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“Competition Policy at the Intensive and  
Extensive Margins in General Equilibrium”

Kenji Fujiwara and Keita Kamei

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# Competition Policy at the Intensive and Extensive Margins in General Equilibrium\*

Kenji Fujiwara<sup>†</sup>

School of Economics, Kwansei Gakuin University

Keita Kamei

Faculty of Literature and Social Sciences, Yamagata University

## Abstract

This paper examines welfare effects of competition policies in a general equilibrium model in which perfectly competitive and oligopolistic industries coexist and compete for a common factor of production. We first show that increasing the number of oligopolistic firms raises welfare if the oligopolists' production technology exhibits non-increasing returns to scale. Then, we address another competition policy modeled by an increase in the portion of perfectly competitive industries, finding that this policy improves welfare if decreasing returns of the oligopolists' technology are strong enough. These results suggest that the degree of returns to scale plays a key role for welfare-enhancing competition policy.

*Keywords:* Competition policy, General oligopolistic equilibrium (GOLE), Welfare, Returns to scale.

*JEL Classifications:* L4, L13.

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<sup>†</sup>Corresponding author: School of Economics, Kwansei Gakuin University. Uegahara 1-1-155, Nishinomiya, Hyogo, 662-8501, Japan. Tel: +81-798-54-7066. Fax: +81-798-51-0944. E-mail: kenjifujiwara@kwansei.ac.jp.

# 1 Introduction

Economic impacts of competition policy have been one of the most important subjects, and received considerable attention in economics. If competition policy is simply defined as an increase in the number of imperfectly competitive firms, it raises welfare because ‘when the number of firms becomes very large, the market price tends to the competitive price.’ (Tirole, 1988, p. 220) Besides this classical view on competition policy, recent evidence suggests that competition policy leads to economic growth by inducing firms to innovate and reduce costs.<sup>1</sup> To sum up, the existing literature allows us to claim that competition policy has a positive effect on economic growth and welfare.<sup>2</sup>

This paper theoretically reexamines the effects of competition policy in a general equilibrium context. While the previous theoretical literature gives useful insights into competition policy, its scope is largely restricted to a partial equilibrium model. The partial equilibrium approach is undoubtedly convenient, but it is difficult to straightforwardly apply its implications to a situation in which the presence of oligopolistic industries is significant. In order to evaluate the economy-wide impacts of competition policy, a general equilibrium model is inevitably called for. Nevertheless, it has been recognized that integrating oligopoly and general equilibrium theories leads to a number of theoretical problems, e.g. non-existence of equilibrium. To our knowledge, Neary (2003, 2016) first overcomes such difficulties by inventing a general oligopolistic equilibrium (GOLE) model.<sup>3</sup> By assuming a contin-

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<sup>1</sup>World Bank (2012) provides a concise but useful review of empirical assessments on the impacts of competition policies.

<sup>2</sup>We should note that there is a literature that finds negative welfare effects of increased competition. For instance, Lahiri and Ono (1988) demonstrate in an asymmetric oligopoly model that increasing the number of inefficient firms can reduce welfare. Mankiw and Whinston (1986) and Suzumura and Kiyono (1987), in addition, show that increasing the number of firms is welfare-reducing in the presence of economies of scale.

<sup>3</sup>Although Neary (2016) is published in 2016, its first working paper version was released in 2002. Colacicco (2015) is a comprehensive survey on the GOLE model and some of its

uum of industries and modeling ‘firms as having market power in their own industry but not in the economy as a whole,’ (Neary, 2016, p. 670) he offers a theoretically consistent model that has a variety of applications. Neary (2003), among others, applies his model to competition policy, showing that an increase in the oligopolistic firms raise welfare when there is a technological difference across industries.<sup>4</sup> Kamei (2014), on the contrary, finds that the same competition policy reduces welfare in the presence of increasing returns to scale.

Our purpose of this paper is to supplement the above strand of literature by extending the GOLE model of the previous studies. While Neary (2003) and Kamei (2014) commonly suppose that all industries are oligopolized, we split the whole industry into a set of perfectly competitive industries and a set of oligopolistic industries. This extension gives us two definitions of competition policy. The first is an increase in the number of oligopolistic firms, and the second is an increase in the share of perfectly competitive industries. We call in this paper the former policy a competition policy at the intensive margin and the latter a competition policy at the extensive margin, following Kreickemeier and Meland (2013). Then, we obtain the following results on the welfare effect of these competition policies. First, the competition policy at the intensive margin raises welfare if the production technology of oligopolistic firms exhibits non-increasing returns to scale. Second, it is impossible to analytically show that the competition policy at the extensive margin improves welfare, but our numerical analysis finds that this policy is beneficial if the oligopolists’ production technology is sufficiently decreasing returns to scale. Accordingly, these results commonly suggest that both kinds of competition policies enhance welfare if oligopolistic firms’ production technology is sufficiently decreasing returns.

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applications.

<sup>4</sup>If all industries are identical, which Neary (2003) calls a ‘featureless economy,’ the competition policy has no effect. Chen and Liu (2014) also obtain a similar result.

This paper is organized as follows. Section 2 presents a model. Section 3 addresses the impacts of the two competition policies. Section 4 concludes.

## 2 Model

Suppose a continuum of goods in a unit interval  $[0, 1]$ , and a representative consumer maximizes utility

$$\int_0^1 \ln x(z) dz, \quad (1)$$

under the budget constraint

$$\int_0^1 p(z)x(z) dz \leq I,$$

where  $x(z)$  is consumption of Good  $z$ ,  $p(z)$  is its price, and  $I$  is nominal income. Thus, letting  $\lambda$  be a Lagrangean multiplier, the first-order condition for utility maximization is  $1/x(z) = \lambda p(z)$ .

All goods are produced from labor only. We then assume perfect competition with a unitary input coefficient in industries  $[0, \tilde{z}]$ , and Cournot competition among  $n \geq 2$  firms in industries  $[\tilde{z}, 1]$ . And, denoting by  $w$  and  $y(z)$  the wage rate and per-firm output in industry  $z$ , the cost function of industry  $z$  is assumed to be  $w[y(z)]^{1/\theta}$ ,  $\theta > 0$ , which is derived from the production function  $y(z) = [l(z)]^\theta$ , where  $l(z)$  is labor input.<sup>5</sup> As mentioned in Introduction, we define competition policy in two ways. The first is an increase in  $n$  (competition policy at the intensive margin) and the second is an increase in  $\tilde{z}$  (competition policy at the extensive margin).

Following Neary (2003, 2016), we normalize marginal utility of income  $\lambda$  to unity for the following reason. In the present model,  $\lambda$  is an endogenous variable that is determined in general equilibrium, and depends on the strategic variable of oligopolistic firms, i.e. output in the present model of

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<sup>5</sup>From this formulation,  $\theta > 1$  (resp.  $\theta < 1$ ) implies increasing (resp. decreasing) returns to scale, and  $\theta = 1$  corresponds to constant returns to scale.

Cournot oligopoly. Hence, truly rational firms would choose output by fully taking account of this effect, ‘which opens a Pandora’s Box of technical difficulties.’ (Neary, 2016, p. 670) But, such difficulties can be resolved by assuming that oligopolists are large in their product market but small in the whole economy because this assumption implies that oligopolists take  $\lambda$  as given in choosing output. Thus, the perceived inverse demand function each firm faces becomes

$$p(z) = \frac{1}{\sum_{i=1}^n y_i(z)}, \quad (2)$$

and the profit of each firm is defined by  $p(z)y_i(z) - w[y_i(z)]^{1/\theta}$ . Under this definition of profit, the first-order condition for profit maximization is

$$\frac{n-1}{n^2 y(z)} = \frac{w}{\theta} [y(z)]^{\frac{1}{\theta}-1},$$

in the symmetric equilibrium where all firms produce the same amount. From this equation, total output in industry  $z$  becomes

$$ny(z) = n \left[ \frac{(n-1)\theta}{n^2 w} \right]^\theta. \quad (3)$$

It follows from the assumption of perfect competition and unitary input coefficient that  $p(z) = w$  and  $Y(z) = 1/w$  in industry  $z$  in industries  $[0, \tilde{z}]$ , where  $Y(z)$  is industry-wide output. Making use of this result and (3), the labor market-clearing condition is

$$L = \int_0^{\tilde{z}} Y(z) dz + \int_{\tilde{z}}^1 n[y(z)]^{\frac{1}{\theta}} dz = \tilde{z} \frac{1}{w} + (1 - \tilde{z}) \frac{(n-1)\theta}{nw},$$

where the first term in the right-hand side is aggregate labor demand in the perfectly competitive industries, and the second term is the counterpart in the oligopolistic industries. Solving this equation for  $w$  gives the equilibrium wage rate:

$$w = \frac{\tilde{z}n + (1 - \tilde{z})(n-1)\theta}{nL}. \quad (4)$$

This completes the description of the model. Once the equilibrium wage rate is determined as above, all of the other endogenous variables are explicitly derived.

### 3 Effects of Competition Policies

Drawing on the model developed in the last section, this section examines the effects of competition policies. At this stage, we make the following restriction on parameters  $\theta$  and  $n$  in order to ensure the non-negative profits of oligopolistic firms.<sup>6</sup>

**Assumption.**  $0 < \theta < n/(n - 1)$ .

Given this assumption, the effect of two competition policies on the equilibrium wage is identified as follows.

**Lemma 1.** *Both the competition policy at the intensive margin and at the extensive margin raises the equilibrium wage.*

*Proof.* By differentiating (4) with respect to  $n$  and  $\tilde{z}$ , we have

$$\frac{\partial w}{\partial n} = \frac{(1 - \tilde{z})\theta}{n^2 L} > 0, \quad \frac{\partial w}{\partial \tilde{z}} = \frac{n - (n - 1)\theta}{nL} > 0,$$

which leads to the result. ||

It is natural that the competition policy at the intensive margin raises the wage rate because it increases aggregate labor demand in the oligopolistic industries and aggregate economy. The positive effect of the competition policy at the extensive margin is, in contrast, a little more complicated than the competition policy at the intensive margin. When  $\tilde{z}$  increases, labor demand in perfectly competitive industries increases by  $1/w$  units, but labor demand in oligopolistic industries decreases by  $(n - 1)\theta/(nw)$  units.<sup>7</sup> Since

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<sup>6</sup>Substituting (3) into the definition of profit, the maximized profit is given by  $[n - (n - 1)\theta]/n^2$ .

<sup>7</sup>This is checked by looking at the labor market-clearing condition in the previous section.

we have assumed  $\theta < n/(n-1)$  for guaranteeing the non-negative profit of oligopolists, an increase in labor demand in perfectly competitive industries outweighs a decrease in labor demand in oligopolistic industries, which eventually results in a higher wage rate.

Note here that the profit maximization condition in the competitive industries  $p(z) = w$  and the above lemma jointly tell that both types of competition policies raise the price of competitive goods.

Let us next examine the effect of the competition policies on the price of oligopolized goods. Substituting (4) into (3), and relating the resulting expression to (2), the price of oligopolized goods depends on primitive parameters as follows.

$$p(z) = n^{-1} \left\{ \frac{(n-1)\theta L}{n [\tilde{z}n + (1-\tilde{z})(n-1)\theta]} \right\}^{-\theta}. \quad (5)$$

Therefore, differentiating (5) with respect to  $n$  and  $\tilde{z}$ , the effect of two competition policies is respectively obtained as

$$\frac{dp(z)}{dn} = \frac{\theta L \Delta}{n^3 [\tilde{z}n + (1-\tilde{z})(n-1)\theta]^2} \left\{ \frac{(n-1)\theta L}{n [\tilde{z}n + (1-\tilde{z})(n-1)\theta]} \right\}^{-1-\theta} \quad (6)$$

$$\Delta \equiv (1-\tilde{z})(n-1)^2\theta^2 + [\tilde{z}(2n^2 - 4n + 1) - (n-1)^2]\theta - \tilde{z}n(n-1)$$

$$\frac{dp(z)}{d\tilde{z}} = \frac{(n-1)[n - (n-1)\theta]\theta^2 L}{n^2 [\tilde{z}n + (1-\tilde{z})(n-1)\theta]^2} \left\{ \frac{(n-1)\theta L}{n [\tilde{z}n + (1-\tilde{z})(n-1)\theta]} \right\}^{-1-\theta}. \quad (7)$$

On the one hand, according to Eq. (7), the competition policy at the extensive margin unambiguously raises the price of oligopolized goods. On the other hand, Eq. (6) suggests that the effect of the competition policy at the intensive margin is generally unclear because the first term in  $\Delta$  is negative and the second term can be both positive and negative. However, Eq. (6) allows us to infer that an increase in  $n$  reduces the product price if non-increasing returns to scale ( $\theta \leq 1$ ) prevail in the oligopolistic industries.



But, it is possible that the same policy can raise the product price if the production technology of oligopolistic industries is subject to sufficiently strong economies of scale. The reason is as follows. When the number of oligopolistic firms increases, each firm reduces output and the goods price tends to fall as a first-order effect. However, as proved in Lemma 1, this policy involves a rise in the wage rate by increasing aggregate labor demand in the whole oligopolistic industries, which puts upward pressure on the product price as a second-order effect. If the degree of increasing returns is sufficiently large, this second-order effect dominates the first-order effect, and thereby raises the price of oligopolized goods.

Having identified the price effect of competition policies, we now address their welfare effect. The welfare effect of increasing the number of oligopolistic firms is summarized as follows.

**Proposition 1.** *The competition policy at the intensive margin improves welfare if and only if*

$$\Gamma \equiv (1 - \tilde{z})(n - 1)^2\theta^2 + [\tilde{z}n(2n - 3) - (n - 1)^2]\theta - \tilde{z}n(n - 1) < 0. \quad (8)$$

*Proof.* In order to compute the impact of an increase in  $n$  on welfare, let us define welfare in the present model. Substituting the demand function  $x(z) = 1/p(z)$  into the direct utility function (1), welfare  $W$  is obtained as

$$W = \int_0^{\tilde{z}} \left[ \frac{1}{p(z)} \right] dz + \int_{\tilde{z}}^1 \left[ \frac{1}{p(z)} \right] dz = -\tilde{z} \ln p_1 - (1 - \tilde{z}) \ln p_2, \quad (9)$$

where  $p_1$  is the price of perfectly competitive goods given by (4), and  $p_2$  is the price of oligopolized goods given by (5).

Taking into account Eqs. (4) and (5), differentiating (9) with respect to  $n$  yields

$$\frac{dW}{dn} = -\tilde{z} \frac{dp_1/dn}{p_1} - (1 - \tilde{z}) \frac{dp_2/dn}{p_2}$$

$$= -\frac{(1 - \tilde{z})\Gamma}{n(n-1)[\tilde{z}n + (1 - \tilde{z})(n-1)\theta]},$$

where  $\Gamma$  is defined by (8). Consequently, we see that  $dW/dn > 0$  if and only if  $\Gamma < 0$ . ||

**(Figure 1 around here)**

While condition (8) contains three parameters  $\theta$ ,  $\tilde{z}$  and  $n$  and somewhat complicated, it is arguably most helpful to look at it as a condition on  $\theta$ . Figure 1 illustrates condition (8) as a condition on  $\theta$ . The left-hand side of (8) is a quadratic function of  $\theta$ , and is negative if  $\theta = 0$  and  $\theta = 1$ . This implies that the competition policy above necessarily improves welfare if the production technology exhibits non-increasing returns to scale ( $\theta \leq 1$ ). However, exactly the same policy can be welfare-reducing if  $\theta$  is larger than  $\tilde{\theta}$ .<sup>8</sup> That is, the competition policy at the intensive margin is welfare-reducing if the production technology of oligopolistic firms is under sufficiently strong economies of scale.

These findings are intuitively interpreted as follows. As noted earlier, when  $n$  increases, the equilibrium wage and the price of competitive goods rise, which has a negative effect on welfare. That is, the competition policy for oligopolistic industries serves as an ‘anti-competitive’ policy for perfectly competitive industries. However, if the oligopolistic firms’ production technology is non-increasing returns ( $\theta \leq 1$ ), the competition policy above leads to a decrease in the price of oligopolized goods, and positively affects welfare. As a result, the overall effect is determined by the interaction of these conflicting effects. The above proposition states that the competition policy at the intensive margin improves welfare if  $\theta \leq 1$  since the positive effect

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<sup>8</sup> $\tilde{\theta}$  is a solution to the quadratic equation that  $\Gamma = 0$ , and derived as

$$\tilde{\theta} = \frac{(n-1)^2 - \tilde{z}n(2n-3) + \sqrt{D}}{2(1-\tilde{z})(n-1)^2},$$

where  $D \equiv [\tilde{z}n(2n-3) - (n-1)^2]^2 + 4\tilde{z}(1-\tilde{z})n(n-1)^3 > 0$ .

dominates the negative effect under  $\theta \leq 1$ . That is, unless  $\theta$  is large enough, the pro-competitive effect on the price of oligopolized goods dominates the anti-competitive effect on the price of competitive goods.

However, the same no longer holds for the case with  $\theta > 1$ . In particular, if the degree of increasing returns is sufficiently high, the competition policy ends up raising the price of all goods, thereby reducing welfare.

Having addressed the competition policy at the intensive margin, we turn to the effect on an increase in  $\tilde{z}$ . Its effect on welfare is formally stated in:

**Proposition 2.** *The welfare effect of the competition policy at the extensive margin is generally ambiguous. However, if the labor endowment is sufficiently large, the welfare effect is positive under  $\theta \rightarrow 0$  and negative under  $\theta \rightarrow n/(n-1)$ .*

*Proof.* Differentiating (9) with respect to  $\tilde{z}$ , a small increase in  $\tilde{z}$  affects welfare as follows.

$$\begin{aligned} \frac{dW}{d\tilde{z}} &= \ln\left(\frac{p_2}{p_1}\right) - \tilde{z}\frac{dp_1/d\tilde{z}}{p_1} - (1-\tilde{z})\frac{dp_2/d\tilde{z}}{p_2} \\ &= \ln\left\{\left[\frac{n}{(n-1)\theta}\right]^\theta \left[\frac{L}{\tilde{z}n + (1-\tilde{z})(n-1)\theta}\right]^{1-\theta}\right\} - \frac{[n - (n-1)\theta][\tilde{z} + (1-\tilde{z})\theta]}{\tilde{z}n + (1-\tilde{z})(n-1)\theta}. \end{aligned}$$

While it is analytically impossible to determine the sign of the right-hand side, its limits are obtained as follows.

$$\begin{aligned} \text{when } \theta \rightarrow 0 & : \frac{dW}{d\tilde{z}} \rightarrow \ln\left(\frac{L}{\tilde{z}n}\right) - 1 > 0 \\ \text{when } \theta \rightarrow \frac{n}{n-1} & : \frac{dW}{d\tilde{z}} \rightarrow \ln\left[\left(\frac{n}{L}\right)^{\frac{1}{n-1}}\right] < 0. \end{aligned}$$

These inequalities allow us to establish the proposition. ||

The ambiguous result above is not surprising by invoking that total effect is decomposed into the following two sub-effects. On the one hand, when the

range of perfectly competitive industries expands and  $\theta$  is close to zero, the consumer can consume more of cheaper goods than before and hence benefits from this policy. But, note that this effect serves to worsen the consumer utility if  $\theta$  is sufficiently large. On the other hand, as shown in Lemma 1 and Eq. (7), this policy has a detrimental effect by raising the price of all goods. All we can say is that the present competition policy reduces welfare if  $\theta$  is close to  $n/(n - 1)$  because both of the two sub-effects above have negative impacts.

**(Figure 2 around here)**

However, we can obtain a little more via a numerical example in Figure 2 that shows the relationship between the welfare effects of competition policy at extensive margin and returns to scale in cases of  $\tilde{z} = 0.8$ ,  $\tilde{z} = 0.5$  and  $\tilde{z} = 0.2$ , respectively.<sup>9</sup> It illustrates that the present type of competition policy raises welfare if  $\theta$  is small enough. The intuition behind this observation is explained as follows. If  $\theta$  is sufficiently small, namely, the degree of decreasing returns is sufficiently strong, both marginal cost and price of oligopolized goods also becomes very high. Therefore, by removing a portion of such inefficient oligopoly industries, the consumer can consume more of low-price goods (perfectly competitive goods) than before, and welfare improves. Conversely, as  $\theta$  is larger, the negative welfare effect through the increase in wage rate and goods prices becomes dominant, and thereby reduces welfare.

## 4 Concluding Remarks

Applying a GOLE model, we have explored the welfare effects of two competition policies. The first is a competition policy at the intensive margin, which is modeled by an exogenous increase in the number of oligopolistic firms. Whether this policy improves welfare highly depends on the degree of

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<sup>9</sup>The parameters are set as  $n = 4$  and  $L = 10$ .

returns to scale of the oligopolistic firms. More concretely, if the production technology is non-increasing returns, the policy raises welfare, but the opposite holds if the production technology exhibits sufficiently strong economies of scale. The second policy is a competition at the extensive margin, i.e. an expansion of the perfectly competitive industries. We have shown through a numerical calculation that this policy enhances welfare if the oligopolists' production technology is subject to sufficiently strong decreasing returns. To summarize, the production technology of the oligopolistic industries plays a key role in the welfare effect of two competition policies.

This paper has hopefully contributed to literature in the sense that the above results are not obtained in the previous studies, but they admittedly rest on a number of restrictive assumptions. First, we have specified relevant functional forms such as a utility function and a production function so as to simplify analysis. Second, our results are purely theoretical, and empirical re-evaluation is required to qualify the welfare effects of competition policies. These are left as future research agenda.

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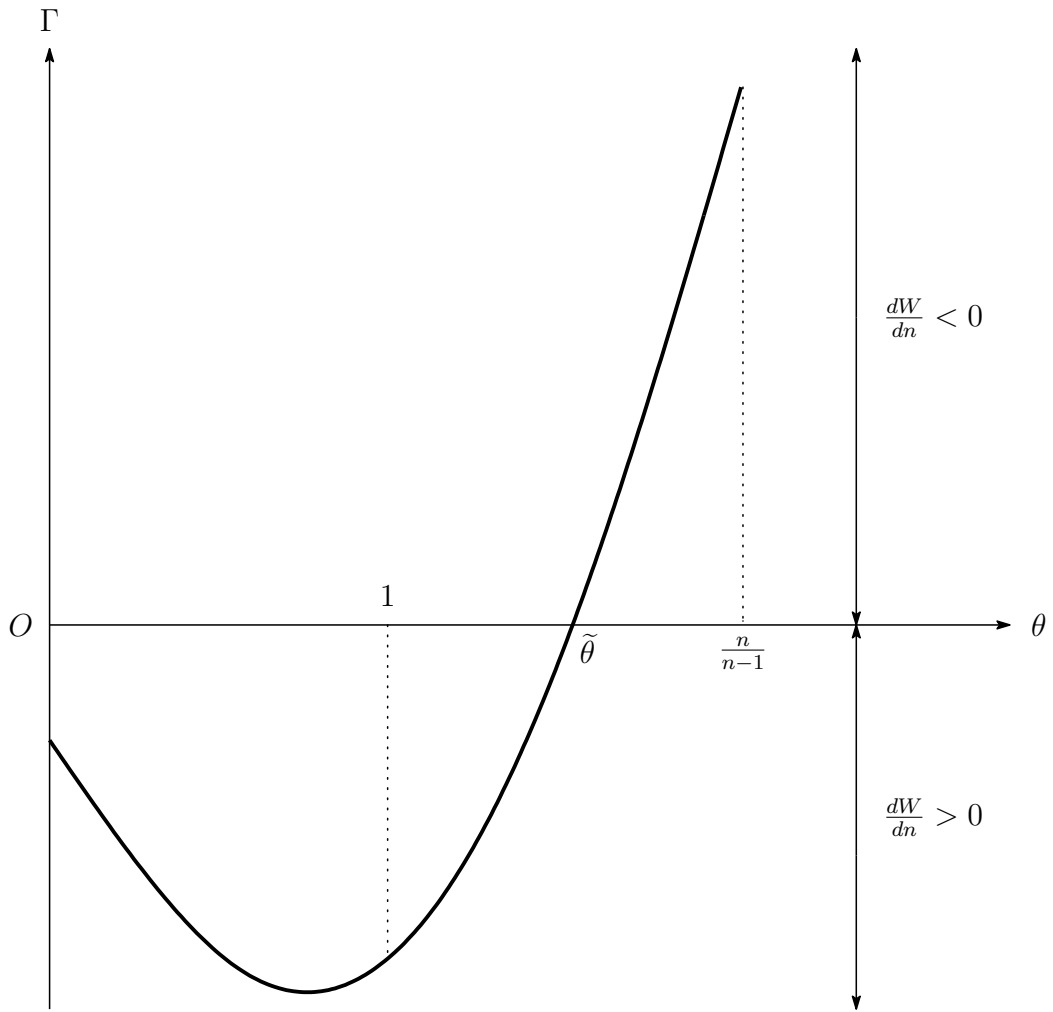


Figure 1: Condition (8)

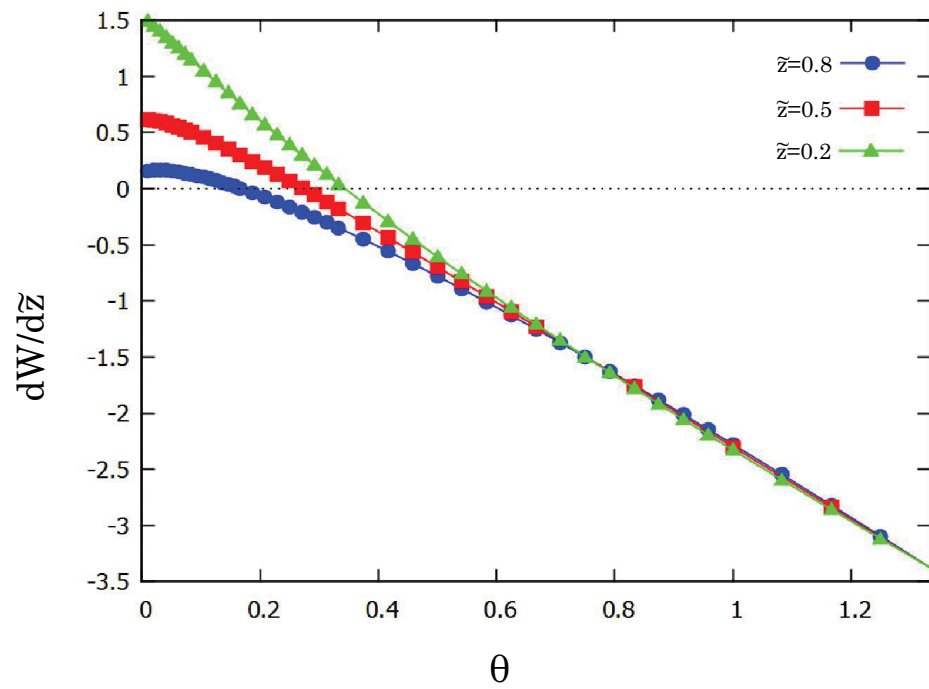


Figure 2: Welfare effects of competition policy at the extensive margins and returns to scale