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"Investment and Capital Structure Decisions under Time-Inconsistent Preferences"

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Investment and capital structure decisions under time-inconsistent preferences[*]

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Abstract Based on a continuous-time model of quasi-hyperbolic discounting, this paper provides an analytically tractable framework of entrepreneurial firms' investment and capital structure decisions with time-inconsistent preferences. We show that the impact of time-inconsistent preferences depends not only on the financing structures (all-equity financing or debt-equity financing), but also on the entrepreneurs' belief regarding their future time-inconsistent behavior (sophisticated or naive). Time-inconsistent preferences delay investment under both all-equity financing and debt-equity financing. However, the impact is weakened under debt-equity financing, because debt financing increases the payoff value upon investment and accelerates investment. Naive entrepreneurs invest later and default earlier than sophisticated entrepreneurs, leading to a shorter operating period. Moreover, we find that naive entrepreneurs may choose higher leverage, while sophisticated entrepreneurs always choose lower leverage, compared to the time-consistent benchmark. These results support the empirical findings in entrepreneurial finance.

Keywords: Investment; Capital structure; Quasi-hyperbolic discounting; Entrepreneurial finance

JEL classification: D92; G02; G11; G32

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1 Introduction

Recently, entrepreneurial finance has received increasing attention, because it replaces the traditional assumptions of corporate finance with behavioral foundations that are more evidence-driven and provides a number of implications for individual entrepreneurs (see Baker and Wurgler, 2012). Empirical evidence reports that firms similar in terms of fundamentals may choose very different leverages. Graham (2000) observes too low leverage compared to the standard capital structure theory, which is called the *debt conservatism puzzle*. Parsons and Titman (2008, 2009) suggest that managerial preferences can conceivably affect leverage choices. Cronqvist, Makhija and Yonker (2012) point out that leverage depends on the personal characteristics of a firm's CEO. They find a positive and robust relation between CEO personal leverage and corporate leverage. Hackbarth (2008, 2009) and Malmendier, Tate and Yan (2011) report that optimistic and/or overconfident managers use leverage more aggressively. To solve the leverage puzzle (i.e., why firms choose different leverages even with similar fundamentals and why firms are under-leveraged compared to the standard capital structure theory), we need to replace the traditional standard assumptions by more evidence-driven ones. In this paper, we focus on the assumption of discounting procedures. In the standard real options and corporate finance models (see, e.g., McDonald and Siegel, 1986; Dixit and Pindyck, 1994; Leland, 1994; Sundaresan and Wang, 2007; Hackbarth and Mauer, 2012), an exponential discounting is assumed. That is, firms are assumed to have a constant rate of time preference. However, empirical evidence on time-varying impatience (see, e.g., Thaler, 1981; Ainslie, 1992; Loewenstein and Prelec, 1992; O'Donoghue and Rabin, 1999) has found that agents are impatient for short-term decisions but are patient for long-term decisions (present-biased preference).

To reflect the empirical evidence, time-inconsistent preferences are modeled with quasi-hyperbolic discount functions. Although the original theoretical literature such as Phelps and Pollak (1968) focuses on the consumption and savings problem, a large new literature has explored the implications of quasi-hyperbolic discounting in many areas, including environmental problem, health economics, as well as corporate finance. For example, Brocas and Carrillo (2004) examine investment by hyperbolic discounting entrepreneurs in a discrete-time framework. Large firms (decisions are made by a large board) may behave time-consistently, while entrepreneurial firms (decisions are made by an individual or a small board) are more likely to behave time-inconsistently.

The objective of this paper is to examine entrepreneurial firms' investment and capital structure decisions with time-inconsistent preferences. The most related literature to our paper is Grenadier and Wang (2007). They consider all-equity entrepreneurial firms with quasi-hyperbolic discounting and find that the impact on investment timing depends not only on whether entrepreneurs are sophisticated or naive (to be defined in Section

2.3), but also on whether the payoff from investment is lump-sum or flow. To solve the leverage puzzle observed in practice, we extend the all-equity financing framework for the flow payoff case in Grenadier and Wang (2007) to consider further debt-equity financed entrepreneurial firms. Under debt-equity financing, entrepreneurs make not only investment decisions but also capital structure decisions (leverage and default decisions). Time-inconsistent preferences as well as the entrepreneurs' characteristics (sophisticated or naive) influence the two interacting decisions.

Grenadier and Wang (2007) find that sophisticated entrepreneurs exercise investment option earlier than naive entrepreneurs under all-equity financing. While investment option is exercised earlier with time inconsistency in the lump-sum payoff case, it is exercised much later with time inconsistency in the flow payoff case. The *qualitative* results are very interesting, but the *quantitative* differences between the time-inconsistent investment threshold and the time-consistent benchmark are very large,[1] which is mainly attributed to the exponential discounting parameter of the time-consistent benchmark. Compared to the exponential discounting with single parameter, there are three parameters in quasi-hyperbolic discounting. In addition to the same exponential discounting parameter, the other two parameters definitely lower the flow payoff value. That is suggested to be the main reason why time inconsistency delays investment so much compared to the time-consistent benchmark. To compare the time-inconsistent results with a reasonable time-consistent benchmark, it is important to adjust the parameters to make the present value of a unit stream of payoff the same under the two discounting procedures.[2]

Based on a continuous-time model of quasi-hyperbolic discounting, this paper provides an analytically tractable framework of entrepreneurial firms' investment and capital structure decisions under time-inconsistent preferences. The questions are two-fold: (i) How does time inconsistency influences entrepreneurial firms' capital structure decisions (leverage and default decisions) as well as investment decisions? Does the impact of time-inconsistent preferences depend on the financing structures (all-equity financing or debt-equity financing) and also on the entrepreneurs' belief regarding their future time-inconsistent behavior (sophisticated or naive)? (ii) Do the qualitative results in Grenadier and Wang (2007) still hold with parameter adjustment, i.e., is investment accelerated with time inconsistency in the lump-sum payoff case but delayed in the flow payoff case? How large are the differences between time-inconsistent results and the time-consistent

[1] Surprisingly, the time-inconsistent investment threshold in the flow payoff case is almost three times of the time-consistent benchmark even with a reasonable parameter setting. We will show the details in Section 5.

[2] Jamison and Jamison (2011) suggest that, when comparing different discounting procedures, the concepts of *amount* and *speed* of a discounting procedure should be disaggregated. They have defined the *amount* of a discounting procedure to be the inverse of the present value of a unit stream of cash flow and compared the *speed* of alternative discounting procedures that accumulate the same amount.

benchmark after parameter adjustment? Are the quantitative differences more reasonable compared to the ones without parameter adjustment?

The main contribution of this paper is to extend Grenadier and Wang (2007) by examining both the investment and capital structure decisions of entrepreneurial firms and providing a more reasonable time-consistent benchmark with parameter adjustment. Regarding the first question above, we find that the impact of time-inconsistent preferences depends not only on the financing structures (all-equity financing or debt-equity financing), but also on the entrepreneurs' belief regarding their future time-inconsistent behavior (sophisticated or naive). Time-inconsistent preferences delay investment under both all-equity financing and debt-equity financing. In fact, time inconsistency influences investment through two channels: (i) earlier investment due to the decrease in option value of waiting and (ii) later investment due to the decrease in payoff value upon investment. The second impact dominates the first one, leading a later investment. However, the second effect is weakened with debt financing, because debt financing increases the payoff value upon investment and accelerates investment. Naive entrepreneurs invest later and default earlier than sophisticated entrepreneurs, leading to a shorter operating period. Moreover, we find that naive entrepreneurs may choose higher leverage, while sophisticated entrepreneurs always choose lower leverage, compared to the time-consistent benchmark. These results are consistent with the empirical findings in Camerer and Lovallo (1999), Malmendier, Tate and Yan (2011), Cronqvist, Makhija and Yonker (2012), and others. By incorporating entrepreneurs' time-inconsistent preferences, we can provide a reasonable interpretation of the observed leverage puzzle. Even with the same fundamentals, entrepreneurial firms may choose very different leverages, depending on the degree of time inconsistency and the entrepreneurs' characteristics (sophisticated or naive). Next, as to the second question, we find that, the qualitative results in Grenadier and Wang (2007) still hold even with parameter adjustment. However, the differences between time-inconsistent results and time-consistent benchmarks are much smaller than those observed in Grenadier and Wang (2007) and therefore understandable.

The rest of this paper is organized as follows. Section 2 describes the setup of our model. Section 3 examines the investment decision in a real options model for an all-equity financed entrepreneurial firm. We first review the time-consistent benchmark, and then consider the time-inconsistent preferences for sophisticated and naive entrepreneurs, respectively. As the main part of this paper, Section 4 extends Section 3 to examine both the investment and capital structure decisions of a debt-equity financed entrepreneurial firm. Section 5 provides several model predictions through numerical examples. In particular, we examine the impact of time inconsistency on investment and capital structure decisions and compare our results to the time-consistent benchmarks without/with parameter adjustment. Finally, Section 6 concludes this paper. Some detailed proofs are given in the Appendix.

2 The model setup

This section describes the model setup. We first illustrate the investment opportunity and then the time preferences, which is the main departure from the standard modeling. Two types of entrepreneurs with time-inconsistent preferences, sophisticated and naive entrepreneurs, are defined. Finally, we give a clear picture of the decision-making stream.

2.1 Investment opportunity

The model is set in a continuous-time, risk-neutral framework. We suppose that a risk-neutral entrepreneur owns a privileged right to undertake a project with an irreversible investment cost I. The potential earnings before interest and taxes (EBIT) generated by the project is given by the geometric Brownian motion process

$$\mathrm{d}X(t) = \mu X(t)\mathrm{d}t + \sigma X(t)\mathrm{d}z(t), \tag{1}$$

where $\mu \geq 0$ and $\sigma > 0$ are constants and $(z(t))_{t\geq 0}$ denotes a standard Brownian motion under risk-neutral measure $\mathbb{P}$. The initial value $X(0)$ is sufficiently low; i.e., the potential EBIT has not yet been favorable enough to undertake the project.

2.2 Time preferences

Our main departure from the standard modeling is the assumption that entrepreneurs have dynamically inconsistent preferences. This assumption is becoming more and more accepted in the light of empirical and experimental evidences (see Frederick, Loewenstein and O'Donoghue, 2002, for a survey). To reflect the empirical evidence, Phelps and Pollak (1968) and Laibson (1997) model time-varying impatience with quasi-hyperbolic discounting by using a discrete-time function. Time is divided into two periods, the present period (discounted exponentially with ρ) and all future periods (first discounted exponentially with ρ and then further discounted by an additional factor $\delta \in [0, 1]$). The quasi-hyperbolic discounting generates a gap between a high short-run discount rate and a low long-run rate. This gap makes the preferences dynamically inconsistent, in the sense that preferences at time t are inconsistent with preferences at time $t + 1$.

Suppose that an entrepreneur receives flow payments P_t and P_{t+1} at time t and time $t + 1$, respectively. The marginal rate of substitution between time t and time $t + 1$ is $(\delta \mathrm{e}^{-\rho t} P_t)/(\delta \mathrm{e}^{-\rho(t+1)} P_{t+1}) = P_t/(\mathrm{e}^{-\rho} P_{t+1})$ from the perspective of the entrepreneur at the current time 0, but is $(\mathrm{e}^{-\rho} P_t)/(\delta \mathrm{e}^{-2\rho} P_{t+1}) = P_t/(\delta \mathrm{e}^{-\rho} P_{t+1})$ from the perspective of the same entrepreneur at the future time $t-1$. That is, the entrepreneur at time 0 views the relative choice between time t and time $t+1$ differently from the one he/she does at time $t-1$. While the entrepreneur at the current time believes that he/she can commit his/her future selves to adopt his/her current preference ordering, he/she is unable to do so at future times. Although the above discrete-time formulation of quasi-hyperbolic discounting captures the present-biased preference observed in practice, it has several drawbacks (see Harris and Laibson, 2004). One main drawback is that the Hamilton–Jacobi–Bellman (HJB) equation that the value function must satisfy is not analytically tractable, in that it contains a non-local term in addition to the standard HJB equation.

In this paper, we follow Harris and Laibson (2004) to model quasi-hyperbolic discounting using a continuous-time formulation. Since the preferences are time-inconsistent, we regard that decisions are made by different selves of the entrepreneur, where each self has a random lifespan. Each self controls the decision-making in the present but also cares about the decision-making of future selves. At the very beginning of the time horizon t_0, we call the entrepreneur self 0. Let t_j be the arrival time of self j $(= 0, 1, 2, \cdots)$. Then, $T_j = t_{j+1} - t_j$ is the lifespan for self j. We assume that the lifespan is exponentially distributed with parameter λ. In other words, the arrival of future selves is modeled as a Poisson process with intensity λ. Let $d_j(t, s)$ denote self j's intertemporal discount function (self j's value at time t of \$1 received at the future time s). Then,

$$d_j(t,s) = \begin{cases} e^{-\rho(s-t)} & \text{if } \ s \in [t_j, t_{j+1}), \\ \delta e^{-\rho(s-t)} & \text{if } s \in [t_{j+1}, \infty), \end{cases} \tag{2}$$

for $s > t$ and $t \in [t_j, t_{j+1})$. That is, time is divided into two periods for each self j, $[t_j, t_{j+1})$ and $[t_{j+1}, \infty)$. At the very beginning, the entrepreneur uses the discount function $d_0(t, s)$ for evaluation. After the arrival of self 1, the entrepreneur uses the discount function $d_1(t, s)$ for evaluation, and so on. For example, cash flow upon time $s \in [t_1, t_2)$ is discounted by $\delta e^{-\rho(s-t)}$, $t \in [t_0, t_1)$, from the perspective of self 0, but is discounted by $e^{-\rho(s-t)}$, $t \in [t_1, t_2)$, from the perspective of self 1. The formulation (2) not only captures the qualitative properties of the original discrete-time quasi-hyperbolic discounting, but also reduces the HJB equation to a system of two stationary ordinary differential equations (ODEs) that characterize the present and future value functions, respectively. The parameter δ in conjunction with the intensity λ determines the degree of the entrepreneur's time inconsistency. In particular, if $\delta = 1$ or $\lambda = 0$, the time-inconsistent problem is reduced to the time-consistent benchmark.

2.3 Sophisticated vs. naive entrepreneurs

In the following, we consider two types of entrepreneurs with time-inconsistent preferences, sophisticated and naive entrepreneurs. By “sophisticated”, we mean that the entrepreneur correctly foresees that his/her future selves act according to their own preferences. That is, self 0 and self 1 do not agree on their decision-making, self 1 and self 2 do not agree on their decision-making, and so on. In other words, the optimal decisions of future selves are only optimal for future selves but suboptimal for the current self. On the other hand, by “naive”, we mean that the entrepreneur mistakenly believes that he/she can commit his/her future selves to behave according to his/her current preferences. That is, the naive entrepreneur acts as if his/her discount function could remain unchanged as $d_0(t, s)$, although future self 1 has the discount function $d_1(t, s)$, future self 2 has the discount function $d_2(t, s)$, and so on.

2.4 Decision-making stream

Let T_i (subscript “i” stands for investment) denote the time that the investment option is exercised. That is, $T_\mathrm{i} = \inf\{t > 0, X(t) \geq x_\mathrm{i}\}$ for the investment threshold x_i (determined optimally later). When the EBIT process $X(t)$ reaches x_i, the entrepreneur decides to exercise the investment option by paying the fixed, irreversible investment cost I, which can be financed by equity and debt. For simplicity, we assume that the issued debt has infinite maturity. The contractual continuous coupon of the perpetual debt is c (determined optimally later), which is tax deductible. Let τ be the corporate tax rate. After engaging in the investment project, the firm receives the EBIT $X(t)$ and pays coupon c to debtholders at each instant. If the EBIT $X(t)$ becomes sufficiently low to hit the default threshold x_d (determined optimally later; subscript “d” stands for default), the firm fails to pay the contractual coupon and goes into default. The corresponding stopping time is denoted by $T_\mathrm{d} = \inf\{t > T_\mathrm{i}, X(t) \leq x_\mathrm{d}\}$, where $x_\mathrm{d} < x_\mathrm{i}$.

To distinguish different stopping times/thresholds of each case, we use the notation listed in Table 1. For example, $\bar{T}_\mathrm{i}^s$ represents the investment timing under all-equity financing (bar is used for all-equity financing) for sophisticated entrepreneurs (“s” stands for sophisticated), x_d^n represents the default threshold under debt-equity financing (no accent is used for debt-equity financing) for naive entrepreneurs (“n” stands for naive), etc.

(Table 1 is inserted here.)

3 All-equity financing

In this section, we consider the investment decision of an all-equity financed firm. In the following, we first briefly review the time-consistent benchmark (see Dixit and Pindyck, 1994). Second, we consider a sophisticated entrepreneur with time-inconsistent preferences. Finally, we examine a naive entrepreneur with time-inconsistent preferences. This section, corresponding to Grenadier and Wang (2007), demonstrates how time inconsistency influences entrepreneurs' investment option exercising. The intuition behind the investment strategy for an all-equity financed firm provides a foundation for the core results in the subsequent sections of this paper.

The entrepreneur's optimal investment problem considered in this section can be formulated generally as

$$(\bar{\mathrm{P}}): \qquad \max_{\bar{x}_{\mathrm{i}}^k} \bar{V}^{ok}(x; \bar{x}_{\mathrm{i}}^k), \tag{3}$$

where $\bar{V}^{ok}$ represents the option value of investment under all-equity financing and $k \in \{*,\ s,\ n\}$ corresponds to the three cases we examine (benchmark, sophisticated, and naive ones, respectively). To solve the problem (3), we need to begin with deriving the value function for each case.

3.1 The time-consistent benchmark

According to the setup, the value of the after-tax, all-equity financed firm after investment ($t \geq \bar{T}_{\mathrm{i}}^*$) is given by

$$\Pi^*(x) = (1-\tau)\mathbb{E}_t\Big[\int_t^\infty \mathrm{e}^{-\rho(u-t)} X(u)\mathrm{d}u\Big] = \frac{1-\tau}{\rho-\mu}x, \tag{4}$$

where $\mathbb{E}_t$ denotes the expectation operator under the risk-neutral measure $\mathbb{P}$, given that $X(t) = x$. For convergence, we assume $\rho > \mu$.

The option value of investment ($t \leq \bar{T}_{\mathrm{i}}^*$) is given by[3]

$$\bar{V}^{o*}(x) = \mathbb{E}_t\left[\mathrm{e}^{-\rho(\bar{T}_{\mathrm{i}}^*-t)}\left[\Pi^*(\bar{x}_{\mathrm{i}}^*) - I\right]\right] = \left[\Pi^*(\bar{x}_{\mathrm{i}}^*) - I\right]\left(\frac{x}{\bar{x}_{\mathrm{i}}^*}\right)^{\beta_1}, \tag{5}$$

where

$$\beta_1 = \frac{1}{2} - \frac{\mu}{\sigma^2} + \sqrt{\left(\frac{\mu}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2\rho}{\sigma^2}} > 1. \tag{6}$$

The option value (5) of investment has two components: (i) the net value obtained upon investment $\Pi^*(\bar{x}_{\mathrm{i}}^*) - I$, and (ii) the investment probability $(x/\bar{x}_{\mathrm{i}}^*)^{\beta_1}$.

With the value representations in Eqs. (4) and (5), the solution to the optimal investment problem (3) with $k = *$ is obtained as follows.

[3]See, e.g., Dixit and Pindyck (1994) for derivation.

Proposition 3.1 (benchmark, all-equity financing). *The investment threshold for time-consistent entrepreneurs under all-equity financing is given by*

$$\bar{x}_{\mathrm{i}}^* = \frac{\beta_1}{\beta_1 - 1}\frac{\rho - \mu}{1 - \tau}I. \tag{7}$$

Moreover, the investment threshold increases with the discount rate ρ, i.e., $\partial \bar{x}_{\mathrm{i}}^*/\partial \rho \geq 0$.

The threshold is larger than the NPV investment threshold $(\rho - \mu)I/(1 - \tau)$ due to the existence of the term $\beta_1/(\beta_1 - 1) > 1$, which expresses the option value of waiting. A larger discount rate reduces the payoff value from investment, leading to a later exercise of the investment option.

3.2 Time-inconsistent preferences

In this subsection, we consider two types of entrepreneurs with time-inconsistent preferences, sophisticated and naive entrepreneurs. Before considering the decision-making with time-inconsistent preferences, we need to first derive the firm value after investment.

Let T $(\geq t)$ denote the arrival time of the future self, which is exponentially distributed with mean $1/\lambda$. With time-inconsistent preferences, the firm value after investment depends on whether the investment option is exercised before the arrival of the future self or not. If the future self arrives first, then the firm value after investment is simply the time-consistent value $\Pi^*(x)$ obtained in (4). If the investment option is exercised before the arrival of the future self, then the firm value after investment is given by

$$\begin{aligned}\Pi(x) &= (1 - \tau)\mathbb{E}_t\left[\int_t^T \mathrm{e}^{-\rho(u-t)}X(u)\mathrm{d}u + \int_T^\infty \delta \mathrm{e}^{-\rho(u-t)}X(u)\mathrm{d}u\right] \\ &= (1 - \theta)\Pi^*(x),\end{aligned} \tag{8}$$

where

$$\theta = \frac{(1 - \delta)\lambda}{\rho - \mu + \lambda} \geq 0. \tag{9}$$

Notice that $\Pi(x) \leq \Pi^*(x)$, because $1 - \theta = (\rho - \mu + \delta\lambda)/(\rho - \mu + \lambda) \leq 1$. Obviously, the future EBIT is discounted more with quasi-hyperbolic discounting, because there are other two parameters $\delta \in [0, 1]$ and $\lambda \in [0, \infty)$ in addition to the single parameter ρ in the exponential discounting.

3.2.1 The sophisticated entrepreneur

In this subsection, we consider the sophisticated entrepreneur with time-inconsistent preferences. The sophisticated entrepreneur correctly foresees that his/her future selves act according to their own preferences.

Since the time horizon of the investment problem we consider is infinite, each self j of the sophisticated entrepreneur faces the same time-invariant option exercising problem:

whether to invest in the current period or the future period. That is, the sophisticated entrepreneur's optimization problem does not depend on j. The stationary solution that we search for is a *fixed-point* to the option exercising problem.

Remember that deriving the option value of investment for $t < \bar{T}_{\mathrm{i}}^{s}$ is to discount the net payoff value upon the investment time $\bar{T}_{\mathrm{i}}^{s}$ to the current time t (see Eq. (5)). Under time-inconsistent preferences, the discounting procedure as well as the net payoff value upon investment depends on whether the investment option is exercised before the arrival of the future self or not. If the investment option is exercised before the arrival of the future self ($\bar{T}_{\mathrm{i}}^{s} \leq T$), then the net value obtained upon investment is $\Pi(\bar{x}_{\mathrm{i}}^{s}) - I$ (Π is defined in Eq. (8)), which should be discounted exponentially with ρ. If the future self arrives first ($\bar{T}_{\mathrm{i}}^{s} > T$), then the net value obtained upon investment is $\Pi^{*}(\bar{x}_{\mathrm{i}}^{s}) - I$ (Π^{*} is defined in Eq. (4)), which should be discounted by the additional factor δ in addition to the exponential discount rate ρ. Thus, the option value of investment for $t < \bar{T}_{\mathrm{i}}^{s}$ is

$$\begin{aligned}\bar{V}^{os}(x) &= \mathbb{E}_{t}\left[\mathbf{1}_{\{\bar{T}_{\mathrm{i}}^{s} \leq T\}} \mathrm{e}^{-\rho(\bar{T}_{\mathrm{i}}^{s}-t)} \left[\Pi(\bar{x}_{\mathrm{i}}^{s}) - I\right] + \mathbf{1}_{\{\bar{T}_{\mathrm{i}}^{s} > T\}} \mathrm{e}^{-\rho(\bar{T}_{\mathrm{i}}^{s}-t)} \delta \left[\Pi^{*}(\bar{x}_{\mathrm{i}}^{s}) - I\right]\right], \\ &= \left[\Pi(\bar{x}_{\mathrm{i}}^{s}) - I\right] \left(\frac{x}{\bar{x}_{\mathrm{i}}^{s}}\right)^{\beta_2} + \delta \left[\Pi^{*}(\bar{x}_{\mathrm{i}}^{s}) - I\right] \left[\left(\frac{x}{\bar{x}_{\mathrm{i}}^{s}}\right)^{\beta_1} - \left(\frac{x}{\bar{x}_{\mathrm{i}}^{s}}\right)^{\beta_2}\right], \end{aligned} \tag{10}$$

where

$$\beta_2 = \frac{1}{2} - \frac{\mu}{\sigma^2} + \sqrt{\left(\frac{\mu}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2(\rho + \lambda)}{\sigma^2}} \geq \beta_1 > 1. \tag{11}$$

See A.1 of the Appendix for the derivation.

The option value of investment for sophisticated entrepreneurs consists of two terms: (i) When the investment option is exercised during the current period ($\bar{T}_{\mathrm{i}}^{s} \leq T$), the arrival time of the future self is still a random variable. Thus, the investment probability $(x/\bar{x}_{\mathrm{i}}^{s})^{\beta_2}$ during the current period should include the influence of the parameter λ (see Eq. (11) for the option factor β_2 corresponding to the discount rate $\rho + \lambda$). (ii) When the investment option is exercised during the future period ($\bar{T}_{\mathrm{i}}^{s} > T$), the arrival time of the future self has already been realized. Thus, the parameter λ has no direct influence on the investment probability $(x/\bar{x}_{\mathrm{i}}^{s})^{\beta_1}$ during the future period (see Eq. (6) for the option factor β_1 corresponding to the discount rate ρ). Since the investment option we consider here can be exercised only once, the $(x/\bar{x}_{\mathrm{i}}^{s})^{\beta_2}$ part is subtracted to exclude the probability that the EBIT process hits the investment threshold $\bar{x}_{\mathrm{i}}^{s}$ again during the future period after its first hitting during the current period.

With the value representations in Eqs. (4), (8) and (10), the solution to the optimal investment problem (3) with $k = s$ is obtained as follows.

Proposition 3.2 (sophisticated entrepreneur, all-equity financing). *The investment threshold for sophisticated entrepreneurs under all-equity financing is given by*

$$\bar{x}_{\mathrm{i}}^{s} = \frac{\rho - \mu}{1 - \tau}\kappa_1 I, \tag{12}$$

where

$$\kappa_1 = \frac{\hat{\beta}}{\delta(\beta_1 - 1) + (1 - \theta - \delta)(\beta_2 - 1)} > 0 \tag{13}$$

and

$$\hat{\beta} = \delta\beta_1 + (1 - \delta)\beta_2 \in [\beta_1, \beta_2]. \tag{14}$$

Moreover, the investment threshold increases with the degree of time inconsistency, i.e., $\partial\bar{x}_{\mathrm{i}}^{s}/\partial\delta \leq 0$ and $\partial\bar{x}_{\mathrm{i}}^{s}/\partial\lambda \geq 0$. If sophisticated entrepreneurs have time-consistent preferences (i.e., $\delta = 1$ or $\lambda = 0$), then $\bar{x}_{\mathrm{i}}^{s} = \bar{x}_{\mathrm{i}}^{*}$.

See A.2 of the Appendix for the proof. By comparing $\bar{x}_{\mathrm{i}}^{*}$ in (7) and $\bar{x}_{\mathrm{i}}^{s}$ in (12), we find that the coefficient κ_1 in (13) of $\bar{x}_{\mathrm{i}}^{s}$ plays the same role as $\beta_1/(\beta_1 - 1)$ in (7) of $\bar{x}_{\mathrm{i}}^{*}$. The numerator of κ_1, $\hat{\beta}$ given by (14), is a weighted average of β_1 and β_2, with δ and $1 - \delta$ as the respective weights. On the other hand, the denominator of κ_1 is a weighted average of $\beta_1 - 1$ and $\beta_2 - 1$, with δ and $1 - \theta - \delta$ as the respective weights. The coefficient $1 - \theta$ stems from the quasi-hyperbolic discounting procedure of the EBIT $X(t)$ (see Eq. (8)).

The next result holds since $\partial\bar{x}_{\mathrm{i}}^{s}/\partial\lambda \geq 0$ and $\bar{x}_{\mathrm{i}}^{s} = \bar{x}_{\mathrm{i}}^{*}$ for $\lambda = 0$.

Corollary 3.1. *Sophisticated entrepreneurs exercise the investment option later than time-consistent entrepreneurs, i.e., $\bar{x}_{\mathrm{i}}^{s} \geq \bar{x}_{\mathrm{i}}^{*}$, where the equality holds when $\delta = 1$ or $\lambda = 0$.*

Corollary 3.1 can be interpreted as follows. Time inconsistency influences the investment through two channels. (i) Option value of waiting: The larger the option value of waiting, the later the investment option is exercised. In this model, the current self of the sophisticated entrepreneur has motivation to exercise before the future self takes control of the exercise decision, because the payoff for the current self from future exercise is discounted by the additional factor δ. Therefore, time inconsistency decreases the option value of waiting and accelerates investment. (ii) Payoff value: The lower the payoff value, the later the investment option is exercised. Since the payoff value is decreased by the factor $(1 - \theta)$ (see Eq. (4)), the investment is not so attractive to the sophisticated entrepreneur. That is, time inconsistency decreases the payoff value and thus delays investment. Grenadier and Wang (2007) find that the second effect dominates the first effect, leading a later investment for sophisticated entrepreneurs with time-inconsistent preferences.

3.2.2 The naive entrepreneur

In this subsection, we consider the naive entrepreneur with time-inconsistent preferences. Unlike the sophisticated entrepreneur, the naive entrepreneur mistakenly believes that he/she can commit his/her future selves to behave according to his/her current preferences.

Notice that the factor δ has impact on the current self's investment threshold, but has no impact on the future self's investment threshold, because the valuation for the whole future period is the δ proportion. Therefore, the naive entrepreneur believes that all future selves invest once the EBIT $X(t)$ hits the threshold $\bar{x}_{\mathrm{i}}^{*}$. Taking the future selves' investment threshold into consideration, the current self of the naive entrepreneur determines $\bar{x}_{\mathrm{i}}^{n}$. Recall that the sophisticated entrepreneur's investment threshold $\bar{x}_{\mathrm{i}}^{s}$ is determined as a fixed-point (the current self's and future selves' investment thresholds are the same, due to the stationary property of the infinite-time problem). The naive entrepreneur's investment problem is more complicated than the sophisticated one.

Based on the discussions in the previous section, we conjecture $\bar{x}_{\mathrm{i}}^{n} \geq \bar{x}_{\mathrm{i}}^{*}$ and verify the inequality *ex post*.[4] The region before investment, $x \leq \bar{x}_{\mathrm{i}}^{n}$, is divided into two cases: the lower one $x \leq \bar{x}_{\mathrm{i}}^{*} \leq \bar{x}_{\mathrm{i}}^{n}$ and the higher one $\bar{x}_{\mathrm{i}}^{*} \leq x \leq \bar{x}_{\mathrm{i}}^{n}$.

For the lower region $x \leq \bar{x}_{\mathrm{i}}^{*} \leq \bar{x}_{\mathrm{i}}^{n}$, the option value of investment is given by

$$\begin{aligned}
\bar{V}_{l}^{on}(x) = & \mathbb{E}_t \left[\mathbf{1}_{\{\bar{T}_{\mathrm{i}}^{n} < T\}} \mathrm{e}^{-\rho(\bar{T}_{\mathrm{i}}^{n} - t)} [\Pi(\bar{x}_{i}^{n}) - I] \right] \\
& + \mathbb{E}_t \left[\mathbf{1}_{\{\bar{T}_{\mathrm{i}}^{n} \geq T \geq \bar{T}_{\mathrm{i}}^{*},\ X(T) \geq \bar{x}_{\mathrm{i}}^{*}\}} \mathrm{e}^{-\rho(T - t)} \delta [\Pi^{*}(X(T)) - I] \right] \\
& + \mathbb{E}_t \left[\mathbf{1}_{\{\bar{T}_{\mathrm{i}}^{n} \geq T \geq \bar{T}_{\mathrm{i}}^{*},\ X(T) < \bar{x}_{\mathrm{i}}^{*}\}} \mathrm{e}^{-\rho(\bar{T}_{\mathrm{i}}^{*} - t)} \delta [\Pi^{*}(\bar{x}_{\mathrm{i}}^{*}) - I] \right] \\
& + \mathbb{E}_t \left[\mathbf{1}_{\{\bar{T}_{\mathrm{i}}^{n} \geq \bar{T}_{\mathrm{i}}^{*} \geq T\}} \mathrm{e}^{-\rho(\bar{T}_{\mathrm{i}}^{*} - t)} \delta [\Pi^{*}(\bar{x}_{\mathrm{i}}^{*}) - I] \right].
\end{aligned} \tag{15}$$

In addition to similar terms in Eq. (10) (see the first and fourth terms in (15)), there exists an intermediate case, i.e., the future self arrives before the current self's investment time but after the future self's investment time ($\bar{T}_{\mathrm{i}}^{n} \geq T \geq \bar{T}_{\mathrm{i}}^{*}$). The second and third terms in Eq. (15) correspond to such an intermediate case: (i) If the EBIT level upon the arrival of the future self is higher than the future self's investment threshold ($X(T) \geq \bar{x}_{\mathrm{i}}^{*}$), the net value obtained upon investment is $\Pi^{*}(X(T)) - I$ (invest immediately at T). (ii) If the EBIT level upon the arrival of the future self is lower than the future self's investment threshold ($X(T) < \bar{x}_{\mathrm{i}}^{*}$), the net value obtained upon investment is $\Pi^{*}(\bar{x}_{\mathrm{i}}^{*}) - I$.

On the other hand, for the higher region $\bar{x}_{\mathrm{i}}^{*} \leq x \leq \bar{x}_{\mathrm{i}}^{n}$, the option value of investment

[4] We have also conjectured $\bar{x}_{\mathrm{i}}^{n} < \bar{x}_{\mathrm{i}}^{*}$, which is rejected through verification.

is given by

$$\begin{aligned}\bar{V}_h^{on}(x) = &\mathbb{E}_t\left[\mathbf{1}_{\{\bar{T}_{\mathrm{i}}^n<T\}}\mathrm{e}^{-\rho(\bar{T}_{\mathrm{i}}^n-t)}[\Pi(\bar{x}_i^n)-I]\right]\\ &+\mathbb{E}_t\left[\mathbf{1}_{\{\bar{T}_{\mathrm{i}}^n\geq T,\ X(T)\geq\bar{x}_{\mathrm{i}}^*\}}\mathrm{e}^{-\rho(T-t)}\delta[\Pi^*(X(T))-I]\right]\\ &+\mathbb{E}_t\left[\mathbf{1}_{\{\bar{T}_{\mathrm{i}}^n\geq T,\ X(T)<\bar{x}_{\mathrm{i}}^*\}}\mathrm{e}^{-\rho(\bar{T}_{\mathrm{i}}^*-t)}\delta[\Pi^*(\bar{x}_{\mathrm{i}}^*)-I]\right].\end{aligned} \tag{16}$$

Since the expressions (15) and (16) depend not only on the first hitting time but also on whether the state $X(T)$ upon the arrival of the future self is higher than $\bar{x}_{\mathrm{i}}^*$ or not, it is complicated to derive the option value of investment directly. In A.3 of the Appendix, we employ a real options approach to derive naive entrepreneurs' investment threshold. The verification of $\bar{x}_{\mathrm{i}}^n \geq \bar{x}_{\mathrm{i}}^*$ is also provided.

Proposition 3.3 (naive entrepreneur, all-equity financing). *The investment threshold $\bar{x}_{\mathrm{i}}^n$ for naive entrepreneurs under all-equity financing satisfies the nonlinear equation*

$$\bar{x}_{\mathrm{i}}^n = \left[\beta_2\frac{\rho+\lambda(1-\delta)}{\rho+\lambda} + \left(\frac{\bar{x}_{\mathrm{i}}^n}{\bar{x}_{\mathrm{i}}^*}\right)^{\gamma_2}\delta\left[\frac{\beta_1(\beta_2-1)(\rho-\mu)}{(\beta_1-1)(\rho+\lambda-\mu)} - \frac{\beta_2\rho}{\rho+\lambda}\right]\right]\frac{\rho+\lambda-\mu}{(1-\tau)(\beta_2-1)}I, \tag{17}$$

where

$$\gamma_2 = \frac{1}{2} - \frac{\mu}{\sigma^2} - \sqrt{\left(\frac{\mu}{\sigma^2}-\frac{1}{2}\right)^2 + \frac{2(\rho+\lambda)}{\sigma^2}} < 0. \tag{18}$$

If naive entrepreneurs have time-consistent preferences (i.e., $\delta = 1$ or $\lambda = 0$), then $\bar{x}_{\mathrm{i}}^n = \bar{x}_{\mathrm{i}}^*$.

The next result is obtained in Grenadier and Wang (2007).

Corollary 3.2. *Under all-equity financing, naive entrepreneurs exercise the investment option later than sophisticated entrepreneurs, who in turn exercise later than time-consistent entrepreneurs, i.e., $\bar{x}_{\mathrm{i}}^n \geq \bar{x}_{\mathrm{i}}^s \geq \bar{x}_{\mathrm{i}}^*$, where the equalities hold when $\delta = 1$ or $\lambda = 0$.*

Intuitively, sophisticated entrepreneurs desire to invest earlier than naive entrepreneurs so as to protect themselves against the suboptimal behavior of future selves.

Finally, we consider $\delta = 0$ and $\lambda \to \infty$ as special cases (see Table 2 for reference). By substituting $\delta = 0$ and $\lambda \to \infty$ into (12) and (17), we obtain the results for the special cases as in Table 3.[5] The economic interpretation is clear. When $\delta = 0$ (ruin occurs once T arrives), the original time-inconsistent investment problem with infinite lifetime becomes a time-consistent one with finite lifetime T. Thus, the future EBIT is discounted by adding the Poisson ruin parameter λ to ρ. This is the reason that the investment threshold is equal to the time-consistent one with discount rate $\rho + \lambda$. When $\lambda \to \infty$ (T arrives immediately), the investment option is never exercised, because of the infinitely

[5] L'Hôpital's rule is employed to obtain the results when $\lambda \to \infty$.

high discount rate $\rho + \lambda$ (as $\lambda \to \infty$) and the assumption that the initial EBIT has not yet been favorable enough to undertake the investment.

(Tables 2 and 3 are inserted here.)

4 Debt-equity financing

From now on, we assume that the firm is partially financed with debt, which is issued upon investment. The contractual continuous coupon of the perpetual debt is denoted by c. We assume that the entrepreneur behaves in equityholders' interests.

Following Leland (1994), we consider equityholders' default on their debt obligations at the first time that the equity value is equal to zero. Let x_{d} denote the default threshold (subscript "d" stands for default). We assume that the default cost is given by a fraction $(1-\alpha)$ of the after-tax unlevered firm value upon default. The parameter $\alpha \in (0,1)$ measures the losses in firm value incurred by default.

Now, we consider the investment and capital structure decisions. The investment decision is characterized by an endogenously determined investment threshold x_{i}, as in Section 3. The capital structure decision involves the choice of a coupon level of debt and an endogenous default threshold. The coupon level c, which is characterized by a trade-off between tax benefits and default costs of debt financing, is determined simultaneously with the investment decision. In contrast, the default threshold x_{d}, which depends on the coupon level, is determined after the investment option is exercised. Note that the three endogenous variables (i.e., x_{i}, c, and x_{d}) in our model form a nested structure, and therefore enable us to examine the interaction between the investment and capital structure decisions.

For the reader's convenience, in this section, we first review the time-consistent benchmark (see Sundaresan and Wang, 2007). Then, we consider the sophisticated entrepreneur with time-inconsistent preferences. Finally, we examine the naive entrepreneur with time-inconsistent preferences.

The whole optimization problem can be formulated as follows:

$$(\mathrm{P}): \qquad \max_{x_{\mathrm{i}}^k, c^k, x_{\mathrm{d}}^k} V^{ok}(x; x_{\mathrm{i}}^k, c^k, x_{\mathrm{d}}^k)\big|_{x=x_{\mathrm{i}}^k}, \tag{19}$$

$$s.t.$$

$$\left.\frac{\partial E^k(x; c^k, x_{\mathrm{d}}^k)}{\partial x}\right|_{x=x_{\mathrm{d}}^k} = 0,$$

$$\left.\frac{\partial V^{ok}(x; x_{\mathrm{i}}^k, c^k, x_{\mathrm{d}}^k)}{\partial x}\right|_{x=x_{\mathrm{i}}^k} = \left.\frac{\partial V^k(x; c^k, x_{\mathrm{d}}^k)}{\partial x}\right|_{x=x_{\mathrm{i}}^k},$$

where E^k and V^k represent the equity value and the firm value (the sum of equity value and debt value) after investment, respectively, and $k \in \{*,\ s,\ n\}$. The two constraints above are known as the smooth-pasting conditions for default threshold x_{d}^k and investment threshold x_{i}^k, respectively, which are implied by value-maximization.[6] To solve the optimal decision problem (P) above, we need to first derive the value function for each case.

4.1 The time-consistent benchmark

We begin with the derivation of value functions. Let $E^*(x)$ be the equity value after investment. Since the instantaneous cash flow received by equityholders is $(1-\tau)(X(t)-c^*)$, the equity value is given by

$$\begin{aligned} E^*(x) &= (1-\tau)\mathbb{E}_t\left[\int_t^{T_{\mathrm{d}}^*} \mathrm{e}^{-\rho(u-t)}(X(u)-c^*)\mathrm{d}u\right] \\ &= \Pi^*(x) - (1-\tau)\frac{c^*}{\rho} - \left[\Pi^*(x_{\mathrm{d}}^*) - (1-\tau)\frac{c^*}{\rho}\right]\left(\frac{x}{x_{\mathrm{d}}^*}\right)^{\gamma_1}, \end{aligned} \tag{20}$$

where

$$\gamma_1 = \frac{1}{2} - \frac{\mu}{\sigma^2} - \sqrt{\left(\frac{\mu}{\sigma^2}-\frac{1}{2}\right)^2 + \frac{2\rho}{\sigma^2}} < 0. \tag{21}$$

The equity value (20) after investment has three components: (i) the present value of future EBIT without default, (ii) the present value of future coupon payments without default, taking into consideration the tax benefits of debt, and (iii) the present value of losses upon default, where $(x/x_{\mathrm{d}}^*)^{\gamma_1}$ is the default probability.

Similarly, the debt value after investment is given by

$$\begin{aligned} D^*(x) &= \mathbb{E}_t\left[\int_t^{T_{\mathrm{d}}^*} \mathrm{e}^{-\rho(u-t)}c^*\mathrm{d}u + \mathrm{e}^{-\rho(T_{\mathrm{d}}^*-t)}(1-\alpha)\Pi(x_{\mathrm{d}}^*)\right] \\ &= \frac{c^*}{\rho}\left[1-\left(\frac{x}{x_{\mathrm{d}}^*}\right)^{\gamma_1}\right] + (1-\alpha)\Pi^*(x_{\mathrm{d}}^*)\left(\frac{x}{x_{\mathrm{d}}^*}\right)^{\gamma_1}. \end{aligned} \tag{22}$$

The firm value after investment is obtained as

$$\begin{aligned} V^*(x) &= E^*(x) + D^*(x) \\ &= \Pi^*(x) + \tau\frac{c^*}{\rho}\left[1-\left(\frac{x}{x_{\mathrm{d}}^*}\right)^{\gamma_1}\right] - \alpha\Pi^*(x_{\mathrm{d}}^*)\left(\frac{x}{x_{\mathrm{d}}^*}\right)^{\gamma_1}. \end{aligned} \tag{23}$$

The option value of investment is given by

$$V^{o*}(x) = [V^*(x_{\mathrm{i}}^*) - I]\left(\frac{x}{x_{\mathrm{i}}^*}\right)^{\beta_1}. \tag{24}$$

With the value representations in Eqs. (20), (23), and (24), the solution set to the optimization problem (P) in (19) with $k = *$ is summarized in the following proposition.

[6] See Ziegler (2004) for the link between the smooth-pasting condition and the determination of the value-maximizing exercise strategy.

Proposition 4.1 (benchmark, debt-equity financing). *The optimal solution set for time-consistent entrepreneurs under debt-equity financing is given by*

$$(x_{\mathrm{i}}^*, x_{\mathrm{d}}^*, c^*) = \left(\frac{\bar{x}_{\mathrm{i}}^*}{\psi^*}, \frac{x_{\mathrm{i}}^*}{h^*}, \frac{\rho}{\rho-\mu} \frac{\gamma_1 - 1}{\gamma_1} x_{\mathrm{d}}^* \right), \tag{25}$$

where

$$\psi^* = 1 + \frac{\tau}{1-\tau} \frac{1}{h^*} > 1 \tag{26}$$

and

$$h^* = \frac{x_{\mathrm{i}}^*}{x_{\mathrm{d}}^*} = \left[1 - \gamma_1 \left(1 - \alpha + \frac{\alpha}{\tau} \right) \right]^{-\frac{1}{\gamma_1}} > 1. \tag{27}$$

Notice that h^*, which is a constant, measures the distance from investment threshold to default threshold. The inequality $h^* > 1$ ensures that $x_{\mathrm{i}}^* > x_{\mathrm{d}}^*$. By noting $\psi^* > 1$, we have $x_{\mathrm{i}}^* \geq \bar{x}_{\mathrm{i}}^*$, which implies that debt financing accelerates investment.

The leverage upon investment is calculated as

$$L^*(x_{\mathrm{i}}^*) = \frac{D^*(x_{\mathrm{i}}^*)}{V^*(x_{\mathrm{i}}^*)} = \frac{\gamma_1 - 1}{\gamma_1} \frac{1 - \xi^*}{(1-\tau)h^* + \tau}, \tag{28}$$

where

$$\xi^* = \left[1 - (1-\alpha)(1-\tau) \frac{\gamma_1}{\gamma_1 - 1} \right] (h^*)^{\gamma_1} \leq 1. \tag{29}$$

By substituting the solution set in (25) into $D^*(x)$ in (22) and $V^*(x)$ in (23) and letting $x = x_{\mathrm{i}}^*$, we have

$$D^*(x_{\mathrm{i}}^*) = \frac{\gamma_1}{\gamma_1 - 1} \frac{1}{\rho - \mu} \frac{x_{\mathrm{i}}^*}{h^*} \left[1 - (h^*)^{\gamma_1} \right] + (1-\alpha) \Pi^*(x_{\mathrm{i}}^*) (h^*)^{\gamma_1 - 1}, \tag{30}$$

and

$$V^*(x_{\mathrm{i}}^*) = \psi^* \Pi^*(x_{\mathrm{i}}^*), \tag{31}$$

respectively. Since both the debt value and firm value upon investment are linear functions of the investment threshold, we find that the leverage is independent of x_{i}^*.

4.2 Time-inconsistent preferences

In parallel to Section 3.2, we consider sophisticated and naive entrepreneurs with time-inconsistent preferences in this subsection. In addition to the investment decision discussed in Section 3.2, entrepreneurs need to consider the capital structure decision.

4.2.1 The sophisticated entrepreneur

If the future self arrives prior to the investment option being exercised, i.e., $T < T_{\mathrm{i}}^s$, the valuation after T is the δ proportion of the time-consistent valuation. In the following, we focus on the valuation for the case $T_{\mathrm{i}}^s \leq T$. As mentioned in Section 3.2.1, the sophisticated entrepreneur's optimization problem does not depend on self j. The stationary solution that we search for is a fixed-point to the option exercising problem.

The equity value after investment ($T \geq t \geq T_{\mathrm{i}}^s$) is given by

$$\begin{aligned}E^s(x) =&(1-\tau)\mathbb{E}_t\left[\mathbf{1}_{\{T_{\mathrm{d}}^s \leq T\}} \int_t^{T_{\mathrm{d}}^s} \mathrm{e}^{-\rho(u-t)}(X(u)-c^s)\mathrm{d}u\right] \\ &+ (1-\tau)\mathbb{E}_t\left[\mathbf{1}_{\{T_{\mathrm{d}}^s > T\}} \left[\int_t^{T} \mathrm{e}^{-\rho(u-t)}(X(u)-c^s)\mathrm{d}u + \int_T^{T_{\mathrm{d}}^s} \mathrm{e}^{-\rho(u-t)}\delta(X(u)-c^s)\mathrm{d}u\right]\right] \end{aligned} \tag{32}$$

$$\begin{aligned}=&(1-\theta)\Pi^*(x) - (1-\tau)(1-\theta_1)\frac{c^s}{\rho} - \left[(1-\theta)\Pi^*(x_{\mathrm{d}}^s) - (1-\tau)(1-\theta_1)\frac{c^s}{\rho}\right]\left(\frac{x}{x_{\mathrm{d}}^s}\right)^{\gamma_2} \\ &- \delta\left[\Pi^*(x) - (1-\tau)\frac{c^s}{\rho}\right]\left[\left(\frac{x}{x_{\mathrm{d}}^s}\right)^{\gamma_1} - \left(\frac{x}{x_{\mathrm{d}}^s}\right)^{\gamma_2}\right],\end{aligned} \tag{33}$$

where

$$\theta_1 = \frac{(1-\delta)\lambda}{\rho+\lambda} \geq 0. \tag{34}$$

See A.4 of the Appendix for the derivation. The right-hand side (RHS) of (32) consists of two terms: (i) When the future self arrives after the default time ($T_{\mathrm{d}}^s \leq T$), the equity value is the EBIT, subtracting the coupon, which is discounted exponentially with ρ until default. (ii) When the future self arrives before the default time ($T_{\mathrm{d}}^s > T$), the equity value is the EBIT, subtracting the coupon, which is discounted exponentially with ρ until the future self arrives, and then further discounted by the factor δ from the arrival of the future self until default.

The equity value (33) after investment for the sophisticated entrepreneur has four components (cf. Eq. (20)): (i) the present value of future uncertain EBIT (reduced by the factor $1-\theta$ due to the time inconsistency), (ii) the present value of future certain coupon payments without default (reduced by the factor $(1-\theta_1)$ due to the time inconsistency), taking the tax benefits of debt into consideration, (iii) the present value of losses upon the current self's default (the default probability is $(x/x_{\mathrm{d}}^*)^{\gamma_2}$ and the corresponding discount factor is $\rho+\lambda$), and (iv) the present value of losses upon the future self's default (the default probability is $(x/x_{\mathrm{d}}^*)^{\gamma_1}$ and the corresponding discount factor is ρ). Since the default option can be exercised only once, the $(x/x_{\mathrm{d}}^s)^{\gamma_2}$ part is subtracted to exclude the probability that the EBIT process first hits x_{d}^s during the current period and then hits x_{d}^s for the second time in the future period, i.e., $(x/x_{\mathrm{d}}^s)^{\gamma_2}\,(x_{\mathrm{d}}^s/x_{\mathrm{d}}^s)^{\gamma_1} = (x/x_{\mathrm{d}}^s)^{\gamma_2}$.

Similarly, the debt value after investment is given by

$$\begin{aligned}D^s(x) =& (1-\theta_1)\frac{c^s}{\rho} + \left[(1-\theta)(1-\alpha)\Pi^*(x_{\mathrm{d}}^s) - (1-\theta_1)\frac{c^s}{\rho}\right]\left(\frac{x}{x_{\mathrm{d}}^s}\right)^{\gamma_2} \\ &+ \delta\left[(1-\alpha)\Pi^*(x_{\mathrm{d}}^s) - \frac{c^s}{\rho}\right]\left[\left(\frac{x}{x_{\mathrm{d}}^s}\right)^{\gamma_1} - \left(\frac{x}{x_{\mathrm{d}}^s}\right)^{\gamma_2}\right].\end{aligned} \tag{35}$$

The firm value after investment is thus obtained as

$$V^s(x) = (1-\theta)\Pi^*(x) + (1-\theta_1)\tau\frac{c^s}{\rho} - \left[\alpha(1-\theta)\Pi^*(x_{\mathrm{d}}^s) + (1-\theta_1)\tau\frac{c^s}{\rho}\right]\left(\frac{x}{x_{\mathrm{d}}^s}\right)^{\gamma_2}$$
$$- \delta\left[\alpha\Pi^*(x_{\mathrm{d}}^s) + \tau\frac{c^s}{\rho}\right]\left[\left(\frac{x}{x_{\mathrm{d}}^s}\right)^{\gamma_1} - \left(\frac{x}{x_{\mathrm{d}}^s}\right)^{\gamma_2}\right]. \tag{36}$$

The representation of the option value of investment is similar to Eq. (10), with Π^*, Π, and $\bar{x}_{\mathrm{i}}^s$ being replaced by V^*, V^s, and x_{i}^s, respectively. That is,

$$V^{os}(x) = [V^s(x_{\mathrm{i}}^s) - I]\left(\frac{x}{x_{\mathrm{i}}^s}\right)^{\beta_2} + \delta\,[V^*(x_{\mathrm{i}}^s) - I]\left[\left(\frac{x}{x_{\mathrm{i}}^s}\right)^{\beta_1} - \left(\frac{x}{x_{\mathrm{i}}^s}\right)^{\beta_2}\right]. \tag{37}$$

With the value representations in Eqs. (33), (36), and (37), the solution set to the optimization problem (P) in (19) with $k = s$ is summarized in the following proposition (cf. Proposition 4.1). The proof is similar to that of Proposition 4.1 and omitted.

Proposition 4.2 (sophisticated entrepreneur, debt-equity financing). *The optimal solution set for sophisticated entrepreneurs under debt-equity financing is given by*

$$(x_{\mathrm{i}}^s, x_{\mathrm{d}}^s, c^s) = \left(\frac{\bar{x}_{\mathrm{i}}^s}{\psi}, \frac{x_{\mathrm{i}}^s}{h^s}, \frac{\rho}{\rho-\mu}\frac{x_{\mathrm{d}}^s}{\kappa_2}\right), \tag{38}$$

where

$$\psi = \frac{(\beta_2-1)(1-\theta)\psi^s - (\beta_2-\beta_1)\delta\psi^*}{(\beta_2-1)(1-\theta)-(\beta_2-\beta_1)\delta}, \tag{39}$$

$$\psi^s = 1 + \frac{\tau}{1-\tau}\frac{1}{(1-\theta)h^s\kappa_2}\left[1-\theta_1-\delta(h^s)^{\gamma_1}-(1-\theta_1-\delta)(h^s)^{\gamma_2}\right]$$
$$- \frac{\alpha}{(1-\theta)h^s}\left[\delta(h^s)^{\gamma_1}+(1-\theta-\delta)(h^s)^{\gamma_2}\right], \tag{40}$$

$$\kappa_2 = \frac{\delta\gamma_1 + (1-\theta_1-\delta)\gamma_2}{\delta(\gamma_1-1)+(1-\theta-\delta)(\gamma_2-1)} > 0, \tag{41}$$

and h^s $(= x_{\mathrm{i}}^s/x_{\mathrm{d}}^s)$ satisfies the following equation:

$$1-\theta_1-\delta(1-\gamma_1)(h^s)^{\gamma_1}-(1-\theta_1-\delta)(1-\gamma_2)(h^s)^{\gamma_2}$$
$$-\alpha\kappa_2\frac{1-\tau}{\tau}\left[\delta(1-\gamma_1)(h^s)^{\gamma_1}+(1-\theta-\delta)(1-\gamma_2)(h^s)^{\gamma_2}\right] = 0. \tag{42}$$

If sophisticated entrepreneurs have time-consistent preferences (i.e., $\delta = 1$ or $\lambda = 0$), the optimal solution set is exactly the same as that of the time-consistent benchmark in Proposition 4.1.

Table 4 summarizes the analytical solutions of the time-consistent benchmark and the sophisticated entrepreneur with time inconsistency. By comparing the default thresholds x_{d}^* for the time-consistent benchmark and x_{d}^s for the sophisticated entrepreneur with time-inconsistent preferences, we find that the coefficient κ_2 in x_{d}^s plays the same role as $\gamma_1/(\gamma_1-$

1) in x_{d}^{*}. The numerator of κ_2 is a weighted average of γ_1 and γ_2, with δ and $1-\theta_1\delta$ as the respective weights. The coefficient $1-\theta_1$ stems from the quasi-hyperbolic discounting procedure of the coupon level c^s (see (33)). On the other hand, the denominator of κ_2 is a weighted average of γ_1-1 and γ_2-1, with δ and $1-\theta-\delta$ as the respective weights. The coefficient $1-\theta$ stems from the quasi-hyperbolic discounting procedure of the EBIT $X(t)$ (see (8)). Moreover, by comparing the investment thresholds $\bar{x}_{\mathrm{i}}^{s}$ and x_{i}^{s} for the sophisticated entrepreneur, we find that x_{i}^{s} under debt-equity financing also resembles $\bar{x}_{\mathrm{i}}^{s}$ under all-equity financing, where δ and $(1-\theta)$ are replaced by $\delta\psi^{*}$ and $(1-\theta)\psi^{s}$, respectively. The similarity stems from the value functions V^{*} in (31) and V^{s} in (44) under debt-equity financing and Π^{*} in (4) and Π in (8) under all-equity financing.

(Table 4 is inserted here.)

By substituting the solution set in (38) into $D^s(x)$ in (35) and $V^s(x)$ in (36) and letting $x = x_{\mathrm{i}}^{s}$, we have

$$\begin{aligned} D^s(x_{\mathrm{i}}^{s}) =& \frac{\kappa_2}{\rho-\mu}\frac{x_{\mathrm{i}}^{s}}{h^s}\left[(1-\theta_1)\left[1-(h^s)^{\gamma_2}\right]-\delta\left[(h^s)^{\gamma_1}-(h^s)^{\gamma_2}\right]\right] \\ &+(1-\theta+\delta)(1-\alpha)\Pi^{*}(x_{\mathrm{i}}^{s})(h^s)^{\gamma_1-1}, \end{aligned} \tag{43}$$

and

$$V^s(x_{\mathrm{i}}^{s}) = \psi^s(1-\theta)\Pi^{*}(x_{\mathrm{i}}^{s}), \tag{44}$$

respectuvely. Since both the debt value and the firm value upon investment are linear functions of the investment threshold, we find that the leverage upon investment $L^s(x_{\mathrm{i}}^{s}) = D^s(x_{\mathrm{i}}^{s})/V^s(x_{\mathrm{i}}^{s})$ is independent of x_{i}^{s}.

4.2.2 The naive entrepreneur

Since the valuation is the δ portion of the valuation in the time-consistent case if $T < T_{\mathrm{i}}^{n}$, we focus on the valuation for the case $T_{\mathrm{i}}^{n} \leq T$ in the following. Recall that the equity value upon default is 0. There is no effect of time inconsistency on payoff. However, the option value of waiting is lower under time-inconsistent preferences, compared to the time-consistent benchmark. Thus, we conjecture that $x_{\mathrm{d}}^{n}(c^n) \geq x_{\mathrm{d}}^{*}(c^n)$ given the same coupon level. As mentioned in Section 3.2.2, the current self of the naive entrepreneur determines $x_{\mathrm{d}}^{n}(c^n)$, taking the future selves' default threshold $x_{\mathrm{d}}^{*}(c^n)$ into consideration.

The equity value after investment ($T \geq t \geq T_\mathrm{i}^n$) is therefore given by

$$
\begin{aligned}
E^n(x) = &(1-\tau)\mathbb{E}_t\left[\mathbf{1}_{\{T_\mathrm{d}^n \leq T\}}\int_t^{T_\mathrm{d}^n} \mathrm{e}^{-\rho(u-t)}(X(u)-c^n)\mathrm{d}u\right] \\
&+ (1-\tau)\mathbb{E}_t\left[\mathbf{1}_{\{T_\mathrm{d}^n > T\}}\left[\int_t^{T} \mathrm{e}^{-\rho(u-t)}(X(u)-c^n)\mathrm{d}u + \int_T^{T_\mathrm{d}^*} \mathrm{e}^{-\rho(u-t)}\delta(X(u)-c^n)\mathrm{d}u\right]\right]
\end{aligned}
\tag{45}
$$

$$
\begin{aligned}
&= (1-\theta)\Pi^*(x) - (1-\theta_1)(1-\tau)\frac{c^n}{\rho} - \left[(1-\theta)\Pi^*(x_\mathrm{d}^n) - (1-\theta_1)(1-\tau)\frac{c^n}{\rho}\right]\left(\frac{x}{x_\mathrm{d}^n}\right)^{\gamma_2} \\
&\quad - \delta\left[\Pi^*(x_\mathrm{d}^*) - (1-\tau)\frac{c^n}{\rho}\right]\left[\left(\frac{x}{x_\mathrm{d}^*}\right)^{\gamma_1} - \left(\frac{x}{x_\mathrm{d}^n}\right)^{\gamma_2}\left(\frac{x_\mathrm{d}^n}{x_\mathrm{d}^*}\right)^{\gamma_1}\right].
\end{aligned}
\tag{46}
$$

See A.4 of the Appendix for the derivation. The equity value (45) resembles (32), where the default time T_d^s is replaced by T_d^n when the future self arrives after the default time and T_d^* when the future self arrives before the default time, respectively.

The third term in Eq. (46) is the present value of losses upon the current self's default (default threshold is x_d^n), where the default probability is $(x/x_\mathrm{d}^n)^{\gamma_2}$ and the corresponding discount factor is $\rho + \lambda$. The fourth term is the present value of losses upon the future self's default (default threshold is x_d^*), where the default probability is $(x/x_\mathrm{d}^*)^{\gamma_1}$ and the corresponding discount factor is ρ. Since the default option can be exercised only once, the $(x/x_\mathrm{d}^s)^{\gamma_2}$ part is subtracted to exclude the probability that the EBIT process first hits x_d^n during the current period and then hits x_d^* in the future period, i.e., $(x/x_\mathrm{d}^n)^{\gamma_2}\,(x_\mathrm{d}^n/x_\mathrm{d}^*)^{\gamma_1}$.

The equity value (46) after investment for the naive entrepreneur also has four components (cf. Eq. (33)), where the current self and the future self's default thresholds are replaced by x_d^n and x_d^*, respectively.

Similarly, the debt value after investment is obtained as

$$
\begin{aligned}
D^n(x) = &(1-\theta_1)\frac{c^n}{\rho} + \left[(1-\theta)(1-\alpha)\Pi^*(x_\mathrm{d}^n) - (1-\theta_1)\frac{c^n}{\rho}\right]\left(\frac{x}{x_\mathrm{d}^n}\right)^{\gamma_2} \\
&+ \delta\left[(1-\alpha)\Pi^*(x_\mathrm{d}^*) - \frac{c^n}{\rho}\right]\left[\left(\frac{x}{x_\mathrm{d}^*}\right)^{\gamma_1} - \left(\frac{x}{x_\mathrm{d}^n}\right)^{\gamma_2}\left(\frac{x_\mathrm{d}^n}{x_\mathrm{d}^*}\right)^{\gamma_1}\right].
\end{aligned}
\tag{47}
$$

The firm value after investment is therefore given by

$$
\begin{aligned}
V^n(x) = &(1-\theta)\Pi^*(x) + (1-\theta_1)\tau\frac{c^n}{\rho} \\
&- \left[\alpha(1-\theta)\Pi^*(x_\mathrm{d}^n) + (1-\theta_1)\tau\frac{c^n}{\rho}\right]\left(\frac{x}{x_\mathrm{d}^n}\right)^{\gamma_2} \\
&- \delta\left[\alpha\Pi^*(x_\mathrm{d}^*) + \tau\frac{c^n}{\rho}\right]\left[\left(\frac{x}{x_\mathrm{d}^*}\right)^{\gamma_1} - \left(\frac{x}{x_\mathrm{d}^n}\right)^{\gamma_2}\left(\frac{x_\mathrm{d}^n}{x_\mathrm{d}^*}\right)^{\gamma_1}\right].
\end{aligned}
\tag{48}
$$

The representations of the option value $V^{on}(x)$ of investment are similar to Eqs. (15) and (16), with Π^*, Π, and $\bar{x}_\mathrm{i}^n$ being replaced by V^*, V^n, and x_i^n, respectively.

Proposition 4.3 (naive entrepreneur, debt-equity financing). *The investment threshold for naive entrepreneurs under debt-equity financing are obtained by solving the smooth-pasting condition* $\left.\frac{\partial V^{on}(x;x_{\mathrm{i}}^n,c^n,x_{\mathrm{d}}^n)}{\partial x}\right|_{x=x_{\mathrm{i}}^n} = \left.\frac{\partial V^{n}(x;c^n,x_{\mathrm{d}}^n)}{\partial x}\right|_{x=x_{\mathrm{i}}^n}$ *for* x_{i}^n.

The default threshold and coupon level are determined by simultaneously solving the following two equations:

$$x_{\mathrm{d}}^n = \frac{\rho-\mu}{\rho}\frac{1}{\gamma_2-1}\frac{c^n}{1-\theta}\left[(1-\theta_1)\gamma_2 - \delta\frac{\gamma_1-\gamma_2}{\gamma_1-1}\left(\frac{x_{\mathrm{d}}^n}{x_{\mathrm{d}}^*}\right)^{\gamma_1}\right] \tag{49}$$

and

$$\begin{aligned}&(1-\theta_1)\tau\left[1-\left(\frac{x_{\mathrm{i}}^n}{x_{\mathrm{d}}^n}\right)^{\gamma_2}\right] + (1-\theta)\alpha\rho(\gamma_2-1)\frac{1-\tau}{\rho-\mu}\left(\frac{x_{\mathrm{i}}^n}{x_{\mathrm{d}}^n}\right)^{\gamma_2}\frac{\mathrm{d}x_{\mathrm{d}}^n}{\mathrm{d}c^n}\\ &+\delta(\gamma_1-1)\left(\alpha(1-\tau)\frac{\gamma_1}{\gamma_1-1}+\tau\right)\left[\left(\frac{x_{\mathrm{i}}^n}{x_{\mathrm{d}}^*}\right)^{\gamma_1} - \left(\frac{x_{\mathrm{i}}^n}{x_{\mathrm{d}}^n}\right)^{\gamma_2}\left(\frac{x_{\mathrm{d}}^n}{x_{\mathrm{d}}^*}\right)^{\gamma_1}\right]\\ &+\left[\delta(\gamma_1-\gamma_2)\left(\alpha(1-\tau)\frac{\gamma_1}{\gamma_1-1}+\tau\right)\left(\frac{x_{\mathrm{d}}^n}{x_{\mathrm{d}}^*}\right)^{\gamma_1} + (1-\theta_1)\tau\gamma_2\right]\left(\frac{x_{\mathrm{i}}^n}{x_{\mathrm{d}}^n}\right)^{\gamma_2}\frac{\mathrm{d}x_{\mathrm{d}}^n}{\mathrm{d}c^n}\frac{c^n}{x_{\mathrm{d}}^n} = 0,\end{aligned} \tag{50}$$

where

$$\frac{\mathrm{d}x_{\mathrm{d}}^n}{\mathrm{d}c^n} = \frac{\rho-\mu}{\rho}\frac{(1-\theta_1)\gamma_2 + \delta(\gamma_1-\gamma_2)\left(x_{\mathrm{d}}^n/x_{\mathrm{d}}^*\right)^{\gamma_1}}{(\gamma_2-1)(1-\theta)+\delta(\gamma_1-\gamma_2)\left(x_{\mathrm{d}}^n/x_{\mathrm{d}}^*\right)^{\gamma_1-1}}, \qquad \frac{\mathrm{d}x_{\mathrm{d}}^*}{\mathrm{d}c^n} = \frac{\rho-\mu}{\rho}\frac{\gamma_1}{\gamma_1-1}. \tag{51}$$

If naive entrepreneurs have time-consistent preferences (i.e., $\delta = 1$ *or* $\lambda = 0$*), the optimal solution set is exactly the same as that of the time-consistent benchmark in Proposition 4.1.*

Since Eqs. (49) and (50) are complicated, it seems difficult to obtain an analytical solution set. We solve the equations numerically in the next section.

By substituting $\delta = 0$ and $\lambda \to \infty$ into (49) and (50) (see Table 2 for reference), we obtain the results for the special cases as in Table 3. The economic interpretation in this case is similar to that in the case of all-equity financing (see the end of Section 3.2.2).

5 Model implications

This section provides several model predictions by examining the properties of the model solutions numerically. In particular, we focus on the impact of time inconsistency on investment and capital structure decisions. The base parameter values are set as $\mu = 0.01$, $\sigma = 0.25$, $\rho = 0.06$, $\tau = 0.15$, $\alpha = 0.25$ (following the typical capital structure models in, e.g., Hackbarth and Mauer, 2012), $\delta = 0.3$ and $\lambda = 0.33$ (following Grenadier and Wang, 2007), and $I = 200$ (can be set freely since the model solutions are all proportional to the fixed investment cost).

5.1 Adjustment of discounting parameter

Figure 1 shows the investment threshold with respect to the time inconsistency parameters δ and λ. As reference points, the time-consistent benchmarks under all-equity and debt-equity financing, $\bar{x}_{\mathrm{i}}^{*} = 27.11$ and $x_{\mathrm{i}}^{*} = 25.72$, are depicted on the right (left) endpoints of the upper (lower) panel for $\delta = 1$ ($\lambda = 0$), respectively. For the base parameter case where $\delta = 0.3$ and $\lambda = 0.33$, we find that $\bar{x}_{\mathrm{i}}^{s} = 77.12$, $\bar{x}_{\mathrm{i}}^{n} = 89.86$, and $x_{\mathrm{i}}^{s} = 73.62$, which are about three times higher than the time-consistent benchmarks under all-equity and debt-equity financing. It is hard to believe that time-inconsistent entrepreneurs exercise investment option only when the EBIT is three times higher than time-consistent entrepreneurs do in reality.

(Figure 1 is inserted here.)

The large difference in the investment decisions is considered to be mainly attributed to the discounting parameters. Comparing the discounting parameter set of the quasi-hyperbolic discounting (δ, λ, ρ) and the single exponential discounting parameter ρ, cash flows are obviously more discounted by quasi-hyperbolic discounting. As Jamison and Jamison (2011) argue, "exponential discounting isolates the concepts of *amount* and *speed* into a single parameter that must be disaggregated in order to characterize nonconstant rate procedures". They suggest the inverse of the present value of a unit stream as a natural measure of the amount of a procedure that discounts the future. It is no doubt that the exponential discounting procedure serves as the time-consistent benchmark in our paper. However, what kind of parameter for the exponential discounting procedure is appropriate for serving as the time-consistent benchmark?

Instead of simply adding the other two parameters δ and λ to the parameter ρ of the exponential discounting, we follow the suggestion given in Jamison and Jamison (2011) to exclude the inequivalence of the amount that the two procedures discount. More precisely, we adjust the corresponding exponential discounting parameter to make the present value of a unit stream of payoff the same as that with the quasi-hyperbolic discounting parameter set (δ, λ, ρ). This parameter adjustment is especially important for the comparison of the results under the two discounting procedures. We will show that the results with parameter adjustment can provide a more reasonable benchmark through numerical examples in the following subsections.

Note that time is divided into two periods for each self of the entrepreneur, the current and future periods. Recalling that the arrival time T of the future self is exponentially distributed with mean $1/\lambda$, the present value of a unit stream of payoff is given by

$$\mathbb{E}_t \left[\int_t^T \mathrm{e}^{-\rho(u-t)} \mathrm{d}u + \int_T^\infty \delta \mathrm{e}^{-\rho(u-t)} \mathrm{d}u \right] = \frac{1 - \theta_1}{\rho}, \tag{52}$$

where θ_1 is defined in Eq. (34).

Remark 5.1. *The present value of a unit stream of payoff under the quasi-hyperbolic discounting parameter set* (δ, λ, ρ) *is the same as that under the exponential discounting parameter* ρ^* *when*

$$\rho^* = \frac{\rho}{1 - \theta_1} \geq \rho, \tag{53}$$

where the equality $\rho^* = \rho$ *holds when* $\delta = 1$ *or* $\lambda = 0$. *Moreover, we have* $\partial\rho^*/\partial\delta \leq 0$ *and* $\partial\rho^*/\partial\lambda \geq 0$.

Remark 5.2. *The present value of an uncertain, continuous cash flow* $X(t)$ *(with a positive drift* $\mu \geq 0$*) under the quasi-hyperbolic discounting parameters* (δ, λ, ρ) *is larger than that under the exponential discounting parameter* ρ^**, i.e.,* $\Pi(x; \delta, \lambda, \rho) \geq \Pi^*(x; \rho^*)$*, where the equality holds when* $\delta = 1$ *or* $\lambda = 0$.[7]

See A.5 of the Appendix for the proof of Remark 5.2. The intuition is as follows. After normalization of the amount that the two procedures discount, entrepreneurs with time-inconsistent preferences use a lower $\rho \leq \rho^*$ to discount the EBIT in the near future, but use an additional factor δ to discount the EBIT in the far future. Since entrepreneurs' preferences are present-biased, the value in the near future is much more important than that in the far future. If the expected growth rate of EBIT is nonnegative (i.e., $\mu \geq 0$), then entrepreneurs with time-inconsistent preferences overestimate the appreciation in the present value of EBIT because of $\rho \leq \rho^*$.[8]

Remark 5.3. *For the case of lump-sum payoff in Grenadier and Wang (2007), the effect of time inconsistency on payoff value does not exist with parameter adjustment. The only effect is the one on the option value of waiting, which is decreased with time inconsistency. Therefore, time inconsistency accelerates investment for the case of lump-sum payoff. In other words, the results in Grenadier and Wang (2007) still hold even after parameter adjustment.*

In the following, we compare our main results (debt-equity financing with time-inconsistent preferences) with four natural benchmarks: (i) all-equity financing with time-inconsistent preferences (Grenadier and Wang, 2007); (ii) the case (i) with parameter adjustment; (iii) debt-equity financing with time-consistent preferences (Sundaresan and Wang, 2007); and (iv) the case (iii) with parameter adjustment.

[7] If the drift is negative, then the present value of an uncertain, continuous cash flow $X(t)$ is lower under quasi-hyperbolic discounting. Since we consider an investment problem in this paper, we focus on the case that $\mu \geq 0$.

[8] On the other hand, if the expected growth rate of EBIT is negative (i.e., $\mu \leq 0$), then entrepreneurs with time-inconsistent preferences overestimate the depreciation in the present value of EBIT.

5.2 Impact of time inconsistency on investment

Figure 2 adds the time-consistent benchmarks $\bar{x}_{\mathrm{i}}^{*}(\rho^{*})$ and $x_{\mathrm{i}}^{*}(\rho^{*})$ with parameter adjustment under all-equity and debt-equity financing to the same panels as in Fig. 1. We write ρ^{*} as an argument of the investment thresholds to distinguish the time-consistent benchmarks with parameter adjustment from those without parameter adjustment ($\bar{x}_{\mathrm{i}}^{*} = 27.11$ and $x_{\mathrm{i}}^{*} = 25.72$). We find that the differences between time-inconsistent investment thresholds and the time-consistent benchmarks with parameter adjustment are much smaller than those without parameter adjustment.

(Figure 2 is inserted here.)

In order to investigate the differences more directly, Fig. 3 plots the ratio of the investment thresholds under time-inconsistent preferences to the time-consistent benchmarks. Without the adjustment of the discounting parameter (corresponding to the investment thresholds under all-equity financing in Grenadier and Wang, 2007 and debt-equity financing in Sundaresan and Wang, 2007), the ratio of investment thresholds increases rapidly with time inconsistency. Taking the base parameter case where $\delta = 0.3$ and $\lambda = 0.33$ for example, the ratios are given by $\bar{x}_{\mathrm{i}}^{s}/\bar{x}_{\mathrm{i}}^{*} = 77.12/27.11 = 2.84$ under all-equity financing and $x_{\mathrm{i}}^{s}/x_{\mathrm{i}}^{*} = 73.62/25.72 = 2.86$ under debt-equity financing. However, after the adjustment of the discounting parameter, the ratio becomes $\bar{x}_{\mathrm{i}}^{s}/\bar{x}_{\mathrm{i}}^{*}(\rho^{*}) = 77.12/53.23 = 1.45$ under all-equity financing and $x_{\mathrm{i}}^{s}/x_{\mathrm{i}}^{*}(\rho^{*}) = 73.62/49.77 = 1.48$ under debt-equity financing, which are more reasonable when considering the differences in investment strategies between large firms with time-consistent preferences and entrepreneurial firms with time-inconsistent preferences. Therefore, the adjustment procedure is considered to be important when comparing the time-inconsistent results and the time-consistent benchmark.

(Figure 3 is inserted here.)

From the upper panel of Fig. 2, we find that $\bar{x}_{\mathrm{i}}^{s} = \bar{x}_{\mathrm{i}}^{n} = \bar{x}_{\mathrm{i}}^{*}(\rho^{*})$ and $x_{\mathrm{i}}^{s} = x_{\mathrm{i}}^{*}(\rho^{*})$ hold when $\delta = 0$, which confirm the results in the third column of Table 3. From the lower panel of Fig. 2, we find that all the investment thresholds increase in λ. The limiting case when $\lambda \to \infty$ is given in the last column of Table 3.

Both panels in Fig. 2 show that the investment thresholds satisfy the inequalities $\bar{x}_{\mathrm{i}}^{n} \geq \bar{x}_{\mathrm{i}}^{s} \geq \bar{x}_{\mathrm{i}}^{*}(\rho^{*}) \geq \bar{x}_{\mathrm{i}}^{*}$ and $x_{\mathrm{i}}^{s} \geq x_{\mathrm{i}}^{*}(\rho^{*}) \geq x_{\mathrm{i}}^{*}$, where the equalities hold when $\delta = 1$ or $\lambda = 0$. Hence, the time inconsistency delays investment under both all-equity financing and debt-equity financing, compared to the respective time-consistent benchmarks.

As mentioned in Section 3.2, for the case of all-equity financing, time inconsistency influences investment through two channels: (i) earlier investment due to the decrease in option value of waiting and (ii) later investment due to the decrease in payoff value upon investment. Investment is delayed because the second effect dominates the first effect. Sophisticated entrepreneurs, who recognize correctly that their future selves do not behave in the current selves' interest, desire to invest earlier than naive entrepreneurs so as to protect themselves against the suboptimal behavior of future selves. Thus, we have $\bar{x}_{\mathrm{i}}^{n} \geq \bar{x}_{\mathrm{i}}^{s} \geq \bar{x}_{\mathrm{i}}^{*}$ under all-equity financing. Even after the adjustment of the parameter for the time-consistent benchmark, the ordering $\bar{x}_{\mathrm{i}}^{n} \geq \bar{x}_{\mathrm{i}}^{s} \geq \bar{x}_{\mathrm{i}}^{*}(\rho^*)$ still holds.

Furthermore, we find from Fig. 2 that $x_{\mathrm{i}}^{s} < \bar{x}_{\mathrm{i}}^{s}$ as well as $x_{\mathrm{i}}^{*} < \bar{x}_{\mathrm{i}}^{*}$, which implies that debt-equity financing accelerates investment compared to all-equity financing, even with time inconsistency. Debt financing increases the payoff value upon investment and thus accelerates investment. Although the impact of the second effect of time inconsistency on investment (later investment due to the decrease in payoff value upon investment) is attenuated by debt financing, it is still dominant, leading $x_{\mathrm{i}}^{s} \geq x_{\mathrm{i}}^{*}(\rho^*) \geq x_{\mathrm{i}}^{*}$. It is difficult to show the comparative analysis of x_{i}^{n} in Fig. 2. However, we conjecture that $x_{\mathrm{i}}^{n} \geq x_{\mathrm{i}}^{s} \geq x_{\mathrm{i}}^{*}(\rho^*) \geq x_{\mathrm{i}}^{*}$, because sophisticated entrepreneurs desire to invest earlier than naive ones.

Prediction 5.1 (comparison of investment thresholds). *Naive entrepreneurs exercise the investment option later than sophisticated entrepreneurs, who in turn exercise later than time-consistent entrepreneurs (even after parameter adjustment), i.e.,* $\bar{x}_{\mathrm{i}}^{n} \geq \bar{x}_{\mathrm{i}}^{s} \geq \bar{x}_{\mathrm{i}}^{*}(\rho^*) \geq \bar{x}_{\mathrm{i}}^{*}$ *under all-equity financing and* $x_{\mathrm{i}}^{n} \geq x_{\mathrm{i}}^{s} \geq x_{\mathrm{i}}^{*}(\rho^*) \geq x_{\mathrm{i}}^{*}$ *under debt-equity financing, where the equalities hold when* $\delta = 1$ *or* $\lambda = 0$.

5.3 Impact of time inconsistency on default and coupon level

Figure 4 shows the default thresholds with respect to the time inconsistency parameters δ and λ, keeping the investment threshold as x_{i}^{s}. As a reference point, the time-consistent benchmark, $x_{\mathrm{d}}^{*} = 7.86$, is depicted on the right (left) endpoint of the upper (lower) panel, respectively.

(Figure 4 is inserted here.)

Prediction 5.2 (comparison of default thresholds). *Naive entrepreneurs exercise the default option earlier than sophisticated entrepreneurs and time-consistent entrepreneurs, i.e.,* $x_{\mathrm{d}}^{n} \geq x_{\mathrm{d}}^{s} \geq x_{\mathrm{d}}^{*}(\rho^*) \geq x_{\mathrm{d}}^{*}$, *where the equalities hold when* $\delta = 1$ *or* $\lambda = 0$.

At first glance, the $x_{\mathrm{d}}^{n} \geq x_{\mathrm{i}}^{s}$ part seems strange, because intuitively sophisticated entrepreneurs desire to default earlier than naive entrepreneurs so as to avoid the suboptimal behavior of future selves. However, the intuition is based on the same coupon level. Figure 5 shows the default threshold with respect to the time inconsistency parameters δ and λ, with the same coupon level being fixed at $c = 20$. In this case, we have

$x_{\mathrm{d}}^{*} \leq x_{\mathrm{d}}^{n} \leq x_{\mathrm{d}}^{s} \leq x_{\mathrm{d}}^{*}(\rho^{*})$ because there is no effect of payoff value but only the effect of option value of waiting.

(Figure 5 is inserted here.)

In fact, default thresholds depend positively on coupon levels. The optimization problem (19) under debt-equity financing is to solve the three endogenous variables x_{i}^{k}, c^{k}, x_{d}^{k} simultaneously. As shown in Fig. 6, sophisticated entrepreneurs choose lower coupon levels ($c^{s} \leq c^{n}$), leading to lower default thresholds. In other words, sophisticated entrepreneurs exercise default option later than naive entrepreneurs. This result is exactly opposite to the exercise of investment option, where sophisticated entrepreneurs exercise investment option earlier than naive entrepreneurs.

(Figure 6 is inserted here.)

Combining with Prediction 5.1, we find that naive entrepreneurs invest later and default earlier than sophisticated entrepreneurs, leading to a shorter operating period. Camerer and Lovallo (1999) report that 61.5% of newly created companies in manufacturing industries are no longer in business after five years. The behavioral finance literature suggests that the short operating period can be explained only by entrepreneurial bounded rationality at the project initiation stage (see Larwood and Whittaker, 1977, for managers who optimistically planning for the future). In our model, naive entrepreneurs' bounded rationality is embodied in their misunderstood belief that they can commit their future selves to behave according to their current preferences. In this sense, our result is consistent with the empirical findings.

Figure 7 plots the ratio of the default thresholds under time-inconsistent preferences to those under the time-consistent benchmarks. Without parameter adjustment (corresponding to the default threshold in Sundaresan and Wang, 2007), the ratio of default thresholds increases rapidly with respect to time inconsistency. Taking the base parameter case where $\delta = 0.3$ and $\lambda = 0.33$ for example, the ratios are given by $x_{\mathrm{d}}^{n}/x_{\mathrm{d}}^{*} = 29.99/7.86 = 3.82$ and $x_{\mathrm{d}}^{s}/\bar{x}_{\mathrm{d}}^{*} = 25.35/7.86 = 3.23$. However, after the adjustment of the discounting parameter, the ratios become $x_{\mathrm{d}}^{n}/x_{\mathrm{d}}^{*}(\rho^{*}) = 29.99/19.90 = 1.50$ and $x_{\mathrm{d}}^{s}/x_{\mathrm{d}}^{*}(\rho^{*}) = 25.35/19.90 = 1.27$, which are more reasonable when considering the differences in default strategies between large firms with time-consistent preference and entrepreneurial firms with time-inconsistent preferences. Once again, the numerical results justify the adjustment procedure for the parameter of the time-consistent benchmark.

(Figure 7 is inserted here.)

Prediction 5.3 (comparison of coupon levels). *Naive entrepreneur choose larger coupon levels than sophisticated entrepreneurs, who in turn choose larger coupon levels* *than time-consistent entrepreneurs, i.e.,* $c^n \geq c^s \geq c^*(\rho^*) \geq c^*$*, where the equalities hold when* $\delta = 1$ *or* $\lambda = 0$*.*

Since the leverage ratio is a better indicator that characterizes the capital structure decision of a firm, we discuss in detail the impact of time inconsistency on the leverage in Section 5.4 instead of that on the coupon level.

It is noteworthy that sophisticated entrepreneurs' decisions on investment, default and coupon level are closer to the time-consistent benchmarks (with/without parameter adjustment) than naive entrepreneurs' decisions.

5.4 Impact of time inconsistency on leverage

Figure 8 depicts the leverage with respect to the time inconsistency parameters δ and λ. As a reference point, the time-consistent benchmark L^* is depicted on the right (left) endpoint of the upper (lower) panel, respectively.

(Figure 8 is inserted here.)

Prediction 5.4 (comparison of leverages). *Sophisticated entrepreneurs choose lower leverages than naive entrepreneurs and time-consistent entrepreneurs with parameter adjustment, i.e.,* $L^s \leq L^n$, $L^s \leq L^*(\rho^*)$*, where the equalities hold when* $\delta = 1$ *or* $\lambda = 0$*. However, naive entrepreneurs may choose higher leverages than time-consistent entrepreneurs with parameter adjustment.*

The result that the leverage choice depends on the entrepreneurs' belief regarding their future time-inconsistent behavior (sophisticated or naive) helps us explain the leverage puzzle observed in practice and better understand the determinants of capital structures. Sophisticated entrepreneur are modest about debt financing, leading conservative leverages, which account for the *debt conservatism puzzle* observed in Graham (2000). However, naive entrepreneurs may choose higher leverages compared to the time-consistent benchmark $L^*(\rho^*)$ with parameter adjustment, and much higher than the time-consistent benchmark L^* without parameter adjustment. It is consistent with Heidhues and Koszegi (2010), who find that naive individuals overborrow in the credit market, and Cronqvist, Makhija and Yonker (2012), who observe empirically a positive relation between CEO personal leverage and corporate leverage. The naive entrepreneur's misunderstanding can be comprehended as too optimistic to believe that he/she can commit to his/her current preferences. In this sense, our result is also consistent with Hackbarth (2010) and Malmendier, Tate and Yan (2011), who report that optimistic and/or overconfident managers use leverage more aggressively.

The lower panel of Fig. 8 shows that entrepreneurs are sensitive to an infinitesimal change of λ from zero. Although the estimation of λ is difficult, there is consensus by empirical evidence that $\lambda > 0$. We find that, even if λ is very small, there exist a drastic difference between entrepreneurial firms' leverage and the time-consistent benchmark. In short, entrepreneurial firms with time inconsistency make decisions very differently from large firms.

6 Conclusions

This paper provides an analytically tractable framework of entrepreneurial firms' investment and capital structure decisions with time-inconsistent preferences. We show that the impact of time-inconsistent preferences depends not only on the financing structures (all-equity financing or debt-equity financing), but also on the entrepreneurs' belief regarding their future time-inconsistent behavior (sophisticated or naive). Time-inconsistent preferences delay investment under both all-equity financing and debt-equity financing. However, the impact is weakened with debt-equity financing, because debt financing increases the payoff value upon investment and accelerates investment. Naive entrepreneurs invest later and default earlier than sophisticated entrepreneurs, leading to a shorter operating period. Moreover, we find that naive entrepreneurs may choose higher leverages, while sophisticated entrepreneurs always choose lower leverages, compared to the time-consistent benchmark. These results support the empirical findings in the entrepreneurial finance.

Finally, as a future research, we point out an important but difficult topic. In our model, we assume that the cash flow is discounted by the same additional factor δ after the arrival of each future self. Moreover, we assume that the lifespan of each self is exponentially distributed with the same parameter λ. These assumptions simplify the model, especially in that sophisticated entrepreneurs' problem becomes a fixed-point problem. However, it may be more plausible to assume that δ depends on each future self and λ is time-dependent.

A Appendix

A.1 Derivation of Eq. (10)

Since T is exponentially distributed with parameter λ, we have

$$\begin{aligned}
&\mathbb{E}_t\left[\mathbf{1}_{\{\bar{T}_{\mathrm{i}}^s \leq T\}} \mathrm{e}^{-\rho(\bar{T}_{\mathrm{i}}^s - t)}\left[\Pi(\bar{x}_{\mathrm{i}}^s) - I\right] + \mathbf{1}_{\{\bar{T}_{\mathrm{i}}^s > T\}} \mathrm{e}^{-\rho(\bar{T}_{\mathrm{i}}^s - t)} \delta\left[\Pi^*(\bar{x}_{\mathrm{i}}^s) - I\right]\right] \\
=&\mathbb{E}_t\left[\int_{\bar{T}_{\mathrm{i}}^s}^{\infty} \mathrm{e}^{-\rho(\bar{T}_{\mathrm{i}}^s - t)}\left[\Pi(\bar{x}_{\mathrm{i}}^s) - I\right] \lambda \mathrm{e}^{-\lambda u} \mathrm{d}u + \int_t^{\bar{T}_{\mathrm{i}}^s} \mathrm{e}^{-\rho(\bar{T}_{\mathrm{i}}^s - t)} \delta\left[\Pi^*(\bar{x}_{\mathrm{i}}^s) - I\right] \lambda \mathrm{e}^{-\lambda u} \mathrm{d}u\right] \\
=&\mathbb{E}_t\left[\mathrm{e}^{-\lambda(\bar{T}_{\mathrm{i}}^s - t)} \mathrm{e}^{-\rho(\bar{T}_{\mathrm{i}}^s - t)}\left[\Pi(\bar{x}_{\mathrm{i}}^s) - I\right] + (1 - \mathrm{e}^{-\lambda(\bar{T}_{\mathrm{i}}^s - t)}) \mathrm{e}^{-\rho(\bar{T}_{\mathrm{i}}^s - t)} \delta\left[\Pi^*(\bar{x}_{\mathrm{i}}^s) - I\right]\right] \\
=&\left[\Pi(\bar{x}_{\mathrm{i}}^s) - I\right]\left(\frac{x}{\bar{x}_{\mathrm{i}}^s}\right)^{\beta_2} + \delta\left[\Pi^*(\bar{x}_{\mathrm{i}}^s) - I\right]\left[\left(\frac{x}{\bar{x}_{\mathrm{i}}^s}\right)^{\beta_1} - \left(\frac{x}{\bar{x}_{\mathrm{i}}^s}\right)^{\beta_2}\right]. \qquad \text{(A.1)}
\end{aligned}$$

The last equality follows from Eq. (5) by replacing ρ with $\rho + \lambda$.

A.2 Proof of Proposition 3.2.

The investment threshold (12) is derived directly by maximizing $\bar{V}^{os}(x)$ in (10). The denominator of κ_1 in (13) is positive, because

$$1 - \theta - \delta = \frac{\rho - \mu}{\rho + \lambda - \mu}(1 - \delta) \geq 0. \qquad \text{(A.2)}$$

Thus, we have confirmed $\bar{x}_{\mathrm{i}}^s > 0$.

Next, we show the comparative statics results $\partial \bar{x}_{\mathrm{i}}^s / \partial \delta \leq 0$ and $\partial \bar{x}_{\mathrm{i}}^s / \partial \lambda \geq 0$. By differentiating $\bar{x}_{\mathrm{i}}^s$ with δ, we have

$$\frac{\partial \bar{x}_{\mathrm{i}}^s}{\partial \delta} = \frac{(\beta_1 - \beta_2)\left[\delta(\beta_1 - 1) + (1 - \theta - \delta)(\beta_2 - 1)\right] - \hat{\beta}(\rho - \mu)\left[\frac{\beta_1 - 1}{\rho - \mu} - \frac{\beta_2 - 1}{\rho + \lambda - \mu}\right]}{\left[\delta(\beta_1 - 1) + (1 - \theta - \delta)(\beta_2 - 1)\right]^2}. \qquad \text{(A.3)}$$

Since the denominator is positive, we need to show that the numerator is nonpositive. The first term of the numerator is nonpositive, because $\beta_1 \leq \beta_2$ and $\delta(\beta_1 - 1) + (1 - \theta - \delta)(\beta_2 - 1) \geq 0$, which has been showed in (A.2). The second term of the numerator is nonnegative if $(\beta_1 - 1)/(\rho - \mu) \geq (\beta_2 - 1)/(\rho + \lambda - \mu)$. This inequality can be confirmed by showing

$$\frac{\partial}{\partial \rho}\left[\frac{\beta(\rho)}{\beta(\rho) - 1}(\rho - \mu)\right] \geq 0, \qquad \text{(A.4)}$$

where the functional mapping from the discount rate ρ to the parameter β is defined by using the following quadratic equation:

$$\frac{\sigma^2}{2}\beta(\rho)(\beta(\rho) - 1) + \mu\beta(\rho) - \rho = 0. \qquad \text{(A.5)}$$

Immediately from (A.5), we have

$$\frac{1}{2}\sigma^2\beta(\rho)^2 + \mu\beta(\rho) = \rho + \frac{1}{2}\sigma^2\beta(\rho) = \frac{\beta(\rho)}{\beta(\rho) - 1}(\rho - \mu).$$

Therefore, we have

$$\frac{\partial}{\partial\rho}\left[\frac{\beta(\rho)}{\beta(\rho)-1}(\rho-\mu)\right]=\frac{\partial}{\partial\rho}\left[\frac{1}{2}\sigma^2\beta(\rho)^2+\mu\beta(\rho)\right]=\left(\sigma^2\beta(\rho)+\mu\right)\frac{\partial\beta(\rho)}{\partial\rho}.$$

Since

$$\frac{\partial\beta(\rho)}{\partial\rho}=\left[\left(\mu-\frac{1}{2}\sigma^2\right)^2+2\sigma^2\rho\right]^{-\frac{1}{2}}>0, \tag{A.6}$$

the inequality (A.4) holds. From the above, the numerator of (A.3) is nonpositive. Thus, we have $\partial\bar{x}_{\mathrm{i}}^s/\partial\delta\leq 0$.

Similarly, by differentiating $\bar{x}_{\mathrm{i}}^s$ with λ, we have

$$\frac{\partial\bar{x}_{\mathrm{i}}^s}{\partial\lambda}=\frac{(1-\delta)\left[\delta(\beta_1-1)+(1-\theta-\delta)(\beta_2-1)\right]\frac{\partial\beta_2}{\partial\lambda}-\hat{\beta}\frac{\partial}{\partial\lambda}\left(\frac{\beta_2-1}{\rho+\lambda-\mu}\right)}{\left[(\beta_2-1)(1-\theta)-(\beta_2-\beta_1)\delta\right]^2}. \tag{A.7}$$

Since

$$\frac{\partial\beta_2}{\partial\lambda}=\frac{\partial\beta_2}{\partial(\rho+\lambda)}=\frac{\partial\beta(\rho)}{\partial\rho}>0 \tag{A.8}$$

from (A.6) and

$$\frac{\partial}{\partial\lambda}\left(\frac{\beta_2-1}{\rho+\lambda-\mu}\right)=\frac{\partial}{\partial(\rho+\lambda)}\left(\frac{\beta_2-1}{\rho+\lambda-\mu}\right)=\frac{\partial}{\partial\rho}\left(\frac{\beta(\rho)-1}{\rho-\mu}\right)\leq 0 \tag{A.9}$$

from (A.4) and (A.6), we have the numerator of (A.7) is positive. Combining the positivity of the denominator, we finally confirmed $\partial\bar{x}_{\mathrm{i}}^s/\partial\lambda\geq 0$.

A.3 Proof of Proposition 3.3

First, we derive the investment threshold $\bar{x}_{\mathrm{i}}^n$. For $x\leq\bar{x}_{\mathrm{i}}^*\leq\bar{x}_{\mathrm{i}}^n$, the option value of investment satisfies the ODE

$$\frac{1}{2}\sigma^2x^2\bar{V}_l^{on\prime\prime}(x)+\mu x\bar{V}_l^{on\prime}(x)-\rho\bar{V}_l^{on}(x)+\lambda\left[\delta\left[\Pi^*(\bar{x}_{\mathrm{i}}^*)-I\right]\left(\frac{x}{\bar{x}_{\mathrm{i}}^*}\right)^{\beta_1}-\bar{V}_l^{on}(x)\right]=0, \tag{A.10}$$

where the first term in the square bracket is the continuation value function for $x\leq\bar{x}_{\mathrm{i}}^*$. For $\bar{x}_{\mathrm{i}}^*\leq x\leq\bar{x}_{\mathrm{i}}^n$, the option value of investment satisfies the ODE

$$\frac{1}{2}\sigma^2x^2\bar{V}_h^{on\prime\prime}(x)+\mu x\bar{V}_h^{on\prime}(x)-\rho\bar{V}_h^{on}(x)+\lambda\left[\delta\left[\Pi^*(x)-I\right]-\bar{V}_h^{on}(x)\right]=0, \tag{A.11}$$

where the first term in the square bracket is the continuation value function for $x\geq\bar{x}_{\mathrm{i}}^*$. We need to solve the ODEs (A.10) and (A.11) with the following boundary conditions

$$\begin{cases}\bar{V}_h^{on}(x)\big|_{x=\bar{x}_{\mathrm{i}}^*}=\bar{V}_l^{on}(x)\big|_{x=\bar{x}_{\mathrm{i}}^*},\\ \bar{V}_h^{on\prime}(x)\big|_{x=\bar{x}_{\mathrm{i}}^*}=\bar{V}_l^{on\prime}(x)\big|_{x=\bar{x}_{\mathrm{i}}^*},\\ \bar{V}_h^{on}(x)\big|_{x=\bar{x}_{\mathrm{i}}^n}=\Pi(\bar{x}_{\mathrm{i}}^n)-I,\\ \bar{V}_h^{on\prime}(x)\big|_{x=\bar{x}_{\mathrm{i}}^n}=\Pi'(x)\big|_{x=\bar{x}_{\mathrm{i}}^n},\end{cases} \tag{A.12}$$

The general solutions for the ODEs (A.10) and (A.11) are given by

$$\begin{cases} \bar{V}_l^{on}(x) = A_l x^{\beta_2} + \delta \left[\Pi^*(\bar{x}_{\mathrm{i}}^*) - I\right] \left(\frac{x}{\bar{x}_{\mathrm{i}}^*}\right)^{\beta_1}, \\ \bar{V}_h^{on}(x) = A_h x^{\beta_2} + B_h x^{\gamma_2} + \lambda\delta \left[\frac{\Pi^*(x)}{\rho+\lambda-\mu} - \frac{I}{\rho+\lambda}\right], \end{cases}$$

respectively. Substituting the general solutions above into the boundary conditions (A.12), we obtain the following four equations:

$$\begin{cases} A_h \left(\bar{x}_{\mathrm{i}}^*\right)^{\beta_2} + B_h \left(\bar{x}_{\mathrm{i}}^*\right)^{\gamma_2} + \lambda\delta \left[\frac{\Pi^*(\bar{x}_{\mathrm{i}}^*)}{\rho+\lambda-\mu} - \frac{I}{\rho+\lambda}\right] = A_l \left(\bar{x}_{\mathrm{i}}^*\right)^{\beta_2} + \delta \left[\Pi^*(\bar{x}_{\mathrm{i}}^*) - I\right], \\ \beta_2 A_h \left(\bar{x}_{\mathrm{i}}^*\right)^{\beta_2} + \gamma_2 B_h \left(\bar{x}_{\mathrm{i}}^*\right)^{\gamma_2} + \lambda\delta \frac{\Pi^*(\bar{x}_{\mathrm{i}}^*)}{\rho+\lambda-\mu} = \beta_2 A_l \left(\bar{x}_{\mathrm{i}}^*\right)^{\beta_2} + \delta\Pi^*(\bar{x}_{\mathrm{i}}^*), \\ A_h \left(\bar{x}_{\mathrm{i}}^n\right)^{\beta_2} + B_h \left(\bar{x}_{\mathrm{i}}^n\right)^{\gamma_2} + \lambda\delta \left[\frac{\Pi^*(\bar{x}_{\mathrm{i}}^n)}{\rho+\lambda-\mu} - \frac{I}{\rho+\lambda}\right] = \Pi(\bar{x}_{\mathrm{i}}^n) - I, \\ \beta_2 A_h \left(\bar{x}_{\mathrm{i}}^n\right)^{\beta_2} + \gamma_2 B_h \left(\bar{x}_{\mathrm{i}}^n\right)^{\gamma_2} + \lambda\delta \frac{\Pi^*(\bar{x}_{\mathrm{i}}^n)}{\rho+\lambda-\mu} = \Pi(\bar{x}_{\mathrm{i}}^n). \end{cases}$$

From the first two equations, we have

$$(\beta_2 - \gamma_2) B_h \left(\bar{x}_{\mathrm{i}}^*\right)^{\gamma_2} + \lambda\delta \left[\frac{(\beta_2 - 1)\Pi^*(\bar{x}_{\mathrm{i}}^*)}{\rho + \lambda - \mu} - \frac{\beta_2 I}{\rho + \lambda}\right] = \delta \left[(\beta_2 - 1)\Pi^*(\bar{x}_{\mathrm{i}}^*) - \beta_2 I\right]. \quad \text{(A.13)}$$

From the last two equations, we have

$$(\beta_2 - \gamma_2) B_h \left(\bar{x}_{\mathrm{i}}^n\right)^{\gamma_2} + \lambda\delta \left[\frac{(\beta_2 - 1)\Pi(\bar{x}_{\mathrm{i}}^n)}{\rho + \lambda - \mu} - \frac{\beta_2 I}{\rho + \lambda}\right] = (\beta_2 - 1)\Pi(\bar{x}_{\mathrm{i}}^n) - \beta_2 I. \quad \text{(A.14)}$$

Dividing (A.14) by (A.13), we have

$$\left(\frac{\bar{x}_{\mathrm{i}}^n}{\bar{x}_{\mathrm{i}}^*}\right)^{\gamma_2} = \frac{\frac{(1-\tau)(\beta_2-1)}{\rho+\lambda-\mu}\bar{x}_{\mathrm{i}}^n - \beta_2 I \frac{\rho+\lambda(1-\delta)}{\rho+\lambda}}{\delta I \left[(\beta_2 - 1)\frac{\beta_1}{\beta_1-1}\frac{\rho-\mu}{\rho+\lambda-\mu} - \beta_2 \frac{\rho}{\rho+\lambda}\right]}.$$

After rearranging the terms in the equation above, we obtain $\bar{x}_{\mathrm{i}}^n$ in (17).

Next, we show that $\bar{x}_{\mathrm{i}}^n \geq \bar{x}_{\mathrm{i}}^*$. Let $m = \bar{x}_{\mathrm{i}}^n / \bar{x}_{\mathrm{i}}^*$, and divide $\bar{x}_{\mathrm{i}}^n$ in (17) by $\bar{x}_{\mathrm{i}}^*$. Then, we have

$$m = \frac{\beta_1 - 1}{\beta_1}\frac{\beta_2}{\beta_2 - 1}\frac{\rho + \lambda - \mu}{\rho - \mu}\frac{\rho + (1 - \delta)\lambda}{\rho + \lambda} + m^{\gamma_2}\delta \left[1 - \frac{\beta_1 - 1}{\beta_1}\frac{\beta_2}{\beta_2 - 1}\frac{\rho + \lambda - \mu}{\rho - \mu}\frac{\rho}{\rho + \lambda}\right]. \quad \text{(A.15)}$$

Let $f(m)$ denote the RHS of (A.15). We search the fixed-point solution for $f(m) = m$, where $m \in [1, \infty)$. Note that

$$f(\infty) = \frac{\beta_1 - 1}{\beta_1}\frac{\beta_2}{\beta_2 - 1}\frac{\rho + \lambda - \mu}{\rho - \mu}\frac{\rho + \lambda(1 - \delta)}{\rho + \lambda} < \infty,$$
$$f'(m) = \gamma_2 m^{\gamma_2 - 1}\delta \left[1 - \frac{\beta_1 - 1}{\beta_1}\frac{\beta_2}{\beta_2 - 1}\frac{\rho + \lambda - \mu}{\rho - \mu}\frac{\rho}{\rho + \lambda}\right] \geq 0,$$

where we have employed (A.4) for the second inequality. Moreover,

$$f(1) = \delta + (1 - \delta)\frac{\beta_1 - 1}{\beta_1}\frac{\beta_2}{\beta_2 - 1}\frac{\rho + \lambda - \mu}{\rho - \mu}.$$

Consider $f(1)$ as a function of λ. When $\lambda = 0$, we have $f(1) = 1$. After tedious algebra, we can show that $\mathrm{d}f(1;\lambda)/\mathrm{d}\lambda \geq 0$. Thus, $f(1) \geq 1$ for $\lambda \geq 0$. Summarizing, we showed that $f(m)$ is an increasing function with $f(1) \geq 1$ and $f(\infty) < \infty$. Therefore, there exists a unique solution $m \in [1, \infty)$ such that $f(m) = m$.

A.4 Derivation of Eqs. (33) and (46)

Since Eq. (46) includes Eq. (33) as a special case by replacing both x_{d}^n and x_{d}^* with x_{d}^s, we derive Eq. (46) in detail. We calculate the two expectations in (45) separately. First,

$$\begin{aligned}&\mathbb{E}_t\left[\mathbf{1}_{\{T_{\mathrm{d}}^n\le T\}}\int_t^{T_{\mathrm{d}}^n}\mathrm{e}^{-\rho(u-t)}(X(u)-c^n)\mathrm{d}u\right]\\ =&\mathbb{E}_t\left[\left(\frac{x}{\rho-\mu}-\frac{c^n}{\rho}\right)\mathrm{e}^{-\lambda(T_{\mathrm{d}}^n-t)}-\left(\frac{x_{\mathrm{d}}^n}{\rho-\mu}-\frac{c^n}{\rho}\right)\mathrm{e}^{-(\rho+\lambda)(T_{\mathrm{d}}^n-t)}\right].\end{aligned}\tag{A.16}$$

Second, we have

$$\begin{aligned}&\mathbb{E}_t\left[\mathbf{1}_{\{T_{\mathrm{d}}^n> T\}}\left[\int_t^{T}\mathrm{e}^{-\rho(u-t)}(X(u)-c^n)\mathrm{d}u+\int_T^{T_{\mathrm{d}}^*}\delta\mathrm{e}^{-\rho(u-t)}(X(u)-c^n)\mathrm{d}u\right]\right]\\
=&\mathbb{E}_t\left[\mathbf{1}_{\{T_{\mathrm{d}}^n> T\}}\left[\frac{x}{\rho-\mu}\left(1-\mathrm{e}^{-(\rho-\mu)(T-t)}\right)-\frac{c^n}{\rho}\left(1-\mathrm{e}^{-\rho(T-t)}\right)\right]\right]\\
&-\delta\mathbb{E}_t\left[\mathbf{1}_{\{T_{\mathrm{d}}^n> T\}}\left[\frac{x}{\rho-\mu}\left(\mathrm{e}^{-(\rho-\mu)(T_{\mathrm{d}}^*-t)}-\mathrm{e}^{-(\rho-\mu)(T-t)}\right)-\frac{c^n}{\rho}\left(\mathrm{e}^{-\rho(T_{\mathrm{d}}^*-t)}-\mathrm{e}^{-\rho(T-t)}\right)\right]\right]\\
=&\mathbb{E}_t\left[\left(\frac{x}{\rho-\mu}-\frac{c^n}{\rho}\right)\left(1-\mathrm{e}^{-\lambda(T_{\mathrm{d}}^n-t)}\right)-\delta\left(\frac{x_{\mathrm{d}}^*}{\rho-\mu}-\frac{c^n}{\rho}\right)\mathrm{e}^{-\rho(T_{\mathrm{d}}^*-t)}\left(1-\mathrm{e}^{-\lambda(T_{\mathrm{d}}^n-t)}\right)\right]\\
&-(1-\delta)\mathbb{E}_t\left[\frac{\lambda}{\rho+\lambda-\mu}\left(\frac{x}{\rho-\mu}-\frac{x_{\mathrm{d}}^n}{\rho-\mu}\mathrm{e}^{-(\rho+\lambda)(T_{\mathrm{d}}^n-t)}\right)-\frac{\lambda}{\rho+\lambda}\frac{c^n}{\rho}\left(1-\mathrm{e}^{-(\rho+\lambda)(T_{\mathrm{d}}^n-t)}\right)\right].\end{aligned}\tag{A.17}$$

By combining (A.16) and (A.17), we obtain

$$\begin{aligned}&\mathbb{E}_t\left[\frac{x}{\rho-\mu}-\frac{c^n}{\rho}-\left(\frac{x_{\mathrm{d}}^n}{\rho-\mu}-\frac{c^n}{\rho}\right)\mathrm{e}^{-(\rho+\lambda)(T_{\mathrm{d}}^n-t)}-\delta\left(\frac{x_{\mathrm{d}}^*}{\rho-\mu}-\frac{c^n}{\rho}\right)\mathrm{e}^{-\rho(T_{\mathrm{d}}^*-t)}\left(1-\mathrm{e}^{-\lambda(T_{\mathrm{d}}^n-t)}\right)\right]\\
&-(1-\delta)\mathbb{E}_t\left[\frac{\lambda}{\rho+\lambda-\mu}\left(\frac{x}{\rho-\mu}-\frac{x_{\mathrm{d}}^n}{\rho-\mu}\mathrm{e}^{-(\rho+\lambda)(T_{\mathrm{d}}^n-t)}\right)-\frac{\lambda}{\rho+\lambda}\frac{c^n}{\rho}\left(1-\mathrm{e}^{-(\rho+\lambda)(T_{\mathrm{d}}^n-t)}\right)\right]\\
=&(1-\theta)\frac{x}{\rho-\mu}-(1-\theta_1)\frac{c^n}{\rho}-\delta\left(\frac{x_{\mathrm{d}}^*}{\rho-\mu}-\frac{c^n}{\rho}\right)\left[\left(\frac{x}{x_{\mathrm{d}}^*}\right)^{\gamma_1}-\left(\frac{x}{x_d^n}\right)^{\gamma_2}\left(\frac{x_d^n}{x_d^*}\right)^{\gamma_1}\right]\\
&-\left((1-\theta)\frac{x_{\mathrm{d}}^n}{\rho-\mu}-(1-\theta_1)\frac{c^n}{\rho}\right)\left(\frac{x}{x_{\mathrm{d}}^n}\right)^{\gamma_2}.\end{aligned}\tag{A.18}$$

By multiplying the above result with $(1-\tau)$ and rearranging the terms, we obtain the result (46).

A.5 Proof of Remark 5.2

Consider θ_1 in Eq. (34) and θ in Eq. (9) as functions of ρ. Then, we have $\theta(\rho)=\theta_1(\rho-\mu)$. Since $\partial\left[\rho/(1-\theta_1)\right]/\partial\rho\ge 1$ and $\mu\ge 0$, we have

$$\frac{\rho-\mu}{1-\theta}\le\frac{\rho}{1-\theta_1}-\mu=\rho^*-\mu.\tag{A.19}$$

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Table 1: Notation index.

notation	meaning
-	all-equity financing
(no accent)	debt-equity financing
$*$	time-consistent benchmark
n	naive entrepreneur
s	sophisticated entrepreneur

Table 2: Parameter values for special cases.

	$\delta = 1$ or $\lambda = 0$	$\delta = 0$	$\lambda \to \infty$
θ	0	$\frac{\lambda}{\rho-\mu}$	$1-\delta$
θ_1	0	$\frac{\lambda}{\rho+\lambda-\mu}$	$1-\delta$
ρ^*	ρ	$\rho+\lambda$	$\frac{\rho}{\delta}$
$\hat{\beta}$	β_1	β_2	∞
κ_1	$\frac{\beta_1}{\beta_1-1}$	$\frac{\beta_2}{\beta_2-1}(1-\theta)$	∞
κ_2	$\frac{\gamma_1}{\gamma_1-1}$	$\frac{\gamma_2}{\gamma_2-1}\frac{1-\theta_1}{1-\theta}$	$\frac{\gamma_1}{\gamma_1-1}$
h^s	h^*	$h^*(\rho^*)$	h^*
ψ^s	ψ^*	$\psi^*(\rho^*)$	ψ^*

Table 3: Solutions for special cases.

	$\delta = 1$ or $\lambda = 0$	$\delta = 0$	$\lambda \to \infty$
$\bar{x}_{\mathrm{i}}^s,\ \bar{x}_{\mathrm{i}}^n$	$\bar{x}_{\mathrm{i}}^*$	$\bar{x}_{\mathrm{i}}^*(\rho+\lambda) = \bar{x}_{\mathrm{i}}^*(\rho^*)$	∞
$x_{\mathrm{i}}^s,\ x_{\mathrm{i}}^n$	x_{i}^*	$x_{\mathrm{i}}^*(\rho+\lambda) = x_{\mathrm{i}}^*(\rho^*)$	∞
$x_{\mathrm{d}}^s,\ x_{\mathrm{d}}^n$	x_{d}^*	$x_{\mathrm{d}}^*(\rho+\lambda) = x_{\mathrm{d}}^*(\rho^*)$	∞
$c^s,\ c^n$	c^*	$c^*(\rho+\lambda) = c^*(\rho^*)$	∞

Table 4: Summary of analytical solutions.

		time-consistent ($*$)		time-inconsistent (s)	
a-e	$\bar{x}_{\mathrm{i}}$	$\dfrac{\beta_1}{\beta_1 - 1}\dfrac{\rho-\mu}{1-\tau}I$	(DP)	$\dfrac{\hat{\beta}}{\delta(\beta_1 - 1) + (1-\theta-\delta)(\beta_2 - 1)}\dfrac{\rho-\mu}{1-\tau}I$	(GW)
d-e	x_{i}	$\dfrac{\beta_1}{\beta_1 - 1}\dfrac{\rho-\mu}{1-\tau}\dfrac{I}{\psi^*}$	(SW)	$\dfrac{\hat{\beta}}{\delta\psi^*(\beta_1 - 1) + ((1-\theta)\psi^s - \delta\psi^*)(\beta_2 - 1)}\dfrac{\rho-\mu}{1-\tau}I$	(KT)
	c	$\dfrac{\gamma_1 - 1}{\gamma_1}\dfrac{\rho}{\rho-\mu}\dfrac{x_{\mathrm{i}}^*}{h^*}$		$\dfrac{1}{\kappa_2}\dfrac{\rho}{\rho-\mu}\dfrac{x_{\mathrm{i}}^s}{h^s}$	
	x_{d}	$\dfrac{\rho-\mu}{\rho}\dfrac{\gamma_1}{\gamma_1 - 1}c^*$		$\dfrac{\rho-\mu}{\rho}\kappa_2 c^s$	

a-e: all-equity fianncing; d-e: debt-equity financing;

DP: Dixit and Pindyck (1994); GW: Grenadier and Wang (2007);

SW: Sundaresan and Wang (2007); KT: this paper.

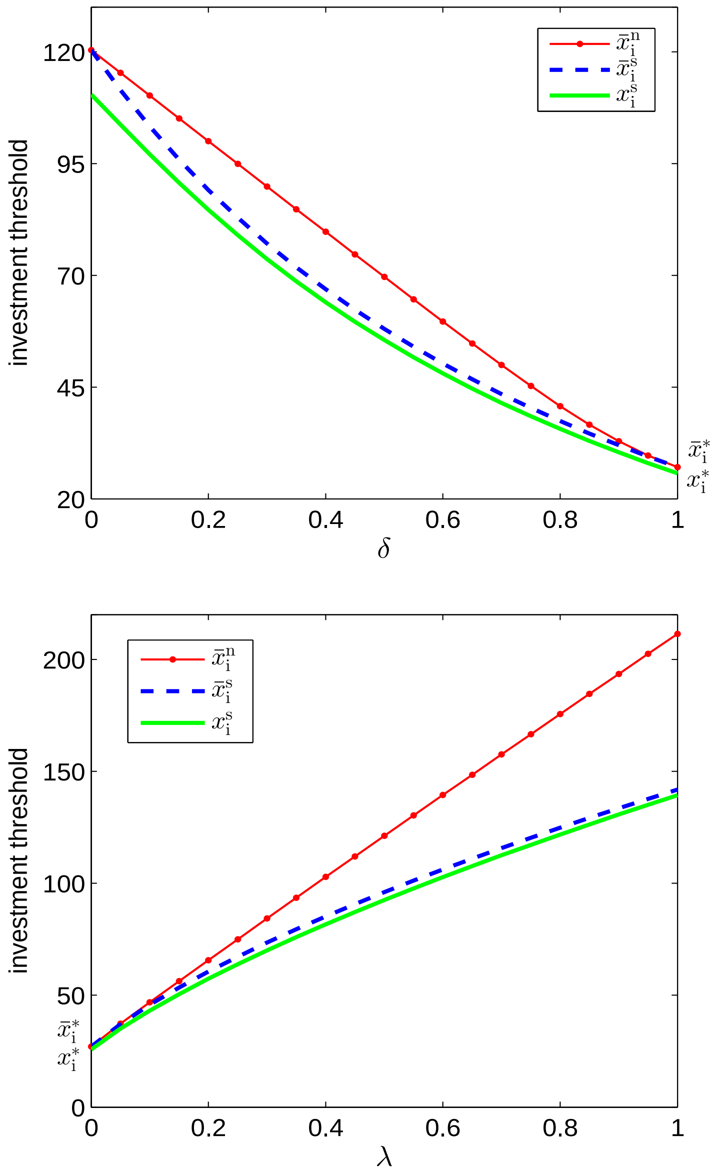
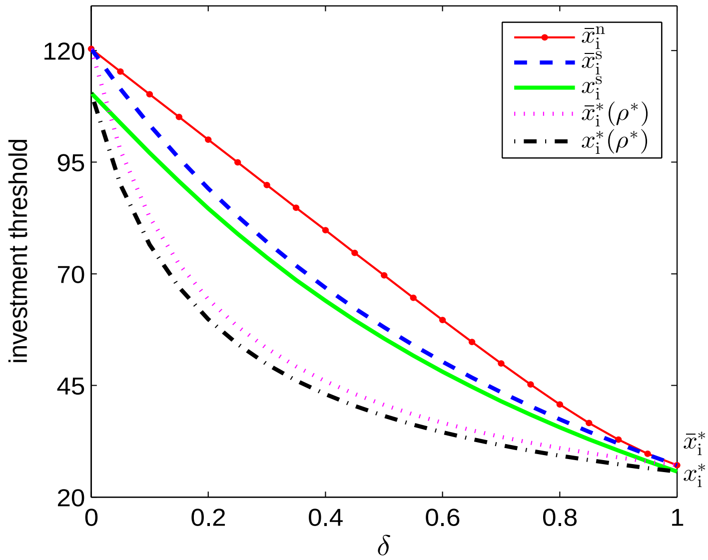


Fig. 1. The impact of time-inconsistent preferences on the investment threshold. These two figures plot the optimal investment thresholds for naive, sophisticated entrepreneurs under all-equity financing, denoted as $\bar{x}_\mathrm{i}^n$ and $\bar{x}_\mathrm{i}^s$, and the optimal investment threshold for sophisticated entrepreneurs under debt-equity financing, denoted as x_i^s, with respect to the parameters $\delta \in [0,1]$ (upper panel) and $\lambda \in [0,1]$ (lower panel) representing the degree of time inconsistency, respectively. The time-consistent benchmarks $\bar{x}_\mathrm{i}^*$ (under all-equity financing) and x_i^* (under debt-equity financing) are depicted on the right (left) endpoints of the upper (lower) panel for $\delta = 1$ ($\lambda = 0$), respectively. It is assumed that $\mu = 0.01,\ \sigma = 0.25,\ \rho = 0.06,\ \tau = 0.15$, and $I = 200$. We set $\lambda = 0.33$ for the upper panel and $\delta = 0.3$ for the lower panel.



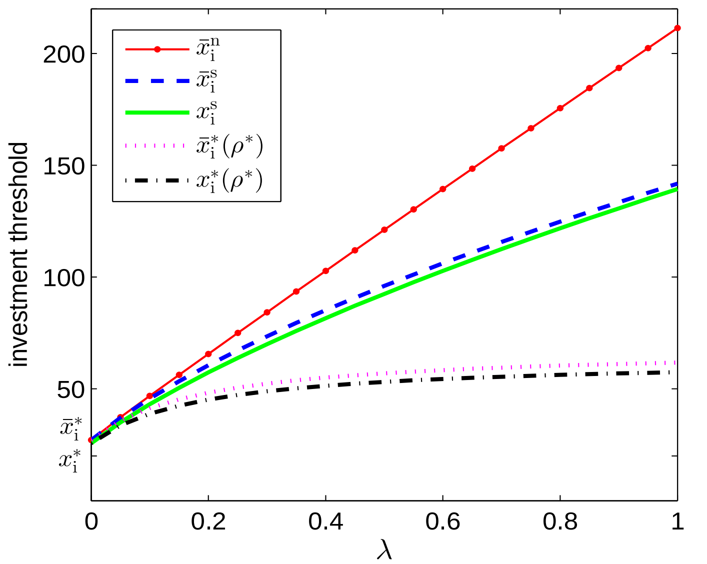


Fig. 2. The impact of time-inconsistent preferences on the investment threshold. These two figures add the time-consistent benchmarks with parameter adjustment under all-equity financing and debt-equity financing, denoted as $\bar{x}_{\rm i}^{*}(\rho^{*})$ and $x_{\rm i}^{*}(\rho^{*})$, respectively, to the same panels as in Fig. 1. It is assumed that $\mu = 0.01$, $\sigma = 0.25$, $\rho = 0.06$, $\tau = 0.15$, and $I = 200$. We set $\lambda = 0.33$ for the upper panel and $\delta = 0.3$ for the lower panel.

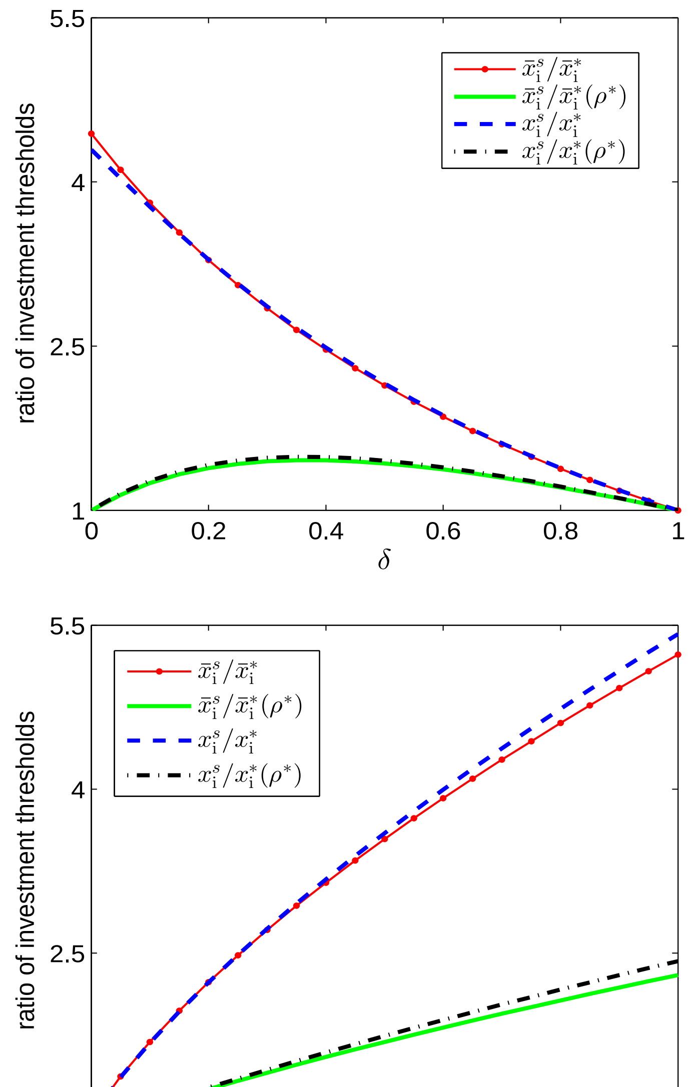


Fig. 3. The ratio of the investment thresholds under time-inconsistent preferences to the time-consistent benchmarks. These two figures plot the ratios of optimal investment thresholds for sophisticated entrepreneurs (without and with parameter adjustment) under all-equity financing, denoted as $\bar{x}_{\rm i}^s/\bar{x}_{\rm i}^*$ and $\bar{x}_{\rm i}^s/\bar{x}_{\rm i}^*(\rho^*)$, and the ratios under debt-equity financing, denoted as $x_{\rm i}^s/x_{\rm i}^*$ and $x_{\rm i}^s/x_{\rm i}^*(\rho^*)$, with respect to the parameters $\delta \in [0,1]$ (upper panel) and $\lambda \in [0,1]$ (lower panel) representing the degree of time inconsistency, respectively. It is assumed that $\mu = 0.01,\ \sigma = 0.25,\ \rho = 0.06,\ \tau = 0.15,$ and $I = 200$. We set $\lambda = 0.33$ for the upper panel and $\delta = 0.3$ for the lower panel.

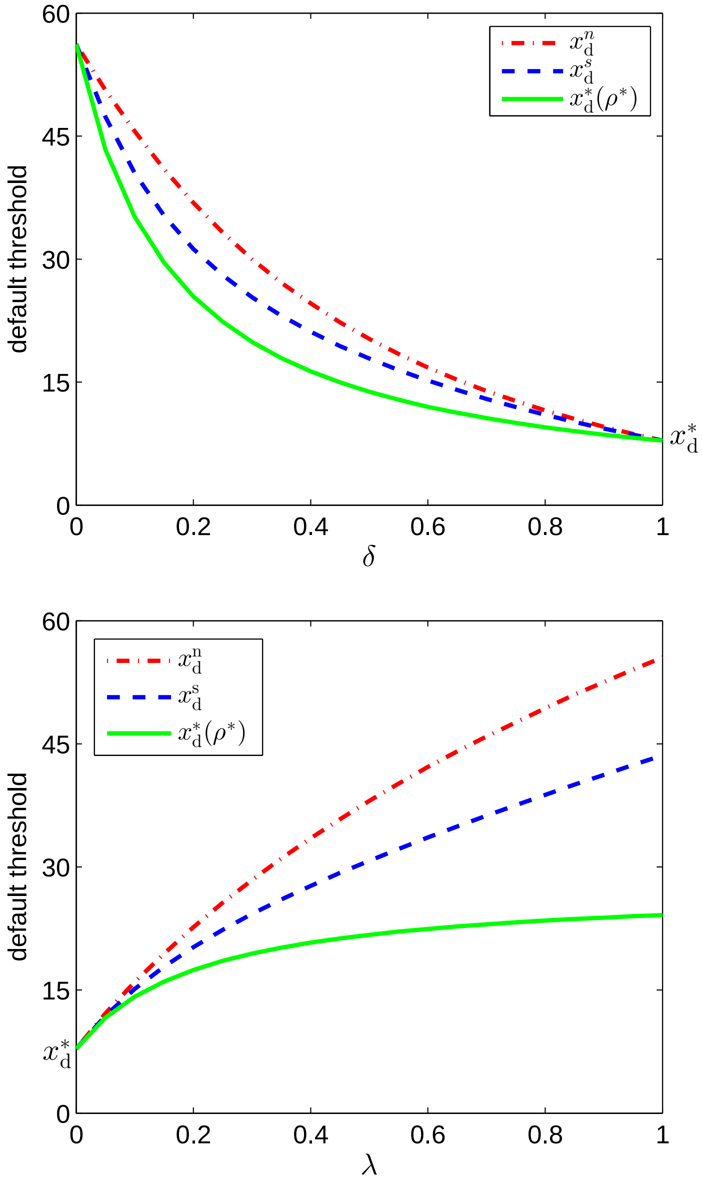


Fig. 4. The impact of time-inconsistent preferences on the default threshold. These two figures plot the default thresholds for naive, sophisticated entrepreneurs and the time-consistent benchmarks with parameter adjustment, denoted as x_d^n, x_d^s, and $x_\mathrm{d}^*(\rho^*)$, with respect to the parameters $\delta \in [0,1]$ (upper panel) and $\lambda \in [0,1]$ (lower panel) representing the degree of time inconsistency, respectively. The time-consistent benchmark x_d^* without parameter adjustment is depicted on the right (left) endpoints of the upper (lower) panel for $\delta = 1$ ($\lambda = 0$), respectively. It is assumed that $\mu = 0.01$, $\sigma = 0.25$, $\rho = 0.06$, $\tau = 0.15$, and $I = 200$. We set $\lambda = 0.33$ for the upper panel and $\delta = 0.3$ for the lower panel. The coupon levels that the default thresholds depend on are also optimized respectively.

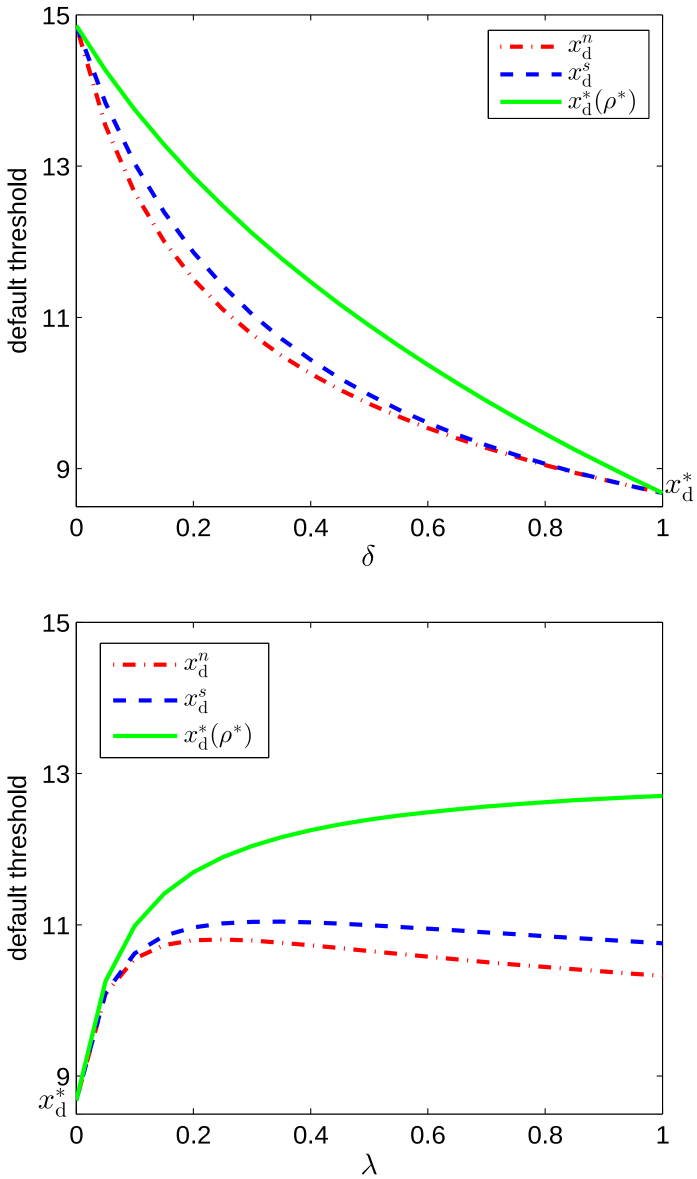


Fig. 5. The impact of time-inconsistent preferences on the default threshold (given the same coupon level). These two figures plot the default thresholds for naive, sophisticated entrepreneurs and the time-consistent benchmarks with parameter adjustment, denoted as x_{d}^n, x_{d}^s, and $x_{\mathrm{d}}^*(\rho^*)$, with respect to the parameters $\delta \in [0,1]$ (upper panel) and $\lambda \in [0,1]$ (lower panel) representing the degree of time inconsistency, respectively. The time-consistent benchmark x_{d}^* without parameter adjustment is depicted on the right (left) endpoints of the upper (lower) panel for $\delta = 1$ ($\lambda = 0$), respectively. It is assumed that $\mu = 0.01$, $\sigma = 0.25$, $\rho = 0.06$, $\tau = 0.15$, and $I = 200$. We set $\lambda = 0.33$ for the upper panel and $\delta = 0.3$ for the lower panel. The coupon level is fixed at $c^n = c^s = c^* = 20$.

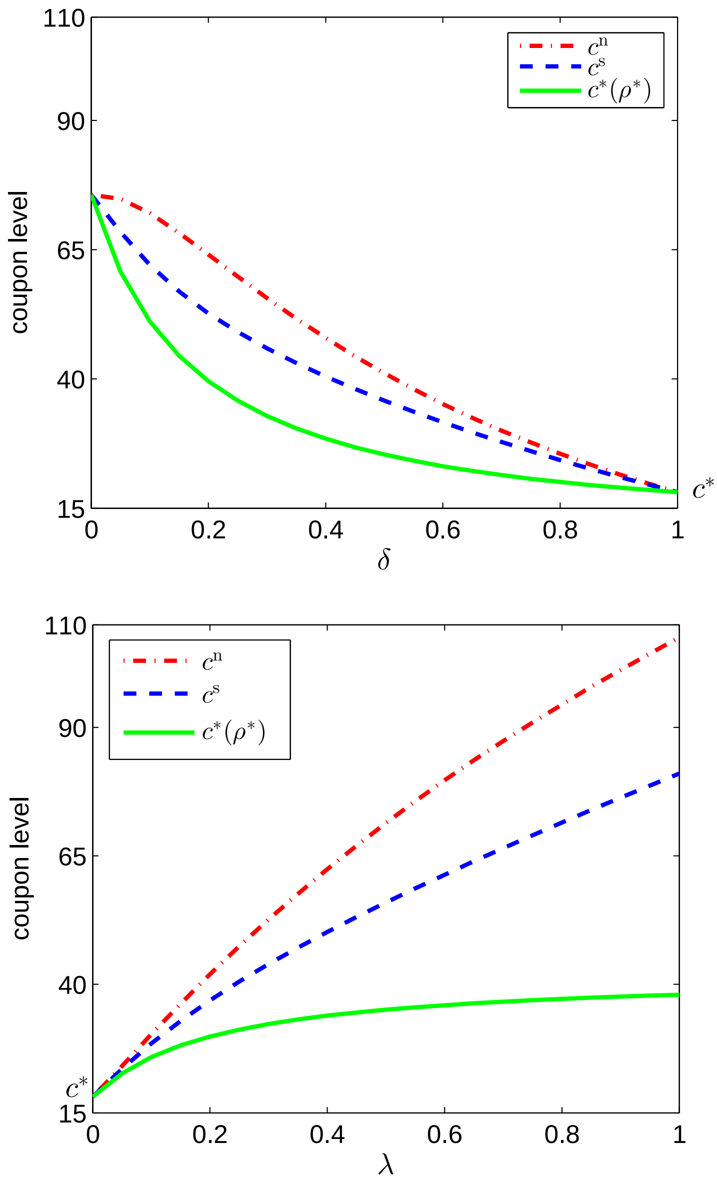


Fig. 6. The impact of time-inconsistent preferences on the coupon level. These two figures plot the coupon levels for naive, sophisticated entrepreneurs and the time-consistent benchmarks with parameter adjustment, denoted as c^n, c^s, and $c^*(\rho^*)$, with respect to the parameters $\delta \in [0, 1]$ (upper panel) and $\lambda \in [0, 1]$ (lower panel) representing the degree of time inconsistency, respectively. The time-consistent benchmark c^* without parameter adjustment is depicted on the right (left) endpoints of the upper (lower) panel for $\delta = 1$ ($\lambda = 0$), respectively. It is assumed that $\mu = 0.01$, $\sigma = 0.25$, $\rho = 0.06$, $\tau = 0.15$, and $I = 200$. We set $\lambda = 0.33$ for the upper panel and $\delta = 0.3$ for the lower panel.

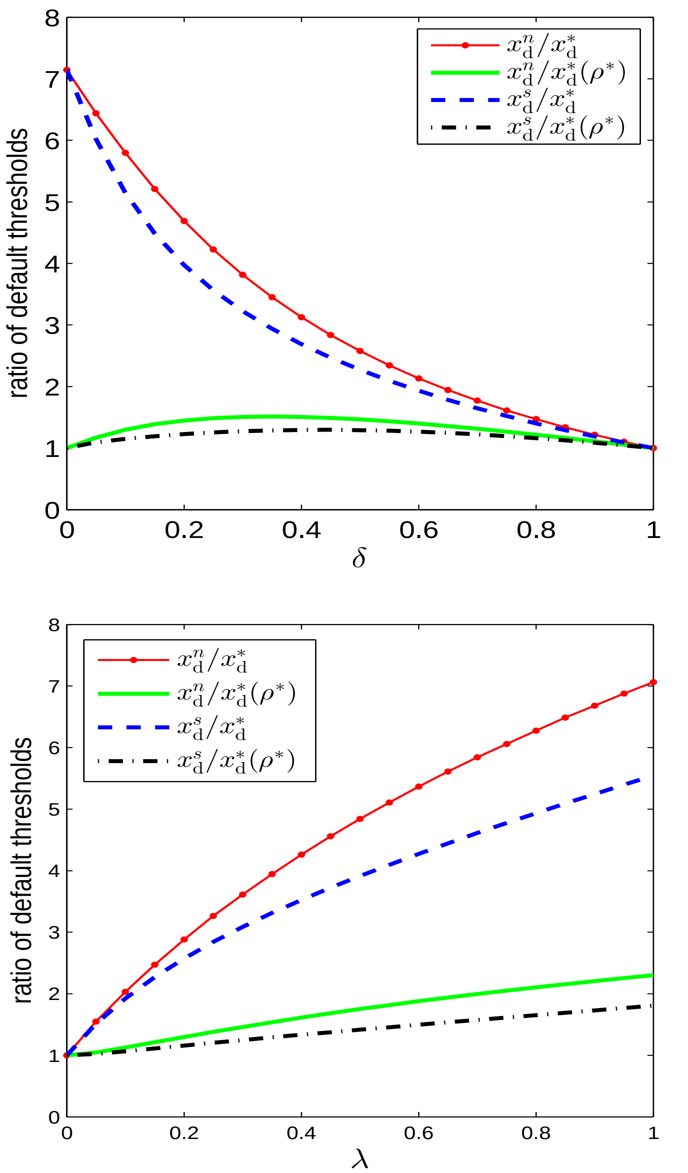
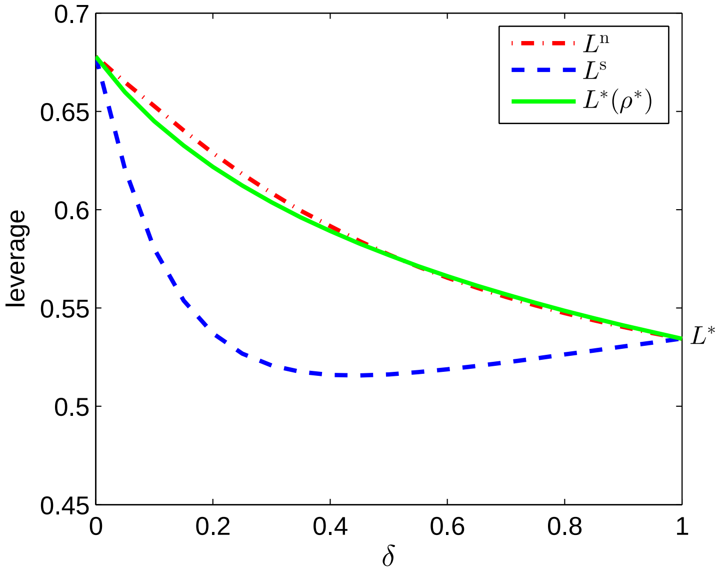


Fig. 7. The ratio of default thresholds under time-inconsistent preferences to the time-consistent benchmarks. These two figures plot the ratios of optimal default thresholds for naive entrepreneurs (without and with parameter adjustment), denoted as $x_{\rm i}^n/x_{\rm i}^*$ and $x_{\rm i}^n/x_{\rm i}^*(\rho^*)$, and the ratios for sophisticated entrepreneurs (without and with parameter adjustment), denoted as $x_{\rm i}^s/x_{\rm i}^*$ and $x_{\rm i}^s/x_{\rm i}^*(\rho^*)$, with respect to the parameters $\delta \in [0,1]$ (upper panel) and $\lambda \in [0,1]$ (lower panel) representing the degree of time inconsistency, respectively. It is assumed that $\mu = 0.01,\ \sigma = 0.25,\ \rho = 0.06,\ \tau = 0.15$, and $I = 200$. We set $\lambda = 0.33$ for the upper panel and $\delta = 0.3$ for the lower panel.



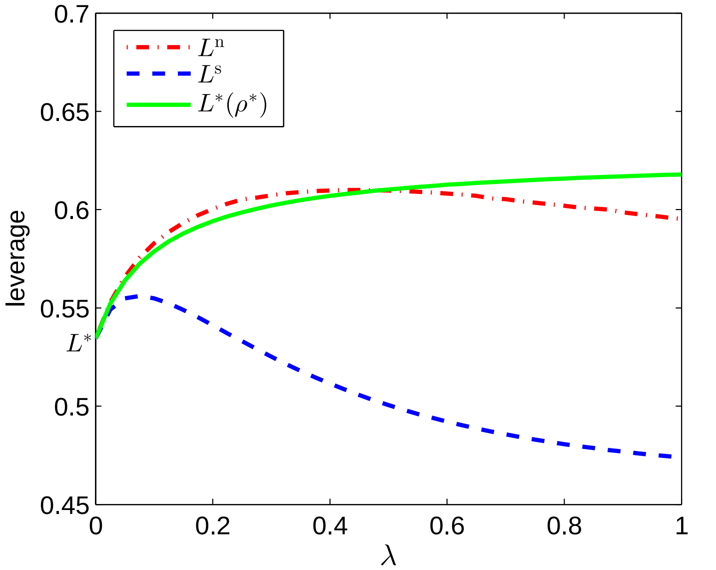


Fig. 8. The impact of time-inconsistent preferences on the leverage. These two figures plot the leverages for naive, sophisticated entrepreneurs and the time-consistent benchmarks with parameter adjustment, denoted as L^n, L^s, and $L^*(\rho^*)$, with respect to the parameters $\delta \in [0, 1]$ (upper panel) and $\lambda \in [0, 1]$ (lower panel) representing the degree of time inconsistency, respectively. The time-consistent benchmark L^* without parameter adjustment is depicted on the right (left) endpoints of the upper (lower) panel for $\delta = 1$ ($\lambda = 0$), respectively. It is assumed that $\mu = 0.01$, $\sigma = 0.25$, $\rho = 0.06$, $\tau = 0.15$, and $I = 200$. We set $\lambda = 0.33$ for the upper panel and $\delta = 0.3$ for the lower panel.