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"Irreversible Investment under Competition with a
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Irreversible Investment under Competition with a Markov Switching Regime*

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Irreversible Investment under Competition with a Markov Switching Regime

Abstract

In this paper, we study an investment problem in which two asymmetric firms face competition and the regime characterizing economic conditions follows Markov switching. We derive the value functions and investment thresholds of a leader and a follower. One of the interesting results is that in contrast to the case of no regime switching, even if the current market size is small, both advantaged and disadvantaged firms have an incentive to become a leader in some parameter settings.

Keywords: Real options; Competition; Timing game; Regime switch

JEL classifications: C73; D43; D81; E32

1 Introduction

The real options approach studies an investment problem where the project value is uncertain in the future and the cost of investment incurs some irreversibility. The theory has been developed by some seminal papers, such as Brennan and Schwartz (1985) and McDonald and Siegel (1986), and there are now a lot of theoretical and empirical research contributions. Note that many existing papers in the literature consider the investment decision of a monopolistic firm. However, as Dixit and Pindyck (1994) point out, it is becoming more and more important to study an investment under competition because not only it enables us to analyze a more realistic situation but also the economy is globalizing under worldwide deregulations and competition becomes fierce as a result.

There are some papers on the investment timing under competition. Among them, Grenadier (1996) is regarded as a pioneering paper. He models a real estate market with two firms using a real options framework and claims that his model explains a US construction boom in 1990s. Other important theoretical papers include Huisman and Kort (1999) and Nielsen (2002). A good survey of game theoretic real options models is written by Huisman et al. (2004). It is worth noting that most of the papers assume that firms are symmetric in irreversible costs and project revenues. Under the assumption of symmetric firms, all firms take the same strategy and thus it is randomly decided which firm becomes a leader or a follower.

Recently, Pawlina and Kort (2006) consider the case where two firms are asymmetric in their irreversible costs and present some theoretical results. Their model has three patterns of equilibrium: preemptive, sequential and simultaneous equilibria. In a preemptive equilibrium, both firms have an incentive to become a leader. On the other hand, in a sequential (simultaneous, resp.) equilibrium, a firm disadvantaged in costs (both firms, resp.) has no incentive to become a leader. Takashima et al. (2008) investigate an electricity market in which two firms are asymmetric in cost parameters and operating options. In these papers, one firm is always a leader while the other is always a follower, depending on the exogenous parameter setting. In other words, a firm who has an advantage in the operation always invest in the project before a disadvantaged firm. Kijima and Shibata (2005) and Bouis et al. (2009) extend such approaches to the framework of three or more symmetric firms.

From another viewpoint, there are several papers that introduce regime uncertainty within a real options analysis to capture the economic cycles. As shown by the worldwide financial crisis after the failure of Lehman Brothers in September 2008, the change of regime can have a huge impact on economic circumstances. Baba and Packer (2009) empirically investigate dislocations in the foreign exchange (FX) swap market between the US dollar and three major European currencies. They report that almost all the FX swap deviates from the covered interest rate parity after the Lehman failure, indicating a big effect caused by the change of economic conditions.

Theoretical papers studying regime shifts within a real options framework include Chapter 9 of Dixit and Pindyck (1994), Hassett and Metcalf (1999), Pawlina and Kort (2005) and Nishide and Nomi (2009). Typically, regime uncertainty is modeled in a way that parameters describing the state variables follow a Markov chain as in Driffill et al. (2003), Guo et al. (2005) and Hackbarth and Miao (2011). Among them, Driffill et al. (2009) study the investment decision for a project whose risk premia and other exogenous parameters are subject to a stationary Markov chain. They derive a simultaneous ordinary differential equation system that can calculate an investment threshold for each regime. Their major finding is that the Markov switching risk causes a delay in the expected timing of the investment.

In this paper, we consider a situation where two asymmetric firms face an investment problem under competition and the market regime is randomly switching. In other words, a Markov switching regime as in Driffill et al. (2009) is introduced into the model of Pawlina and Kort (2006). To the authors' best knowledge, this paper is the first attempt to combine the competitive real options model with a Markov switching regime. Our analysis is interesting because not only the model is an extension of the previous papers to a more general and realistic setup but also it enables us to describe various patterns of the market entry. For example, it is possible that the type of equilibrium changes from the preemptive equilibrium to the sequential equilibrium when the economic situation becomes worse, which cannot occur in Pawlina and Kort (2006). That is, our model can describe regime-dependent equilibria.

Some of the interesting findings are as follows:

1. A wide variety of strategies can be described in a unified framework;

2. The investment thresholds of one firm as a leader and a follower are more different in a recession than in a boom;

3. Sequential investment is more possible in a recession than in a boom;

4. The equity risk premium in our model is consistent with other theoretical and empirical works.

For the first finding, recall that, in previous contributions, if both firms wait for the investment at the initial state, only an advantaged firm has an incentive to invest earlier and can always become a leader. This means that in a newly developing market, a less profitable firm never invests and enters the market before a more profitable firm. However, in our model, it is possible that both firms have an incentive to invest earlier, or that both firm simultaneously invest even if both firms wait for the investment at the initial state.

Second, we investigate the difference between the thresholds of one firm as a leader and a follower. The difference is measured by the ratio of the threshold in the case of a follower to that for a leader. We find that investment strategies in a bad regime are more different than in a good regime, and that investment strategy of an advantaged firm is more different than that of a disadvantaged firm.

Third, as in Pawlina and Kort (2006), we calculate the condition for which type of equilibrium to happen. From the calculation results, we find that a sequential equilibrium is more possible in a bad regime than in a good one, especially when the transition probability from a bad regime to a good one is high.

Forth and final, we find that the firm's beta in a recession is higher than that in a boom. While Aguerrevere (2009) presents a result in a one-regime setting that the beta is higher when the demand factor is lower. We show several results on how the regime shift affects the beta of a firm. As we shall see, our findings echo previous theoretical and empirical papers.

This paper is organized as follows. In the next section, we concisely review the model and results of Pawlina and Kort (2006) as a benchmark case. Section 3 presents our model that introduces Markov regime switching into the case of asymmetric firms. In Section 4, we implement a numerical analysis and show interesting findings as mentioned above. Section 5 provides some concluding remarks.

2 The Model

2.1 Cash Flow and Market Settings

Consider a situation where two firms are facing an investment decision. Superscript $i \in \{1, 2\}$ denotes the identity of a firm. Each firm has the same potential investment project whose state variable (demand shock) is denoted by P_t. Prior to making an investment, firms generate no cash flow. It can be interpreted that two firms are considering the entry in a market whose size is represented by P_t. We assume that P_t follows a stochastic differential equation as

$$\mathrm{d}P_t = \mu_{\epsilon(t)} P_t \mathrm{d}t + \sigma_{\epsilon(t)} P_t \mathrm{d}z_t,$$

with initial value $P_0 = P$. Here, the expected growth rate μ and the volatility σ are dependent on $\epsilon(t)$, the regime at time t. We assume that there are only two regimes in the economy, so that we have

$$(\mu_\epsilon, \sigma_\epsilon) = \begin{cases} (\mu_1, \sigma_1) & \text{if } \epsilon = 1, \\ (\mu_2, \sigma_2) & \text{if } \epsilon = 2. \end{cases}$$

The key assumption is that the regime $\{\epsilon(t)\}$ follows a stationary Markov chain as

$$\begin{aligned} 1 \to 2 &\quad \text{with intensity } \lambda_1, \\ 2 \to 1 &\quad \text{with intensity } \lambda_2. \end{aligned}$$

In later discussions, we regard regime 1 as a good state (boom) and regime 2 as a bad one (recession).

Suppose that firm i enters a market and earns a cash flow from the project after paying the investing cost K^i. Let τ_L^i denote the investment timing of firm i when the firm is a leader, and τ_F^i the timing in the case of a follower. If firm i becomes a leader, the firm receives $D^i(1)P_t$ until the other firm participates in the market, and $D^i(2)P_t$ after the investment by the other firm. Here, we assume $D^i(1) > D^i(2)$, implying that the entry of the other firm causes a decrease in the cash flow due to the competition. Thus the instantaneous cash flow of a leader from the project can be expressed as

$$\mathbf{1}_{\{\tau_L^i \le t < \tau_F^{\hat{i}}\}} D^i(1) P_t + \mathbf{1}_{\{t \ge \tau_F^{\hat{i}}\}} D^i(2) P_t, \tag{1}$$

where $\hat{i} = 3 - i$. When firm i decides to be a follower, the firm receives the instantaneous cash flow $D^i(2)P_t$ after the investment. The cash flow in this case is given by

$$\mathbf{1}_{\{t \geq \tau_F^i\}} D^i(2) P_t. \tag{2}$$

Finally, the discount rate r is assumed to be constant for simplicity.

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2.2 The Asymmetric Case without Regime Shift

In this subsection, we quickly review the investment problem of asymmetric firms without regime switching, considered by Pawlina and Kort (2006). The setup corresponds to the case $\mu \equiv \mu_1 = \mu_2$ and $\sigma \equiv \sigma_1 = \sigma_2$.[2]

Suppose first that firm i is a follower and let V_F^i and τ_F^i denote the value function and the investment timing of firm i, respectively. The optimal investment time takes the form

$$\tau_F^i = \inf\{t \geq 0;\ P_t \geq \bar{P}_F^i\}$$

and the value function is given by

$$V_F^i(P) = \mathbf{1}_{\{P \geq \bar{P}_F^i\}} \left(\frac{D^i(2)P}{r - \mu} - K^i \right) + \mathbf{1}_{\{P < \bar{P}_F^i\}} \left(\frac{P}{\bar{P}_F^i} \right)^{\alpha} \left(\frac{D^i(2)\bar{P}_F^i}{r - \mu} - K^i \right).$$

Here, the optimal threshold as a follower is

$$\bar{P}_F^i = \frac{\alpha}{\alpha - 1}(r - \mu)\frac{K^i}{D^i(2)}.$$

Let denote G_L^i the net present value of the project for firm i as a leader for $t < \tau_F^{\hat{i}}$. Then G_L^i, is calculated as

$$\begin{aligned}
G_L^i(P) :=& \mathbb{E}\left[\int_t^{\tau_F^{\hat{i}}} \mathrm{e}^{-r(u-t)} D^i(1) P_u \mathrm{d}u + \int_{\tau_F^{\hat{i}}}^{\infty} \mathrm{e}^{-r(u-t)} D^i(2) P_u \mathrm{d}u \,\middle|\, P_t = P \right] \\
=& D^i(1)\mathbb{E}_P\left[\int_t^{\infty} \mathrm{e}^{-r(u-t)} P_u \mathrm{d}u \right] - (D^i(1) - D^i(2))\mathbb{E}_P\left[\int_{\tau_F^{\hat{i}}}^{\infty} \mathrm{e}^{-r(u-t)} P_u \mathrm{d}u \right] \\
=& \frac{D^i(1)P}{r - \mu} - \left(\frac{P}{\bar{P}_F^{\hat{i}}} \right)^{\alpha} \frac{(D^i(1) - D^i(2))\bar{P}_F^{\hat{i}}}{r - \mu}.
\end{aligned}$$

Following the equilibrium notion of Fudenberg and Tirole (1985), we suppose that firm i has an incentive to invest in the project when $G_L^i(P) - K^i \geq V_F^i(P)$. In other words, denoting by $\bar{P}_L^i$ the investment threshold of firm i as a leader, $\bar{P}_L^i$ satisfies the equation

$$\frac{D^i(1)\bar{P}_L^i}{r - \mu} + \left(\frac{\bar{P}_L^i}{\bar{P}_F^{\hat{i}}} \right)^{\alpha} \frac{(D^i(2) - D^i(1))\bar{P}_F^{\hat{i}}}{r - \mu} - K^i = \left(\frac{\bar{P}_L^i}{\bar{P}_F^{\hat{i}}} \right)^{\alpha} \left(\frac{D^i(2)\bar{P}_F^{\hat{i}}}{r - \mu} - K^i \right).$$

The above equation can be rearranged into

$$\frac{D^i(1)\bar{P}_L^i}{r-\mu} - \left(\frac{\bar{P}_L^i}{\bar{P}_F^{\hat{i}}}\right)^\alpha \frac{D^i(1)\bar{P}_F^{\hat{i}}}{r-\mu} - \left[1 - \left(\frac{\bar{P}_L^i}{\bar{P}_F^{\hat{i}}}\right)^\alpha\right] K^i = 0. \tag{3}$$

Throughout the following analysis, we lose no generality in assuming that $\bar{P}_F^1 < \bar{P}_F^2$. Hereafter we say that firms 1 and 2 are advantaged and disadvantaged, respectively, if this inequality holds. As we shall see later, when the threshold levels are distinct between the two firms, the threshold as a leader may not exist. More concretely, when $\bar{P}_F^1 < \bar{P}_F^2$, there is always a unique $\bar{P}_L^1$ but there are totally three cases for the existence of $\bar{P}_L^2$, depending on the parameter setting: (i) there exist two real numbers that satisfy (3), (ii) there exist a unique value that satisfies (3), (iii) there exist no value that satisfies (3), for $i = 2$.

In what follows we consider only the case where $\bar{P}_L^1 < \bar{P}_L^2$ in addition to $\bar{P}_F^1 < \bar{P}_F^2$.[3] The following proposition describes the strategies of both firms, depending on the three cases.

Proposition 1 (Pawlina and Kort, 2006) *In the case of asymmetric firms and no regime switch, each firm takes the following strategy, depending on $\bar{P}_L^2$ and the initial value of P.*

- *Preemptive investment: In the case where there exist two real numbers that satisfy (3), denote the two values by $\bar{P}_L^2$ and $\tilde{P}_L^2$ with $\bar{P}_L^2 < \tilde{P}_L^2$.*
  - *For $P < \bar{P}_L^1$, both firms wait for the investment.*
  - *For $\bar{P}_L^1 \leq P < \bar{P}_L^2$, only firm 1 has an incentive to invest immediately in the project as a leader. Firm 2 has no incentive to invest and becomes a follower.*
  - *For $\bar{P}_L^2 \leq P < \tilde{P}_L^2$, both firms have an incentive of immediate investment. If one of the firms invests in the project, the other firm becomes a follower and waits for the investment.*
  - *For $\tilde{P}_L^2 \leq P < \bar{P}_F^2$, only firm 1 has an incentive to invest immediately in the project. Firm 2 has no incentive to invest and wait to invest.*
  - *For $P \geq \bar{P}_F^2$, both firms simultaneously invest in the project.*
- *Sequential investment: Otherwise, the strategy of each firm is described by the following:*[4]
  - *For $P < \bar{P}_L^1$, both firms wait for the investment.*

- *For $\bar{P}_L^1 \leq P < \bar{P}_F^2$, only firm 1 has an incentive to invest immediately in the project as a leader. Firm 2 has no incentive to invest and becomes a follower.*
- *For $P \geq \bar{P}_F^2$, both firms simultaneously invest in the project.*

Remark 1 In this paper, we focus on which strategy for each firm to take and what the market is like as a consequence. In other words, we pay no attention to which firm actually becomes a leader. We also exclude the case of coordination failure in which both firms simultaneously enter the market although it is not optimal. On the timing game and the results, refer to Fudenberg and Tirole (1985) for a general concept and Huisman and Kort (1999) for related topics in a real options analysis.

Hereafter, firm 2 is said to be fully disadvantaged if there exists no real number that satisfies (3) for $i = 2$. In other words, firm 2 has no incentive to become a leader if firm 2 is fully disadvantaged. Otherwise, we call firm 2 partly disadvantaged.

We observe from Proposition 1 that firm 1 is always a leader when the state variable starts at a low level. In other words, if investments in a newly developing market are considered within this setup, a firm that is profitable or has an advanced technology in costs can always enter the market before the other. However, in actual markets, there are some cases that a firm that seems less profitable enters a market way before an advantaged firm. In the next section, we present a model that can explain this fact. That is, a disadvantaged firms may enter a new market before an advantaged firm in our model.

3 The Asymmetric Case with Markov Regime Switching

In this section, we propose our model that introduces a Markov switching regime into Pawlina and Kort (2006), and show how results are different from the case of no regime switch. As in the previous section, we assume that firm 1 is advantaged for all regimes.

3.1 Follower's Problem

We firstly consider the problem of the follower's investment decision. Denote by $V_{F\epsilon}^i$ the value function of firm i for regime ϵ and by $G_{F\epsilon}^i$ the net present value of the immediate investment.

Recall that many papers report that uncertainty is negatively related to the economic conditions.[5] Following this empirical finding, we here assume $\mu_1 > \mu_2$ and $\sigma_1 < \sigma_2$, implying that regimes 1 and 2 represent a boom and a recession, respectively. When the parameters are modulated by a Markov chain with two possible states, there are two thresholds $\bar{P}^i_{F1}$ and $\bar{P}^i_{F2}$ with $\bar{P}^i_{F1} < \bar{P}^i_{F2}$.[6] Suppose that $\bar{P}^i_{F1} \leq P < \bar{P}^i_{F2}$ and the regime shifts from 2 to 1. Then the follower firm has an incentive to invest in the project all at once. Note that the investment is irreversible in the sense that the firm cannot cancel the project if the regime becomes 2 again. Figure 1 describes how the project values changes, depending on the value of P and the regime.

[Figure 1 is inserted here.]

We need to take the regime shift into account to derive the value function for each regime. The procedure is exactly the same as Driffill et al. (2009) and thus refer to their paper for a detailed discussion.

First, for $P \geq \bar{P}^i_{F2}$, firm i immediately invest in the project regardless of the realized regime. Hence the value function $V^i_{F\epsilon}$ is equal to the net present value of the project minus the cost $G^i_{F\epsilon} - K^i$, and $G^i_{F\epsilon}$ satisfies the following simultaneous ordinary differential equation (ODE hereafter) system:

$$\begin{cases} \dfrac{\sigma_1^2}{2}P^2\dfrac{\mathrm{d}^2 G^i_{F1}}{\mathrm{d}P^2} + \mu_1 P\dfrac{\mathrm{d}G^i_{F1}}{\mathrm{d}P} - rG^i_{F1} + \lambda_1(G^i_{F2} - G^i_{F1}) + D^i(2)P = 0, \\ \dfrac{\sigma_2^2}{2}P^2\dfrac{\mathrm{d}^2 G^i_{F2}}{\mathrm{d}P^2} + \mu_2 P\dfrac{\mathrm{d}G^i_{F2}}{\mathrm{d}P} - rG^i_{F2} + \lambda_2(G^i_{F1} - G^i_{F2}) + D^i(2)P = 0. \end{cases} \tag{4}$$

The last terms of (4) represent the received cash flow of the follower in regime ϵ because it has already entered in the market, and the fourth terms represent the possibility of regime shift from one to the other.

Since $G^i_{F\epsilon}$ evidently includes no option value, we conjecture that the function takes a linear form

$$G^i_{F\epsilon}(P) = \pi_\epsilon D^i(2)P.$$

Substituting it into the simultaneous ODEs, we have

$$\pi_\epsilon = \frac{r + \lambda_\epsilon + \lambda_{\hat{\epsilon}} - \mu_{\hat{\epsilon}}}{(r + \lambda_\epsilon - \mu_\epsilon)(r + \lambda_{\hat{\epsilon}} - \mu_{\hat{\epsilon}}) - \lambda_\epsilon \lambda_{\hat{\epsilon}}}, \tag{5}$$

where $\hat{\epsilon} = 3 - \epsilon$.

Second, we consider the case $\bar{P}^i_{F1} \le P < \bar{P}^i_{F2}$. When $\epsilon = 1$, the follower firm immediately invests in the project and value function is equal to $\pi_1 D^i(2)P - K^i$ with coefficient π_1 given by (5). On the other hand, the value function in regime 2 satisfies the following ODE:

$$\frac{\sigma_2^2}{2}P^2\frac{\mathrm{d}^2V^i_{F2}}{\mathrm{d}P^2} + \mu_2 P\frac{\mathrm{d}V^i_{F2}}{\mathrm{d}P} - rV^i_{F2} + \lambda_2(G^i_{F1} - K^i - V^i_{F2}) = 0.$$

We conjecture that the candidate function takes the form

$$V^i_{F2}(P) = b^i_{21}P^{\alpha_1} + b^i_{22}P^{\alpha_2} + b^i_{23}P + b^i_{24}. \tag{6}$$

The first two terms of (6) represent the option value to wait for the investment in the project, while the last two terms are the net present value of the cash flow after investment due to the regime shift. Substituting it into the ODE, we obtain

$$b^i_{23} = \frac{\lambda_2}{r + \lambda_2 - \mu_2}\pi_1 D^i(2), \quad b^i_{24} = -\frac{\lambda_2}{r + \lambda_2}K^i$$

and find that α_1 and α_2 are the roots of the quadratic equation

$$\frac{\sigma_2^2}{2}\alpha(\alpha - 1) + \mu_2\alpha - (r + \lambda_2) = 0. \tag{7}$$

Note that the value function must satisfy

$$V^i_{F2}(\bar{P}^i_{F2}) = G^i_{F2}(\bar{P}^i_{F2}) - K^i,$$

and

$$\lim_{P\uparrow\bar{P}^i_{F2}}\frac{\mathrm{d}V^i_{F2}}{\mathrm{d}P}(P) = \lim_{P\downarrow\bar{P}^i_{F2}}\frac{\mathrm{d}G^i_{F2}}{\mathrm{d}P}(P)$$

as value-matching and smooth-pasting conditions, respectively.

Third, for $P < \bar{P}^i_{F1}$, the value functions satisfy the following ODE system:

$$\begin{cases} \frac{\sigma_1^2}{2}P^2\frac{\mathrm{d}^2V^i_{F1}}{\mathrm{d}P^2} + \mu_1 P\frac{\mathrm{d}V^i_{F1}}{\mathrm{d}P} - rV^i_{F1} + \lambda_1(V^i_{F2} - V^i_{F1}) = 0, \\ \frac{\sigma_2^2}{2}P^2\frac{\mathrm{d}^2V^i_{F2}}{\mathrm{d}P^2} + \mu_2 P\frac{\mathrm{d}V^i_{F2}}{\mathrm{d}P} - rV^i_{F2} + \lambda_2(V^i_{F1} - V^i_{F2}) = 0. \end{cases} \tag{8}$$

The candidate function of $V^i_{F\epsilon}$ is conjectured to be

$$V^i_{F\epsilon}(P) = c^i_{\epsilon 1}P^{\gamma_1} + c^i_{\epsilon 2}P^{\gamma_2}, \quad k = 1, 2. \tag{9}$$

In contrast to (6), (9) contains only the option value since the regime shift does not induce an immediate investment. Substituting (9) into (8) leads to the four equations:

$$\begin{cases} \left(\frac{\sigma_1^2}{2}\gamma_1(\gamma_1-1)+\mu_1\gamma_1-(r+\lambda_1)\right)c_{11}^i+\lambda_1 c_{21}^i=0, \\ \left(\frac{\sigma_1^2}{2}\gamma_2(\gamma_2-1)+\mu_1\gamma_2-(r+\lambda_1)\right)c_{12}^i+\lambda_1 c_{22}^i=0, \\ \left(\frac{\sigma_2^2}{2}\gamma_1(\gamma_1-1)+\mu_2\gamma_1-(r+\lambda_2)\right)c_{21}^i+\lambda_2 c_{11}^i=0, \\ \left(\frac{\sigma_2^2}{2}\gamma_2(\gamma_2-1)+\mu_2\gamma_2-(r+\lambda_2)\right)c_{22}^i+\lambda_2 c_{12}^i=0. \end{cases}$$

Since $\lim_{P\downarrow 0} V_{F\epsilon}^i(P)=0$, γ_1 and γ_2 must be the positive roots of the following quartic equation:

$$\left[\frac{\sigma_1^2}{2}\gamma(\gamma-1)+\mu_1\gamma-(r+\lambda_1)\right]\left[\frac{\sigma_2^2}{2}\gamma(\gamma-1)+\mu_2\gamma-(r+\lambda_2)\right]=\lambda_1\lambda_2. \tag{10}$$

The threshold in regime 1, denoted by $\bar{P}_{F1}^i$, satisfies

$$V_{F1}^i(\bar{P}_{F1}^i)=G_{F1}^i(\bar{P}_{F1}^i)-K^i$$

and

$$\lim_{P\uparrow\bar{P}_{F1}^i}\frac{\mathrm{d}V_{F1}^i}{\mathrm{d}P}(P)=\lim_{P\downarrow\bar{P}_{F1}^i}\frac{\mathrm{d}G_{F1}^i}{\mathrm{d}P}(P)$$

as value-matching and smooth-pasting conditions. Similarly, in regime 2, the continuity and high-contact conditions are given by

$$\lim_{P\uparrow\bar{P}_{F1}^i}V_{F2}^i(P)=\lim_{P\downarrow\bar{P}_{F1}^i}V_{F2}^i(P)$$

and

$$\lim_{P\uparrow\bar{P}_{F1}^i}\frac{\mathrm{d}V_{F2}^i}{\mathrm{d}P}(P)=\lim_{P\downarrow\bar{P}_{F1}^i}\frac{\mathrm{d}V_{F2}^i}{\mathrm{d}P}(P),$$

respectively.

We now summarize the result as a proposition.

Proposition 2 *The value function of the follower firm in regime 1 is given by*

$$V_{F1}^i(P)=\begin{cases}\pi_1 D^i(2)P-K^i & \textit{for } P\geq\bar{P}_{F1}^i, \\ c_{11}^i P^{\gamma_1}+c_{12}^i P^{\gamma_2} & \textit{otherwise}\end{cases}$$

and in regime 2 by

$$V_{F2}^i(P) = \begin{cases} \pi_2 D^i(2)P - K^i & \textit{for } P \geq \bar{P}_{F2}^i, \\ b_{21}^i P^{\alpha_1} + b_{22}^i P^{\alpha_2} + \frac{\lambda_2}{r+\lambda_2-\mu_2}\pi_1 D^i(2)P - \frac{\lambda_2}{r+\lambda_2}K^i & \textit{for } \bar{P}_{F1}^i \leq P < \bar{P}_{F2}^i, \\ c_{21}^1 P^{\gamma_1} + c_{22}^i P^{\gamma_2} & \textit{otherwise.} \end{cases}$$

The coefficients and the investment thresholds are determined by the simultaneous equation system:

$$\pi_2 D^i(2)\bar{P}_{F2}^i - K^i = b_{21}^i(\bar{P}_{F2}^i)^{\alpha_1} + b_{22}^i(\bar{P}_{F2}^i)^{\alpha_2} + \frac{\lambda_2}{r+\lambda_2-\mu_2}\pi_1 D^i(2)\bar{P}_{F2}^i - \frac{\lambda_2}{r+\lambda_2}K^i,$$

$$\pi_2 D^i(2) = \alpha_1 b_{21}^i(\bar{P}_{F2}^i)^{\alpha_1-1} + \alpha_2 b_{22}^i(\bar{P}_{F2}^i)^{\alpha_2-1} + \frac{\lambda_2}{r+\lambda_2-\mu_2}\pi_1 D^i(2),$$

$$c_{11}^i(\bar{P}_{F1}^i)^{\gamma_1} + c_{12}^i(\bar{P}_{F1}^i)^{\gamma_2} = \pi_1 D^i(2)\bar{P}_{F1}^i - K^i,$$

$$\gamma_1 c_{11}^i(\bar{P}_{F1}^i)^{\gamma_1-1} + \gamma_2 c_{12}^i(\bar{P}_{F1}^i)^{\gamma_2-1} = \pi_1 D^i(2),$$

$$c_{21}^i(\bar{P}_{F1}^i)^{\gamma_1} + c_{22}^i(\bar{P}_{F1}^i)^{\gamma_2} = b_{21}^i(\bar{P}_{F1}^i)^{\alpha_1} + b_{22}^i(\bar{P}_{F1}^i)^{\alpha_2} + \frac{\lambda_2}{r+\lambda_2-\mu_2}\pi_1 D^i(2)\bar{P}_{F1}^i - \frac{\lambda_2}{r+\lambda_2}K^i,$$

$$\gamma_1 c_{21}^i(\bar{P}_{F1}^i)^{\gamma_1-1} + \gamma_2 c_{22}^i(\bar{P}_{F1}^i)^{\gamma_2-1} = \alpha_1 b_{21}^i(\bar{P}_{F1}^i)^{\alpha_1-1} + \alpha_2 b_{22}^i(\bar{P}_{F1}^i)^{\alpha_2-1} + \frac{\lambda_2}{r+\lambda_2-\mu_2}\pi_1 D^i(2),$$

$$\left(r + \lambda_1 - \mu_1\gamma_1 - \frac{\sigma_1^2}{2}\gamma_1(\gamma_1 - 1)\right)c_{11}^i = \lambda_1 c_{21}^i,$$

$$\left(r + \lambda_1 - \mu_1\gamma_2 - \frac{\sigma_2^2}{2}\gamma_2(\gamma_2 - 1)\right)c_{12}^i = \lambda_2 c_{22}^i.$$

Since there are totally 8 unknowns $\bar{P}_{F1}^i$, $\bar{P}_{F2}^i$, b_{21}^i, b_{22}^i, c_{11}^i, c_{12}^i, c_{21}^i and c_{22}^i while the system has eight equations, the system is theoretically solvable. However, it seems hard to obtain a closed-form solution. Therefore, we shall numerically calculate the simultaneous equations to solve and analyze the investment strategies.

3.2 Leader's Problem

In this subsection, we consider the investment decision of firm i as a leader. Let $G_{L\epsilon}^i$ denote the net present value of the project for a leader in regime ϵ after investment. Note that the function $G_{L\epsilon}^i$ is dependent on the thresholds of the follower firm $P_{F\epsilon}^{\hat{i}}$, since the cash flow varies according to the investment by the follower. Taking this into consideration, the NPVs of an immediate investment by the leader are described as Figure 2.

[Figure 2 is inserted here.]

We derive the functions $G_{L\epsilon}^i$ by noting these regime relations.

Consider first the case $P \geq P^{\hat{i}}_{F2}$. In this situation both firms are willing to immediately enter the market and so $G^i_{L\epsilon}(P) = G^i_{F\epsilon}(P) = \pi_\epsilon D^i(2)P$, where π_ϵ are given by (5).

For $P^{\hat{i}}_{F1} \leq P < P^{\hat{i}}_{F2}$, $G^i_{L1} = \pi_1 D^i(2)P$ since both firms immediately invest and receives the cash flow in regime 1. On the other hand, we easily verify that G^i_{L2} satisfies the following ODE

$$\frac{\sigma_2^2}{2}P^2\frac{\mathrm{d}^2G^i_{L2}}{\mathrm{d}P^2} + \mu_2 P\frac{\mathrm{d}G^i_{L2}}{\mathrm{d}P} - rG^i_{L2} + \lambda_2(G^i_{F1} - G^i_{L2}) + D^i(1)P = 0. \tag{11}$$

Note that (11) includes G^i_{F1} and that it is already solved in the previous discussions. The last term of (11) represents the cash flow of firm i as a leader. Let the candidate function of G^i_{L2} be conjectured as

$$G^i_{L2}(P) = e^i_{21}P^{\alpha_1} + e^i_{22}P^{\alpha_2} + e^i_{23}P. \tag{12}$$

The first two term describe the (negative) option value for the regime shift at which the other firm enters the market, while the last term is equal to the net present value of the cash flow in the future. Substituting the particular solution $e^i_{23}P$ into the ODE yields

$$e^i_{23} = \frac{D^i(1) + \lambda_2\pi_1 D^i(2)}{r + \lambda_2 - \mu_2}.$$

In the case of the leader firm, only the value-matching condition at $\bar{P}^{\hat{i}}_{F2}$ holds, i.e.,

$$G^i_{L2}(P^{\hat{i}}_{F2}) = G^i_{F2}(P^{\hat{i}}_{F2})$$

and any smooth-pasting condition is not necessary (see Driffill et al., 2009).

For $P < P^{\hat{i}}_{F1}$, the ODEs of $G^i_{L\epsilon}$ are given by

$$\begin{cases} \dfrac{\sigma_1^2}{2}P^2\dfrac{\mathrm{d}^2G^i_{L1}}{\mathrm{d}P^2} + \mu_1 P\dfrac{\mathrm{d}G^i_{L1}}{\mathrm{d}P} - rG^i_{L1} + \lambda_1(G^i_{L2} - G^i_{L1}) + D^i(1)P = 0, \\ \dfrac{\sigma_2^2}{2}P^2\dfrac{\mathrm{d}^2G^i_{L2}}{\mathrm{d}P^2} + \mu_2 P\dfrac{\mathrm{d}G^i_{L2}}{\mathrm{d}P} - rG^i_{L2} + \lambda_2(G^i_{L1} - G^i_{L2}) + D^i(1)P = 0. \end{cases} \tag{13}$$

The candidate function of $G^i_{L\epsilon}$ is conjectured to be

$$G^i_{L\epsilon}(P) = h^i_{\epsilon 1}P^{\gamma_1} + h^i_{\epsilon 2}P^{\gamma_2} + h^i_{\epsilon 3}P. \tag{14}$$

We can provide an interpretation for (14) in a similar way to the one for (6). Substituting the particular solution $h^i_{\epsilon 3}P$ into the ODEs, we obtain

$$h^i_{\epsilon 3} = \pi_\epsilon D^i(1).$$

In regime 1, the value-matching condition at $\bar{P}^{\hat{i}}_{F1}$ is given by

$$G^i_{L1}(P^{\hat{i}}_{F1}) = G^i_{F1}(P^{\hat{i}}_{F1}).$$

In regime 2, we have continuity and high-contact conditions as

$$\lim_{P\uparrow P^{\hat{i}}_{F1}} G^i_{L2}(P) = \lim_{P\downarrow P^{\hat{i}}_{F1}} G^i_{L2}(P)$$

and

$$\lim_{P\uparrow P^{\hat{i}}_{F1}} \frac{\mathrm{d}G^i_{L2}(P)}{\mathrm{d}P} = \lim_{P\downarrow P^{\hat{i}}_{F1}} \frac{\mathrm{d}G^i_{L2}(P)}{\mathrm{d}P}.$$

The following proposition summarizes the case of the leader.

Proposition 3 *The NPV of cash flow for firm i as a leader is given by*

$$G^i_{L1}(P) = \begin{cases} \pi_1 D^i(2)P & \textit{for } P \geq \bar{P}^{\hat{i}}_{F1}, \\ h^i_{11}P^{\gamma_1} + h^i_{12}P^{\gamma_2} + \pi_1 D^i(1)P & \textit{otherwise} \end{cases}$$

in regime 1 and

$$G^i_{L2}(P) = \begin{cases} \pi_2 D^i(2)P & \textit{for } P \geq \bar{P}^{\hat{i}}_{F2}, \\ e^i_{21}P^{\alpha_1} + e^i_{22}P^{\alpha_2} + \frac{D^i(1)+\lambda_2\pi_2 D^i(2)}{r+\lambda_2-\mu_2}P & \textit{for } \bar{P}^{\hat{i}}_{F1} \leq P < \bar{P}^{\hat{i}}_{F2}, \\ h^i_{21}P^{\gamma_1} + h^i_{22}P^{\gamma_2} + \pi_2 D^i(1)P & \textit{otherwise} \end{cases}$$

in regime 2. The coefficients and the investment thresholds are determined by the simultaneous equation system:

$$e^i_{21}(P^{\hat{i}}_{F2})^{\alpha_1} + e^i_{22}(P^{\hat{i}}_{F2})^{\alpha_2} + \frac{D^i(1)+\lambda_2\pi_2 D^i(2)}{r+\lambda_2-\mu_2}P^{\hat{i}}_{F2} = \pi_2 D^i(2)P^{\hat{i}}_{F2},$$
$$h^i_{11}(P^{\hat{i}}_{F1})^{\gamma_1} + h^i_{12}(P^{\hat{i}}_{F1})^{\gamma_2} + \pi_1 D^i(1)P^{\hat{i}}_{F1} = \pi_1 D^i(2)P^{\hat{i}}_{F1},$$
$$h^i_{21}(P^{\hat{i}}_{F1})^{\gamma_1} + h^i_{22}(P^{\hat{i}}_{F1})^{\gamma_2} + \pi_2 D^i(1)P^{\hat{i}}_{F1} = e^i_{21}(P^{\hat{i}}_{F1})^{\alpha_1} + e^i_{22}(P^{\hat{i}}_{F1})^{\alpha_2} + \frac{D^i(1)+\lambda_2\pi_2 D^i(2)}{r+\lambda_2-\mu_2}P^{\hat{i}}_{F1},$$
$$\gamma_1 h^i_{21}(P^{\hat{i}}_{F1})^{\gamma_1-1} + \gamma_2 h^i_{22}(P^{\hat{i}}_{F1})^{\gamma_2-1} + \pi_2 D^i(1)$$
$$= \alpha_1 e^i_{21}(P^{\hat{i}}_{F1})^{\alpha_1-1} + \alpha_2 e^i_{22}(P^{\hat{i}}_{F1})^{\alpha_2-1} + \frac{D^i(1)+\lambda_2\pi_2 D^i(2)}{r+\lambda_2-\mu_2},$$
$$\left(r+\lambda_1-\mu_1\gamma_1-\frac{\sigma_1^2}{2}\gamma_1(\gamma_1-1)\right)h^i_{11} = \lambda_1 h^i_{21}$$
$$\left(r+\lambda_1-\mu_1\gamma_2-\frac{\sigma_1^2}{2}\gamma_2(\gamma_2-1)\right)h^i_{12} = \lambda_1 h^i_{22}$$

Finally $\bar{P}^i_{L\epsilon}$, the threshold of the leader in regime ϵ, can be obtained by the condition that $G^i_{L\epsilon}(\bar{P}^i_{L\epsilon}) - K^i = V^i_{F\epsilon}(\bar{P}^i_{L\epsilon})$.

3.3 Existence of the thresholds as a leader

Before examining the results in detail, we shall examine whether or not the thresholds $\bar{P}^i_{L\epsilon}$ exist. Similarly to Proposition 1, we show that a disadvantaged firm has an incentive to become a leader for some cases but does not have for the other cases, depending on the parameter values.

Suppose that $K^1 < K^2$, $D^1(1) > D^2(1)$ and $D^1(2) > D^2(2)$. Then firm 2 is apparently disadvantaged in this setting for both regimes. In other words, $\bar{P}^1_{F\epsilon} < \bar{P}^2_{F\epsilon}$ for $\epsilon = 1, 2$.

First, we present Figure 3, describing the value function of firm 1.

[Figure 3 is inserted here.]

Although the equation system is rather complicated, the shape of the value function is quite similar to the case of no regime switch. When the current regime is ϵ and $\bar{P}^1_{L\epsilon} \leq P < \bar{P}^2_{F\epsilon}$, the net present value of immediate investment is greater than the value to wait for the investment. This means that an advantaged firm always has an incentive to invest in the project for some interval.

Second, the value function of a partly disadvantaged firm is depicted in Figure 4. The figure indicates that firm 2 is not fully disadvantaged and has an incentive to become a leader for some set of the state variable.

[Figure 4 is inserted here.]

More concretely, we observe from the figure that $G^2_{L\epsilon} - K^2 \geq V^2_{F\epsilon}$ for $\bar{P}^2_{L\epsilon} \leq P < \tilde{P}^2_{L\epsilon}$. This indicates that firm 2 has an incentive to immediately invest in the project to become a leader for P to be in the interval.

Third, we provide Figure 5, describing the value function of firm 2 in the case where the firm is fully disadvantaged.

[Figure 5 is inserted here.]

In this example, there does not exist a value of P that satisfies $G^2_{L\epsilon} - K^2 \geq V^2_{F\epsilon}$. In other words, firm 2 never has an incentive to become a leader, and firm 1 always invests in the project before firm 2.

With these examples, we observe that the results in Proposition 1 still hold in each regime. However, taking the regime shift into account, it can be the case that a disadvantaged firm can invest in the project before the other firm at the time of regime shift even when the initial value of P is low. In what follows, we shall show that it can actually happen and present what conditions lead to such situations.

4 Numerical Analyses

In this section, we study with numerical examples how each firm chooses its investment strategy, depending on the strategy of the other firm. Also some comparative static results are provided. It will be observed that our numerical results show a stark contrast to the results obtained in the previous literature.

4.1 Various Investment Strategies

We first present three example to show that our model is rich and flexible enough to explain many actual situations within a unified framework.

4.1.1 Case 1: Benchmark case

The parameter values in Table 1 are used for the numerical analysis as a benchmark case. Firm 1 is advantaged in both regimes.

[Table 1 is inserted here.]

With these parameter values, we obtain $\bar{P}^1_{L1} = 0.991$, $\bar{P}^1_{L2} = 1.487$, $\bar{P}^1_{F1} = 2.561$, $\bar{P}^1_{F2} = 4.439$, $\bar{P}^2_{L1} = 1.574$, $\tilde{P}^2_{L1} = 2.239$, $\bar{P}^2_{F1} = 3.687$ and $\bar{P}^2_{F2} = 6.392$. The numerical results actually shows that $\bar{P}^1_{F\epsilon} < \bar{P}^2_{F\epsilon}$ for $\epsilon = 1, 2$.

Figure 6 describes the value function and the investment thresholds of firm 2.

[Figure 6 is inserted here.]

In regime 1, firm 2 is partly disadvantaged, while the firm is fully disadvantaged in regime 2. Table 2 summarizes the investment strategies that each firm chooses, depending on the level of the state variable P.

[Table 2 is inserted here.]

Numbers in the table represent the label of the investing firm, and a blank cell indicates that both firms wait for investing. The situation where both firms have an incentive to invest and only one of them can become a leader is represented by $\times$. For example, for

$\bar{P}^1_{L1} \leq P < \bar{P}^1_{L2}$, firm 1 becomes a leader and firm 2 a follower in regime 1, while both firms wait for investing in regime 2. For $P \geq \bar{P}^2_{F2}$, both firms immediately and simultaneously invest.

In this case, firm 1 always has an incentive to become a leader for $P \geq \bar{P}^1_{L2}$. However, firm 2 has the incentive only for $\bar{P}^2_{L1} \leq P < \tilde{P}^2_{L1}$ in regime 1 and can never become a leader in regime 2. We observe that in this parameter setting, only firm 1 can be a leader when the state variable starts at a lower level like previous theoretical papers.

4.1.2 Case 2: Unknown winner

In this case, we choose $K^2 = 11$ and assume that the other parameters remain the same. Then, the thresholds are calculated as $\bar{P}^1_{L1} = 1.010$, $\bar{P}^1_{L2} = 1.536$, $\bar{P}^1_{F1} = 2.561$, $\bar{P}^1_{F2} = 4.439$, $\bar{P}^2_{L1} = 1.306$, $\tilde{P}^2_{L1} = 2.417$, $\bar{P}^2_{L2} = 2.183$, $\tilde{P}^2_{L2} = 4.171$, $\bar{P}^2_{F1} = 3.380$ and $\bar{P}^2_{F2} = 5.860$. A major difference from case 1 is that firm 2 is partly disadvantaged in both regimes 1 and 2.

Table 3 presents the investment strategies that each firm takes for each regime.

[Table 3 is inserted here.]

An interesting observation is as follows. Suppose that the current regime is a recession ($\epsilon = 2$) and the initial value P is in $[\bar{P}^2_{L1}, \bar{P}^1_{L2})$. Then both firms do not invest immediately and wait until the state variable becomes higher. However, when the regime suddenly changes from 2 to 1, both firms have an incentive to invest and enter the market simultaneously.

This result shows a stark contrast to Pawlina and Kort (2006). That is, in their model without regime switching in the economic conditions, a firm that is more profitable than the other always becomes a leader and enters the market before the other when the initial value of P is low. On the contrary, our model produces a situation where a disadvantaged firm may be a leader in a newly developing market, just by introducing a Markov chain in the exogenous parameters.

4.1.3 Case 3: Simultaneous investment

In this case, we choose $K^2 = 11$, $\lambda_1 = 0.001$, $\mu_2 = -0.1$, $\sigma_2 = 0.8$ and set the other parameters to be the same. With these parameter values, we obtain $\bar{P}^1_{L1} = 0.823$, $\bar{P}^1_{L2} = 2.938$, $\bar{P}^1_{F1} = 2.212$, $\bar{P}^1_{F2} = 8.386$, $\bar{P}^2_{L1} = 1.104$, $\tilde{P}^2_{L1} = 2.080$, $\bar{P}^2_{L2} = 4.160$, $\tilde{P}^2_{L2} = 7.879$,

$\bar{P}^2_{F1} = 2.920$, $\bar{P}^2_{F2} = 11.069$. Firm 2 is still partly disadvantaged both in regimes 1 and 2. A important difference from case 2 is $\bar{P}^2_{F1} < \bar{P}^1_{L2}$, so that all the thresholds in regime 1 are lower than those in regime 2.

Table 4 presents the investment strategies that each firm takes for each regime.

[Table 4 is inserted here.]

For $\bar{P}^2_{L1} \leq P < \tilde{P}^2_{L1}$, we obtain the same situation as in case 2. Another interesting observation is the following. Suppose that the current regime is a recession ($\epsilon = 2$) and that $\bar{P}^2_{F1} \leq P < \bar{P}^1_{L2}$. Then, both firms do not invest immediately and wait until the state variable becomes higher. However, when the regime changes from 2 to 1, both firms do not care about the decision of the other and simultaneously invest in the project. The result is an extreme version of case 2. The situation happens when regime 2 is critically bad for investment or when regime 1 is significantly good for investment. This is because more profitable firm can not enter at the lower level of demand in a recession while the economic condition is good for both firms enough to invest when the regime shifts to a boom. Recall again that previous theoretical papers of competitive real options approach cannot give such a scenario.

In summary, we have found from the numerical examples that our model is quite rich and flexible to explain many actual situations within a unified framework.

4.2 Difference in the Investment Strategies

We here investigate how the investment strategy differs, depending on whether a firm is a leader or a follower. We measure the difference of firm i's strategy in regime ϵ by $\bar{P}^i_{F\epsilon}/\bar{P}^i_{L\epsilon}$, the ratio of the threshold in the case of a follower to that for a leader. What we are interested in is how the difference is affected by the regime and the rates of regime shift λ_1 and λ_2 We use the base case parameter set in Table 1 except for $D^1(2) = D^2(2) = 0.5$. Table 5 shows the calculation results.

[Table 5 is inserted here.]

We find from the figure that the differences are larger in a recession and are smaller in a boom than in the one regime case for both firms and all sets of (λ_1, λ_2). This implies that when a boom–recession cycle is considered, an investment strategy in a recession is more affected by the cyclical structure. Another finding is that the differences of firm 1 are larger than those of firm 2, indicating that the investment strategy of an advantaged firm is more different than that of a disadvantaged firm.

Additionally, even though the impacts of λ_1 and λ_2 on the ratio are quite small, the difference of firm 1 in a recession with $\lambda_2 = 0.4$ is the largest. This means that λ_2 has a big impact in a recession than λ_1.

An intuition of the observations is as follows. As we said in page 9, the economy is more volatile and uncertain in a recession than in a boom. A firm optimally takes the uncertainty into account and set the investment threshold higher in a recession. In other words, the option value to wait is higher in a recession. This causes the more difference of strategies in regime 2.

4.3 Equilibrium Types

Pawlina and Kort (2006) examine the conditions for equilibria, depending on the parameter setting. They call the preemptive equilibrium if one of the firms is partially disadvantaged and can have an incentive to invest as a leader, and the sequential equilibrium if one of the firms is fully disadvantaged and always become a follower. We follow their analysis and examine which type of equilibrium happens.

To compare our result to Pawlina and Kort (2006), we suppose that $D(1) := D^i(1) = D^i(1)$ and $D(2) := D^i(2) = D^i(2)$. The first-mover advantage and cost asymmetry are defined by $\omega = D(1)/D(2)$ and $\kappa = K^2/K^1$, respectively. Pawlina and Kort (2006) show that a sequential equilibrium happens if $\kappa > \kappa^*$, where

$$\kappa^* = \left[\frac{\omega^\alpha - 1}{\alpha(\omega - 1)}\right]^{\frac{1}{\alpha-1}}.$$

Otherwise, a preemptive equilibrium happens and a disadvantaged firm can be a leader. While a closed-form expression of κ^* is obtained in the one-regime case, κ^* in our model needs to be found numerically. We use the base case parameter set in Table 1 again, except for $D^1(2) = D^2(2) = 0.5$.

Table 6 shows the calculation results as κ^*–ω mapping for various λ_1 and λ_2.

Table 6 is inserted here.]

The values of κ^* in a boom or a recession for various λ_1 and λ_2 are given in the table. In a boom, κ^*'s are larger than in the one regime case, while κ^*'s in a recession are smaller than those in the one regime case. Therefore, sequential investment is more possible in a recession than in a boom, especially when $\lambda_2 = 0.4$. Similar to the previous subsection, results in a recession with $\lambda_2 = 0.4$ are different from other results.

The above observation leads to an important implication. We see from the numerical results that the impact of introducing Markov switching regime is asymmetric on a boom and a recession and the sequential equilibrium is more likely when the economy is in a recession. Recall again that the economy in a recession is more uncertain and the option value of the follower firm becomes higher. On the other hand, the investment strategy as a follower is determined only by the NPV of the investment. Therefore it becomes more likely that $G^2_{L2} - K^2 < V^2_{F2}$ for all P. The implication echoes the intuition presented in the previous discussion.

4.4 Equity Risk Premium

In this subsection, we present a numerical analysis related to other theoretical papers on the equity risk premium. Following Carlson et al. (2004) and Aguerrevere (2009), we define the beta of firm i's equity as a leader in regime ϵ to be

$$\beta^i_{L\epsilon}(P) = \frac{\mathrm{Cov}_{P,\epsilon}[(\mathrm{d}P/P), (\mathrm{d}G^i_{L\epsilon}/G^i_{L\epsilon})]}{\mathrm{Var}_{P,\epsilon}[(\mathrm{d}P/P)]} = \frac{P}{G^i_{L\epsilon}(P)} G^{i\,\prime}_{L\epsilon}(P) \tag{15}$$

and that as a follower to be

$$\beta^i_{F\epsilon}(P) = \frac{\mathrm{Cov}_{P,\epsilon}[(\mathrm{d}P/P), (\mathrm{d}V^i_{F\epsilon}/V^i_{F\epsilon})]}{\mathrm{Var}_{P,\epsilon}[(\mathrm{d}P/P)]} = \frac{P}{V^i_{F\epsilon}(P)} V^{i\,\prime}_{F\epsilon}(P), \tag{16}$$

where $\mathrm{Var}_{P,\epsilon}$ and $\mathrm{Cov}_{P,\epsilon}$ are the variance and covariance operators conditional on (P, ϵ), respectively.[7] In this analysis, we select the parameter values to the ones estimated by Bhamra et al. (2009) except for K^i, $D(1)$ and $D(2)$ to match actual economic conditions. The parameters setting and the thresholds in this setup is given in Tables 7 and 8, respectively.

[Table 8 is inserted here.]

Table 9 shows the betas of both firms for a variety of P.

[Table 9 is inserted here.]

First, we observe from the table that a lower value of P leads to a higher beta. Recall that Aguerrevere (2009) shows the same finding in a one-regime setting and presents an economic interpretation of the result. Mathematically, if P is low, a negative option value associate with the potential entry of the other firm becomes high. This structure leads to a negative relationship between P and the beta.

Second, the betas of both firm in a recession are higher than that in a boom. This is a natural result because the volatility is higher in a recession. Note that Hoberg and Phillips (2010) empirically find that systematic risk decreases for industry booms in competitive industries. The result is consistent with our observation with regime switching.

Third, the betas of firm 2 are higher than the betas of firm 1, except for the case $P = 2$ and $\epsilon = 1$. The intuitive explanation is as follows. Suppose that firm 2 is a leader. As we have seen, the threshold of firm 1 for the investment as a follower is low compared to firm 2. Therefore, reflecting the potential entry of firm 1, the negative option value for firm 2 is more sensitive to P, implying a higher beta in most cases.

Next we compare the beta as a leader with that as a follower for firm 1 in Table 10. The double line in the table indicates that the threshold as a leader lies in the interval.

[Table 10 is inserted here.]

The betas as a leader are much higher than those of the follower before the leader's investment. However, it becomes lower after the investment and lower than that as a follower when P takes a higher value. This implies that the leader is riskier before investment than after investment. Carlson et al. (2011) show a similar result in a one-regime setting and present an economic intuition. The observation also explains why an advantaged firm does not always enter a newly developing market. That is, when a firm takes its business risk into consideration, it is better the firm allows another to firstly enter the market to avoid an additional risk. This finding with various equilibrium types sheds a new light on the analysis of an investment decision with competition.

5 Conclusion

In this paper, we introduce a switching regime as Driffill et al. (2003) into the model of Pawlina and Kort (2006) to consider the investment problem of asymmetric firms with regime uncertainty. In the case of no regime switch, a profitable firm always becomes a leader in the investment, and a disadvantaged firm never has an incentive to become a leader in a newly developing market. However, if there is uncertainty in regime, there are some parameter settings in which both firms can be a leader even when the initial state variable is in a lower level. This finding shows a stark contrast to Pawlina and Kort (2006) in that our model can provide richer results within a unified framework.

From the numerical calculation, we find several interesting results. First, the investment strategies of a leader and a follower are more different in a recession than in a boom when a boom–recession cycle is considered, and that the difference of an advantaged firm is more significant than that of a disadvantaged firm. Second, sequential investment is more likely in a recession than in a boom, especially when the transition rate from a recession to a boom is high. Third, several implications on the equity risk premium of a firm as a leader and a follower are obtained.

As a future study, it seems important to consider the changes of profitability and cost invoked by the regime. It is natural that the firm's profitability and cost are better in a boom than a recession. By doing this, we will be able to explain more complicated economic behavior of the firms facing the entry race under uncertainty.

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Endnotes

1 We do not consider the case where the discount rate r is modulated by a Markov chain because it produces no qualitative difference.

2 Pawlina and Kort (2006) consider the case where only cost parameters are asymmetric. The results in this subsection are essentially the same as theirs despite the difference.

3 The sufficient condition for $\bar{P}_L^1 < \bar{P}_L^2$ and $\bar{P}_F^1 < \bar{P}_F^2$ is that $D^1(1)/K^1 \geq D^2(1)/K^2$ and $D^1(2)/K^1 > D^2(2)/K^2$. In this case, firm 2 cannot be advantaged.

4 If (3) has exactly one solution for firm 2, firm 2 is at this point indifferent between being the leader and the follower and strictly prefers being the follower for the remaining values of P. Therefore, it always weakly prefers to be the follower.

5 See, for example, Bloome (2009).

6 From numerical implementation with a wide variety of parameter settings, $\bar{P}_{F1}^i$ is always lower than $\bar{P}_{F2}^i$ if $\mu_1 > \mu_2$ and $\sigma_1 < \sigma_2$.

7 Even if P does not fully reflect the macroeconomic risk, the beta must be proportional to (15) or (16).

Tables

Table 1: Parameter setting in the benchmark case.

μ_1	μ_2	σ_1	σ_2	r	λ_1	λ_2	K^1	K^2	$D^1(1)$	$D^2(1)$	$D^1(2)$	$D^2(2)$
0.05	0	0.2	0.5	0.1	0.1	0.1	10	12	1	1	0.6	0.5

Table 2: Investment strategies of each firm in case 1.

$\epsilon = 1$	1	1	$\times$	1	1	1, 2	1, 2	1, 2
$\epsilon = 2$		1	1	1	1	1	1	1, 2
	$\bar{P}^1_{L1}$	$\bar{P}^1_{L2}$	$\bar{P}^2_{L1}$	$\tilde{P}^2_{L1}$	$\bar{P}^1_{F1}$	$\bar{P}^2_{F1}$	$\bar{P}^1_{F2}$	$\bar{P}^2_{F2}$

Table 3: Investment strategies of each firm in case 2.

$\epsilon = 1$	1	$\times$	$\times$	$\times$	1	1	1, 2	1, 2	1, 2	1, 2
$\epsilon = 2$			1	$\times$	$\times$	$\times$	$\times$	1	1	1, 2
	$\bar{P}^1_{L1}$	$\bar{P}^2_{L1}$	$\bar{P}^1_{L2}$	$\bar{P}^2_{L2}$	$\tilde{P}^2_{L1}$	$\bar{P}^1_{F1}$	$\bar{P}^2_{F1}$	$\tilde{P}^2_{L2}$	$\bar{P}^1_{F2}$	$\bar{P}^2_{F2}$

Table 4: Investment strategies of each firm in case 3.

$\epsilon = 1$	1	$\times$	1	1	1, 2	1, 2	1, 2	1, 2	1, 2	1, 2
$\epsilon = 2$						1	$\times$	1	1	1, 2
	$\bar{P}^1_{L1}$	$\bar{P}^2_{L1}$	$\tilde{P}^2_{L1}$	$\bar{P}^1_{F1}$	$\bar{P}^2_{F1}$	$\bar{P}^1_{L2}$	$\bar{P}^2_{L2}$	$\tilde{P}^2_{L2}$	$\bar{P}^1_{F2}$	$\bar{P}^2_{F2}$

Table 5: The ratio of the investment thresholds for various λ_1 and λ_2.

		firm 1		firm 2	
λ_1	λ_2	boom	recession	boom	recession
0.1	0.1	3.150	3.672	2.760	2.967
0.2	0.1	3.047	3.611	2.701	2.959
0.3	0.1	2.992	3.572	2.664	2.950
0.4	0.1	2.961	3.545	2.639	2.942
0.1	0.2	3.186	3.846	2.773	3.048
0.1	0.3	3.213	3.969	2.783	3.119
0.1	0.4	3.233	4.055	2.791	3.180
one regime		3.374	3.431	2.863	2.884

Table 6: κ^*–ω mapping for various λ_1 and λ_2.

	one regime		$\lambda_1 = \lambda_2 = 0.1$		$\lambda_1 = 0.4$, $\lambda_2 = 0.1$		$\lambda_1 = 0.1$, $\lambda_2 = 0.4$	
$\omega \backslash \kappa^*$	boom	reces.	boom	reces.	boom	reces.	boom	reces.
1.1	1.050	1.050	1.050	1.050	1.051	1.050	1.050	1.050
1.2	1.099	1.099	1.100	1.099	1.102	1.099	1.100	1.099
1.3	1.149	1.148	1.151	1.148	1.154	1.148	1.150	1.148
1.4	1.198	1.197	1.201	1.197	1.206	1.197	1.200	1.196
1.5	1.247	1.246	1.251	1.245	1.259	1.245	1.250	1.243
1.6	1.295	1.294	1.301	1.293	1.312	1.293	1.300	1.289
1.7	1.344	1.343	1.352	1.341	1.365	1.340	1.350	1.333
1.8	1.392	1.391	1.402	1.388	1.418	1.387	1.399	1.377
1.9	1.441	1.439	1.452	1.434	1.471	1.433	1.449	1.420
2.0	1.489	1.487	1.503	1.481	1.524	1.480	1.499	1.462

Table 7: Parameter values. We follow Bhamra et al. (2009) except for K and D.

μ_1	μ_2	σ_1	σ_2	r	λ_1	λ_2	K^1	K^2	$D(1)$	$D(2)$
0.0782	-0.0401	0.0834	0.1334	0.1	0.2718	0.4928	10	12	1	0.5

Table 8: Thresholds of the firms.

$\bar{P}^1_{L1}$	$\bar{P}^1_{L2}$	$\bar{P}^1_{F1}$	$\bar{P}^1_{F2}$	$\bar{P}^2_{L1}$	$\bar{P}^2_{L2}$	$\bar{P}^2_{F1}$	$\bar{P}^2_{F2}$
0.715655	0.823626	2.240123	2.546976	0.972813	1.120081	2.688148	3.056372

Table 9: The betas of both firms given P.

	firm 1		firm 2	
P	boom	recession	boom	recession
1.0	2.555	3.669	5.764	27.155
1.5	1.284	1.596	1.539	2.224
2.0	0.719	0.997	0.425	1.003

Table 10: Firm 1's beta as a leader and a follower for given P.

P		0.7	0.8	0.9	1.0	1.5
boom	leader	12.762	4.988	3.307	2.555	1.284
	follower	2.047	2.047	2.047	2.047	2.047
recession	leader		13.905	5.541	3.669	1.596
	follower		2.047	2.047	2.047	2.047

Figures

Figure 1: The regime shift and the value functions for the follower firm.

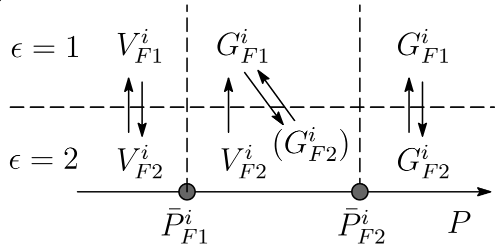


Figure 2: The regime shift and the NPV of the leader firm.

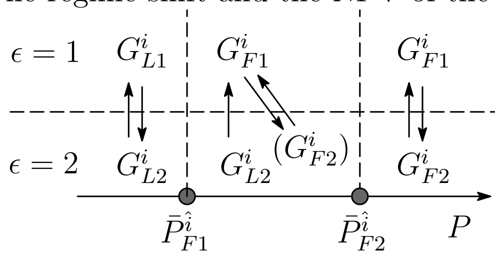


Figure 3: The value function of an advantaged firm.

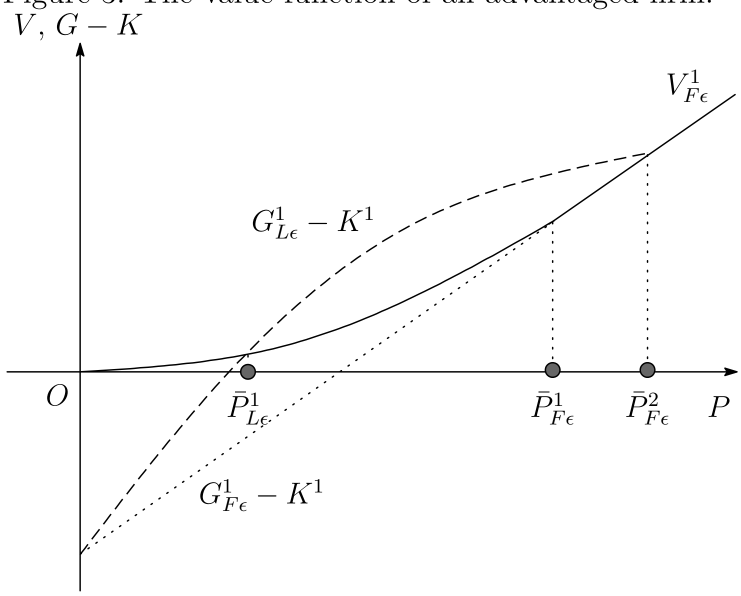


Figure 4: The value function of a partly disadvantaged firm.

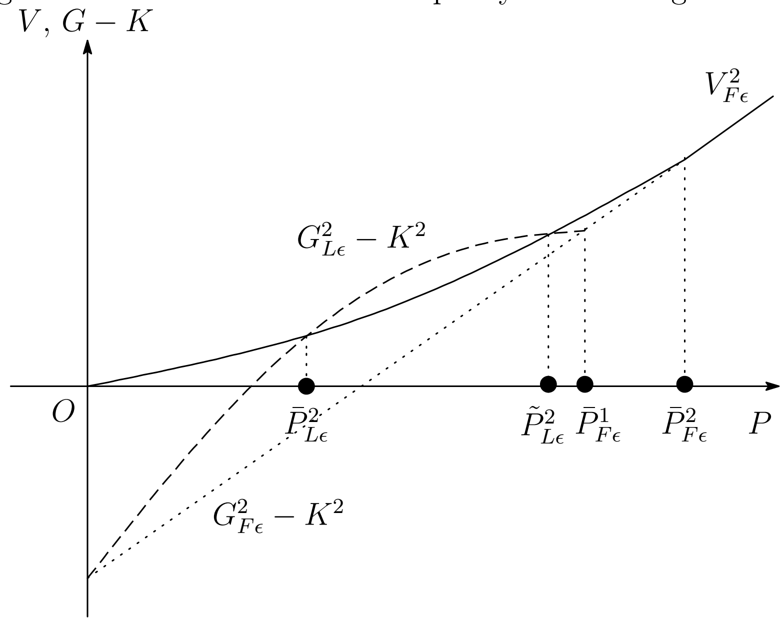


Figure 5: The value function of a fully disadvantaged firm.

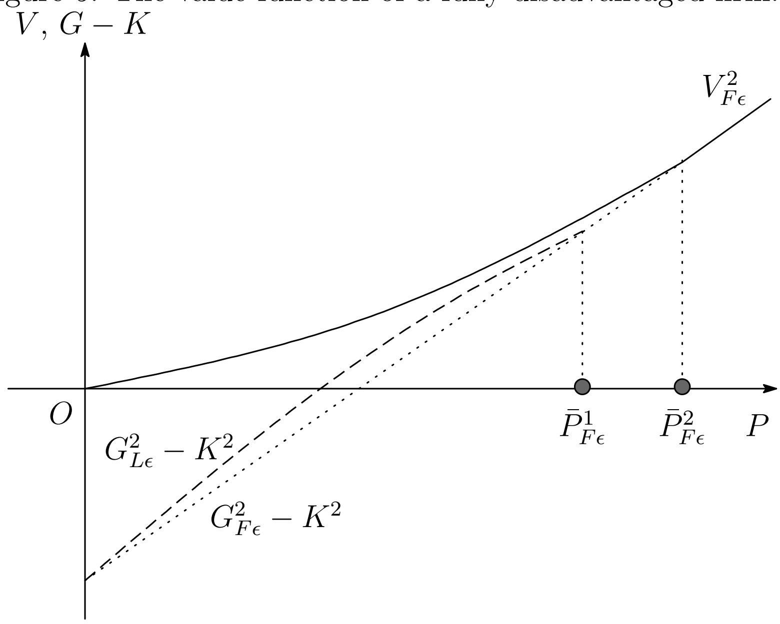
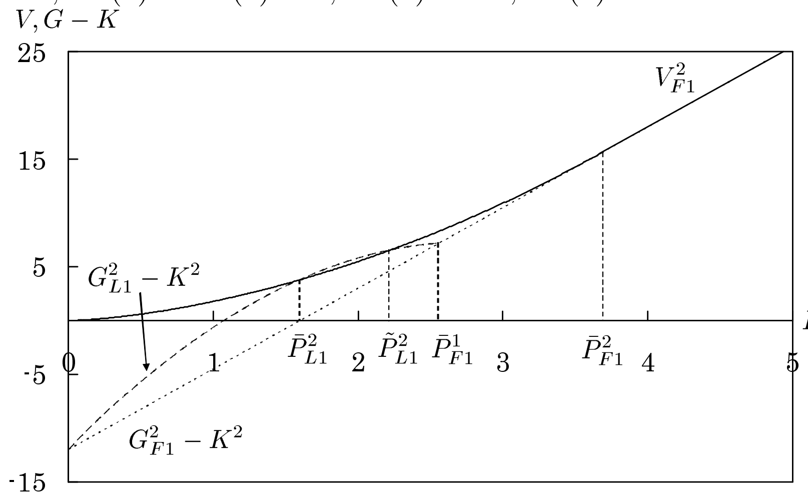
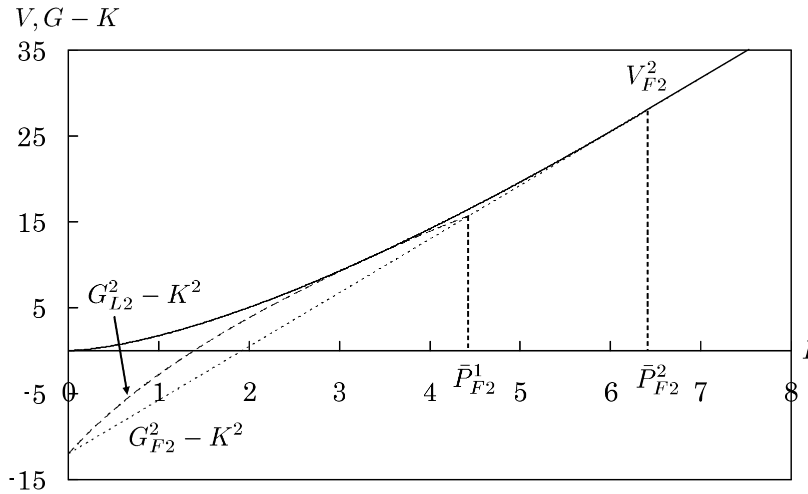


Figure 6: The value function and the investment thresholds of firm 2 in both regimes for the following set of parameter values: $\mu_1 = 0.05$, $\mu_2 = 0$, $\sigma_1 = 0.2$, $\sigma_2 = 0.5$, $r = 0.1$, $K^1 = 10$, $K^2 = 12$, $D^1(1) = D^2(1) = 1$, $D^1(2) = 0.6$, $D^2(2) = 0.5$.



(Regime 1)



(Regime 2)