KIER DISCUSSION PAPER SERIES

KYOTO INSTITUTE
OF
ECONOMIC RESEARCH

Discussion Paper No.800

"The Welfare Effects of Third-Degree Price Discrimination in a Differentiated Oligopoly"

Takanori Adachi
Noriaki Matsushima

December 2011

KYOTO UNIVERSITY

KYOTO, JAPAN

The Welfare Effects of Third-Degree Price Discrimination in a Differentiated Oligopoly

Takanori Adachi* Noriaki Matsushima†

December 5, 2011

Abstract

This paper studies the welfare effects of third-degree price discrimination under oligopolistic competition with horizontal product differentiation. We derive a necessary and sufficient condition for price discrimination to improve social welfare: the degree of substitution must be sufficiently greater in the "strong" market (where the discriminatory price is higher than the uniform price) than in the "weak" market (where it is lower). It is verified, however, that consumer surplus is never improved; social welfare improves solely due to an increase in the firms' profits.

Keywords: Third-Degree Price Discrimination; Oligopoly; Social Welfare; Horizontal Product Differentiation; Substitutability; Complementarity.

JEL classification: D43; L11; L13.

*School of Economics, Nagoya University, Chikusa, Nagoya 464-8601, Japan. E-mail: adachi.t@soec.nagoya-u.ac.jp

†Institute of Social and Economic Research, Osaka University, 6-1 Mihogaoka, Ibaraki, Osaka 567-0047, Japan. E-mail: nmatsush@iser.osaka-u.ac.jp

1 Introduction

Product differentiation is one of the main reasons why firms can enjoy market power; it enables them to sell products that are no longer perfect substitutes. For example, Coca Cola and PepsiCo sell similar types of soda, though it is arguable that they differ in taste. Each firm thereby attracts some consumers over another. It is often the case that firms' differentiation of their products leads consumers to value the variety. Examples of complementary products abound and include such products as "breakfast cereal and milk" and "cars and petrol."

If firms have some control over the price that consumers pay, they naturally want to take advantage of it. *Third-degree price discrimination* is one marketing technique that is widely used in imperfectly competitive markets. In third-degree price discrimination, the seller uses identifiable signals (e.g., age, gender, location, and time of use) to categorize buyers into different segments or submarkets, each of which is given a constant price per unit. Behind the recent trend toward third-degree price discrimination is rapid progress in information-processing technology, notably including the widespread use of the Internet in the past two decades.[1]

This paper examines the welfare effects of *oligopolistic* third-degree price discrimination, explicitly considering product differentiation as a source of *market power and strategic interaction.* In a story related to the example in the first paragraph, the *New York Times* once reported (October 28, 1999)[2] that Coca Cola was testing a vending machine that would automatically raise prices in hot weather. Although the article triggered nationwide controversy and Coca Cola had to abandon the project as a result, the plan could have changed the regime of uniform pricing to one of price discrimination in the soda market. How would the resulting change affect consumer welfare and firms' profits? In other words, is third-degree price discrimination is good or bad? Answering this question is important because it helps antitrust authorities to evaluate price discrimination in two important characteris-

[1] See Shy (2008) concerning how advances in the information technology have made 'fine-tailored' pricing tactics more practicable for sellers.

[2] http://www.nytimes.com/1999/10/28/business/variable-price-coke-machine-being-tested.html (retrived December 2011)

tics of market: *oligopoly and product differentiation*.

In this paper, we focus on *horizontal* product differentiation to consider *substitutability* as well as *complementarity*.[3] By assuming a linear-quadratic utility function of a representative consumer and focusing on the symmetric equilibrium of a pricing game, we characterize the conditions relating to such demand properties as substitutability and complementarity required for price discrimination to improve social welfare. More specifically, we derive a necessary and sufficient condition for price discrimination to improve social welfare: the degree of substitution must be sufficiently greater in the "strong" market (where the discriminatory price is higher than the uniform price) than in the "weak" market (where it is lower). It is also shown that aggregate output must increase for price discrimination to improve social welfare. We verify, however, that consumer surplus is *never* improved by price discrimination: welfare improvement from price discrimination is solely due to an increase in firms' profits.

Why does allowing price discrimination improve social welfare? Notice that there are two forces that determinine equilibrium prices in price-setting oligopoly: a larger market size raises the price, while a larger substitutability parameter lowers the price. For price discrimination to raise social welfare, it is necessary that a high degree of substitutability mitigates an increase in the discriminatory price in the larger market. Therefore, although the price rises with discrimination in this market, the market size effect is not significant. The welfare gain in the weak market outweighs the welfare loss in the strong market when an increase in the discriminatory price in the strong market is kept low.

Since Pigou's (1920) seminal work, the central question in the analysis of third-degree price discrimination is about its welfare effects: what are the effects of third-degree price discrimination on consumer surplus and Marshallian social welfare (the sum of consumer surplus and firms' profits)? In the literature, however, little has

[3] With *horizontal* product differentiation, some consumers prefer product A to B while others prefer B to A. On the other hand, *vertical* product differentiation captures the situation where all consumers agree on the ranking of products. See, for example, Belleflamme and Peitz (2010, Ch.5) for further discussion of its distinction.

been reported about the welfare effects of *oligopolistic* third-degree price discrimination since the publication of a seminal paper by Holmes (1989), which analyzes the output effects of third-degree price discrimination in oligopoly, but not the welfare effects.[4] On the other hand, the welfare effects of *monopolistic* third-degree price discrimination are relatively well known. Since the work by Robinson (1933), it has been well known that when all submarkets are served under uniform pricing,[5] price discrimination must decrease social welfare unless aggregate output increases. This implies that an increase in aggregate output is a *necessary* condition for social welfare to be improved by third-degree price discrimination.[6] In particular, price discrimination *necessarily decreases social welfare if demands are linear* because aggregate output remains constant.[7] The welfare consequences of *oligopolistic* third-degree price discrimination, however, remain largely unknown. It is therefore important to study oligopolistic third-degree price discrimination, because only a small number of goods are supplied by monopolists in the real world and an increasing number of firms use price discrimination for their products and services.

This paper investigates the relationship between product differentiation and change in social welfare associated with the regime change from uniform pricing

[4] See Armstrong (2006) and Stole (2007) for comprehensive surveys of price discrimination with imperfect competition, and Liu and Serfes (2010) for a brief survey. In contrast to Holmes' (1989) focus on a *symmetric* Nash equilibrium (where *all firms behave identically*), an important work by Corts (1998) relaxes the requirement for symmetry to show that *asymmetry* in firms' best response functions is necessary for unambiguous welfare effects (when prices drop in all markets, the result is unambiguous welfare improvement, and when these prices jump, the result is unambiguous welfare deterioration). Our focus on a *symmetric* equilibrium is based on the assumption that all firms agree in their ranking in pricing (see Stole (2007) for details), and is motivated by our recognition that this situation is more natural than the asymmetric cases in many examples of third-price discrimination.

[5] Under uniform pricing, firms may be better off by refusing supply to some submarkets. See, for example, Hausman and MacKie-Mason (1988) regarding this issue.

[6] Aguirre, Cowan, and Vickers (2010) offer a comprehensive analysis, finding sufficient conditions relating the curvatures of direct and indirect demand functions in separate markets. While they allow nonlinear demand functions, they, like many researchers, restrict an endogenous event: all markets are simply assumed to be open. Cowan (2007) offers a similar analysis by restricting a class of demand functions.

[7] For example, Schmalensee (1981), Varian (1985), Schwartz (1990), and Bertoletti (2004). In contrast to these studies, Adachi (2002, 2005) shows that, when there are consumption externalities, price discrimination can increase social welfare even if aggregate output remains the same (see also Ikeda and Nariu (2009) and Okada and Adachi (2011)). Ikeda and Toshimitsu (2010) show that if quality is endogenously chosen, price discrimination necessarily improves social welfare.

to price discrimination when all submarkets are open under uniform pricing.[8] To model price competition with product differentiation, we adopt the Chamberlin-Robinson approach (named by Vives (1999, p.243)): a "representative" consumer (i.e., a virtual individual that is an aggregation of an infinitesimal number of identical consumers) is assumed to value the variety of goods. In this paper, we consider the (fully parameterized) linear demand structure to obtain an explicit solution as well as an explicit condition for all submarkets to be open under uniform pricing. The benefit of this specification is that *we do not have to simply assume such endogenous events as a market opening*. In addition, while Holmes (1989) assumes substitutability of products, our formulation allows inclusion of complementarity in a welfare analysis.

One important difference between monopoly and oligopoly is that in a monopoly, the price elasticity of demand in each submarket has a one-to-one relationship with the optimal discriminatory price: the larger the price elasticity, the lower the discriminatory price is. In oligopoly, however, this may not be the case because *strategic interaction* affects the pricing decision of each firm. In particular, the price elasticity that a firm faces in a discriminatory market is generally different from the elasticity that the firms *as a whole* (i.e., in a collusive oligopoly) face. In this paper, we show that *in equilibrium* this "firm-level" price elasticity has a simple expression in terms of product differentiation. More specifically, it is verified that, as in Holmes (1989), *in equilibrium* the firm-level price elasticity decomposes into "market-level" and "strategic-related" elasticity (the precise meanings are given in the text). The latter elasticity simply coincides with the degree of product differentiation.[9] It is observed from numerical and graphical analysis that this "strategic" elasticity plays

[8] In a similar study of third-degree price discrimination in price-setting oligopoly, Dastidar (2006), by focusing on symmetric Nash equilibrium, as Holmes (1989) and this paper do, provides sufficient conditions for output, profit and welfare to be higher or lower under discrimination than under uniform pricing. Our study complements Dastidar's (2006) analysis in the sense that while Dastidar (2006) utilizes such endogenous objects as price differentials in each submarket in characterizing the conditions, ours explicitly takes into account such demand properties as substitutability and complementarity to characterize the conditions under which price discrimination improves social welfare.

[9] Our analysis below shows that Holmes' (1989) decomposition also holds for the case of complementarity with linear demands in the specification of our paper.

an important role in the determination of discriminatory prices and social welfare. One benefit of using linear demands is that we can evaluate welfare without the complications associated with demand concavity/convexity. In particular, whether a change in aggregate output by price discrimination is positive is simply expressed only by the parameters of product differentiation.

The rest of the paper is organized as follows. The next section presents a model and preliminary results. Section 3 presents the welfare analysis. Section 4 concludes the paper. Technical arguments are relegated to the Appendix.

2 The Model

In this section, we first set up the model and then provide the preliminary results necessary for the welfare analysis in the next section.

2.1 Setup

Firms produce (horizontally) differentiated products and compete in price[10] to sell their products (directly) to consumers. A firm sells only one type of product, which can therefore also be interpreted as a *brand*. Markets are partitioned according to identifiable signals (e.g., location, time of use, age, and gender).[11] The qualifier "horizontally" denotes that firms differentiate by targeting consumer heterogeneity in taste rather than quality. For simplicity, we assume that all firms have the same constant marginal cost, $c \geq 0$. Resale among consumers must be impossible,

[10]Stole (2008, pp.2233-4) argues that if quantity (Cournot) competition is considered, third-degree price discrimination does *not* change aggregate output when demands are linear (and when all markets are served). This is reminiscent of the result in the case of monopoly (see Introduction), and it results from the fact that Cournot competition is a "generalization" of monopoly to strategic situations. For example, Neven and Phlips (1985) consider a duopoly model of quantity competition with homogenous products, linear demands, and quadratic costs, in the context of international trade (transportation costs between submarkets are considered), and show that social welfare is always lower under price discrimination. Cheng and Wang (1997) derive the same result by allowing different costs among firms. We thank Iñaki Aguirre and Simon Cowan for this point.

[11]There are no interdependencies between separate markets. Layson (1998) and Adachi (2002) study the welfare effects of monopolistic third-degree price discrimination in the presence of interdependencies.

otherwise some consumers would be better off buying the good at a lower price from other consumers (arbitrage).

Following Robinson (1933) and most subsequent papers in the literature, we suppose that the whole market is divided into two subgroups: "*strong*" and "*weak*" markets. Loosely put, a strong (weak) market is a "larger" ("smaller") market.[12] Consumer preference in market $m \in \{s, w\}$ (s denotes (the set of) the strong markets and w the weak markets) is represented by the following quasi-linear utility function:

$$U_m(q_m^A, q_m^B) \equiv \alpha_m \cdot (q_m^A + q_m^B) - \frac{1}{2}\left(\beta_m [q_m^A]^2 + 2\gamma_m q_m^A q_m^B + \beta_m [q_m^B]^2\right),$$

where $|\gamma_m| < \beta_m$ denotes the degree of horizontal product differentiation in market m, q_m^j is the amount of consumption/output produced by firm j for market m ($j \in \{A, B\}$), and $\beta_m > 0$.[13] Notice that this specification allows the cross-partial derivative to be expressed by just one parameter: $\partial U_m / \partial q_m^A \partial q_m^B = -\gamma_m$. If $\gamma_m > 0$, the goods in market m are called *substitutes*. On the other hand, they are called *complements* if $\gamma_m < 0$. If $\gamma_m = 0$, they are *independent*.

Notice that the direction of the sign is associated with the usual definitions of complementarity/substitutability: when the firms' goods are substitutes (complements), the marginal utility from consuming an additional unit of the good purchased from one firm is lower (higher) when a consumer consumes more units of the good from the other firms. Note that the lower the value of γ_m, the *more* differentiated firms' products are.[14] The ratio $\gamma_m/\beta_m \in (-1, 1)$ is interpreted as a *(normalized) measure of horizontal product differentiation* in market m (see Belle-

[12]More precisely, following the literature, we define a strong (weak) market as one in which the price is increased (decreased) by price discrimination. Notice that this definition is based on an "equilibrium"result from optimizing behavior (either in monopoly or oligopolistic pricing). Appendices A1 and A2 show the parametric restrictions by which a market is strong or weak in the model presented below.

[13]More precisely, we assume that the utility function has a quasi-linear form of $U_m(q_m^A, q_m^B) + q_0$, where q_0 is the "composite" good (produced by the competitive sector) whose (competitive) price is normalized at one. Thus, there are no income effects on the determination of demands in the markets that are focused, validating partial equilibrium analysis. This quadratic utility function is a standard one that justifies linear demands (see Vives (1999, p.145), for example). Here, symmetry between firms is additionally imposed.

[14]In the case of independence in market m ($\gamma_m = 0$), each firm behaves as a monopolist of its own brand. Hence, the results from studies of monopolistic third-degree price discrimination with linear demands apply.

flamme and Peitz (2010, p.65)). The greater γ_m/β_m, the greater the degree of substitution is (interpreting complementarity as negative substitutability). As we see in Section 3, γ_m/β_m plays an important role in interpreting the equilibrium prices, outputs and social welfare under price discrimination.

Note also that it is assumed that γ_m can vary across submarkets. This setting is not unnatural in the context of *horizontal* product differentiation. For example, a consumer may care less about brands in summer bay resorts (where temperature is high) than in urban areas. In another case where submarkets are separated by seniority, elderly consumers may care less about brands.

Utility maximization by the representative consumer yields the inverse demand function for firm j in each market m, $p_m^j(q_m^j, q_m^{-j}) = \alpha_m - \beta_m q_m^j - \gamma_m q_m^{-j}$. The demand functions in market m are thus given by

$$\begin{cases} q_m^A(p_m^A, p_m^B) = \dfrac{\alpha_m}{\beta_m + \gamma_m} - \dfrac{\beta_m}{\beta_m^2 - \gamma_m^2} p_m^A + \dfrac{\gamma_m}{\beta_m^2 - \gamma_m^2} p_m^B \\ \qquad\qquad\qquad = \dfrac{1}{(1-\theta_m^2)\beta_m}\left[\alpha_m(1-\theta_m) - p_m^A + \theta_m p_m^B\right] \\ q_m^B(p_m^A, p_m^B) = \dfrac{1}{(1-\theta_m^2)\beta_m}\left[\alpha_m(1-\theta_m) - p_m^B + \theta_m p_m^A\right], \end{cases} \tag{1}$$

where $\theta_m \equiv \gamma_m/\beta_m$. Notice here that the symmetry in firms' demands, $q_m^A(p', p'') = q_m^B(p'', p')$. As stated above, we follow Holmes (1989) and many others to focus on a symmetric Nash equilibrium where all firms set the same price in one market.[15] With little abuse of notation, let $q_m(p) = q_m^A(p, p)$. For a simpler exposition, there are two firms and two discriminatory markets. These numbers can be arbitrary and the results presented below hold as long as we focus on a symmetric Nash equilibrium.

Social welfare in market m is defined by

$$SW_m(q_m^A, q_m^B) \equiv U_m(q_m^A, q_m^B) - c \cdot (q_m^A + q_m^B)$$

and thus the aggregate social welfare is given by

$$SW(\{q_m^A, q_m^B\}_m) \equiv \sum_m SW_m(q_m^A, q_m^B).$$

[15] See Corts (1998) for interesting issues that arise from the asymmetric equilibrium.

We measure social efficiency by this aggregate social welfare. We can also define aggregate consumer surplus by

$$CS(\{p_m^A, p_m^B\}_m) \equiv \sum_m CS_m(p_m^A, p_m^B)$$

where consumer surplus in market m is

$$CS_m(p_m^A, p_m^B) \equiv U_m[q_m^A(p_m^A, p_m^B), q_m^B(p_m^A, p_m^B)] - p_m^A q_m^A(p_m^A, p_m^B) - p_m^B q_m^B(p_m^A, p_m^B),$$

and denote aggregate corporate surplus (profit) by

$$\Pi(\{p_m^A, p_m^B\}_m) \equiv \sum_m \sum_j (p_m^j - c) q_m^j(p_m^A, p_m^B)$$

so that the aggregate social welfare is divided in the following way:

$$SW(\{q_m^A(p_m^A, p_m^B), q_m^B(p_m^A, p_m^B)\}_m) = CS(\{p_m^A, p_m^B\}_m) + \Pi(\{p_m^A, p_m^B\}_m).$$

We consider two regimes, *uniform pricing* ($r = U$) and *price discrimination* ($r = D$): under uniform pricing, firms set a common unit price for all separate markets. Under price discrimination, they can set a different price in each market. Throughout this paper, we restrict our attention to the case where all markets are served under both regimes.

Because the profit of firm $j \in \{A, B\}$ is given by

$$\pi^j(p_s^A, p_s^B, p_w^A, p_w^B) \equiv \sum_m (p_m^j - c) q_m^j(p_m^A, p_m^B),$$

we know that under symmetry, the symmetric equilibrium price under uniform pricing, p^*, is given by

$$[q_s(p^*) + q_w(p^*)] + (p^* - c)\left[\frac{\partial q_s^A(p^*, p^*)}{\partial p_m^A} + \frac{\partial q_w^A(p^*, p^*)}{\partial p_m^A}\right] = 0, \tag{2}$$

while the equilibrium prices in market m under price discrimination, p_m^*, are determined by the following first-order condition:

$$q_m(p_m^*) + (p_m^* - c)\frac{\partial q_m^A(p_m^*, p_m^*)}{\partial p_m^A} = 0. \tag{3}$$

Again, one caveat here is the well-known problem in the literature concerning third-degree price discrimination: *under uniform pricing, when a market is sufficiently small, it may not be served by either firm.* While many papers in the literature simply assume that all markets are open under uniform pricing, we provide a more specific structure in the next subsection to guarantee this and to proceed further the analysis.

2.2 Best Response Functions

Note also that the relationship between Corts' (1998) study and this paper.[16] Let $BR^j_m(p^k_m) \equiv \arg\max_{p^j}(p^j - c)q^j_m(p^j, p^{-j}_m)$ be firm j's best response function in market m under price discrimination, given firm k's price in market m, p^k_m. Corts (1998) makes four assumptions concerning the profit functions and the best response functions. In our settings, Assumptions 1 and 3 in Corts (1998) are satisfied,[17] although Assumption 2, which assumes strategic complementarity, is violated. This is because Corts (1998) assumes that the cross-partial derivative of the profit function in each submarket,

$$\frac{\partial^2 \pi^j_m(p^j, p^{-j}_m)}{\partial p^j_m \partial p^{-j}_m}$$

where $\pi^j_m(p^j, p^{-j}_m) \equiv (p^j - c)q^j_m(p^j, p^{-j}_m)$, is positive, while in our setting it is equal to θ_m, and it can be negative (i.e., strategic substitutability) if the firms' products are *complements*.[18] Assumption 4 in Corts (1998), which guarantees that a firm's best response functions never cross (i.e., $BR^j_m(p^{-j}) > BR^j_{-m}(p^{-j})\ \forall p^{-j}$), does not necessarily hold. To see this, note first that firm j's best response functions are given by

$$BR^j_m(p^{-j}_m) = \frac{1}{2}\theta_m p^{-j}_m + \frac{1}{2}\left[\alpha_m(1-\theta_m) + c\right],$$

while under uniform pricing, they are

$$BR^j_m(p^{-j}) = \eta \cdot BR^j_s(p^{-j}) + (1-\eta) \cdot BR^j_w(p^{-j})$$

[16] We are grateful to Hajime Sugeta for this point.

[17] Assumption 1 in Corts (1998) ensures the uniqueness of the best response, and Assumption 3 the equilibrium stability.

[18] The logic of this result dates back to Cournot (1838, Chapter 9).

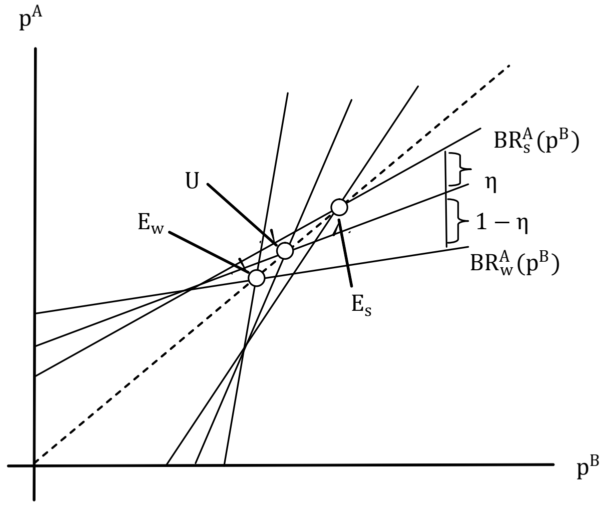

Figure 1: Crossing Best Response Curves

where

$$\eta = \frac{\beta_w(1-\theta_w^2)}{\beta_s(1-\theta_s^2)+\beta_w(1-\theta_w^2)} \in (0,1).$$

Here, firm j's best response curve in submarket m under price discrimination is upward sloping (resp. downward sloping) if the firms' products are substitutes (resp. complements). If $\theta_s \neq \theta_w$, then the two curves can cross as Figure 1 depicts (E_s corresponds to the symmetric equilibrium price in the strong market, E_s to the equilibrium price in the weak market, and U to the equilibrium price under uniform pricing). This occurs if and only if $\alpha_s/\alpha_w < (1-\theta_w)/(1-\theta_s)$. In this sense, our model specification puts fewer restrictions (except linearity) on the economic fundamentals than Corts' (1998) study does.

2.3 Solutions and Preliminary Results

As an innocuous normalization, we set the constant marginal cost to zero, $c = 0$.[19] Given that

$$\frac{\partial q_m^A(p_m^A, p_m^B)}{\partial p_m^A} = -\frac{\beta_m}{\beta_m^2 - \gamma_m^2}$$

from (1) and that $q_m(p) = (\alpha_m - p)/(\beta_m + \gamma_m)$ is the symmetric demand function, the equilibrium discriminatory prices are (from (3))

$$p_m^* = \frac{\alpha_m(\beta_m - \gamma_m)}{2\beta_m - \gamma_m}$$

and from (2) the equilibrium uniform price is

$$p^* = \frac{(\beta_w - \gamma_w)(\beta_s - \gamma_s)[(\beta_w + \gamma_w)\alpha_s + (\beta_s + \gamma_s)\alpha_w]}{(2\beta_w - \gamma_w)(\beta_s^2 - \gamma_s^2) + (2\beta_s - \gamma_s)(\beta_w^2 - \gamma_w^2)} \; (\equiv p^*(\boldsymbol{\gamma}, \boldsymbol{\alpha}, \boldsymbol{\beta}))$$

under the regime of uniform pricing (where $\boldsymbol{\gamma} \equiv (\gamma_s, \gamma_w)$, $\boldsymbol{\alpha} \equiv (\alpha_s, \alpha_w)$ and $\boldsymbol{\beta} \equiv (\beta_s, \beta_w)$) *if both markets are open.* See Appendix A1 for derivation of the associated output in each market under price discrimination and under uniform pricing. Appendix A2 shows that the weak market must be sufficiently large for neither firm to have an incentive to deviate to close it. It must also be small enough for the strong market to remain strong (i.e., the equilibrium prices under price discrimination are higher than under uniform pricing; see Footnote 12). Thus, we restrict the relative size in intercepts, $\alpha_w/\alpha_s \in (\underline{\alpha_w/\alpha_s}, \overline{\alpha_w/\alpha_s})$. These upper and lower bounds are functions of $\boldsymbol{\gamma}$ and $\boldsymbol{\beta}$, and their precise expressions are given in Appendix A2.[20]

Notice that $\partial p_m^*/\partial(\gamma_m/\beta_m) = -\alpha_m/[2 - (\gamma_m/\beta_m)]^2 < 0$, which implies that as γ_m/β_m becomes larger the discriminatory prices decreases. In addition, the uniform price and the discriminatory prices converge to the marginal cost because $\lim_{\tilde{\gamma}_m \uparrow 1} p_m^* = 0 = \lim_{\gamma_m \uparrow \min(\beta_m)} p^*$ for all m.

[19] Notice the innocuousness of the zero marginal cost assumption: it is equivalent to assuming a constant marginal cost if prices and consumers' willingness to pay are interpreted as the net cost (interpreting it as $\alpha_m - c$ as α_m).

[20] The "weak" market is smaller than the "strong" market in the sense that the marginal willingness to pay $\partial U_m(q_m^A, q_m^B)/\partial q_m^j$ at $(q_m^A, q_m^B) = (0, 0)$ is greater in the strong market.

3 Welfare Analysis

This section consists of two subsections. The first subsection presents analytical properties that are useful for welfare analysis. We then investigate the welfare effects of price discrimination in the second subsection.

3.1 Analytical Properties

In symmetric equilibrium, social welfare under regime $r \in \{D, U\}$ is written as

$$SW^r = 2(\alpha_s q_s^r + \alpha_w q_w^r) - (\beta_s + \gamma_s)[q_s^r]^2 - (\beta_w + \gamma_w)[q_w^r]^2$$

where $q_m^D = q_m(p_m^*)$ and $q_m^U = q_m(p^*)$ are the equilibrium quantities in market m under the regimes of price discrimination and of uniform pricing, respectively (see Appendix A1 for the actual functional forms). Let ΔSW^* be defined by the equilibrium difference $SW^D - SW^U$. It is then given by

$$\begin{aligned}
\Delta SW^* &= \Delta SW^*(\boldsymbol{\gamma}, \boldsymbol{\alpha}, \boldsymbol{\beta}) \\
&\equiv 2\left[\alpha_s(q_s^D - q_s^U) + \alpha_w(q_w^D - q_w^U)\right] \\
&\quad -(\beta_s + \gamma_s)\left(q_s^D - q_s^U\right)\left(q_s^D + q_s^U\right) - (\beta_w + \gamma_w)\left(q_w^D - q_w^U\right)\left(q_w^D + q_w^U\right) \\
&= \Delta q_s^*[2\alpha_s - (\beta_s + \gamma_s)(q_s^D + q_s^U)] + \Delta q_w^*[2\alpha_w - (\beta_w + \gamma_w)(q_w^D + q_w^U)],
\end{aligned}$$

where $\Delta q_m^* \equiv q_m^D - q_m^U$. It is further shortened, and thus we have the following lemma (see Appendix A3 for a proof):

Lemma 1. *The equilibrium difference $\Delta SW^* = \Delta SW^*(\boldsymbol{\gamma}, \boldsymbol{\alpha}, \boldsymbol{\beta})$ is given by*

$$\Delta SW^*(\boldsymbol{\gamma}, \boldsymbol{\alpha}, \boldsymbol{\beta}) = -\sum_{m \in \{s,w\}} \frac{\Delta p_m^*}{\beta_m + \gamma_m} \cdot (p_m^* + p^*),$$

where $\Delta p_m^* \equiv p_m^* - p^*$.

This expression has the following graphical interpretation. Figure 2 shows the relationship between Δp_m^* and Δq_m^*. As Appendix A1 demonstrates, we have $\Delta p_m^* = -(\beta_m + \gamma_m)\Delta q_m^*$. This relationship can be interpreted as the situation where *in* *symmetric equilibrium* any firm faces the "virtual" inverse demand function, $p_m = \alpha_m - (\beta_m + \gamma_m)q_m$, in market m (notice the difference from the original inverse demand function, $p_m^j(q_m^j, q_m^{-j}) = \alpha_m - \beta_m q_m^j - \gamma_m q_m^{-j}$). Notice that the "virtual" inverse demand function is not defined in the whole domain of q_m except for the bold part of the curve: more precisely, it is defined only on the equilibrium points (for uniform pricing and for price discrimination). The welfare change in market m is depicted by the shaded trapezoid in Figure 2 (in this example, it is a welfare gain). Thus, its size is calculated by the sum of the upper and bottom segments $(p_m^* + p^*)$ multiplied by height $(\Delta q_m^* = -\Delta p_m^*/(\beta_m + \gamma_m))$, divided by two. Noting that two identical firms exist in market m, we have $-\Delta p_m^*(p_m^* + p^*)/(\beta_m + \gamma_m)$ as a welfare change in market m.

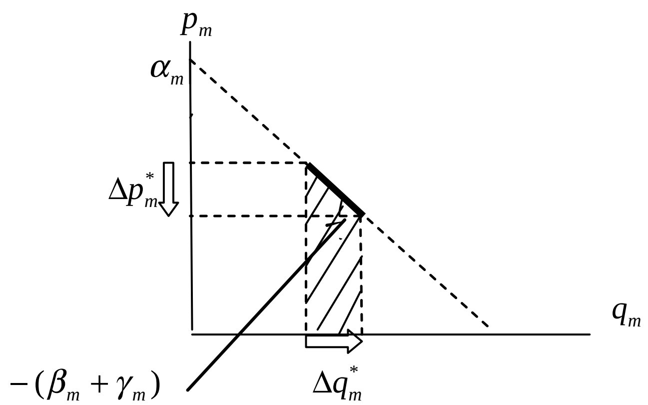

Figure 2: Equilibrium Changes in Quantity and Price in Market m (for any firm)

If it is positive (when $\Delta p_m^* < 0$), then it is a welfare *gain*. Similarly, if it is negative (when $\Delta p_m^* > 0$), then it is a welfare *loss*. Other things being equal, the greater the value of γ_m (more substitutable) the steeper (and hence less elastic) the equilibrium inverse demand curve becomes.

We have the following property of the price elasticity under price discrimination. A simple calculation leads to the following lemma:

Lemma 2. *Let the equilibrium price elasticity of demand in market m under price* *discrimination be defined by*

$$\varepsilon_m(p_m^*) \equiv \left| -\frac{dq_m(p_m^*)}{dp_m^*}\frac{p_m^*}{q_m^D} \right|,$$

where $q_m(p_m^*) = (\alpha_m - p_m^*)/(\beta_m + \gamma_m)$. *Then, it is expressed by*

$$\varepsilon_m(p_m^*) = \underbrace{1}_{\textit{market elasticity}} + \underbrace{\left(-\frac{\gamma_m}{\beta_m}\right)}_{\textit{cross-price elasticity}}. \tag{4}$$

Notice that $\varepsilon_m(p_m^*)$ is a constant, and does not depend on either q_m^D or even the intercept, α_m. This decomposition is a special result of Holmes' (1989, p.246) general result: *firm-level* elasticity is the sum of the *market* elasticity and the *cross-price* elasticity.[21]

The *market* elasticity of demand is a unit-free measure of responsiveness of the firms *as a whole*. However, *strategic interaction* distinguishes it from the elasticity on which each firm bases its decision making: the *cross-price* elasticity measures of how much each firm "damages" the other firm *in equilibrium*. In our model, strategic interaction is created by the very fact that firms (horizontally) differentiate their products or services. Notice that the market elasticity is exactly one as in the case of a one-good monopoly with a linear demand curve (remember that price elasticity of demand is one when the marginal revenue curve crosses the constant marginal cost curve (i.e., the horizontal axis)).

As we mention in Section 2, the ratio $\gamma_m/\beta_m \in (-1, 1)$ is interpreted as the normalized measure of horizontal product differentiation in market m. The cross-price elasticity in Holmes (1989) is simply expressed by the negative of the ratio alone. From (4), we have the relationship, $\varepsilon_m(p_m^*) \lesseqqgtr 1$ if and only if $\gamma_m \gtrless 0$. That is, if the brands are substitutes ($\gamma_m > 0$), then the firm-level elasticity in equilibrium is *less* than one, meaning that raising the price by one percent (as joint decision) creates a *less than* one percent decrease in its demand, and thus an *increase* in revenue (hence in profit). On the other hand, a one percent price cut creates *more*

[21] Holmes (1989) shows the decomposition under the assumption of symmetric demands between firms: it also holds off equilibrium. The term "market elasticity" is borrowed from Stole (2007) (Holmes (1989) originally called it the "industry-demand elasticity").

than a one percent increase in its demand if the brands are complements ($\gamma_m < 0$). These facts imply that substitutability (resp. complementarity) in market m keeps the equilibrium prices relatively low (resp. high).

As to changes in equilibrium aggregate output, ΔQ^* (see Appendix A1 for the derivation), it is shown that if the aggregate output is not increased by price discrimination, then social welfare deteriorates (i.e., $\Delta Q^* \leq 0 \Rightarrow \Delta SW^* < 0$), as verified by Bertoletti (2004) in the case of monopoly with linear and nonlinear demands[22] and by Dastidar (2006, p.244) in the case of oligopoly with linear and nonlinear demands (see Appendix A4 for the derivation in our settings).

Given that market s is strong ($\alpha_s/\alpha_w > \underline{\alpha_s/\alpha_w}$), we have the following relationship:

$$\Delta Q^* \gtreqless 0 \quad \Leftrightarrow \quad \frac{\gamma_s}{\beta_s} \gtrless \frac{\gamma_w}{\beta_w} \quad \Leftrightarrow \quad \varepsilon_s(p_s^*) \lesseqgtr \varepsilon_w(p_w^*),$$

which is a special case of Holmes' (1989) result that includes nonlinear demands.[23] Notice that, as Dasidar (2006, p.240) also points out, shifting from uniform pricing to price discrimination can increase aggregate output under oligopoly with linear demands while under monopoly, it remains constant.

Holmes (1989, p.247) shows that a change in the aggregate output resulting from price discrimination is positive if and only if the sum of the two terms, the "adjusted-concavity condition" and "elasticity-ratio condition", is positive. As its name implies, the first term is related to the demand curvature, and in our case of linear demands, it is zero. The second term is written as

$$\frac{\text{cross-price elasticity in market } s}{\text{market elasticity in market } s} - \frac{\text{cross-price elasticity in market } w}{\text{market elasticity in market } w},$$

which is equivalent to $\gamma_s/\beta_s - \gamma_w/\beta_w$ from Lemma 1. If this is positive, a change in the aggregate output is positive. This is consistent with the relationship mentioned above. We can understand this relationship by focusing on the elasticities: $\varepsilon_s(p_s^*) < \varepsilon_w(p_w^*)$ means that an increase in discriminatory price takes place in the market

[22] Bertoletti's (2004) result is a generalization of the well-known result of Varian (1985) and Schwartz (1990) who state that $\Delta Q^* < 0 \Rightarrow \Delta SW^* < 0$.

[23] Aguirre (2011) points out that Footnote 6 of Holmes (1989) is incorrect: his analysis does not apply to the family of constant elasticity demand functions.

where it creates more revenue, and that a decrease in discriminatory price takes place in the market where it doesn't. Notice also that the relationship, together with Proposition 2, implies that the degree of substitution must be larger in the strong market than in the weak market for a positive change in social welfare. There are two sources of an increase in aggregate output: one is a large increase in the weak market, and the other is a small decrease in the strong market. Noting the equilibrium discriminatory price in market m is a decreasing function of γ_m/β_m (see subsection 2.2), $\gamma_s/\beta_s > \gamma_w/\beta_w$ means that the latter effect must be small, and thus the degree of substitution must be larger in the strong, rather than the weak, market. The result that the output change, ΔQ^*, can be positive in oligopoly is in sharp contrast with the case in monopoly where the output change is *always* zero with linear demand.[24] In the next subsection, we explore the possibility of $\Delta SW^* > 0$ in the differentiated oligopoly. However, a positive change in social welfare is solely due to an improvement in the firms' profits. This is because a change in aggregate consumer surplus is always negative. Let ΔCS^* be defined by the equilibrium difference between aggregate consumer surpluses under price discrimination and under uniform pricing ($CS^D - CS^U$). We then have the following result (see Appendix A5 for the proof).

Proposition 1. *Price discrimination always deteriorates aggregate consumer surplus (i.e., $\Delta CS^* < 0$ for all exogenous parameters).*

3.2 Welfare-Improving Price Discrimination

We now explore the possibility of $\Delta SW^*(\boldsymbol{\gamma}, \boldsymbol{\alpha}, \boldsymbol{\beta}) > 0$. To proceed further, we now reduce the number of the parameters. More specifically, we assume that $\alpha_s = 1 > \alpha_w > 0$. This is because price discrimination never improves welfare if $\alpha_s = \alpha_w$ (the formal proof is available upon request). Thus, $\alpha_s/\alpha_w > 1$ is necessary for social welfare to improve. In the following analysis, we first consider the case of symmetry in product differentiation in the strong and weak markets ($\gamma_s/\beta_s = \gamma_w/\beta_w$). We then

[24] Dastidar (2006, p.240) also points out this difference in price-setting models of oligopoly (see also Footnote 10).

allow asymmetric product differentiation. To do so, we first construct an intuitive argument why price discrimination improves social welfare. Given the equilibrium discriminatory price is higher (lower) than the uniform price in the strong (weak) market, we know that (note that, $\Delta q_m^* = -\Delta p_m^*/(\beta_m + \gamma_m)$)

$$\Delta SW^* > 0 \Leftrightarrow \Delta q_w^* \cdot (p^* + p_w^*) > \Delta q_s^* \cdot (p^* + p_s^*).$$

For the latter inequality to hold, (1) Δq_w^* or $(p^* + p_w^*)$ is sufficiently *large*, and/or (2) $|\Delta q_s^*|$ or $(p^* + p_s^*)$ is sufficiently *small*. Figure 3 shows the asymmetry between the strong and the weak markets. Notice that the upper segment of the trapezoid of the welfare loss in the strong market and the bottom segment of the trapezoid of the welfare gain in the weak market have the same length (p^*). Thus, the larger $|\Delta q_s^*|$, the larger $(p^* + p_s^*)$ is. On the other hand, the larger Δq_w^*, the smaller $(p^* + p_w^*)$ is. Hence, *the smaller* $|\Delta q_s^*|$*, the better it is for welfare improvement, while* Δq_w^* *should not be too small or too large.*

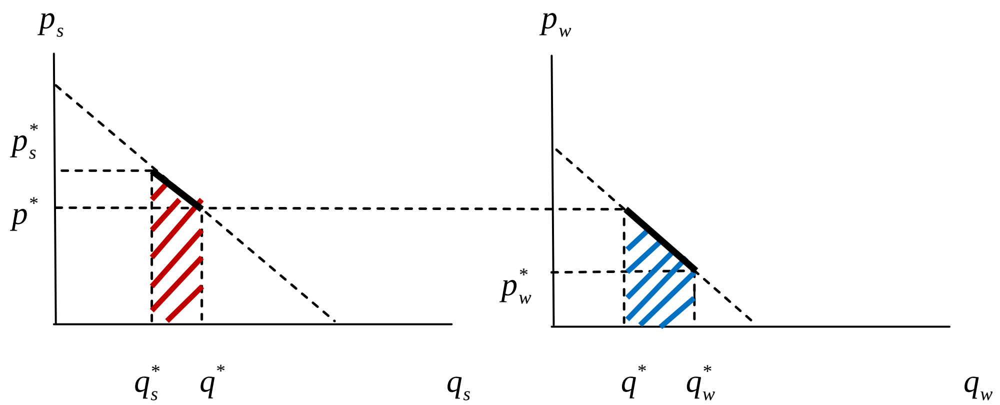

Figure 3: Asymmetry between the Strong and the Weak Markets

Lastly, we consider the relationship between ΔSW^* and $\Delta\Pi^*$. Remember that an increase in aggregate output is necessary for price discrimination to improve social welfare. It is shown that its necessary condition is an increase in profit (i.e.,

$\Delta SW^* > 0 \Rightarrow \Delta Q^* > 0 \Rightarrow \Delta\Pi^* > 0$; see Appendix A6 for the verification). It means that whenever price discrimination improves social welfare, firms always have an incentive to (jointly) switch to the regime of price discrimination from the regime of uniform pricing.

3.2.1 The Case of Symmetric Product Differentiation

Let the situation be called *symmetric* product differentiation if the normalized measures of horizontal product differentiation coincide in the two markets (i.e., $\gamma_s/\beta_s = \gamma_w/\beta_w$). In this case, the two markets are "similar" in the sense that the only difference in the two markets is in the intercepts of the inverse demand curves. It is shown that if $\gamma_s = \gamma_w$ and $\beta_s = \beta_w$, then $\Delta Q^* = 0$ (see Appendix A1).[25] This means that $|\Delta q_w^*| = |\Delta q_s^*|$. Because p_s^* is greater than p_w^* (which comes from the assumption $\alpha_s > \alpha_w$), the loss in the strong market is always larger than the gain in the weak market. Thus, in the case of symmetric product differentiation, social welfare is never improved by price discrimination (i.e., $\Delta SW^* < 0$ for all exogenous parameters). We therefore need to consider the case of $\gamma_s\beta_w \neq \gamma_w\beta_s$, which is called *asymmetric* product differentiation, to study the possibility that $\Delta SW^* > 0$.[26]

3.2.2 The Case of Asymmetric Product Differentiation

To simplify the analysis, we assume that $\beta_s = \beta_w$. By so doing, we are able to focus on the effects of (γ_s, γ_w) on social welfare. More specifically, we allow γ_s and γ_w to differ, letting $\beta \equiv \beta_s = \beta_w$ to avoid unnecessary complications (Appendix A8 gives an analysis with $\gamma_s = \gamma_w$ to show the effects of $\beta_s = \beta_w$ on social welfare). We present numerical and graphical arguments on the domains (γ_s, γ_w) that make $\Delta SW^* > 0$ for fixed values of $(\alpha_w, \beta_s, \beta_w)$.

Figures 4 and 5 depict the region of $\Delta SW^* > 0$ (with $\alpha_w = 0.85$ for Figure 4 and

[25] Cowan (2011) also verifies this result with linear demands that are not derived from the representative consumers' utility functions,

[26] Note that we do not necessarily assume that the strong market has a larger intercept ($\alpha_s > \alpha_w$). However, Appendix A7 verifies that if $\alpha_s = \alpha_w$, price discrimination never improves social welfare.

$\alpha_w = 0.95$ for Figure 5) when $\beta = 1.0$ (the shaded area).[27] Consider first the case of substitutable goods ($\gamma_s > 0$ and $\gamma_w > 0$). Remember from Proposition 2 that for the total social surplus to be improved by price discrimination it is necessary that $\Delta Q^* > 0 \Leftrightarrow \gamma_s/\beta_s > \gamma_w/\beta_w$, that is, $\gamma_s > \gamma_w$ in this specification. Substitutability in the strong market must be larger than that in the weak market for a welfare improvement. Remember that the slope of the equilibrium demand in the strong market $-(\beta_s+\gamma_s)$ is steeper than that in the weak market. This is associated with a larger increase in output in the weak market rather than a decrease in output in the strong market. Why is there a bottom right boundary of the region for $\Delta SW^* > 0$? It derives from the restriction that markets s and w are strong and weak markets respectively: $\alpha_w/\alpha_s < \overline{\alpha_w/\alpha_s}(\boldsymbol{\gamma}, \boldsymbol{\beta})$ (the details are available upon request). In the unshaded southeastern area, this inequality does not hold. That is, $p_s^* < p^* < p_w^*$ in equilibrium. In other words, markets s and w are actually "weak" and "strong" markets, respectively. In this area, the substitutability in the actual strong market (market w) is lower than that in the actual weak market (market s), i.e., $\gamma_w < \gamma_s$. As mentioned earlier, $\Delta SW^* > 0$ *only if* the actual strong market is more elastic. Therefore, $\Delta SW^* < 0$ in the unshaded southeastern area.

An intuitive explanation for welfare improvement is as follows. The greater the level of competition is in the weak market, the more welfare gain is expected in the weak market. This welfare gain in the weak market outweighs the welfare loss in the strong market when an increase in the discriminatory price in the strong market is kept low. Thus, competition must be sufficiently fiercer in the strong market than in the weak market because the strong market has a larger market size. Note that the distance between the line $\gamma_w = \gamma_s$ and the left-hand side line of the area where $\Delta SW^* > 0$ is wider as γ_w becomes lower. This is because for a low γ_w the discriminatory price in the weak market is close to the uniform price so that the welfare gain in this market is small. This inefficiency is greater for a lower value of γ_w. Thus, the level of competition in the strong market that is necessary to offset

[27] It is verified that all of the model parameters in the analysis below satisfy the restriction conditions provided in Appendix A2. Appendix A9 provides figures allowing β_s and β_w to be different: one sees no significant differences.

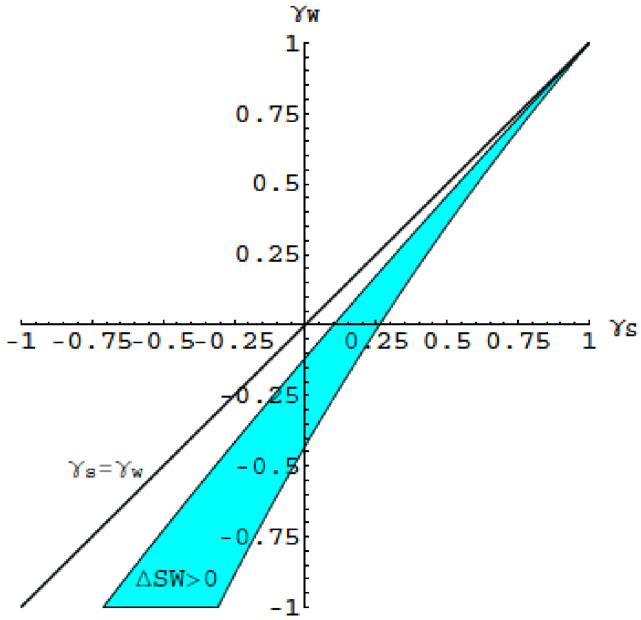

Figure 4: The Region of $\Delta SW^* > 0$ for the Case of $\alpha_w = 0.85$ and $\beta = 1.0$

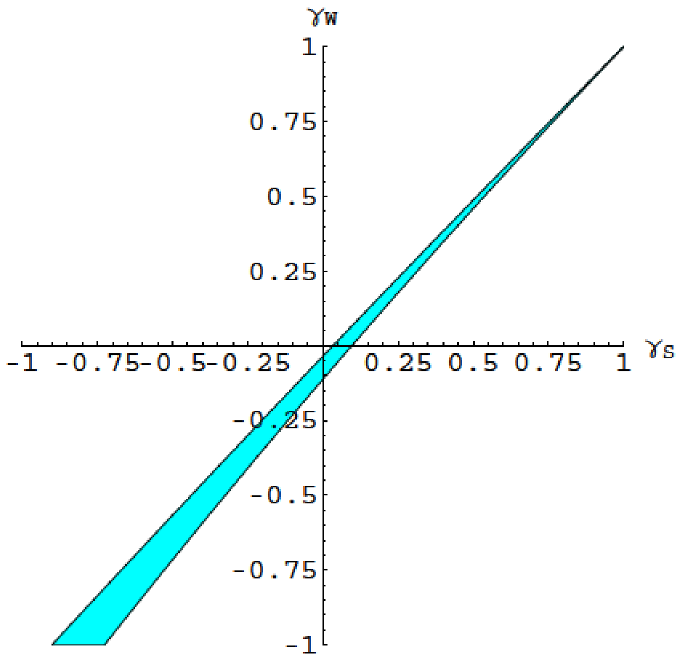

Figure 5: The Region of $\Delta SW^* > 0$ for the Case of $\alpha_w = 0.95$ and $\beta = 1.0$

this inefficiency loss is greater.

Notice that price discrimination *never* improves social welfare in the second quadrant ($\gamma_s < 0 < \gamma_w$). That is, *if the two brands are complementary in the strong market* ($\gamma_s < 0$) *while the firms sell substitutable goods in the weak market* ($\gamma_w > 0$), *then price discrimination necessarily deteriorates social welfare.* This result seems to hold for other parameter values because $\Delta SW^* \leq 0$ if $\beta_s = \beta_w$ and $\gamma_s = \gamma_w$: in the northwestern region separated by $\gamma_s = \gamma_w$, social welfare would be negative. The intuitive reason is that complementarity in the strong market makes the price change caused by discrimination more responsive, which creates more inefficiency, while substitutability in the weak market makes the price change less responsive. The latter positive effect is not sufficiently large to outweigh the former negative effect.

On the other hand, it is possible that price discrimination improves social welfare *if the firms' brands are substitutes in the strong market* ($\gamma_s > 0$) *and are complements in the weak market* ($\gamma_w < 0$). Figures 4 and 5 also show that the combination of a high degree of complementarity in the weak market and a low degree of complementarity in the weak market (i.e., $|\gamma_w|$ larger than $|\gamma_s|$) is suited to welfare gain. This result is as expected: strong complementarity in the weak market keeps the discriminatory price low enough to offset the loss from the price increase in the strong market. However, it has been verified that consumer surplus is *never* improved by price discrimination (Proposition 1).

Analytical arguments for these results are provided as follows. Fix $\gamma_w \in (-1, 1)$, and notice that $\Delta SW^* = 0$ if (γ_s, γ_w) satisfies $\overline{\alpha_w/\alpha_s}(\boldsymbol{\gamma}, \boldsymbol{\beta}) = \alpha_w/\alpha_s$. This equality is rewritten as

$$\begin{aligned}\frac{\alpha_w}{\alpha_s} &= \frac{(\beta_s - \gamma_s)(2\beta_w - \gamma_w)}{(\beta_w - \gamma_w)(2\beta_s - \gamma_s)} \\ \Leftrightarrow \ \gamma_s &= \frac{\beta_s[2(\alpha_s - \alpha_w)\beta_w - (\alpha_s - 2\alpha_w)\gamma_w]}{(2\alpha_s - \alpha_w)\beta_w - (\alpha_s - \alpha_w)\gamma_w} \equiv \overline{\gamma}_s(\gamma_w, \boldsymbol{\alpha}, \boldsymbol{\beta}).\end{aligned}$$

Substituting $\overline{\gamma}_s$ into $\partial \Delta SW^* / \partial \gamma_s$, we have:

$$\left.\frac{\partial \Delta SW^*}{\partial \gamma_s}\right|_{\gamma_s = \overline{\gamma}_s} < 0,$$

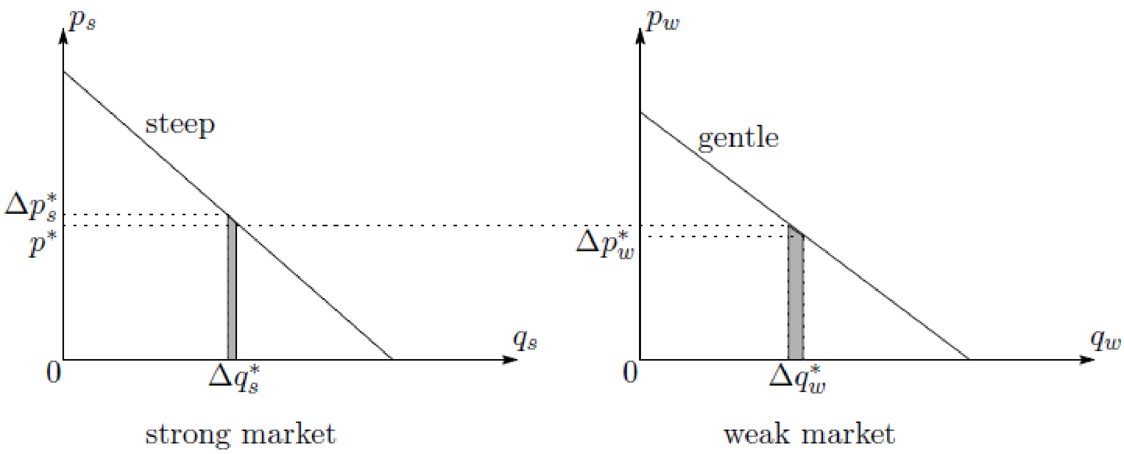

Figure 6: Welfare Changes in the Strong and the Weak Markets

which leads to the following proposition (the formal proof is in Appendix A10):

Proposition 2. *Fix* $\boldsymbol{\alpha}$ *and* $\boldsymbol{\beta}$ *that satisfy the restrictions stated above. For any* $\gamma_w \in (-1, 1)$, *there exists* γ_s' *such that* $\Delta SW^* > 0$ *for* $\gamma_s \in (\gamma_s', \overline{\gamma}_s(\gamma_w, \boldsymbol{\alpha}, \boldsymbol{\beta}))$.

In other words, given the values of γ_w and γ_s that satisfy $\overline{\alpha_w/\alpha_s}(\boldsymbol{\gamma}, \boldsymbol{\beta}) = \alpha_w/\alpha_s$, a slight decrease in γ_s from $\overline{\gamma}_s(\gamma_w)$, keeping γ_w constant, enhances ΔSW^*. This is consistent with the result in Figures 4 and 5. In relation to the example in the Introduction, if an introduction of price discrimination by Coca Cola had triggered Pepsi to adopt the same strategy, it would have improved social welfare if the degree of substitution was greater in the resort area (i.e., the strong market) than in the city area (i.e., the weak market) so that (γ_s, γ_w) falls into the region where $\Delta SW^* > 0$. This is likely because consumers would care less about the brands if temperature is high. Another example is when submarkets are grouped by seniority. If the market for elderly consumers is strong and if they care less about brands, then (γ_s, γ_w) satisfy the requirement for welfare improvement.

Figure 6 provides a graphical exposition of Proposition 2. Consider a small decrease in γ_s (keeping γ_w constant) from any point (γ_s, γ_w) that satisfies $\overline{\alpha_w/\alpha_s}(\boldsymbol{\gamma}, \boldsymbol{\beta})$ $= \alpha_w/\alpha_s$. First, note that $\gamma_s > \gamma_w$. Second, for very small price changes Δp_s^* and Δp_w^* caused by the small decrease in γ_s, Δq_w^* is greater than $|\Delta q_s^*|$ because the gra-

	$(\gamma_s, \gamma_w) =$	
	$(0.1, -0.1)$	$(-0.1, 0.1)$
p^*	0.4588	0.4663
p_s^* $(\Delta p_s^*/p^*)$	0.4737 (3%)	0.5238 (12%)
p_w^* $(\Delta p_w^*/p^*)$	0.4452 (−3%)	0.4026 (−14%)
Δq_s^* $(\Delta q_s^*/q_s^*(p^*))$	−0.0136 (−3%)	−0.0640 (−11%)
Δq_w^* $(\Delta q_w^*/q_w^*(p^*))$	0.0150 (3%)	0.0578 (17%)
ΔSW^*	0.0009	−0.0131
ΔCS_s^*	−0.0145	−0.0646
ΔCS_w^*	0.0120	0.0481
$\Delta\Pi^*$	0.0034	0.0034
ΔQ^*	0.0014	−0.0061

Table 1: Asymmetric Product Differentiation ($\alpha_w = 0.85$ and $\beta = 1.0$)

dient in the weak market has a less steep slope. Note that the less γ_m, the gentler the slope of the "virtual" inverse demand function, $p_m = \alpha_m - (\beta_m + \gamma_m)q_m$ (see Subsection 3.1). While the changes in output have first-order effects on the welfare changes, the price changes has second-order effects ($\Delta p_m^* \times \Delta q_m^*$) and thus can be ignored. Therefore, when γ_s slightly decreases from any point (γ_s, γ_w) that satisfies $\overline{\alpha_w/\alpha_s}(\boldsymbol{\gamma}, \boldsymbol{\beta}) = \alpha_w/\alpha_s$, price discrimination improves social welfare. This argument would be applied to non-linear demands.

Next, we focus on one case of asymmetric product differentiation. Table 1 shows the result for the case of $\alpha_w = 0.85$ and $\beta_s = \beta_w = 1.0$. The first case, where the two brands are substitutes in the strong market while they are complementary goods in the weak market, has smaller changes in both prices and output than the second case has. Social welfare is improved by price discrimination in the first case. The price differentials in the latter case are greater: what happens after the regime change from uniform pricing to price discrimination is that while competition in the weak market becomes fiercer due to substitutability, complementarity softens the competition to increase discriminatory price in the strong market.

Lastly, we also provide a graphical analysis for $\Delta\Pi^* > 0$ (remember that $\Delta\Pi^* > 0$ is necessary for $\Delta SW > 0$). Figure 7 depicts the area of $\Delta\Pi^* > 0$ when $\alpha_w = 0.85$

and $\beta_s = \beta_w = 1.0$. The bottom right boundary comes from, as in Figure 4, the restriction that markets s and w are strong and weak markets respectively. The top center boundary is set for both markets to be open under uniform pricing. Then, where does the left curve come from? Notice that the excluded area roughly corresponds to "$\gamma_s < \gamma_w$", which leads to a relatively high discriminatory price in the strong market. Thus, the strong market is the market where firms want to lower the price. However, price discrimination "forces" them to raise the price in the strong market. Thus, price discrimination makes the firms worse off, and the lower γ_s, the more significant this negative effect is.

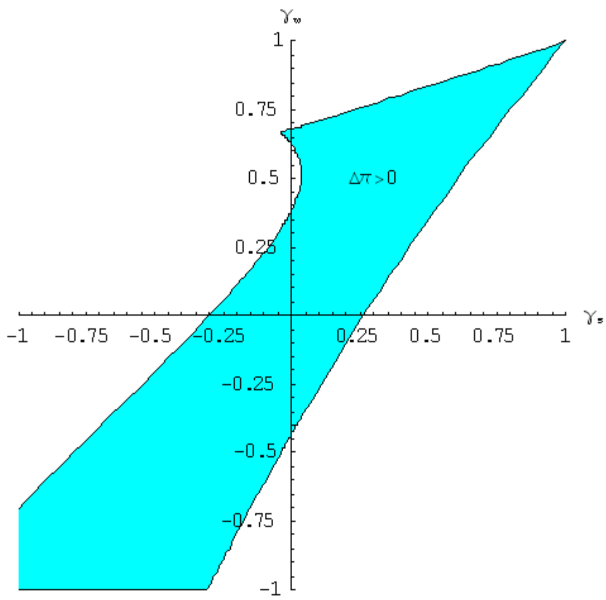

Figure 7: The Region of $\Delta\Pi^* > 0$ for the Case of $\alpha_w = 0.85$ and $\beta_s = \beta_w = 1.0$

4 Concluding Remarks

In this paper, we study the relationship between horizontal product differentiation and the welfare effects of third-degree price discrimination in oligopoly with linear demands. By deriving linear demands from a representative consumer's utility and focusing on symmetric equilibrium in a pricing game, we characterize conditions relating to such demand properties as substitutability and complementarity for price discrimination to improve social welfare. More specifically, we derive a necessary and sufficient condition for price discrimination to improve social welfare: the degree of substitution must be sufficiently greater in the strong market than in the weak market. It is shown that price discrimination never improves social welfare if firms' brands are complements in the strong market and are substitutes in the weak market. An increase in aggregate output is necessary for welfare improvement. We verify, however, that consumer surplus is *never* improved by price discrimination: welfare improvement by price discrimination is solely the result of an increase in the firms' profits.

In the present paper, we focus only on symmetric equilibrium of the pricing game to gain analytical insight. In particular, the equilibrium amount of output is common for all firms under either uniform pricing or price discrimination.[28] This limitation would be particularly unappealing if one wished to compare the equilibrium predictions from our model with empirical data.[29] It is also important to consider nonlinear demands in oligopoly with price discrimination.[30] These and other interesting issues await future research.

[28] Galera and Zaratiegui (2006) consider duopolistic third-degree price discrimination with *heterogeneity in constant marginal cost* and show that price discrimination can improve social welfare even if the total output does *not* change. This favors the low-cost firm to cut its prices significantly, and this cost saving may overcome the welfare losses from price discrimination.

[29] Recent empirical studies on price discrimination under competition in final-product markets include **Asplund, Eriksson, and Strand (2008)**, Grennan (2011) and Hendel and Nevo (2011).

[30] See, e.g., Weyl and Fabinger (2009), Aguirre, Cowan, and Vickers (2010), and Cowan (2012) for recent advances in the study of monopolistic third-degree price discrimination with nonlinear demands.

Appendices

A1. Changes in Equilibrium Prices and Quantities by Price Discrimination

Equilibrium quantities produced by each firm under price discrimination in market m are

$$q_m(p_m^*) = \frac{\alpha_m \beta_m}{(2\beta_m - \gamma_m)(\beta_m + \gamma_m)},$$

where the denominator is positive because $|\gamma_m| < \beta_m$.

Under uniform pricing, *if both markets are open* (see Appendix A2 for the verification of market opening), then tedious calculation shows that the equilibrium uniform price is

$$p^* = \frac{(\beta_m - \gamma_m)(\beta_{m'} - \gamma_{m'})[\alpha_m(\beta_{m'} + \gamma_{m'}) + \alpha_{m'}(\beta_m + \gamma_m)]}{\Phi^U}, \tag{A1}$$

where $m \neq m'$ $(m, m' \in \{s, w\})$ and $\Phi^U \equiv \sum_{m \neq m'} (\beta_m^2 - \gamma_m^2)(2\beta_{m'} - \gamma_{m'})$. The denominator and the numerator are also found to be positive because $|\gamma_m| < \beta_m$. One can verify that the equilibrium quantities under uniform pricing in market $m \neq m'$ are then given by

$$q_m(p^*) = \frac{\alpha_m[\beta_m(\beta_{m'}^2 - \gamma_{m'}^2) + \beta_{m'}(\beta_m^2 - \gamma_m^2)] + (\alpha_m - \alpha_{m'})(\beta_m^2 - \gamma_m^2)(\beta_{m'} - \gamma_{m'})}{(\beta_m + \gamma_m)\Phi^U}. \tag{A2}$$

Now, let

$$\Delta p_m^* \equiv p_m^* - p^* = \frac{(\beta_m^2 - \gamma_m^2)[\alpha_m(\beta_m - \gamma_m)(2\beta_{m'} - \gamma_{m'}) - \alpha_{m'}(\beta_{m'} - \gamma_{m'})(2\beta_m - \gamma_m)]}{(2\beta_m - \gamma_m)\Phi^U}$$

be defined as the changes in the equilibrium prices resulting from a move uniform pricing to price discrimination in each market. Thus, if we define the *strong* (*weak*) market as that where the equilibrium price increases (decreases) by price discrimination, then market m is strong if and only if

$$\alpha_m > \frac{(\beta_{m'} - \gamma_{m'})(2\beta_m - \gamma_m)}{(\beta_m - \gamma_m)(2\beta_{m'} - \gamma_{m'})}\alpha_{m'}.$$

This implies that, in contrast to the case of monopoly with inter-market dependencies (see Adachi (2002)), the condition on the intercepts, $\alpha_m > \alpha_{m'}$, is not exactly the necessary and sufficient condition for market m to be strong: *if* $\gamma_{m'}\beta_m$ *is much larger than* $\gamma_m\beta_{m'}$ (note that either or both can be negative), *then market* m *with* $\alpha_m > \alpha_{m'}$ *can be weak.* Of course, if $\beta_m = \beta_{m'}$ and $\gamma_m = \gamma_{m'}$, then $\alpha_m > \alpha_{m'}$ is the necessary and sufficient condition for market m to be strong.

Turning our attention to output, we have

$$\begin{aligned}\Delta q_m^* &\equiv q_m(p_m^*) - q_m(p^*) \\ &= -\frac{(\beta_m - \gamma_m)[\alpha_m(\beta_m - \gamma_m)(2\beta_{m'} - \gamma_{m'}) - \alpha_{m'}(\beta_{m'} - \gamma_{m'})(2\beta_m - \gamma_m)]}{(2\beta_m - \gamma_m)\Phi^U}.\end{aligned} \tag{A3}$$

as the equilibrium changes in output resulting from a more from uniform pricing to price discrimination for each firm in strong and weak markets, respectively. It is then verified that Δp_m^* and Δq_m^* are related in the following way:

$$\Delta p_m^* = -(\beta_m + \gamma_m)\Delta q_m^*, \tag{A4}$$

so that we have $q_m(p_m^*) > q_m(p^*)$ if and only if $p_m^* < p^*$. One can also derive the change in equilibrium aggregate output:

$$\Delta Q^* \equiv \Delta q_s^* + \Delta q_w^* = \frac{(\beta_w\gamma_s - \beta_s\gamma_w)[\alpha_s(\beta_s - \gamma_s)(2\beta_w - \gamma_w) - \alpha_w(\beta_w - \gamma_w)(2\beta_s - \gamma_s)]}{(2\beta_s - \gamma_s)(2\beta_w - \gamma_w)\Phi^U},$$

which does not necessarily coincides with zero, as opposed to the case of monopoly with linear demands.

Now, although market m is strong even if $\alpha_m = \alpha_{m'}$ as long as $(\beta_m - \gamma_m)(2\beta_{m'} - \gamma_{m'}) > (\beta_{m'} - \gamma_{m'})(2\beta_m - \gamma_m)$, we assume that $\alpha_m \neq \alpha_{m'}$. This is because if $\alpha_m = \alpha_{m'}$, then we have

$$\Delta Q^* = -\frac{\alpha_m(\beta_{m'}\gamma_m - \beta_m\gamma_{m'})^2}{(2\beta_m - \gamma_m)(2\beta_{m'} - \gamma_{m'})\Phi^U} \leq 0,$$

$$\Delta p_m^* = \frac{\alpha_m(\beta_m^2 - \gamma_m^2)(\beta_m\gamma_{m'} - \beta_{m'}\gamma_m)}{(2\beta_m - \gamma_m)\Phi^U}$$

and most importantly, ΔSW^*, the difference in social welfare under price discrimination and under uniform pricing (introduced in Section 3), can never be positive (the formal proof is upon request). Thus, *unequal values of intercepts of the two markets are necessary for price discrimination to improve social welfare.* Hence, for markets s and w to be strong and weak, respectively, it is necessary for the weak market to be sufficiently small:

$$\frac{\alpha_w}{\alpha_s} < \min\left[\frac{(\beta_s-\gamma_s)(2\beta_w-\gamma_w)}{(\beta_w-\gamma_w)(2\beta_s-\gamma_s)}, 1\right].$$

The reason why this is not a sufficient condition is that we must verify the parameter restrictions *for market w to be sufficiently large to be open under uniform pricing.* We verify them in Appendix A2.

For later use (Appendix A3), we also calculate the sum of a firm's output under uniform pricing and the under price discrimination in each market $m \in \{s, w\}$:

$$q_m(p_m^*) + q_m(p^*) = \frac{\alpha_m(3\beta_m-\gamma_m)}{(\beta_m+\gamma_m)(2\beta_m-\gamma_m)} - \frac{p^*}{\beta_m+\gamma_m}. \tag{A5}$$

A2. Market Opening under Uniform Pricing

Remember that the symmetric equilibrium under uniform pricing in the main text and Appendix A1 is obtained, *given that* both markets are supplied by either firm under uniform pricing ($q_s(p^*) > 0$ and $q_w(p^*) > 0$). In this appendix, we obtain a (sufficient) condition guaranteeing that in equilibrium each firm supplies to the weak market under uniform pricing. To do so, we consider one firm's incentive not to deviate from the equilibrium by stopping its supply to the weak market.

Suppose firm j supplies only to the strong market, given that the rival firm supplies both markets with the equilibrium price, p^* (see Appendix A1). Let firm j's price when deviating from the equilibrium price under the uniform pricing regime be denoted by p'. Then, when firm j closes the weak market, its profit is written by[31]

$$\widetilde{\pi}(p', p^*) = p' \cdot q_s^j(p', p^*)$$

where

$$q_s^j(p', p^*) = \frac{\alpha_s}{\beta_s + \gamma_s} - \frac{\beta_s}{\beta_s^2 - \gamma_s^2} p' + \frac{\gamma_s}{\beta_s^2 - \gamma_s^2} p^*.$$

Now, it is verified that

$$\arg\max_{p' \neq p^*} \widetilde{\pi}(p^*) = \frac{\alpha_s(\beta_s - \gamma_s)}{2} + \frac{\gamma_s}{2} p^* \; (\equiv p'').$$

Note that firm j's profit function when it deviates to any price other than the equilibrium price would not necessarily be (globally) concave because it would be kinked at the threshold price where the weak market closes, as depicted in Figure 8.

If p'' attains the local maximum as in Panels (1) and (2) in Figure 8, then one needs to solve for the restriction on the set of parameters guaranteeing that the equilibrium profit when both markets are open

$$p^* \left(\frac{\alpha_s - p^*}{\beta_s + \gamma_s} + \frac{\alpha_w - p^*}{\beta_w + \gamma_w} \right)$$

is no smaller than the maximized profit when firm j deviates to close the weak market

$$\max_{p' \neq p^*} \widetilde{\pi}(p^*).$$

It is, however, too complicated to obtain the set of parameters from this inequality. Thus, we instead focus on the case that corresponds to Panel (3) in Figure 8. This gives a *sufficient* condition for the weak market to open. Notice that by

[31] Given p^*, the upper bound of p' such that $q_s^j(p', p^*) \geq 0$ is larger than that such that $q_w^j(p', p^*) \geq 0$ if and only if $\alpha_s > (\beta_w - \gamma_w)(2\beta_s - \gamma_s)\alpha_w / ((\beta_s - \gamma_s)(2\beta_w - \gamma_w))$. That is, for any p' such that $q_w^j(p', p^*) \geq 0$, $q_s^j(p', p^*) \geq 0$. In other words, given p^*, the strong market opens if the weak market does. The upper bound of p' such that $q_s^j(p', p^*) \geq 0$ is $((\beta_s - \gamma_s)\alpha_s + \gamma_s p^*)/\beta_s$. The upper bound of p' such that $q_w^j(p', p^*) \geq 0$ is $((\beta_w - \gamma_w)\alpha_w + \gamma_w p^*)/\beta_w$. The former minus the latter is

$$\frac{[(\beta_s - \gamma_s)(2\beta_w - \gamma_w)\alpha_s - (\beta_w - \gamma_w)(2\beta_s - \gamma_s)\alpha_w](\beta_s(\beta_w^2 - \gamma_w^2) + \beta_w(\beta_s^2 - \gamma_s^2))}{\beta_s\beta_w((\beta_s^2 - \gamma_s^2)(2\beta_w - \gamma_w) + (\beta_w^2 - \gamma_w^2)(2\beta_s - \gamma_s))} > 0.$$

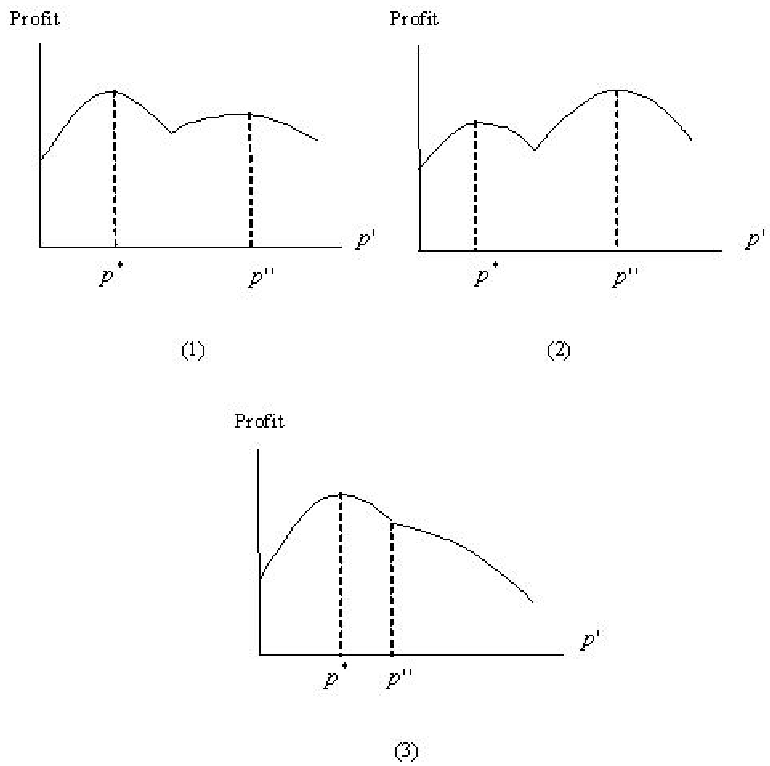

Figure 8: Profit when Deviating from the Equilibrium Price under Uniform Pricing

definition, p'' must satisfy $q_w^j(p'', p^*) \le 0$, given p^*. If this is violated, then it is the case of Panel (3) in Figure 8. It shortens to

$$\alpha_w > \frac{(\beta_s - \gamma_s)[2(\beta_w - \gamma_w)(\beta_w^2 - \gamma_w^2)\beta_s + (2\beta_w - \gamma_w)(\beta_s^2 - \gamma_s^2)\beta_w]}{(\beta_w - \gamma_w)[2(2\beta_s - \gamma_s)(\beta_w^2 - \gamma_w^2)\beta_s + (4\beta_s - \gamma_s)(\beta_s^2 - \gamma_s^2)\beta_w]}\alpha_s.$$

Together with the argument in Appendix A1, throughout the paper, we assume the relative size of the intercept of the weak market satisfies $\alpha_w/\alpha_s \in (\underline{\alpha_w/\alpha_s}, \overline{\alpha_w/\alpha_s})$, where

$$\begin{aligned}\underline{\frac{\alpha_w}{\alpha_s}} &= \underline{\frac{\alpha_w}{\alpha_s}}(\boldsymbol{\gamma}, \boldsymbol{\beta}) \\ &\equiv \frac{(\beta_s - \gamma_s)[2(\beta_w - \gamma_w)(\beta_w^2 - \gamma_w^2)\beta_s + (2\beta_w - \gamma_w)(\beta_s^2 - \gamma_s^2)\beta_w]}{(\beta_w - \gamma_w)[2(2\beta_s - \gamma_s)(\beta_w^2 - \gamma_w^2)\beta_s + (4\beta_s - \gamma_s)(\beta_s^2 - \gamma_s^2)\beta_w]},\end{aligned}$$

and

$$\begin{aligned}\overline{\frac{\alpha_w}{\alpha_s}} &= \overline{\frac{\alpha_w}{\alpha_s}}(\boldsymbol{\gamma}, \boldsymbol{\beta}) \\ &\equiv \min\left[\frac{(\beta_s - \gamma_s)(2\beta_w - \gamma_w)}{(\beta_w - \gamma_w)(2\beta_s - \gamma_s)}, 1\right].\end{aligned}$$

These restrictions are sufficient for markets s and w to be actually strong and weak *and* to be open under uniform pricing.

A3. Proof of Lemma 1 (Calculating ΔSW^* as a Function of p^*, p_s^* and p_w^*)

By using equations (A1-A5), we can calculate

$$\begin{aligned}\Delta SW^* &= \sum_{m\in\{s,w\}} [2\alpha_m(q_m(p_m^*) - q_m(p^*)) - (\beta_m + \gamma_m)[q_m(p_m^*) - q_m(p^*)][q_m(p_m^*) + q_m(p^*)]] \\ &= \sum_{m\in\{s,w\}} \frac{\Delta p_m^*}{\beta_m + \gamma_m}[-2\alpha_m + (\beta_m + \gamma_m)(q_m(p_m^*) + q_m(p^*))] \\ &= \sum_{m\in\{s,w\}} \frac{\Delta p_m^*}{\beta_m + \gamma_m}\left[-2\alpha_m + (\beta_m + \gamma_m)\left[\frac{\alpha_m(3\beta_m - \gamma_m)}{(\beta_m + \gamma_m)(2\beta_m - \gamma_m)} - \frac{p^*}{\beta_m + \gamma_m}\right]\right] \\ &= \sum_{m\in\{s,w\}} \frac{\Delta p_m^*}{\beta_m + \gamma_m}\left[-\frac{\alpha_m(\beta_m - \gamma_m)}{2\beta_m - \gamma_m} - p^*\right] \\ &= -\sum_{m\in\{s,w\}} \frac{\Delta p_m^*}{\beta_m + \gamma_m}(p_m^* + p^*).\end{aligned}$$

A4. $\Delta Q^* \leq 0 \Rightarrow \Delta SW^* < 0$

Using the explicit forms for Δp_m^* and for ΔQ^*(derived in Appendix A1), we have

$$\Delta Q^* = -2\left(\frac{\Delta p_s^*}{\beta_s + \gamma_s} + \frac{\Delta p_w^*}{\beta_w + \gamma_w}\right),$$

which implies that $\Delta Q^* \leq 0$ if and only if

$$\frac{\Delta p_s^*}{\beta_s + \gamma_s} \geq -\frac{\Delta p_w^*}{\beta_w + \gamma_w}.$$

Now, suppose that $\Delta Q^* \leq 0$. Then, we have

$$\begin{aligned}
\Delta SW^* &= -\frac{\Delta p_s^*}{\beta_s + \gamma_s}(p_s^* + p^*) - \frac{\Delta p_w^*}{\beta_w + \gamma_w}(p_w^* + p^*) \\
&\leq -\frac{\Delta p_s^*}{\beta_s + \gamma_s}(p_s^* + p^*) + \frac{\Delta p_s^*}{\beta_s + \gamma_s}(p_w^* + p^*) \\
&= -\frac{\Delta p_s^*(p_s^* - p_s^*)}{\beta_s + \gamma_s}(p_s^* + p^*) < 0.
\end{aligned}$$

A5. Proof of Proposition 1

We have

$$\Delta CS^* = -\frac{YZ}{(2\beta_s - \gamma_s)^2(2\beta_w - \gamma_w)^2[(\beta_s^2 - \gamma_s^2)(2\beta_w - \gamma_w) + (\beta_w^2 - \gamma_w^2)(2\beta_s - \gamma_s)]^2},$$

where

$$\begin{aligned}
Y &\equiv [\alpha_s(\beta_s - \gamma_s)(2\beta_w - \gamma_w) - \alpha_w(\beta_w - \gamma_w)(2\beta_s - \gamma_s)], \\
Z &\equiv \{\alpha_s(\beta_s - \gamma_s)(2\beta_w - \gamma_w) \\
&\qquad \times [(\beta_s^2 - \gamma_s^2)(3\beta_s - \gamma_s)(2\beta_w - \gamma_w)^2 \\
&\qquad\quad + (2\beta_s - \gamma_s)(\beta_w^2 - \gamma_w^2)(4\beta_s\beta_w - 4\beta_s\gamma_w + 2\beta_s\beta_w - \gamma_s\beta_w + \gamma_s\gamma_w)] \\
&\qquad - \alpha_w(2\beta_s - \gamma_s)(\beta_w - \gamma_w) \\
&\qquad\quad \times [(\beta_s^2 - \gamma_s^2)(2\beta_w - \gamma_w)(4\beta_s\beta_w - 4\beta_w\gamma_s + 2\beta_s\beta_w - \gamma_w\beta_s + \gamma_s\gamma_w) \\
&\qquad\qquad + (2\beta_s - \gamma_s)^2(\beta_w^2 - \gamma_w^2)(3\beta_w - \gamma_w)]\}.
\end{aligned}$$

Both Y and Z are decreasing in α_w. Remember from Appendix A2 that

$$\alpha_w < \min\left[\frac{\alpha_s(\beta_s - \gamma_s)(2\beta_w - \gamma_w)}{(\beta_w - \gamma_w)(2\beta_s - \gamma_s)}, \alpha_s\right],$$

and it is verified that

$$\min\left[\frac{\alpha_s(\beta_s-\gamma_s)(2\beta_w-\gamma_w)}{(\beta_w-\gamma_w)(2\beta_s-\gamma_s)},\alpha_s\right]=\begin{cases}\alpha_s & \text{if and only if } -\gamma_s\beta_w+\beta_s\gamma_w>0,\\ \frac{\alpha_s(\beta_s-\gamma_s)(2\beta_w-\gamma_w)}{(\beta_w-\gamma_w)(2\beta_s-\gamma_s)} & \text{otherwise.}\end{cases}$$

If Y and Z are positive for any α_w that satisfies the above inequality, ΔCS^* is negative.

First, if $-\gamma_s\beta_w+\beta_s\gamma_w>0$, the upper bound of α_w is α_s. If Y and Z are non-negative for $\alpha_w=\alpha_s$, Y and Z are positive for any α_w. When $\alpha_w=\alpha_s$, Y and Z are

$$\begin{aligned}Y &= \alpha_s^2(-\gamma_s\beta_w+\beta_s\gamma_w)\\ Z &= [(\beta_s^2-\gamma_s^2)(2\beta_w-\gamma_w)(2\beta_s\beta_w+(\beta_s-\gamma_s)\gamma_w)\\ &\qquad +(\beta_w^2-\gamma_w^2)(2\beta_s-\gamma_s)(2\beta_s\beta_w+(\beta_w-\gamma_w)\gamma_s)].\end{aligned}$$

Both Y and Z are positive. This implies that ΔCS^* is negative.

Second, if $-\gamma_s\beta_w+\beta_s\gamma_w<0$, the upper bound of α_w is $\alpha_s(\beta_s-\gamma_s)(2\beta_w-\gamma_w)/((\beta_w-\gamma_w)(2\beta_s-\gamma_s))$. If Y and Z are non-negative for $\alpha_w=\alpha_s(\beta_s-\gamma_s)(2\beta_w-\gamma_w)/((\beta_w-\gamma_w)(2\beta_s-\gamma_s))$, Y and Z are positive for any α_w. When $\alpha_w=\alpha_s(\beta_s-\gamma_s)(2\beta_w-\gamma_w)/((\beta_w-\gamma_w)(2\beta_s-\gamma_s))$, Y and Z are

$$\begin{aligned}Y &= 0,\\ Z &= 2\alpha_s(\beta_s-\gamma_s)(2\beta_w-\gamma_w)(\gamma_s\beta_w-\beta_s\gamma_w)\\ &\qquad\times[(\beta_s^2-\gamma_s^2)(2\beta_w-\gamma_w)+(\beta_w^2-\gamma_w^2)(2\beta_s-\gamma_s)].\end{aligned}$$

Y is zero and Z is positive. This implies that $\Delta CS^*<0$ for $\alpha_w<\alpha_s(\beta_s-\gamma_s)(2\beta_w-\gamma_w)/((\beta_w-\gamma_w)(2\beta_s-\gamma_s))$.

A6. $\Delta Q^*>0\Rightarrow\Delta\Pi^*>0$

It is verified that $\Delta\Pi^*>0$ if and only if

$$\begin{aligned}\frac{\alpha_w}{\alpha_s} &< [(2\beta_s-\gamma_s)(\beta_w-\gamma_w)(2\beta_s^2\beta_w(-2\gamma_s\beta_w+2\beta_w^2+\gamma_s\gamma_w-2\gamma_w^2)+\beta_s^3(4\beta_w^2-\gamma_w^2)\\ &\quad +\gamma_s^2\beta_w(4\gamma_s\beta_w+\beta_w^2-2\gamma_s\gamma_w-\gamma_w^2)+\beta_s\gamma_s(-4\beta_w^3+4\beta_w\gamma_w^2+\gamma_s(-4\beta_w^2+\gamma_w^2))]^{-1}\end{aligned}$$

$$\times(\beta_s-\gamma_s)(2\beta_w-\gamma_w)[\beta_s^3(2\beta_w-\gamma_w)^2+4\beta_s^2(\beta_w-\gamma_w)^2(\beta_w+\gamma_w)$$
$$-\gamma_s^2\beta_w(\beta_w^2-\gamma_w^2)-\beta_s\gamma_s(-2\beta_w^2\gamma_s+2\gamma_w^3+\gamma_s(-2\beta_w+\gamma_w)^2)].$$

The difference between the right hand side and the upper bound for α_w/α_s,

$$\min\left[\frac{(\beta_s-\gamma_s)(2\beta_w-\gamma_w)}{(\beta_w-\gamma_w)(2\beta_s-\gamma_s)},1\right]$$

is:

$$\begin{aligned}
&[(2\beta_s-\gamma_s)(\beta_w-\gamma_w)(2\beta_s^2\beta_w(-2\gamma_s\beta_w+2\beta_w^2+\gamma_s\gamma_w-2\gamma_w^2)+\beta_s^3(4\beta_w^2-\gamma_w^2)\\
&+\gamma_s^2\beta_w(4\gamma_s\beta_w+\beta_w^2-2\gamma_s\gamma_w-\gamma_w^2)+\beta_s\gamma_s(-4\beta_w^3+4\beta_w\gamma_w^2+\gamma_s(-4\beta_w^2+\gamma_w^2))]^{-1}\\
&\times(\beta_s-\gamma_s)(2\beta_w-\gamma_w)[\beta_s^3(2\beta_w-\gamma_w)^2+4\beta_s^2(\beta_w-\gamma_w)^2(\beta_w+\gamma_w)\\
&-\gamma_s^2\beta_w(\beta_w^2-\gamma_w^2)-\beta_s\gamma_s(-2\beta_w^2\gamma_s+2\gamma_w^3+\gamma_s(-2\beta_w+\gamma_w)^2)]\\
&-\frac{(\beta_s-\gamma_s)(2\beta_w-\gamma_w)}{(\beta_w-\gamma_w)(2\beta_s-\gamma_s)},
\end{aligned}$$

which is positive if and only if $\beta_w\gamma_s-\beta_s\gamma_w>0$. That is, if $\gamma_s/\beta_s-\gamma_w/\beta_w>0$, the upper bound of α_w/α_s such that $\Delta\Pi^*$ is positive is larger than

$$\min\left[\frac{(\beta_s-\gamma_s)(2\beta_w-\gamma_w)}{(\beta_w-\gamma_w)(2\beta_s-\gamma_s)},1\right].$$

Note that

$$\frac{(\beta_s-\gamma_s)(2\beta_w-\gamma_w)}{(\beta_w-\gamma_w)(2\beta_s-\gamma_s)}$$

is decreasing in γ_s. This expression takes value 1 when $\gamma_s=\beta_s\gamma_w/\beta_w$. Therefore, this is smaller than 1 when $\gamma_s/\beta_s>\gamma_w/\beta_w$.

A7. $\Delta SW^*\leq 0$ for $\alpha_m=\alpha_{m'}$

We can proceed as follows:

$$\begin{aligned}
\Delta SW^* &= \frac{(\beta_w+\gamma_w)(p^{*2}-p_s^{*2})+(\beta_s+\gamma_s)(p^{*2}-p_w^{*\,2})}{(\beta_s+\gamma_s)(\beta_w+\gamma_w)}\\
&= -\frac{(\beta_w+\gamma_w)(p^*+p_s^*)\Delta p_s^*+(\beta_s+\gamma_s)(p^*+p_w^*)\Delta p_w^*}{(\beta_s+\gamma_s)(\beta_w+\gamma_w)}\\
&= -(p^*+p_s^*)\frac{\alpha_m(\beta_s\gamma_w-\beta_w\gamma_s)(\beta_s^2-\gamma_s^2)}{(\beta_s+\gamma_s)(2\beta_s-\gamma_s)\Phi^U}-(p^*+p_w^*)\frac{\alpha_m(\beta_w\gamma_s-\beta_s\gamma_w)(\beta_w^2-\gamma_w^2)}{(\beta_w+\gamma_w)(2\beta_w-\gamma_w)\Phi^U}
\end{aligned}$$

$$
\begin{aligned}
= & -\frac{\alpha_m(\beta_s\gamma_w-\beta_w\gamma_s)}{(\beta_s+\gamma_s)(\beta_w+\gamma_w)(2\beta_s-\gamma_s)(2\beta_w-\gamma_w)\Phi^U} \\
& \times\big[(p^*+p_w^*+p_s^*-p_w^*)(\beta_s^2-\gamma_s^2)(\beta_w+\gamma_w)(2\beta_w-\gamma_w) \\
& \qquad\qquad -(p^*+p_w^*)(\beta_w^2-\gamma_w^2)(\beta_s+\gamma_s)(2\beta_s-\gamma_s)\big] \\
= & -\frac{\alpha_m(\beta_s\gamma_w-\beta_w\gamma_s)}{(\beta_s+\gamma_s)(\beta_w+\gamma_w)(2\beta_s-\gamma_s)(2\beta_w-\gamma_w)\Phi^U} \\
& \times\big[(p^*+p_w^*)\{(\beta_s^2-\gamma_s^2)(\beta_w+\gamma_w)(2\beta_w-\gamma_w)-(\beta_w^2-\gamma_w^2)(\beta_s+\gamma_s)(2\beta_s-\gamma_s)\} \\
& \quad +(p_s^*-p_w^*)(\beta_s^2-\gamma_s^2)(\beta_w+\gamma_w)(2\beta_w-\gamma_w)\big] \\
= & -\frac{\alpha_m(\beta_s\gamma_w-\beta_w\gamma_s)}{(\beta_s+\gamma_s)(\beta_w+\gamma_w)(2\beta_s-\gamma_s)(2\beta_w-\gamma_w)\Phi^U} \\
& \times[(p^*+p_w^*)(\beta_s+\gamma_s)(\beta_w+\gamma_w)(\beta_s\gamma_w-\beta_w\gamma_s) \\
& \quad +(p_s^*-p_w^*)(\beta_s^2-\gamma_s^2)(\beta_w+\gamma_w)(2\beta_w-\gamma_w)\big] \\
= & -\frac{\alpha_m(\beta_s\gamma_w-\beta_w\gamma_s)}{(\beta_s+\gamma_s)(\beta_w+\gamma_w)(2\beta_s-\gamma_s)(2\beta_w-\gamma_w)\Phi^U} \\
& \times[(p^*+p_w^*)(\beta_s+\gamma_s)(\beta_w+\gamma_w)(\beta_s\gamma_w-\beta_w\gamma_s) \\
& +\frac{\alpha_m(\beta_s\gamma_w-\beta_w\gamma_s)((\beta_s^2-\gamma_s^2)(2\beta_w-\gamma_w)+(\beta_w^2-\gamma_w^2)(2\beta_s-\gamma_s))}{(2\beta_s-\gamma_s)\Phi^U}(\beta_s^2-\gamma_s^2)(\beta_w+\gamma_w)\bigg] \\
= & -\frac{\alpha_m(\beta_s\gamma_w-\beta_w\gamma_s)^2}{(\beta_s+\gamma_s)(\beta_w+\gamma_w)(2\beta_s-\gamma_s)(2\beta_w-\gamma_w)\Phi^U} \\
& \times[(p^*+p_w^*)(\beta_s+\gamma_s)(\beta_w+\gamma_w) \\
& +\frac{\alpha_m(\beta_s^2-\gamma_s^2)(\beta_w+\gamma_w)((\beta_s^2-\gamma_s^2)(2\beta_w-\gamma_w)+(\beta_w^2-\gamma_w^2)(2\beta_s-\gamma_s))}{(2\beta_s-\gamma_s)\Phi^U}\bigg]\leq 0.
\end{aligned}
$$

It is easy to see that the equality holds if and only if $\beta_s\gamma_w-\beta_w\gamma_s=0$.

A8. Welfare Analysis when γ_m is Common

Let $\gamma\equiv\gamma_s=\gamma_w$. We allow β_s and β_w to differ and provide numerical analysis to contrast substitutability with complementarity for a fixed value of $(\alpha_w,\beta_s,\beta_w)$, and graphical arguments on the domains (β_s,β_w) for $\Delta SW^*>0$, with the value of (γ,α_w) fixed.

Table 2 shows the result for the case of $\alpha_w=0.85$. The first and the second column corresponds to the case of substitutability ($\gamma=0.3$), while the third and the fourth correspond to the case of complementarity ($\gamma=-0.3$). The difference between the first and the second (the third and the fourth in the case of

	$(\gamma, \beta_s, \beta_w) =$			
	$(0.3, 1.0, 0.75)$	$(0.3, 0.75, 1.0)$	$(-0.3, 1.0, 0.75)$	$(-0.3, 0.75, 1.0)$
p^*	0.3582	0.3644	0.5235	0.5423
p_s^* $(\Delta p_s^*/p^*)$	0.4118 (15%)	0.3750 (3%)	0.5652 (8%)	0.5833 (8%)
p_w^* $(\Delta p_w^*/p^*)$	0.3188 (−11%)	0.3500 (−4%)	0.4958 (−5%)	0.4804 (−11%)
Δq_s^* $(\Delta q_s^*/q_s^*(p^*))$	−0.9412 (−8%)	−0.0101 (−2%)	−0.0596 (−9%)	−0.0912 (−9%)
Δq_w^* $(\Delta q_w^*/q_w^*(p^*))$	0.0375 (8%)	0.0111 (3%)	0.0615 (8%)	0.0884 (20%)
ΔSW^*	−0.0063	0.0005	−0.0022	−0.0123
ΔCS_s^*	−0.0507	−0.0127	−0.0543	−0.0797
ΔCS_w^*	0.0384	0.0109	0.0419	0.0598
$\Delta \Pi^*$	0.0060	0.0023	0.0102	0.0076
ΔQ^*	−0.0037	0.0009	0.0019	−0.0028

Table 2: Substitutability versus Complementarity with $\beta_s \neq \beta_w$ ($\alpha_w = 0.85$)

complementarity) columns is whether the own slope of the inverse demand curve in the strong market is greater than that in the weak market (i.e., $\beta_s > \beta_w$). Notice that price discrimination improves social welfare only in the second case ($(\gamma, \beta_s, \beta_w) = (0.3, 0.75, 1.0)$). In this case, $|\Delta q_s^*|/q_s^*(p^*)$ is particularly small (2%), while $\Delta q_w^*/q_w^*(p^*)$, is also not too large (3%), in comparison with the other three cases.

First, consider the case of substitutable goods ($\gamma > 0$). Notice that when $\beta_s > \beta_w$, the strong market has a higher value of price elasticity than the weak market (see equation (4)). The equilibrium price in the strong market p_s^*, however, is at a *higher* level than in the case of $\beta_s < \beta_w$ (0.4118 vs. 0.3750). This seemingly paradoxical result is due to strategic effects: the firms want to "cooperate" because they are afraid of retaliation when the market is more price elastic. Now, if the market is "integrated" (i.e., uniform pricing is forced), then the market price in the strong market is expected to drop to a larger extent than in the case of $\beta_s < \beta_w$, because the strong market has a higher value of price elasticity (it is more competitive) than the weak market when $\beta_s > \beta_w$. In Table 2, we see the price in the strong market drop from 0.4118 to 0.3582 (−6%) when $(\gamma, \beta_s, \beta_w) = (0.3, 1, 0.75)$, while p_s drops from 0.3750 to 0.3644 (−3%) when $(\gamma, \beta_s, \beta_w) = (0.3, 0.75, 1.0)$. To summarize, *when the strong market is less price elastic, the regime of uniform pricing does not* *lower the price in the strong market sufficiently.* As a result, uniform pricing may harm social welfare. In other words, *price discrimination may improve welfare.*

Even though the products are complements, a similar logic can apply to the property of price discrimination. When the products are complements, the price changes and the associated production changes are large due to the greater elasticity created by complementarity. In fact, welfare loss is larger in the fourth case (where the strong market has a higher value of price elasticity than the weak market does) than in the third case ($|-0.0123] > |-0.0022|$). As to the changes in equilibrium aggregate output, it is positive in our second and third cases but negative in the other two. These results are consistent with Proposition 2: *an increase in the aggregate output is necessary for welfare to be improved by price discrimination*, as in the case of monopoly.

The difference between substitutability and complementarity is further investigated graphically. Figures 9 and 10 depict the region of $\Delta SW^* > 0$ for the cases of substitutability ($\gamma = 0.3$) and of complementarity ($\gamma = -0.3$), respectively (with $\alpha_w = 0.85$). Notice that $(\beta_s, \beta_w) = (0.75, 1.0)$ in Table 2 is contained in the shaded region of Figure 9. The result for the case of substitutability is expected from the argument above. For the case of complementarity, the combination of "high β_s and low β_w" works for welfare improvement, the reverse of the situation in the case of substitutability. Notice that complementarity makes the demand in each market more price elastic. With elasticity already sufficiently high, a higher value of β_s raises the uniform price, and thus the price change introduced by price discrimination is reduced because of the high value of β_s, reducing the inefficiency of price discrimination in the strong market.

In Figure 9, the white area around the top right corner violates the condition that $\alpha_s/\alpha_w > \underline{\alpha_s/\alpha_w}$. The violation means that the discriminatory price in the strong market with α_s is *lower* than that in the weak market with α_w (note that $\alpha_s > \alpha_w$). In other words, the discriminatory price at the market with a higher intercept (α_s) is lower than that in the market with a lower intercept (α_w). Following the definition of a "strong" market in Section 2, we now redefine the former as the

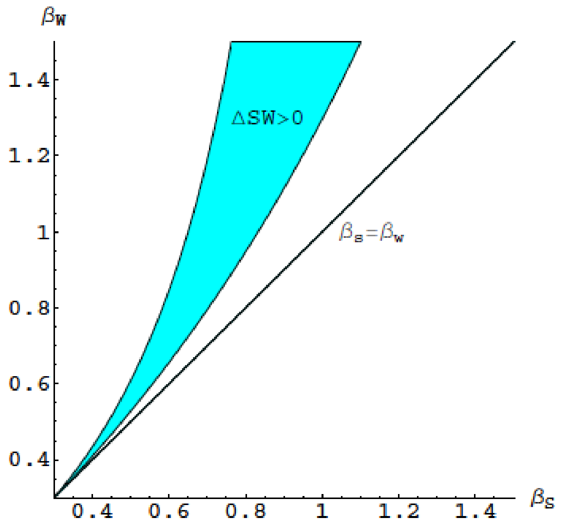

Figure 9: Substitutability ($\gamma = 0.3$) in the Case of $\alpha_w = 0.85$

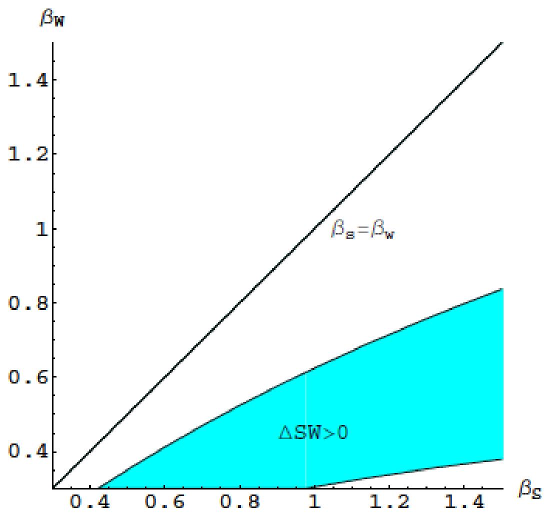

Figure 10: Complementarity ($\gamma = -0.3$) in the Case of $\alpha_w = 0.85$

"weak market" and the latter as the "strong' market." On this white area where $\beta_s < \beta_w$ holds, the redefined "weak" market with a higher intercept is *more* elastic than the redefined "strong" market with a lower intercept. As mentioned above, when the "weak" market is elastic, the increase in quantity in the weak market is not high enough to offset the loss from the decrease in quantity in the strong market; that is, $\Delta Q < 0$. In fact, in this white area, price discrimination deteriorates the total social surplus.

Lastly, it is verified that consumer surplus is never improved by price discrimination in the cases of $(\gamma, \alpha_w) = (0.3, 0.85)$ and of $(\gamma, \alpha_w) = (-0.3, 0.85)$. Thus, this and other numeral results suggest that welfare improvement from price discrimination is solely due to an increase in the firms' profits. In particular, it means that there is little or no chance that firms will suffer from "prisoners' dilemma"; that is, firms are mostly or always better by switching from uniform pricing to price discrimination.

A9. Areas of Welfare Improvement when $\beta_s \neq \beta_w$

Figures 11 and 12 depict the region of $\Delta SW^* > 0$ (with $\alpha_w = 0.95$) for the case of $\beta_s = 1.2$ and $\beta_w = 1.0$ (Figure 11) and for the case of $\beta_s = 1.0$ and $\beta_w = 1.2$.

A10. Proof of Proposition 2

Substituting $\overline{\gamma}_s$ into $\partial \Delta SW^* / \partial \gamma_s$, we have

$$\left.\frac{\partial \Delta SW^*}{\partial \gamma_s}\right|_{\gamma_s = \overline{\gamma}_s} = -\frac{2(\alpha_s - \alpha_w)\alpha_w(\beta_w - \gamma_w)[\alpha_s\beta_w + (\alpha_s - \alpha_w)(\beta_w - \gamma_w)]^3}{\alpha_s\beta_s(2\beta_w - \gamma_w)^3 M}$$

where

$$\begin{aligned} M &= (2\beta_w - \gamma_w)(\beta_w + \gamma_w)\alpha_s^2 \\ &\quad + \alpha_s\alpha_w\{2\beta_s(2\beta_w - \gamma_w) - (\beta_w - \gamma_w)(\beta_w + \gamma_w)\} - 3\beta_s\alpha_w^2(\beta_w - \gamma_w) \end{aligned}$$

Notice that $\alpha_s > \alpha_w$, $\beta_w > \gamma_w$, $\alpha_s\beta_w + (\alpha_s - \alpha_w)(\beta_w - \gamma_w) > 0$, and $2\beta_w - \gamma_w$. It can be shown that M is a concave function of α_w. For $\alpha_w \in [0, \alpha_s]$, this function is locally

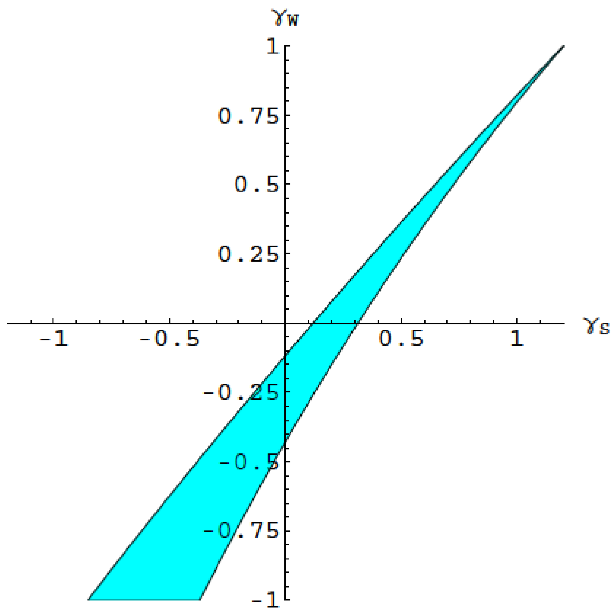

Figure 11: The Region of $\Delta SW^* > 0$ for the Case of $\alpha_w = 0.95$, $\beta_s = 1.2$ and $\beta_w = 1.0$

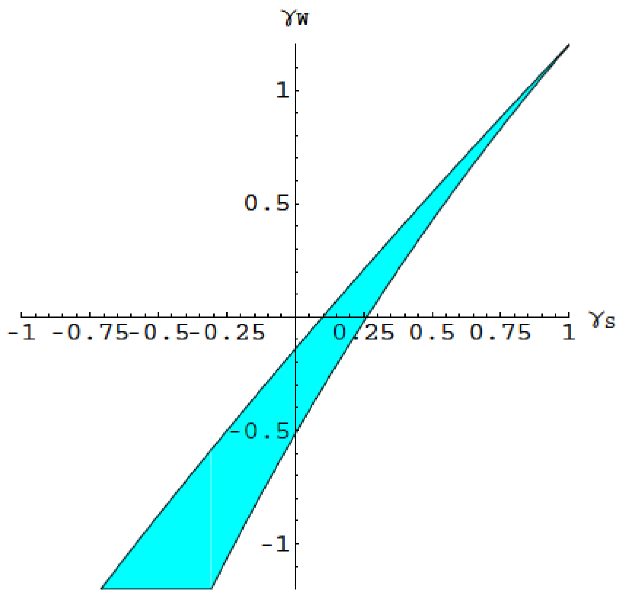

Figure 12: The Region of $\Delta SW^* > 0$ for the Case of $\alpha_w = 0.95$, $\beta_s = 1.0$ and $\beta_w = 1.2$

maximized at $\alpha_w = 0$ or $\alpha_w = \alpha_s$. When $\alpha_w = 0$, $M = (2\beta_w - \gamma_w)(\beta_w + \gamma_w)\alpha_s^2 > 0$. When $\alpha_w = \alpha_s$, $M = \alpha_s^2(\beta_s + \beta_w)(\beta_w + \gamma_w) > 0$. Therefore,

$$\left.\frac{\partial \Delta SW^*}{\partial \gamma_s}\right|_{\gamma_s = \overline{\gamma}_s} < 0.$$

Thus, there exists γ_s' such that $\Delta SW^* > 0$ for $\gamma_s \in (\gamma_s', \overline{\gamma}_s)$.

Acknowledgement

We are grateful to Iñaki Aguirre, Takao Asano, Jan Bouckaert, Simon Cowan, Takanori Ida, Hiroaki Ino, Domenico Menicucci, Makoto Okamura, Hajime Sugeta, and seminar and conference participants at Kwansei Gakuin University, Nagoya University, Shinshu University, Tokyo Institute of Technology, the University of Tokyo, the 9th International Industrial Organization Conference (IIOC), the Japanese Economic Association, the Japan Association for Applied Economics, and the European Association for Research in Industrial Economics (EARIE) for helpful comments. Adachi also acknowledges a Grant-in-Aid for Scientific Research (A) (23243049) and a Grant-in-Aid for Young Scientists (B) from the Japanese Ministry of Education, Science, Sports, and Culture (21730184). Matsushima acknowledges a Grant-in-Aid for Scientific Research (B) from the Ministry (21730193). Any remaining errors are our own.

References

[1] Adachi, Takanori. 2002. "A Note on 'Third-Degree Price Discrimination with Interdependent Demands'." *Journal of Industrial Economics*, 50 (2): 235.

[2] —. 2005. "Third-Degree Price Discrimination, Consumption Externalities and Social Welfare." *Economica,* 72 (1): 171-178.

[3] Aguirre, Iñaki. 2011. Personal Communication.

[4] —, Simon Cowan, and John Vickers. 2010. "Monopoly Price Discrimination and Demand Curvature." *American Economic Review*, 100 (4): 1601-1615.

[5] Armstrong, Mark. 2006. "Recent Developments in the Economics of Price Discrimination." In *Advances in Economics and Econometrics, 9th World Congress: Theory and Applications*. Vol. 2, ed. Richard Blundell, Whitney K. Newey, and Torsten Persson, 97-141. Cambridge, UK: Cambridge University Press.

[6] Asplund, Marcus, Rickard Eriksson, and Niklas Strand. 2008. "Price Discrimination in Oligopoly: Evidence from Regional Newspapers." *Journal of Industrial Economics*, 56 (2): 333-346.

[7] Belleflamme, Paul, and Martin Peitz. 2010. *Industrial Organization: Markets and Strategies*. Cambridge, UK: Cambridge University Press.

[8] Bertoletti, Paolo. 2004. "A Note on Third-Degree Price Discrimination and Output." *Journal of Industrial Economics*, 52 (3): 457.

[9] Cheung, Francis K., and Xinghe Wang. 1997. "The Output Effect under Oligopolistic Third Degree Price Discrimination." *Australian Economic Papers*, 36 (68), 23-30.

[10] Corts, Kenneth S. 1998. "Third-Degree Price Discrimination in Oligopoly: All-Out Competition and Strategic Commitment." *RAND Journal of Economics*, 29 (2): 306-323.

[11] Cournot, Augustin. 1838. *Researches into the Mathematical Principles of the Theory of Wealth*, (Macmillan, New York); English translation, N. Bacon, 1897.

[12] Cowan, Simon. 2007. "The Welfare Effects of Third-Degree Price Discrimination with Nonlinear Demand Functions." *RAND Journal of Economics*, 38 (2): 419-428.

[13] —. 2011. "Competitive Price Discrimination." Unpublished lecture notes, Oxford University.

[14] —. 2012. "Third-Degree Price Discrimination and Consumer Surplus." *Journal of Industrial Economics*, Forthcoming.

[15] Dastidar, Krishnendu Ghosh. 2006. "On Third-Degree Price Discrimination in Oligopoly." *The Manchester School*, 74 (2): 231-250.

[16] Galera, Francisco, and Jesús M. Zaratiegui. 2006. "Welfare and Output in Third-Degree Price Discrimination: A Note." *International Journal of Industrial Organization*, 24 (3): 605-611.

[17] Grennan, Matthew. 2011. "Price Discrimination and Bargaining: Empirical Evidence from Medical Devices." Unpublished manuscript.

[18] Hausman, Jerry A., and Jeffrey K. MacKie-Mason. 1988. "Price Discrimination and Patent Policy." *RAND Journal of Economics* 19 (2): 253-265.

[19] Hendel, Igal, and Aviv Nevo. 2011. "Intertemporal Price Discrimination in Storable Goods Markets." NBER Working Paper 16988.

[20] Holmes, Thomas J. 1989. "The Effects of Third-Degree Price Discrimination in Oligopoly." *American Economic Review*, 79 (1): 244-250.

[21] Ikeda, Takeshi, and Tatsuhiko Nariu. 2009. "Third-Degree Price Discrimination in the Presence of Asymmetric Consumption Externalities." *Journal of Industry, Competition and Trade*, 9(3): 251-261.

[22] —, and Tsuyoshi Toshimitsu. 2010. "Third-Degree Price Discrimination, Quality Choice, and Welfare." *Economics Letters* 106 (1): 54-56.

[23] Layson, Stephen K. 1988. "Third-Degree Price Discrimination, Welfare and Profits: A Geometrical Analysis." *American Economic Review*, 78 (5): 1131-1132.

[24] —. 1998. "Third-Degree Price Discrimination with Interdependent Demands." *Journal of Industrial Economics*, 46 (4): 511-524.

[25] Liu, Qihong, and Konstaninos Serfes. 2010. "Third-Degree Price Discrimination." *Journal of Industrial Organization Education* 5 (1): Article 5.

[26] Neven, Damien, and Louis Phlips. 1985. "Discriminating Oligopolists and Common Markets." *Journal of Industrial Economics* 34 (2): 133-149.

[27] Okada, Tomohisa, and Takanori Adachi. 2011. Third-Degree Price Discrimination, Consumption Externalities, and Market Opening. Unpublished manuscript..

[28] Pigou, Arthur C. 1920. *The Economics of Welfare*. London, UK: Macmillan.

[29] Robinson, Joan. 1933. *The Economics of Imperfect Competition*. London, UK: Macmillan.

[30] Schmalensee, Richard. 1981. "Output and Welfare Implications of Monopolistic Third-Degree Price Discrimination." *American Economic Review*, 71 (2): 242-247.

[31] Schwartz, Marius. 1990. "Third-Degree Price Discrimination ant Output: Generalizing a Welfare Result." *American Economic Review*, 80 (5): 1259-1262.

[32] Shy, Oz. 2008. *How to Price: A Guide to Pricing Techniques and Yield Management*. Cambridge, UK: Cambridge University Press.

[33] Stole, Lars A. 2007. "Price Discrimination and Competition." In *Handbook of Industrial Organization*, Vol. 3, ed. Mark Armstrong and Robert H. Porter, 2221-2299. Amsterdam, The Netherlands: Elsevier Science Publishers B.V.

[34] Varian, Hal R. 1985. "Price Discrimination and Social Welfare." *American Economic Review*, 75 (4): 870-875.

[35] Vives, Xavier. 1999. *Oligopoly Pricing: Old Ideas and New Tools*. Cambridge, MA: The MIT Press.

[36] Weyl, E. Glen, and Michal Fabinger. 2009. "Pass-Through as an Economic Tool." Unpublished manuscript.

Additional Appendices (Not for Publication)

Restrictions on α_w/α_s in Figure 4

Assume that $\alpha_s = 1.0$, $\alpha_w = 0.85$, and $\beta_s = \beta_w = 1.0$. Then, the regions (γ_s, γ_w) that satisfy $\alpha_w/\alpha_s > \underline{\alpha_s/\alpha_w}(\boldsymbol{\gamma}, \boldsymbol{\beta})$ and $\alpha_w/\alpha_s < \overline{\alpha_w/\alpha_s}(\boldsymbol{\gamma}, \boldsymbol{\beta})$ are respectively drawn in Figures AA 1 and AA 2. One can see that $\alpha_w/\alpha_s = \overline{\alpha_w/\alpha_s}(\boldsymbol{\gamma}, \boldsymbol{\beta})$ binds for $\Delta SW^* > 0$.

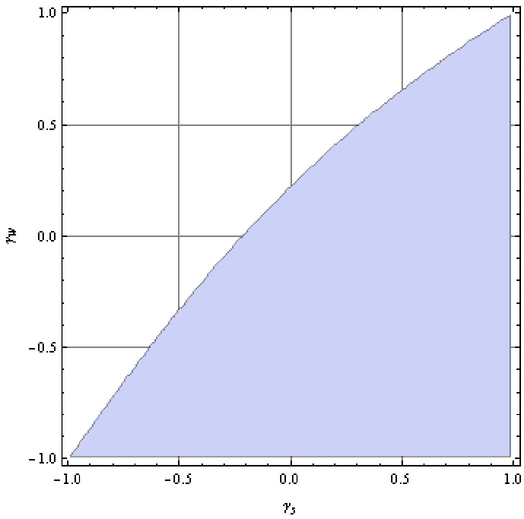

Figure AA 1: The weak market must be large enough: $\alpha_w/\alpha_s > \underline{\alpha_s/\alpha_w}(\boldsymbol{\gamma}, \boldsymbol{\beta})$

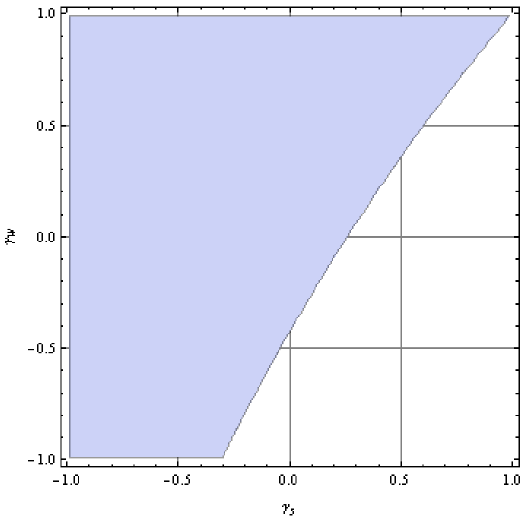

Figure AA 2: The weak market must be small enough: $\alpha_w/\alpha_s < \overline{\alpha_w/\alpha_s}(\boldsymbol{\gamma}, \boldsymbol{\beta})$

Numerical Calculation (Tables 1 and 2)

We use Mathematica to obtain numerical results. For the case of symmetric product differentiation, the following results are used to obtain numerical values in Table 1.

	$(\gamma_s, \gamma_w) =$	
	$(0.1, -0.1)$	$(-0.1, 0.1)$
$q_s^*(p^*)$	0.4920	0.5931
$q_w^*(p^*)$	0.4347	0.3489

Table AA 1: $q_s^*(p^*)$, $q_w^*(p^*)$ and ΔQ^* with $\beta_s = \beta_w = 1.0$ ($\alpha_w = 0.85$)

The following table is for Table 2.

	$(\gamma, \beta_s, \beta_w) =$			
	$(0.3, 1, 0.75)$	$(0.3, 0.75, 1.0)$	$(-0.3, 1.0, 0.75)$	$(-0.3, 0.75, 1.0)$
$q_s^*(p^*)$	0.4937	0.6053	0.6807	1.0171
$q_w^*(p^*)$	0.4684	0.3735	0.7255	0.4396

Table AA 2: $q_s^*(p^*)$, $q_w^*(p^*)$ and ΔQ^* with $\beta_s \neq \beta_w$ ($\alpha_w = 0.85$)