), and the parameters for transport and communication cost, τ and c.[36]

Regarding the number of equilibria, our simulation results indicate that there are at most two stable equilibria along each of the OA and the $O'A'$ segments on the Lorenz curves.[37] Figures 4 and 5 present typical numerical examples showing the relation between the degree of interpersonal skill inequality and that of population and skill agglomeration. We consider two ways to increase the Gini index. The first is to increase the aggregate skill, $\mathbf{s}$, along with the highest skill level, $\overline{s}$, while keeping the lowest skill level, $\underline{s}$, fixed. The second is to expand the skill range $[\underline{s}, \overline{s}]$, while keeping the aggregate (and average) skill level constant. In the first case a higher value of the Gini index is necessarily associated with a higher value of $\mathbf{s}$, while in the second case a higher value for the Gini index corresponds to a mean preserving spread in the skill distribution obtained by raising $\overline{s}$ and lowering $\underline{s}$ by the same amount. Figure 4 describes how the equilibrium configurations behave when inequality changes in the first way, while Figure 5 depicts the second case. The horizontal axis in each diagram in Figure 4 [resp., 5] measures the value of $\mathbf{s}$ [resp., $\overline{s}$], which, as explained above, represents the degree of interpersonal inequality in this case. The vertical axis in Diagram (a) [resp., (b)] in each figure measures the share of skilled population [resp., the aggregate skill share] in the high-skill region (which is assumed to be region a). In each diagram, different curves correspond to different values of τ, while other parameter values are indicated in the figures. In both diagrams in Figure 4, for ease of understanding, the unstable equilibria are drawn by dotted curves when there are two stable equilibria for given parameter values.[38]

Figure 4

Consider first Figure 4. There, the minimum skill level is fixed at $\underline{s} = 1$, while the aggregate skill $\mathbf{s}$ is varied from 1.0 to 25.0.[39] Diagram (a) indicates that if transport costs are sufficiently low ($\tau = 0.8513, \tau = 0.8525$), then the population share of the high-skill region is likely to rise as the interpersonal inequality increases (i.e., $\mathbf{s}$ increases). The relation between the extent of agglomeration and that of

[35] One of the important implications of this setting is that when interpersonal inequality increases, the boundaries of feasible domain M (refer to Figure 2) expands monotonically, i.e., each point on the upper [resp., lower] Lorenz curve moves upward [resp., downward].

[36] The effect of the substitution elasticity σ can be deduced straightforwardly from the result of the standard core-periphery model: a smaller σ (higher degree of differentiation) is associated with higher population (i.e., variety) concentration (in our context, in the high-skill region). It is because the sales is less dependent on transport cost when the goods are more differentiated from one another.

[37] In fact, these equilibria bifurcate from the concentrated and symmetric equilibria of the core-periphery model. As $\underline{s}$ and c approach $\overline{s}$ and 0 respectively, these segmented equilibria converge to the equilibria of the core-periphery model.

[38] The unstable equilibria are omitted in Figure 5. But, whenever there are two stable equilibria, there is one unstable equilibrium between them.

[39] All the numerical examples in Figure 4 start from the symmetric equilibrium at $\mathbf{s} = \underline{s} = 1.0$, where there is no interpersonal inequality. For τ close to 1, our simulation indicates that the possible stable equilibrium is instead only the concentration when $\mathbf{s} = \underline{s}$ under Assumption 2, and that the concentration continues to be a stable equilibrium for all $\mathbf{s} > \underline{s}$.

inequality seems however highly nonlinear. We see indeed that the share of population in the high-skill region may actually fall when the interpersonal inequality is small. Diagram (b) indicates that when the transport costs are low, the aggregate skill share in the high-skill region is likely to rise as the interpersonal inequality increases ($\tau = 0.8513, \tau = 0.8525$). Recall that the segmented equilibrium is located along the segment OA or $O'A'$ of the Lorenz curves shown in Figure 3(b). It follows that if interpersonal inequality increases in such a way that the feasible domain M expands monotonically at each point on the Lorenz curves (as in the case of our simulation setting), then we can unambiguously say that skill agglomeration will go up when the rise in inequality induces more population agglomeration; though the impact is ambiguous when the population agglomeration instead decreases. The simulation results show that for small transport costs ($\tau = 0.8513, \tau = 0.8525$), the skill agglomeration rises even when the population agglomeration is falling, while it may decrease with the share of population in the region when the transport costs are high ($\tau = 0.8, \tau = 0.85$). From these results we can argue that *when higher interpersonal inequality is associated with a growth in the aggregate human capital and transport costs are low, skill and population tend to agglomerate in the same region as this inequality rises.*[40]

We can interpret this behavior of the equilibrium configurations as follows. When an increase in interpersonal inequality is associated with the growth of overall human capital, the fraction of skilled workers whose earnings are less dependent on location rises with the degree of interpersonal inequality. As a result, the population would tend to move into the high-skill region where the cost of living is lower. Hence, the agglomeration of skill and that of population tend to coincide when inequality is sufficiently large. Not surprisingly, greater dominance of communication costs over transport costs, and hence, less dependence of overall costs for transit on the sales value, will enhance this tendency of co-agglomeration of population and skill. These results are consistent with the continuing urbanization over the centuries in the world economy where income, human capital, and often inequality have been rising (Bairoch [2]).

Figure 5

In Figure 5, the aggregate skill is fixed at $\mathbb{s} = 25.0$, and the highest skill $\overline{s}$ is varied from 25.0 (namely, the value of $\underline{s}$) to 49.0, at which $\underline{s} = 1$.[41] A glance at the diagrams is sufficient to conclude that *when higher interpersonal inequality is generated by a mean preserving spread in the skill distribution, the agglomeration of skill becomes less correlated with that of population as inequality rises.* The underlying mechanism is simple. A greater interpersonal inequality under a given aggregate skill in the economy implies a greater concentration of skill to a fewer number of workers. Thus, the earnings of most of the workers heavily depend on the size of the local market, which depends in turn on the degree of local

[40] From the simulation results in Figure 4 we also notice that the range of interpersonal inequality in which the region with a larger population and that with larger human capital do not match is smaller when transport costs are low.

[41] The equilibria at $\mathbb{s} = 25.0$ in Figure 4 correspond to those at $\overline{s} = 49.0$ in Figure 5.

competition. Consequently, one may observe a large population also in a region with a small aggregate skill endowment. This result may help to interpret some stylized facts that are observed in cross country comparisons. For instance, if we compare the urban structure in countries with similar income per capita such as the US and Japan, cities tend on average to be more populated in Japan, where the extent of interpersonal inequality is smaller.[42]

8 Concluding remarks

Economic agents are not all equal. This is a basic fact of life. Some workers, sellers, entrepreneurs are more skillful, brilliant, or simply luckier than others. In a world of unequal abilities and fortunes, we necessarily observe rich and poor places: wealth and human capital are not evenly represented across towns, regions or states. This paper addresses these points formally. Our economy is populated by sellers that differ in their skills, thus performing differently in the marketplace. We show that modifying the well-known core-periphery model of location in such a way that agent heterogeneity is allowed has major analytical implications. The sustainability of a symmetric location pattern, which is a common feature of the existing "new economic geography" models, breaks. Regional inequality is inevitable. Most skilled agents are attracted by wealthier locations, those where human capital and wealth are more abundant.

[42]In 1990, the largest three metropolitan areas account for 14% of the total population in the US. The corresponding numbers are 21%, 21% and 31% in France, Italy, and Japan, respectively (United Nations [39]).

A Appendix

A.1 Proof of Proposition 2(ii)

Without loss of generality, assume all workers are located in region a ($s_a = n_a = 1; s_b = n_b = 0$). In this case, the expressions for the relative price index (16) and the relative local firm demand (17) boils down to $P = (1 - c/\tilde{s})^{\frac{1}{1-\sigma}}/\tau$ and $X = \frac{\sigma-1}{\sigma+1}\frac{\tau^{1-\sigma}}{1-c/\tilde{s}}$, respectively. By substituting these expressions into (15), we obtain

$$u(s) = (1 - c/\tilde{s})^{\frac{1}{\sigma-1}} \frac{(\sigma-1)\tau^{2-\sigma} + (\sigma+1)(1-c/s)(1-c/\tilde{s})\tau^{\sigma}}{(\sigma+1)(1-c/\tilde{s}) + (\sigma-1)(1-c/s)}. \tag{23}$$

From the investigation of (23), we can show the following. If $\sigma \leq 2$, then $u(\cdot) = 0$ at $\tau = 0$, and $u(\cdot)$ is increasing in $\tau > 0$. If $\sigma > 2$, then $u(\cdot)$ approaches ∞ as τ approaches 0, and as τ increases from 0, it first decreases, then increases (in particular, it is convex for $\tau > 0$). Statement (ii) follows from this result together with statement (i). Q.E.D.

A.2 Proof of Proposition 3

We can classify the global dynamics of regional population and aggregate skill, (n_a, s_a), into four typical patterns: cases of (a) low transport cost ($\tilde{\tau} < \tau < 1$), (b) high transport cost ($0 < \tau < \tau^c$), and (c) intermediate transport cost ($\tau^c < \tau < \tilde{\tau}$) and high skill inequality (large M), (c') intermediate transport cost ($\tau^c < \tau < \tilde{\tau}$) and low skill inequality (small M) .

These patterns are mutually exclusive, and also exhaustive as far as the possible types of equilibria are concerned. That is, they are not exhaustive only in cases (b,c') where the actual number of segmented equilibria may be different depending on parameter values.

Depending on the relative home market size X and the relative price index P, the feasible domain M can be decomposed as follows. Given a contiguous domain $N \subset M$, we say that it is of *type* Ω_a [resp., Ω_b] if $u(s) < 1$ [resp., $u(s) > 1$] for all s at each point in N; *type* Ω_c [resp., Ω_d] if there exists $z \in (\underline{s}, \overline{s})$ such that $u(z) = 1$, $u(s) > 1$ [resp., $u(s) < 1$] for all $s > z$, and $u(s) < 1$ [resp., $u(s) > 1$] for all $s < z$ (i.e., $X < 1$ [resp., $X > 1$] by (20)) at each point in N.

Regarding the migration pattern in each domain type, we can show the following:

Lemma 2 (*Direction of local adjustments*) *At each point in a domain of type $\Omega_a, \Omega_b, \Omega_c$, and Ω_d, the direction of local adjustment $(\dot{n}_a, \dot{s}_a)$ under the dynamics (13) is in, respectively, the first quadrant (figure 2), the third quadrant, the domain below the 45° line through the origin, and the domain above the 45° line through the origin, where $\dot{n}_a$ and $\dot{s}_a$ are the time derivatives of n_a and s_a.*[43]

Proof: Since in any domain of type Ω_a [resp., Ω_b], we have $u(s) < 1$ [resp., $u(s) > 1$] for all s, the adjustment at each point in the domain takes place so that both the population and the aggregate skill in region a [resp., b] increases.

[43] In fact, we can narrow down the adjustment direction in type Ω_a and type Ω_b domains even more. Namely, the steepest increment [resp., decrement] of the aggregate skill in each region is $\overline{s}$ [resp., $\underline{s}$]per unit increase in the population. So, the adjustment direction is restricted between the vectors $(1, s_a\overline{s})$ and $(1, s_a\underline{s})$ [resp., $(-1, -s_a\overline{s})$ and $(-1, -s_a\underline{s})$] in the area of type Ω_a [resp., Ω_b].

Next, consider any domain of type Ω_c. Since $X < 1$ at each point in the domain, we know that $u(s) > u(s')$ if $s > s'$, which implies that workers with skill level $s < z$ want to locate in region a, while those with $s > z$ in region b. Then, the direction of the adjustment vector $(\dot{n}_a, \dot{s}_a)$ at each point in the domain has an acute angle to the vector $(1, z_o)$ normal to $(1, z)$ (refer to Diagram (a) in Figure 6). Note that for $0 <\underline{s}< \overline{s} < \infty$, vector $(1, z_o)$ is directed to the southeast (i.e., in the forth quadrant) for any threshold skill level, $z \in (\underline{s}, \overline{s})$. It follows that the adjustment at each point projected on the direction of vector (1,-1) is positive. In other words, the direction of adjustment at each point in the domain is below 45° line passing the point. (that is, toward either the east-northeast, southeast, or south-southwest). It can be shown in a similar manner that the direction of the adjustment at each point in any domain of type Ω_d is above the 45° line passing the point (that is, toward either the north-northeast, northwest, or west-southwest) (refer to Diagram (c) in Figure 6). Q.E.D.

Figure 6

Note that if O [resp., O'] is contained in a domain of type Ω_a [resp., Ω_b], then it is a stable equilibrium. Note also that if the segmented equilibria exist, they must be on the part of the upper [resp., lower] Lorenz curve which is contained in a domain of type Ω_d [resp., Ω_c].

Using Propositions 1, 3, Lemmas 1 and 2, we can decompose domain M into types introduced above. The result is shown in Figure 7. Each diagram in the figure depicts the same Lorenz curves we have in Figure 5. Diagrams (a,b) in Figure 5 correspond to Diagrams (a,b) in Figure 7. Figure 7 also shows the case of intermediate transport cost ($\tau^c < \tau <$ b) in Diagrams (c,c').

Figure 7

Lemma 3 (*Decomposition of feasible domain M*) *(i) Area $A'EB$ [resp., AEB'] (including the boundary) is of type Ω_a [resp., Ω_b]. (ii) Area AEB [resp., $A'EB'$] consists either of a type Ω_c [resp., Ω_d] domain entirely, or of domains of type Ω_a and/or type Ω_b isolated by a type Ω_c domain. (iii) The line segments $A'E$ and BE [resp., AE and $B'E$] are contained in a type Ω_a [resp, Ω_b] domain except for the end point E. (iv) The neighborhood of O' [resp., O] (of positive measure) is either of type Ω_b or of type Ω_d [resp., Ω_a or Ω_c] if $\tau < \tau^c$.*

Proof: By (19), we know that $u(s) < 1$ [resp. $u(s) > 1$] for all s at each point in area $A'EB$ [resp., AEB'] (including the boundary) in all the diagrams in Figure 7. Hence, this area is of type Ω_a [resp., Ω_b], and statement (i) is proved.

By (20), in Diagrams (a,c,c'), in the area above [resp., below] the $(X = 1)$-line (AA'), we have the area $X < 1$ [resp., $X > 1$] at any point. It follows that $u(s) > u(s')$ [resp., $u(s) < u(s')$] if $s > s'$ at any point in this area. On the other hand, by (20), again, in Diagram (b), in the area above [resp., below] the $(X = 1)$-line (AA'), we have $X > 1$ [resp., $X < 1$] at any point. It follows that $u(s) < u(s')$ [resp., $u(s) > u(s')$] if $s > s'$ at any point in this area. Note that within the area of $X < 1$ and that of $X > 1$, a type Ω_a and a type Ω_b domains cannot have a non-empty intersection except for the symmetric equilibrium, E.

Let us first deduce types of the rest of domain M in Diagram (a). The continuity of $u(\cdot)$ in n_a and s_a, and the fact that a type Ω_a and a type Ω_b domains cannot have common points except E imply that we can find a continuous curve contained in area AEB connecting E, and a point, say, C, on the segment AB of the upper Lorenz curve, such that $u(\overline{s}) = 1$ and $u(s) < 1$ for all $s < \overline{s}$ along curve CE, and another continuous curve in area AEC, connecting E and a point, say, D, on the segment AC of the upper Lorenz curve, such that $u(\underline{s}) = 1$ and $u(s) > 1$ for all $s > \underline{s}$ along DE.[44] Moreover, curves CE and DE can be chosen so that the entire area BEC is of type Ω_a, and area AED is of type Ω_b.[45]

By the symmetry of the adjustment dynamics (13) with respect to the symmetric equilibrium, E, we can also find at the symmetric location a pair of curves $C'E$ and $D'E$ connecting E and the segment $A'B'$ of the lower Lorenz curve. We have that $u(\underline{s}) = 1$ and $u(s) < 1$ for all $s > \underline{s}$ along curve $D'E$, while $u(\overline{s}) = 1$ and $u(s) > 1$ for all $s < \overline{s}$ along $C'E$, and that area $A'ED'$ is of type Ω_a, and area $B'EC'$ is of type Ω_b.

Since $u(s) > u(s')$ [resp., $u(s) < u(s')$] if $s > s'$, at each point in area CED [resp., $C'ED'$], and since a domain of type Ω_a and that of type Ω_b cannot have common points except E in an area with a common sign of $X - 1$, it follows that area CED [resp., $C'ED'$] must consist either of a domain of type Ω_c [resp., Ω_d] entirely, or of isolated domains of type Ω_a and/or type Ω_b surrounded by a domain of type Ω_c [resp., Ω_d].

In a similar manner, in Diagrams (b,c,c'), we can find corresponding points C and D [resp., C' and D'] on the segment AOB [resp., $A'O'B'$] of the Lorenz curve, where curves $CE, DE, C'E$, and $D'E$ have the same meaning as those in Diagram (a). Area CED [resp., $C'ED'$] in Diagram (c) and area CED [resp., $C'ED'$] in Diagram (b,c') has the same properties as area CED [resp., $C'ED'$] in Diagram (a). Thus, (ii) and (iii) have been proved.

Area CED and $C'ED'$ in Diagrams (b,c,c') can be further decomposed. Recall that the concentrated distribution is an equilibrium if and only if $\tau^c < \tau < 1$. That is, point O [resp., O'] is contained in a domain of type Ω_b [resp., Ω_a] in Diagrams (c,c'). Since the concentration is not an equilibrium for $0 < \tau < \tau^c$, point O [resp., O'] is contained in a domain of type Ω_a or Ω_c [resp., Ω_b or Ω_d] in Diagram (c), which proves (iv). Q.E.D.

Under the decomposition of domain M given by Lemma 3, and the migration pattern in each type of domains given by Lemma 2, we can prove statement (iv) of the proposition:

Lemma 4 *No cycle is possible.*

Proof: First, by Lemma 2, it is evident that a cycle is impossible in areas $A'EB$ and AEB' which consists entirely of type Ω_a and type Ω_b domains, respectively. It is also obvious from the same lemma that there is no cycle within a type Ω_c or type Ω_d area.

[44] In Diagrams (a,b,c) points C and D describe the evolution of the same intersections as τ changes. For instance, point D passed O as we move from Diagram (c) to (b) as τ gets smaller. In that sense, points C and D in Diagram (c') are not the exact extension of C and D from Diagram (a). But, here, we tried to maintain the consistency of each curve-segment in terms of the adjustment process surrounding these curves. Note that the division of domain types by curves $CE, DE, C'E, D'E$ are consistent throughout the diagrams.

[45] The possibility of having more curves like CE and DE between the depicted CE and DE cannot be rejected, due to the non-linearity of $\Omega(\cdot)$ in n_a and s_a.

By Lemma 3(ii) and (iii), area AEB consists of a type Ω_c domain and isolated domains of type Ω_a and/or type Ω_b. Suppose there exists a cycle passing a point in a type Ω_a area within AEB. Then the dynamic trajectory starting the point must exit the type Ω_a area and enter the type Ω_c area before it comes back to the same point again. Note that at the boundary of each type Ω_a area surrounded by a type Ω_c area, the direction of the adjustment is between $(1, s_a\underline{s})$ and $(1, s_a\overline{s})$ (refer to footnote 43). But, recall that at each point in a type Ω_c area the adjustment is directed into the domain below the 45° line through the point. It follows by the continuity of the adjustment process that in order for the trajectory to enter again the same type Ω_a area it left, there must be a period in which the adjustment in the type Ω_c area is directed into the domain above the 45° line through the point: a contradiction. Hence, a cycle is not possible in area AEB. The impossibility of a cycle in the case of $A'EB'$ can be similarly proved. Q.E.D.

Using the above lemmas, we can describe the global dynamics of (n_a, s_a) as presented in Proposition 3. In the following, we prove all of its statements.

Proof of statement (i):

By Lemmas 2 and 3, any segmented distribution cannot be an equilibrium in this range of τ. And by Proposition 1 and Lemma 4, we know that there is neither equilibria (except E) nor cycles in area CED and $C'ED'$, and by the continuity of the migration process (13), any dynamic trajectory starting in these areas eventually either reaches E, or enters area CED' or $C'ED$. We also know that the adjustment is directed toward the northeast [resp., southwest] in area CED' [resp., $C'ED$]. Then, by the continuity of the adjustment process, it follows that a dynamic trajectory starting from any point in area CED' [resp., $C'ED$] eventually reaches the concentrated equilibrium at O' [resp., O]. Hence, besides the unstable symmetric equilibrium, we have two (stable) concentrated equilibria, and no other equilibrium. Q.E.D.

Proof of statement (ii):

In this case, we have two possibilities, one in which the segment AO [resp., $A'O'$] of the lower [resp., upper] Lorenz curve is contained entirely in a domain of type Ω_b [resp., Ω_a] (Diagram (c)), the other in which a part of the segment is contained in a domain of type Ω_c [resp., Ω_d] (Diagram (c')).[46]

In the former case, it can be easily shown that the global dynamics is qualitatively the same as that for $\tau \in (\mathsf{b}, 1)$ derived above. In the latter case, however, we have a different adjustment process. An example is shown in Diagram (c'). Here, curve FG [resp., $F'G'$] has the same property as curve DE [resp., $D'E$], where the size of the segment DF [resp., $D'F'$] is of strictly positive measure (refer to the proof of Lemma 3 for the construction of curves DE and $D'E$). Note that in the interior of the segment

[46]Here, it is possible to have a segment along AO which is contained in domain of type Ω_{a}. But, in that case, we can always find another segment along AO which is contained in a type Ω_{c} area. To see this, note that along AO, $Y < 1$, which means $\Omega(s)$ is increasing in s. It in turn implies that type Ω_{a} and Ω_{b} areas cannot be adjacent, i.e., we need to have a type Ω_{c} area between them if Ω_{a} should appear between A and O. Similarly, if there is a segment contained in a domain of type Ω_{b} along $A'O'$, then we also have another segment contained in the Ω_{d} domain along $A'O'$.

DF [resp., $D'F'$], there is at least one contiguous segment of strictly positive measure which is contained in a domain of type Ω_c [resp., Ω_d], since otherwise the entire segment must be contained in a domain of type Ω_b [resp., Ω_a] which is the case of Diagram (c).

Since the adjustment is toward the southwest [resp., northeast] on the segments OF and AD [resp., $O'F'$ and $A'D'$] of the lower [resp., upper] Lorenz curve, then by the continuity of u and that of the adjustment process, it must be true that in the type Ω_c [resp., type Ω_d] part of DF [resp., $D'F'$], the adjustment must change directions at least twice. It follows that when the adjustment is restricted to be on the Lorenz curve, we have at least one stable and one unstable segmented distributions in this part of DF [resp., $D'F'$]. But, by the adjustment direction in a domain of type Ω_c [resp., type Ω_d] stated in Lemma 2, we can deduce that the adjustment is directed toward this type Ω_c [resp., type Ω_d] part of DF [resp., $D'F'$] in its neighborhood. Hence, the segmented distributions which are stable on DF and $D'F'$ are indeed stable equilibria.

It can be shown in a similar manner as in the case of $b < \tau < 1$ above that a dynamic trajectory starting from any point in domain M eventually reaches either the segmented equilibria on DF or $D'F'$, or the concentrated equilibria at O and O', or the symmetric equilibrium at E. Hence, in this case, we have the concentrated equilibria and at least a stable and an unstable segmented equilibria on DF and on $D'F'$. The number of segmented equilibria may differ depending on the parameter values. Though we cannot formally show under what conditions the two cases (c) and (c') arise, our simulations indicate that the coexistence of segmented equilibria and concentrated equilibria (i.e., (c') is more likely when the heterogeneity of skill levels is smaller so that we have a smaller feasible domain M. It is due to the fact that the standard core-periphery model has both concentrated and symmetric equilibria simultaneously for the intermediate range of τ. When our model is close to the core-periphery model (i.e., small interpersonal skill inequality), then we have also equilibria which bifurcated from these. But, as we depart from the core-periphery model further, we may loose some of these equilibria. Also, it is not surprising given the fact that the migration direction at each (n_a, s_a) does not basically depend on the size of M as long as it is in the interior of M. Thus, it is more likely for the curve ED' [resp., ED] in Diagram (c) to hit $O'A'$ [resp., OA] on the upper [resp., lower] Lorenz curve if the two Lorenz curves locate closely (i.e., the size of M is smaller). Q.E.D.

Proof of statement (iii):

Diagram (b) corresponds to this case. By Lemma 1, we know that for this value of τ, the concentration cannot be an equilibrium. It follows that O [resp., O'] must be in the interior of areas different from type Ω_a [resp., Ω_b]. This also means that D [resp., D'] cannot be on the upper [resp., lower] Lorenz curve as C [resp., C'] (refer to the proof of Lemma 3 for the construction of curves CE and $C'E$). By similar reasoning used for the case shown in Diagram (c') in the proof of statement (ii) above, we can show that there is at least a stable segmented equilibrium on the segments OD and on $O'D'$ on the Lorenz curve. It is to be noted that unlike the case of Diagram (c'), we do not necessarily have an unstable segmented equilibrium. Along the segment OA of the lower Lorenz curve, we may have the transition of domains from Ω_a to Ω_c to Ω_b, or from Ω_c to Ω_b. Note that adjustment along OA must change directions at least once, but not necessarily more. Thus, we can only know the existence of one stable segmented equilibria on the Lorenz curve. Similar reasoning applies on the segment $O'A'$ on the upper Lorenz curve. Q.E.D.

References

[1] Armington, P.S., "A theory of demand for products distinguished by place of production," IMF Staff Papers 16, 159-176 (1969).

[2] Bairoch, P., *De Jéricho à Mexico: Villes et économie dans l'histoire*: Editions Gillimard *(1985)* Translated by C. Braider to *Cities and Economic Development: From the Dawn of History to the Present*, Chicago: The University of Chicago Press (1988).

[3] Bénabou, R., "Equity and efficiency in human capital investment: the local connection," *Review of Economic Studies* 63(2), 237-264 (1996).

[4] Bénabou, R., "Heterogeneity, stratification, and growth: macroeconomic implications of community structure and school finance," *American Economic Review*, 86(3), 584-609 (1996).

[5] Black, D., "Local knowledge spillovers and inequality," mimeo. (1999).

[6] Black, D. and V. Henderson, "Urban evolution in the USA," mimeo. (1998).

[7] Black, D. and V. Henderson, "A theory of urban growth", *Journal of Political Economy*, 107, 252-284 (1999).

[8] Borjas, G.J., S.G. Bronars and S.J. Trejo, "Self-selection and internal migration," *Journal of Urban Economics*, 32, 159-185 (1992).

[9] Ciccone, A., "Dynamic externalities and the spatial distribution of wages in the US", mimeo, University of California, Berkeley.

[10] Ciccone, A., and G.Peri, "Human capital and externalities in cities", CEPR Discussion Paper No. 2599 (2000).

[11] Ciccone, A., G. Peri and D. Almond, "Capital, wages, and growth: Theory and evidence," CEPR Discussion Paper No. 2199 (1999).

[12] Ciccone, A., and R. E. Hall, "Productivity and the density of economic activity", *American Economic Review*, 86, 54-70 (1996).

[13] Davis, D., and R. Weinstein, "Market access, economic geography and comparative advantage", mimeo. (1998).

[14] Dixit, A.K. and J.E. Stiglitz, "Monopolistic competition and optimum product diversity," *American Economic Review,* 67 (June), 297-308 (1977).

[15] Dobkins, L.H. and Y.M. Ioannides, "Spatial interactions among US cities," mimeo. (1996).

[16] Dobkins, L.H. and Y.M. Ioannides, "Dynamic evolution of the size distribution of US cities, in J.M. Huriot and J.-F. Thisse (eds.), *Economics of Cities*, Cambridge: Cambridge University Press (2000)

[17] Dumais, G., G. Ellison, and E. Glaeser, "Geographic concentration as a dynamic process," NBER Working Paper No.6270 (1997).

[18] Fernandez, R., "Sorting, education and inequality", NBER Working Paper No. W8101 (2001).

[19] Forslid R., and I. Wooton, "Comparative advantage and the location of production," CEPR Discussion Paper No. 2118 (1999).

[20] Fujita, M., "Spatial interactions and agglomeration in urban economics," in M. Chatterji and R.E. Kunne (eds.), *New Frontiers in Regional Science*, 185-221, London: Macmillan Publishers (1990).

[21] Fujita, M., P. Krugman and A.J. Venables, *The Spatial Economy: Cities, Regions, and International Trade*, Cambridge, MA: MIT Press (1999).

[22] Fujita, M. and T.E. Smith, "Additive-interaction models of spatial agglomeration," *Journal of Regional Science,* 30, 51-74 (1990).

[23] Gaspar, J. and E. Glaeser, "Information technology and the future of cities," *Journal of Urban Economics, 43, 136-156* (1998).

[24] Glaeser, E., "Are cities dying?," *Journal of Economic Perspectives*, 12(2), 139-160 (1998).

[25] Glaeser, E., H. Kallal, J. Scheinkman, and A. Shleifer, "Growth in cities," *Journal of Political Economy*, 100, 1126-1152 (1992).

[26] Glaeser, E., J. Scheinkman, and A. Shleifer, "Economic growth in a cross-section of cities," *Journal of Monetary Economics*, 36, 117-143 (1995).

[27] Helliwell, J.F., "National borders, trade and migration," NBER Working Paper No.6027 (1997).

[28] Jacobs, J., *The Economy of Cities*, New York: Vintage Books (1968).

[29] Japan Statistics Bureau, *Population Census of Japan* (1990)

[30] Krugman, P., "Increasing returns and economic geography," *Journal of Political Economy*, 99, 183-199 (1991).

[31] Lucas, R., "On the mechanics of economic development," *Journal of Monetary Economics*, 22, 3-42 (1988)

[32] Manasse, P. and A. Turrini, "Trade, wages, and "superstars"," CEPR Discussion Paper No.2262, *Journal of International Economics, 54, 1, 97-117* (2001).

[33] Marshall, A., *Principles of Economics*, London: McMillan (1890).

[34] Matsuyama, K., "Why are there rich and poor countries?: symmetry-breaking in the world economy," *Journal of the Japanese and International Economies* 10, 419-439 (1996).

[35] Nakamura, R. and T. Tabuchi, *Urban and Regional Economics*, Ch.12, Tokyo: Yuhikaku Books (1996).

[36] Ottaviano, G.I.P., Dynamic and Strategic Considerations in International and Interregional Trade, Ph.D. Dissertation, CORE, Louvain-la-Neuve (1998).

[37] Rauch, J., "Productivity gains from geographic concentration of human capital: Evidence from the cities", *Journal of Urban Economics*, 34, 3, 380-400 (1993).

[38] Rosen, S., "The economics of superstars," *American Economic Review,* 71, 845-858 (1981).

[39] United Nations, *World Urbanization Prospects, The 1992 Revision*, New York: United Nations (1993).

[40] Volterra, V., *Theory of Functionals and Integral and Integro-Differential Equations*, New York: Dover (1959).

[41] Wolff, E., "International comparisons of inequality," *Review of Income and Wealth*, 42, 433-451 (1996).

[42] Yamada, H. and K. Tokuoka, "Standard Metropolitan Employment Area in Japan," *Keizai Ronso*, 132(3,4), 145-173 (1983).

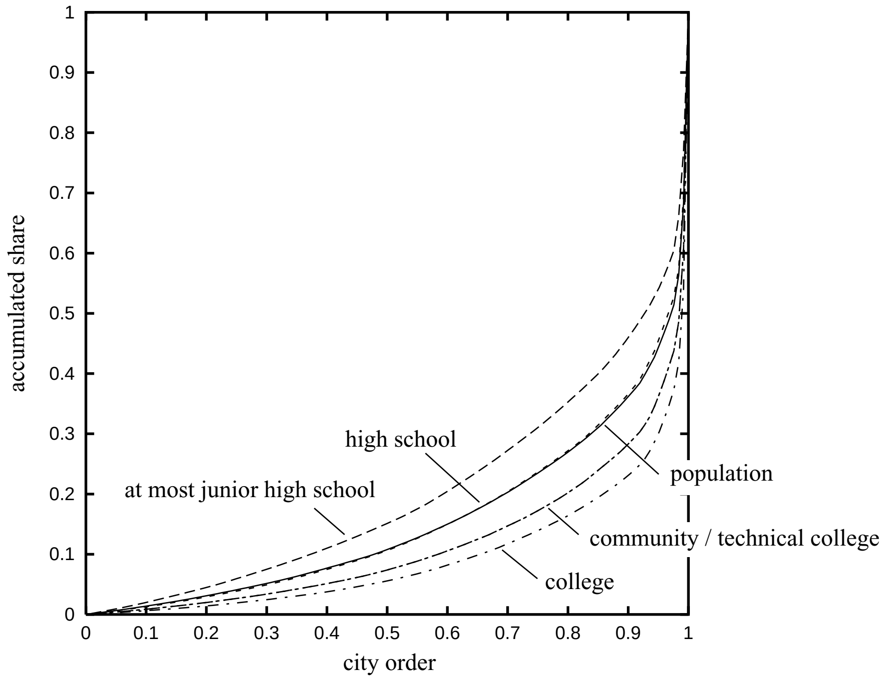


Figure 1. Lorenz curves of inter-city inequality in education level

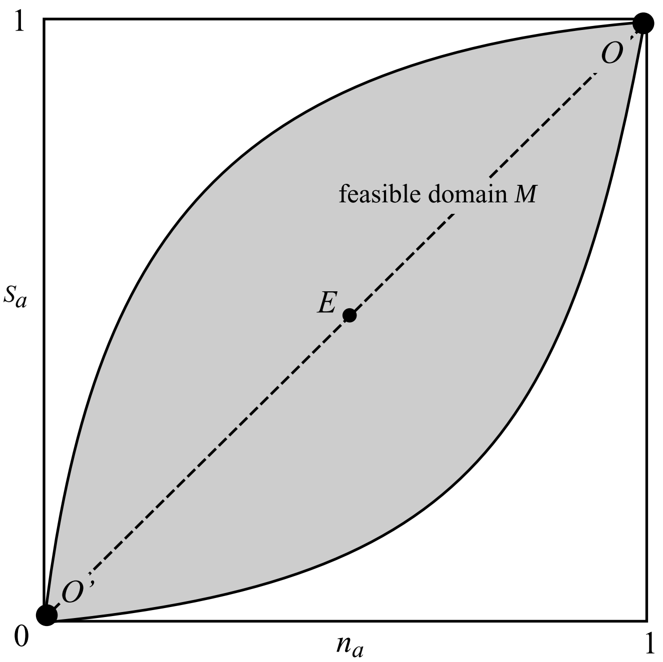


Figure 2. Interpersonal distribution of skills and
feasible interregional distribution of population and skill

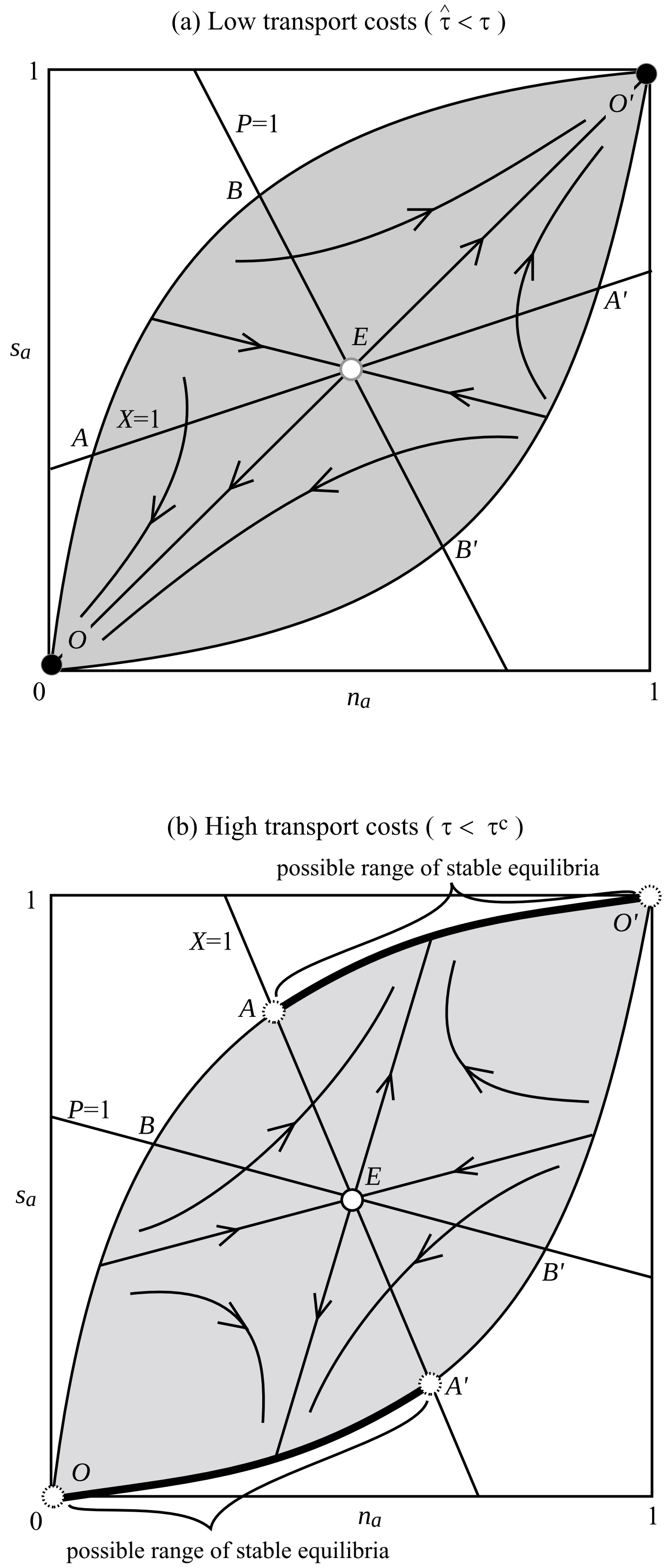


Figure 3. Global dynamics of interregional population and skill distribution

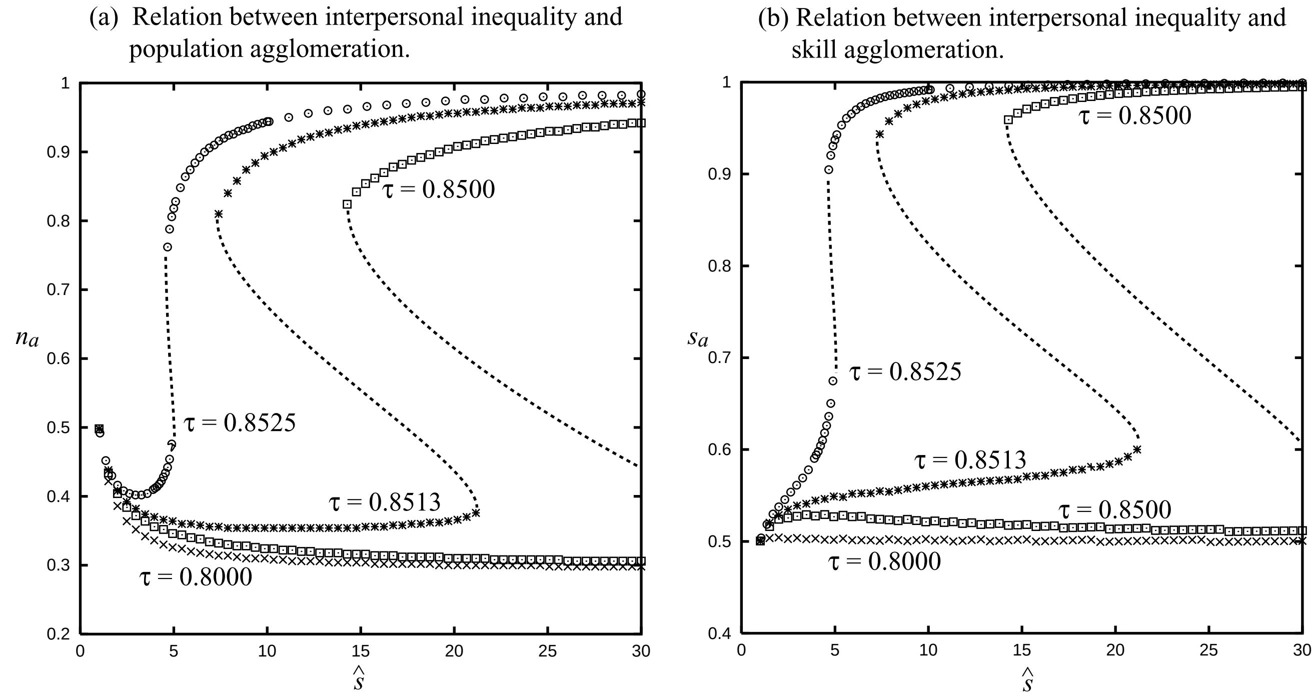


Figure 4. Agglomeration and interpersonal inequality when the interpersonal inequality is associated with the growth of aggregate skill in the economy ($\underline{s}$ = 1.0, σ = 5.0, c = 0.05)

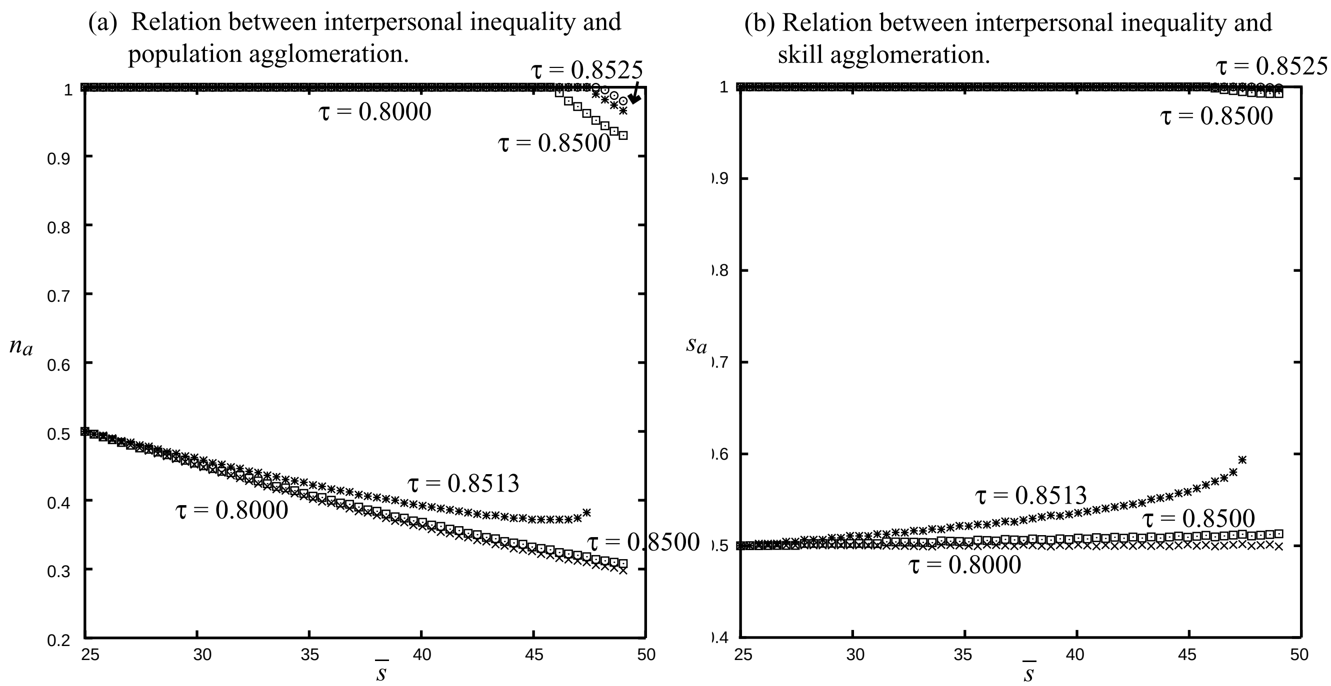
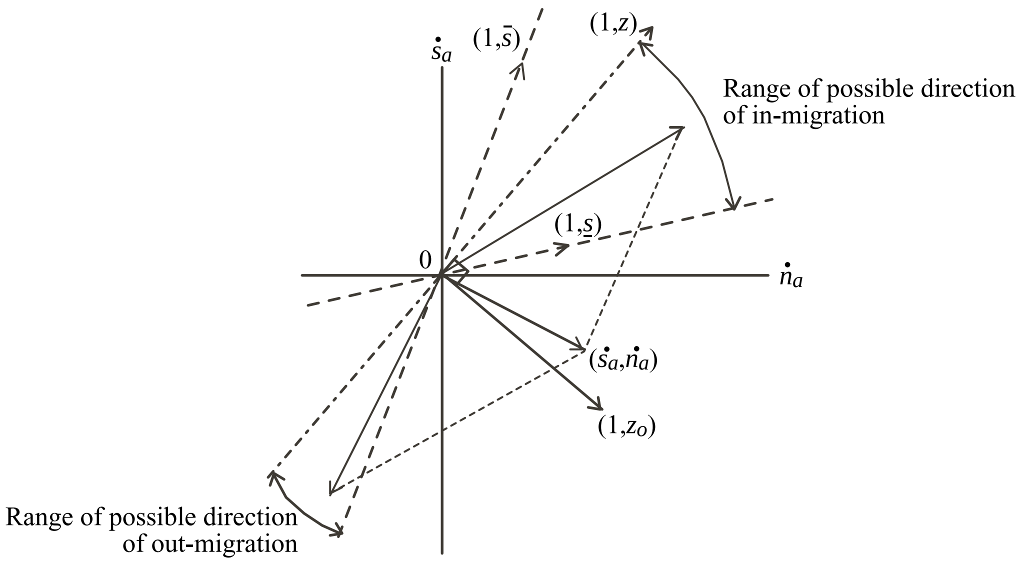
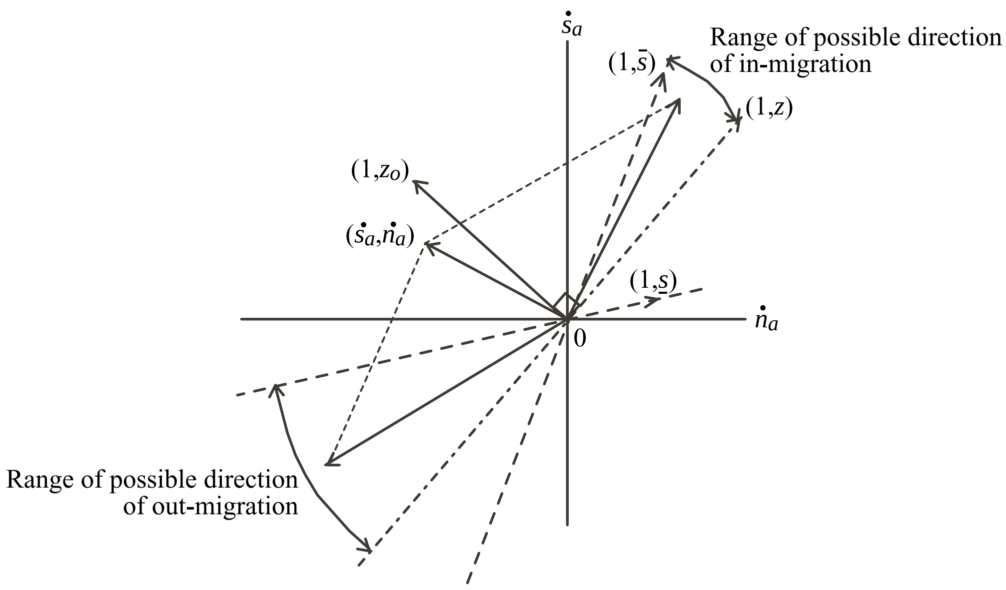


Figure 5. Agglomeration and interpersonal inequality when the aggregate skill in the economy is constant ($\hat{s}$ = 25.0, σ = 5.0, c = 0.05)



(a) Domain of type Ω_c



(b) Domain of type Ω_d

Figure 6. Adjustment direction of (n_a,s_a)

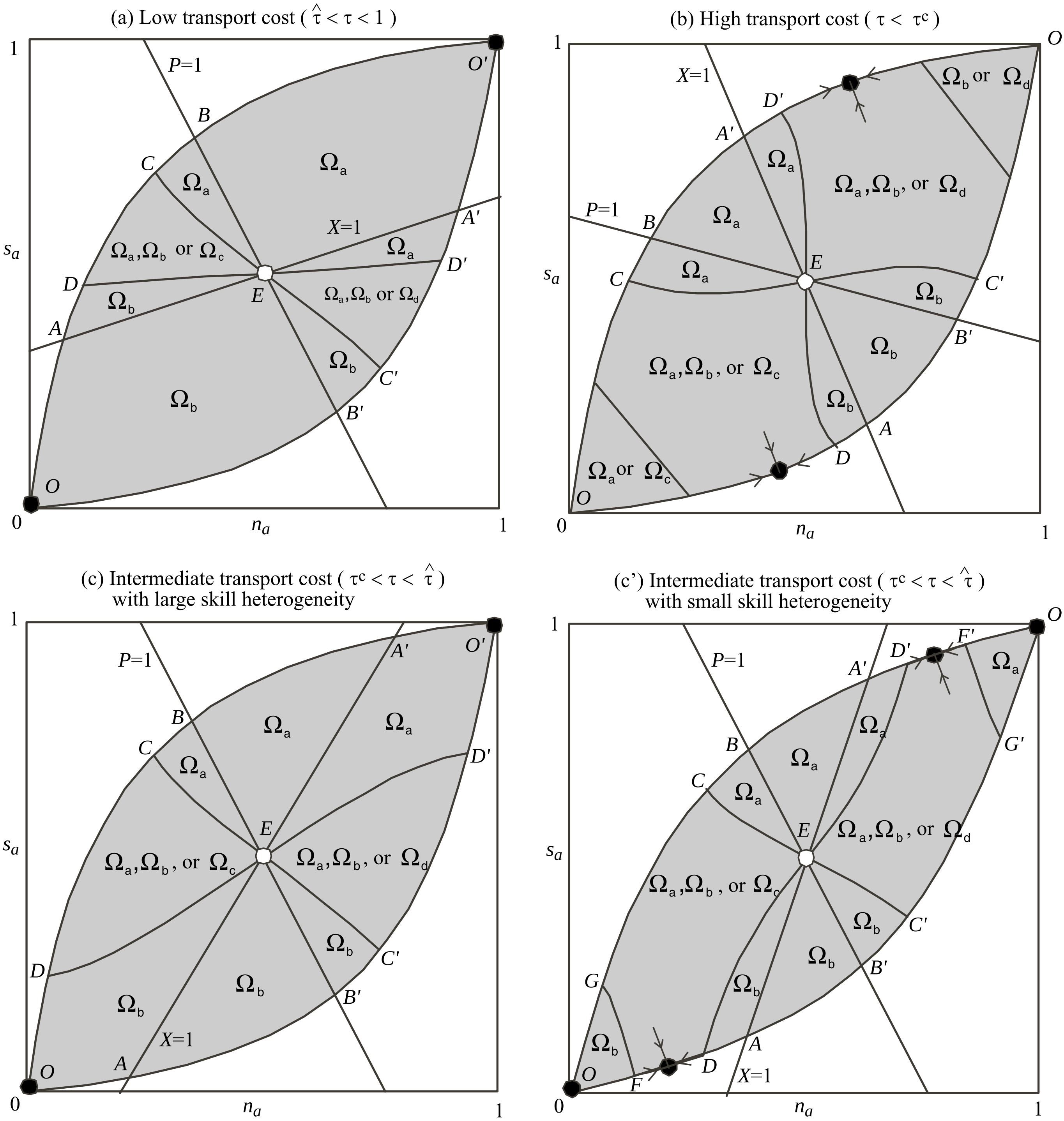


Figure 7. Decomposition of the feasible domain M, and global dynamics of interregional population/skill distribution