KIER DISCUSSION PAPER SERIES

KYOTO INSTITUTE
OF
ECONOMIC RESEARCH

Discussion Paper No.903

"Overconfidence, Underconfidence, and Welfare"

Takao Asano, Takuma Kunieda, and Akihisa Shibata

September 2014

KYOTO UNIVERSITY

KYOTO, JAPAN

Overconfidence, Underconfidence, and Welfare*

Takao Asano†, Takuma Kunieda‡ and Akihisa Shibata§

September 16, 2014

Abstract

Using a simple framework of Cooper and John (1988) and Cooper (1999), this paper derives the conditions under which overconfidence and underconfidence of agents lead to Pareto improvement. We show that an agent's overconfidence in a game exhibiting strategic complementarity and positive spillovers and an agent's underconfidence in a game exhibiting strategic complementarity and negative spillovers can lead to Pareto improvement.

JEL classification: D62, C72
Keywords: overconfidence, underconfidence, strategic complementarity, strategic substitutability, positive spillover, negative spillover

---

*We are grateful to two anonymous referees for their detailed comments and suggestions. This research is financially supported by Grant-in-Aid for Specially Promoted Research (No. 23000001), Grant-in-Aid for Scientific Research (No. 23330060) and the Joint Research Program of KIER.

†Faculty of Economics, Okayama University, 3-1-1, Tsushimanaka, Kita-ku, Okayama 700-8530, Japan. E-mail: asano@e.okayama-u.ac.jp

‡P7318, Academic Building, City University of Hong Kong, 83 Tat Chee Avenue, Kowloon Tong, Hong Kong, Phone: +852 3442-7960, Fax: +852 3442-0195, E-mail: tkunieda@cityu.edu.hk

§Institute of Economic Research, Kyoto University, Yoshida, Sakyo-ku, Kyoto 606-8501, Japan, Email: shibata@kier.kyoto-u.ac.jp

1 Introduction

Recently, many researchers have emphasized that the misperception of agents about themselves such as overconfidence or underconfidence about their own abilities may have non-negligible effects on economic outcomes.[1] For example, Chu (2007), Compte and Postlewaite (2004), Dubra (2004), Gervais and Goldstein (2007), and Weinberg (2009) analyze the role of overconfidence in economic activities. Among them, Chu (2007) and Gervais and Goldstein (2007) show that the overconfidence of agents can have positive effects on the aggregate economy. In Chu's (2007) model, if a decentralized equilibrium exhibits underinvestment in research and development (R&D) activity owing to the activity's strong positive externalities, a small degree of entrepreneurial overconfidence about the probability of R&D success corrects this inefficiency and improves social welfare by stimulating R&D investment and thereby promoting economic growth, although a large degree of overconfidence decreases social welfare by causing overinvestment in R&D activity. Gervais and Goldstein (2007) show that in a situation where there is a free-riding problem, the agents who overestimate their skills work harder and mitigate the free-riding problem. More precisely, when the complementarities among agents are sufficiently strong, the overconfidence of some agents induces all of them to work harder and thereby brings about Pareto improvement in the economy.

When strategic interactions occur in production activities, under what conditions do overconfidence and underconfidence increase the welfare of agents? This paper derives the conditions under which the misperception of agents about their own abilities leads to Pareto improvement by using a

[1]Many experimental studies have shown that overconfidence is a widely observed phenomenon. A recent study by Benoît and Dubra (2011), however, points out that we should be cautious in interpreting these empirical findings. They show that, even under rational learning, most people situate themselves above the median. Nevertheless, investigating Svenson's (1981) study, Benoît and Dubra (2011) conclude that American data exhibit the presence of overconfidence.

simple framework of Cooper and John (1988) and Cooper (1999).

2 The model

We employ a simplified version of Cooper and John's (1988) model. The economy consists of two agents engaged in a certain economic activity. The individual payoff of each agent is assumed to be related to the other agent's activity and its own ability. Agent i selects its action $e_i \in [0, 1]$ so as to maximize its payoff. The payoff of agent i is given by

$$\sigma^i(e_i, e_j, \theta_i) \quad (i \neq j),$$

where e_j is the other agent's action and θ_i agent i's ability. Furthermore, σ^i is twice continuously differentiable and $\sigma^i_{11} < 0$. We assume that the marginal payoff, σ^i_1, is increasing with respect to θ_i; that is, $\sigma^i_{13} > 0$. Let the true value of θ_i be $\theta_i = \bar{\theta}$ for $i = 1, 2$. By restricting our attention to interior symmetric Nash equilibria, we can easily derive agent i's best response as follows:

$$e_i = \underset{x \in [0,1]}{\arg\max}\ \sigma^i(x, e_j, \bar{\theta}) \quad i \neq j,$$

indicating that e_i satisfies

$$\sigma^i_1(e_i, e_j, \bar{\theta}) = 0. \tag{1}$$

In a symmetric Nash equilibrium, we have

$$\sigma^i_1(e, e, \bar{\theta}) = 0.$$

2.1 Strategic complementarity and spillover

If in a model an increase (decrease) in one agent's action leads to a corresponding increase (decrease) in the other agent's action, that is, if the reaction curve has a positive (negative) slope, we say that the model exhibits strategic complementarity (substitutability). Totally differentiating (1), we have

$$\frac{de_i}{de_j} = -\frac{\sigma^i_{12}}{\sigma^i_{11}} =: \nabla^i.$$

Hence, if $\sigma^i_{12} > 0$ ($\sigma^i_{12} < 0$), the sign of ∇^i is positive (negative); that is, the model exhibits strategic complementarity (substitutability).

Now, consider the case of strategic complementarity; that is, the case of $\sigma^i_{12} > 0$ and $\nabla^i > 0$. In this case, the reaction curve slopes upward, indicating the possibility of multiple equilibria. Typical situations with multiple equilibria are depicted in Figure 1. Let us here introduce the notion of *spillover* as proposed by Cooper and John (1988). If an increase in one agent's action raises (lowers) the other agent's payoff, that is, if $\sigma^i_j > 0$ ($\sigma^i_j < 0$) for $i \neq j$, we say that the model exhibits positive (negative) spillovers. As Cooper and John (1988) show, when a game has multiple symmetric Nash equilibria and exhibits positive spillovers, the equilibria are Pareto-ranked and the Nash equilibrium with high agent activity Pareto-dominates the Nash equilibrium with lower agent activity. In Figure 1, three Nash equilibria are Pareto-ranked: equilibrium A gives the highest payoffs, equilibrium B gives the median payoffs and equilibrium C gives the lowest payoffs.

For examples of strategic complementarities and positive spillovers, see Brunello and Ishikawa (1999) and Redding (1996). The models developed in these studies consist of individuals who invest in education and firms who choose either high technology or low technology. If the individuals attain higher levels of education, the firms employ high technology, and if the firms adopt high technology, the individuals invest in higher education. Thus, multiple Pareto-ranked equilibria emerge. A typical example of strategic complementarities and negative spillovers is the Bertrand competition model with differentiated goods, as is pointed out by Froot et al. (1992) in their footnote 7. In this model, if a firm lowers its price, other firms follow and they too lower their prices (strategic complementarities); thus, a firm's more proactive pricing behavior reduces the profit of other firms (negative spillovers). Moreover, De Bondt and Veugelers (1991) develop a duopoly investment model with a positive or negative spillover effect, and show that the model can strategically be either a complement or substitute, depending on the parameter value representing the degree of the spillover effect. This model can yield cases of both strategic complementarity with positive spillovers and strategic substitutability with negative spillovers.

For simplicity, we restrict our attention to the case of a stable, unique Nash equilibrium in this paper. We can easily show that in the case of strategic complementarity, the condition for a unique Nash equilibrium to be stable is $0 < \nabla^i < 1$ for $i = 1, 2$ in equilibrium, from which we obtain $1/\nabla^1 > \nabla^2$, or equivalently,

$$\sigma^1_{11}\sigma^2_{11} - \sigma^1_{12}\sigma^2_{12} > 0$$

in equilibrium.

[Figure 1 around here.]

3 Overconfidence and welfare

3.1 Strategic complementarity and overconfidence

Let us now introduce the overconfidence of agents about their own abilities. Although the true ability of both agents remains $\bar{\theta}$, agent 1 is overconfident about its ability; that is, the agent believes that its ability is greater than its true ability by an amount of $d\tilde{\theta}_1$:

$$\theta_1 = \bar{\theta} + d\tilde{\theta}_1. \tag{2}$$

Meanwhile, agent 2 has a correct understanding of its true ability. As can be easily verified from Eq. (1), the effect of agent 1's overconfidence is:

$$\left.\frac{de_1}{d\tilde{\theta}_1}\right|_{\theta_1=\bar{\theta}} = \frac{-\sigma^2_{11}\sigma^1_{13}}{\sigma^1_{11}\sigma^2_{11} - \sigma^1_{12}\sigma^2_{12}}, \tag{3}$$

$$\left.\frac{de_2}{d\tilde{\theta}_1}\right|_{\theta_1=\bar{\theta}} = \frac{\sigma^2_{12}\sigma^1_{13}}{\sigma^1_{11}\sigma^2_{11} - \sigma^1_{12}\sigma^2_{12}}. \tag{4}$$

Thus, the impact on both agent's actions depends on the signs of σ^2_{12}, σ^1_{13}, and $\sigma^1_{11}\sigma^2_{11} - \sigma^1_{12}\sigma^2_{12}$.

Since we assume that $\sigma^i_{12} > 0$, $\sigma^i_{11} < 0$, and $\sigma^1_{13} > 0$ and consider the case of a stable, unique Nash equilibrium ($\sigma^1_{11}\sigma^2_{11} - \sigma^1_{12}\sigma^2_{12} > 0$), the right-hand sides of Eqs. (3) and (4) are positive, implying that the overconfidence of agent 1 stimulates both the agents to act and shifts agent 1's own reaction curve upward. This situation is depicted in Figure 2.[2] Of course, our outcome follows directly from the strategic complementarity between the two agents. This can be confirmed from

$$\left.\frac{de_2}{d\tilde{\theta}_1}\right|_{\theta_1=\bar{\theta}} = \nabla^2 \left.\frac{de_1}{d\tilde{\theta}_1}\right|_{\theta_1=\bar{\theta}} > 0.$$

[Figure 2 around here.]

Now, we consider the first-order welfare effect on each agent. First, we obtain the following proposition.

Proposition 1 *Suppose that there exist strategic complementarity and a positive (negative) spillover. Then, the overconfidence of agent 1 increases (decreases) agent 2's welfare in a symmetric Nash equilibrium.*

[2]This situation corresponds to that analyzed by Gervais and Goldstein (2007), who show that the overconfidence of some agents makes all of them work harder, leading to Pareto improvement.

Proof. In order to analyze the first-order effect on agent 2's payoff in the Nash equilibrium, we totally differentiate agent 2's payoff to obtain

$$\frac{d}{d\tilde{\theta}_1}\sigma^2(e_2, e_1, \bar{\theta}) = \sigma_1^2(e_2, e_1, \bar{\theta})\frac{de_2}{d\tilde{\theta}_1} + \sigma_2^2(e_2, e_1, \bar{\theta})\frac{de_1}{d\tilde{\theta}_1}.$$

Since $\sigma_1^2(e, e, \bar{\theta}) = 0$, it follows that in a symmetric Nash equilibrium,

$$\frac{d}{d\tilde{\theta}_1}\sigma^2(e, e, \bar{\theta}) = \sigma_2^2(e, e, \bar{\theta})\frac{de_1}{d\tilde{\theta}_1},$$

which depends on the sign of σ_2^2 because $de_1/d\tilde{\theta}_1 > 0$ from strategic complementarity and the presence of agent 1's overconfidence. If the game exhibits a positive spillover, that is, if σ_2^2 is positive, the sign is positive. Thus, the welfare effect on agent 2 is positive. On the other hand, if the game exhibits a negative spillover, that is, if σ_2^2 is negative, the sign is negative. Hence, the welfare effect on agent 2 is negative. This proves the proposition. ■

Next, we consider the first-order welfare effect on agent 1 and obtain the following proposition.

Proposition 2 *Suppose that there exist strategic complementarity and a positive (negative) spillover. Then, the overconfidence of agent 1 increases (decreases) agent 1's own welfare in a symmetric Nash equilibrium.*

Proof. In order to analyze the first-order effect on agent 1's payoff in the Nash equilibrium, we totally differentiate agent 1's payoff to obtain

$$\frac{d}{d\tilde{\theta}_1}\sigma^1(e_1, e_2, \bar{\theta}) = \sigma_1^1(e_1, e_2, \bar{\theta})\frac{de_1}{d\tilde{\theta}_1} + \sigma_2^1(e_1, e_2, \bar{\theta})\frac{de_2}{d\tilde{\theta}_1} + \sigma_3^1(e_1, e_2, \bar{\theta})\frac{d\bar{\theta}}{d\tilde{\theta}_1}.$$

Since $\sigma_1^1(e, e, \bar{\theta}) = 0$, it follows that in a symmetric Nash equilibrium,

$$\frac{d}{d\tilde{\theta}_1}\sigma^1(e, e, \bar{\theta}) = \sigma_2^1(e, e, \bar{\theta})\frac{de_2}{d\tilde{\theta}_1} + \sigma_3^1(e, e, \bar{\theta})\frac{d\bar{\theta}}{d\tilde{\theta}_1}. \quad (5)$$

Since the true ability of agent 1 remains unchanged, it follows that

$$\frac{d\bar{\theta}}{d\tilde{\theta}_1} = 0,$$

and therefore

$$\frac{d}{d\tilde{\theta}_1}\sigma^1(e,e,\bar{\theta}) = \sigma_2^1(e,e,\bar{\theta})\frac{de_2}{d\tilde{\theta}_1}.$$

The sign of this depends on the sign of σ_2^1 because $de_2/d\tilde{\theta}_1 > 0$ from strategic complementarity and the presence of agent 1's overconfidence. If the game exhibits a positive spillover, that is, if σ_2^1 is positive, the sign is positive. Thus, the welfare effect on agent 1 is positive. On the other hand, in the case of negative spillover, that is, if σ_2^1 is negative, the sign is negative. Hence, the welfare effect on agent 1's payoff is negative. This proves the proposition. ■

Propositions 1 and 2 indicate that under strategic complementarity and positive spillover, one agent's overconfidence improves the welfare of both agents. Thus, we have the following proposition.

Proposition 3 *Suppose that there exist strategic complementarity and a positive spillover. Then, the overconfidence of an agent Pareto-improves welfare.*

Proposition 3 clarifies the basic mechanism behind Gervais and Goldstein's (2007) result and confirms that their analysis can be applied to a broader class of games with strategic complementarities and positive spillovers, including the models of Brunello and Ishikawa (1999) and Redding (1996). The intuition behind Proposition 3 is simple. Suppose that I consider myself a high type. Now, I will raise the level of my action, which will in turn increase the other player's welfare through positive externality and stimulate the other agent's action through strategic complementarity. This increase in the other agent's action will have a positive externality effect on me and stimulate my action further through strategic complementarity, which will further increase the other agent's welfare and action, and so on.

3.1.1 Discrete changes in beliefs

Thus far, we restricted our analysis to a marginal introduction of overconfidence. We now discuss the case of discrete changes in beliefs.

From Eq. (5), our result depends on the fact that the slope of an agent's payoff function with respect to its action is zero at optimal action, that is, $\sigma_1^i = 0$ at the correct belief, and therefore marginal deviations from the optimal action can cause only second-order losses (the envelope theorem), a point emphasized by Akerlof and Yellen (1985), Mankiw (1985), and Parkin (1986). However, since discrete changes in beliefs can lead to corresponding large deviations from the optimal point, we can not apply the envelope theorem to Eq. (5) because it would lead to first-order welfare losses. In the current setting, an agent's strong overconfidence leads to a large increase in its action beyond the optimal level, and thus $\sigma_1^i < 0$. Then, we have

$$\frac{d}{d\tilde{\theta}_1}\sigma^1 = \underset{(-)}{\sigma_1^1}\underset{(+)}{\frac{de_1}{d\tilde{\theta}_1}} + \sigma_2^1\frac{de_2}{d\tilde{\theta}_1}. \tag{6}$$

From this equation, it is clear that even under strategic complementarity and positive spillovers, the sign of welfare effect cannot be identified because the first term on the right-hand side of Eq. (6) is negative whereas the second term is positive; in the case of discrete changes in beliefs, a trade-off exists between the welfare gain through positive spillovers and welfare loss through excessive action.

3.1.2 Simultaneous overconfidence

We next examine the case in which agent 2 becomes overconfident simultaneously with agent 1. In this case, the results are essentially the same as before. While overconfidence shifts the reaction curve of agent 2 rightward, it moves the reaction curve of agent 1 in the upward direction, as depicted in Figure 3. Thus, the activities of both agents are stimulated to a greater magnitude (see point C in Figure 3) compared to the case in which only agent 1 becomes overconfident (point B). This case corresponds to the situation analyzed by Chu (2007), who shows that when all entrepreneurs become overconfident to a small degree, R&D investment is stimulated and social welfare is enhanced.[3]

3.1.3 The case of underconfidence

Suppose that agent 1 is underconfident about its own ability. Underconfidence can be easily introduced into the model by replacing $d\tilde{\theta}_1$ with $-d\tilde{\theta}_1$ in Eq.(2) as follows:

$$\theta_1 = \bar{\theta} - d\tilde{\theta}_1. \tag{7}$$

From Eqs. (1) and (7), we obtain the following inequality:

$$\begin{aligned} \left.\frac{de_1}{d\tilde{\theta}_1}\right|_{\theta_1=\bar{\theta}} &= \frac{\sigma^2_{11}\sigma^1_{13}}{\sigma^1_{11}\sigma^2_{11} - \sigma^1_{12}\sigma^2_{12}} < 0, \\ \left.\frac{de_2}{d\tilde{\theta}_1}\right|_{\theta_1=\bar{\theta}} &= \frac{-\sigma^2_{12}\sigma^1_{13}}{\sigma^1_{11}\sigma^2_{11} - \sigma^1_{12}\sigma^2_{12}} < 0. \end{aligned}$$

The analysis is straightforward because it is almost the same as in the case of overconfidence. The situation is depicted in Figure 2. As the figure shows, both e_1 and e_2 decrease as a result of agent 1's underconfidence. For the first-order welfare effect on agent 2, we obtain the following proposition.

Proposition 4 *Suppose that there exist strategic complementarity and a positive (negative) spillover. Then, the underconfidence of agent 1 decreases (increases) agent 2's welfare in a symmetric Nash equilibrium.*

[3] As mentioned in the Introduction, the effect with respect to social welfare in Chu's (2007) model depends on the degree of overconfidence. While a small degree of overconfidence increases social welfare, a large degree of overconfidence decreases social welfare. This result consistently corresponds to our above argument about discrete changes in beliefs.

Proof. The proof of this is similar to that of Proposition 1. The first-order welfare effects in the Nash equilibrium can be obtained as follows:

$$\frac{d}{d\tilde{\theta}_1}\sigma^2(e,e,\bar{\theta}) = \sigma^2_2(e,e,\bar{\theta})\frac{de_1}{d\tilde{\theta}_1},$$

which depends on the sign of σ^2_2 because $de_1/d\tilde{\theta}_1 < 0$ from strategic complementarity and the presence of agent 1's underconfidence. If the game exhibits a positive spillover, that is, if σ^2_2 is positive, the sign is negative. Thus, the welfare effect on agent 2 is negative. On the other hand, in the case of negative spillover, that is, if σ^2_2 is negative, the sign is positive. Hence, the welfare effect on agent 2's payoff is positive. This proves the proposition. ■

For the first-order welfare effect on agent 1, we obtain the following proposition.

Proposition 5 *Suppose that there exist strategic complementarity and a positive (negative) spillover. Then, the underconfidence of agent 1 decreases (increases) the agent's own welfare in a symmetric Nash equilibrium.*

Proof. The proof of this proposition is similar to that of Proposition 2. The first-order welfare effects in the Nash equilibrium can be obtained as follows:

$$\frac{d}{d\tilde{\theta}_1}\sigma^1(e,e,\bar{\theta}) = \sigma^1_2(e,e,\bar{\theta})\frac{de_2}{d\tilde{\theta}_1},$$

which depends on the sign of σ^1_2 because $de_2/d\tilde{\theta}_1 < 0$ from strategic complementarity and the presence of agent 1's underconfidence. If the game exhibits a positive spillover, that is, if σ^2_2 is positive, the sign is negative. Thus, the welfare effect on agent 1 is negative. On the other hand, in the case of negative spillover, that is, if σ^2_2 is negative, the sign is positive. Hence, the welfare effect on agent 1's payoff is positive. This proves the proposition. ■

By combining Propositions 4 and 5, we directly obtain the following proposition.

Proposition 6 *Suppose that there exist strategic complementarity and a negative spillover. Then, the underconfidence of an agent Pareto-improves welfare.*

This result can be directly applied to a Bertrand competition model with differentiated products or a case of De Bondt and Veugelers' (1991) model because these models exhibit strategic complementarity and negative spillovers. The intuition behind Proposition 6 is also simple: Suppose that I consider myself a low type. Now, I will decrease my action and thereby raise the other player's welfare through a negative spillover and dampen the other agent's action through strategic complementarity. This decrease of the other agent's action raises my welfare through negative spillover and lowers my action through strategic complementarity, and so on.

3.2 The case of strategic substitutability

Thus far, we investigated the case of strategic complementarity. In this section, we consider the case of strategic substitutability. In the case of strategic substitutability, the reaction curve slopes downward because $\sigma_{12}^i < 0$:

$$\frac{de_i}{de_j} = -\frac{\sigma_{12}^i}{\sigma_{11}^i} =: \nabla^i < 0.$$

As in the case of strategic complementarity, we focus on a Nash equilibrium that is unique and stable. The condition for a unique Nash equilibrium to be stable is given by $-1 < \nabla^i < 0$ in equilibrium. From this, it follows that $1/\nabla^1 < \nabla^2$, or equivalently,

$$\sigma_{11}^1 \sigma_{11}^2 - \sigma_{12}^1 \sigma_{12}^2 > 0.$$

Note that this inequality is the same as that in the case of strategic complementarity.

3.2.1 The case of overconfidence

The welfare analysis in this case is almost the same as that for overconfidence under strategic complementarity except that $\sigma^i_{12} < 0$. From Eqs. (3) and (4), we have

$$\left.\frac{de_1}{d\tilde{\theta}_1}\right|_{\theta_1=\bar{\theta}} = \frac{-\sigma^2_{11}\sigma^1_{13}}{\sigma^1_{11}\sigma^2_{11} - \sigma^1_{12}\sigma^2_{12}} > 0,$$
$$\left.\frac{de_2}{d\tilde{\theta}_1}\right|_{\theta_1=\bar{\theta}} = \frac{\sigma^2_{12}\sigma^1_{13}}{\sigma^1_{11}\sigma^2_{11} - \sigma^1_{12}\sigma^2_{12}} < 0.$$

Now, for the first-order welfare effect on agent 2, we obtain the following proposition.

Proposition 7 *Suppose that there exist strategic substitutability and a positive (negative) spillover. Then, the overconfidence of agent 1 increases (decreases) agent 2's welfare in a symmetric Nash equilibrium.*

Proof. The proof of this proposition is similar to that of Proposition 1. The first-order welfare effects in the Nash equilibrium can be obtained as follows:

$$\frac{d}{d\tilde{\theta}_1}\sigma^2(e, e, \bar{\theta}) = \sigma^2_2(e, e, \bar{\theta})\frac{de_1}{d\tilde{\theta}_1},$$

which depends on the sign of σ^2_2 because $de_1/d\tilde{\theta}_1 > 0$ from strategic substitutability and the presence of agent 1's overconfidence. If the game exhibits a positive spillover, that is, if σ^2_2 is positive, the sign is positive. Thus, the welfare effect on agent 2 is positive. On the other hand, if the game exhibits a negative spillover, that is, if σ^2_2 is negative, the sign is negative. Hence, the welfare effect on agent 2 is negative. This proves the proposition. ■

On the other hand, for the first-order welfare effect on agent 1, we obtain the following proposition.

Proposition 8 *Suppose that there exist strategic substitutability and a positive (negative) spillover. Then, the overconfidence of agent 1ecreases (increases) agent 1's own welfare in a symmetric Nash equilibrium.*

Proof. The proof of this proposition is similar to that of Proposition 2. The first-order welfare effects in a symmetric Nash equilibrium can be obtained as follows:

$$\frac{d}{d\tilde{\theta_1}}\sigma^1(e,e,\bar{\theta}) = \sigma_2^1(e,e,\bar{\theta})\frac{de_2}{d\tilde{\theta_1}}.$$

The sign of this depends on the sign of σ_2^1 because $de_2/d\tilde{\theta_1} < 0$ from strategic substitutability and the presence of agent 1's overconfidence. If the game exhibits a positive spillover, that is, if σ_2^1 is positive, the sign is negative. Thus, the welfare effect on agent 1 is negative. On the other hand, in the case of negative spillover, that is, if σ_2^1 is negative, the sign is positive. Hence, the welfare effect on agent 1's payoff is positive. This proves the proposition. ■

Propositions 7 and 8 indicate that in the case of positive spillovers, the first-order welfare effect of agent 1's overconfidence contributes to agent 2's welfare improvement but reduces agent 1's own welfare on account of strategic substitutability.

3.2.2 The case of underconfidence

Recall that in contrast to the case of strategic complementarity, we have $\sigma_{12}^i < 0$ for the case of strategic substitutability. Other than this, the analysis is the same as that for underconfidence under strategic complementarity. From Eqs. (1) and (7), we obtain

$$\left.\frac{de_1}{d\tilde{\theta_1}}\right|_{\theta_1=\bar{\theta}} = \frac{\sigma_{11}^2\sigma_{13}^1}{\sigma_{11}^1\sigma_{11}^2 - \sigma_{12}^1\sigma_{12}^2} < 0, \tag{8}$$

$$\left.\frac{de_2}{d\tilde{\theta_1}}\right|_{\theta_1=\bar{\theta}} = \frac{-\sigma_{12}^2\sigma_{13}^1}{\sigma_{11}^1\sigma_{11}^2 - \sigma_{12}^1\sigma_{12}^2} > 0. \tag{9}$$

For the first-order welfare effect on agent 2, we obtain the following proposition.

Proposition 9 *Suppose that there exist strategic substitutability and a positive (negative) spillover. Then, the underconfidence of agent 1 decreases (increases) agent 2's welfare in a symmetric Nash equilibrium.*

Proof. The proof of this proposition is similar to that of Proposition 4. The first-order welfare effects in a Nash equilibrium can be obtained as follows:

$$\frac{d}{d\tilde{\theta}_1}\sigma^2(e, e, \bar{\theta}) = \sigma^2_2(e, e, \bar{\theta})\frac{de_1}{d\tilde{\theta}_1},$$

which depends on the sign of σ^2_2 because $de_1/d\tilde{\theta}_1 < 0$ from strategic substitutability and the presence of agent 1's underconfidence. If the game exhibits a positive spillover, that is, if σ^2_2 is positive, the sign is negative. Thus, the welfare effect on agent 2 is negative. On the other hand, in the case of negative spillover, that is, if σ^2_2 is negative, the sign is positive. Hence, the welfare effect on agent 2's payoff is positive. This proves the proposition. ■

On the other hand, for the first-order welfare effect on agent 1, we obtain the following proposition.

Proposition 10 *Suppose that there exist strategic substitutability and a positive (negative) spillover. Then, the underconfidence of agent 1 increases (decreases) the agent 1's own welfare in a symmetric Nash equilibrium.*

Proof. The proof of this proposition is similar to that of Proposition 5. The first-order welfare effects in a Nash equilibrium can be obtained as follows:

$$\frac{d}{d\tilde{\theta}_1}\sigma^1(e, e, \bar{\theta}) = \sigma^1_2(e, e, \bar{\theta})\frac{de_2}{d\tilde{\theta}_1},$$

which depends on the sign of σ^1_2 because $de_2/d\tilde{\theta}_1 > 0$ from strategic substitutability and the presence of agent 1's underconfidence. If the game exhibits a positive spillover, that is, if σ^2_2 is positive, the sign is positive. Thus, the welfare effect on agent 1 is positive. On the other hand, in the case of negative spillover, that is, if σ^2_2 is negative, the sign is negative. Hence, the welfare effect on agent 1's payoff is negative. This proves the proposition. ■

Propositions 9 and 10 indicate that in the case of positive spillovers, the underconfidence of agent 1 increases the agent's own welfare through strategic substitutability but reduces agent 2's welfare because agent 1 becomes less active.

Since we can very easily analyze cases of negative discrete changes in beliefs and simultaneous underconfidence in exactly the same way as in the previous subsection, we do not repeat the analysis here.

4 Conclusion

In this study, we analyzed the conditions under which overconfidence and underconfidence of agents in a game increase the welfare of agents. The main result of this study is as follows: If the game exhibits strategic complementarity and positive spillovers, the overconfidence of an agent can lead to Pareto improvement; similarly, if the game exhibits strategic complementarity and negative spillovers, the underconfidence of an agent can lead to Pareto improvement. The other results of this study are summarized in Tables 1 and 2.

Table1: The first-order welfare effect under positive spillovers

Agent 1's	Overconfidence	Underconfidence
Complementarity	agent 1 +	agent 1 -
	agent 2 +	agent 2 -
Substitutability	agent 1 -	agent 1 +
	agent 2 +	agent 2 -

Table 2: The first-order welfare effect under negative spillovers

Agent 1's	Overconfidence	Underconfidence
Complementarity	agent 1 -	agent 1 +
	agent 2 -	agent 2 +
Substitutability	agent 1 +	agent 1 -
	agent 2 -	agent 2 +

References

[1] Akerlof, G. A. and J. Yellen (1985): "Can Small Deviations from Rationality Make Significant Differences to Economic Equilibria?" *American Economic Review* Vol.75, 708-721

[2] Benoît, J.-P. and J. Dubra (2011): "Apparent Overconfidence," *Econometrica* Vol. 79, 1591-1625.

[3] Brunello, G. and T. Ishikawa (1999): "Elite Schools, High Tech Jobs and Economic Welfare," *Journal of Public Economics* Vol.72, 395-419.

[4] Chu, Angus C. (2007): "Confidence-Enhanced Economic Growth," *The B. E. Journal of Macroeconomics* (Topics) Vol.7, Article 13. DOI: 10.2202/1935-1690.1495.

[5] Compte, O. and A. Postlewaite (2004): "Confidence-Enhanced Performance," *American Economic Review* Vol. 94, 1536-1557.

[6] Cooper, R. W. (1999): *Coordination Games*, Cambridge University Press.

[7] Cooper, R. W. and A. John (1988): "Coordinating Coordination Failures in Keynesian Models," *Quarterly Journal of Economics* Vol. 103, 441-463.

[8] De Bondt, R. and R. Veugelers (1991): "Strategic Investment with Spillovers," *European Journal of Political Economy* Vol. 7, 345-366.

[9] Dubra, J. (2004): "Optimism and Overconfidence in Search," *Review of Economic Dynamics* Vol. 7, 198-218.

[10] Froot, K. A., D. S. Scharfstein, and J. C. Stein (1992): "Herd on the Street: Informational Inefficiencies in a Market with Short-term Speculation," *The Journal of Finance* Vol. 47, 1461-1484.

[11] Gervais, S. and I. Goldstein (2007): "The Positive Effects of Biased Self-Perceptions in Firms," *Review of Finance* Vol. 11, 453-496.

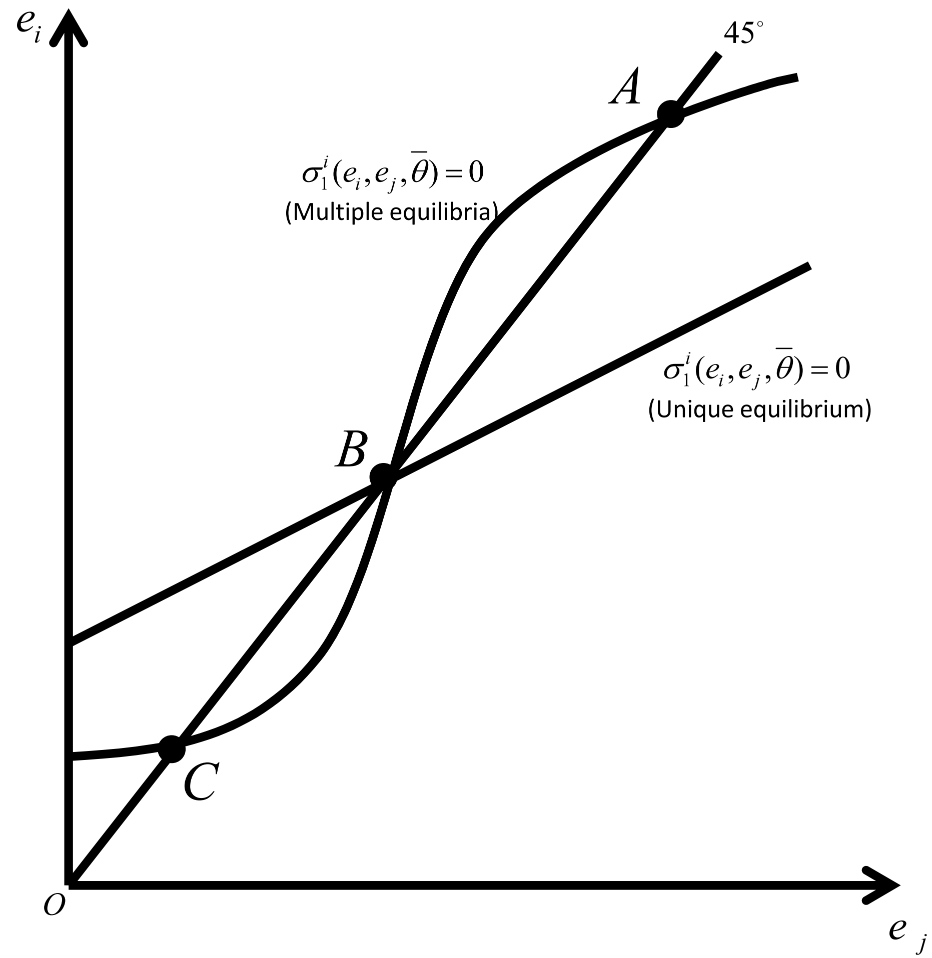
[12] Mankiw, N. G. (1985): "Small Menu Costs and Large Business Cycles: A Macroeconomic Model of Monopoly," *Quarterly Journal of Economics* Vol. 100, 529-37.

[13] Parkin, M. (1986): "The Output-Inflation Tradeoff When Prices Are Costly to Change," *Journal of Political Economy* Vol. 94, 200-224.

[14] Redding, S. (1996): "The Low-skill, Low-quality Trap: Strategic Complementarities between Human Capital and R & D," *The Economic Journal* Vol.106, 458-470.

[15] Svenson, O. (1981): "Are We All Less Risky and More Skillful Than Our Fellow Drives?" *Acta Psychologica* Vol.47, 143-148.

[16] Weinberg, B. (2009): "A Model of Overconfidence," *Pacific Economic Review* Vol. 14, 502-15.



e_i
45°
A
$\sigma_1^i(e_i, e_j, \overline{\theta}) = 0$
(Multiple equilibria)
$\sigma_1^i(e_i, e_j, \overline{\theta}) = 0$
(Unique equilibrium)
B
C
O
e_j

Figure 1

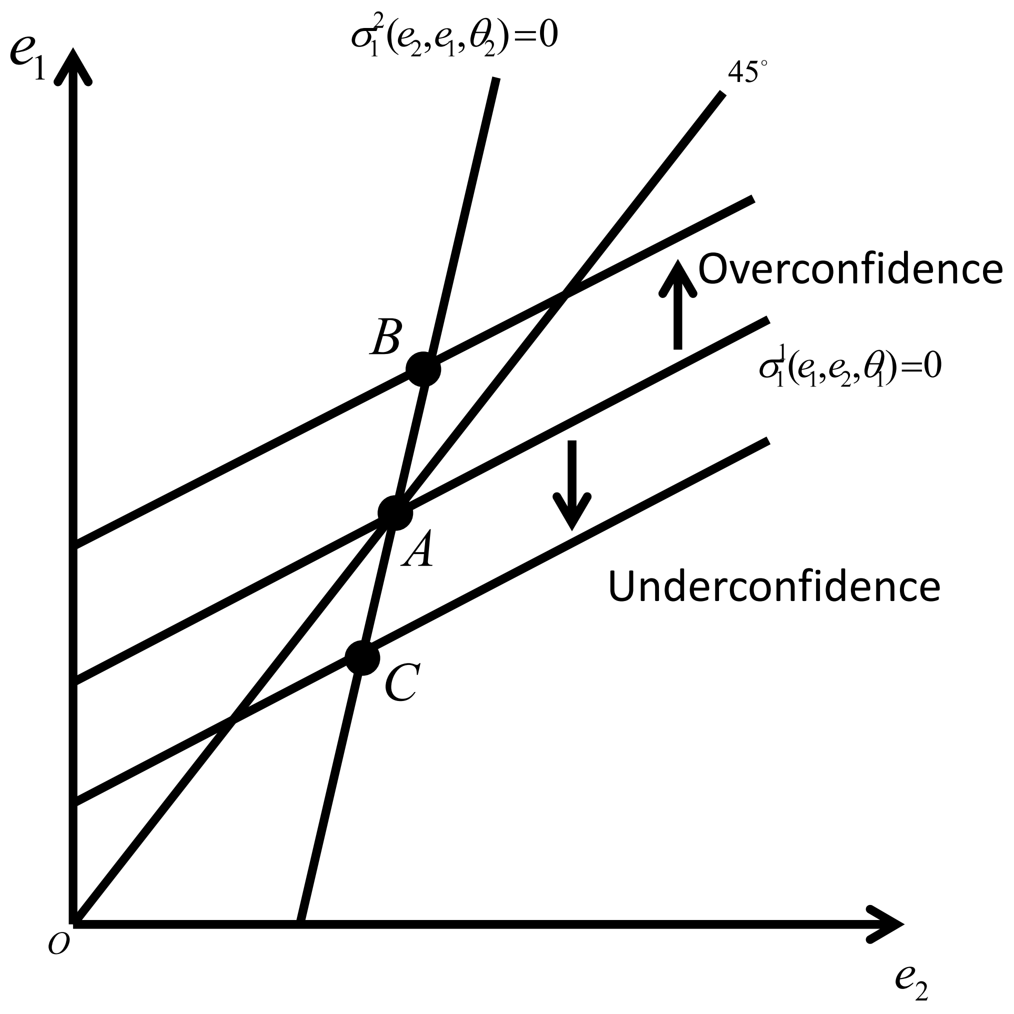
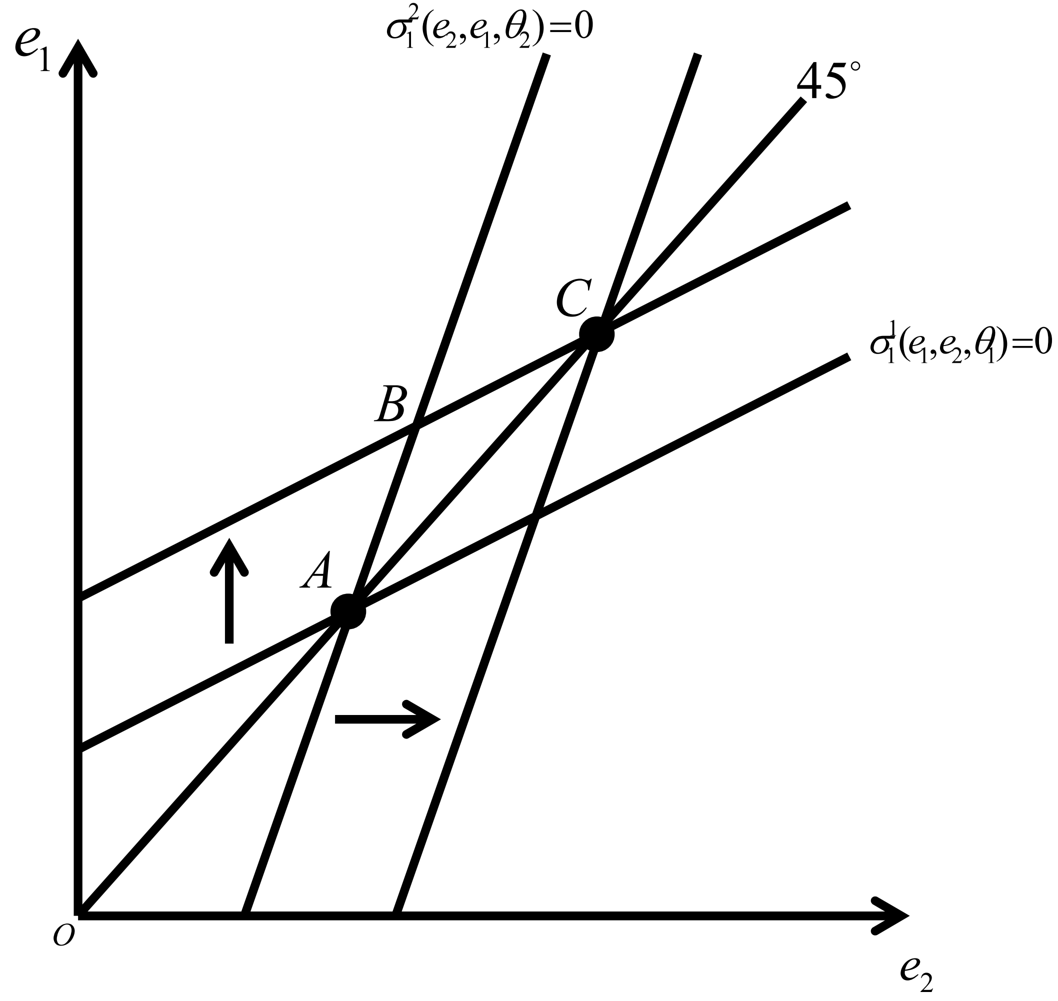

e_1
$\sigma_1^2(e_2,e_1,\theta_2)=0$
45°
Overconfidence
$\sigma_1^1(e_1,e_2,\theta_1)=0$
Underconfidence
B
A
C
o
e_2

Figure 2



e_1
$\sigma_1^2(e_2, e_1, \theta_2) = 0$
45°
C
$\sigma_1^1(e_1, e_2, \theta_1) = 0$
B
A
o
e_2

Figure 3