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"Contracting for Multiple Goods under Asymmetric Information: The Two-goods Case"

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Contracting for Multiple Goods under Asymmetric Information: The Two-goods Case*

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Abstract

This paper investigates how a buyer and a seller exchanging two goods should write the contract, where the seller makes sequences of unobservable relation-specific investments and the buyer privately learns valuations for goods which are stochastically influenced by the investments and these two types of asymmetric information cause inefficiency in trading. Three types of contract structures are possible. In a dynamic contract, the goods are traded sequentially and the order for the second good can be canceled to restore efficiency for the first good. In separate contracts, two goods are treated independently, whereas the two goods are bundled as a single good in bundled contracts. It will be shown that the dynamic contract is suboptimal and that the second-best contract is either a separate or a bundle contract, depending on the costs of investments.

Keywords: bilateral trading, cooperative investment, dynamic contract, hidden action, hidden information.

Journal of Economic Literature Classification Numbers: C72, D23, D82, D86.

1 Introduction

It is efficient for a buyer (him) to procure goods from a seller (her), because the seller usually has a technical advantage over the buyer in production. This efficiency is improved by a specific investment that creates value inside the relationship. Often, the seller's relation-specific

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investment directly benefits the buyer, in which case the investment is called *cooperative*.[1] However, a hold-up problem arises with cooperative investment.[2]

Consider the situation of a buyer ordering a custom-built dining table set (one table and one chair) from a seller. The quality of the set depends on the quality of the materials and the seller's effort. These are components of the seller's cooperative investment, which stochastically improve the buyer's valuation of the dining set. However, it is difficult for the buyer to tell the quality of the materials used to create the dining set or how carefully the seller built it. Thus, the level of the investment is the seller's private information. Also, even if the set looks beautiful, it not might be what the buyer was looking for. Thus, the valuation of the dining set is the buyer's private information.

Efficiency is achieved when the seller invests and the buyer reports his valuation of the dinning set truthfully so that only a valuable dining set is delivered to him. However, when both the seller's investment and the buyer's valuation are their private information, there is a hold-up problem even when the buyer and the seller can sign an enforceable contract. This hold-up problem under asymmetric information arises because there is a conflict between the incentive motivating the seller to invest and that motivating the buyer to tell the truth. To induce the buyer's truth-telling, the payment amount made by the buyer to the seller when the dining set is traded between them should be constant, whereas payment should be made only when the value of the dining set is high to provide sufficient investment incentive. Because the parties cannot satisfy the efficiency and the incentive requirements simultaneously, they must give up trading low-value dining set to provide the investment incentive, even at the cost of the surplus it would have created. Thus, there is a trade-off between trade efficiency and the investment.

Schmitz (2002a), Hori (2006), and Zhao (2008a) have studied this hold-up problem in a simple situation.[3] In their analysis, the parties can invest and trade only in single goods. It remains unclear whether the hold-up problem is still inevitable in a more general environment. The current paper extends their analysis to a multiple goods situation, for example, handling the chair (good 1) and the table (good 2) separately. In the separate case, the seller and buyer can invest and trade in the table after investing and trading in the chair. This sequential setting may create additional incentives from their mutual monitoring, as canceling the order for the table when one party suspects the other of wrongdoing during dealing the chair becomes possible. This paper begins by considering whether dynamic contracts, which utilize this sequential setting, can solve the hold-up problem.

It will be shown that dynamic contracts cannot improve the efficiency of the transaction. In order to motivate investment in the chair (good 1), the parties have to give up trading the chair, burn some money which the seller could have received in return for the chair, or cancel the order for the table (good 2) when the value of the chair is low. When the order for the

[1] *Cooperative investment* is unrelated to cooperative behavior. Although this wording may be inappropriate, it has already become common in this literature.

[2] It is often noted that this problem occurs when contracts are incomplete (Klein *et al.,* 1978 and Che and Hausch, 1999), but the problem may also exist even when contracts are complete.

[3] Under a *selfish investment* model, where the seller's investment stochastically determines the seller's cost, an efficient mechanism exists, as in Konakayama *et al.* (1986), Rogerson (1992), and Schmitz (2002b).

table (good 2) is cancelled, the total surplus from the table is sacrificed and the buyer cannot receive the benefit from trading the table. In the buyer's truth-telling condition for the chair (good 1), his expected benefit from the table (good 2) substitutes for his monetary payment for the chair. In order to maintain the buyer's incentive compatibility for the chair (good 1), his monetary payment for the chair must be reduced by his benefit from the table (good 2). Also, cancellation of the order for the table removes the seller's benefit from trading the table. The seller, consequently, has to give up both the buyer and seller's benefit from the table, and retains the investment incentive compatibility for the chair (good 1) in order that the seller cannot receive a benefit equal to the total surplus from trading. Burning one unit of money creates one unit of incentive and destroys one unit of total welfare. The marginal ratio of the incentive to the welfare loss from the cancellation of the order for the table and that from the burning of money are the same. On the other hand, the investment incentive for the chair (good 1) created by giving up the chance to trade a low-value chair is affected by the nature of information rent. The removal of the chance to trade a low-value chair creates an incentive of magnitude equal to the value of a high-value chair while decreasing total welfare by an amount equal to the value of the low-value chair, which is more efficient than canceling the order for the table or burning money. Since parties always choose not to cancel the order for the table, the distinction of the contract being dynamic is not utilized in the solving of the hold-up problem.

Fuchs (2007) and Chan and Zheng (2011) have considered similar problems. They considered models in which a principal hires an agent for multiple periods. In each period, the agent makes a hidden investment which stochastically affects output, which the principal privately observes. They showed the inefficiency of the result and the optimality of the contracts in which the agent is never fired. Nevertheless, their optimal contract is not unique: the contract which fires the agent can be equivalently optimal.[4] In the model that considered herein, the optimal contract is unique. This difference arises because the principal can utilize the output without trade in their model. The principal and the agent consequently cannot be motivated by giving up the chance to trade the goods instead of burning money or by being fired. The current analysis shows that the inefficiency resulting from asymmetric information cannot be mitigated even by contracts that can control more variables, such as the chance to trade.

However, there are dynamic solutions that solve the contractual incompleteness hold-up problem.[5] Che and Sákovics (2004) have considered an alternative bargaining situation. They showed that first-best outcomes can be achieved when additional bargaining and investment allows the parties to reach agreement. Neher (1999) showed that hold-up can be overcome when the investor can build up collateral by sequential investment. Pitchford and

[4]Fuchs (2007) showed the optimality of review contracts in which the agent is fired if his performance is not satisfactory after reviewing several periods in the infinite periods case. Its uniqueness is not discussed.

[5]The results that will be presented herein are different from folk theorems, which are that efficiency results when a same stage game is played infinitely many times. Telser (1980) and Klein and Leffler (1981) showed that the contractual incompleteness hold-up problem can be solved in the case of repeated interaction. When the same stage game with a hidden action and hidden information is repeated infinitely, the existence of contracts induces efficient action and trade, as has been proved by Fuchs (2007).

Snyder (2004) showed that sequential investment creates a dynamic threat and solves the hold-up problem.

Also of interest are the properties of second-best contracts. Contracts that do not utilize a sequential setting are either separate or bundled contracts. Separate contracts treat the chair and the table separately, whereas bundled contracts treat the table and the chair as a dining table set. It will be shown that the optimal bundled contract is superior to the optimal separate contract when the investment cost is small. In such a case, the bundled contract utilizes two effects. The first is the *spillover effect*, that when two goods are bundled together, the incentive for the seller to make a hidden investment in one good also creates an incentive for the other good. This spillover effect is similar to what Laux (2001) and Zhao (2008b) have considered in multi-task moral hazard problems,[6] although the agent is rewarded only if he demonstrates good performance in every task in their optimal contract, while the seller is punished only if all goods have poor quality in the current task because their objective is to minimize the reward to the agent whereas the goal in the problem examined in the current study is to minimize the loss of total welfare. The second effect utilized in the bundled contract is the *information rent effect*, that the investment incentive created by reducing trade is more effective than burning money, due to information rent.

When the investment cost is small, the bundled contract is the optimal one because the seller can be motivated to invest by reducing trade, and this involves both the spillover and information rent effects. When the investment cost is large, a reduction of trade is not enough to motivate the seller to invest in the bundled contract and some money has to be burnt. The bundled contract consequently involves only the spillover effect, while the separate contract involves only the information rent effect. The separate contract is superior to the bundled contract when its information rent effect sufficiently overwhelms the bundled contract's spillover effect.

Dana (1993) and Mookherjee and Tsumagari (2004) have also argued the superiority of bundled contracts over separate contracts when goods are substitutes and private information is distributed independently, which is the case considered herein.[7] Their analyses are, however, different from the one conducted herein in that their analyses include no hidden investment and it is the conflict between the incentive compatibility and the participation constraints that creates a problem. Consequently, they focused on the effects of information rent on monetary transfers between a seller and a buyer. The analysis performed herein, on the other hand, focuses on the effects of information rent on the marginal monetary transfers which affect the seller's investment incentive.

In the remainder of the paper, first Section 2 lays out the model and analyzes dynamic contracts, bundled contracts, and separate contracts, and then Section 3 concludes.

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[6]The other notable examples in the literature that focus on this problem are Holström and Milgram (1991), Itoh (1992), and Che and Yoo (2001), which considered the case when signals from multiple tasks are stochastically correlated, and Schmitz (2005), which considered the case when tasks are performed sequentially.

[7]Dana (1993) also showed the superiority of the separate contract when the private information is positively correlated. Baron and Besanko (1992), Gilbert and Riordan (1995), and Mookherjee and Tsumagari (2004) studied the optimal contract when goods are compliments.

2 Model and Analysis

Consider a buyer contracting for two goods, $i = \{1, 2\}$, supplied by a seller. When they sign the contract, the buyer's valuation of each good, θ_i, is uncertain and both parties only know that it will be either $\underline{\theta}_i$ or $\bar{\theta}_i$ $(> \underline{\theta}_i > 0)$, distributed independently. Assume for simplicity that the seller's valuation of each good is zero and that the seller's investment stochastically determines θ_i. When the seller invests in good i, $\bar{\theta}_i$ is realized with probability p_i^h, while it is realized with lower probability p_i^l $\left(< p_i^h\right)$ when she does not invest. Although investment increases the value of goods, the seller privately incurs cost $e_i > 0$.

Both parties are risk neutral. The buyer has a linear preference over the value of goods and monetary transfer. The seller has a linear preference over monetary transfer and the investment cost. For simplicity, there is no time discounting.

Total surplus is the sum of the buyer's surplus and the seller's surplus. When total surplus is maximized, the solution is most efficient. There are two kinds of efficiency. First, the goods should always be produced and sold to the buyer because this produces an ex post surplus. Also, when $\bar{\theta}_i - \underline{\theta}_i \geq \frac{e_i}{p_i^h - p_i^l}$, the seller should invest to increase the ex ante total surplus. The first-best outcome is realized when both efficiencies are achieved.

The efficiency attainable is unknown, when there are the following two kinds of information asymmetry. First, investment is the seller's private information. Second, only the buyer can know the realized value of each good. Thus, the contract cannot be contingent on such private information, but only on messages between the seller and the buyer.

Contracts

When there are multiple goods, the contract can be either *dynamic* or *bundled*. In dynamic contracts, the first and second goods are traded sequentially and the parties can cancel the order for the second good after trading the first good. This class of contracts includes one such that the order for the second good is never cancelled and contractual provisions for the two goods are independent of each other. Such a contract is equivalent to a *separate* contract in which two goods are traded separately. In bundled contracts, multiple goods are treated as one bundled good.

2.1 Dynamic Contracts

In dynamic contracts, the parties trade goods sequentially. The buyer and the seller sign a contract at time 0. The buyer pays a non-contingent (fixed) payment to the buyer at that time.[8] Because this non-contingent payment does not have any effect on incentive constraints at subsequent times, it is not explicitly considered in the present analysis. At time 1, the seller chooses to invest in good 1 or not and the buyer learns its value, θ_1. After this sequence (say, at time 1.5, i.e., between time 1 and time 2), the buyer sends message

[8] Participation constraints are not considered. However, the parties can appropriately choose non-contingent payment to satisfy their participation constraints whenever a contract creates a net total surplus because they have symmetric information at time 0; *cf.* Myerson and Satterthwaite (1983).

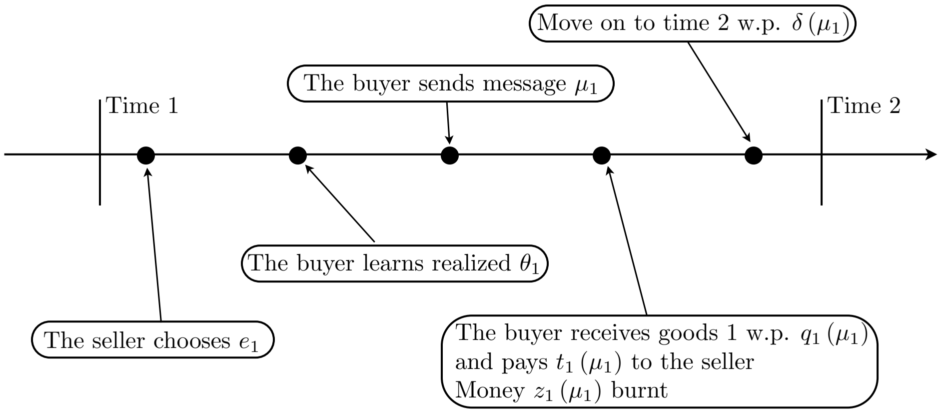

Figure 1: Timeline.

μ_1. A contract describes the probability that good 1 is sold to the buyer, $q_1(\mu_1) \in [0,1]$, with payment to the seller, $t_1(\mu_1) \in \mathbb{R}$, contingent on the message.[9] Part of the payment is burned before the seller receives it, $z_1(\mu_1) \in \mathbb{R}$. A contract also specifies whether to cancel the order for goods 2, $\delta(\mu_1) \in [0,1]$. When the order for goods 2 is canceled, $\delta(\mu_1) = 0$, the contract is terminated after time 1.5. When they do not cancel, $\delta(\mu_1) = 1$, they move on to time 2, and the seller chooses to invest in good 2 and the buyer learns its value, θ_2. At time 2.5, the buyer sends message μ_2. Contingent on μ_1 and μ_2, good 2 is traded with probability $q_2(\mu_1, \mu_2) \in [0,1]$, and the buyer pays $t_2(\mu_1, \mu_2) \in \mathbb{R}$ to the seller and part of it is burned, $z_2(\mu_1, \mu_2) \in \mathbb{R}$, as specified in the contract.

The contract can also be contingent on a message from the seller, but this message cannot be utilized, because the seller's utility does not satisfy the single-crossing property on investment level. Without loss of generality, the current analysis is restricted to truth-telling contracts from a version of the revelation principle.

When the investment cost is high, it is optimal not to induce investment. Throughout this paper, investment cost is assumed to be sufficiently small,

Assumption 1.
$$\frac{\left(p_i^h - p_i^l\right)\left(\bar{\theta}_i - \underline{\theta}_i\right)\bar{\theta}_i}{\left(p_i^h - p_i^l\right)\bar{\theta}_i + \left(1 - p_i^h\right)\underline{\theta}_i} > \frac{e_i}{p_i^h - p_i^l}.$$

This assumption guarantees that the following separate contract induces investment. A contract such that $\delta(\mu_1) = 1$ for all μ_1 and $q_2(\mu_1, \mu_2)$, $t_2(\mu_1, \mu_2)$, and $z_2(\mu_1, \mu_2)$ are constant with respect to μ_1 is equivalent to a separate contract, in which good i is traded to the buyer with probability $\hat{q}_i(\mu_i) \in [0,1]$, with payment to the seller, $\hat{t}_i(\mu_i) \in \mathbb{R}$, and money $\hat{z}_i(\mu_i) \in \mathbb{R}$ burnt, contingent on the message. The optimal separate contract is duplication

[9]When q_1 and t_1 are contingent on μ_2, this is equivalent to a bundled contract. See footnote 11.

of the optimal contract in the single good case which has been analyzed by Schmitz (2002a), Hori (2006), and Zhao (2008).

Lemma 1. *(Proposition 2 of Schmitz (2002a)) The optimal separate contract is*

$$\begin{aligned}\hat{q}_i\left(\bar{\theta}_i\right) &= 1,\\ \hat{q}_i\left(\underline{\theta}_i\right) &= 1-\frac{e_i}{\left(p_i^h-p_i^l\right)\bar{\theta}_i},\\ \hat{z}_i\left(\bar{\theta}_i\right) &= \hat{z}_i\left(\underline{\theta}_i\right)=0,\end{aligned}$$

and it induces each investment.[10]

Dynamic contracts are now analyzed. First, consider the problem of maximizing total welfare from time 2 onward.

$$V\left(\mu_1\right)\equiv p_2^h\left\{\bar{\theta}_2 q_2\left(\mu_1,\bar{\theta}_2\right)-z_2\left(\mu_1,\bar{\theta}_2\right)\right\}+\left(1-p_2^h\right)\left\{\underline{\theta}_2 q_2\left(\mu_1,\underline{\theta}_2\right)-z_2\left(\mu_1,\underline{\theta}_2\right)\right\}-e_2.$$

At time 2.5, the buyer reports truthfully when

$$\begin{aligned}\underline{\theta}_2 q_2\left(\mu_1,\underline{\theta}_2\right)-t_2\left(\mu_1,\underline{\theta}_2\right)&\geq\underline{\theta}_2 q_2\left(\mu_1,\bar{\theta}_2\right)-t_2\left(\mu_1,\bar{\theta}_2\right),\\ \bar{\theta}_2 q_2\left(\mu_1,\bar{\theta}_2\right)-t_2\left(\mu_1,\bar{\theta}_2\right)&\geq\bar{\theta}_2 q_2\left(\mu_1,\underline{\theta}_2\right)-t_2\left(\mu_1,\underline{\theta}_2\right).\end{aligned}$$

These conditions are equivalent to

$$\bar{\theta}_2\left\{q_2\left(\mu_1,\bar{\theta}_2\right)-q_2\left(\mu_1,\underline{\theta}_2\right)\right\}\geq t_2\left(\mu_1,\bar{\theta}_2\right)-t_2\left(\mu_1,\underline{\theta}_2\right)\geq\underline{\theta}_2\left\{q_2\left(\mu_1,\bar{\theta}_2\right)-q_2\left(\mu_1,\underline{\theta}_2\right)\right\},\quad(1)$$

and imply $q_2\left(\mu_1,\bar{\theta}_2\right)\geq q_2\left(\mu_1,\underline{\theta}_2\right)$.

At time 2, the seller invests e_2 when

$$\begin{aligned}&p_2^h\left\{t_2\left(\mu_1,\bar{\theta}_2\right)-z_2\left(\mu_1,\bar{\theta}_2\right)\right\}+\left(1-p_2^h\right)\left\{t_2\left(\mu_1,\underline{\theta}_2\right)-z_2\left(\mu_1,\underline{\theta}_2\right)\right\}-e_2\\ &\quad\geq p_2^l\left\{t_2\left(\mu_1,\bar{\theta}_2\right)-z_2\left(\mu_1,\bar{\theta}_2\right)\right\}+\left(1-p_2^l\right)\left\{t_2\left(\mu_1,\underline{\theta}_2\right)-z_2\left(\mu_1,\underline{\theta}_2\right)\right\}\\ &\quad\Leftrightarrow t_2\left(\mu_1,\bar{\theta}_2\right)-t_2\left(\mu_1,\underline{\theta}_2\right)-z_2\left(\mu_1,\bar{\theta}_2\right)+z_2\left(\mu_1,\underline{\theta}_2\right)\geq\frac{e_2}{p_2^h-p_2^l}.\end{aligned}$$

The left-hand side of this condition is the *investment incentive* induced by this contract. From (1), the upper bound of the investment incentive is

$$\bar{\theta}_2\left\{q_2\left(\mu_1,\bar{\theta}_2\right)-q_2\left(\mu_1,\underline{\theta}_2\right)\right\}-z_2\left(\mu_1,\bar{\theta}_2\right)+z_2\left(\mu_1,\underline{\theta}_2\right)\geq\frac{e_2}{p_2^h-p_2^l}.\quad(2)$$

This condition implies that investment cannot be induced when trade is efficient, $q_2\left(\mu_1,\bar{\theta}_2\right)=q_2\left(\mu_1,\underline{\theta}_2\right)=1$ and $z_2\left(\mu_1,\bar{\theta}_2\right)=z_2\left(\mu_1,\underline{\theta}_2\right)=0$. The trade-off between investment and ex post efficiency exists at time 2 and the first-best outcome cannot be achieved.

[10]Note that $\bar{\theta}_i-\underline{\theta}_i>\frac{\left(p_i^h-p_i^l\right)\left(\bar{\theta}_i-\underline{\theta}_i\right)\bar{\theta}_i}{\left(p_i^h-p_i^l\right)\bar{\theta}_i+\left(1-p_i^h\right)\underline{\theta}_i}$.

Lemma 2. *The contract maximizing* $V(\mu_1)$ *chooses* $q_2\left(\mu_1, \bar{\theta}_2\right) = 1$, $q_2\left(\underline{\theta}_2\right) = 1 - \frac{e_2}{\left(p_2^h - p_2^l\right)\bar{\theta}_2}$, *and* $z_2\left(\mu_1, \bar{\theta}_2\right) = z_2\left(\mu_1, \underline{\theta}_2\right) = 0$.

Lemma 2 follows straightforwardly from Lemma 1. This analysis shows that total welfare in this case is bounded from above. Let $\mathcal{V}$ be the set of $V(\mu_1)$, which is implementable. Note that this set is independent of θ_1 because θ_1 and θ_2 are independent and $\sup \mathcal{V} \geq 0$.

Next, consider the problem at time 1. Let

$$V^B(\mu_1) \equiv p_2^h\left\{\bar{\theta}_2 q_2\left(\mu_1, \bar{\theta}_2\right) - t_2\left(\mu_1, \bar{\theta}_2\right)\right\} + \left(1 - p_2^h\right)\left\{\underline{\theta}_2 q_2\left(\mu_1, \underline{\theta}_2\right) - t_2\left(\mu_1, \underline{\theta}_2\right)\right\},$$

and

$$V^S(\mu_1) \equiv p_2^h\left\{t_2\left(\mu_1, \bar{\theta}_2\right) - z_2\left(\mu_1, \bar{\theta}_2\right)\right\} + \left(1 - p_2^h\right)\left\{t_2\left(\mu_1, \underline{\theta}_2\right) - z_2\left(\mu_1, \underline{\theta}_2\right)\right\} - e_2,$$

be the buyer's and the seller's expected utilities from time 2 onwards, after the buyer reports μ_1 at time 1.5, so that $V(\mu_1) = V^B(\mu_1) + V^S(\mu_1)$.

The buyer reports truthfully at time 1.5 when

$$\begin{aligned}
\underline{\theta}_1 q_1\left(\underline{\theta}_1\right) - t_1\left(\underline{\theta}_1\right) + \delta\left(\underline{\theta}_1\right) V^B\left(\underline{\theta}_1\right) &\geq \underline{\theta}_1 q_1\left(\bar{\theta}_1\right) - t_1\left(\bar{\theta}_1\right) + \delta\left(\bar{\theta}_1\right) V^B\left(\bar{\theta}_1\right), \\
\bar{\theta}_1 q_1\left(\bar{\theta}_1\right) - t_1\left(\bar{\theta}_1\right) + \delta\left(\bar{\theta}_1\right) V^B\left(\bar{\theta}_1\right) &\geq \bar{\theta}_1 q_1\left(\underline{\theta}_1\right) - t_1\left(\underline{\theta}_1\right) + \delta\left(\underline{\theta}_1\right) V^B\left(\underline{\theta}_1\right),
\end{aligned}$$

which can be rewritten as

$$\begin{aligned}
&\bar{\theta}_1\left\{q_1\left(\bar{\theta}_1\right) - q_1\left(\underline{\theta}_1\right)\right\} \\
&\qquad \geq \left\{t_1\left(\bar{\theta}_1\right) - \delta\left(\bar{\theta}_1\right) V^B\left(\bar{\theta}_1\right)\right\} - \left\{t_1\left(\underline{\theta}_1\right) - \delta\left(\underline{\theta}_1\right) V^B\left(\underline{\theta}_1\right)\right\} \\
&\qquad\qquad \geq \underline{\theta}_1\left\{q_1\left(\bar{\theta}_1\right) - q_1\left(\underline{\theta}_1\right)\right\}. \qquad (3)
\end{aligned}$$

This condition implies that $q_1\left(\bar{\theta}_1\right) \geq q_1\left(\underline{\theta}_1\right)$.

The seller invests at time 1 when

$$\begin{aligned}
&p_1^h\left\{t_1\left(\bar{\theta}_1\right) - z_1\left(\bar{\theta}_1\right) + \delta\left(\bar{\theta}_1\right) V^S\left(\bar{\theta}_1\right)\right\} + \left(1 - p_1^h\right)\left\{t_1\left(\underline{\theta}_1\right) - z_1\left(\underline{\theta}_1\right) + \delta\left(\underline{\theta}_1\right) V^S\left(\underline{\theta}_1\right)\right\} - e_1 \\
&\geq p_1^l\left\{t_1\left(\bar{\theta}_1\right) - z_1\left(\bar{\theta}_1\right) + \delta\left(\bar{\theta}_1\right) V^S\left(\bar{\theta}_1\right)\right\} + \left(1 - p_1^l\right)\left\{t_1\left(\underline{\theta}_1\right) - z_1\left(\underline{\theta}_1\right) + \delta\left(\underline{\theta}_1\right) V^S\left(\underline{\theta}_1\right)\right\}.
\end{aligned}$$

Rewrite this condition as

$$\left\{t_1\left(\bar{\theta}_1\right) - z_1\left(\bar{\theta}_1\right) + \delta\left(\bar{\theta}_1\right) V^S\left(\bar{\theta}_1\right)\right\} - \left\{t_1\left(\underline{\theta}_1\right) - z_1\left(\underline{\theta}_1\right) + \delta\left(\underline{\theta}_1\right) V^S\left(\underline{\theta}_1\right)\right\} \geq \frac{e_1}{p_1^h - p_1^l}.$$

From this condition and (3), the upper bound on this investment incentive is

$$\bar{\theta}_1\left\{q_1\left(\bar{\theta}_1\right) - q_1\left(\underline{\theta}_1\right)\right\} - z_1\left(\bar{\theta}_1\right) + z_1\left(\underline{\theta}_1\right) + \delta\left(\bar{\theta}_1\right) V\left(\bar{\theta}_1\right) - \delta\left(\underline{\theta}_1\right) V\left(\underline{\theta}_1\right) \geq \frac{e_1}{p_1^h - p_1^l}. \qquad (4)$$

Under constraint (4) and $V(\cdot) \in \mathcal{V}$, consider the optimal contract that maximizes total welfare. Here the analysis is restricted to that of the investment induced at time 1. If investment cannot be induced at time 1, then dynamic contracts are dominated by the optimal separate contract under Assumption 1.

The problem considered here is

$$\max_{q_1(\cdot),\delta(\cdot),V(\cdot)} p_1^h \left\{\bar{\theta}_1 q_1\left(\bar{\theta}_1\right) - z_1\left(\bar{\theta}_1\right) + \delta\left(\bar{\theta}_1\right) V\left(\bar{\theta}_1\right)\right\} + \left(1-p_1^h\right)\left\{\underline{\theta}_1 q_1\left(\underline{\theta}_1\right) - z_1\left(\underline{\theta}_1\right) + \delta\left(\underline{\theta}_1\right) V\left(\underline{\theta}_1\right)\right\} - e_1,$$

subject to (4) and $V(\cdot) \in \mathcal{V}$.

Proposition 1. *The optimal dynamic contract is*

$$\begin{aligned}
q_1\left(\bar{\theta}_1\right) &= q_2\left(\mu_1, \bar{\theta}_2\right) = \delta\left(\bar{\theta}_1\right) = \delta\left(\underline{\theta}_1\right) = 1, \\
q_1\left(\underline{\theta}_1\right) &= 1 - \frac{e_1}{\left(p_1^h - p_1^l\right)\bar{\theta}_1}, \\
q_2\left(\mu_1, \underline{\theta}_2\right) &= 1 - \frac{e_2}{\left(p_2^h - p_2^l\right)\bar{\theta}_2}, \\
z_1\left(\bar{\theta}_1\right) &= z_1\left(\underline{\theta}_1\right) = z_2\left(\mu_1, \bar{\theta}_2\right) = z_2\left(\mu_1, \underline{\theta}_2\right) = 0,
\end{aligned}$$

for all $\mu_1 \in \left\{\underline{\theta}_1, \bar{\theta}_1\right\}$, which is equivalent to the optimal separate contract.

Proof. If $q_1\left(\bar{\theta}_1\right)$ and $\delta\left(\bar{\theta}_1\right) V\left(\bar{\theta}_1\right)$ are large and $z_1\left(\bar{\theta}_1\right)$ is small, then constraint (4) is satisfied and the outcome is an increase in total welfare. It is optimal to set $q_1\left(\bar{\theta}_1\right) = \delta\left(\bar{\theta}_1\right) = 1$ and $z_1\left(\bar{\theta}_1\right) = 0$ and to set $\delta\left(\bar{\theta}_1\right) V\left(\bar{\theta}_1\right)$ as large as possible. On the other hand, the constraint is met when $q_1\left(\underline{\theta}_1\right)$ and $\delta\left(\underline{\theta}_1\right) V\left(\underline{\theta}_1\right)$ are small or $z_1\left(\underline{\theta}_1\right)$ is large, which results in a decrease in total welfare. The marginal effects on (4) and total welfare change due to reducing $q_1\left(\underline{\theta}_1\right)$ are $\bar{\theta}_1$ and $-\left(1-p_1^h\right)\underline{\theta}_1$. Those from reducing $\delta\left(\underline{\theta}_1\right) V\left(\underline{\theta}_1\right)$ are 1 and $-\left(1-p_1^h\right)$, which are identical to those from increasing $z_1\left(\underline{\theta}_1\right)$. Because the rate of decrease of total welfare with respect to increases in the investment incentive is lower when reducing $q_1\left(\underline{\theta}_1\right)$ than when reducing $\delta\left(\underline{\theta}_1\right) V\left(\underline{\theta}_1\right)$ and/or increasing $z_1\left(\underline{\theta}_1\right)$, the optimal dynamic contract should reduce $q_1\left(\underline{\theta}_1\right)$ before reducing $\delta\left(\underline{\theta}_1\right) V\left(\underline{\theta}_1\right)$ and/or increasing $z_1\left(\underline{\theta}_1\right)$.

Note that, when $q_1\left(\underline{\theta}_1\right) = z_1\left(\underline{\theta}_1\right) = z_1\left(\bar{\theta}_1\right) = 0$, $q_1\left(\bar{\theta}_1\right) = \delta\left(\bar{\theta}_1\right) = \delta\left(\underline{\theta}_1\right) = 1$, and $V\left(\bar{\theta}_1\right) = V\left(\underline{\theta}_1\right) = \sup \mathcal{V}$, the left-hand side of (4) is $\bar{\theta}_1$. Because

$$\bar{\theta}_1 > \bar{\theta}_1 - \underline{\theta}_1 > \frac{e_1}{p_1^h - p_1^l},$$

where the last inequality follows from Assumption 1, investment at time 1 can be induced by reducing solely $q_1\left(\underline{\theta}_1\right)$.

To make $\delta\left(\bar{\theta}_1\right) V\left(\bar{\theta}_1\right)$ and $\delta\left(\underline{\theta}_1\right) V\left(\underline{\theta}_1\right)$ as large as possible, either $V\left(\bar{\theta}_1\right)$ and $V\left(\underline{\theta}_1\right)$ should be positive and $\delta\left(\bar{\theta}_1\right) = \delta\left(\underline{\theta}_1\right) = 1$, or $0 \geq \delta\left(\bar{\theta}_1\right) V\left(\bar{\theta}_1\right)$ and $0 \geq \delta\left(\underline{\theta}_1\right) V\left(\underline{\theta}_1\right)$ should hold. The second-best contract is $q_1\left(\bar{\theta}_1\right) = \delta\left(\bar{\theta}_1\right) = \delta\left(\underline{\theta}_1\right) = 1$, $q_1\left(\underline{\theta}_1\right) = 1 - \frac{e_1}{\left(p_1^h - p_1^l\right)\bar{\theta}_1}$, $z_1\left(\bar{\theta}_1\right) = z_1\left(\underline{\theta}_1\right) = 0$, and $V\left(\bar{\theta}_1\right)$ and $V\left(\underline{\theta}_1\right)$ as large as possible. Lemma 1 gives the optimal contract at time 2.

□

	Incentive	Total welfare	Marginal ratio
$q_1(\underline{\theta}_1)$	$-\bar{\theta}_1$	$\left(1-p_1^h\right)\underline{\theta}_1$	$\left(1-p_1^h\right)\frac{\bar{\theta}_1}{\underline{\theta}_1}$
$\delta(\underline{\theta}_1)V(\underline{\theta}_1)$	-1	$\left(1-p_1^h\right)$	$\left(1-p_1^h\right)$
$z_1(\underline{\theta}_1)$	1	$-\left(1-p_1^h\right)$	$\left(1-p_1^h\right)$

Table 1: Marginal effects on the investment incentive and total welfare.

From the preceding analysis, it is clear that the investment incentive at time 1 is provided only by reducing trade for good 1, rather than by canceling the order for good 2 or burning money. When good 1 valued at $\underline{\theta}_1$ is not traded, payment is $\bar{\theta}_1$ less than if traded and an investment incentive of size $\bar{\theta}_1$ is induced by sacrificing trade efficiency by $\underline{\theta}_1$; in this case, the losses in the investment incentive and total welfare created by canceling the order for good 2 or burning money are equal. The marginal ratio of the incentive to the welfare loss is $\left(1-p_1^h\right)\frac{\bar{\theta}_1}{\underline{\theta}_1}$ for $q_1(\underline{\theta}_1)$, while that for $\delta(\underline{\theta}_1)V(\underline{\theta}_1)$ and $z_1(\underline{\theta}_1)$ are $\left(1-p_1^h\right)$. Inducing the investment incentive by canceling the order for good 2 or burning money is inefficient because of information rent.

The optimal dynamic contract is equivalent to separate contracts in which each good is traded separately. A dynamic contracts arrangement cannot improve efficiency.

2.2 Bundled Contracts

This subsection describes bundled contracts in which two goods are bundled as one. A bundled contract specifies the probability, $\tilde{q}(\cdot)$, that the bundle of the goods will be traded, with payment by the buyer to the seller of $\tilde{t}(\cdot)$ and money burnt $\tilde{z}(\cdot)$. The buyer can send multiple messages at different times; however, it is without loss of generality to consider only contracts where the buyer sends no more than one message evaluating the bundle because the buyer cares only about the value of the bundle, $\theta_1+\theta_2$, and his payment.[11]

For simplicity, it is assumed that $\bar{\theta}_1=\bar{\theta}_2\equiv\bar{\theta}$ and $\underline{\theta}_1=\underline{\theta}_2\equiv\underline{\theta}$. The bundle value is either θ_H, θ_M, or θ_L, where $\theta_H\equiv 2\bar{\theta}$, $\theta_M\equiv\bar{\theta}+\underline{\theta}$, and $\theta_L\equiv 2\underline{\theta}$. It is also assumed that $p^h\equiv p_1^h=p_2^h$, $p^l\equiv p_1^l=p_2^l$, and $e\equiv e_1=e_2$.

An optimal bundled contract that induces both investments is considered.[12] The optimal

[11] A version of separate contract $q_1(\mu_1,\mu_2), t_1(\mu_1,\mu_2), q_2(\mu_1,\mu_2), t_2(\mu_1,\mu_2)$ is equivalent to a bundled contract such that

$$(\theta_1+\theta_2)\,\tilde{q}(\mu_1,\mu_2)\equiv\theta_1 q_1(\mu_1,\mu_2)+\theta_2 q_2(\mu_1,\mu_2),$$
$$\tilde{t}(\mu_1,\mu_2)\equiv t_1(\mu_1,\mu_2)+t_2(\mu_1,\mu_2).$$

In this contract, $\delta(\cdot)=1$.

[12] Bundled contracts inducing only one investment are discussed in Appendix.

bundled contract maximizes total welfare,

$$\left(p^h\right)^2\left\{\theta_H\tilde{q}\left(\theta_H\right)-\tilde{z}\left(\theta_H\right)\right\}+2p^h\left(1-p^h\right)\left\{\theta_M\tilde{q}\left(\theta_M\right)-\tilde{z}\left(\theta_M\right)\right\}$$
$$+\left(1-p^h\right)^2\left\{\theta_L\tilde{q}\left(\theta_L\right)-\tilde{z}\left(\theta_L\right)\right\}-2e,$$

given the following constraints.

First, the buyer's incentive compatibility constraints are the local downward constraints,

$$\begin{aligned}\theta_H\tilde{q}\left(\theta_H\right)-\tilde{t}\left(\theta_H\right)&\geq\theta_H\tilde{q}\left(\theta_M\right)-\tilde{t}\left(\theta_M\right),\\ \theta_M\tilde{q}\left(\theta_M\right)-\tilde{t}\left(\theta_M\right)&\geq\theta_M\tilde{q}\left(\theta_L\right)-\tilde{t}\left(\theta_L\right),\end{aligned}\tag{5}$$

and the local upward constraints,

$$\begin{aligned}\theta_M\tilde{q}\left(\theta_M\right)-\tilde{t}\left(\theta_M\right)&\geq\theta_M\tilde{q}\left(\theta_H\right)-\tilde{t}\left(\theta_H\right),\\ \theta_L\tilde{q}\left(\theta_L\right)-\tilde{t}\left(\theta_L\right)&\geq\theta_L\tilde{q}\left(\theta_M\right)-\tilde{t}\left(\theta_M\right).\end{aligned}\tag{6}$$

It is well known that the local downward constraints are necessary and sufficient for global incentive compatibility.[13]

Next, the seller makes a second investment given that she has made the first investment if

$$\left(p^h\right)^2\left\{\tilde{t}\left(\theta_H\right)-\tilde{z}\left(\theta_H\right)\right\}+2p^h\left(1-p^h\right)\left\{\tilde{t}\left(\theta_M\right)-\tilde{z}\left(\theta_M\right)\right\}+\left(1-p^h\right)^2\left\{\tilde{t}\left(\theta_L\right)-\tilde{z}\left(\theta_L\right)\right\}-e$$
$$\geq p^hp^l\left\{\tilde{t}\left(\theta_H\right)-\tilde{z}\left(\theta_H\right)\right\}+\left\{p^h\left(1-p^l\right)+p^l\left(1-p^h\right)\right\}\left\{\tilde{t}\left(\theta_M\right)-\tilde{z}\left(\theta_M\right)\right\}$$
$$+\left(1-p^l\right)\left(1-p^h\right)\left\{\tilde{t}\left(\theta_L\right)-\tilde{z}\left(\theta_L\right)\right\},$$

and she makes the first investment if

$$p^hp^l\left\{\tilde{t}\left(\theta_H\right)-\tilde{z}\left(\theta_H\right)\right\}+\left\{p^h\left(1-p^l\right)+p^l\left(1-p^h\right)\right\}\left\{\tilde{t}\left(\theta_M\right)-\tilde{z}\left(\theta_M\right)\right\}$$
$$+\left(1-p^l\right)\left(1-p^h\right)\left\{\tilde{t}\left(\theta_L\right)-\tilde{z}\left(\theta_L\right)\right\}-e$$
$$\geq\left(p^l\right)^2\left\{\tilde{t}\left(\theta_H\right)-\tilde{z}\left(\theta_H\right)\right\}+2p^l\left(1-p^l\right)\left\{\tilde{t}\left(\theta_M\right)-\tilde{z}\left(\theta_M\right)\right\}+\left(1-p^l\right)^2\left\{\tilde{t}\left(\theta_L\right)-\tilde{z}\left(\theta_L\right)\right\}.$$

Rearranging, these constraints can be written as

$$\begin{aligned}&p^h\left\{\tilde{t}\left(\theta_H\right)-\tilde{t}\left(\theta_M\right)-\tilde{z}\left(\theta_H\right)+\tilde{z}\left(\theta_M\right)\right\}+\left(1-p^h\right)\left\{\tilde{t}\left(\theta_M\right)-\tilde{t}\left(\theta_L\right)-\tilde{z}\left(\theta_M\right)+\tilde{z}\left(\theta_L\right)\right\}\\&\qquad\geq\frac{e}{p^h-p^l},\\&p^l\left\{\tilde{t}\left(\theta_H\right)-\tilde{t}\left(\theta_M\right)-\tilde{z}\left(\theta_H\right)+\tilde{z}\left(\theta_M\right)\right\}+\left(1-p^l\right)\left\{\tilde{t}\left(\theta_M\right)-\tilde{t}\left(\theta_L\right)-\tilde{z}\left(\theta_M\right)+\tilde{z}\left(\theta_L\right)\right\}\\&\qquad\geq\frac{e}{p^h-p^l}.\end{aligned}\tag{7}$$

[13] See, for example, Theorem 23 of Stole (1999).

To maximize total welfare, the constraints of (5) should bind. If the second constraint is not binding, a larger $\tilde{q}(\theta_L)$ yields a higher efficiency. If the first constraint is not binding, increasing $\tilde{q}(\theta_M)$ and $\tilde{q}(\theta_L)$ improves total welfare. These constraints also imply that $\tilde{q}(\theta_H) \geq \tilde{q}(\theta_M) \geq \tilde{q}(\theta_L)$ and (6) is not binding. Binding constraints (7) with (5) can be rewritten as

$$\begin{aligned} &p^h \left[\theta_H \left\{\tilde{q}(\theta_H) - \tilde{q}(\theta_M)\right\} - \tilde{z}(\theta_H) + \tilde{z}(\theta_M)\right] \\ &\qquad + \left(1 - p^h\right) \left[\theta_M \left\{\tilde{q}(\theta_M) - \tilde{q}(\theta_L)\right\} - \tilde{z}(\theta_M) + \tilde{z}(\theta_L)\right] \geq \frac{e}{p^h - p^l}, \end{aligned} \tag{8}$$

and

$$\begin{aligned} &p^l \left[\theta_H \left\{\tilde{q}(\theta_H) - \tilde{q}(\theta_M)\right\} - \tilde{z}(\theta_H) + \tilde{z}(\theta_M)\right] \\ &\qquad + \left(1 - p^l\right) \left[\theta_M \left\{\tilde{q}(\theta_M) - \tilde{q}(\theta_L)\right\} - \tilde{z}(\theta_M) + \tilde{z}(\theta_L)\right] \geq \frac{e}{p^h - p^l}. \end{aligned} \tag{9}$$

The investment incentive, the left-hand side of the constraints, increases when $\tilde{q}(\theta_L)$ or $\tilde{q}(\theta_M)$ is reduced, or $\tilde{z}(\theta_L)$ or $\tilde{z}(\theta_M)$ is increased. Now the problem is that of maximizing total welfare subject to (8), (9), and $\tilde{q}(\theta_H) \geq \tilde{q}(\theta_M) \geq \tilde{q}(\theta_L)$.

Proposition 2. *The optimal bundled contract which induces both investments is,*

- *when* $\left(1 - p^h\right)\theta_M \geq \frac{e}{p^h - p^l}$,

$$\begin{aligned} \tilde{q}(\theta_H) &= \tilde{q}(\theta_M) = 1, \\ \tilde{q}(\theta_L) &= 1 - \frac{e}{(1 - p^h)(p^h - p^l)\theta_M}, \\ \tilde{z}(\theta_H) &= \tilde{z}(\theta_M) = \tilde{z}(\theta_L) = 0, \end{aligned}$$

- *when* $\frac{e}{p^h - p^l} > \left(1 - p^h\right)\theta_M$,

$$\begin{aligned} \tilde{q}(\theta_H) &= \tilde{q}(\theta_M) = 1, \\ \tilde{q}(\theta_L) &= 0, \\ \tilde{z}(\theta_H) &= \tilde{z}(\theta_M) = 0, \\ \tilde{z}(\theta_L) &= \frac{e}{(1 - p^h)(p^h - p^l)} - \theta_M. \end{aligned}$$

Proof. The following analysis proceeds without constraint (9) for the moment. To maximize total welfare under constraint (8), the optimal contract should set $\tilde{q}(\theta_H) = 1$ and $\tilde{z}(\theta_H) = 0$. Then, the question is whether to decrease $\tilde{q}(\theta_M)$ or $\tilde{q}(\theta_L)$, or to increase $\tilde{z}(\theta_M)$ or $\tilde{z}(\theta_L)$. The marginal effects of $\tilde{q}(\theta_M), \tilde{q}(\theta_L), \tilde{z}(\theta_M)$ and $\tilde{z}(\theta_L)$ on the left-hand side of (8) are $-p^h\theta_H + \left(1 - p^h\right)\theta_M$, $-\left(1 - p^h\right)\theta_M$, $p^h - \left(1 - p^h\right)$, and $1 - p^h$. The marginal effects on total welfare are $2p^h\left(1 - p^h\right)\theta_M$, $-\left(1 - p^h\right)\theta_L$, $-2p^h\left(1 - p^h\right)$, and $-\left(1 - p^h\right)^2$, respectively.

The marginal ratio of the investment incentive to the loss of total welfare from decreasing $\tilde{q}(\theta_L)$ is $\frac{\theta_M}{(1-p^h)\theta_L}$, which is higher than that from increasing $\tilde{z}(\theta_L)$, $\frac{1}{(1-p^h)}$, decreasing $\tilde{q}(\theta_M)$, $\frac{p^h\theta_H-(1-p^h)\theta_M}{2p^h(1-p^h)\theta_M}$, and increasing $\tilde{z}(\theta_M)$, $\frac{p^h-(1-p^h)}{2p^h(1-p^h)}$. Note that $\frac{1}{(1-p^h)} > \frac{p^h\theta_H-(1-p^h)\theta_M}{2p^h(1-p^h)\theta_M} > \frac{p^h-(1-p^h)}{2p^h(1-p^h)}$. From these marginal ratios, the optimal contract minimizes $\tilde{q}(\theta_L)$ and then increases $\tilde{z}(\theta_L)$ if necessary.

When the investment cost is moderate, $\left(1-p^h\right)\theta_M \geq \frac{e}{p^h-p^l}$, the seller is motivated to invest by only sacrificing $\tilde{q}(\theta_L)$. Therefore, the optimal contract is

$$\begin{aligned}
\tilde{q}(\theta_H) &= \tilde{q}(\theta_M) = 1, \\
\tilde{q}(\theta_L) &= 1 - \frac{e}{(1-p^h)(p^h-p^l)\theta_M}, \\
\tilde{z}(\theta_H) &= \tilde{z}(\theta_M) = \tilde{z}(\theta_L) = 0.
\end{aligned}$$

When $\frac{e}{p^h-p^l} > \left(1-p^h\right)\theta_M$, because solely a reduction of $\tilde{q}(\theta_L)$ is not enough to motivate the seller to invest and it is necessary to increase $\tilde{z}(\theta_L)$, the optimal contract is

$$\begin{aligned}
\tilde{q}(\theta_H) &= \tilde{q}(\theta_M) = 1, \\
\tilde{q}(\theta_L) &= 0, \\
\tilde{z}(\theta_H) &= \tilde{z}(\theta_M) = 0, \\
\tilde{z}(\theta_L) &= \frac{e}{(1-p^h)(p^h-p^l)} - \theta_M.
\end{aligned}$$

These contracts satisfy (9). □

The intuition can be well understood by considering the method of punishing the seller rather than rewarding her to motivate investment. The seller is punished by not trading the bundle or burning money, and the punishment is always associated with an efficiency loss. In order to minimize this efficiency loss, two effects are considered. The first effect is the marginal investment incentive of the seller's utility. Because the seller's marginal utility of burning money is -1, the marginal investment incentive of the seller's utility is the opposite of the marginal investment incentive of burning money. Comparison of the marginal investment incentive of $\tilde{z}(\theta_L)$, $\tilde{z}(\theta_M)$, and $\tilde{z}(\theta_H)$ shows that the investment incentive can be provided the most effectively by reducing the utility when θ_L is realized. The second effect considered is the seller's marginal utility of the probability of trading the bundle or burning money. While the seller's marginal utility of burning money is -1, that of the probability of trading the bundle is determined by information rent. Combining the two effects, reduction of $\tilde{q}(\theta_L)$ creates the investment incentive the most efficiently: an incentive of size $\left(1-p^h\right)\theta_M$ is created by sacrificing $\left(1-p^h\right)^2\theta_L$ total surplus. But there is a limit to how much incentive can be provided by reducing $\tilde{q}(\theta_L)$. An increase of $\tilde{z}(\theta_L)$ is the second most efficient solution. When a large incentive is needed, it is created by reducing $\tilde{q}(\theta_L)$ as much as possible and increasing $\tilde{z}(\theta_L)$.

In this subsection, we consider only the case of bundled contracts that induce both investments. Bundled contracts inducing one or no investment can be the optimal bundled contract when the investment cost is high. However, these contracts are not considered here, because they are inferior to the optimal separate contract and cannot be the second-best contract, which is considered in the next subsection.

2.3 Second-best contract

Because neither the separate contract nor the bundled contract can achieve the first-best outcome, it follows that the first-best outcome must be impossible. This subsection characterizes the second-best contract. First, note that contracts inducing a single investment are inferior to the optimal separate contract. If a single investment is induced, the upper bound on total welfare which bundled contracts can implement is less than that which separate contracts can implement.[14] Because separate contracts that induce only one investment cannot be the optimal separate contract under Assumption 1, bundled contracts that induce a single investment are also inferior to the optimal separate contract. Next, bundled and separate contracts which induce no investment are also inferior to the optimal separate contract under Assumption 1. Consequently, the optimal separate and bundled contracts inducing both investments are compared.

Proposition 3. *When $\frac{e}{p^h-p^l} > \frac{(1-p^h)\bar{\theta}(\bar{\theta}-\underline{\theta})}{\bar{\theta}-2\underline{\theta}}$ and $\bar{\theta}-2\underline{\theta} > 0$, the separate contract is second-best. Otherwise, the second-best contract is the bundled contract.*

Proof. Let

$$W^{FB} \equiv 2p^h\bar{\theta} + 2\left(1-p^h\right)\underline{\theta} - 2e = \left(p^h\right)^2\theta_H + 2p^h\left(1-p^h\right)\theta_M + \left(1-p^h\right)^2\theta_L - 2e.$$

Total welfare induced by the optimal separate contract is

$$2p^h\bar{\theta} + 2\left(1-p^h\right)\underline{\theta}\hat{q}\left(\underline{\theta}\right) - 2e = W^{FB} - 2\left(1-p^h\right)\underline{\theta}\frac{e}{\left(p^h-p^l\right)\bar{\theta}}. \tag{10}$$

When $\left(1-p^h\right)\theta_M \geq \frac{e}{p^h-p^l}$, the optimal bundled contract yields total welfare

$$\begin{aligned}&\left(p^h\right)^2\theta_H + 2p^h\left(1-p^h\right)\theta_M + \left(1-p^h\right)^2\theta_L\tilde{q}\left(\theta_L\right) - 2e\\ &\qquad = W^{FB} - \left(1-p^h\right)^2\theta_L\frac{e}{\left(1-p^h\right)\left(p^h-p^l\right)\theta_M}.\end{aligned} \tag{11}$$

Some calculation shows that (11) is larger than (10). The optimal bundled contract can therefore induce investment more efficiently than the optimal separate contract.

[14]See Appendix.

When $\frac{e}{p^h - p^l} > \left(1 - p^h\right)\theta_M$, total welfare induced by the optimal bundled contract is

$$\begin{aligned}
&\left(p^h\right)^2\theta_H + 2p^h\left(1-p^h\right)\theta_M + \left(1-p^h\right)^2\tilde{z}\left(\theta_L\right) - 2e \\
&\qquad = W^{FB} - \left(1-p^h\right)^2\left\{\theta_L + \frac{e}{\left(1-p^h\right)\left(p^h-p^l\right)} - \theta_M\right\}. \qquad (12)
\end{aligned}$$

Because (10) minus (12) is

$$\left(1-p^h\right)\left[\frac{\left(\bar{\theta}-2\underline{\theta}\right)e}{\bar{\theta}\left(p^h-p^l\right)} - \left(1-p^h\right)\left(\bar{\theta}-\underline{\theta}\right)\right],$$

the optimal separate contract is superior to the optimal bundled contract when $\frac{e}{p^h-p^l} > \frac{(1-p^h)\bar{\theta}(\bar{\theta}-\underline{\theta})}{\bar{\theta}-2\underline{\theta}}$ and $\bar{\theta}-2\underline{\theta} > 0$. $\square$

To provide intuition for Proposition 3, consider a unit increase in the investment cost $\frac{e}{p^h-p^l}$ for both goods. First, suppose contracts can be adjusted only in terms of the amount of money burnt. In bundled contracts, $\tilde{z}\left(\theta_L\right)$ is increased by the amount $\frac{e}{\left(1-p^h\right)} \equiv \Delta\tilde{z}\left(\theta_L\right)$ from(8) and the total welfare loss is $\left(1-p^h\right)^2\Delta\tilde{z}\left(\theta_L\right) = \left(1-p^h\right)e$. In separate (dynamic) contracts, from (2) and (4), both $z_1\left(\underline{\theta}_1\right)$ and $z_2\left(\underline{\theta}_2\right)$ are increased by the amount $\frac{e}{p^h-p^l} \equiv \Delta z_1\left(\underline{\theta}_1\right) = \Delta z_2\left(\underline{\theta}_2\right)$ and the welfare loss is $\left(1-p^h\right)\left\{\Delta z_1\left(\underline{\theta}_1\right) + \Delta z_2\left(\underline{\theta}_2\right)\right\} = 2\left(1-p^h\right)e$. The welfare loss in bundled contracts is half of that in separate contracts due to the *spillover effect*; therefore, an increase of punishment creates an incentive for both investments.

Next, consider reducing the probability of trading goods. Due to the *information rent effect*, reduction of the probability of trading goods creates the investment incentive more efficiently than burning money.

Reduction of $\tilde{q}\left(\theta_L\right)$ in bundled contracts is the most efficient because it creates incentive through both the information rent effect, which is $\frac{\theta_M}{\theta_L}$ times more efficient than burning money, and the spillover effect. However, reduction of $\tilde{q}\left(\theta_L\right)$ can create enough incentive only if the investment cost is moderate, $\left(1-p^h\right)\theta_M \geq \frac{e}{p^h-p^l}$. When $\frac{e}{p^h-p^l} > \left(1-p^h\right)\theta_M$, with bundled contracts, it is necessary to increase $\tilde{z}\left(\theta_L\right)$, besides minimizing $\tilde{q}\left(\theta_L\right)$, to create further incentive, and there is only the spillover effect, while in the case of separate contracts, incentive is created through reduction of $\hat{q}\left(\theta\right)$ and there is only the information rent effect. Because the information rent effect in the latter case creates $\frac{\bar{\theta}}{\underline{\theta}}$ times more efficiency than burning money in separate contracts, while the former spillover effect creates twice as much efficiency, the marginal loss of total welfare through the investment cost is larger in bundled contracts than in separate contracts when $\bar{\theta}-2\underline{\theta} > 0$. If this condition holds, the gross total welfare loss is larger in the optimal bundled contract than in the optimal separate contract when $\frac{e}{\left(p^h-p^l\right)} > \frac{\left(1-p^h\right)\bar{\theta}\left(\bar{\theta}-\underline{\theta}\right)}{\bar{\theta}-2\underline{\theta}}$.

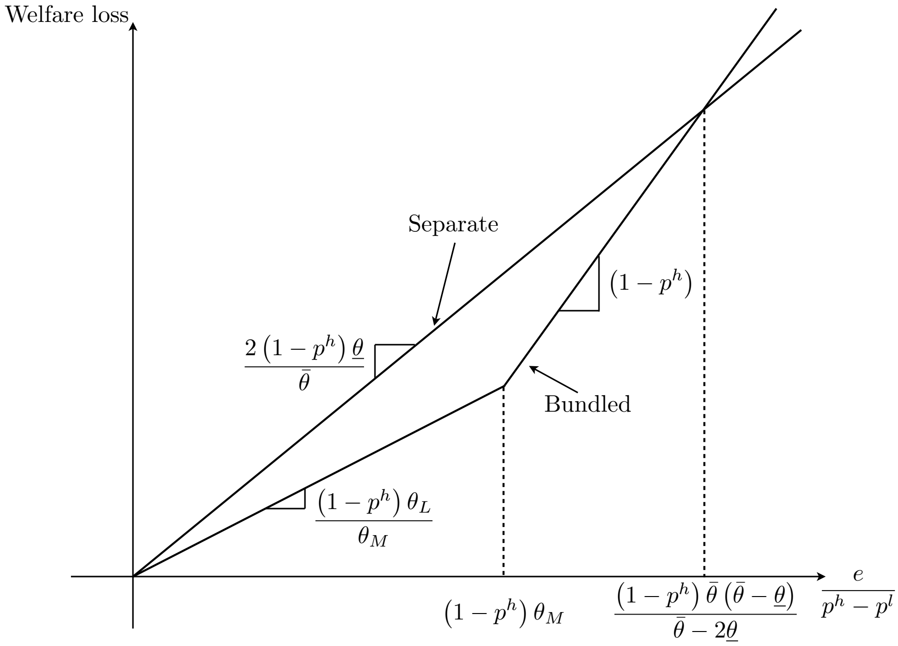

Figure 2: The total welfare loss $\left(\bar{\theta}-2\underline{\theta}>0\right)$.

3 Conclusion

This paper has analyzed the optimal contracting problem for multiple goods where asymmetric information causes a hold-up problem. The second-best contract has been shown to be a bundled contract when the investment cost is sufficiently small. Where this condition does not hold, either a separate or a bundled contract is second-best. Dynamic contracts cannot be second-best under any conditions. The first contribution of our paper is to corroborate previous research. Although Schmitz (2002a), Hori (2006), and Zhao (2008a) investigated only the single good case, the optimality of the bundled contract proves that their results hold in more general environments. The hold-up problem cannot easily be mitigated.

The second contribution is providing new insight into problems resulting from asymmetric information. It is often argued that problems can be partially solved by mandatory information disclosure. The superiority of the bundled contract suggests that too much information disclosure, as in dynamic or separate contracts, leads to antagonism. Although information disclosure may have an advantage when the parties want to implement a complicated outcome, in the simple situation discussed in this paper, information disclosure harms the relationship between the buyer and seller. An arm's length relationship is justified.

The current analysis does not yet solve the following problem. Consider a seller who offers a finite amount of divisible goods to a buyer. They can divide the goods into some set of infinite and converging sequences. Although Fuchs (2007) showed that the first-best contract is asymptotically possible when there are infinitely many identical goods, it is not yet clear whether this result still holds for converging sequences. This analysis is left for future investigations.

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A Appendix

Contracts that induce only one investment are considered here. It is assumed that $\bar{\theta}_1 = \bar{\theta}_2 \equiv \bar{\theta}$, $\underline{\theta}_1 = \underline{\theta}_2 \equiv \underline{\theta}$, $p^h \equiv p_1^h = p_2^h$, $p^l \equiv p_1^l = p_2^l$, and $e \equiv e_1 = e_2$. Define $\theta_H \equiv 2\bar{\theta}$, $\theta_M \equiv \bar{\theta} + \underline{\theta}$, and $\theta_L \equiv 2\underline{\theta}$.

Without loss of generality, consider contracts that induce investment only in good 1. From the above analysis, dynamic contracts are of no use. The optimal contract that induces only one investment is either separate or bundled. The optimal separate contract inducing only one investment is

$$\begin{aligned}\hat{q}_1\left(\bar{\theta}\right) &= \hat{q}_2\left(\bar{\theta}\right) = \hat{q}_2\left(\underline{\theta}\right) = 1,\\ \hat{q}_1\left(\underline{\theta}\right) &= 1 - \frac{e}{\left(p^h - p^l\right)\bar{\theta}}.\end{aligned}$$

This contract attains total welfare

$$W^o - \left(1 - p^h\right)\underline{\theta}\frac{e}{\left(p^h - p^l\right)\bar{\theta}}, \tag{13}$$

where $W^o \equiv p^h\bar{\theta} + \left(1 - p^h\right)\underline{\theta} + p^l\bar{\theta} + \left(1 - p^l\right)\underline{\theta} - e = p^h p^l \theta_H + \left\{p^h\left(1 - p^l\right) + p^l\left(1 - p^h\right)\right\}\theta_M + \left(1 - p^h\right)\left(1 - p^l\right)\theta_L - e$.

Next, bundled contracts that maximize total welfare W^o under constraints (5), (6), and the two constraints

$$\begin{aligned}p^h\left\{\tilde{t}\left(\theta_H\right) - \tilde{t}\left(\theta_M\right) - \tilde{z}\left(\theta_H\right) + \tilde{z}\left(\theta_M\right)\right\} + \left(1 - p^h\right)\left\{\tilde{t}\left(\theta_M\right) - \tilde{t}\left(\theta_L\right) - \tilde{z}\left(\theta_M\right) + \tilde{z}\left(\theta_L\right)\right\}&\\ \leq \frac{e}{p^h - p^l},&\\ p^l\left\{\tilde{t}\left(\theta_H\right) - \tilde{t}\left(\theta_M\right) - \tilde{z}\left(\theta_H\right) + \tilde{z}\left(\theta_M\right)\right\} + \left(1 - p^l\right)\left\{\tilde{t}\left(\theta_M\right) - \tilde{t}\left(\theta_L\right) - \tilde{z}\left(\theta_M\right) + \tilde{z}\left(\theta_L\right)\right\}&\\ \geq \frac{e}{p^h - p^l},&\end{aligned}$$

are considered. The final new constraints allows only one investment to be made. When total welfare is maximized, constraint (5) is binding, while constraint (6) is not, as in Section 2.1, and these constraints can be rewritten as

$$\begin{aligned}&p^h\left[\theta_H\left\{\tilde{q}\left(\theta_H\right) - \tilde{q}\left(\theta_M\right)\right\} - \tilde{z}\left(\theta_H\right) + \tilde{z}\left(\theta_M\right)\right]\\ &\quad + \left(1 - p^h\right)\left[\theta_M\left\{\tilde{q}\left(\theta_M\right) - \tilde{q}\left(\theta_L\right)\right\} - \tilde{z}\left(\theta_M\right) + \tilde{z}\left(\theta_L\right)\right] \leq \frac{e}{p^h - p^l},\end{aligned} \tag{14}$$

and

$$\begin{aligned}&p^l\left[\theta_H\left\{\tilde{q}\left(\theta_H\right) - \tilde{q}\left(\theta_M\right)\right\} - \tilde{z}\left(\theta_H\right) + \tilde{z}\left(\theta_M\right)\right]\\ &\quad + \left(1 - p^l\right)\left[\theta_M\left\{\tilde{q}\left(\theta_M\right) - \tilde{q}\left(\theta_L\right)\right\} - \tilde{z}\left(\theta_M\right) + \tilde{z}\left(\theta_L\right)\right] \geq \frac{e}{p^h - p^l}.\end{aligned} \tag{15}$$

Under these constraints and $\tilde{q}\left(\theta_H\right) \geq \tilde{q}\left(\theta_M\right) \geq \tilde{q}\left(\theta_L\right)$, the welfare maximizer is, if $\left(1 - p^l\right)\theta_M \geq \frac{e}{p^h - p^l}$,

$$\begin{aligned}\tilde{q}\left(\theta_L\right) &= 1 - \frac{e}{\left(1 - p^l\right)\left(p^h - p^l\right)\theta_M},\\ \tilde{q}\left(\theta_H\right) &= \tilde{q}\left(\theta_M\right) = 1,\\ \tilde{z}\left(\theta_H\right) &= \tilde{z}\left(\theta_M\right) = \tilde{z}\left(\theta_L\right) = 0,\end{aligned}$$

and, if $\frac{e}{p^h - p^l} > \left(1 - p^l\right) \theta_M$,

$$\begin{aligned}
\tilde{q}\left(\theta_H\right) &= \tilde{q}\left(\theta_M\right) = 1, \\
\tilde{q}\left(\theta_L\right) &= 0, \\
\tilde{z}\left(\theta_H\right) &= \tilde{z}\left(\theta_M\right) = 0, \\
\tilde{z}\left(\theta_L\right) &= \frac{e}{\left(1 - p^l\right)\left(p^h - p^l\right)} - \theta_M.
\end{aligned}$$

Total welfare under these contracts is, if $\left(1 - p^l\right)\left(p^h - p^l\right)\theta_M \geq e$,

$$W^o - \left(1 - p^h\right)\left(1 - p^l\right)\theta_L \frac{e}{\left(1 - p^l\right)\left(p^h - p^l\right)\theta_M}, \tag{16}$$

and if $p^l\left(p^h - p^l\right)\theta_H \geq e > \left(1 - p^l\right)\left(p^h - p^l\right)\theta_M$,

$$W^o - \left(1 - p^h\right)\left(1 - p^l\right)\left\{\theta_L + \frac{e}{\left(1 - p^l\right)\left(p^h - p^l\right)} - \theta_M\right\}. \tag{17}$$

Because (13) is larger than both (16) and (17), the separate contract is superior to the bundled contract when one investment is induced.