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Side Payments Contract”

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Abstract

We present a non-cooperative sequential bargaining game for side payments contracting. Players voluntarily participate in negotiations. If any player does not participate, then renegotiation will take place in the next round, given an on-going contract. We show that if the stopping probability of negotiations is sufficiently small, then there exists an efficient Markov perfect equilibrium where all players immediately participate in negotiations and agree to the Nash bargaining solution. The efficiency result is strengthened by the asymptotically efficient one that in every Markov perfect equilibrium, all players participate in negotiations through a process of renegotiations in the long run with probability one. Finally, we illustrate international negotiations for climate change as an application of the result.

JEL classification: C71, C72, C78

Key words: Coase theorem, contract, efficiency, externality, Nash bargaining solution, non-cooperative bargaining, side payments,

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1 Introduction

The so-called Coase (1960) "theorem" says that if property rights are well-defined and there are no transaction costs, rational and fully informed agents agree to a contract generating an efficient outcome.[1] An intuitive idea (or belief) supporting the "theorem" is that if an agreement is inefficient, rational agents renegotiate it towards a Pareto-improving one. We call it the idea of Coasian bargaining. Recently, Jackson and Wilkie (2005) and Ellingsen and Paltseva (2016) examine the relevancy of the Coase theorem in the framework of non-cooperative game theory. They show that voluntary side contracting does not necessarily leads to an efficient outcome, and that the outcome of a contracting process critically depends on the number of agents. The outcome in the case of two agents is very different from that in the case of more than two agents. Ellingsen and Paltseva (2016) argued that the Coase theorem does not hold except the case of two agents.

In this paper, we consider the Coase theorem in the light of two points which are not studied by the previous works mentioned. One point is the problem of distributional fairness, and the other is that of renegotiations. Before we state our main results, let us illustrate each point by an example.

Jackson and Wilkie (2005) present a two-stage model of side contracting where agents make enforceable offers of side payments contingent on actions to each other before they play a game in strategic form. In their model, agents can unilaterally commit themselves to payoff transfer plans. While the possibility of commitments plays a critical role in negotiations, the model does not include that of agreement, which is a fundamental element in many bargaining situations where any contract can be binding only based on the unanimous consent among all parties involved. In the commitment model of Jackson and Wilkie (2005), players can effectively refuse payoff transfers from other players only by returning the same amounts of payoffs to them. An unilateral com-

[1] For a detailed account of the Coase theorem, see Cooter (1989).

mitment may leads to an efficient, but unfair payoff allocation. We illustrate this point by a classic problem of one-sided externality.

Example 1. One sided externalities

This is a special case of one sided externalities problem discussed by Jackson and Wilkie (2005). Consider the Coase's example of a steel mill affecting a laundry. Let $x_1 \in [0, 1]$ denote the output of the steel mill.[2] The payoff functions of the steel mill and the laundry are $v_1(x_1) = 1 - (1 - x_1)^2$ and $v_2(x_1) = 1 - x_1$, respectively. The steel mill's production affects the laundry's payoff. There is an unique Nash equilibrium $x_1^n = 1$ generating the payoff allocation $(1, 0)$, and $x_1^* = 0.5$ is an unique efficient production under transferability and it generates the payoff allocation $(0.75, 0.5)$. In the Jackson and Wilkie's (2005) game, it is optimal for the laundry to offer the side payments contract that the laundry gives 0.25 to the steel mill to compensate it for choosing the efficient output $x^* = 0.5$ and otherwise does nothing. As a result, the laundry can exploit the net surplus 0.25 while the steel mill gets the same payoff as in the Nash equilibrium. The underlying situation is that the steel mill has the legal right to choose any output without the consent of the laundry, and thus the threat point of the bargaining problem between the steel mill and the laundry is the Nash equilibrium payoffs $(1, 0)$. If any contract can be enforceable only based on the mutual consent, the natural outcome seems to be a fair allocation such as the Nash (1950) bargaining solution $(1.125, 0.125)$, not $(1, 0.25)$. In this paper, we consider a non-cooperative bargaining game and whether it is possible for the steel mill and the laundry to agree to the Nash bargaining solution.

Ellingsen and Paltseva (2016) consider a non-cooperative model of contract negotiation. In their model, players decide to participate in negotiations, or not. All participants make contract proposals and thereafter they decide in-

[2] Jackson and Wilkie (2005) consider the output x_2 of the laundry. For simplicity, we assume that the output of the laundry is fixed.

dependently which contract proposals to sign. A contract can be binding if and only if it is signed by all players who may pay or receive transfers under the contract. Finally, players play the underlying game under the agreed-upon contract. Since the signing stage has a coordination problem, it has an inefficient Nash equilibrium where players fail to sign desirable contracts. To eliminate such an inefficient Nash equilibrium, Ellingsen and Paltseva (2016) employ the strong solution concept of a consistent equilibrium (Bernheim and Ray 1989) that applies Pareto-dominant selection recursively. Ellingsen and Paltseva (2016) point out that players may have an incentive not to participate in negotiations. This point is illustrated by the following example given by them.

Example 2. Public goods

There are four players, each with an endowment M money units. They choose independently how much of the endowments to contribute to a public good. Let x_i be a contribution of every player $i = 1, \cdots, 4$. Player i's payoff is given by $0.4 \sum_i x_i + M - x_i$. It is the dominant action for each player to contribute nothing. If all four players participate in negotiations for the joint provision of the public good, the efficient provision is attained by the full contribution profile $(M, \cdots, M)$ with payoffs $(1.6M, \cdots, 1.6M)$. Suppose that one player, say 1, does not participate and contribute nothing. The payoff of every remaining player $i \neq 1$ is given by $0.4 \sum_{i \neq 1} x_i + M - x_i$. The total payoff is $0.2 \sum_{i \neq 1} x_i + 3M$, and this is maximized by the full contribution profile (M, M, M). This gives payoffs $1.2M$ to each of the three participants and does payoff $2.2M$ to non-participant 1. Thus, player 1 has an incentive not to participate in negotiations, free-riding on the public good provided by the other players. This, however, is not the end of the story. Since the total payoff of the three participants and of one free-rider is $5.8M$ which is smaller than the efficient outcome $6.4M$, there exists a Pareto-improving outcome. Thus, the three contributors and a free-rider have an incentive to renegotiate for their contributions, given that the payoff profile $(2.2M, 1.2M, 1.2M, 1.2M)$ is the threat point of the bargaining problem. For example, all of them may agree to split the surplus $0.6M$ equally, which results in an efficient payoff $(2.35M, 1.35M, 1.35M, 1.35M)$. This is the idea of Coasian bargaining. In this paper, we examine whether the idea of Coasian bargaining can be justified by a non-cooperative bargaining model with voluntary participation and renegotiations.

The incentive to non-participation in the context of public goods has been a central problem by a classic argument called "the second-order dilemma of public goods" (Oliver 1980 and Ostrom 1990). It says that since any mechanism which achieves an efficient provision of public goods is itself a kind of public goods, players have an incentive to free ride on the mechanism. Thus, the mechanism may fail. Putting this dilemma into contract negotiations, players may have an incentive not to participate in negotiations and to free ride on a contract made by others. Non-participation of some (or all) players results in an inefficient outcome. However, as we suggest in the example above, the Coasian idea may work. If a contract is inefficient due to non-participation, then all parties involved may have an incentive to renegotiate it towards an Pareto-improving one so that (at least some) non-participants are motivated to participate. If this is true, then by repeating renegotiations, an efficient outcome may result in the end.

We summarize the results in this paper as follows. We first present a basic model of contract negotiations where all players are assumed to participate. The model is a two-stage process. Different from the models of Jackson and Wilkie (2005) and of Ellingsen and Paltseva (2016), players play a Rubinstein (1982)-type sequential bargaining game in the first stage. Specifically, at the beginning of each round, one player is selected as a proposer according to a predetermined probability distribution θ over the set of players. A proposer proposes a side payments contract, and thereafter all other players either accept or reject it sequentially. If all accept it, then negotiations stop and the underlying game is played under the agreed-upon contract. If any responder rejects the proposal, two events may happen. With a positive probability ϵ, negotiations may stop and the underlying game is played under no contract. With probability $1 - \epsilon$, negotiations may continue in the next round by the same rule.

We show that for every $\epsilon > 0$, there exists a stationary subgame perfect equilibrium (SSPE) of the two stage game where the expected payoff profile for players is equal to an asymmetric Nash bargaining solution $NB(\theta, u(a))$ of the underlying game where the weights of players is equal to the probability distribution θ and the disagreement point $u(a)$ is a payoff profile under some Nash equilibrium a of the underlying game (Theorem 1). The equilibrium payoff profile of players in the second-stage game converges to $NB(\theta, u(a))$, independent of a proposer, in the limit that the stopping probability ϵ of negotiations goes to zero. Whenever negotiations fail, a Nash equilibrium of the underlying game must be played in a subgame perfect equilibrium. Given a Nash equilibrium played when negotiations fail, we show that the Nash bargaining solution with the selected Nash equilibrium payoffs as the threat point is a unique SSPE outcome of the two-stage game when the stopping probability of negotiations is sufficiently small (Theorem 2). By the results of the basic model, we conclude that an efficient and fair allocation of payoffs (the Nash bargaining solution) can be attained through a process of voluntary contracting, provided that all players participate in negotiations. The assumption of participation can be justified in some economic situations. They include pure exchange markets of private goods and provision of excludable goods. In these situations, no agents have an incentive to free ride. In real situations, non-participants are often punished by participants (with no provision of public goods, for example). Kosfeld et al. (2009) report experimental evidence in a four-person institution formation game with public goods which show that subjects are reluctant to implement institutions if there exist non-participants and the majority (on average, around 75 percent) of successful institutions are the largest one.

We next extend the basic model so that it includes voluntary participation and renegotiations. In the extended model, at the beginning of each round $k = 1, 2, \cdots$, all players who have not participated in negotiations decide independently to participate, and thereafter negotiations take place among a new set S_{k+1} of all the incumbent and new participants. The bargaining rule is the same as in the basic model. If a side payments contract t^{k+1} within S_{k+1} is agreed, negotiations may stop with probability ϵ and the underlying game is played under the final agreement t^{k+1}. All non-participants choose their default actions (no-trade or no-contribution, for example). With probability $1-\epsilon$, negotiations continue in the the next round $k+1$ with the on-going contract t^{k+1} by the same rule as in round k. The pair (S_{k+1}, t^{k+1}) of the set of participants and the on-going contract composes a state of round $k+1$. Negotiations stop if all players participate and make an agreement. We prove that there exists an efficient Markov perfect equilibrium of the extended model where all players participate in negotiations in the first period and agree to the Nash bargaining solution if the stopping probability ϵ is sufficiently small (Theorem 3). The equilibrium strategy does not include inefficient punishments in the sense that all non-participants participate immediately off equilibrium play. To strengthen our efficiency result, we prove that all players participate in negotiations and attain an efficient outcome with probability one in the long run in every Markov perfect equilibrium under a certain condition of supperadditivity of coalitional values of players (Theorem 4). Finally, we illustrate international negotiations on climate change as an application of the result.

Before presenting the models and results, let us review briefly the relationship between our work and other works in the literature.

As mentioned above, this paper is inspired by the works of Jackson and Wilkie (2005) and Ellingsen and Paltseva (2016). Employing a Rubinstein-type sequential bargaining game on side payments contract, we obtain more positive results than theirs. We show that voluntary contracting can lead to an efficient and fair allocation (the Nash bargaining solution) either without or with voluntary participation. In the case of voluntary participation, renegotiations are effective in attaining the efficiency of allocations. The idea of Coasian bargaining can be supported in our framework. Our result does not depend on the number of agents. This paper is also related to recent literature on non-cooperative coalitional bargaining. This literature shows that the Coase theorem does not always hold due to the formation of subcoalitions in characteristic function games (Chatterjee et al. 1993, Okada 1996 and 2011, Ray 2007, Compte and Jehiel 2010 among others) and in partition function games with externality (Ray and Vohra 1999). The literature also shows that the opportunity of renegotiations reinstates the efficiency in characteristic function games (Seidmann and Winter 1998, Okada 2000 and Hyndman and Ray 2007), in partition function games (Gomes 2005 and Bloch and Gomes 2006) and in a class of social interactions (Gomes and Jehiel 2005). In most coalitional bargaining games, a player can propose a coalition that he want to form, and voluntary participation is not the main focus in their studies. They also assume that any contract on actions and allocations within a coalition of players is binding if all members accept it.[3] Maruta and Okada (2012) consider a sequential bargaining game with voluntary participation in a special case of the repeated n-person prisoner's dilemma. In their model, a set of participants negotiate for self-binding contracts (subgame perfect equilibrium strategies in the repeated game). They show that an efficient group of cooperators is formed after a finite number of renegotiations in every Markov perfect equilibrium. The literature on voluntary participation is sizeable (Selten 1973, Palfrey and Rosenthal 1984, Dixit and Olson 2000, Ellingsen and Paltseva 2012, for example).[4] Most models are analysed in static setups without renegotiation.

[3]The assumption on binding contracts on actions seems strong at first sight, but it is not very much so as long as players can write any enforceable contract of side payments. As Lemma 1 shows, any action profile in a coalition can be implemented under a suitably designed side payments contract.

[4]To the best of our knowledge, Selten (1973) first considered the problem of voluntary participation in a formal model of a non-cooperative game. He considered the formation of

A vast literature on the theory of incomplete contract (property right) usually assume that ex post renegotiation is efficient (Hart and Moore 1988). It is also standard to apply the Nash bargaining solution to ex post renegotiation (Grossman and Hart 1986). See Segal and Whinston (2013) for a survey on the theory of property rights. This paper complements the works in this literature. The issue of renegotiation is also discussed as a way of eliminating inefficient punishments off the equilibrium play in implementation literature (Maskin and Moore 1999) and in repeated games (Farrell and Maskin 1989). Our result is irrelevant to this aspect of renegotiation. The efficient MPE strategy constructed in Theorem 3 does not include inefficient punishments, and in the asymptotic efficiency result of Theorem 4, renegotiation takes place on the equilibrium play and as a result, the set of participants is expanded.

The remainder of the paper is organized as follows. Section 2 presents the basic model of contract negotiations. Section 3 characterizes an SSPE of the model. Section 4 presents the repeated bargaining model with voluntary participation, and provides the efficiency result. Section 4 illustrates international negotiations for climate change as an application of the result. Section 5 discusses the result. All proofs are given in Appendix.

2 The Model

Players interact in a two-stage process. In the first stage, they bargain for a side payments contract which is a payoff transfer plan contingent on actions. Unlike Jackson and Wilkie (2005) who allow players to commit to payoff transfers unilaterally, we assume that a side payment contract can be binding only based on unanimous agreement among players. Namely, we consider unanimous bargaining for a side payments contract. Any player can refuse a payoff transfer plan simply by rejecting it. In the commitment model of Jackson and Wilkie (2005), players can effectively refuse payoff transfers from other players only

cartel by oligopolistic firms which plays a role of public goods for non-member firms.

by returning the same amounts of payoffs to them. A detailed negotiation process is given in a formal definition of a model below. In the second stage, players choose actions given a side payments contract possibly agreed in the first stage.

We now provide formal definitions. Let $G = (N, \{A_i\}_{i\in N}, \{u_i\}_{i\in N})$ be an n-person game in strategic form. $N = \{1, \cdots, n\}$ is the set of players. A_i is a finite set of player i's pure actions a_i.[5] Let $A = \Pi_{i\in N} A_i$. A is the set of all pure action profiles $a = (a_1, \cdots, a_n)$ for players. For each $i \in N$, a_{-i} denotes the pure action profile in a except a_i. Whenever it is convenient, we employ the notation $a = (a_i, a_{-i})$. Player i's payoff function u_i is a real-valued function on A. $\Delta(A_i)$ denotes the set of all probability distributions on A_i. An element μ_i in $\Delta(A_i)$ is called a mixed action for player i. Let $\Delta = \Pi_{i\in N}\Delta(A_i)$. For a mixed action profile $\mu = (\mu_1, \cdots, \mu_n) \in \Delta$ of players, the expected payoff $Eu_i(\mu)$ for player i is given in a usual way. For a subset S of N, the cardinality of S is denoted by s. R^s denotes the s-dimensional Euclidean space.

An action profile a^* is *efficient* if it maximizes the payoff sum $\sum_{i\in N} u_i(a)$ over A. Let M be the maximum payoff sum. A *side payments contract* $t = (t_1, \cdots, t_n)$ is a vector of functions where $t_i : A \to R$ satisfies the balancedness condition

$$\sum_{i\in N} t_i(a) = 0.$$

Player i receives a side payment $t_i(a)$ as a function of an action profile a played in the second stage. We denote by $t^0 = (t_1^0, \cdots, t_n^0)$ the null side payments contract such that $t_i^0(a) = 0$ for all i and all a. As in Jackson and Wilkie (2005), an alternative (and more detailed) formulation of a side payments contract specifies senders of and receivers of payoff transfers such that $t_{ij}(a)$ is

[5] The result of the paper can be extended to the case of continuum action sets without much difficulty.

a payoff transfer from player i to player j. In this formulation, it holds that

$$t_i(a) = \sum_{j \in N, j \neq i} t_{ji}(a) - t_{ij}(a).$$

To obtain the results of this paper, it does not matter which formulation is employed. For simplicity of exposition, we employ the formulation $t = (t_1, \cdots, t_n)$ as a side payments contract among players.[6] In the following , we will call a side payments contract simply a contract.

We describe a two-stage process of contracting.

Stage 1.

The negotiation process in the first stage is a Rubinstein-type sequential bargaining game with random proposers. It consists of possibly infinitely many bargaining rounds. The precise rule is as follows.

(1) At the beginning of the first round, a player $i \in N$ is randomly selected as a proposer according to a predetermined probability distribution $\theta = (\theta_1, \cdots, \theta_n)$ over the player set N where θ_i is the probability that player i is selected. In real situations, a probability distribution for proposers may be determined by institutional and cultural factors as well as formal protocols. For example, in legislative bargaining, a political party with more seats tends to have more opportunities to make proposals. Some countries have a social norm of seniority that older persons make proposals with higher likelihoods than younger ones. The model of random proposers attempts to capture a risk that a player may not become a proposer. In the principal-agent model where only the principal can make a proposal, the probability distribution is degenerate in that the principal is selected as a proposer with probability one. It turns out that the probability for a player to be selected as a proposer generates his bargaining power.

(2) Proposer i chooses a contract t, and thereafter all other players either accept or reject it sequentially. The order of responders does not affect the

[6] Ellingsen and Paltseva (2016) employ the same formulation as ours.

result of the paper in any critical way.

(3) If all responders accept a proposal t, then it is binding. We assume that players can write a binding contract costlessly by unanimous consent. Information is complete. Players choose actions in the second-stage game given t. If any responder rejects the proposal, two events may happen. Bargaining may stop with a positive probability $\epsilon > 0$. If this event happens, players choose actions independently in the underlying game G without any contract (that is, under the null contract t^0). With probability $1 - \epsilon$, negotiations continue in the next round by the same rule as above. Whenever players make choices, they know a history of play perfectly. When negotiations continue forever with no agreement,[7] it is assumed that players choose actions in the second-stage game under the null contract t^0.

Stage 2.

Every player i chooses his action $a_i \in A_i$ independently either with or without a contract. Suppose that a contract $t = (t_1, \cdots, t_n)$ is agreed in the first stage. The payoff of player i is given by

$$u_i(a, t) = u_i(a) + t_i(a).$$

Let $NE(t)$ be the set of (pure and mixed) Nash equilibria when players' payoff functions are defined above. If there is no contract, player i receives $u_i(a)$.

We denote the whole game defined above by Γ^ϵ where $\epsilon > 0$ is the stopping probability of negotiations when a proposal is rejected. A behavior strategy profile σ for Γ^ϵ prescribes a randomized choice of every player at his every move in the first and second stages, depending on a history of play.

The aim of our analysis is to characterize a stationary subgame perfect equilibrium (SSPE) of Γ^ϵ when the stopping probability ϵ is sufficiently small.[8]

[7]Although the probability of this event is zero as long as the stopping probability ϵ of negotiations is positive, this assumption is made for completeness of modelling.

[8]It is well-known that the set of non-stationary subgame perfect equilibrium outcomes in a broad class of Rubinstein-type sequential multilateral bargaining games including our game is large when there are more than two players (see Osborne and Rubinstein 1990).

An SSPE is a subgame perfect equilibrium where every player's behavior in every bargaining round of the first stage does not depend on a history of play. We note that a player's equilibrium action in the second stage may depend on a history of negotiations in the first-stage game. When a contract has been agreed in the first- stage game, players' actions in the second-stage game surely depend on the contract. When there is no agreement in the first-stage game, such a history may affect players' actions in the second-stage game.

We also note that any randomized choice of players in the first-stage game is unnecessary to characterize an SSPE of Γ^ϵ as is well-known in the literature of non-cooperative sequential bargaining theory with complete information. In equilibrium, a randomized choice may be needed in the second-stage game where there exists no pure Nash equilibrium in the game with or without a contract. This gives an analytical simplicity to our bargaining model compared with the commitment model of Jackson and Wilkie (2005) where a mixed strategy over transfer functions in the first-stage game may be necessary in equilibrium.

The central questions in this paper are which side payments contract is agreed, and whether the possibility of contracting leads to efficiency. Specifically, we examine whether the Nash bargaining solution can be attained through side payments when the stopping probability of negotiations is sufficiently small.

We consider an asymmetric Nash bargaining solution for a strategic game G with side payments. Recall that M is the maximum value of $\sum_{i\in N} u_i(a)$ over $a \in A$. Let $\omega = (\omega_1, \cdots, \omega_n) \in R^n$ satisfy $\sum_{i\in N} \omega_i = 1$ and $\omega_i > 0$ for all i, and let $d = (d_1, \cdots, d_n)$ satisfy $d_i = u_i(a)$ for all i and for some (pure or mixed) action profile a in the underlying game G. A payoff vector $x^* = (x_1^*, \cdots, x_n^*)$ is an *asymmetric Nash bargaining solution* of G with a weight vector ω and a disagreement point d, denoted by $\mathrm{NB}(\omega, d)$, if it is a solution of the program

For example, every allocation in an n-person bargaining problem of splitting a pie of fixed-size can be supported by a non-stationary subgame perfect equilibrium when the stopping probability of negotiations is sufficiently small.

$$\begin{array}{lll} \max & \Pi_{i \in N}(x_i - d_i)^{\omega_i} \\ \text{subject to} & (1) \displaystyle\sum_{i \in N} x_i = M \\ & (2)\ x_i \geq d_i \quad \text{for all} \quad i = 1, \cdots, n. \end{array}$$

It is straightforward to see that the solution x^* satisfies

$$\frac{x_1^* - d_1}{\omega_1} = \cdots = \frac{x_n^* - d_n}{\omega_n}.^9$$

(2.5) with $\sum_{i \in N} x_i^* = M$ and $\sum_{i \in N} \omega_i = 1$ implies that for all $i \in N$

$$x_i^* = d_i + \omega_i\ (M - \sum_{j \in N} d_j).$$

The asymmetric Nash bargaining solution $NB(\omega, d)$ allocates the net surplus $M - \sum_{i \in N} d_i$ to players in proportional to the weight vector ω in addition to their disagreement payoffs. The weight vector ω reflects the bargaining power of players.

3 The Agreement of Nash Bargaining Solution

We first prove the existence of an SSPE of the two-stage game Γ^ϵ for every ϵ. The SSPE has the following properties. On equilibrium play, a contract is agreed in the first round, independent of a proposer, and an efficient action profile is played in the second stage given the agreed-upon contract. When negotiations break down off equilibrium play, some (pure or mixed) Nash equilibrium a in the underlying game G is played. The expected payoff profile (due

[9] If $\sum_{i \in N} d_i = M$, then it is clear that d is a unique feasible solution of the program and thus that $x^* = d$. (2.5) is trivially satisfied in this case. If $\sum_{i \in N} d_i < M$, constraint (2) is non-binding, and thus (2.5) is obtained from FOC.

to random proposers) of players in the whole game is equal to an asymmetric Nash bargaining solution $NB(\theta, u(a))$ where the weight vector of players is equal to the probability distribution θ choosing proposers and a disagreement point is given by the payoff vector of the Nash equilibrium a of G that is played when negotiations break down. Moreover, when the stopping probability ϵ goes to zero, the equilibrium payoff profile of players given the contract converges to the Nash bargaining solution $NB(\theta, u(a))$, independent of a proposer.

When the underlying game G has an efficient Nash equilibrium a^*, the existence of an SSPE of Γ^ϵ satisfying the properties above is trivial. In equilibrium, every player proposes the null contract t^0 and all other players accept t^0. In the second stage, the Nash equilibrium a^* of G is played when t^0 is agreed (and also negotiations fail). When a contract $t \neq t^0$ is agreed off equilibrium play, any Nash equilibrium $a(t)$ in $NE(t)$ is played.[10] When a contract $t \neq t^0$ is proposed off equilibrium play, every responder i accepts it if $u_i(a(t), t) \geq u_i(a^*)$. In this case, we note that the Nash bargaining solution $NB(\theta, u(a^*))$ is equal to the disagreement point $u(a^*)$.

In the following, we assume:

Assumption 1. Every (pure or mixed) Nash equilibrium in G is inefficient.

We provide a useful result to construct an SSPE.[11]

Lemma 1. For any pure action profile a of the underlying game G and any payoff profile $x \in R^n$ satisfying $\sum_{i \in N} x_i = \sum_{i \in N} u_i(a)$, there exists a contract t satisfying (1) $x_i = u_i(a, t)$ for all $i \in N$ and (2) a is a unique Nash equilibrium in the second-stage game given t.

The intuition for the lemma is straightforward. When an action profile

[10] Recall that $NE(t)$ is the set of Nash equilibria in the underlying game G given a contract t.

[11] Jackson and Wilkie (2005, p.563) show a similar result.

a is played, a contract t is designed so that players receive the payoff profile x. Such a payoff transfer is possible since $\sum_{i\in N} x_i = \sum_{i\in N} u_i(a)$. When any action profile $a' \neq a$ is played, the contract t prescribes every player i deviating from a_i to pay a large amount penalty M to all other players. Under t, it is a strictly dominant action for player i to choose a_i. Thus, condition (2) is satisfied. We denote by $t(x, a)$ the contract given in Lemma 1.

We now present the existence of an SSPE in the two-stage game Γ^ϵ where the Nash bargaining solution is attained through side payments contracts, independent of a proposer, in the limit that the stopping probability of negotiations goes to zero.

Let a be a Nash equilibrium of the underlying game G, and let θ be a probability distribution to select a proposer in the first stage of negotiations.

Theorem 1. For every $\epsilon > 0$, there exists an SSPE of the two stage game Γ^ϵ where the expected payoff profile for players is equal to the Nash bargaining solution $NB(\theta, u(a))$ with the weight vector θ and the disagreement point $u(a)$. The equilibrium payoff profile of players in the second-stage game converges to $NB(\theta, u(a))$, independent of a proposer, in the limit that the stopping probability ϵ of negotiations goes to zero.

We construct an SSPE strategy profile in the theorem by the following idea. Let $x^* = NB(\theta, u(a))$ be the Nash bargaining solution with a disagreement point $u(a)$ which is a Nash equilibrium payoff profile in the underlying game. First, an SSPE strategy prescribes a Nash equilibrium in the second-stage game given any contract. Specifically, players choose the Nash equilibrium a under the null contract, namely, when negotiations break down in the first-stage game. Secondly, to obtain the Nash bargaining solution, players must choose an efficient action profile e in the second-stage game. Since e is not a Nash equilibrium of the underlying game (by Assumption 1), we need some appropriate contract under which e becomes a Nash equilibrium. Lemma 1 guarantees such a contract. Furthermore, any payoff profile can be attained through payoff transfers, keeping that e is a Nash equilibrium. Which payoff profile should be attained? It depends on who is a proposer. Since a proposer has a strategic advantage, it is not optimal to propose simply the Nash bargaining solution x^* if the stopping probability ϵ of negotiations is positive.

Temporarily, suppose that the expected equilibrium payoffs for players in the whole game are equal to the Nash bargaining solution x^*. The key factor to determine an equilibrium contract is the *continuation payoff after rejection* that a player receives in the whole game when negotiations break down. Since an equilibrium is stationary, the continuation payoff c_j for player j is given by $c_j = (1-\epsilon)x_j^* + \epsilon \cdot u_j(a)$. Every player i proposes the contract under which he receives payoff $\sum_{k\in N} u_k(e) - \sum_{j\neq i} c_j$, giving continuation payoffs c_j to all other players j. If all such contracts are agreed, it can be shown that the expected payoffs for players (due to random proposers) are actually equal to the Nash bargaining solution x^*. The equilibrium response rule for every player i is to accept any contract t if $u_i(b(t), t) \geq c_i$ where $b(t)$ is the action profile played in the second-stage game given t. It remains to show the optimality of the equilibrium contract that every player i proposes. Suppose that he proposes any other contract t to obtain a payoff higher than $\sum_{k\in N} u_k(e) - \sum_{j\neq i} c_j$. Since the equilibrium contract gives an efficient payoff profile for players, there must exist some player $j \neq i$ who receives a payoff smaller than his continuation payoff c_j given t. Such a player j rejects t, and player i becomes worse-off since his continuation payoff c_i is smaller than $\sum_{k\in N} u_k(e) - \sum_{j\neq i} c_j$. Finally, when the stopping probability ϵ of negotiations goes to zero, all player s' continuation payoffs c_i converge to the Nash bargaining solution x^*, and thus the equilibrium payoff profile in the second stage given the equilibrium contract does so, too, independent of a proposer.

We now move to the issue of uniqueness of an SSPE outcome in the two stage game. Theorem 1 states that for every (pure or mixed) Nash equilibrium a of the underlying game there exists an SSPE in the two-stage game which supports the Nash bargaining solution with the disagreement point $u(a)$. Thus, the two stage game has multiple SSPE outcomes if the underlying game G has multiple Nash equilibria. We, however, prove that there exists no other SSPE outcomes in the two-stage game. Specifically, if the underlying game has a unique Nash equilibrium, the two-stage game has a unique SSPE outcome.

Theorem 2. Let a be a Nash equilibrium of the underlying game G. For every $\epsilon > 0$, there exists a unique SSPE outcome of the two-stage game Γ^ϵ where a is played when negotiations break down.

The logic for the uniqueness of an SSPE outcome in the two-stage game is basically the same as that in many multilateral sequential bargaining games in the literature. The only difference is that a non-cooperative game is played in the second-stage game after an agreement is made in the first-stage game. If there are multiple Nash equilibria in the second-stage game given a contract, the contract cannot determine a final payoff profile. Lemma 1 overcomes this difficulty. It guarantees that every action profile and every payoff profile generated by it through payoffs transfer can be supported as a unique Nash equilibrium under some suitably designed contract.

Specifically, the proof of Theorem 2 proceeds in two steps. First, it is shown that every player i's proposal is accepted in an SSPE. The intuition for this is as follows. Let v_j be the expected equilibrium payoff for every player j. If responder j rejects i's proposal, then j receives his continuation payoff $c_j = (1-\epsilon)v_j + \epsilon \cdot u_j(a)$ since the equilibrium is stationary. We have assumed that the Nash equilibrium is inefficient. Thus, the continuation payoff vector $c = (c_j)$ is also inefficient. Then, there exists some payoff vector $x \in R^n$ such that players can attain x through payoff transfers by playing an efficient action profile e in the underlying game, and that $x_j > c_j$ for every $j \in N$. Let $t(x, e)$ be the contract under which e is a unique Nash equilibrium generating payoffs x in the second-stage game. If i proposes $t(x, e)$, then it can be shown by backward induction that all responders j accept it, since $x_j > c_j$. This means that every player i's equilibrium proposal must be accepted. Secondly, every proposer i optimally proposes the side payments contract $t(y^{i,\epsilon}, e)$ under which the efficient action profile e is played in the second-stage game. The final payoff profile $y^{i,\epsilon}$ is such that all responders j obtain their continuation payoffs c_j and proposer i does $M - \sum_{j \neq i} c_j$ where $M = \sum_{i \in N} u_i(e)$. It implies that every player i's equilibrium expected payoff v_i satisfies

$$v_i = \theta_i \{M - \sum_{j \neq i} ((1-\epsilon)v_j + \epsilon \cdot u_j(a))\} + (1-\theta_i)((1-\epsilon)v_i + \epsilon \cdot u_i(a)).$$

These n equations solve $v_i = u_i(a) + \theta_i \ (M - \sum_{j \in N} u_j(a))$, which is the asymmetric Nash bargaining solution $NB(\theta, u(a))$ with the weight vector $\theta = (\theta_i)$ and the disagreement point $u(a)$. Thus, given that a fixed Nash equilibrium of the underlying game is played when negotiations fail, an SSPE outcome of the two-stage game is unique.

4 Repeated Bargaining with Voluntary Participation

In the last section, we have shown that the efficient and fair outcome (the Nash bargaining solution) is attained in a two-stage process of side payments contract. It is assumed that all players participate in negotiations. As we have discussed in the introduction, players may have an incentive not to participate in negotiations and to free ride on a contract made by others. Nonparticipation of some (or all) players results in an inefficient outcome. The idea of Coasian bargaining suggests that if a contract is inefficient due to nonparticipation, then all parties involved may have an incentive to renegotiate it towards a Pareto-improving one so that (at least some) non-participants are motivated to participate. If this is true, then by repeating renegotiations, an efficient outcome may result in the end.

In this section, we present a formal model of renegotiation process and examine whether or not inefficiency caused by non-participation can be overcome by renegotiations.[12] We will show that an answer to the question is affirmative. Specifically, we will show the two results. First, there exists an efficient Markov perfect equilibrium of the repeated bargaining game with voluntary participation where all players participate in negotiations in the first round, provided that the probability of renegotiations is sufficiently high. Second, all players participate in negotiations and attain an efficient outcome with probability one in the long run in every Markov perfect equilibrium if the coalitional values of players in the underlying game satisfy a certain condition of supper-additivity.

We now describe a formal model of renegotiations with voluntary participation. In the model, the two-stage game Γ^ϵ in Section 2 is repeated until all players participate in negotiations and make an agreement.

For every $k = 1, 2, \cdots$, let $\omega_k = (S_k, t^k)$ be a state in round k where $S_k \subset N$ and a transfer contract $t^k = (t_i^k)_{i \in S^k}$ is a vector of functions where $t_i^k : \Pi_{j \in S_k} A_j \to R$ satisfies the balancedness condition

$$\sum_{i \in S_k} t_i^k(a^k) = 0$$

for all action profiles a^k in $\Pi_{j \in S_k} A_j$.

The interpretation of $\omega_k = (S_k, t^k)$ is that S_k is the set of all players who have participated in negotiations before round k and t^k is an "on-going" side payments contract agreed by S_k. Let the initial state $\omega_1 = (S_1, t^1)$ satisfy $S_1 = \emptyset$ and $t^1 = 0$ (null contract). Each round k is played by the following rule.

[12] Anderlini and Felli (2001) consider a renegotiation model based on the Rubinstein's alternating-offers model different from ours. In their model, players may forget a previous history of play with a positive probability at the end of every round, and they will restart negotiations in the next period. They show that such a possibility of renegotiation induces the inefficiency (no agreement) with the additional rule that the players decide simultaneously to pay participation costs at the beginning of each round.

Round k.

Stage 0.

All non-participants $i \notin S_k$ decide independently to participate in negotiations, or not. Let P_k be the set of new participants. If P_k is the empty set, then the next stage is vacuous and let $\omega_{k+1} = \omega_k$.

Stage 1.

Negotiations take place among incumbent participants S_k and new participants P_k. Let $S_{k+1} = S_k \cup P_k$, and let θ^k be a predetermined probability distribution over S_{k+1}. Each player $i \in S_{k+1}$ is randomly selected as a proposer according to the probability distribution θ^k. Proposer i chooses a side payments contract t among S_{k+1}, and thereafter all other participants in S_{k+1} either accept or reject it sequentially according to a predetermined order. If all accept the proposal t, then t becomes an on-going agreement, replacing t^k. In this case, we say that coalition S_{k+1} forms. If t is rejected by any responder, t^k remains the on-going contract. When P_k is the empty set, stage 1 is vacuous and t^k remains the on-going contract. At the end of stage 1, there is a random choice that determines whether or not negotiations stop. With probability $1 - \epsilon$, negotiations continue in the next round $k + 1$ and the same process is repeated with a new state ω_{k+1} determined by

$$\omega_{k+1} = \begin{cases} (S_{k+1}, t^{k+1}) & \text{if } t^{k+1} \text{ is agreed by } S_{k+1} \text{ in period } k, \\ \omega_k & \text{otherwise.} \end{cases}$$

With probability $\epsilon > 0$, negotiations stop and the on-going contract becomes the final agreement. Once the largest coalition N is formed, negotiations stop with probability one.

Stage 2.

When negotiations stop, the underlying game G is played under the final agreement of payoff transfer. All non-participants choose their default actions (non-contribution or no-trade, for example).

We denote by $\Gamma^{\epsilon,\infty}$ the dynamic bargaining game defined above. Every player can know a history of play perfectly whenever he makes a choice.

A behavior strategy profile σ for $\Gamma^{\epsilon,\infty}$ is defined in a standard manner. It prescribes a randomized choice to every player, depending on a history of play. For a behavior strategy profile σ, we denote by $Eu_i(\sigma)$ the expected payoff for player i in $\Gamma^{\epsilon,\infty}$.[13]

We consider a Markov perfect equilibrium in the game $\Gamma^{\epsilon,\infty}$. A behavior strategy profile for $\Gamma^{\epsilon,\infty}$ is a Markov perfect equilibrium if it is a subgame perfect equilibrium where every player's choice in every round k depends only on a state variable ω_k.[14] There exists a trivial Markov perfect equilibrium where in every round, no players participate in negotiations and all players play a Nash equilibrium of the underlying game G. To eliminate such a trivial equilibrium, we employ a refinement that a Markov perfect equilibrium prescribes a strict Nash equilibrium (if any) in the participation stage on equilibrium play.[15] The trivial equilibrium describes above prescribes a non-strict Nash equilibrium in the participation stage since every player is indifferent to participate or not, given that any other player does not participate. We remark that the strictness property is applied only on equilibrium play.

To characterize a Markov perfect equilibrium of $\Gamma^{\epsilon,\infty}$, we assume the following.

Assumption 2. (i) An efficient action profile of the underlying game G does not include any player's default action a_i^0. (ii) The default action profile $a^0 = (a_1^0, \cdots, a_n^0)$ is a Nash equilibrium of G,

This assumption is not restrictive. It holds in many economic games such

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[13]The probability that a play continues forever in $\Gamma^{\epsilon,\infty}$ is zero as long as the stopping probability ϵ of negotiations is positive.

[14]Every responder's choice in stage 1 of negotiations certainly depends on a proposal.

[15]A strict Nash equilibrium is a Nash equilibrium where every player has a unique best response to all other players' choices.

as exchange markets of private goods, team production and voluntary contribution games of public goods when the default actions mean no-trade or no-contribution.

We first prove that there exists an efficient Markov perfect equilibrium in the game $\Gamma^{\epsilon,\infty}$ when the stopping probability ϵ is sufficiently small.

Theorem 3. There exists an efficient Markov perfect equilibrium of $\Gamma^{\epsilon,\infty}$ where all players participate in negotiations in the first round and they agree to the Nash bargaining solution $x^* = NB(\theta, u(a^0))$ if the stopping probability ϵ of negotiations is sufficiently small.

In the theorem, we construct an efficient Markov perfect equilibrium of $\Gamma^{\epsilon,\infty}$ as follows. In round 1, all players participate in negotiations and they behave according to the SSPE σ^* of Γ^ϵ constructed in Theorem 1. The expected payoff profile of players is equal to the Nash bargaining solution $x^* = NB(\theta, u(a^0))$ of the underlying game G where a^0 is the default action profile. When negotiations stop with no agreement off equilibrium play, all players choose their default actions. By Assumption 2(ii), a^0 is a Nash equilibrium of G. Suppose that the game reaches round $k(> 1)$ with a state $\omega_k = (S_k, t^k)$ off the equilibrium play. Let $\tilde{a}^{t^k}_{S_k} = (a^{t^k}_{S_k}, a^0_{N-S_k})$ be an action profile in the underlying game G where all participants S_k play a Nash equilibrium $a^{t^k}_{S_k}$ of G under the side payments contract t^k, given that non-participants $N - S_k$ choose their default actions $a^0_{N-S_k}$. Let $u(t^k)$ be the payoff profile of all players for $\tilde{a}^{t^k}_{S_k}$, and let $x^*(t^k) = NB(\theta, u(t^k))$ be the Nash bargaining solution of G with the disagreement point $u(t^k)$. In round k, all players outside S_k participate in negotiations. In the negotiation stage, all players behave in the same way as σ^* and the expected payoff profile of them is equal to the Nash bargaining solution $x^*(t^k) = NB(\theta, u(t^k))$. The only difference is that the disagreement point is $u(t^k)$ instead of $u(a^0)$. Assumption 2(i) is needed in order for us to guarantee that every proposer makes an accepted proposal.

The central question in Theorem 3 is why all players participate in negotiations in the equilibrium. The answer to this is as follows. Suppose that any player h does not participate in negotiations, while all other players do. Then, negotiations take place among the players in $S = N - \{h\}$. For every contract w for S, the action profile $\tilde{a}_S^w$ can be defined in the same way as $\tilde{a}_{S_k}^{t^k}$. Let $x^*(w) = NB(\theta, u(w))$ be the Nash bargaining solution of G where the disagreement point $u(w)$ is the payoff profile attained by $\tilde{a}_S^w$. In negotiations, the continuation payoff after rejection for every player $i \in S$ is $(1-\epsilon)x_i^* + \epsilon u_i(a^0)$. Note that the Nash bargaining solution $x^* = NB(\theta, u(a^0))$ will be agreed in the next round if negotiations may continue with probability $1-\epsilon$. On the other hand, if any contract w is agreed, every player $j \in S$ receives the expected payoff $(1-\epsilon)x_j^*(w) + \epsilon u_j(w)$ since renegotiations take place in the next round, given the contract w, and the Nash bargaining solution $x^*(w) = NB(\theta, u(w))$ with the disagreement point $u(w)$ will be agreed. Thus, every proposer i's (potentially) optimal contract t is such that all responders j receive payoffs $u_j(t)$ satisfying $(1-\epsilon)x_j^*(t) + \epsilon u_j(t) = (1-\epsilon)x_i^* + \epsilon u_i(a^0)$, and they accept it in equilibrium. If the optimal contract t is agreed, then proposer i receives payoff $(1-\epsilon)x_j^*(t) + \epsilon(M^S - \sum_{j\in S, j\neq i} u_j(t))$ and non-participant h receives payoff $(1-\epsilon)x_h^*(t) + \epsilon u_h(e^S, a_h^0)$. A critical point is whether or not it is indeed optimal for proposer i to propose the contract t. It is so if

$$(1-\epsilon)x_i^*(t) + \epsilon(M^S - \sum_{j\in S, j\neq i} u_i(t)) \geq (1-\epsilon)x_i^* + \epsilon u_i(a^0).$$

Since $\sum_{j\in N} x_j^*(t) = \sum_{j\in N} x_j^* = M$ where M is the maximum value of the payoff sum for all players in the underlying game G, there exists a trade-off of payoffs for participants and non-participant h between $x^*(t)$ and x^*. Roughly, if $x_i^*(t) > x_i^*$ so that proposer i makes the optimal proposal t for sufficiently small $\epsilon > 0$, then it holds that $x_h^*(t) < x_h^*$,[16] which means that non-participant

[16]To be precise, we will show that the payoff differences $x_j^*(t) - x_j^*$ have the same sign for all participants. See the proof in Appendix.

h is worse off by the optimal contract t for S. Conversely, if non-participant h is better off by the optimal contract t, then it should be actually optimal for proposer i to make an unacceptable proposal, and as a result, non-participant h becomes worse off. Whichever happens, the non-participant is worse off by deviating from her equilibrium strategy.

Theorem 3 shows the existence of an efficient Markov perfect equilibrium of the repeated bargaining game $\Gamma^{\epsilon,\infty}$. We next strengthen the efficiency result of Theorem 3 and show that all n players participate in negotiations in the long run in every Markov perfect equilibrium of $\Gamma^{\epsilon,\infty}$. For every subset S of N, let $M^S = max_{a \in A} \sum_{i \in S} u_i(a)$ subject to $a_i = a_i^0$ for every $i \notin S$, and let $e^S \in A$ be the action profile attaining M^S. We call M^S the *coalitional value* of S. We assume the following.

Assumption 3. For every two subsets S and T of N with $S \subset T$, $M^S + \sum_{i \in T-S} u_i(e^S) < M^T$.

This assumption says that if a set of players participate in a group S, then the coalitional value of the extended group T is higher than the sum of the coalitional value of S and their total payoff before participation. It implies that the participation of some players in a group increases the total payoffs of the incumbent members and the new members. In this sense, Assumption 3 is interpreted as the supper-additivity of coalitional values.

Example 3.

Consider an n-person game of voluntary contributions to a public good. Every player has an endowment one monetary unit. Let $x_i \in [0,1]$ be every player i's contribution and his payoff is given by $u_i(x_1, \cdots, x_n) = 1 - x_i + a \sum_{j=1}^{n} x_j$. Parameter a represents the marginal per capita return from contributing to the public good. We assume that $1/2 < a < 1$. Under this condition, it is the dominant action for every player i to choose $x_i = 0$ and it is optimal for every coalition S of players with $s \geq 2$ to choose the full contribution $x_i = 1$ by its members. Thus, $M^S = as^2$ and free-riders receive payoffs $1 + as$. This game satisfies Assumption 3 since

$$M^S + \sum_{i \in T-S} u_i(e^S) = as^2 + (t-s)(as+1) = ats + t - s < at^2 = M^T$$

for every $t > s$.

Theorem 4. Under Assumption 3, all n players participate in negotiations and attain an efficient outcome with probability one in the long run in every Markov perfect equilibrium of $\Gamma^{\epsilon,\infty}$.

The logic for this result is as follows. When the set S_k of participants is smaller than the largest group N at the beginning of some round k, there exists at least one player who may participate in negotiations with a positive probability η on the play of every Markov perfect equilibrium σ. This fact can be explained intuitively by the following reason. Let t^k be an on-going contract in round k. If no players participate in negotiations, the game may end with probability ϵ and t^k becomes the final agreement. In this case, players receive the payoff profile $u(t^k)$ where all participants choose a Nash equilibrium of the underlying game G under the contract t^k while non-participants choose their default actions. With probability $1 - \epsilon$, the game may continue in the next round $k+1$ with the same state as in round k. Since the equilibrium σ satisfies the Markov property, it induces the same play in round $k+1$ as in round k. Thus, every player i receives the payoff $u_i(t^k)$ in the subgame of $\Gamma^{\epsilon,\infty}$ starting in round k. If any one player i participates in negotiations, the coalitional value of the extended group $S = S_k \cup \{i\}$ is larger than $\sum_{j \in S} u_j(t^k)$ by Assumption 3. This means that a beneficial contract can be agreed in negotiations,[17] and thus

[17]To be precise, we should take into account players' continuation payoffs after a contract is agreed. See the formal proof given in Appendix.

every player in S obtains an expected payoff larger than in $u(t^k)$. Specifically, the new participant i can be better off by participation. This implies that in the participation stage, σ prescribes either a strict pure Nash equilibrium where at least one player participates in negotiations, or a mixed Nash equilibrium where at least one player may participate with a positive probability. Given the fact above, Theorem 4 can be proved roughly as follows. By round n, all n players may participate in negotiations at least with probability η^n. Thus, for any integer $r = 1, 2, \cdots$ all players may not participate in negotiations by round rn at most with probability $(1 - \eta^n)^r$. In the limit that $r \to \infty$, the probability that all players do not participate in negotiations converges to zero.

5 Application

We illustrate international negotiations on climate change as an application of the two-stage model in Section 2.[18] The following illustration is based on our previous work of Okada (2007).

The Kyoto Protocol of 1997 is an international agreement that developed countries commit to reducing their greenhouse gas (GHG) emissions. The contents of the Protocol is that developed countries as a whole reduce emissions by 5.2 percent below 1990 levels between 2008 and 2012, and that reduction commitments are assigned to developed countries. For example, the reduction rates of major countries are: Russia and Ukraine 0 percent, Japan 6 percent, the Unites States 7 percent, and European Union 8 percent. The Protocol also includes emissions trading as a flexible mechanism for international emissions transfer. Focusing on negotiations for reduction commitments and emissions trading, the two-stage model in Section 2 can be applied to international negotiations for the Protocol. In the first stage, developing countries play the bargaining game. Given an agreement of reduction commitments, they play a

[18]Harstad (2012 and 2015) considers a dynamic game of climate contract where countries contribute to emissions and invest in technologies.

game of emissions trading in the second stage. We assume that emissions trading take place in a competitive market, and thus that the efficient reduction can be attained across the countries.

Let $N = \{1, \cdots, n\}$ be the set of countries. For every $i \in N$, we denote by E_i country i's current level of CO_2 emissions. Let x_i denote country i's reduction of CO_2 emissions where $0 \leq x_i \leq E_i$. The CO_2 abatement cost function of country i is denoted by $C_i(x_i)$. $C_i(x_i)$ is a differentiable, strictly convex and monotonically increasing function on R_+. Let $MC_i(x_i)$ denote the marginal abatement cost function of country i.

In the first stage, n countries negotiate for an allocation $\omega = (\omega_1, \cdots, \omega_n) \in R^n_+$ of emissions satisfying $\overline{\omega} = \sum_{i=1}^n \omega_i$, where ω_i denotes an amount of emission permits allocated to country i and $\overline{\omega}$ is a total emission target. For simplicity of analysis, we assume that the total emission target is predetermined by a scientific committee.[19] Country i has to reduce $E_i - \omega_i$ amount of emissions if emissions trading is impossible.

In the second stage, emissions trading takes place in a competitive market. Let p be a price of emissions and let x_i be an actual level of emissions reduction by country i after the trading. A competitive equilibrium of emissions trading with an initial emission allocation $\omega = (\omega_1, \cdots, \omega_n) \in R^n_+$ is defined to be a vector $(p^*, x_1^*, \cdots, x_n^*) \in R^{n+1}_+$ satisfying

$$x_i^* \in \text{argmin } \{C_i(x_i) + p^*(E_i - x_i - \omega_i) \ \mid \ 0 \leq x_i \leq E_i\} \quad \text{for any} \quad i \in N,$$
$$\sum_{i \in N}(E_i - x_i^*) = \sum_{i \in N} \omega_i.$$

The competitive equilibrium reduction cost for country i is given by

$$c_i^e(\omega_i) = C_i(x_i^*) + p^*(E_i - x_i^* - \omega_i).$$

[19] In a more elaborate bargaining model, a total emission target $\overline{\omega}$ is also an agenda of negotiations.

A competitive equilibrium of emissions trading satisfies the principle of marginal cost pricing, $p^* = MC_i(x_i^*)$ for all $i \in N$, and the equilibrium emission reduction minimizes the total reduction cost for n countries. We denote by $c(\bar{\omega})$ the minimum value of the total emissions reduction costs given the emission target $\bar{\omega}$.

Anticipating the competitive equilibrium outcome of emissions trading, countries negotiate for an emission allocation. Let d_i be the costs that country i has to burden when negotiations fail, referred to as the "business as usual" case. The "business as usual" costs for countries are caused by the delay of prevention of global warming, and are highly uncertain in character. For an illustration, we simply assume here that an estimation of them is available to negotiating countries. The cost vector $d = (d_1, \cdots, d_n)$ consists of the disagreement point of the bargaining problem of emission reductions. Assuming $\sum_{i \in N} d_i > c(\bar{\omega})$, Theorem 1 implies that n countries agree to an asymmetric Nash bargaining solution $\omega^* = (\omega_1^*, \cdots, \omega_n^*)$ with a weight vector $\theta = (\theta_1, \cdots, \theta_n)$ where the competitive equilibrium reduction cost $c_i^e(\omega_i^*)$ of country i is given by

$$c_i^e(\omega_i^*) = d_i + \theta_i(c(\bar{\omega}) - \sum_{i=1}^{n} d_i).$$

An interesting question is whether the Kyoto protocol can be justified by an asymmetric Nash bargaining solution. In Okada (2007), we examine this question using actual data when three different weights of equality, GDP and population are considered. We show that the Kyoto protocol can be justified by an asymmetric Nash bargaining solution with equal weights and population weights, provided that the EU and the US estimate their "business as usual" costs high. For numerical results, see Okada (2007).

6 Discussion

We have characterised a stationary subgame perfect equilibrium outcome when players negotiate for side payments contracts according to a Rubinstein-type sequential bargaining game. When players voluntarily participate in negotiations, we have characterised a Markov perfect equilibrium outcomes for a repeated bargaining game where incumbent participants renegotiate repeatedly for their side payments contracts with new participants (if any).

The basic conclusion of the paper can be summarised as follows. First, when all players participate in negotiations, they can attain an efficient and fair outcome (the Nash bargaining solution) immediately, given a Nash equilibrium played in the failure of negotiations. Second, when players voluntarily participate in negotiations, there exists an efficient Markov perfect equilibrium outcome with the Nash bargaining agreement, and moreover the efficient outcome that all players participate in a contract can be attained in the long run with probability one in every Markov perfect equilibrium through a process of renegotiations. In view of the results above, the Coase "Theorem" holds true in our framework.

Let us discuss some of implications and restrictions of our results.

The nature of the Coase theorem

The Coase "Theorem" is not a mathematical theorem. It has not been formalized in a vigorous way. Coase (1960) himself did not call it a theorem.[20] Economists' intuition behind the theorem may be called the efficiency principle which says that "if people are able to bargain together effectively and can effectively implement and enforce their decisions, then the outcomes of economic activity will tend to be efficient (at least for the parties to the bargain)"(Mil-

[20] Coase regards the case of zero transaction costs unrealistic. He writes: "while consideration of what happen in a world of zero transaction costs can give us valuable insights, these insights are, in my view, without value except as steps on the way to the analysis of the real world of positive transaction costs." (Coase 1981)

grom and Roberts 1992, p.24). It is not meaningful to ask such questions as whether the theorem is right or false, and whether the theorem always holds true. Rather, in our view, an appropriate question is in what cases the theorem holds, or to what extent an economist's intuition supporting the theorem can be justified. In this paper, we show *a* case where the theorem holds. It is quite possible that the theorem does not hold in other cases. In fact, Jackson and Wilkie (2005) show that the theorem does not always hold where players make binding offers of side payments. Perhaps, one of the most simple cases where the theorem does not hold is as follows. Two players simultaneously propose allocations in the division problem of a fixed size. An allocation is agreed and enforced if and only if their proposals coincide. This model is a reduced form of the bargaining game of Ellingsen and Paltseva (2016) where players simultaneously make contract proposals, and choose at most one contract to be signed, and a contract is enforced by unanimous consent of affected players. Due to its coordination character, the bargaining model above have many Nash equilibria. All efficient and inefficient allocations can be sustained by Nash equilibria.

Modelling negotiations

As we have discussed above, it is critical for us how to model negotiations for the examination of the Coase theorem. Negotiation in real world is often complex and unstructured. As Schelling (1960) illustrates, many elements such as bluffing, cheating, coalitions, commitment, communications, coordination, delegation, focal points, promise, threat, etc. are involved. Obviously, a theoretical model is abstract and simple for tractability, and cannot include all these elements. Based on the Rubinstein's alternating-offers model, our bargaining model focuses a sequence of offers and responses in negotiations. A special character of our model is that of random proposers. It turns out that the probability of each player's being selected as a proposer is a source of his bargaining power. Specifically, it determines the player's weight in an asymmetric Nash bargaining solution. Jackson and Wilkie (2005) emphasise players' commitments or promises in negotiations. If we compare our proposer-response model with Jackson and Wilkie's (2005) unilateral promise model, we can say that it is harder for players to attain an efficient outcome in their model since players can refuse another player's promised transfers only by announcing a transfer that returns the other player's transfer. Players use transfers to try to manipulate other players' behavior. In our model, every player can refuse any transfer simply by rejecting it. In our view, it depends on a context which of bargaining models is more appropriate. For example, the unilateral promise model of Jackson and Wilkie (2005) fits well to international trade negotiations where developed countries promises monetary transfer (investments) to developing countries, and a principal-agent relationship where a principal promises a compensation schedule to an agent. Our proposal-response model is suitable for the analysis of international treaty negotiations such as Kyoto protocol.

Disagreement points

One version of the Coase Theorem says that "the initial allocation of legal entitlements does not matter from an efficiency perspective so long as they can be freely exchanged" (Cooter 1989). The initial allocation of legal entitlements determines the set of feasible actions for players, and thus induces an outcome in the failure of negotiations, which corresponds to a disagreement point of the Nash bargaining solution. Thus, our result confirms this version of the theorem. The legal entitlements of property rights does not affect the efficiency in negotiations, but does a payoff allocation. Our asymptotic efficiency result critically depends on the assumption that a threat point of renegotiation is an on-going contract by incumbent participants. Although this assumption seems to us reasonable since players can write such a renegotiation rule in the initial contract, this does not always the case. If our assumption of a threat point in renegotiation is not satisfied, for example, if a contract is effective only in one period and players restart their negotiations in every round, the asymptotic efficiency result in Theorem 4 does not hold. Gomes and Jehiel

(2005) show the persistence of inefficiency in a general renegotiation process under externality.

Towards future works

We close by noting a few future works. As Coase (1981) emphasizes, a clear insight on negotiations in the case of zero transaction costs should be a first step on the way to the analysis of the real world of positive transaction costs. It is interesting to develop a bargaining model for a broad class of mechanisms ranging from direct (and indirect) mechanisms contingent on players types (and messages) under incomplete information to relational contracts (repeated game strategies). A theory of mechanism bargaining in decentralized environments can contribute to theories of mechanism design and implementation which assume the existence of a social planner (or a principal) to choose and implement a mechanism. Endogenous formation of institutions to enforce a contract is also in a list of future works.

Appendix

Proof of Lemma 1. Let $L = 1 + max_{i,a',a''}\{|u_i(a') - u_i(a'')|, |x_i - u_i(a')|\}$. We construct a side payments contract t as follows. For any action profile a' and any i, let $k_i(a')$ be the number of players $j \neq i$ who choose $a'_j \neq a_j$. Define

$$t_i(a') = \begin{cases} x_i - u_i(a) & \text{if } a = a', \\ k_i(a')L & \text{if } a_i = a'_i \text{ and } k_i(a') > 0 \\ k_i(a')L - (n-1)L & \text{otherwise.} \end{cases}$$

The side payments contract t above is defined according to the rule that any player i pays a large amount L to every other player if he deviates from a_i. It holds that $\sum_{i \in N} t_i(a') = 0$ for every $a' \in A$. By definition, $x_i = u_i(a) + t_i(a)$ for all $i \in N$. Thus, condition (1) in the lemma is clearly satisfied. Under t, it is a strictly dominant action for each player i to play a_i. To check this, consider the following two cases. Let $a'_i \neq a_i$ be player i's any action other than a_i, and let a'_{-i} be any action profile for all players except i.

Case 1. $a'_{-i} = a_{-i}$. In this case, $u_i(a, t) - u_i((a'_i, a_{-i}), t) = x_i - u_i(a'_i, a_{-i}) + (n-1)L$. This value is positive by the definition of L.

Case 2. $a'_{-i} \neq a_{-i}$. In this case, $u_i((a_i, a'_{-i}), t) - u_i((a'_i, a'_{-i}), t) = u_i(a_i, a'_{-i}) - u_i(a'_i, a'_{-i}) + (n-1)L$. This value is positive by the definition of L, too. □

Proof of Theorem 1. Let $x^* = NB(\theta, u(a))$. It holds that $x^*_i = u_i(a) + \theta_i(M - \sum_{k \in N} u_k(a))$ where M is the maximum value of $\sum_{i \in N} u_i(a)$ over $a \in A$. For every $\epsilon > 0$ and every $i \in N$, define the payoff vector $y^{i,\epsilon} \in R^n$ such that

$$y^{i,\epsilon}_j = (1-\epsilon)x^*_j + \epsilon \cdot u_j(a) \quad \text{for all} \quad j \neq i \tag{A.1}$$

$$y^{i,\epsilon}_i = M - \sum_{j \neq i} y^{i,\epsilon}_j. \tag{A.2}$$

Let $e \in A$ be an efficient action profile of the underlying game. Let $t(y^{i,\epsilon}, e)$

be the side payment contract where $y_k^{i,\epsilon} = u_k(e, t(y^{i,\epsilon}, e))$ for every k and e is a unique Nash equilibrium in the second-stage game under $t(y^{i,\epsilon}, e)$. The existence of $t(y^{i,\epsilon}, e)$ is proved in Lemma 1.

We construct an SSPE strategy profile σ for the two-stage game Γ^ϵ as follows.

Stage 1:

Player i proposes the side payments contract $t(y^{i,\epsilon}, e)$ if he is selected as a proposer. When player i responds to a side payments contract t, he accepts it if and only if $u_i(b(t), t) \geq (1-\epsilon)x_i^* + \epsilon \cdot u_i(a)$ where $b(t)$ is an action profile played in the second-stage game when t is agreed in the first-stage game.

Stage 2:

When a side payments contract t is agreed in the first-stage game, an action profile $b(t)$ in the second stage is defined by the following rule: play the efficient action profile e if $t = t(y^{i,\epsilon}, e)$ for any i, and otherwise play any Nash equilibrium under t. When negotiations break down in the first stage, play the Nash equilibrium a in the underlying game.

If σ is played, the side payments contract $t(y^{i,\epsilon}, e)$ is agreed in the first-stage game if player i is selected as a proposer, and the efficient action profile e is played in the second-stage game. Thus, when σ is played, the expected payoff $Eu_i(\sigma)$ for every player i in the whole game is given by

$$\begin{aligned}
Eu_i(\sigma) &= \sum_{j\in N} \theta_j \cdot y_i^{j,\epsilon} \\
&= \theta_i\{M - \sum_{j\neq i}((1-\epsilon)x_j^* + \epsilon u_j(a))\} + (1-\theta_i)((1-\epsilon)x_i^* + \epsilon u_i(a)) \\
&= \theta_i\{M - \sum_{k\in N}((1-\epsilon)x_k^* + \epsilon u_k(a))\} + (1-\epsilon)x_i^* + \epsilon u_i(a) \\
&= \epsilon\theta_i(M - \sum_{k\in N} u_k(a)) + (1-\epsilon)x_i^* + \epsilon u_i(a) \quad (by \sum_{k\in N} x_k^* = M) \\
&= \epsilon(x_i^* - u_i(a)) + (1-\epsilon)x_i^* + \epsilon u_i(a) \quad \text{(by the definition of } x_i^*) \\
&= x_i^*.
\end{aligned}$$

In the first-stage game, if player i rejects a side payments contract t proposed by player j, then negotiation goes to the next round with probability $1-\epsilon$, and i receives the expected payoff $Eu_i(\sigma) = x_i^*$ in the following subgame. With probability ϵ, negotiations break down and the Nash equilibrium a is played in the second-stage game by σ. Thus, player i receives $y_i^{j,\epsilon} = (1-\epsilon)x_i^* + \epsilon \cdot u_i(a)$ by rejecting t. If player i accepts t, then i receives either payoff $u_i(b(t), t)$ or $y_i^{j,\epsilon}$. Payoff $y_i^{j,\epsilon}$ will be attained if any responder after i rejects t. Thus, i's response rule in σ, accepting t if and only if $u_i(b(t), t) \geq (1-\epsilon)x_i^* + \epsilon \cdot u_i(a)$, is his optimal choice.

We need only show that it is optimal for every player i to propose the side payments contract $t(y^{i,\epsilon}, e)$. If i proposes it, then it is accepted in σ and thus the payoff profile $y^{i,\epsilon}$ is attained in the second-stage game. Suppose that i proposes $t \neq t(y^{i,\epsilon}, e)$ such that $u_i(b(t), t) > y_i^{i,\epsilon}$. Since the payoff vector $y^{i,\epsilon}$ is efficient, there exists some player $j \neq i$ such that $u_j(b(t), t) < y_j^{i,\epsilon}$. Player j rejects t in σ, and thus player i receives $(1-\epsilon)x_i^* + \epsilon \cdot u_i(a)$, which is smaller than $y_i^{i,\epsilon}$. Player i is worse-off by proposing t.

Finally, it can be easily seen that for every i $y^{i,\epsilon}$ converges to the Nash bargaining solution $x^* = NB(\theta, a)$ in the limit that ϵ goes to zero. $\square$

Proof of Theorem 2. Let σ be an SSPE of the two-stage game Γ^ϵ where the Nash equilibrium a is played when negotiations break down. Theorem 1 guarantees the existence of such an SSPE. Let $v_i = Eu_i(\sigma)$ be the expected payoff for every player i in Γ^ϵ when σ is played. Suppose that player i is selected as a proposer. If every responder $j \neq i$ rejects a proposal of i, then j receives the expected payoff $c_j = (1-\epsilon)v_j + \epsilon \cdot u_j(a)$ since σ is stationary. By assumption, the Nash equilibrium a is inefficient, and thus the payoff vector $c = (c_j)$ is inefficient, too. Then, there exists some payoff vector $x \in R^n$ such that (1) $\sum_{j\in N} x_j = \sum_{j \in N} u_j(e)$ where $e \in A$ is an efficient action profile in the underlying game G, and (2) $x_j > c_j$ for every $j \in N$. Let $t(x, e)$ be a side payments contract satisfying (1) $x_j = u_j(e, t)$ for all $j \in N$ and (2) e is a unique Nash equilibrium in the second-stage game given $t(x, e)$. Lemma 1 guarantees the existence of $t(x, e)$. If proposer i proposes $t(x, e)$, then by backward induction it is shown that $t(x, e)$ is agreed in the SSPE σ. Therefore, every proposer i can make an accepted proposal in the SSPE σ. In fact, proposer i optimally proposes the side payments contract $t(y^{i,\epsilon}, e)$ and it is accepted in σ where the payoff vector $y^{i,\epsilon}$ is defined by (A.1) and (A.2) with $x_i^* = v_i$ for all $i \in N$.[21] It implies that the expected payoffs $v = (v_i)$ for σ satisfy for all $i \in N$

$$v_i = \theta_i\{M - \sum_{j \neq i}((1-\epsilon)v_j + \epsilon \cdot u_j(a))\} + (1-\theta_i)((1-\epsilon)v_i + \epsilon \cdot u_i(a)) \quad \text{(A.3)}$$

Let $\bar{v} = \sum_{j \in N} v_j$ and $\bar{d} = \sum_{j \in N} u_j(a)$. Then, (A.3) can be arranged as

$$v_i = \theta_i\{M - (1-\epsilon)\bar{v} - \epsilon \cdot \bar{d}\} + (1-\epsilon)v_i + \epsilon \cdot u_i(a). \quad \text{(A.4)}$$

By summing both sides of (A.4) for all $i \in N$, we obtain $\bar{v} = M$. By substituting this into (A.4), we obtain

$$v_i = u_i(a) + \theta_i \ (M - \sum_{j \in N} u_j(a)).$$

Thus, the expected payoffs for the SSPE σ is equal to the asymmetric Nash bargaining solution $NB(\theta, u(a))$ with the weight vector $\theta = (\theta_i)$ and the disagreement point $u(a)$. In σ, the expected payoff profile and every player's payoff obtained by an (accepted) equilibrium proposal are uniquely determined. The equilibrium payoff profile in the second-stage game given the equilibrium side payments contract converges to the Nash bargaining solution $NB(\theta, u(a))$ when the stopping probability ϵ goes to zero, independent of a proposer. $\square$

Proof of Theorem 3. We first introduce several notations. Let $e \in A$ be an efficient action profile in the underlying game G, and let $x^* = NB(\theta, u(a^0))$ be

[21] While all responders are indifferent to accepting or rejecting the proposal, we have shown that they accept it in equilibrium.

the Nash bargaining solution of G with the disagreement point $u(a^0)$ where a^0 is the default action profile. Let $y^{i,\epsilon}$ is the payoff vector defined by (A.1) and (A.2) where $a = a^0$. For every subset S of N, let $M^S = max_{a \in A} \sum_{i \in S} u_i(a)$ subject to $a_i = a_i^0$ for all $i \notin S$, and let $e^S \in A$ be the action profile attaining M^S. Let $G(S)$ be the s-person strategic form game obtained from the underlying game G under the assumption that all non-participants $j \notin S$ choose their default actions a_j^0. For an action profile $a_S \in \Pi_{i \in S} A_i$ for S and a payoff vector $x \in R^s$ satisfying $\sum_{i \in S} x_i = \sum_{i \in S} u_i(a_S, a_{N-S}^0)$, let $t(x, a_S)$ be the side payments contract proved in Lemma 1 with respect to the game $G(S)$. For every contract t for S, choose a Nash equilibrium a_S^t of $G(S)$ under t,[22] and let $\tilde{a}_S^t$ be the action profile in G that prescribes a_S^t for the members of S and does their default actions for all non-participants. Define a payoff vector $u(t) \in R^n$ such that $u_i(t) = u_i(\tilde{a}_S^t) + t_i(a_S^t)$ for every $i \in S$ and $u_j(t) = u_j(\tilde{a}_S^t)$ for every $j \notin S$. Let $x^*(t) = NB(\theta, u(t))$ be the Nash bargaining solution of G with the disagreement point $u(t)$.

We construct an equilibrium strategy profile σ^* of $\Gamma^{\epsilon,\infty}$ as follows. For every $k = 1, 2, \cdots$, let $\omega_k = (S_k, t^k)$ be a state in round k.

Case 1. $S_k = \emptyset$.

- All players in N participate in negotiations.
- Let S be a set of participants either on or off the equilibrium play. When $S = N$, every player $i \in N$ employs the SSPE strategy constructed in the proof of Theorem 1. When $S \neq N$, every player $i \in S$ proposes $t = t(z^{i,\epsilon}, e^S)$ such that a payoff vector $z^{i,\epsilon} \in R^s$ is given by

$$
\begin{aligned}
z_j^{i,\epsilon} &= \frac{y_j^{i,\epsilon} - (1-\epsilon)x_j^*(t)}{\epsilon} \quad \text{for all } j \in S, j \neq i && \text{(A.5)} \\
z_i^{i,\epsilon} &= M^S - \sum_{j \in S, j \neq i} z_j^{i,\epsilon}, && \text{(A.6)}
\end{aligned}
$$

[22] If there exists no pure Nash equilibrium, choose a mixed Nash equilibrium. The following proof is not affected in any critical way.

where $y_j^{i,\epsilon}$ is defined by (A.1) with $a = a^0$, if $(1-\epsilon)x_i^*(t) + \epsilon z_i^{i,\epsilon} \geq y_i^{j,\epsilon}$, and otherwise makes an unacceptable proposal. Note that $y_i^{j,\epsilon}$ is independent of $j(\neq i)$. When a contract w for S is proposed, every responder $j \in S$ accepts it if and only if $(1-\epsilon)x_j^*(w) + \epsilon u_j(w) \geq y_j^{i,\epsilon}$. When negotiations stop with the agreement w for the set S of participants, $\tilde{a}_S^w$ is played.

Case 2. $S_k \neq \emptyset$.
Let t^k be an on-going contract among participants in S^k.

- All players in $N - S_k$ participate in negotiations.
- Let S be a set of participants either on or off the equilibrium play. When $S = N$, every player $i \in N$ proposes $t(z^{i,\epsilon}, e)$ where a payoff vector $z^{i,\epsilon} \in R^n$ is given by

$$z_j^{i,\epsilon} = (1-\epsilon)x_j^*(t^k) + \epsilon u_j(t^k) \quad \text{for all } j \in S, j \neq i \tag{A.7}$$

$$z_i^{i,\epsilon} = M - \sum_{j \in N, j \neq i} z_j^{i,\epsilon}. \tag{A.8}$$

When a contract w for N is proposed, every responder $j \in N$ accepts it if and only if $u_j(w) \geq z_j^{i,\epsilon}$. When $S \neq N$, every player $i \in S$ proposes $t = t(z^{i,\epsilon}, e^S)$ where a payoff vector $z^{i,\epsilon} \in R^s$ is given by

$$z_j^{i,\epsilon} = \frac{\epsilon u_j(t^k) + (1-\epsilon)(x_j^*(t^k) - x_j^*(t))}{\epsilon} \quad \text{for all } j \in S, j \neq i \tag{A.9}$$

$$z_i^{i,\epsilon} = M^S - \sum_{j \in S, j \neq i} z_j^{i,\epsilon}, \tag{A.10}$$

if $(1-\epsilon)x_i^*(t) + \epsilon z_i^{i,\epsilon} \geq (1-\epsilon)x_i^*(t^k) + \epsilon u_i(t^k)$, and otherwise makes an unacceptable proposal. When a contract w for S is proposed, every responder $j \in S$ accepts it if and only if $(1-\epsilon)x_j^*(w) + \epsilon u_j(w) \geq (1-\epsilon)x_j^*(t^k) + \epsilon u_j(t^k)$. When negotiations stop with the agreement w for the set S of participants, $\tilde{a}_S^w$ is played.

When σ^* is played, all players participate in negotiations in round 1, and they behave according to the SSPE defined in the proof of Theorem 1. That is, every player's proposal is accepted and the expected payoff profile of players is equal to the Nash bargaining solution $x^* = NB(\theta, u(a^0))$ with the disagreement point $u(a^0)$. The game ends in round 1.

When the game starts in round $k(> 1)$ with a state $\omega_k = (S_k, t^k)$ off the play of σ^*, all players participate in negotiations, and every player's proposal is accepted. The game ends in round k. The expected payoff of every player $i \in N$ evaluated at the beginning of round k is given by

$$
\begin{aligned}
Eu_i(\sigma^*) &= \sum_{j\in N} \theta_j \cdot z_i^{j,\epsilon} \\
&= \theta_i\{M - \sum_{j\in N, j\neq i} ((1-\epsilon)x_j^*(t^k) + \epsilon u_j(t^k))\} + (1-\theta_i)((1-\epsilon)x_i^*(t^k) + \epsilon u_i(t^k)) \\
&= \theta_i\{M - \sum_{j\in N}((1-\epsilon)x_j^*(t^k) + \epsilon u_j(t^k))\} + (1-\epsilon)x_i^*(t^k) + \epsilon u_i(t^k) \\
&= \epsilon\theta_i(M - \sum_{j\in N} u_j(t^k)) + (1-\epsilon)x_i^*(t^k) + \epsilon u_i(t^k) \quad (by \sum_{j\in N} x_j^*(t^k) = M) \\
&= \epsilon(x_i^*(t^k) - u_i(t^k)) + (1-\epsilon)x_i^*(t^k) + \epsilon u_i(t^k) \quad \text{(by the definition of } x_i^*(t^k)) \\
&= x_i^*(t^k).
\end{aligned}
$$

The expected payoff profile of players is equal to the Nash bargaining solution $x^*(t^k) = NB(\theta, u(t^k))$ of the underlying game G with the disagreement point $u(t^k)$.

It is clear that σ^* satisfies the Markov property. When negotiations stop with an agreement w for the set S of participants, σ^* prescribes the Nash equilibrium $\tilde{a}_S^w$ of $G(S)$ under w while all non-participants choose their default actions. To prove that σ^* is a subgame perfect equilibrium, it remains to show that σ^* prescribes the optimal choice of every player at his every move in the two stages of participation and of negotiations in every round, given that σ^* will be played in all future moves. We prove this in each case.

Case 1. When the set of participants is N, σ^* prescribes the SSPE constructed in the proof of Theorem 1, and thus it satisfies the optimality of every player's choice in the stage of negotiations. Suppose that the set of participants is $S \neq N$. When every player $i \in S$ proposes a contract w for S, every responder j receives the payoff $(1-\epsilon)x_j^*(w) + \epsilon u_j(w)$ if w is accepted, and otherwise, does the payoff $y_j^{i,\epsilon}$. Thus, σ^* prescribes the optimal response rule for j. Given the optimal response rules for all other participants, the optimal proposal for i must be the contract $t = t(z^{i,\epsilon}, e^S)$ defined by (A.4) and (A.5) if its acceptance makes i better off than rejection. If t is accepted, proposer i receives payoff $(1-\epsilon)x_i^*(t) + \epsilon z_i^{i,\epsilon}$ while i receives payoff $y_i^{j,\epsilon}$ by rejection. Thus, it is optimal for i to propose $t = t(z^{i,\epsilon}, e^S)$ if $(1-\epsilon)x_i^*(t) + \epsilon z_i^{i,\epsilon} \geq y_i^{j,\epsilon}$. Otherwise, it is optimal for i to make an unaccepted proposal.

Finally, consider the participation stage. Suppose that any player $h \in N$ deviates from σ^* and does not participate in negotiations. Then, the set of participants is $S = N - \{h\}$. We shall examine what would happen in negotiations among S. Every player $i \in S$ proposes the equilibrium contract $t = t(z^{i,\epsilon}, e^S)$ defined by (A.5) and (A.6) if its acceptance makes her better off than rejection. When t is implemented, the payoff profile $u(t) = (z^{i,\epsilon}, u_h(e^S, a_h^0))$ is attained. For every $j \in S$ with $j \neq i$, (A.5) implies

$$\begin{aligned} y_j^{i,\epsilon} &= \epsilon z_j^{i,\epsilon} + (1-\epsilon)x_j^*(t) \\ &= \epsilon z_j^{i,\epsilon} + (1-\epsilon)\{z_j^{i,\epsilon} + \theta_j(M - \sum_{m \in S} z_m^{i,\epsilon} - u_h(e^S, a_h^0))\} \\ &= \epsilon z_j^{i,\epsilon} + (1-\epsilon)\{z_j^{i,\epsilon} + \theta_j(M - M^S - u_h(e^S, a_h^0))\}. \end{aligned} \tag{A.11}$$

The last equality holds by (A.6). By definition of $y_j^{i,\epsilon}$ in (A.1) and x_j^*, it holds that

$$y_j^{i,\epsilon} = \epsilon u_j(a^0) + (1-\epsilon)\{u_j(a^0) + \theta_j(M - \sum_{m \in N} u_m(a^0)\}. \tag{A.12}$$

It follows from (A.11) and (A.12) that

$$z_j^{i,\epsilon} - u_j(a^0) = (1-\epsilon)\theta_j\{M^S + u_h(e^S, a_h^0) - \sum_{m \in N} u_m(a^0)\}. \tag{A.13}$$

Since

$$\epsilon u_j(a^0) + (1-\epsilon)x_j^* = y_j^{i,\epsilon} = \epsilon z_j^{i,\epsilon} + (1-\epsilon)x_j^*(t), \tag{A.14}$$

(A.13) implies that

$$x_j^* - x_j^*(t) = \epsilon\theta_j\{M^S + u_h(e^S, a_h^0) - \sum_{m\in N} u_m(a^0)\}.$$

Since j can be any element of $S = N - \{h\}$, three cases may be possible: (i) $x_j^* > x_j^*(t)$ for every $j \in S$, (ii) $x_j^* < x_j^*(t)$ for every $j \in S$, and (iii) $x_j^* = x_j^*(t)$ for every $j \in S$. In case (i), $x_i^* > x_i^*(t)$ for proposer $i \in S$. Then, for any sufficiently small $\epsilon > 0$, it holds that

$$(1-\epsilon)x_i^*(t) + \epsilon z_i^{i,\epsilon} < (1-\epsilon)x_i^* + \epsilon u_i(a^0) = y_i^{i,\epsilon}.$$

Thus, it is not optimal for i to propose t, and negotiations fail. Non-participant h receives payoff $(1-\epsilon)x_h^* + \epsilon u_h(a^0)$, which is smaller than x_h^*. In case (ii), it is optimal for i to propose t for any sufficiently small $\epsilon > 0$, and it is accepted in σ^*. Since $x^*(t) = NB(\theta, u(t))$ and $x^* = NB(\theta, u(a^0))$, it holds that $\sum_{m\in N} x_m^*(t) = \sum_{m\in N} x_m^* = M$. Since $x_j^* < x_j^*(t)$ for every $j \in N$ with $j \neq h$, it must be that $x_h^* > x_h^*(t)$. Non-participant h receives payoff $(1-\epsilon)x_h^*(t) + \epsilon u_h(t)$, which is smaller than x_h^* for sufficiently small $\epsilon > 0$. In case (iii), it holds that $x_j^* = x_j^*(t)$ for all $j \in N$ since $\sum_{m\in N} x_m^*(t) = \sum_{m\in N} x_m^* = M$. Also, it follows from (A.14) that $z_j^{i,\epsilon} = u_j(a^0)$ for every $j \in N$ with $j \neq h$. Since x^* and $x^*(t)$ are the Nash bargaining solutions with disagreement points $u(a^0)$ and $u(t) = (z^{i,\epsilon}, u_h(e^S, a_h^0))$, respectively, it must be that $u_h(a^0) = u_h(e^S, a_h^0)$. Thus, regardless of the outcome of negotiations, non-participant h receives payoff $(1-\epsilon)x_h^* + \epsilon u_h(a^0)$, which is smaller than x_h^*. In all three cases, non-participant h is strictly worse off by deviating from σ^*. This means that σ^* prescribes a strict Nash equilibrium in the participation stage.

Case 2. Let $\omega_k = (S_k, t^k)$ be a state in round k. Suppose that the set of participants is N. When every player $i \in N$ proposes a contract w for N, every responder j receives the payoff $u_j(w)$ if w is accepted, and otherwise, does the payoff $z_j^{i,\epsilon} = (1-\epsilon)x_j^*(t^k) + \epsilon u_j(t^k)$. Note that negotiations stop with probability one if w is accepted. Thus, σ^* prescribes the optimal response rule for j. It is optimal for i to propose the contract $t(z^{i,\epsilon}, e)$ defined by (A.9) and (A.10) since

$$\begin{aligned}
z_i^{i,\epsilon} &= M - \sum_{j \in N, j \neq i} z_j^{i,\epsilon} \\
&= M - \sum_{j \in N, j \neq i} \{(1-\epsilon)x_j^*(t^k) + \epsilon u_j(t^k)\} \\
&= M - \sum_{j \in N} \{(1-\epsilon)x_j^*(t^k) + \epsilon u_j(t^k)\} + (1-\epsilon)x_i^*(t^k) + \epsilon u_i(t^k) \\
&= \epsilon(M - \sum_{j \in N} u_j(t^k)) + (1-\epsilon)x_i^*(t^k) + \epsilon u_i(t^k) \quad (\text{by} \sum_{j \in N} x_j^*(t^k) = M) \\
&> (1-\epsilon)x_i^*(t^k) + \epsilon u_i(t^k) = z_i^{j,\epsilon} \quad (\text{by Assumption 2 (i)}).
\end{aligned}$$

Next, suppose that the set of participants is $S \neq N$. When every player $i \in S$ proposes a contract w for S, every responder j receives the payoff $(1-\epsilon)x_j^*(w) + \epsilon u_j(w)$ if w is accepted, and otherwise, does the payoff $(1-\epsilon)x_j^*(t^k) + \epsilon u_j(t^k)$. Thus, σ^* prescribes the optimal response rule for j. Given the optimal response rules for all other participants, the optimal proposal for i must be the contract $t = t(z^{i,\epsilon}, e^S)$ defined by (A.9) and (A.10) if its acceptance makes i better off than rejection. If w is accepted, proposer i receives payoff $(1-\epsilon)x_i^*(w) + \epsilon z_i^{i,\epsilon}$ while i receives payoff $(1-\epsilon)x_i^*(t^k) + \epsilon u_i(t^k)$ by rejection. Thus, it is optimal for i to propose $t = t(z^{i,\epsilon}, e^S)$ if $(1-\epsilon)x_i^*(t) + \epsilon z_i^{i,\epsilon} \geq (1-\epsilon)x_i^*(t^k) + \epsilon u_i(t^k)$. Otherwise, it is optimal for i to make an unaccepted proposal.

Finally, consider the participation stage. By the same arguments as in case 1, we shall show that σ^* prescribes a strict Nash equilibrium in the participation stage. Suppose that any player $h \in N$ deviates from σ^* and does not participate in negotiations. The set of participants is $S = N - \{h\}$. Every player $i \in S$ proposes the equilibrium contract $t = t(z^{i,\epsilon}, e^S)$ defined by (A.9) and (A.10) if its acceptance makes her better off than rejection. For every $j \in S$ with $j \neq i$, (A.9) implies

$$\epsilon z_j^{i,\epsilon} + (1-\epsilon)x_j^*(t) = \epsilon u_j(t^k) + (1-\epsilon)x_j^*(t^k). \tag{A.15}$$

By definitions of $x_j^*(t)$ and $x_j^*(t^k)$, it holds that

$$\begin{aligned} \epsilon z_j^{i,\epsilon} \;&+\; (1-\epsilon)\{z_j^{i,\epsilon} + \theta_j(M - M^S - u_h(e^S, a_h^0))\} \\ &= \;\epsilon u_j(t^k) + (1-\epsilon)\{u_j(t^k) + \theta_j(M - \sum_{m\in N} u_m(t^k))\}. \end{aligned}$$

By arranging the equation above, we obtain

$$z_j^{i,\epsilon} - u_j(t^k) = (1-\epsilon)\theta_j\{M^S + u_h(e^S, a_h^0) - \sum_{m\in N} u_m(t^k)\}. \tag{A.16}$$

It follows from (A.15) and (A.16) that

$$x_j^*(t^k) - x_j^*(t) = \epsilon\theta_j\{M^S + u_h(e^S, a_h^0) - \sum_{m\in N} u_m(t^k)\}.$$

Similarly to case 1, three cases may be possible: (i) $x_j^*(t^k) > x_j^*(t)$ for every $j \in S$, (ii) $x_j^*(t^k) < x_j^*(t)$ for every $j \in S$, and (iii) $x_j^*(t^k) = x_j^*(t)$ for every $j \in S$. In case (i), $x_i^*(t^k) > x_i^*(t)$ for proposer $i \in S$. Then, for any sufficiently small $\epsilon > 0$, it holds that

$$(1-\epsilon)x_i^*(t) + \epsilon z_i^{i,\epsilon} < (1-\epsilon)x_i^*(t^k) + \epsilon u_i(t^k).$$

Thus, it is not optimal for i to propose t, and negotiations fail. Non-participant h receives payoff $(1-\epsilon)x_h^*(t^k) + \epsilon u_h(t^k)$, which is smaller than $x_h^*(t^k)$. In case (ii), it is optimal for i to propose t for any sufficiently small $\epsilon > 0$, and it is accepted in σ^*. Since $x^*(t) = NB(\theta, u(t))$ and $x^*(t^k) = NB(\theta, u(t^k))$, it holds that $\sum_{m\in N} x_m^*(t) = \sum_{m\in N} x_m^*(t^k)$. Since $x_j^*(t^k) < x_j^*(t)$ for every $j \in N$ with $j \neq h$, it must be that $x_h^*(t^k) > x_h^*(t)$. Non-participant h receives payoff $(1-\epsilon)x_h^*(t)+\epsilon u_h(t)$, which is smaller than $x_h^*(t^k)$ for sufficiently small $\epsilon > 0$. In case (iii), it holds that $x_j^*(t^k) = x_j^*(t)$ for all $j \in N$ since $\sum_{m\in N} x_m^*(t) = \sum_{m\in N} x_m^*(t^k)$. Also, it follows from (A.15) that $z_j^{i,\epsilon} = u_j(t^k)$ for every $j \in N$ with $j \neq h$. Since $x^*(t^k)$ and $x^*(t)$ are the Nash bargaining solutions with disagreement points $u(t^k)$ and $u(t) = (z^{i,\epsilon}, u_h(e^S, a_h^0))$, respectively, it must be that $u_h(t^k) = u_h(e^S, a_h^0)$. Thus, regardless of the outcome of negotiations, non-participant h receives payoff $(1-\epsilon)x_h^*(t^k)+\epsilon u_h(t^k)$, which is smaller than $x_h^*(t^k)$. In all three cases, non-participant h is strictly worse off by deviating from σ^*. $\square$

Proof of Theorem 4. Let σ be any Markov perfect equilibrium of $\Gamma^{\epsilon,\infty}$. We use the same notations introduced in the proof of Theorem 3. Specifically, for a subset S of N and a contract t for S, $u(t) \in R^n$ is the payoff vector for all n players where all participants in S choose the Nash equilibrium of the game $G(S)$ assigned by σ under t and all non-participants choose their default actions.

We will show that in every round k with a state $\omega_k = (S_k, t^k)$ satisfying $S_k \neq N$, σ does not prescribe that no players outside S_k participate, given that σ is played in the following subgame of $\Gamma^{\epsilon,\infty}$. By way of contradiction, suppose that σ prescribes so. If no players participate in round k, then the negotiation stage is vacuous, and the game may stop with probability ϵ. If this happens, t^k becomes the final agreement and all players receive payoffs $u(t^k)$. With probability $1-\epsilon$, negotiations may continue in the next period $k+1$ with the same state $\omega_{k+1} = (S_k, t^k)$ as in round k. Since σ has the Markov property, it induces the same play in round $k+1$ as in round k. This means that the expected payoff profile for players in σ from round $k+1$ (and also from round k) is equal to $u(t^k)$.

Suppose that any one player $h \notin S_k$ deviates from σ and participate in negotiations. Then, negotiations take place among $S = S_k \cup \{h\}$. Suppose further that every player $i \in S$, if selected as a proposer, proposes the contract

$t = t(z^{i,\delta}, e^S)$ where a payoff vector $z^{i,\delta} \in R^s$ is given by

$$\begin{aligned} z_j^{i,\delta} &= u_j(t^k) + \delta \ \text{ for all } \ j \notin S, j \neq i \\ z_i^{i,\delta} &= M^S - \sum_{j \in S, j \neq i} z_j^{i,\delta}, \end{aligned}$$

where $\delta > 0$ is any sufficiently small positive number. If t is accepted, every responder j receives payoff $(1-\epsilon)u_j(\sigma|(S,t)) + \epsilon z_j^{i,\delta}$ where $u_j(\sigma|(S,t))$ is the payoff that j receives according to σ in the subgame of $\Gamma^{\epsilon,\infty}$ starting at round $k+1$ with a state (S,t), that is, $u_j(\sigma|(S,t))$ is j's continuation payoff by σ after t is agreed by S. Since $u_j(\sigma|(S,t)) \geq u_j(t) = z_j^{i,\delta} > u_j(t^k)$, it holds that $(1-\epsilon)u_j(\sigma|(S,t)) + \epsilon z_j^{i,\delta} > u_j(t^k)$. Thus, every j accepts t since his payoff is $u_j(t^k)$ by rejection. Then, proposer i receives payoff

$$\begin{aligned} & (1-\epsilon)u_i(\sigma|(S,t)) + \epsilon(M^S - \sum_{j \in S, j \neq i} z_j^{i,\delta}) \\ = \ & (1-\epsilon)u_i(\sigma|(S,t)) + \epsilon\{M^S - \sum_{j \in S, j \neq i} u_j(t^k) - (s-1)\delta\} \\ = \ & (1-\epsilon)u_i(\sigma|(S,t)) + \epsilon\{M^S - \sum_{j \in S} u_j(t^k) - (s-1)\delta\} + \epsilon u_i(t^k) \\ > \ & u_i(t^k). \end{aligned}$$

The last inequality comes from the fact that $M^S - \sum_{j \in S} u_j(t^k) - (s-1)\delta > 0$ for any sufficiently small $\delta > 0$ by Assumption 3 applied to $S = S_k \cup \{h\}$, together with $u_i(\sigma|(S,t)) > u_i(t^k)$. Therefore, proposer i optimally makes an accepted proposal (which may be different from the t above) and receives a higher payoff than $u_i(t^k)$. The arguments above show that every player i in S can receive an expected payoff higher than $u_i(t^k)$ in the negotiation stage. Thus, player $h \notin S_k$ is better off by participating in negotiations in σ. This contradicts that σ prescribes a Nash equilibrium in the participation stage.

By the proof above, σ assigns either a strict pure Nash equilibrium where at least one player participate in negotiations, or a mixed Nash equilibrium where at least one player may participate in negotiations with a positive probability. In either case, in every round k with a state $\omega_k = (S_k, t^k)$ satisfying $S_k \neq N$, at least one player outside S_k may participate in negotiations with a positive probability $\eta > 0$ on the play of σ. It implies that all n players may participate in negotiations by round n at least with probability η^n.[23] Thus, all n players may not participate in negotiations by round rn at most with probability $(1 - \eta^n)^r$. In the limit that $r \to \infty$, the probability that all players do not participate in negotiations converges to zero. When all players participate in negotiations, they agree to an efficient payoff allocation with probability one. $\square$

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[23]To be precise, the probability η may depend on a state variable of a round. Since the set of state variables reached by the equilibrium play is finite, we can choose the minimum value of possible η's, if necessary.

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