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"Increased Regressivity of the Optimal Capital Tax under
a Welfare Constraint for Newborn Children"

Yosuke Furukawa

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Yosuke Furukawa*

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Abstract

In this paper, we develop a three-period model that incorporates parents' heterogeneous skills and a welfare constraint for newborn children. Our numerical analysis shows how the optimal tax system is affected by the weight attached to the newborn child by a social planner. The main finding is that an increase in the guaranteed welfare level for newborn children makes the optimal capital income tax rate more regressive. This result is closely related to the trade-off between incentives for parents and insurance for the newborn child.

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*Graduate School of Economics, Kyoto University

1 Introduction

Reducing inequality is an important topic for researchers and a worthy objective for policymakers. In particular, society is responsible for protecting the living standards of the newborn child. In countries with high government debt, there is substantial intergenerational inequality because newborn children face the prospect of repaying large debts. Although fiscal policy can be used to reduce this inequality, this may adversely affect the current generation. The purpose of this paper is to design an optimal mechanism for the "current generation" that can be implemented by a social planner whose aim is to reduce intergenerational inequality. Although there are many ways of developing such a policy, the mechanism we use is based on optimal tax theory, known as "New Dynamic Public Finance."

In recent decades, many studies of optimal tax theory have emerged. Ramsey (1927) proposes that the tax system be used not only to finance government purchases but also to maximize social welfare. Developing this idea further, Mirrlees (1971) analyzes an economy in which there are heterogeneous agents whose skills are private information. Judd (1985) and Chamley (1986) extend Ramsey's work to dynamic economies.

During this century, these strands of literature merged into "New Dynamic Public Finance." In these dynamic economic models, agents' skills are private information that follow arbitrary stochastic processes. Golosov, Kocherlakota and Tsyvinski (2003) show that the intertemporal optimality condition is distorted as agents are discouraged from saving. This means that agents' marginal benefits of investing in capital exceed the marginal costs of doing so under the constraint of efficient allocation. This optimality condition is known as the inverse Euler equation or reciprocal Euler equation, and the associated distortion is termed the capital wedge. Golosov, Tsyvinski and Werning (2007) develop a two-period model that incorporates this wedge. The wedge is consistent with a tax on capital income. Kocherlakota (2005) and Albanesi and Sleet (2006) design tax policies that generate constrained efficient allocation. Kocherlakota (2005) shows that the optimal capital income tax is regressive.

There is another strand of literature on optimal tax systems. Tax systems are also used to reduce income inequality. Farhi and Werning (2007) develop a social discounting model in which intergenerational inequality is analyzed. Their study is extended by Farhi and Werning (2010), who analyze insurance for the newborn child. The planner solves a Pareto problem in which the average utility of the newborn child must exceed a certain level. They show that the intertemporal optimality condition has a negative wedge, which implies that each agent is encouraged to leave a bequest. They term this wedge an implicit estate tax. This wedge is consistent with a negative estate tax; i.e., a subsidy on bequests. Moreover, Farhi and Werning confirm that the implicit estate tax should be progressive, so that parents leaving larger bequests earn lower net returns on their bequests. By incorporating agents' skills that follow arbitrary stochastic processes, their model exhibits the discouraged savings problem. However, the relationship between the capital wedge and the implicit estate tax is not explained clearly. They are vague on how the weight attached to the newborn child by the planner affects the capital wedge and the optimal tax system that is designed to deliver constrained efficient allocation.

In this paper, we develop a three-period model that incorporates two types of wedges. A continuum of parents live for two periods and each couple produces a single child who lives for one period. Parents have heterogeneous productivity levels acquired at the beginning of each period. This framework enables us to analyze the relationship between the two types of wedges. This is because the capital wedge and the implicit estate tax emerge separately from the model. We first consider the planner's problem. We assume that the planner's objective function is that of a utilitarian. The three constraints in the planner's problem are the incentive constraint, the resource constraint and the newborn child's welfare constraint. In Section 3, we define the capital wedge and the implicit estate tax, both of which represent the difference between the intertemporal marginal rate of substitution and the marginal rate of transformation. We then derive the inverse Euler equations based on perturbation argument. We show how our optimal conditions are related to those of Golosov, Tsyvinski and Werning (2007) and Farhi and Werning (2010). Then, by following the approach suggested by Kocherlakota (2005), we attempt to design an explicit tax system that generates constrained efficient allocation.

We also conduct a numerical analysis that shows how the weight attached to the newborn child by the planner affects allocation and the optimal tax system. The main finding is that the difference between the capital income tax rates of high- and low-productivity parents increases with the weight attached to the newborn child by the planner. This means that the optimal capital income tax becomes more regressive. The capital wedge also increases with the weight attached to the newborn child. Mean reversion may explain these results. Not only does the planner's policy for newborn children reduce intergenerational inequality, it also reduces intragenerational inequality. Therefore, the incentive constraint of high-productivity parents tightens and the planner must provide them with stronger incentives. This result is troublesome. Policymakers may baulk at introducing a regressive optimal capital tax because of opposition from the current generation.

2 A Three-period Economy

2.1 Preferences

A continuum of parents live in periods $t = 0$ and $t = 1$. Each couple produces a single child who lives in period $t = 2$. Parents work and consume in each period, whereas their children only consume. At the beginning of periods $t = 0$ and $t = 1$, parents learn their productivity or skill level, θ_t, which is private information. They then produce y_t units of the consumption good, which requires y_t/θ_t units of work effort in $t = 0, 1$. Let the productivity realization in period 0 be $\theta_0(i)$ for $i = 1, 2, \cdots, N_0$. Let $\pi_0(i)$ denote the ex ante probability distribution, which, by the law of large numbers, is equivalent to the ex post distribution in the population. In period 1, productivity becomes $\theta_1(i,j)$ for $j = 1, 2, \cdots, N_1(i)$, where $\pi_1(j|i)$ is the conditional probability distribution for parents of skill type j, whose skill type in period 0 is given by i. We assume that the probability distribution of productivity, π, is common knowledge. The lifetime utility of parents with productivity of $\theta_0(i), \theta_1(i,j)$ is given by

$$\begin{aligned} V_0(\theta_0(i), \theta_1(i,j)) &\equiv u(c_0(\theta_0(i))) - h(\frac{y_0(\theta_0(i))}{\theta_0(i)}) \\ &+ \beta[u(c_1(\theta_1(i,j))) - h(\frac{y_1(\theta_1(i,j))}{\theta_1(i,j)})] + \beta^2 V_2(\theta_0(i), \theta_1(i,j)), \end{aligned}$$

with $\beta < 1$. Children's utility, $V_2(\cdot)$, is

$$V_2(\theta_1(i,j)) \equiv u(c_2(\theta_1(i,j))).$$

The utility function $u(\cdot)$ is increasing, concave, differentiable and satisfies Inada's conditions; the disutility function $h(\cdot)$ is increasing, convex and differentiable. In the rest of this paper, we write $c_0(\theta_0(i))$, $c_1(\theta_1(i,j))$, $c_2(\theta_1(i,j))$, $y_0(\theta_0(i))$, $y_1(\theta_1(i,j))$ as $c_0(i)$, $c_1(i,j)$, $c_2(i,j)$, $y_0(i)$, $y_1(i,j)$, respectively.

2.2 Resource Constraints

We assume that production is linear in efficiency units of labor supplied by parents. We also assume linear savings technology with rate of return R. Then, the resource constraints are

$$\sum_i c_0(i)\pi_0(i) + K_1 \le \sum_i y_0(i)\pi_0(i) + K_0,$$

$$\sum_{i,j} c_1(i,j)\pi_1(j|i)\pi_0(i) + K_2 \le \sum_{i,j} y_1(i,j)\pi_1(j|i)\pi_0(i) + RK_1,$$

$$\sum_{i,j} c_2(i,j)\pi_1(j|i)\pi_0(i) \leq RK_2,$$

where K_1 is the capital stock held between $t = 0$ and $t = 1$, K_2 is capital stock held between $t = 1$ and $t = 2$, and K_0 is the endowed level of capital. Solving for K_2 and K_1 yields the following present-value resource constraint:

$$\sum_{i,j}[c_0(i) + \frac{1}{R}c_1(i,j) + \frac{1}{R^2}c_2(i,j)]\pi_1(j|i)\pi_0(i)$$

$$\leq \sum_{i,j}[y_0(i) + \frac{1}{R}y_1(i,j)]\pi_1(j|i)\pi_0(i) + K_0. \tag{1}$$

2.3 Incentive Constraints

We assume that the planner's objective function is that of a utilitarian. Because parents' productivity levels are private information, the allocation that maximizes social welfare is constrained by informational friction. According to the revelation principle, the best allocation can always be achieved directly if parents report their productivity levels truthfully to the planner. Parents report productivity levels of i_r and j_r to the planner in the first and second periods, respectively. Then, for all alternative feasible reporting strategies i_r and $j_r(j)$, the allocation must satisfy the following incentive constraints:

$$u(c_0(i)) - h(\frac{y_0(i)}{\theta_0(i)}) + \beta\sum_j[u(c_1(i,j)) - h(\frac{y_1(i,j)}{\theta_1(i,j)}) + \beta u(c_2(i,j))]\pi_1(j|i)$$

$$\geq u(c_0(i_r)) - h(\frac{y_0(i_r)}{\theta_0(i)}) + \beta\sum_j[u(c_1(i_r,j_r(j))) - h(\frac{y_1(i_r,j_r(j))}{\theta_1(i,j)}) + \beta u(c_2(i_r,j_r(j)))]\pi_1(j|i). \tag{2}$$

2.4 The Planning Problem

The planner is also constrained by the welfare of the newborn child. Let $\underline{V_2}$ be the minimal level of average utility for the newborn child guaranteed by the planner. This constraint is formalized as follows:

$$E[V_2(\theta_1(i,j))] \geq \underline{V_2}. \tag{3}$$

Thus, the constrained efficient planning problem is

$$\max E[V_0(\theta_0(i), \theta_1(i,j))] \tag{4}$$

subject to (1), (2) and (3).

If equation (3) is not binding, our planning problem (4) is similar to that of Golosov, Tsyvinski and Werning (2007). If productivity does not follow a stochastic process (i.e., if $\theta_0(i) = \theta_1(i,j)$ for all i, j), then the problem described by (4) is similar to that of Farhi and Werning (2010).

3 The Intertemporal Wedge

Let (c_0^*,c_1^*,c_2^*,y_0^*,y_1^*) be the constrained efficient allocation of the planning problem. We first define the intertemporal wedges, termed the capital wedge and the implicit estate tax. We then derive the necessary conditions for the planning problem and characterize the properties of these wedges.

3.1 The Capital Wedge

The capital wedge $\tau_{0,1}$ is defined as the intertemporal wedge between $t = 0$ and $t = 1$, as follows:

$$(1 + \tau_{0,1}(i))u'(c_0^*(i)) \equiv \beta R \sum_j u'(c_1^*(i,j))\pi_1(j|i)$$

for all i.

3.2 The Implicit Estate Tax

The implicit estate tax $\tau_{1,2}$ is defined as the intertemporal wedge between $t = 1$ and $t = 2$, as follows:

$$(1 + \tau_{1,2}(i,j))u'(c_1^*(i,j)) \equiv \beta R u'(c_2^*(i,j)),$$

for all i, j.

Note that when $\tau_{0.1}$ and $\tau_{0,2}$ are zero, the standard Euler equations are satisfied.

3.3 Necessary Conditions: The Inverse Euler Equations

We analyze the planning problem given by (4), and derive the necessary conditions for optimality; i.e., the inverse Euler equations. We assume that the newborn child's welfare constraint (3) is binding. Given a constrained efficient allocation, we consider a class of perturbations that are all incentive compatible. Atkinson and Stiglitz's (1976) result is derived by similar means. Hence, we obtain the following proposition.

Proposition 1 *A constrained efficient allocation satisfies the following inverse Euler equations:*

$$\frac{1}{u'(c_0(i))} = \frac{1}{\beta R}\sum_j \frac{1}{u'(c_1(i,j))}\pi_1(j|i), \tag{5}$$

for all i, and

$$\frac{1}{u'(c_1(i,j))} = \frac{1}{\beta R u'(c_2(i,j))} - \frac{R}{\beta}\frac{\nu}{\mu}, \tag{6}$$

for all i,j.

If consumption is stochastic, equation (5) implies that the standard Euler equations are not satisfied. This result follows directly from applying Jensen's inequality to the reciprocal function $1/x$ in equation (5). We obtain

$$u'(c_0(i)) < \beta R \sum_j u'(c_1(i,j))\pi_1(j|i)$$

for all i. This implies that $\tau_{0,1}(i) > 0$ for all i. This result corresponds to that obtained by Golosov, Tsyvinski and Werning (2007), who show that it is optimal to introduce a positive wedge into savings that discourages saving. This means that agents' marginal benefits of investing in capital exceed the corresponding marginal costs at the constrained efficient allocation under imperfect risk sharing.

Because of our assumption that the newborn child's welfare constraint is binding, the Lagrange multiplier ν is positive. Thus, the implicit estate tax $\tau_{1,2}(i,j)$ must be negative for all i, j. This result corresponds to that of Farhi and Werning (2010), who show that constrained efficient allocation generates a negative wedge in bequests, and this wedge implicitly encourages agents to leave bequests. This is consistent with a negative estate tax, or simply, a subsidy on bequests. Moreover, equation (6) implies that $c_1(i,j)$ and $c_2(i,j)$ are positively correlated at the constrained efficient allocation. The incentive constraint implies that $c_1(i,j)$ and $c_2(i,j)$ are both nondecreasing in $\theta_1(i,j)$. Hence, $\tau_{1,2}(i,j)$ is strictly negative and increasing in $\theta_1(i,j)$ for all i. This property is referred to by Farhi and Werning (2010) as the progressivity of the implicit estate tax. The following proposition summarizes these properties.

Proposition 2 *Suppose that the optimal allocation has a strictly positive consumption plan and that the consumption plan is stochastic. Suppose also that the Lagrange multiplier ν is strictly positive. Then, the capital wedge $\tau_{0,1}(i)$ is positive for all i. Moreover, the implicit estate tax $\tau_{1,2}(i,j)$ is strictly negative and takes the following form:*

$$\tau_{1,2}(i,j) = -R^2\frac{\nu}{\mu}u'(c_2(i,j)), \tag{7}$$

for all i, j.

4 Implementation

4.1 Important Assumptions

In this section, we follow Kocherlakota (2005) to design the optimal tax system, which generates constrained efficient allocation for the planning problem (4). We show that a nonlinear tax system incorporating labor income tax, capital income tax and an estate tax can deliver constrained efficient allocation.

Before we design an explicit tax system, we introduce some definitions and assumptions. Let DOM_0 be a subset of R defined as follows: y_0 is in DOM_0 if and only if there exists a $\theta_0(i)$ such that

$$y_0 = y_0^*(\theta_0(i)).$$

Similarly, let DOM_1 be a subset of R^2 defined as follows: (y_0, y_1) is in DOM_1 if and only if a $\theta_0(i)$ and $\theta_1(i,j)$ exist such that

$$(y_0, y_1) = (y_0^*(\theta_0(i)), y_1^*(\theta_1(i.j))).$$

Given this definition, implementation of the tax system does not to require direct reporting of agents' productivity levels but merely requires information on levels of production; i.e., (y_0, y_1).

We also make the following assumption.

Assumption 1 *There exists a sequence of functions $\hat{c}^* = (\hat{c_0^*}, \hat{c_1^*}, \hat{c_2^*})$, where $\hat{c_t^*} : DOM_t \to R_+$,*

$$\hat{c_0^*}(y_0^*(\theta_0(i))) = c_0^*(i),$$

$$\hat{c_1^*}(y_0^*(\theta_0(i)), y_1^*(\theta_1(i,j))) = c_1^*(i,j),$$

$$\hat{c_2^*}(y_0^*(\theta_0(i)), y_1^*(\theta_1(i,j))) = c_2^*(i,j),$$

for all i, j, where c_0^*, c_1^* and c_2^* represent the constrained efficient allocation for the planning problem given by (4).

In this model, it is possible for two agents whose levels of productivity differ over time to have the same sequence of production levels. The role of Assumption 1 is to prevent this inconvenient case (for our tax system) from arising. This case also invalidates the estate tax system designed by Farhi and Werning (2010), which is affected by the size of bequests. This is because agents with different levels of production over time can bequeath the same amounts. Therefore, our estate tax system is quite different. We design estate tax rates that depend on the evolution of production levels over time.

4.2 The Optimal Tax System

An allocation is generated through nonlinear labor income taxes $\psi_0(y_0)$,$\psi_1(y_0, y_1)$, a capital income tax $\tau_1(y_0, y_1)$ and an estate tax $\tau_2(y_0, y_1)$ if, for all i, $(c_0^*(i)$,$c_1^*(i,j)$,$c_2^*(i,j)$,$y_0^*(i)$,$y_1^*(i,j))$ solves

$$\max \sum_j \left[u(c_0) - h(\frac{y_0}{\theta_0(i)}) + \left\{ \beta \left(u(c_1) - h(\frac{y_1}{\theta_1(i,j)}) \right) + \beta^2 u(c_2) \right\} \right] \pi_1(j|i),$$

subject to

$$c_0 + k_1 \leq Rk_0 + y_0 - \psi_0(y_0),$$

$$c_1 + k_2 \leq R(1 - \tau_1(y_0, y_1))k_1 + y_1$$
$$- \psi_1(y_0, y_1) - \tau_2(y_0, y_1)k_2,$$

$$c_2 \leq Rk_2.$$

Proposition 3 *Under Assumption 1, there exist nonlinear labor income taxes* $\psi_0(y_0)$,$\psi_1(y_0, y_1)$, *a capital tax* $\tau_1(y_0, y_1)$ *and an estate tax* $\tau_2(y_0, y_1)$ *that generate constrained efficient allocation* $(c_0^*(i)$,$c_1^*(i,j)$,$c_2^*(i,j)$,$y_0^*(i)$,$y_1^*(i,j))$ *for all* i, j.

The proof is in the Appendix. If the optimal allocation has a strictly positive consumption plan, we can derive the following first-order conditions:

$$\left(1 + \frac{\beta R \sum_j \tau_1(y_0, y_1) u'(c_1) \pi_1(j|i)}{u'(c_0)} \right) u'(c_0) = \beta R \sum_j u'(c_1) \pi_1(j|i),$$

$$(1 + \tau_2(y_0, y_1)) u'(c_1) = \beta R u'(c_2).$$

Given Propositions 1 and 2, to generate the constrained efficient allocation, the capital and estate tax rates must satisfy:

$$\tau_{0,1}(i) = \frac{\beta R \sum_j \tau_1(y_0, y_1) u'(c_1) \pi_1(j|i)}{u'(c_0)} > 0,$$

$$\tau_{1,2}(i,j) = \tau_2(y_0, y_1) = -R^2 \frac{\nu}{\mu} u'(c_2) < 0,$$

where $\tau_{0.1}(i)$ is the capital wedge and $\tau_{1,2}(i,j)$ is the implicit estate tax defined in Sections 3.1 and 3.2, respectively. In this scheme, the implicit estate tax is the same as the estate tax rate.

5 Numerical Analysis

How does the level of the newborn child's welfare under the constraint $\underline{V_2}$ affect the optimal tax system? We consider an example similar to those used by Kocherlakota (2005) and Kocherlakota (2010). Despite the simplicity of our model, as is the case with all models under the heading "New Dynamic Public Finance," it is difficult to determine the constrained efficient allocation analytically. Moreover, our model includes complex nonlinear equations. Hence, we solve the following example numerically.

Let $u(c) = \ln(c)$, $h(l) = \frac{l^{1+1/\eta}}{1+1/\eta}$, $\beta = 1$, $R = 1$ and $k_0 = 0$. The parameter η is the Frisch elasticity of labor supply. We suppose that $\eta = 0.5$. Suppose also that $\Theta_0 = \{1\}$, $\Theta_1 = \{0, 1\}$ and $\Pr(\theta_1 = 1) = 1/2$. Note that $l_1 = 0$ if θ_1 is 0. Then, we rewrite the planner's problem as follows:

$$\max_{c,y} \ln(c_0) - \frac{y_0^{1+1/\eta}}{1+1/\eta} + \frac{\ln(c_{1h})}{2} - \frac{y_{1h}^{1+1/\eta}}{2(1+1/\eta)} + \frac{\ln(c_{1l})}{2} + \frac{\ln(c_{2h})}{2} + \frac{\ln(c_{2l})}{2},$$

subject to

$$c_0 + \frac{1}{2}(c_{1h} + c_{1l} + c_{2h} + c_{2l}) \leq k_0 + y_0 + \frac{y_{1h}}{2},$$

$$\ln(c_0) - \frac{y_0^{1+1/\eta}}{1+1/\eta} + \frac{\ln(c_{1h})}{2} - \frac{y_{1h}^{1+1/\eta}}{2(1+1/\eta)} + \frac{\ln(c_{2h})}{2} \geq \ln(c_0) - \frac{y_0^{1+1/\eta}}{1+1/\eta} + \frac{\ln(c_{1l})}{2} + \frac{\ln(c_{2l})}{2},$$

$$\frac{1}{2}(\ln(c_{2h}) + \ln(c_{2l})) \geq \underline{V_2}$$

for some $\underline{V_2}$.

Let $(c_0^*, c_{1h}^*, c_{1l}^*, c_{2h}^*, c_{2l}^*, y_0^*, y_1^*)$ be the constrained efficient allocation. Define $(\tau_k^{1h}, \tau_k^{1l}, \tau_k^{2h}, \tau_k^{2l}, \psi_y^{1h}, \psi_y^{2l})$ as

$$\frac{(1-\tau_k^{1h})}{c_{1h}^*} = \frac{1}{c_0^*}, \ \frac{(1-\tau_k^{1l})}{c_{1l}^*} = \frac{1}{c_0^*},$$

$$\tau_k^{2h} = -\frac{W}{c_{1h}^*}, \ \tau_k^{2l} = -\frac{W}{c_{1l}^*},$$

$$\psi_y^{1h} = (1-\tau_k^{1h})k_1^* + y_{1h}^* - c_{1h}^* - (1+\tau_k^{2h})k_{2h}^*,$$

$$\psi_y^{1l} = (1-\tau_k^{1l})k_1^* - c_{1l}^* - (1+\tau_k^{2l})k_{2l}^*$$

, respectively, where $W \equiv \nu/\mu$, $k_1^* \equiv k_0 + y_0^* - c_0^*$, $k_{2h}^* \equiv c_{2h}^*$, and $k_{2l}^* \equiv c_{2l}^*$.

Note that W is determined by the guaranteed level of average utility for the newborn child, $\underline{V_2}$. Indeed, in our model, how much the planner cares for children can be measured either by $\underline{V_2}$ or by W. To examine how the constrained efficient allocation and the corresponding optimal tax system are affected by how much weight the planner attaches to the newborn child, it is convenient to examine how the planner reacts to a change in W

rather than to a change in $\underline{V_2}$. Thus, the following figures illustrate how the constrained efficient allocation and the optimal tax system are related to $W \in [0, 1]$.

Note that $W = 0$ corresponds to the case in which the planner does not care about the newborn child, in which case our model reduces to the single-generation model of Golosov, Tsyvinski and Werning (2007). The higher the value of W, the more weight the planner attaches to the welfare of the newborn child.

5.1 The Optimal Allocation

First, we consider the features of the optimal allocation. Figure 1 illustrates the constrained efficient allocation. Higher values of W generate lower consumption levels and higher production levels at $t = 0$ and $t = 1$. This means that the planner designs a tax system to deliver a guaranteed welfare level for the newborn child, $\underline{V_2}$, and this system gives parents a greater incentive to work and increase their savings.

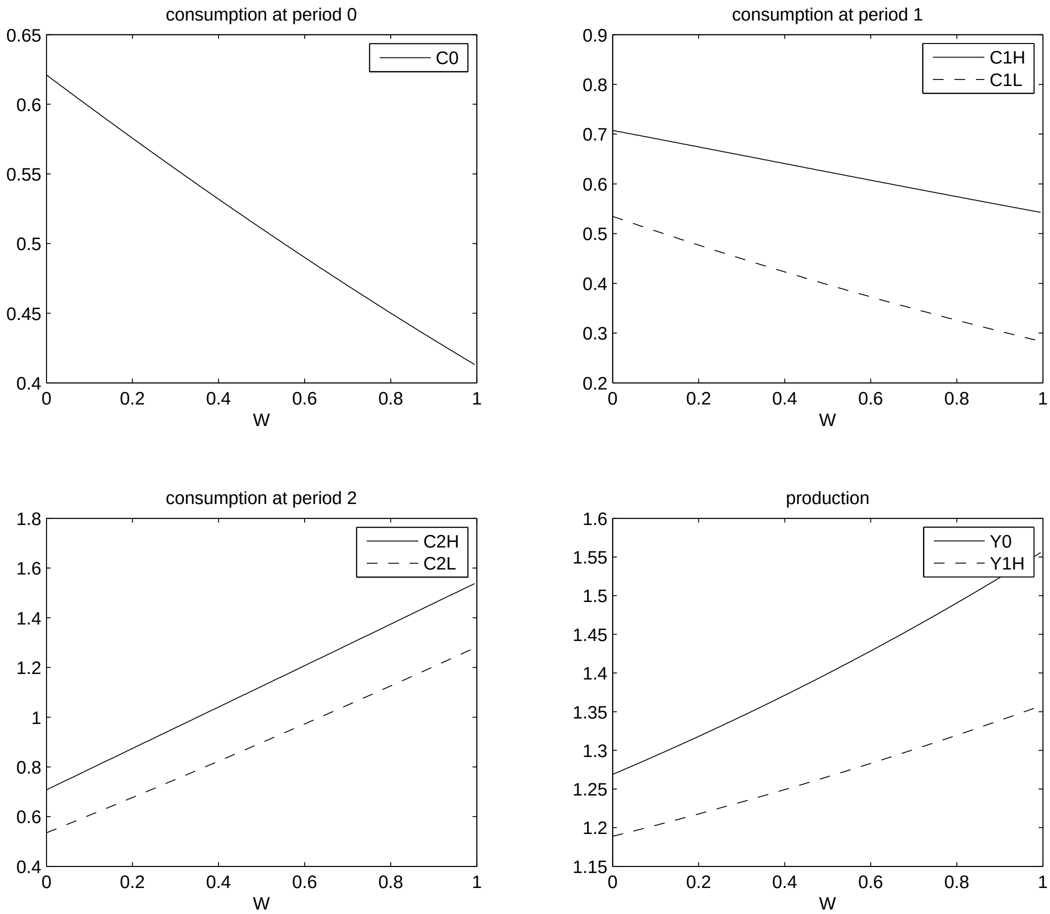

Figure 1: The Constrained Efficient Allocation

5.2 Insurance versus Incentives

Figure 2 plots the behavior of the optimal taxes and the capital wedge. Our central concern is how the capital tax rates τ_k^{1h} and τ_k^{1l} are related to the weight attached to the newborn child by the planner. It is clear than an increase in W makes capital tax rates more regressive. That is, the ex post rate of return on savings increases for high-productivity agents and decreases for low-productivity agents. Note that there is no effect on the ex ante rate of return on savings. This implies that the weight attached to children by the planner does not explain Kocherlakota's (2005) so-called zero-expected-wealth-taxes result. The interpretation of the increasing capital wedge $\tau_{0,1}$ is that at period $t = 0$, parents have an incentive to discourage saving.

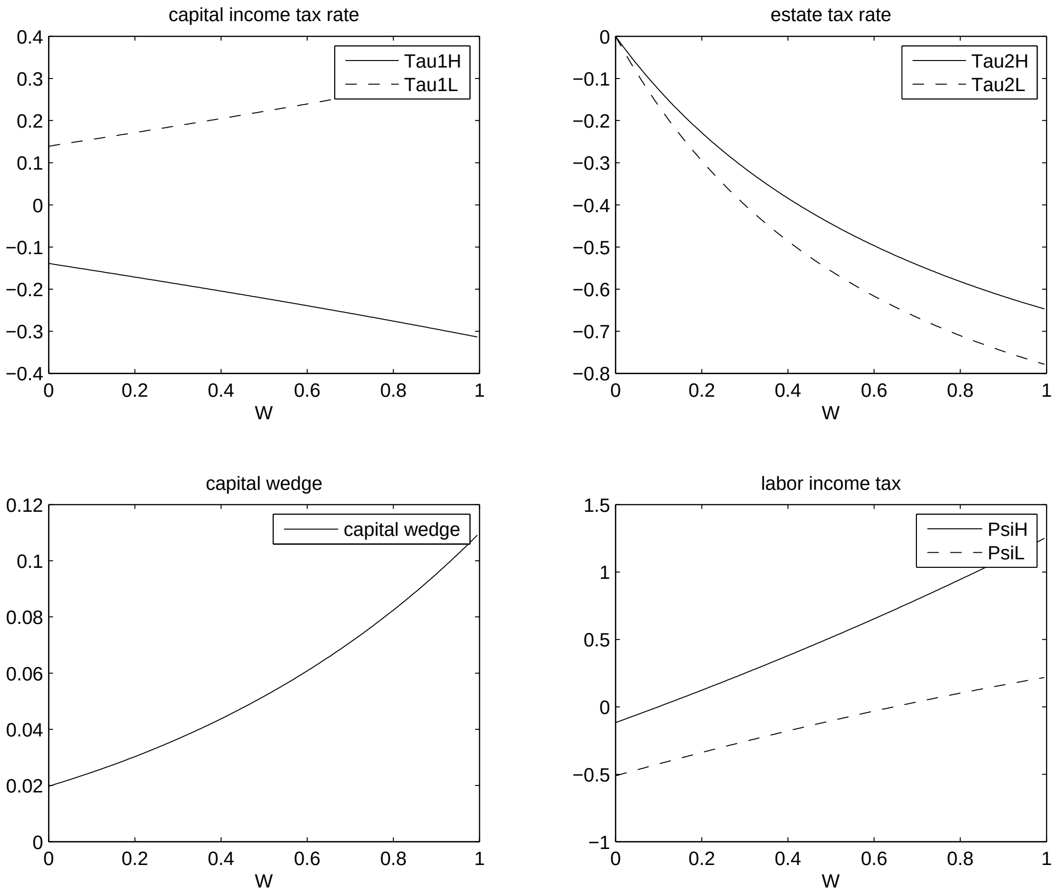

Figure 2: Optimal Taxes and the Capital Wedge

Why does an increase in the weight attached to the newborn child by the planner raise the capital wedge? Two types of processes explain this: the production effect and the mean reversion effect.[1] Both of these effects are related to the incentive constraint. Because the incentive constraint is binding at the optimum, we obtain

$$u(c_{1h}) - u(c_{1l}) = h(y_{1h}/\theta_{1h}) - (u(c_{2h}) - u(c_{2l})). \tag{8}$$

It is clear why the production effect arises: we have already shown that production levels are increasing in W. Hence, an increase in W causes the right-hand side of equation (8) to increase.

The mean reversion effect is important because it can be interpreted as the result of a trade-off between parents' incentives and insurance for the newborn child. This is related to the behavior of the estate tax rate. The estate tax rate behaves in such a way that an increase in W gives parents higher subsidies for their bequests. This happens because the planner requires parents to bequeath large amounts to their newborn child. Moreover, the estate tax rate is progressive. Specifically, low-productivity agents get a high rate of return on their bequests. This progressivity serves to reduce inequality faced by the newborn child.

Because the mean reversion effect reduces the inequality of the newborn child, the right-hand side of equation (8) increases. The production effect and the mean reversion effect combine to cause an increase in W to tighten the incentive constraint. Thus, the planner must offer greater incentives to high-productivity parents to induce them to work more.

In our model, it is possible to control high-productivity parents' willingness to work by changing the capital income tax rates τ_k^{1h} and τ_k^{1l}. By setting a high regressive capital income tax, the planner makes saving in period $t = 0$ more risky. Parents then reduce their savings in period $t = 0$ and increase their consumption. This fall in savings raises parents' marginal utility of consumption in period $t = 1$, to which high-productivity parents respond by working more. Reduced savings in period $t = 0$ also generate a high capital wedge. This is why a higher regressive capital income tax is optimal.

Figure 2 illustrates labor income taxes. Both agents face negative labor income taxes when $W = 0$. The extent to which labor income taxes rise more quickly for high-productivity agents than for low-productivity agents increases with the weight attached to the newborn child by the planner.

[1] The result obtained by Farhi and Werning (2007) is explained by the mean reversion effect. The mean reversion effect is that an increase in the weight attached to the newborn child by the planner reduces intra-generational inequality suffered by the newborn child.

6 Conclusion

In this paper, we provided a brief intuitive explanation of the optimal mechanism to implement by a planner who is considering reducing intergenerational inequality for the "current generation". Our finding of the increased regressivity of the optimal capital tax is closely related to the trade-off between incentives for parents and insurance for the newborn child. This trade-off arises because of not only the production effect but also the mean reversion effect. This raises a dilemma for the policymaker because the current generation must accept greater intragenerational inequality to provide insurance for newborn children.

A Appendix

A.1 Proof of Proposition 1

The Lagrangian of the planning problem without incentive constraints is

$$\begin{aligned}
L &= \sum_{i,j}[u(c_0(i)) - h(\frac{y_0(i)}{\theta_0(i)}) + \beta[u(c_1(i,j)) - h(\frac{y_1(i,j)}{\theta_1(i,j)})] + \beta^2 u(c_2(i,j))]\pi_1(j|i)\pi_0(i) \\
&+ \mu[K_0 + \sum_{i,j}[y_0(i) + \frac{1}{R}y_1(i,j) - c_0(i) - \frac{1}{R}c_1(i,j) - \frac{1}{R^2}c_2(i,j)]\pi_1(j|i)\pi_0(i)] \\
&+ \nu[\sum_{i,j} u(c_2(i,j))\pi_1(j|i)\pi_0(i) - \underline{V_2}].
\end{aligned}$$

We fix the value of any first-period realization i. We then increase utility in period $t = 1$, $u(c_1(i,j))$, by the same amount across all second-period realizations j. That is, we define $u(c_1^\epsilon(i,j;\epsilon)) \equiv u(c_1(i,j)) + \epsilon$ for some small ϵ. To compensate, we decrease utility in period $t = 0$ by $\beta\epsilon$. That is, we define $u(c_0^\epsilon(i,\epsilon)) \equiv u(c_0(i)) - \beta\epsilon$. The key point is that such variations affect neither the objective function nor the incentive constraints in the planning problem. A first-order necessary condition is that, when evaluated at the constrained efficient allocation, the derivative of L with respect to ϵ is zero. Based on this perturbation from $t = 0$ to $t = 1$, the following Lagrangian can be defined for all i:

$$\begin{aligned}
L_{0,1} &= \sum_{i,j}[u(c_0^\epsilon(i)) - h(\frac{y_0(i)}{\theta_0(i)}) + \beta[u(c_1^\epsilon(i,j)) - h(\frac{y_1(i,j)}{\theta_1(i,j)})] + \beta^2 u(c_2(i,j))]\pi_1(j|i)\pi_0(i) \\
&+ \mu[K_0 + \sum_{i,j}[y_0(i) + \frac{1}{R}y_1(i,j) - c_0^\epsilon(i) - \frac{1}{R}c_1^\epsilon(i,j) - \frac{1}{R^2}c_2(i,j)]\pi_1(j|i)\pi_0(i)] \\
&+ \nu[\sum_{i,j} u(c_2(i,j))\pi_1(j|i)\pi_0(i) - \underline{V_2}].
\end{aligned}$$

Differentiating $L_{0,1}$ with respect to ϵ, and evaluating the result at $\epsilon = 0$, yields

$$\frac{1}{u'(c_0(i))} = \frac{1}{\beta R}\sum_j \frac{1}{u'(c_1(i,j))}\pi_1(j|i)$$

for all i.

Similarly, we can fix any first- and second-period (i,j), We then define $u(c_2^\epsilon(i,j;\epsilon)) \equiv u(c_2(i,j)) + \epsilon$ for some small ϵ. To compensate, we define $u(c_1^\epsilon(i,j,\epsilon)) \equiv u(c_1(i,j)) - \beta\epsilon$. A first-order necessary condition is that the derivative of L with respect to ϵ is zero. A perturbation from $t = 1$ to $t = 2$ changes the Lagrangian into the following form:

$$\begin{aligned}
L_{1,2} &= \sum_{i,j}[u(c_0(i)) - h(\frac{y_0(i)}{\theta_0(i)}) + \beta[u(c_1^\epsilon(i,j)) - h(\frac{y_1(i,j)}{\theta_1(i,j)})] + \beta^2 u(c_2^\epsilon(i,j)]\pi_1(j|i)\pi_0(i) \\
&+ \mu[K_0 + \sum_{i,j}[y_0(i) + \frac{1}{R}y_1(i,j) - c_0(i) - \frac{1}{R}c_1^\epsilon(i,j) - \frac{1}{R^2}c_2^\epsilon(i,j)]\pi_1(j|i)\pi_0(i)] \\
&+ \nu[\sum_{i,j} u(c_2^\epsilon(i,j))\pi_1(j|i)\pi_0(i) - \underline{V_2}]
\end{aligned}$$

for all i, j. Differentiating $L_{1,2}$ with respect to ϵ and evaluating the result at $\epsilon = 0$ yields

$$\frac{1}{u'(c_1(i,j))} = \frac{1}{\beta R u'(c_2(i,j))} - \frac{R}{\beta}\frac{\nu}{\mu}$$

for all i, j.

A.2 Proof of Proposition 2

We have already shown that $\tau_{0,1}(i)$ is positive. The other part of Proposition 2 can be straightforwardly derived from equation (6).

A.3 Proof of Proposition 3

Given Assumption 1, there is a function $\hat{c_2^*} : DOM_1 \to R_+$ such that $\hat{c_2^*}(y_0^*(i), y_1^*(i,j)) = c_2^*(i,j)$. Let the estate tax τ_2^* be defined so that it satisfies

$$\tau_2^*(y_0^*(i), y_1^*(i,j)) = -R^2 \frac{\mu}{\nu} u'(c_2^*(i,j)),$$

for all i, j, if (y_0, y_1) in DOM_1. Otherwise, $\tau_2^* = 0$. Next, let the capital tax τ_1^* be defined so that it satisfies the ex post optimal condition that

$$\beta R(1 - \tau_1^*(y_0^*(i), y_1^*(i,j)))u'(c_1^*(i,j)) = u'(c_0^*(i))$$

for all i, j, if (y_0, y_1) in DOM_1. Otherwise, $\tau_1^* = 1$. Let the first-period labor income tax ψ_0^* be defined such that

$$c_0^*(i) + k_1^*(i) = Rk_0 + y_0^*(i) - \psi_0^*(y_0^*(i))$$

for all i, if y_0 in DOM_0. Otherwise, $\psi_0^* = Rk_0 + y_0$. Similarly, let the second-period labor income tax ψ_1^* be defined such that

$$\begin{aligned} c_1^*(i,j) + k_2^*(i,j) = R(1 - \tau_1^*(y_0^*(i), y_1^*(i,j)))k_1^*(i) + y_1^*(i,j) \\ -\psi_1^*(y_0^*(i), y_1^*(i,j)) - \tau_2^*(y_0^*(i), y_1^*(i,j))k_2^*(i,j) \end{aligned}$$

for all i, j, if (y_0, y_1) in DOM_1. Otherwise, $\psi_1^* = y_1$

We now show that the tax system represented by $\psi_0^*, \psi_1^*, \tau_1^*, \tau_2^*$ generates the optimal allocation. When $(y_0, y_1) = (y_0^*(i), y_1^*(i,j)) \in DOM_1$ for each agent, optimality requires $c_0 = \hat{c_0^*}(y_0), c_1 = \hat{c_1^*}(y_0, y_1), c_2 = \hat{c_2^*}(y_0, y_1)$. This is because this configuration satisfies both the standard Euler equations under the tax system and the resource constraint. If there exists another reporting strategy, σ_t, such that $(y_0', y_1') = (y_0^*(\sigma_0(\theta_0(i))), y_1^*(\sigma_1(\theta_1(i,j)))) \in$

DOM_1, then optimality requires $c_0' = \hat{c_0^*}(y_0'), c_1' = \hat{c_1^*}(y_0', y_1'), c_2' = \hat{c_2^*}(y_0', y_1')$, for the same reason. However, the incentive constraint implies that the utility derived by each agent from the allocation $(y_0', y_1', c_0', c_1', c_2')$ cannot exceed that derived from $(y_0, y_1, c_0, c_1, c_2)$. Because $(y_0, y_1) \notin DOM_1$ are never selected by agents because of the heavy punishments they attract from the planner, the allocation $(y_0, y_1, c_0, c_1, c_2)$ is the optimal allocation.

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