Optimal Contracts for Central Banker and Public Debt Policy*

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*Any views expressed in this paper are solely those of the authors, and do not necessarily represent any of the institutions.

Abstract

We examine the institutional arrangements which achieve the second-best allocation corresponding to an optimal rule under the policy commitment of a fiscal authority and a central bank, whose policies interact through a consolidated government budget constraint, under the assumption that those policy makers are unable to commit themselves to their optimal policies and they ignore the strategic interaction between their policies. Our results suggest that the practically best institutional arrangement is an instrument-independent central bank which controls for money supply to determine the rate of inflation and commits itself to some numerical inflation target that depends on fiscal variables. Although the second-best allocation could be supported by an instrument-independent central bank with a performance contract, it is practically difficult to implement a lump-sum transfer payment for a central banker. Furthermore, the second-best allocation cannot be attained by a performance contract or a targeting scheme for the fiscal authority alone. These results indicate that the numerical targets for the budget deficit together with the independent central bank, observed in the United States and the EU countries, do not ensure the good performance of the economy. The simpler and better solution is to have an independent central bank with an inflation target.

Key Words : inflation contract, inflation targeting, independent central bank.
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1. Introduction

The idea that the most important prerequisite for the conduct of monetary policy is an independent central bank now becomes a global standard among both academic researchers and policy makers. This idea provides a basis for many institutional changes including the establishment of European Central bank and the reforms of central bank laws in many countries such as England, New Zealand, Japan, and Korea as well as Transitional Economies. The influential models supporting the idea of an independent central bank in macroeconomics after the seminal paper of Rogoff [1985] are built on the models of dynamic inconsistency by Kydland and Prescott [1977] and Barro and Gordon [1983]: a monetary authority faces an incentive to expand output above the equilibrium level so that an inflationary bias exists without policy commitment. Among others, the important work of Walsh [1995] shows that there is no trade-off between the inflation bias and the stabilization of output once the society offers a linear performance contract based on the realized rate of inflation to the central banker. Svensson [1997] suggests that an inflation target, which is actually adopted in recent years by a number of industrial countries including Australia, Canada, Finland, Israel, New Zealand, Spain, Sweden, and United Kingdom, is regarded as a counterpart of the linear performance contract à la Walsh in the real world. Since Walsh [1995] and Svensson [1997] assume a stable relationship between the growth rate of money supply and the rate of inflation on average, the models of inflation targeting theoretically extend the idea of uncontingent monetary targeting advocated by Friedman [1960] and Kydland and Prescott [1977] into the contingent optimal policy plan.

Nevertheless, it remains an unresolved question of how the role of an independent central bank is affected by a strategic interaction between a fiscal authority and a central bank in the absence of their policy commitment. The purpose of this paper is to consider whether or not the practically best solution to achieve the second-best allocation corresponding to an optimal rule under the policy commitment of the two policy makers is still an independent central bank with a performance contract or a targeting scheme even if the strategic interaction between the two policy makers without their policy commitment is explicitly accounted for.

We explore the following three interesting issues by examining the strategic interaction between the two policy makers without their policy commitment. First, the fiscal authority as well as the central bank has an incentive to expand output above the equilibrium level. Thus, we need to investigate whether or not a performance contract or a targeting scheme is effective to restore the efficiency of the economy in the absence of the policy commitment of both the fiscal authority and the central bank.

Second, in addition to the non-commitment problem of the policy makers, we need to tackle a

non-coordination problem between the fiscal authority and the central bank. This problem adds new distortions if the optimal combinations of fiscal and monetary policies are not attained. Thus, we also need to have another investigation of whether a performance contract or a targeting scheme can eliminate the distortions caused by the non-coordination between the fiscal authority and the central bank.

Finally, we can discuss the relationship between the studies of central bank independence and the studies regarding budget rules (see Poterba [1997] for recent review) for the fiscal authority. The conventional idea for justifying an operationally independent central bank separated from the fiscal authority is based on the historical experience that the government pursues inflationary policy for many reasons; and as a result, the society would be better off by having a mechanism that prevents the government from issuing excessive fiat money. One plausible institutional solution to achieve this objective is an operationally independent central bank that is committed to monetary targeting or gold standard before the World War I. This view is strongly supported by the studies of public choice (for example, Buchanan and Wagner [1981]), long before the recent studies of central bank independence have appeared. Even though such a view is correct, we might well wonder if we do not need to have an independent central bank once we can impose some rules for the fiscal authority which constrain its inflationary policy. Indeed, in the EU countries, Japan, and the United States, they do have operationally independent central banks and some numerical targets for the budget deficit simultaneously. Could budget rules substituted for an independent central bank, or do we need both of them to achieve the efficient resource allocation of the economy?

To answer these questions, we extend the analysis of Beetsma and Bovenberg [1997] by allowing for the possibility that the society can offer a performance contract or a targeting scheme to the monetary authority. We model a strategic interaction between the central bank which controls for monetary policy and the fiscal authority which determines the tax rate, the government spending, and the path of public debt. In this model, we view the source of a supply shock in Walsh [1995] as a policy shock by the fiscal authority and make the consolidated government budget constraint explicit. In analyzing the model, we are concerned with two kinds of strategic interactions between the fiscal authority and the central bank in the absence of their policy commitment. One is the case in which the fiscal authority and the central bank are integrated. We call this case the “integrated agency without commitment”. The other is the case in which these two policy makers are independent of one another and they cannot coordinate their policy decisions. This case is called the “non-coordination without commitment”.

The results obtained in this paper are summarized as follows. First, the second-best allocation is still achieved by a performance contract in each of the “integrated agency without commitment”

and the “non-coordination without commitment”. The performance contract for the “integrated agency without commitment” depends on both the realized levels of the inflation rate and the public debt level, and the performance contract for the central bank under the assumption of “non-coordination without commitment” is based only on the realized inflation rate. However, the coefficient on the realized inflation rate in the second period, or the penalty to increase one percent in the rate of inflation in the second period, negatively correlates with the government financing requirements, which depends on the level of public debt, in both cases. Therefore, both the realized levels of the inflation rate and the public debt level affect the penalty schedules determined by the performance contracts.

Second, the second-best allocation is also attained by a targeting scheme consisting of both an inflation target and a public debt target under the “integrated agency without commitment”, and by a targeting scheme consisting of an inflation target alone under the “non-coordination without commitment”. The optimal inflation target also negatively correlates with the government financing requirements in each of these cases. In practice, the targeting scheme is a much simpler policy institutional arrangement than the performance contract. In particular, the targeting scheme does not require a policy instrument for financing contract transfer payments; but the performance contract does. Hence, this result implies that the targeting scheme is a more useful policy instrument than the performance contract. Furthermore, the optimal targeting scheme under the “integrated agency without commitment” involves both the inflation rate and the public debt level, while the optimal targeting scheme under the “non-coordination without commitment” involves only the inflation rate. Since it is practically difficult for the society not only to find someone who feels losses from the excessive amount of debt but also to implement the targets of the inflation rate and the public debt level simultaneously, this finding suggests that an independent central bank with an inflation target, separated with the fiscal authority, is better than a central bank integrated with the fiscal authority in order to solve for the problem of an inflationary bias. Our view is consistent with the reasoning based on the historical experience that the excessive accumulation of government debt becomes generally possible as a result of the political pressure and the lack of an operationally independent central bank which constrains such a government funding; therefore, the debt target is not credible and it is better to have an operationally independent central bank from the government financing requirement.

Finally, the second-best allocation cannot be attained by a contract for the fiscal authority under the “non-coordination without commitment”. Thus, a well disciplined fiscal authority in this sense cannot become a substitute for an independent central bank. This result shows that the numerical targets for the budget deficit together with the independent central bank, observed in the

EU countries, Japan, and the United States, do not ensure the good performance of the economy. The simpler and better solution is to have an independent central bank with an inflation target that negatively correlates with the government financing requirements to control for the excessive public spending.

Our research is related to several strands of literature. The topics of the inflation contract and targeting are rapidly emerging (see Persson and Tabellini [1993], Walsh [1995], Svensson [1997], the special issue of Journal of Monetary Economics [1997], and Jonsson [1997]). Our work differs from theirs in that it explicitly examines the government budget constraint and the strategic interaction between the fiscal authority and the central bank. Moreover, we consider the situation where the private sector, rather than the fiscal authority, can offer a performance contract or a targeting scheme for the central bank, although in some countries, the Minister of Finance negotiates with the central bank regarding the level of the inflation target.

Beetsma and Bovenberg [1997] discusses the strategic interaction between the fiscal authority and the central bank. However, they do not consider a performance contract or an inflation target. Instead, they study only the combination of the debt target and the weight-conservative, independent central bank *à la* Rogoff [1985] as a plausible optimal institutional design. On the other hand, our analysis shows that not only the performance contract but also the "inflation-target- and debt-target-conservative" integrated agency (or the "inflation-target-conservative" central bank) *à la* Svensson [1997] achieve the second-best allocation under the "integrated agency without commitment" (or the "non-coordination without commitment"). Furthermore, the "inflation-target-conservative" central bank is superior to the "inflation-target- and debt-target-conservative" integrated agency because it is practically difficult for the policy makers to implement a public debt targeting and to find someone who prefers a lower rate of inflation.

Sargent and Wallace [1981] argues that if the fiscal and monetary authorities are independent of one another, it is important to allow the monetary authority to impose a lower rate of debt accumulation on the fiscal authority, in order to assure that the monetary authority can control for the rate of inflation. Our results are much stronger than their recommendation, because our "inflation-target-conservative" central bank can achieve the second-best allocation by controlling for inflation. Since the fiscal authority takes the decision of the central bank as given if the instrument independence of the central bank is guaranteed, the fiscal authority needs to make the government spending consistent with the rate of inflation chosen by the central bank even though the fiscal authority chooses the amount of debt. This result is attributed to the assumption that the private sector (or the representative of the private sector), rather than the fiscal authority, offers a performance contract or an inflation target to the central bank.

The rest of out paper is organized as follows. Section 2.1 describes the basic model. Section 2.2 characterizes a solution of the model under the assumption that the fiscal authority and the central bank are integrated and are credibly committed to their policy announcements. This is the benchmark case and called the second-best. Section 2.3 gives a solution under the "integrated agency without commitment", where the integrated agency cannot be committed to their policy announcements. Section 2.4 studies the case of the "non-coordination without commitment", where neither their policy coordination nor their commitments to their policy announcements are possible. On the basis of the models presented in section 2, section 3 develops our main analysis. Section 3.1 discusses whether or not a performance contract for the integrated agency (or the central bank) *à la* Walsh can lead to the second-best allocation under the "integrated agency without commitment" (or the "non-coordination without commitment"). Section 3.2 examines the issue of an optimal targeting scheme in each of these two cases. Section 3.3 evaluates the practical advantages of the performance contract and targeting schemes under each of the integrated agency and the independent central bank, and investigates which combination of the policy instrument and the authority organization is more practically advantageous. Section 4 concludes our paper.

2. The Model

In this section, we analyze a two period model based on Beetsma and Bovenberg [1997], and derive the conditions for an optimal solution.

2.1. The basic model

Let us consider a game between three agents: the private sector, the fiscal authority (government), and the monetary authority (central bank).

In the private sector, nominal wages are concluded before the policy makers choose their policies. Thus, unless the policy makers announce an inflation rate and commit themselves to it at the beginning of each period before nominal wages are set, they can cause unexpected inflation to boost the economy; and the private sector acts as a Stackelberg leader for the policy makers by expecting their actions in advance. Using the arguments of Beetsma and Bovenberg [1997], we can then characterize the behavior of the private sector in period t by a Lucas supply function with government taxation:[1]

$$x_t = \nu(\pi_t - \pi_t^e - \tau_t), \quad t = 1, 2, \tag{1}$$

where x_t denotes the (normalized) output in period t, ν the constant parameter, π_t the inflation

rate in period *t*, π_t^e the inflation rate expected by the private sector in period *t*, and τ_t the tax rate in period *t*. If there exist no tax distortions (τ_t=0), the (normalized) output x_t is reduced to 0 in a rational expectations equilibrium because of $\pi_t = \pi_t^e$. Thus, this (normalized) output level corresponds to the natural rate of employment. In fact, the socially desirable (normalized) output $\tilde{x}_t$ without tax distortions in period *t* is allowed to be positive because the socially desirable employment level is assumed to exceed the natural rate of employment. Furthermore, since we allow for non-tax distortions due to union power in the labor market or monopoly power in the commodity market, we can take $\tilde{x}_t$ as the output level attained in the second-best allocation.[2]

The government budget constraint in period *t* is given by[3]

$$g_t + (1+\rho)d_{t-1} \le \tau_t + \kappa\pi_t + d_t, \qquad t = 1, 2. \tag{2}$$

Here, g_t indicates the government spending as a share of the output realized without tax distortions or inflation surprises in period *t*, ρ the real interest rate, κ the revenues from real money holdings as a share of the output realized without tax distortions or inflation surprises, and d_t the amount of newly issued public debt as a share of the output realized without tax distortions or inflation surprises in period *t*. From now on, we will assume that $0 < \kappa < 1$ throughout this paper.[4] We also assume that all public debt is indexed and matures after one period. Since d_t expresses the amount of newly issued public debt (as a share of the output realized without tax distortions or inflation surprises) in period *t*, d_{t-1} denotes the amount of public debt (as a share of the output realized without tax distortions or inflation surprises) carried over from period *t*-1 to period *t*. Because of the two period economy, all public debt is paid off at the end of period 2 so that $d_2 = 0$. Furthermore, d_0 is assumed to be given exogenously.

For convenience of the analysis, we rewrite inequality (2) to define the "government financing requirement":

$$GFR_t(d_{t-1}) \equiv \tilde{K}_t + (1+\rho)d_{t-1} - d_t \le (\tau_t + \frac{\tilde{x}_t}{\eta}) + \kappa\pi_t + (\tilde{g}_t - g_t), \tag{3}$$

where $\tilde{K}_t \equiv \tilde{g}_t + \tilde{x}_t / \eta$.

The government has perfect control over the tax rate, the government spending, and the amount of newly issued public debt in each period, whereas the central bank has perfect control over the inflation rate in each period. This assumption implies that the government can choose τ_t, g_t, and d_t in each period while the central bank can choose π_t in each period.

The society has the social loss function *V*, which is represented by

$$V = \frac{1}{2}\sum_{t=1}^{2} \beta^{t-1} V_t \equiv \frac{1}{2}\sum_{t=1}^{2} \beta^{t-1}[\alpha_\pi \pi_t^2 + (x_t - \tilde{x}_t)^2 + \alpha_g (g_t - \tilde{g}_t)^2], \qquad (4)$$

where $\alpha_\pi > 0$, $\alpha_g > 0$, and $0 < \beta \leq 1$. Here, β denotes the discount factor, $\tilde{x}_t$ the socially desirable (normalized) output in period *t*, and $\tilde{g}_t$ the government spending target as the optimal share of the output realized without tax distortions or inflation surprises in period *t*. We assume for simplicity that both the government and the central bank have the same loss function as the society.

In the subsequent analysis, we will discuss three cases to model the various aspects of the strategic interaction between the government and the central bank. First, we deal with the situation in which the government and the central bank are integrated and are credibly committed to their policy announcements. The credible commitment particularly implies that the policy makers announce an inflation rate and commit themselves to the announced rate at the beginning of each period before nominal wages are concluded. This is called the "second-best" or the benchmark case. Secondly, we investigate the case in which the government and the central bank are still integrated but are not able to commit themselves to their policy announcements. Since nominal wages are set before the policy makers choose their policies, the private sector in this case acts as a Stackelberg leader for the policy makers. As a result, when choosing the decisions, the policy makers must make do with taking the inflation expectations of the private sector as exogenously given. We call this the "integrated agency without commitment". Finally, we consider the "non-coordination without commitment". In this case, the two policy makers are independent of one another. Furthermore, they can neither coordinate their policy decisions nor commit themselves to their policy announcements. The "non-coordination without commitment" can be interpreted such that the government delegates monetary policy to a central bank with both goal and instrument independence: in other words, the government delegates monetary policy to an instrument-independent central bank that is assigned a particular loss function.[5]

2.2. Second-best: benchmark case

In this subsection, we analyze the most desirable case, where the two policy makers are integrated and are committed to their policy announcements. To solve for the two period decision problem, we use the backward induction method. Thus, we begin with solving for the solution in the second period and then proceed to solve for the solution in the first period.

2.2.1. Rational expectations equilibrium in the second period

Let us first describe the second period problem of the integrated agency.

Because of the social loss function (4), the social loss in the second period is represented by

$$\frac{1}{2}[a_{\pi}\pi_2^2 + (x_2 - \tilde{x}_2)^2 + a_g(g_2 - \tilde{g}_2)^2]. \quad (5)$$

The constraints of the second period problem consist of the Lucas supply function (1) in period 2, the government financing requirement (3) in period 2, and the restriction generated by the rational expectations formation of the private sector. To make the mathematical expression easier, let $-f_t/\nu \equiv \tau_t + \tilde{x}_t/\nu$ denote the explicit and implicit tax revenues in period t,[6] and $-h_t \equiv \tilde{g}_t - g_t$ denote the government spending gap in period t, respectively. Now, for $t = 2$, we substitute (1) into (5) and rearrange the resulting second period social loss and the government financing requirement (3) with $f_t = -\nu\tau_t - \tilde{x}_t$ and $h_t = -(\tilde{g}_t - g_t)$. Under the rational expectations of the private sector ($\pi_2^e = \pi_2$), the optimal decision problem of the integrated agency in the second period is then specified as follows:

$$\min_{\pi_2, f_2, h_2} \frac{1}{2}(a_{\pi}\pi_2^2 + f_2^2 + a_g h_2^2), \quad (6)$$

$$\textit{subject to } GFR_2(d_1) \equiv \tilde{K}_2 + (1+\rho)d_1 \le -\frac{f_2}{\nu} + \kappa\pi_2 - h_2, \quad (7)$$

$$\pi_2^e = \pi_2. \quad (8)$$

In this optimal decision problem, the strategic variables controlled by the integrated agency are made up of the inflation rate in period 2, π_2, the explicit and implicit tax revenues in period 2, $-f_2/\nu$, and the government spending gap in period 2, $-h_2$. Note that d_2 is always set to zero because all public debt is paid off at the end of period 2. Since the integrated agency announces π_2 and can commit itself to the rate, the inflation rate expected by the private sector, π_2^e, is given by π_2 in (8) under the rational expectations of the private sector. This implies that the integrated agency is prevented from causing unexpected inflation to boost the economy.

The first-order conditions for the choice of π_2, f_2, and h_2 are

$$-a_{\pi}\pi_2 + \lambda_{S2}\kappa = 0, \quad (9)$$

$$-f_2 + \lambda_{S2}\left(-\frac{1}{\nu}\right) = 0, \quad (10)$$

$$-a_g h_2 - \lambda_{S2} = 0, \qquad (11)$$

where λ_{S2} is the nonnegative Lagrange multiplier associated with (7). Let $(\pi_2, f_2, h_2) = (\pi_2^*(d_1), f_2^*(d_1), h_2^*(d_1))$ denote the solution to this problem, which satisfies (7) and (9)-(11).

2.2.2. Rational expectations equilibrium in the first period

Given the optimal decision making in the second period, $(\pi_2^*(d_1), f_2^*(d_1), h_2^*(d_1))$, we next describe the first period problem of the integrated agency by substituting (1) into the social loss function (4) and rearranging the resulting social loss function and the government financing requirement (3) in period 1 with $f_t = -\tau_t - \tilde{x}_t$ and $h_t = -(\tilde{g}_t - g_t)$:

$$\min_{\pi_1, f_1, h_1, d_1} \frac{1}{2}(a_\pi \pi_1^2 + f_1^2 + a_g h_1^2) + \frac{1}{2}\beta\{a_\pi [\pi_2^*(d_1)]^2 + [f_2^*(d_1)]^2 + a_g [h_2^*(d_1)]^2\}, \qquad (12)$$

$$\textit{subject to } GFR_1(d_0) \equiv \tilde{K}_1 + (1+\rho)d_0 - d_1 \le -\frac{f_1}{\tau} + \kappa\pi_1 - h_1, \qquad (13)$$

$$\pi_1^e = \pi_1, \qquad (14)$$

$$\pi_2^e = \pi_2^*(d_1). \qquad (15)$$

The integrated agency in period 1 can choose the inflation rate in period 1, π_1, the explicit and implicit tax revenues in period 1, $-f_1/\tau$, the government spending gap in period 1, $-h_1$, and the newly issued public debt in period 1, d_1, given $(\pi_2^*(d_1), f_2^*(d_1), h_2^*(d_1))$. The integrated agency also announces π_1 and $\pi_2^*(d_1)$, and can commit itself to them. Thus, under the rational expectations of the private sector, the inflation rates expected by the private sector, π_1^e and π_2^e, are provided by π_1 in (14) and $\pi_2^*(d_1)$ in (15), respectively. As a result, the integrated agency is again prevented from causing unexpected inflation to boost the economy.

The first-order conditions with respect to π_1, f_1, h_1, and d_1 are

$$-a_\pi \pi_1 + \lambda_{S1}\kappa = 0, \qquad (16)$$

$$-f_1 + \lambda_{S1}\left(-\frac{1}{\tau}\right) = 0, \qquad (17)$$

$$-a_g h_1 - \lambda_{S1} = 0, \qquad (18)$$

$$-\beta\left[\alpha_\pi \pi_2^*(d_1)\frac{\partial \pi_2^*(d_1)}{\partial d_1} + f_2^*(d_1)\frac{\partial f_2^*(d_1)}{\partial d_1} + \alpha_g h_2^*(d_1)\frac{\partial h_2^*(d_1)}{\partial d_1}\right] + \lambda_{S1} = 0, \qquad (19)$$

where λ_{S1} is the nonnegative Lagrange multiplier associated with (13). The rational expectations equilibrium in period 1 is then characterized by (13) and (16)-(19) relative to d_0, given $(\pi_2^*(d_1), f_2^*(d_1), h_2^*(d_1))$ that are determined by (7) and (9)-(11). As mentioned in the subsection 2.2., this equilibrium is second-best.

2.3. Integrated agency without commitment

In this subsection, we retain the assumption that the government and the central bank are integrated. However, we drop the assumption that the integrated agency announces an inflation rate and commits itself to the announced rate. Thus, the integrated agency cannot avoid a temptation to cause unexpected inflation to raise the output of the economy to the bliss point even though in the long run such an expansion is not feasible. Since nominal wages are set before the integrated agency chooses its decisions, the integrated agency takes the private sector's inflation expectation as given when choosing its decisions. Hence, the private sector acts as a Stackelberg leader for the integrated agency.

2.3.1. Rational expectations equilibrium in the second period

Because the integrated agency takes the private sector's inflation expectation π_2^e as exogenously given in the second period problem, we substitute (1) into (5) for given π_2^e and rearrange the resulting second period social loss and the government financing requirement (3) with $f_t = -\nu_t - \tilde{x}_t$ and $h_t = -(\tilde{g}_t - g_t)$ for $t = 2$. Then, we describe the decision problem of the integrated agency choosing the optimal policies in the second period:

$$\min_{\pi_2, f_2, h_2} \frac{1}{2}(\alpha_\pi \pi_2^2 + [\nu(\pi_2 - \pi_2^e) + f_2]^2 + \alpha_g h_2^2), \qquad (20)$$

$$\textit{subject to } GFR_2(d_1) \equiv \tilde{K}_2 + (1+\rho)d_1 \le -\frac{f_2}{\nu} + \kappa\pi_2 - h_2, \qquad (21)$$

$$\pi_2^e: \text{given}. \qquad (22)$$

Note that the expected inflation rate π_2^e is not usually equal to the actual inflation rate π_2 at the stage of the second period optimization problem of the integrated agency in this case.

The first-order conditions with respect to π_2, f_2, and h_2 are written by

$$-a_{\pi}\pi_2 - [\nu(\pi_2 - \pi_2^e) + f_2]\nu + \lambda_{I2}\kappa = 0, \tag{23}$$

$$-[\nu(\pi_2 - \pi_2^e) + f_2] + \lambda_{I2}(-\frac{1}{\nu}) = 0, \tag{24}$$

$$-a_g h_2 - \lambda_{I2} = 0, \tag{25}$$

where λ_{I2} is the nonnegative Lagrange multiplier associated with (21). Let $(\pi_2, f_2, h_2) = (\pi_2^{**}(d_1), f_2^{**}(d_1), h_2^{**}(d_1))$ denote the solution to this problem, which satisfies (21) and (23)-(25).

In fact, under the rational expectations equilibrium, we see $\pi_2^e = \pi_2$. Thus, we can replace (23) and (24) by the following conditions under the rational expectations equilibrium:

$$-a_{\pi}\pi_2 - f_2\nu + \lambda_{I2}\kappa = 0, \tag{23´}$$

$$-f_2 + \lambda_{I2}(-\frac{1}{\nu}) = 0. \tag{24´}$$

Note that $(\pi_2^{**}(d_1), f_2^{**}(d_1), h_2^{**}(d_1))$ does not depend on the expected inflation rate π_2^e because (21), (23´), (24´), and (25) are independent of π_2^e.

Comparing the system of (21), (23´), (24´), and (25) with that of (7) and (9)-(11), we find an extra term $-f_2\nu = (\nu t_2 + \tilde{x}_2)\nu$ in (23´). This term captures an *inflation bias in the second period under the integrated agency without commitment* because of $f_2 < 0$ from (24´) and $\lambda_{I2} > 0$.[7] The inflation bias arises from the discretionary policies chosen by the integrated agency in the second period because the integrated agency takes π_2^e as given and is induced to increase the output of the economy.

2.3.2. Rational expectations equilibrium in the first period

Given the optimal decision making in the second period, $(\pi_2^{**}(d_1), f_2^{**}(d_1), h_2^{**}(d_1))$, we now represent the first period problem of the integrated agency by substituting (1) into the social loss function (4) for given π_1^e and $\pi_2^e = \pi_2^{**}(d_1)$ and rearranging the resulting social loss function and the government financing requirement (3) in period 1 with $f_t = -\nu t_t - \tilde{x}_t$ and $h_t = -(\tilde{g}_t - g_t)$:

$$\begin{aligned}\min_{\pi_1, f_1, h_1, d_1} & \frac{1}{2}(a_{\pi}\pi_1^2 + [\nu(\pi_1 - \pi_1^e) + f_1]^2 + a_g h_1^2) \\ & + \frac{1}{2}\beta\{a_{\pi}[\pi_2^{**}(d_1)]^2 + [f_2^{**}(d_1)]^2 + a_g[h_2^{**}(d_1)]^2\},\end{aligned} \tag{26}$$

$$\textit{subject to } GFR_1(d_0) \equiv \tilde{K}_1 + (1+\rho)d_0 - d_1 \le -\frac{f_1}{\nu} + \kappa\pi_1 - h_1, \tag{27}$$

$$\pi_1^e : \text{given}, \tag{28}$$

$$\pi_2^e = \pi_2^{**}(d_1). \tag{29}$$

Several remarks on the decision problem (26) are in order. First, the integrated agency is not able to be committed to the announced rate in the first period, whereas nominal wages in the first period are set before the integrated agency chooses its decisions. Thus, the integrated agency must take the expected inflation rate π_1^e in the first period as given when choosing its decisions. Hence, the integrated agency is induced to cause unexpected inflation to boost the economy in the first period. Second, the integrated agency can choose the policy decisions in the first period before nominal wages in the second period are set. Thus, the integrated agency can anticipate the inflation rate $\pi_2^{**}(d_1)$ expected by the private sector in the second period under the rational expectations equilibrium and affect it through the policies chosen in the first period. In particular, the expected inflation rate in the second period depends on the government debt chosen by the integrated agency in the first period.

The first-order conditions with respect to π_1, f_1, h_1, and d_1 are described by

$$-a_\pi \pi_1 - [\nu(\pi_1 - \pi_1^e) + f_1]\nu + \lambda_{I1}\kappa = 0, \tag{30}$$

$$-[\nu(\pi_1 - \pi_1^e) + f_1] + \lambda_{I1}\left(-\frac{1}{\nu}\right) = 0, \tag{31}$$

$$-a_g h_1 - \lambda_{I1} = 0, \tag{32}$$

$$-\beta\left[a_\pi \pi_2^{**}(d_1)\frac{\partial \pi_2^{**}(d_1)}{\partial d_1} + f_2^{**}(d_1)\frac{\partial f_2^{**}(d_1)}{\partial d_1} + a_g h_2^{**}(d_1)\frac{\partial h_2^{**}(d_1)}{\partial d_1}\right] + \lambda_{I1} = 0, \tag{33}$$

where λ_{I1} is the nonnegative Lagrange multiplier associated with (27). The optimal solution to the problem (26) is then represented by (27) and (30)-(33) relative to π_1^e, given $(\pi_2^{**}(d_1), f_2^{**}(d_1), h_2^{**}(d_1))$ that are determined by (21), (23′), (24′), and (25).

Because of $\pi_1^e = \pi_1$ under the rational expectations equilibrium, we can actually replace (30) and (31) by

$$-a_\pi \pi_1 - f_1\nu + \lambda_{I1}\kappa = 0, \tag{30′}$$

$$-f_1 + \lambda_{I1}\left(-\frac{1}{\nu}\right) = 0. \qquad (31')$$

Then, we should notice that this solution does not depend on the expected inflation rate π_1^e because these equations and inequalities are independent of π_1^e. Thus, we only have to mention that π_1^e is given by the rational expectations of the private sector under the rational expectations equilibrium.

Comparing the system of (27), (30´), (31´), (32), and (33) with that of (13) and (16)-(19), we understand that there exists an extra term $-f_1\nu = (\nu t_1 + \tilde{x}_1)\nu$ in (30´). As in the second period case, this term again indicates an *inflation bias in the first period under the integrated agency without commitment*, given $f_1 < 0$ from (31´) and $\lambda_{I1} > 0$.[8] The inflation gap is caused from the discretionary policies chosen by the integrated agency in the first period because the integrated agency takes π_1^e as given and is induced to increase the output of the economy.

2.4. Non-coordination without commitment

We now discuss the case of the "non-coordination without commitment", in which the two policy makers are not integrated; furthermore, they can neither coordinate their policy decisions nor commit themselves to their policy announcements. Since the two policy makers cannot coordinate their policy decisions, we need to consider the decision making of the government and the central bank separately.

2.4.1. Rational expectations equilibrium in the second period

First, let us examine the behavior of the government. Since the government is assumed to be able to choose τ_t, d_t, and g_t in each period, the decision variables of the government consist of f_2 and h_2 in period 2, given $d_2 = 0$. The second period problem for the government is thus specified by

$$\min_{f_2, h_2} \frac{1}{2}\left(a_\pi \pi_2^2 + [\nu(\pi_2 - \pi_2^e) + f_2]^2 + a_g h_2^2\right), \qquad (34)$$

$$\text{subject to } GFR_2(d_1) \equiv \tilde{K}_2 + (1+\rho)d_1 \le -\frac{f_2}{\nu} + \kappa\pi_2 - h_2, \qquad (35)$$

$$\pi_2,\ \pi_2^e: \text{given}. \qquad (36)$$

Note that the government takes not only the expected inflation rate π_2^e as given but also the realized inflation rate π_2 as given because π_2 is determined by the central bank.

The first-order conditions for the choice of f_2 and h_2 are

$$-[\nu(\pi_2 - \pi_2^e) + f_2] + \lambda_{N2}(-\frac{1}{\nu}) = 0, \tag{37}$$

$$-a_g h_2 - \lambda_{N2} = 0, \tag{38}$$

where λ_{N2} is the nonnegative Lagrange multiplier associated with (35).

Next, we investigate the behavior of the central bank. Since the central bank is assumed to be able to choose π_t in each period without taking into account of the government financing requirement and the government fiscal decisions, the central bank's decision problem is represented by

$$\min_{\pi_2} \frac{1}{2}(a_\pi \pi_2^2 + [\nu(\pi_2 - \pi_2^e) + f_2]^2 + a_g h_2^2), \tag{39}$$

$$\textit{subject to } \pi_2^e \text{: given.} \tag{40}$$

The first-order condition with respect to π_2 is then

$$-a_\pi \pi_2 - [\nu(\pi_2 - \pi_2^e) + f_2]\nu = 0. \tag{41}$$

Since $\pi_2^e = \pi_2$ under the rational expectations equilibrium, we can replace equations (37) and (41) by

$$-f_2 + \lambda_{N2}(-\frac{1}{\nu}) = 0, \tag{37′}$$

$$-a_\pi \pi_2 - f_2 \nu = 0. \tag{41′}$$

The rational expectations equilibrium in the second period is now given by (35), (37′), (38), and (41′). Let $(\pi_2, f_2, h_2) = (\pi_2^{***}(d_1), f_2^{***}(d_1), h_2^{***}(d_1))$ denote a solution that satisfies (35), (37′), (38), and (41′). Note that $(\pi_2^{***}(d_1), f_2^{***}(d_1), h_2^{***}(d_1))$ does not depend on the expected inflation rate π_2^e because (35), (37′), (38), and (41′) are independent of π_2^e.

Comparing the system of (35), (37′), (38), and (41′) with that of (7) and (9)-(11), we see that the differences are reduced to only the differences in the second terms of (9) and (41′). Thus, if $\kappa = 1$, the first-order conditions of these two cases are identical because it follows from (10) and (37′) that $\lambda_{N2} = -f_2\nu = \lambda_{S2}$. Indeed, we assume $0 < \kappa < 1$ due to the statistical findings. Hence, these differences exist and reflect *an inflation bias in the second period under the non-coordination without commitment*. The inflation bias is caused by the lack of not only commitment but also policy coordination.

2.4.2. Rational expectations equilibrium in the first period

Given $(\pi_2^{***}(d_1), f_2^{***}(d_1), h_2^{***}(d_1))$, the government in the first period faces the following problem:

$$\min_{f_1,h_1,d_1} \frac{1}{2}(a_\pi \pi_1^2 + [\nu(\pi_1 - \pi_1^e) + f_1]^2 + a_g h_1^2) + \frac{1}{2}\beta\{a_\pi[\pi_2^{***}(d_1)]^2 + [f_2^{***}(d_1)]^2 + a_g[h_2^{***}(d_1)]^2\}, \tag{42}$$

$$\text{subject to } GFR_1(d_0) \equiv \tilde{K}_1 + (1+\rho)d_0 - d_1 \le -\frac{f_1}{\nu} + \kappa\pi_1 - h_1, \tag{43}$$

$$\pi_1,\ \pi_1^e : \text{given}, \tag{44}$$

$$\pi_2^e = \pi_2^{***}(d_1). \tag{45}$$

Several remarks on the decision problem (42) are in order. First, the government cannot choose an inflation rate in the first period while nominal wages in the first period are set before the government chooses his decision. Thus, the government takes both the realized inflation and expected inflation rates in the first period as given when choosing his decisions. Second, the government selects the policy decisions in the first period before nominal wages in the second period are set. Thus, the government anticipates the inflation rate expected by the private sector in the second period under the rational expectations equilibrium. In particular, the government can affect the expected inflation rate in the second period by adjusting the amount of the first period government debt.

The first-order conditions with respect to f_1, h_1, and d_1 are

$$-[\nu(\pi_1 - \pi_1^e) + f_1] + \lambda_{N1}(-\frac{1}{\nu}) = 0, \tag{46}$$

$$-a_g h_1 - \lambda_{N1} = 0, \tag{47}$$

$$-\beta\left[a_\pi \pi_2^{***}(d_1)\frac{\partial \pi_2^{***}(d_1)}{\partial d_1} + f_2^{***}(d_1)\frac{\partial f_2^{***}(d_1)}{\partial d_1} + a_g h_2^{***}(d_1)\frac{\partial h_2^{***}(d_1)}{\partial d_1}\right] + \lambda_{N1} = 0, \tag{48}$$

where λ_{N1} is the nonnegative Lagrange multiplier associated with (43).

On the other hand, given $(\pi_2^{***}(d_1), f_2^{***}(d_1), h_2^{***}(d_1))$, the central bank's decision problem in

the first period is characterized by

$$\min_{\pi_1} \frac{1}{2}(a_\pi \pi_1^2 + [\nu(\pi_1 - \pi_1^e) + f_1]^2 + a_g h_1^2) \tag{49}$$
$$+ \frac{1}{2}\beta\{a_\pi [\pi_2^{***}(d_1)]^2 + [f_2^{***}(d_1)]^2 + a_g [h_2^{***}(d_1)]^2\},$$

$$\textit{subject to}\ \ \pi_1^e : \text{given}, \tag{50}$$

$$\pi_2^e = \pi_2^{***}(d_1). \tag{51}$$

The remarks similar to those on the government decision problem (42) can also be applied to this problem, except that the central bank can select the inflation rate in period 1.

The first-order condition with respect to π_1 is then

$$-a_\pi \pi_1 - [\nu(\pi_1 - \pi_1^e) + f_1]\nu = 0. \tag{52}$$

Because of $\pi_1^e = \pi_1$ under the rational expectations equilibrium, the conditions (46) and (52) can be substituted by

$$-f_1 + \lambda_{N1}(-\frac{1}{\nu}) = 0, \tag{46´}$$

$$-a_\pi \pi_1 - f_1\nu = 0. \tag{52´}$$

The rational expectations equilibrium in the first period is now characterized by (43), (46´), (47), (48), and (52´) relative to π_1^e, given $(\pi_2^{***}(d_1), f_2^{***}(d_1), h_2^{***}(d_1))$ that are determined by (35), (37´), (38), and (41´). We should notice that this solution does not depend on the expected inflation rate π_1^e because these equations and inequalities are independent of π_1^e. Thus, we only have to state that π_1^e is determined by the rational expectations of the private sector under the rational expectations equilibrium.

Comparing the system of (43), (46´), (47), (48), and (52´) with that of (13) and (16)-(19), we see that the differences between these two cases are reduced to the differences in the second terms of (16) and (52´). Since (17) and (46´) show that $\lambda_{N1} = -f_1\nu = \lambda_{S1}$, the first-order conditions of the second-best and those of "non-coordination without commitment" are identical if $\kappa = 1$. However, as explained before, based on the empirical evidence, we assume $0 < \kappa < 1$. Hence, these differences exist and indicate *an inflation bias in the first period under the non-coordination without commitment*. The inflation bias is caused by the lack of not only commitment but also policy coordination.

3. Optimal Performance Contracts and Targeting Schemes for Policy Makers

In the preceding section, we have discussed the features of the model of Beetsma and Bovenberg [1997], which is concerned with the strategic interaction between the government (fiscal authority) and the central bank. Beetsma and Bovenberg [1997] shows that the second-best equilibrium allocation in this economy is restored by the combination of an optimal debt target and a decentralized, independent central banker with the appropriate degree of conservatism à la Rogoff [1985]. Instead, in this section, we first consider whether or not the second-best equilibrium allocation can be attained by a performance contract à la Walsh [1995] offered by the representative of the private sector (or the Congress) to the integrated agency in the case of the "integrated agency without commitment"[9] or to the central bank in the case of the "non-coordination without commitment". We next explore whether or not the second-best equilibrium allocation can also be achieved by a targeting scheme in these two distortionary cases. Finally, for practical reasons, we argue that the best solution among these schemes including the one suggested by Beetsma and Bovenberg [1997] is a decentralized, independent central bank with an inflation target.

3.1. Optimal performance contracts

Since the loss function is quadratic, we only have to examine a simple class of performance contracts. Let $w_{It} = w_{It}^{a} + w_{It}^{\pi}\pi_t + w_{It}^{d}d_t + w_{It}^{d^2}d_t^2$ denote a contract transfer payment from the government budget to the integrated agency in the case of the "integrated agency without commitment", and $w_{Nt} = w_{Nt}^{a} + w_{Nt}^{\pi}\pi_t$ denote a contract transfer payment from the government budget to the central bank in the case of the "non-coordination without commitment", respectively.

In fact, since all the variables in the model are verified, we can show that there are many contracts that would achieve the desired result: for example, the second-best allocation can be achieved by any contract that imposes a large penalty on the integrated agency (or the central bank) if (π_t, d_t) (or π_t alone) deviates from the desired level.[10] However, as Walsh [1995] has argued, such knife-edge solutions are of little practical interest. Although our model does not assume uncertainty in the aggregate supply shock, these knife-edge solutions cause the Congress to have difficulty specifying a complete set of rules to follow under all contingencies if the Congress cannot verify the aggregate supply shock correctly in the actual economy. We therefore focus on the class of linear or quadratic performance contracts with respect to the inflation rate and the public

debt level because this class of contracts has practical advantages over the other complicated contracts.

We also assume that the contract transfer payment is completely financed by the lump-sum tax τ_{It}^{C} (τ_{Nt}^{C}) under the "integrated agency without commitment" ("non-coordination without commitment"). Since $w_{kt} = \tau_{kt}^{C}$ for $k = I, N$ and $t = 1, 2$, we still represent the government financing requirement constraints as (21) and (27) ((35) and (43)) under the "integrated agency without commitment" ("non-coordination without commitment").

The performance contract adds benefits contingent on the inflation rate and the public debt amount (or the inflation rate alone) to the loss function of the integrated agency (or the central bank). We assume that the integrated agency and the central bank care about both the transfer they receive and the social loss generated by inflation, output, and government spending fluctuations. We also assume that the preferences of the integrated agency and the central bank are separable in income and social loss, and that these agents are risk neutral with respect to income. Then, the integrated agency is assigned by the following utility function: $U \equiv \sum_{t=1}^{2} \beta^{t-1}[w_{It} - \frac{1}{2}V_t]$. Similarly, the central bank has the following utility function: $U \equiv \sum_{t=1}^{2} \beta^{t-1}[w_{Nt} - \frac{1}{2}V_t]$.

Since we assume that the reservation utility level of the integrated agency or the central bank is normalized to zero in each period, we must consider the participation constraint for the integrated agency or the central bank which motivates them to participate in the performance contract. This requirement is given by

$$U = \sum_{t=1}^{2} \beta^{t-1}[w_{kt} - \frac{1}{2}V_t] \geq 0, \qquad k = I \text{ or } N. \tag{53}$$

The participation constraint is only used to determine the constant term of the performance contract, w_{kt}^{a} for $k = I, N$.

Given the modified objective function and the participation constraint (53), we now discuss how the Congress can design a performance contract to attain the second-best equilibrium allocation in each of the cases of the "integrated agency without commitment" and the "non-coordination without commitment". To this end, let us notice that the government and the central bank under a performance contract with lump-sum taxes τ_{kt}^{C} face the same constraints as those in the absence of any performance contracts, and that the participation constraint under a performance contract is only used to determine the constant term of the performance contract.

Due to these remarks, under the "integrated agency without commitment", the integrated agency to which a performance contract is offered solves the following problems in periods 1 and 2:

$$\min_{p_2, f_2, h_2} \{\frac{1}{2}(a_p p_2^2 + [n(p_2 - p_2^e) + f_2]^2 + a_g h_2^2) - w_{I2}^p p_2\}, \tag{54}$$

subject to (21) and (22);

and

$$\min_{p_1, f_1, h_1, d_1} \{\frac{1}{2}\{a_p p_1^2 + [v(p_1 - p_1^e) + f_1]^2 + a_g h_1^2\} - w_{I1}^p p_1 - w_{I1}^d d_1 - w_{I1}^{d^2} d_1^2 \tag{55}$$

$$+\frac{1}{2}b\{a_p [p_{C2}^{**}(d_1)]^2 + [f_{C2}^{**}(d_1)]^2 + a_g [h_{C2}^{**}(d_1)]^2\} - b w_{I2}^p p_{C2}^{**}(d_1)\},$$

subject to (27), (28), and (29),

where $(p_{C2}^{**}(d_1), f_{C2}^{**}(d_1), h_{C2}^{**}(d_1))$ is an optimal solution to (54).[11] Note that the minimization of the loss function of the integrated agency is equivalent to the maximization of the utility function of the integrated agency.

Similarly, under the "non-coordination without commitment", the central bank to which a performance contract is offered solves the following problems in periods 1 and 2:

$$\min_{p_2}\{\frac{1}{2}(a_p p_2^2 + [n(p_2 - p_2^e) + f_2]^2 + a_g h_2^2) - w_{N2}^p p_2\}, \tag{56}$$

subject to (40);

and

$$\min_{p_1} \{\frac{1}{2}\{a_p p_1^2 + [v(p_1 - p_1^e) + f_1]^2 + a_g h_1^2\} - w_{N1}^p p_1 \tag{57}$$

$$+ \frac{1}{2}b\{a_p [p_{C2}^{***}(d_1)]^2 + [f_{C2}^{***}(d_1)]^2 + a_g [h_{C2}^{***}(d_1)]^2\} - b w_{N2}^p p_{C2}^{***}(d_1)\},$$

subject to (50) and (51),

where $(p_{C2}^{***}(d_1), f_{C2}^{***}(d_1), h_{C2}^{***}(d_1))$ is an optimal solution to (56).[12] Again, note that the minimization of the loss function of the central bank is equivalent to the maximization of the utility function of the central bank. We should also notice that in this case, the government's problems in periods 1 and 2 are the same as those of the minimization problems (34) and (42), respectively.[13]

Solving these problems by the procedure explained in the Appendix A, we see that the second-best allocation can be attained using the performance contract $(\hat{w}_{I1}^{\rho}, \hat{w}_{I1}^{d}, \hat{w}_{I1}^{d^2}, \hat{w}_{I2}^{\rho})$ under the "integrated agency without commitment" and the performance contract $(\hat{w}_{N1}^{\rho}, \hat{w}_{N2}^{\rho})$ under the "non-coordination without commitment":

$$\hat{w}_{I2}^{\rho} = -\frac{GFR_2(d_1)}{C}, \tag{58}$$

$$\hat{w}_{I1}^{\rho} = -\beta(1+r)\delta\frac{\tilde{F}}{C}, \tag{59}$$

$$\hat{w}_{I1}^{d} = \frac{2\kappa}{a_{\rho}C^2}\beta(1+r)\tilde{K}_2, \tag{60}$$

$$\hat{w}_{I1}^{d^2} = \frac{\kappa}{a_{\rho}C^2}\beta(1+r)^2, \tag{61}$$

$$\hat{w}_{N2}^{\rho} = -(1-\kappa)\frac{GFR_2(d_1)}{C}, \tag{62}$$

$$\hat{w}_{N1}^{\rho} = -(1-\kappa)\beta(1+r)\delta\frac{\tilde{F}}{C}, \tag{63}$$

where

$$C = \frac{\kappa^2}{a_{\rho}} + \frac{1}{\nu^2} + \frac{1}{a_g}, \tag{64}$$

$$\delta = \frac{1+r}{\beta(1+r)^2+1}, \tag{65}$$

$$\tilde{F} = (1+r)d_0 + \tilde{K}_1 + \frac{\tilde{K}_2}{1+r} = GFR_1(d_0) + \frac{GFR_2(d_1)}{1+r}. \tag{66}$$

Note that the constant term $\hat{w}_{It}^{a}$ ($\hat{w}_{Nt}^{a}$) for t = 1, 2 is determined so as to satisfy the participation constraint for the integrated agency (central bank). Since we have two unknown variables in the single equation, we can only determine the intertemporal ratio of the constant terms, $\hat{w}_{I2}^{a}/\hat{w}_{I1}^{a}$ ($\hat{w}_{N2}^{a}/\hat{w}_{N1}^{a}$). From now on, the performance contract that achieves the second-best allocation in each distortionary case is defined as an optimal performance contract.

Several remarks on the optimal performance contracts are in order. First, equations (58), (59), and (62)-(66) show that the coefficient on the inflation rate in period 2 is negatively related to the government financing requirement in period 2; and the coefficient on the inflation rate in period 1 is also negatively related to the discounted present value of the government financing requirements from period 1 to period 2. This implies that, as the government financing requirement is larger, the slope of the optimal performance contract with respect to the inflation rate becomes steeper. Note that, since we assume $0 < \kappa < 1$, this finding holds regardless of whether the Congress deals with the situation of the "integrated agency without commitment" or the "non-coordination without commitment".[14] The optimal performance contracts suggested by Walsh [1995] and Svensson [1997] do not share this feature because they consider neither the fiscal authority nor the government financing requirement constraint.

Second, equations (60), (61), and (64)-(66) indicate that the coefficients on both the linear and quadratic terms of the debt level in period 1 are nonzero under the "integrated agency without commitment". Furthermore, given the definition of $GFR_2(d_1)$ (see (7)), it also follows from (60) and (61) that

$$\hat{w}_{I1}^{d} + 2\hat{w}_{I1}^{d^2} d_1 = \frac{2\kappa}{a_\pi C}\beta(1+\rho)\frac{GFR_2(d_1)}{C}. \qquad (67)$$

This equation might imply that under the "integrated agency without commitment", the slope of the optimal performance contract with respect to the debt level in period 1 is positively related to the government financing requirement in period 2. Indeed, this 'direct' effect cancels out the effect of a change in d_1 on the contract transfer payment with respect to the inflation rate in period 2, $\hat{w}_{I2}^{\pi}\pi_{C2}^{**}(d_1)$, as shown in (A15) in the Appendix A.

Finally, these optimal performance contracts depend on several parameters that are estimated and calculated by actual data: the government financing requirement in each period, $GFR_t(d_{t-1})$, the preference parameters of the society, (a_π, a_g), the inverse of the velocity of money, κ, the production elasticity of labor, $\theta \equiv \eta/(1+\eta)$,[15] the discount factor of the society, β, and the interest rate, ρ.

3.2. Implementation of performance contract through targeting schemes

The analysis in subsection 3.1 makes theoretically clear that the second best allocation could be supported by the performance contract. However, there is neither a central bank governor nor government official who actually faces such a contract. Svensson [1997] suggests that the performance contract proposed by Walsh [1995] is implemented by the inflation targeting, which is

becoming very common in the real world. In this subsection, we would like to examine how the suggestion of Svensson [1997] is modified in our model. In particular, we consider if we can regard the optimal performance contracts we have examined in subsection 3.1 as targeting schemes. We admit that there could be other ways to achieve the second best allocation once we discuss general classes of targeting schemes. We will not try to prove that our targeting scheme is the unique institutional arrangement which achieves the second best allocation within the general classes of targeting schemes. Rather, we show that the optimal performance contract considered in the previous section could be implemented by a simple targeting scheme, which is practically more interesting. We believe that our restriction to a class of targeting scheme is justified because the targeting schemes we will analyze in the subsequent analysis can always support the second best allocation of resources.

Under the "integrated agency without commitment", it follows from the discussions of the Appendix B with (58)-(61) that the second-best outcome is achieved by an integrated agency which solves the following targeting problem in each period:

$$\min_{\pi_2, f_2, h_2} \frac{1}{2}\{ a_\pi (\pi_2 - \tilde{\pi}_{I2})^2 + [\nu(\pi_2 - \pi_2^e) + f_2]^2 + a_g h_2^2 - \frac{(\hat{w}_{I2}^\pi)^2}{a_\pi} \}, \qquad (68)$$

subject to (21) and (22);

and

$$\min_{\pi_1, f_1, h_1, d_1} \frac{1}{2}\{ a_\pi (\pi_1 - \tilde{\pi}_{I1})^2 - Q_1 (d_1 - \tilde{d}_{I1})^2 + [\nu(\pi_1 - \pi_1^e) + f_1]^2 + a_g h_1^2 \} + Q_2 \qquad (69)$$

$$+ \frac{1}{2}\beta\{ a_\pi [\pi_{T2}^{**}(d_1) - \tilde{\pi}_{I2}]^2 + [f_{T2}^{**}(d_1)]^2 + a_g [h_{T2}^{**}(d_1)]^2 \};$$

subject to (27), (28), and (29).

Here, $(\pi_{T2}^{**}(d_1),\ f_{T2}^{**}(d_1), h_{T2}^{**}(d_1))$ is an optimal solution to (68),[16]

$$Q_1 = \hat{w}_{I1}^{d^2} + \frac{\beta(1+\rho)^2}{2a_\pi C^2},$$

$$Q_2 = -\frac{(\hat{w}_{I1}^\pi)^2}{2a_\pi} + \frac{[\hat{w}_{I1}^d + \frac{\beta(1+\rho)\tilde{K}_2}{a_\pi C^2}]^2}{4[\hat{w}_{I1}^{d^2} + \frac{\beta(1+\rho)^2}{2a_\pi C^2}]} - \frac{\beta(\tilde{K}_2)^2}{2a_\pi C^2};$$

and the relevant inflation target for $t = 1, 2$ and the relevant debt target for $t = 1$ are

$$\tilde{\pi}_{It} = \frac{\hat{w}^{\pi}_{It}}{a_{\pi}}, \qquad t = 1, 2, \tag{70}$$

$$\tilde{d}_{I1} = -\frac{\hat{w}^{d}_{I1} + \frac{\beta(1+\rho)\tilde{K}_2}{a_{\pi}C^2}}{2[\hat{w}^{d^2}_{I1} + \frac{\beta(1+\rho)^2}{2a_{\pi}C^2}]} = -\frac{\tilde{K}_2}{1+\rho} = d_1^* + \frac{C\hat{w}^{\pi}_{I1}}{(1+\rho)^2\beta} = d_1^* + \frac{a_{\pi}C\tilde{\pi}_{I1}}{(1+\rho)^2\beta}, \tag{71}$$

where d_1^* is the second-best level of debt and $-\tilde{K}_2/(1+\rho)$ is the first-best level of debt, which is derived from (3) at $t = 2$ under the evaluation of $d_2 = \tau_2 = \tilde{x}_2 = \pi_2 = 0$ and $g_2 = \tilde{g}_2$. Note that $\hat{w}^{\pi}_{I1}$, $\hat{w}^{d}_{I1}$, and $\hat{w}^{d^2}_{I1}$ consist of the exogenous parameters, as shown in (59)-(61). We should also notice that the integrated agency takes the residual term in (68), $-(\hat{w}^{\pi}_{I2})^2/a_{\pi}$, as exogenous when choosing its decision in period 2. The same remark can be applied to Q_2 of (69). Furthermore, the derivation procedure in the Appendix B ensures that the second-period targeting problem (68) is dynamically consistent with the first-period targeting problem (69).

Under the "non-coordination without commitment", it follows from (62) and (63) that the second-best outcome is achieved by a central bank that solves the following targeting problem in each period:

$$\min_{\pi_2} \frac{1}{2}\{a_{\pi}(\pi_2 - \tilde{\pi}_{N2})^2 + [\nu(\pi_2 - \pi_2^e) + f_2]^2 + a_g h_2^2 - \frac{(\hat{w}^{\pi}_{N2})^2}{a_{\pi}}\}, \tag{72}$$

subject to (40);

and

$$\min_{\pi_1} \frac{1}{2}\{a_{\pi}(\pi_1 - \tilde{\pi}_{N1})^2 + [\nu(\pi_1 - \pi_1^e) + f_1]^2 + a_g h_1^2 - \frac{(\hat{w}^{\pi}_{N1})^2}{a_{\pi}}\} \tag{73}$$

$$+ \frac{1}{2}\beta\{a_{\pi}[\pi^{***}_{T2}(d_1) - \tilde{\pi}_{N2}]^2 + [f^{***}_{T2}(d_1)]^2 + a_g[h^{***}_{T2}(d_1)]^2 - \frac{(\hat{w}^{\pi}_{N2})^2}{a_{\pi}}\},$$

subject to (50) and (51).

Here, $(\pi^{***}_{T2}(d_1), f^{***}_{T2}(d_1), h^{***}_{T2}(d_1))$ is an optimal solution to (72);[17] and the relevant inflation target is

$$\tilde{\pi}_{Nt} = (1-\kappa)\frac{f_t\eta}{a_{\pi}} = \frac{\hat{w}^{\pi}_{Nt}}{a_{\pi}}, \qquad t = 1, 2. \tag{74}$$

We should add several comments on these targeting schemes. First, the relevant inflation target under the "integrated agency without commitment", $\tilde{\pi}_{It}$, becomes negative because $f_t < 0$ from (24′), (31′), and $\lambda_{It} > 0$.[18] Similarly, given the assumption of $0 < k < 1$, the relevant inflation target under the "non-coordination without commitment", $\tilde{\pi}_{Nt}$, is also negative due to $f_t < 0$ from (37′), (46′), and $\lambda_{Nt} > 0$.[19] These findings suggest that the negative inflation target scheme should be used under both the "integrated agency without commitment" and the "non-coordination without commitment".

Second, Svensson [1997] shows that the constant inflation target eliminates the constant inflation bias à la Walsh [1995] and attains the second-best outcome. On the other hand, our targeting solutions in the two distortionary cases share the same characteristics as those in the lagged employment case of Svensson [1997]; that is, the relevant inflation target changes over time. More specifically, it follows from (58), (59), (62)-(66), (70), and (74) that the optimal inflation target in period 2 inversely varies with the government financing requirement in period 2; and that the optimal inflation target in period 1 also inversely varies with the discounted present value of the government financing requirements from period 1 to period 2. This is so because the integrated agency (central bank) cares about both the inflation-output trade off and the government spending gap under the "integrated agency without commitment" ("non-coordination without commitment"). Thus, although our loss function implies that the zero inflation rate is the first-best inflation rate, these findings lead us to conclude that the integrated agency (central bank) should choose a time-varying negative inflation target under the "integrated agency without commitment" ("non-coordination without commitment").

Third, the final equation of (71) suggests that the relevant debt target level under the "integrated agency without commitment" is equal to (second-best level of debt) + (constant)$\times$(negative inflation target). This implies that the optimal debt targeting level in this case is less than the second-best level of debt: in particular, the optimal debt targeting level is exactly equal to the first-best level of debt.

Finally, the most important finding in these targeting schemes is that under the "integrated agency without commitment", not only the inflation targeting scheme but also the debt targeting scheme is needed to attain the second-best allocation; on the other hand, under the "non-coordination without commitment", only the inflation target scheme is required to achieve the second-best allocation.

3.3. Discussion

We have shown in subsection 3.1 that we can achieve the second-best allocation by a performance contract for each of the integrated agency and the independent central bank. The results of subsection 3.2 also imply that the performance contracts we have considered in subsection 3.1 could be interpreted either as the debt and inflation target for the integrated agency or the inflation target for the independent bank. In this subsection, we argue that the inflation target for the independent central bank is the most attractive among those four institutional frameworks which achieve the second-best allocation. We first point out that the targeting schemes are more attractive than the performance contracts. We then investigate the pros and cons of the debt and inflation target for the integrated agency versus the inflation target for the independent bank.

We begin by arguing that the targeting schemes are practically easier to be implemented than the performance contracts because our performance contract schemes require that the contract transfer payments must be completely financed by lump-sum taxes. Since it is practically impossible to impose lump-sum taxes, the Congress must actually have distortionary taxes for financing the contract transfer payments. This tax requirement may prevent the integrated agency or the central bank from attaining the second-best allocation. In contrast, none of our targeting schemes make it necessary for the Congress to consider the financing requirements. Our analysis thus suggests that the targeting scheme has another financial advantage over the performance contract scheme in addition to the practical and political difficulties mentioned in the previous literature (see McCallum [1995] and King [1997]).

We next discuss that the inflation target by the central bank under the "non-coordination without commitment" is more practically attractive than the inflation and debt target by the "integrated agency without commitment". First, the integrated agency must implement both the inflation and debt targets simultaneously. Thus, it has difficulty obtaining the credibility of its policy from the general public because these two targets are mutually interacted, and because historical experiences indicate that the debt target is subject to various political pressures. Second, in order to attain the second-best resource allocation by the debt target rule (71), we need to find someone who feels losses from the excessive amount of debt in comparison with the first-best level of debt.

In contrast, our instrument-independent central bank, which is separated from the government financing requirement, only announces a numerical inflation target and needs to be accountable only for the realized rate of inflation. Therefore, it is more likely that such a transparent and simple framework of policy making is socially desirable.

Some readers might wonder whether such an inflation target can ever be successful if the central bank takes the decision of the fiscal authority as given because in our model the debt level affects

the rate of inflation in equilibrium. Indeed, Sargent and Wallace [1981] clearly indicates that the central bank is likely to lose the credibility of the monetary control and the inflation target if the central bank must use an inflation tax to accommodate fiscal expenditures. However, in our setting, while the instrument independent central bank takes the public debt accumulation as given, the fiscal authority also takes the rate of inflation as given. The fiscal authority thus knows that it cannot spend public expenditures as much as it wants in the second period. In this way, our instrument-independent central bank can successfully constrain the fiscal authority and can even eliminate the inflation bias. To make this arrangement more plausible, our institutional regime requires that the inflation target must be known to the Congress in advance; and that the transparency of monetary policy does matter not only for the sake of controlling for inflation but also constraining excessive fiscal expenditures. The model here generalizes the idea of classic public choice theory (see Buchanan and Wagner [1981] for example) in the context of the optimal contract literature and the studies on the optimal institutional design of a central bank.

We have not so far examined an optimal contract or targeting for the fiscal agency under the "non-coordination without commitment". To conclude this section, we discuss whether or not the second-best allocation is attained by a performance contract or a targeting rule offered from the Congress to the government.

Let $z_{Nt} = z_{Nt}^{a} + z_{Nt}^{f} f_t + z_{Nt}^{f^2} f_t^2 + z_{Nt}^{h} h_t + z_{Nt}^{h^2} h_t^2 + z_{Nt}^{d} d_t + z_{Nt}^{d^2} d_t^2$ denote a contract transfer payment from the government budget to the government itself under the "non-coordination without commitment". We assume that the performance contract for the government must be completely financed by the lump-sum tax, t_{Nt}^{F}. Since the government can choose only f_t, h_t, and d_t under the "non-coordination without commitment", this kind of contract is the most comprehensive one in the class of quadratic performance contracts for the government in our model.

In fact, we can show that none of the quadratic performance contracts for the government can achieve the second-best allocation under the "non-coordination without commitment". Similarly, none of the targeting schemes for the government can attain the second-best allocation. This finding suggests that the nominal debt target, as is commonly observed in the EU countries, Japan, and the United States, is not sufficient enough to lead the economy to the optimal allocation of resources within the class of model considered here if such countries have a positive amount of debt and take the central bank as an instrument-independent one. Even though the fiscal authority is well disciplined by a quadratic performance contract or a targeting scheme, it cannot be a substitute for an instrument-independent central bank to achieve the second best allocation of

resources.

4. Conclusion

The results obtained here show that in order to deal with the problem of dynamic inconsistency in monetary and fiscal policies practically, we must ask for an instrument-independent central bank which controls for money supply to determine the rate of inflation and commits itself to some numerical inflation target that depends on fiscal variables. One might wonder if the best solution could be a strong integrated agency that dictates both fiscal and monetary policies as we have seen under the "integrated agency without commitment". However, it is practically difficult for the policy makers to find someone who feels losses from the excessive amount of debt, and to implement an inflation target and a public debt target simultaneously. Furthermore, throughout the history of monetary economies, inflation generally happens because of the excessive government spending together with the lending of the central bank to the government which makes the spending possible. It therefore becomes the wisdom of the democratic society to separate the fiscal agency and the central bank as we have supposed under the "non-coordination without commitment". Nonetheless, since the central bank is given the power to issue fiat money, it must not misuse such a privilege and must be committed to some numerical targets to explain the behavior for the general public. This idea has been proposed by Simons [1936] after the collapse of the international gold standard system that effectively constrains the excessive issue of fiat money. These considerations suggest that the instrument-independent and inflation-target conservative central bank which is free from the governmental financial constraint is one of the most attractive institutional designs, as we have seen under the "non-coordination without commitment".

There are some issues that we do not analyze in this paper. First, as McCallum [1995] criticized, we do not explicitly consider why the policy makers cannot credibly be committed to their optimal policies. Second, we do not specify the political process for explaining how our fiscal or monetary authority is appointed. Nevertheless, we believe that our model should be a useful first step to discuss the implications of performance contract and targeting schemes based on some observable policy variables in the presence of strategic interaction between a fiscal authority and a central bank.

Appendix A

We prove that the second-best allocation can be achieved by the performance contract $(\hat{w}^{\mathsf{p}}_{I1}, \hat{w}^{d}_{I1}, \hat{w}^{d^2}_{I1}, \hat{w}^{\mathsf{p}}_{I2})$ under the "integrated agency without commitment" and by the performance contract $(\hat{w}^{\mathsf{p}}_{N1}, \hat{w}^{\mathsf{p}}_{N2})$ under the "non-coordination without commitment".

Under the "integrated agency without commitment", we solve the minimization problems (54) and (55) with respect to the control variables for the performance contract $(\hat{w}^{\mathsf{p}}_{I1}, \hat{w}^{d}_{I1}, \hat{w}^{d^2}_{I1}, \hat{w}^{\mathsf{p}}_{I2})$ defined by (58)-(61), and obtain the first-order conditions evaluated under the rational expectations equilibrium ($\mathsf{p}^{e}_{t} = \mathsf{p}_{t}$ for t = 1, 2). The first-order conditions for a solution to the minimization problem (54) are

$$-\mathsf{p}_2 + \frac{-f_2\mathsf{n} + \hat{w}^{\mathsf{p}}_{I2}}{a_{\mathsf{p}}} = -\frac{\mathsf{l}^{C}_{I2}\mathsf{k}}{a_{\mathsf{p}}}, \tag{A 1}$$

$$-f_2 = \frac{\mathsf{l}^{C}_{I2}}{\mathsf{n}}, \tag{A 2}$$

$$-h_2 = \frac{\mathsf{l}^{C}_{I2}}{a_g}, \tag{A 3}$$

$$\tilde{K}_2 + (1+\mathsf{r})d_1 = -\frac{f_2}{\mathsf{n}} + \mathsf{k}\mathsf{p}_2 - h_2, \tag{A 4}$$

where l^{C}_{I2} is the nonnegative Lagrange multiplier associated with (21). Similarly, the first-order conditions for a solution to the minimization problem (55) are

$$-\mathsf{p}_1 + \frac{-f_1\mathsf{n} + \hat{w}^{\mathsf{p}}_{I1}}{a_{\mathsf{p}}} = -\frac{\mathsf{l}^{C}_{I1}\mathsf{k}}{a_{\mathsf{p}}}, \tag{A 5}$$

$$-f_1 = \frac{\mathsf{l}^{C}_{I1}}{\mathsf{n}}, \tag{A 6}$$

$$-h_1 = \frac{\mathsf{l}^{C}_{I1}}{a_g}, \tag{A 7}$$

$$-\mathsf{b}\left[a_{\mathsf{p}}\mathsf{p}^{**}_{C2}(d_1)\frac{\partial \mathsf{p}^{**}_{C2}(d_1)}{\partial d_1} + f^{**}_{C2}(d_1)\frac{\partial f^{**}_{C2}(d_1)}{\partial d_1} + a_g h^{**}_{C2}(d_1)\frac{\partial h^{**}_{C2}(d_1)}{\partial d_1}\right] \tag{A 8}$$

$$+\hat{w}_{I1}^{d}+2\hat{w}_{I1}^{d^2}d_1+\beta\left[\hat{w}_{I2}^{\pi}\frac{\partial\pi_{C2}^{**}(d_1)}{\partial d_1}+\frac{\partial\hat{w}_{I2}^{\pi}}{\partial d_1}\pi_{C2}^{**}(d_1)\right]+\lambda_{I1}^{C}=0,$$

$$\tilde{K}_1+(1+r)d_0-d_1=-\frac{f_1}{\nu}+k\pi_1-h_1, \tag{A 9}$$

where λ_{I1}^{C} is the nonnegative Lagrange multiplier associated with (27); and $(\pi_{C2}^{**}(d_1), f_{C2}^{**}(d_1), h_{C2}^{**}(d_1))$ denotes a solution to the minimization problem (54), which satisfies (A1)-(A4). Note that $\hat{w}_{I2}^{\pi}$ depends on d_1 because $GFR_2(d_1)$ is a function of d_1.

If $\hat{w}_{It}^{\pi}=f_t\nu$ for t = 1, 2 and if $\hat{w}_{I1}^{d}+2\hat{w}_{I1}^{d^2}d_1+\beta\{\hat{w}_{I2}^{\pi}[\partial\pi_{C2}^{**}(d_1)/\partial d_1]+[\partial\hat{w}_{I2}^{\pi}/\partial d_1]\pi_{C2}^{**}(d_1)\}=0$, we can prove that these first-order conditions are equivalent to those for the minimization problems (6) and (12) in the second-best case. In other words, if $\hat{w}_{It}^{\pi}=f_t\nu$ for t = 1, 2 and if $\hat{w}_{I1}^{d}+2\hat{w}_{I1}^{d^2}d_1+\beta\{\hat{w}_{I2}^{\pi}[\partial\pi_{C2}^{**}(d_1)/\partial d_1]+[\partial\hat{w}_{I2}^{\pi}/\partial d_1]\pi_{C2}^{**}(d_1)\}=0$, we can show that any solution to the minimization problem (6) ((12)) in the second-best case is replicated by a solution to the minimization problem (54) ((55)) with the performance contract $(\hat{w}_{I1}^{\pi}, \hat{w}_{I1}^{d}, \hat{w}_{I1}^{d^2}, \hat{w}_{I2}^{\pi})$ defined by (58)-(61). The performance contract $(\hat{w}_{I1}^{\pi}, \hat{w}_{I1}^{d}, \hat{w}_{I1}^{d^2}, \hat{w}_{I2}^{\pi})$ defined by (58)-(61) can then achieve the second-best allocation if $\hat{w}_{It}^{\pi}=f_t\nu$ for t = 1, 2 and if $\hat{w}_{I1}^{d}+2\hat{w}_{I1}^{d^2}d_1+\beta\{\hat{w}_{I2}^{\pi}[\partial\pi_{C2}^{**}(d_1)/\partial d_1]+[\partial\hat{w}_{I2}^{\pi}/\partial d_1]\pi_{C2}^{**}(d_1)\}=0$.

We now proceed to prove $\hat{w}_{It}^{\pi}=f_t\nu$ for t = 1, 2 and $\hat{w}_{I1}^{d}+2\hat{w}_{I1}^{d^2}d_1+\beta\{\hat{w}_{I2}^{\pi}[\partial\pi_{C2}^{**}(d_1)/\partial d_1]+[\partial\hat{w}_{I2}^{\pi}/\partial d_1]\pi_{C2}^{**}(d_1)\}=0$. To this end, we substitute (A2) into (A1) and obtain

$$\pi_2=\frac{\hat{w}_{I2}^{\pi}}{a_\pi}+\frac{(1+k)\lambda_{I2}^{C}}{a_\pi}. \tag{A 10}$$

Substituting (A2), (A3), and (A10) into (A4) and rearranging the resulting equation, we have

$$\left(1+\frac{k}{a_\pi C}\right)\lambda_{I2}^{C}=\frac{GFR_2(d_1)}{C}-\frac{k}{a_\pi C}\hat{w}_{I2}^{\pi}, \tag{A 11}$$

where $GFR_2(d_1)$ is defined by (21). Inspecting (A11) with (58) and (A2) immediately yields $\hat{w}_{I2}^{\pi}=-\lambda_{I2}^{C}=f_2\nu$.

Given $\hat{w}_{I2}^{\pi}=-\lambda_{I2}^{C}$, we next substitute (58) into (A1)-(A3) and differentiate the resulting equations with respect to d_1. Then,

$$\frac{\partial\pi_{C2}^{**}(d_1)}{\partial d_1}=\frac{k}{a_\pi}\frac{1+r}{C}, \tag{A 12}$$

$$\frac{\partial f_{C2}^{**}(d_1)}{\partial d_1} = -\frac{1}{\nu}\frac{1+r}{C}, \tag{A 13}$$

$$\frac{\partial h_{C2}^{**}(d_1)}{\partial d_1} = -\frac{1}{a_g}\frac{1+r}{C}. \tag{A 14}$$

Note that $(p_{C2}^{**}(d_1), f_{C2}^{**}(d_1), h_{C2}^{**}(d_1))$ is a solution to the minimization problem (54), which satisfies (A1)-(A4). Inspecting the term $\hat{w}_{I1}^{d} + 2\hat{w}_{I1}^{d^2} d_1 + \beta\{\hat{w}_{I2}^{p}[\partial p_{C2}^{**}(d_1)/\partial d_1] + [\partial \hat{w}_{I2}^{p}/\partial d_1] \times p_{C2}^{**}(d_1)\}$ with (58), (60), (61), (A10), (A12), and $\hat{w}_{I2}^{p} = -\lambda_{I2}^{C}$ leads us to show that

$$\hat{w}_{I1}^{d} + 2\hat{w}_{I1}^{d^2} d_1 + \beta\left[\hat{w}_{I2}^{p}\frac{\partial p_{C2}^{**}(d_1)}{\partial d_1} + \frac{\partial \hat{w}_{I2}^{p}}{\partial d_1} p_{C2}^{**}(d_1)\right] = 0. \tag{A 15}$$

The remaining problem is to verify that $\hat{w}_{I1}^{p} = f_1\nu$. Substituting (A12)-(A15) into (A8) and rearranging it with (A4), we see

$$d_1 = \frac{C\lambda_{I1}^{C}}{(1+r)^2\beta} - \frac{\tilde{K}_2}{1+r}. \tag{A 16}$$

Further substituting (A5)-(A7) and (A16) into (A9), we obtain

$$\left[1 + \frac{\kappa}{a_p C}\beta(1+r)\delta\right]\lambda_{I1}^{C} = \beta(1+r)\delta\frac{\tilde{F}}{C} - \frac{\kappa}{a_p C}\beta(1+r)\delta\hat{w}_{I1}^{p}. \tag{A 17}$$

Now, it follows from (59) and (A6) that $\hat{w}_{I1}^{p} = -\lambda_{I1}^{C} = f_1\nu$, which finally establishes that the first-order conditions (A1)-(A9) are equivalent to those derived in the second-best case.

Under the "non-coordination without commitment", we solve the minimization problems (56) and (57). Then, applying the above procedure to the resulting first-order conditions with (62) and (63), we can show that the performance contract defined by (62) and (63) can achieve the second-best allocation.

Appendix B

Since the second-best allocation is attained by the optimal performance contract, it is also achieved by the optimal targeting rule if the contracting problems (54) and (55) ((56) and (57)) for the optimal performance contract $(\hat{w}_{I1}^{p}, \hat{w}_{I1}^{d}, \hat{w}_{I1}^{d^2}, \hat{w}_{I2}^{p})$ $((\hat{w}_{N1}^{p}, \hat{w}_{N2}^{p}))$ defined by (58)-(61) ((62) and (63)) are reduced to the targeting problems (68) and (69) ((72) and (73)) and if the optimal inflation target derived from the targeting problem (68) ((72)) is consistent with the optimal

inflation target in period 2 derived from the targeting problem (69) ((73)).

Here, we only rewrite the contracting problem (55) for the optimal performance contract $(\hat{w}^{\pi}_{I1}, \hat{w}^{d}_{I1}, \hat{w}^{d^2}_{I1}, \hat{w}^{\pi}_{I2})$ as the targeting problem (69) because we can rewrite the other contracting problems as the corresponding targeting problems by applying similar discussions. Now, for the optimal performance contract $(\hat{w}^{\pi}_{I1}, \hat{w}^{d}_{I1}, \hat{w}^{d^2}_{I1}, \hat{w}^{\pi}_{I2})$, the objective function of the contracting problem (55) is modified by

$$\frac{1}{2}\{a_{\pi}\pi_1^2+[\nu(\pi_1-\pi_1^e)+f_1]^2+a_g h_1^2\}-\hat{w}^{\pi}_{I1}\pi_1-\hat{w}^{d}_{I1}d_1-\hat{w}^{d^2}_{I1}d_1^2$$

$$+\frac{1}{2}\beta\{a_{\pi}[\pi^{**}_{C2}(d_1)]^2+[f^{**}_{C2}(d_1)]^2+a_g[h^{**}_{C2}(d_1)]^2\}-\beta\hat{w}^{\pi}_{I2}\pi^{**}_{C2}(d_1)$$

$$=\frac{1}{2}\{a_{\pi}(\pi_1-\frac{\hat{w}^{\pi}_{I1}}{a_{\pi}})^2-\frac{(\hat{w}^{\pi}_{I1})^2}{a_{\pi}}+[\nu(\pi_1-\pi_1^e)+f_1]^2+a_g h_1^2\}-\hat{w}^{d}_{I1}d_1-\hat{w}^{d^2}_{I1}d_1^2$$

$$+\frac{1}{2}\beta\{a_{\pi}[\pi^{**}_{C2}(d_1)-\frac{\hat{w}^{\pi}_{I2}}{a_{\pi}}]^2-\frac{(\hat{w}^{\pi}_{I2})^2}{a_{\pi}}+[f^{**}_{C2}(d_1)]^2+a_g[h^{**}_{C2}(d_1)]^2\}. \qquad \text{(B1)}$$

Substituting (58) with (7) into the term $-(\hat{w}^{\pi}_{I2})^2/a_{\pi}$ of (B1) and rearranging the right-hand side of (B1), we have

$$\text{(right-hand side of (B1))} = \frac{1}{2}\{a_{\pi}(\pi_1-\frac{\hat{w}^{\pi}_{I1}}{a_{\pi}})^2-\frac{(\hat{w}^{\pi}_{I1})^2}{a_{\pi}}+[\nu(\pi_1-\pi_1^e)+f_1]^2+a_g h_1^2\}$$

$$-[\hat{w}^{d^2}_{I1}+\frac{\beta(1+\rho)^2}{2a_{\pi}C^2}]\{d_1+\frac{\hat{w}^{d}_{I1}+\frac{\beta(1+\rho)\tilde{K}_2}{a_{\pi}C^2}}{2[\hat{w}^{d^2}_{I1}+\frac{\beta(1+\rho)^2}{2a_{\pi}C^2}]}\}^2+\frac{[\hat{w}^{d}_{I1}+\frac{\beta(1+\rho)\tilde{K}_2}{a_{\pi}C^2}]^2}{4[\hat{w}^{d^2}_{I1}+\frac{\beta(1+\rho)^2}{2a_{\pi}C^2}]}-\frac{\beta(\tilde{K}_2)^2}{2a_{\pi}C^2}$$

$$+\frac{1}{2}\beta\{a_{\pi}[\pi^{**}_{C2}(d_1)-\frac{\hat{w}^{\pi}_{I2}}{a_{\pi}}]^2+[f^{**}_{C2}(d_1)]^2+a_g[h^{**}_{C2}(d_1)]^2\}.$$

Then, choose the inflation and public debt targets such that

$$\tilde{\pi}_{It}=\frac{\hat{w}^{\pi}_{It}}{a_{\pi}}, \qquad t = 1, 2, \qquad \text{(B 2)}$$

$$\tilde{d}_{I1}=-\frac{\hat{w}^{d}_{I1}+\frac{\beta(1+\rho)\tilde{K}_2}{a_{\pi}C^2}}{2[\hat{w}^{d^2}_{I1}+\frac{\beta(1+\rho)^2}{2a_{\pi}C^2}]}. \qquad \text{(B 3)}$$

Since we can show that the optimal inflation target derived from the targeting problem (68) is

identical with $\tilde{\rho}_{I2}$ given by (B2), we see that the objective function of the contracting problem (55) is exactly transformed into the objective function of the targeting problem (69) for the optimal performance contract $(\hat{w}^{\rho}_{I1}, \hat{w}^{d}_{I1}, \hat{w}^{d^2}_{I1}, \hat{w}^{\rho}_{I2})$ if $\tilde{d}_{I1} = -\tilde{K}_2/(1+r)$. Because the constraints of the contracting problem (55) are the same as those of the targeting problem (69), this implies that the contracting problem (55) is equivalent to the targeting problem (69) for the optimal performance contract $(\hat{w}^{\rho}_{I1}, \hat{w}^{d}_{I1}, \hat{w}^{d^2}_{I1}, \hat{w}^{\rho}_{I2})$ if $\tilde{d}_{I1} = -\tilde{K}_2/(1+r)$.

We next prove the relations represented by (71). Substituting (60) and (61) into (B3), we obtain

$$\tilde{d}_{I1} = -\frac{\tilde{K}_2}{1+r}. \tag{B 4}$$

Furthermore, since the performance contract $(\hat{w}^{\rho}_{I1}, \hat{w}^{d}_{I1}, \hat{w}^{d^2}_{I1}, \hat{w}^{\rho}_{I2})$ achieves the second-best allocation, the second-best level of debt, d_1^*, is given by (A16) in the Appendix:

$$d_1^* = \frac{C l^{C}_{I1}}{(1+r)^2 b} - \frac{\tilde{K}_2}{1+r}. \tag{B 5}$$

Substituting the relation $\hat{w}^{\rho}_{I1} = -l^{C}_{I1}$ into (B5), we rewrite (B4) by

$$\tilde{d}_{I1} = -\frac{\tilde{K}_2}{1+r} = d_1^* + \frac{C\hat{w}^{\rho}_{I1}}{(1+r)^2 b}.$$

Finally, given $\hat{w}^{\rho}_{I1} = a_{\rho}\tilde{\rho}_{I1}$ from (70), it is straightforward to see

$$\tilde{d}_{I1} = d_1^* + \frac{a_{\rho} C \tilde{\rho}_{I1}}{(1+r)^2 b}.$$

Notes

1. If the production function is given by $Y_t = L_t^{\theta}$, the representative firm maximizes the profits $P_t L_t^{\theta}(1-\tau_t) - W_t L_t$ with respect to L_t, where L_t, P_t, and W_t denote the labor input, the price level, and the nominal wage rate, respectively. We assume that workers (unions) aim at a target real wage rate, and that the logarithm of the real wage rate is normalized to zero. Then, the logarithm of the nominal wage rate in period t is set equal to the logarithm of the expected price level in period t. The logarithm of the output in period t is now represented by $y_t = [\theta/(1-\theta)](p_t - p_t^e - \tau_t + \log\theta)$. The normalized output x_t in period t is then defined by $x_t \equiv y_t - [\theta/(1-\theta)]\log\theta$; and the constant ν is defined by $\nu \equiv [\theta/(1-\theta)]$. See Beetsma and Bovenberg [1997] for details.

2. Beetsma and Bovenberg [1997] supposes that $\tilde{x}_t$ is the first-best output level. However, following the convention of the literature on inflation contracts and targeting, we assume that $\tilde{x}_t$ corresponds to the second-best output level.

3. For the derivation procedure of the government financing requirement constraint (2), see Beetsma and Bovenberg [1997], which proves that this constraint is a good approximation if the output level realized without tax distortions or inflation surprises is not so different from the anti-log of x_t.

4. We assume that $\kappa X^* = M_t/P_t$, where X^* is the level of output which is consistent with the natural rate of unemployment, and M is nominal money supply. This assumption can be understood either as the Cambridge equation of quantity theory of money or as a cash in advance constraint. If X^* is constant, this assumption implies that the central bank can determine the rate of inflation by controlling for the growth rate of money supply. Although it is impossible to measure the level of X^* precisely, we can state that the empirical counterpart of κ is the inverse of the velocity of money. The inverse of the velocity of money can be different from one and slowly changing over time mainly due to the effect of financial sophistication as Bordo and Joung [1988] has shown. The series of American data for the ratio of M1 to nominal GDP have a range of 0.161 to 0.188, and those for the ratio of currency to nominal consumption have a range of 0.064 to 0.078 during the period of 1988-1995. The corresponding series of Japanese data have a range of 0.278 to 0.357 and 0.140 to 0.160, respectively. For these data, see The Bank of Japan [1996]. Since the government financing requirement constraint (2) assumes that the government can impose the inflation tax on monetary assets in this model, it is plausible to consider the monetary assets as either M1 or Cash. Hence, the plausible value of κ is at most 0.19 for the United States and 0.36

for Japan, which are strictly less than one.

5. For the goal- and instrument-independent central bank, see Debelle and Fischer [1994].

6. Since $\tilde{x}_t$ measures the deviation from the first-best output level that is caused by the non-tax distortions, it can be interpreted as an implicit tax on output. Furthermore, it follows from (1) that the output subsidy $-\tilde{x}_t/\theta$ can raise output towards the second-best output level $\tilde{x}_t$. Thus, $-\tilde{x}_t/\theta$ can be taken as implicit tax revenues.

7. Since (21) is always binding with equality, λ_{I2} is almost always positive.

8. Since (27) is always binding with equality, λ_{I1} is almost always positive.

9. In this case, the contract transfer payment to the integrated agency can be divided between the government and the central bank in accordance with their internal arrangements. The government may hold several administrative positions in the central bank instead of receiving a part of the contract transfer payment.

10. This remark also holds true in the other previous studies of the inflation contract and targeting rule.

11. The constraint (29) is evaluated by $\pi_2^*(d_1) = \pi_{C2}^{**}(d_1)$.

12. The constraint (51) is evaluated by $\pi_2^{***}(d_1) = \pi_{C2}^{**}(d_1)$.

13. The minimization problem (42) is evaluated by $(\pi^{**}(\ _1)\ f_2^{***}(d_1), h_2^{***}(d_1)) = (\pi_{C2}^{***}(d_1), f_{C2}^{***}(d_1), h_{C2}^{***}(d_1))$.

14. Although we do not assume $\kappa = 1$, this parametric case needs some comment. If $\kappa = 1$, the allocation of the economy happens to coincide with the second-best allocation even though neither the government nor the central bank coordinates their policy decisions or commits themselves to their policy announcements. Thus, we need not to design any performance contracts if $\kappa = 1$.

15. See note 1.

16. The constraint (29) is evaluated by $\pi_2^{**}(d_1) = \pi_{T2}^{**}(d_1)$.

17. The constraint (51) is evaluated by $\pi_2^{***}(d_1) = \pi_{T2}^{***}(d_1)$.

18. See notes 7 and 8.

19. Since (35) and (43) are always binding with equality, λ_{N1} and λ_{N2} are almost always positive.

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