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"Price Competition or Tacit Collusion"

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Price Competition or Tacit Collusion[1]

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Abstract

Every now and then, we observe a fierce price war in a real world market, through which competing firms end up with a Bertrand-like price competition equilibrium. Despite this, very little has been known in the existing literature as to why a price competition market is formed. We address this question in the context of a choice between engaging in price competition and holding a price leader. Focusing on a duopoly market, we demonstrate that if supply is tight relative to demand, and if the cost differential between firms is reasonably large, a price competition market is formed non-cooperatively.

Keywords: price competition, price leader, market organization game

JEL Classification Numbers: D21, D43, L11, L13.

1 Introduction

Every now and then, we observe a fierce price war in a real world market, through which competing firms end up with a Bertrand-like price competition equilibrium. A good example may be the U.S. airline industry, in which small companies occasionally enter the market and engage in harsh price-cutting competition with incumbents before reaching a Bertrand-like equilibrium. Prevalent as this phenomenon is, little has been known in the existing literature as to why a market is formed in which Bertrand-like price competition takes place.

While, as is discussed above, off-equilibrium price cutting competition appears to lead to a Bertrand-like equilibrium in some cases, in other cases, it appears to lead to the formation of a price leader, which enables firms to form tacit collusion. As this shows, the formation of a price competition market and that of a price leader market may be closely related.

With these considerations, the present study addresses the endogenous formation of a price competition market in the context of a firm's choice between engaging in price competition and holding a price leader. Focusing on a market for homogenous products, we demonstrate that if supply is tight relative to demand, and if the cost differential between the firms is reasonably large, a price competition market is formed non-cooperatively. Either if supply is not sufficiently tight or if the cost differential is not sufficiently large, a market with a price leader is formed.

This study develops a two-stage game in which duopolistic firms non-cooperatively form a particular market organization in the first stage and set their respective prices in the second stage to sell their products. In building the first-stage game, we follow Hamilton and Slutsky (1990) by assuming that a Stackelberg price leader market will be formed if a leader-follower role assignment can be made non-cooperatively in the first-stage game. If not, a price competition market will be formed. In order to describe price competition in the second-stage game, we adopt Bertrand-Chamberlin competition, in which each firm is assumed to sell as much as the existing demand at the price that it sets (Bertrand, 1883, and Chamberlin, 1933).[1]

This study reveals that if supply is tight relative to demand, under Chamberlin's demand specification, a sharp conflict of interest exists between

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[1]Edgeworth (1897) develops an alternative specification under which a firm can sell any amount not exceeding the market demand. See, for example, Kreps and Scheinkman (1983), Allen and Hellwig (1986), and Davidson and Deneckere (1986) for modern treatments of Bertrand-Edgeworth price competition. For more recent treatments of price competition, see Yano (2005), who describes a contestable market as a Nash equilibrium, and Yano (2008), who builds a core theoretic approach to price competition.

duopolistic firms in their choice of a price leader. As the cost differential between duopolistic firms becomes larger, this conflict of interest becomes more severe and reaches a point at which the conflict of interest is too large for either firm to accept the role of a price follower. In this case, firms end up with price competition; i.e., the price competition market is formed as a Nash equilibrium in our market organization game.

If, in contrast, the cost differential becomes sufficiently small, the conflict of interest becomes ignorable relative to the profit losses from engaging in price competition. In this case, either firm would not object to serve as a price follower so long as its competitor serves as the price leader. In this case, multiple equilibrium market organizations emerge in which the cost superior firm serves as the leader and in which the cost inferior firm serves as the leader.

If the cost differential is too small to induce price competition and too large to make a price leader indeterminate, the conflict of interest becomes non-ignorable only for the cost superior firm. In that case, the cost inferior firm is still willing to take the follower's role, thereby forming a market in which the cost superior firm serves as the price leader.

The case in which supply is not tight relative to demand corresponds to the cases studied by Ono (1978) and Yano and Komatsubara (2006). In those cases, the interest of the firms are coordinated in such a way that both firms agree on the choice of a price leader. In that case, as those studies show, the cost superior firm tends to be chosen as the price leader through the cooperation of the firms.

One important contribution of the present study is to demonstrate that a Bertrand-like price competition market may be formed in a non-cooperative fashion through the role choices of firms. In the existing literature, such a result has not been known except Bertrand's original argument, in which firms with identical and constant average costs are likely to end up with price competition. Since then, however, the endogenous formation of a price competition market has scarcely been studied in the existing literature.

This void may be attributable to the difficulty in describing Bertrand-like price competition in a simple manner as a pure strategy equilibrium in a model with increasing marginal cost curves. In order to overcome this difficulty, we rely on Chamberlin's demand specification (Chamberlin, 1933), under which a firm sells as much as the demand existing in the market at the price at which that firm sets.[2] In the modern literature, since Bulow, Geanakoplos and Klemperer (1985), Dixon (1990), and Vives (1990), an

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[2]See Vives (1999) for a textbook explaining Bertrand equilibria under Edgeworth's and Chamberlin's specifications.

increasing number of studies have recognized the importance of this specification for the analysis of price competition (see Dastidar (1995, 1997, 2001), Ray Chaudhuri (1996), Wambach (1999), Novshek and Roy Chowdhury (2003), Roy Chowdhury and Sengupta (2004), Roy Chowdhury (2009), Bagh (2010), Boccard and Wauthy (2010)). In this paper, we adopt, in particular, the specification of Dastidar (1995) in characterizing a price competition equilibrium by pure strategies.

It has not been known in the existing literature that a price competition market may be supported as a Nash equilibrium in the Hamilon-Slutsky game, which has commonly been adopted to explain the endogenous formation of a price leader.[3] Deneckere and Kovenock (1992) develop another framework, which is followed by Ishibashi (2008). Yano (2001), Tasnádi (2003), Komatsubara (2008), and Hirata and Matsumura (2011) incorporate Bertrand-Edgeworth price competition into the Hamilton-Slutsky game. Yano and Komatsubara (2006) incorporate Bertrand-Chamberlin price competition. Moreover, Amir and Stepanova (2006) and van Damme and Hurkens (2004) incorporate Bertrand competition for differentiated products. Those studies show that results depend critically on model specifications.

The organization of this paper is as follows: In Section 2, we construct the basic model. In Section 3, we explain why a price competition market may be formed endogenously. In Section 4, we explain the endogenous formation of a price competition market by means of the tightness of the market and the cost differential between the firms. In Section 5, we study different market organizations in terms of consumers' welfare.

2 Market Organization Game

In the present study, we address the endogenous formation of a price competition market in the context of a choice between engaging in price competition and holding a price leader. Toward this end, we adopt the Hamilton-Slutsky framework for the first-stage game of forming a particular market organization and incorporate Bertrand-Chamberlin price competition, which takes place at the second stage. Think of the market for a homogenous product supplied by two firms S and I. Firm S can produce at a marginal cost cheaper than (or at the same marginal cost as) firm I. Denote as $C_i(y)$ the total cost function of firm $i = S, I$. Assume $C_i(0) = 0$, $C_i' > 0$, $C_i'' > 0$, and

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[3]See Ono (1978), who studies the endogenous formation of a price leader in a cooperative framework.

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$C'_S(y) \leq C'_I(y)$ for all y. Call S the cost superior firm (low cost firm) and I the cost inferior firm (high cost firm).

Assume that, as an outcome of the first-stage game, three different types of market organizations can be formed in the second-stage game: (i) A Stackelberg superior price leader market, in which S acts as the price leader; (ii) a Stackelberg inferior price leader market, in which I acts as the price leader; and (iii) a Bertrand-Chamberlin price competition market, in which S and I engage in price competition.

In the first-stage game, we assume that each firm $(i = S, I)$ can choose either to act as the price leader (strategy $x_i = L$) or to act as the price follower (strategy $x_i = F$) in the second-stage product market. If the two firms choose mutually different roles, each firm will play in the second stage the exact role it chooses. That is, S will act as the price leader if $(x_S, x_I) = (L, F)$; I will act as the price leader if $(x_S, x_I) = (F, L)$. If they choose the same role as each other (i.e., if either $(x_S, x_I) = (L, L)$ or (F, F)), they will engage in price competition.

We focus on the market that satisfies Chamberlin's specification, i.e., each firm is committed to satisfy the entire demand that it faces at the price that it sets. In describing the demand side of this market, we follow Dastidar (1995) by assuming that if a firm sets its price equal to its competitor's price, each firm receives one half of the market demand and sells that amount (also see Yano, 2001). In the case in which a firm sets its price above its competitor's price, it can sell none. In the case in which a firm undercuts its competitor's price, under Chamberlin's specification, that firm must sell as much as the market demand. Assume that a firm matches its competitor's price either if it can make the same profit by matching its competitor's price as by undercutting that price, or if it makes the same profit by matching its competitor's price as by raising its price above the competitor's price.

If, as is well known, a firm is to undercut its competitor's price, there may be no optimal price for that firm; the closer to its competitor's price a firm sets its price, the larger its profit. In order to describe optimal pricing, therefore, we assume that there is a minimum unit for pricing, ε; that is, a firm must choose its price from the set $\mathbb{P}^\varepsilon = \{p : p = n\varepsilon,\, n \in \mathbb{N}\}$, where $\mathbb{N}$ is the set of natural numbers (including 0).

Let $D(p)$ be the market demand curve, defined on $\mathbb{R}_+$ (the set of non-negative real numbers). Assume that the demand curve is downward sloping $(D' < 0)$, that the marginal revenue is decreasing in the quantity to be sold, and that $\lim_{p \to \infty} D(p) = 0$ and $\lim_{p \to 0} pD(p) = 0$. Let $\bar{p}$ be the smallest upperbound for p such that $D(p) > 0$; if $D(p) > 0$ for all $p > 0$, denote $\bar{p} = \infty$.

In order to analyze price competition, it is useful to write down each

firm's profit as a function of the market price. Think of the following profit functions:

$$\pi_i^H(p) = \frac{1}{2}pD(p) - C_i\left(\frac{1}{2}D(p)\right); \tag{1}$$

$$\pi_i^M(p;\delta) = (p - \delta)D(p - \delta) - C_i\left(D(p - \delta)\right). \tag{2}$$

These functions imply that each firm must satisfy the demand that it faces at the price it sets. In (1), the firm is supposed to match its competitor's price, p. In (2), it is supposed to undercut its competitor's price, p, by δ. If δ is equal to the minimum pricing unit ($\delta = \varepsilon$), we denote $\pi_i^M(p) = \pi_i^M(p;\varepsilon)$.

In Figure 1, the dotted curve, π_i^H, illustrates that of $\pi_i^H(p)$; the solid curve, π_i^M, illustrates that of $\pi_i^M(p)$. (In this figure, $\bar{p} < \infty$ is assumed.) Under Chamberlin-Dastidar's specification, as the next lemma shows, both profit functions, π_i^H and π_i^M, are unimodal in price.

Lemma 1 *There are $\hat{p}_i^{H^o} \in \mathbb{R}_+$ and $\hat{p}_i^{M^o} \in \mathbb{R}_+$ such that $\pi_i^H(p)$ and $\pi_i^M(p;0)$, respectively, are monotone increasing in $n \in \mathbb{N}$ if $p < \hat{p}_i^{H^o}$ and $p < \hat{p}_i^{M^o}$ and are monotone decreasing in $n \in \mathbb{N}$ if $p > \hat{p}_i^{H^o}$ and $p > \hat{p}_i^{M^o}$.*

Proof. Since we assume that the marginal revenue, $d(pD)/dD$, is decreasing in D, there is a unique local maximum either for $\pi_i^H(p)$ or for $\pi_i^M(p;0)$. This implies the lemma. ∎

3 Endogenous Price Competition

In this section, we will demonstrate that, under certain conditions, a price competition market may be formed endogenously as a Nash equilibrium in the first-stage game. This may be explained by the existence of a fundamental conflict of interest, which prompts both firms to insist on acting as the price leader, thereby ending up with price competition.

At the outset, we define several critical price levels. First, denote as $\tilde{p}_i^k$ ($k = M, H$) the minimum at which the profit, π_i^k, is non-negative. That is,

$$\tilde{p}_i^k = \min\{p \in \mathbb{P}^\varepsilon : \pi_i^k(p) \geq 0\}. \tag{3}$$

Next, denote as $\hat{p}_i^k$ the price maximizing the profit function $\pi_i^k(p)$. In the case in which $\max_{p \in \mathbb{P}^\varepsilon} \pi_i^k(p) = \pi_i^k(p_i) = \pi_i^k(p_i + \varepsilon)$, we set $\hat{p}_i^k$ to be the smaller of p_i and $p_i + \varepsilon$. That is,

$$\hat{p}_i^k = \min\{\arg\max_{p \in \mathbb{P}^\varepsilon} \pi_i^k(p)\}. \tag{4}$$

Finally, denote as p_i^{**} the price at the intersection between these two curves, π_i^M and π_i^H. Then, p_i^{**} satisfies

$$p_i^{**} = \max\{p_i \in \mathbb{P}^\varepsilon : \pi_i^H(p_i) \geq \pi_i^M(p_i)\}. \tag{5}$$

Since we assume $C_I'(y) \geq C_S'(y)$, it holds that

$$\tilde{p}_I^H \geq \tilde{p}_S^H, \ \hat{p}_I^M \geq \hat{p}_S^M, \ \text{and} \ p_I^{**} \geq p_S^{**}. \tag{6}$$

Critical values $\tilde{p}_i^k$, $\hat{p}_i^k$, and p_i^{**} are chosen from $\mathbb{P}^\varepsilon$. Since, by Lemma 1, all profit curves are unimodal, as Figure 1 shows, $\tilde{p}_i^k$ approximates the lower horizontal intercept of curve π_i^k, that $\hat{p}_i^k$ approximates the peak of curve π_i^k, and that p_i^{**} approximates the intersection between curves π_i^H and π_i^M.

Figure 1 illustrates the case in which a price competition market is formed. This figure captures the case in which the following is satisfied:

$$\text{(i)} \quad \hat{p}_S^H < \tilde{p}_I^H; \tag{7}$$

$$\text{(ii)} \quad \tilde{p}_I^H < p_S^{**}; \tag{8}$$

$$\text{(iii)} \quad p_S^{**} < \hat{p}_I^H. \tag{9}$$

The result in this section is based on (7), (8), and (9). In the subsequent sections, we will demonstrate that these conditions can be satisfied in a standard model setting.

The next lemma characterizes the set of Bertrand-Chamberlin prices; a Bertrand-Chamberlin price is defined as the price that consumers pay in a Bertrand-Chamberlin equilibrium.

Lemma 2 *Suppose that condition* (8) *is satisfied. Define*

$$Q^B = \{p : \tilde{p}_I^H \leq p \leq p_S^{**}\}. \tag{10}$$

Then, Q^B *is the set of Bertrand-Chamberlin prices.*

Proof. Suppose that both firms are setting their respective prices at a p between $\tilde{p}_I^H$ and p_S^{**}; this is possible by (8). Then, both firms are making positive profits, which are shown by curves π_S^H and π_I^H, respectively, at p in Figure 1. For either firm, therefore, it is not optimal to raise its price above p. Suppose that firm S were to undercut I's price by $\delta = n\varepsilon$, $n \geq 1$. Since curve π_S^M lies below curve π_S^H on interval $\tilde{p}_I^H < p < p_S^{**}$, S's profit would fall to $\pi_S^M(p; \delta)$. Thus, undercutting I's price is not optimal for S. If firm I were to undercut S's price by $\delta = n\varepsilon$, $n \geq 1$, it would reduce its profit

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since $p < p_I^{**}$. Then, p is a Bertrand-Chamberlin price. It is clear from these arguments, $\tilde{p}_I^H$ and p_S^{**} are also Bertrand-Chamberlin prices.

In order to complete the proof, suppose that there is $p \notin Q^B$ that is a Bertrand-Chamberlin price. Let $p \leq \tilde{p}_I^H - \varepsilon$. There is no incentive for I to sell products in this price range. Thus, S sells at p. It is optimal for S to match I's price whereas I sets its price above S's price so as to avoid selling at a loss. Thus, p cannot be a Bertrand-Chamberlin price. Thus, $p \geq p_S^{**} + \varepsilon$. Since consumers do not pay a price $p \geq \bar{p}$ (or they buy none), $p_S^{**} + \varepsilon \leq p \leq \bar{p} - \varepsilon$. In this price range, it is optimal for S to undercut I's price whereas it is optimal for I either to match or to undercut S's price; see Figure 1. Thus, $p \geq p_S^{**} + \varepsilon$ cannot be a Bertrand-Chamberlin price. ∎

The next lemma characterizes the equilibrium that holds in the superior price leader market.

Lemma 3 *Suppose that conditions (7) and (8) are satisfied. If firm S acts as the price leader, it will set its price at $\tilde{p}_I^H$.*

Proof. Suppose that S sets its price p_S above p_I^{**}. Since, as is shown in Figure 1, it is optimal for I to undercut its competitor's price, S will obtain no profit. Suppose that S sets its price p_S between, or at one of, $\tilde{p}_I^H$ and p_I^{**}. Since it is optimal for I to match S's price, S will obtain a profit $\pi_S^H(p_S)$. Suppose, finally, that S sets its price p_S below $\tilde{p}_I^H$. Since it is optimal for firm I to set its price above S's price, p_S, S will obtain a profit $\pi_S^M(p_S; 0)$, which captures S's profit if I raises its price above p_S. As is shown in Figure 1, by (8), $\pi_S^H(\tilde{p}_I^H) > \pi_S^M(p_S; 0)$ for any $p_S < \tilde{p}_I^H$. Moreover, by (7), $\pi_S^H(\tilde{p}_I^H) > \pi_S^H(p_S)$ for any p_S such that $\tilde{p}_I^H < p_S \leq p_I^{**}$. Thus, $\tilde{p}_I^H$ is the price that S chooses as the price leader. ∎

Finally, the next lemma characterizes the equilibrium that holds in the inferior price leader market.

Lemma 4 *Suppose that conditions (8) and (9) are satisfied. If firm I acts as the price leader, it will set its price at p_S^{**}.*

Proof. Suppose that I sets its price p_I above p_S^{**}. As is shown in Figure 1, it is optimal for S to undercut I's price, in which case I will have no profit. Suppose that I sets its price below $\tilde{p}_I^H$. Since it is optimal for S either to match I's price (in the case of $p_I \geq \tilde{p}_S^H$) or to set its price above I's price (in the case of $p_I < \tilde{p}_S^H$). In either case, I's profit will be negative because the graphs of functions $\pi_I^H(p)$ and $\pi_I^M(p; 0)$ lie below the horizontal axis for $p < \tilde{p}_I^H$. Finally, suppose that I sets its price in $\tilde{p}_I^H \leq p_I \leq p_S^{**}$, which is

non-empty by (8). In that case, S will match I's price, and I will make a positive profit, since curve π_I^H lies above the horizontal line. Since curve π_I^H is increasing on $\tilde{p}_I^H \leq p_I \leq p_S^{**}$ by (9), it is optimal for I to set its price at p_S^{**}. ∎

We are now able to show that a price competition market may be formed in our market organization game, in which both firms insist to be price leaders. Note that there are multiple Bertrand-Chamberlin equilibria, Q^B, as Lemma 2 shows. In order to determine the first-stage game, it is necessary to assume that each firm has an expectation on the realization of an equilibrium price in the case in which the price competition market will be formed in the second stage. Our first main result below shows that, under a certain condition, the price competition market will be formed regardless of probability distributions on Q^B that the firms may have.

Denote as $\Pi_i^{L^j}$ firm i's profit in the case in which firm j acts as the price leader. Then, by Lemmas 3 and 4,

$$\Pi_i^{L^S} = \pi_i^H(\tilde{p}_I^H) \text{ and } \Pi_i^{L^I} = \pi_i^H(p_S^{**}). \tag{11}$$

Our first main result is as follows:

Theorem 1 *In the case in which conditions* (7), (8), *and* (9) *are satisfied,* $(x_S, x_I) = (L, L)$ *is a Nash equilibrium in the market organization game (so that a Bertrand-Chamberlin price competition market is formed in the second stage). This is a unique equilibrium market organization unless the firms believe, for certain, that the price at an end point of* Q^B, $\tilde{p}_I^H$ *or* p_S^{**}, *will hold under price competition.*

Proof. By Lemma 1, (10) and (11) imply that for any $p \in Q^B$, $\pi_i^H(p) \geq \Pi_i^{L^j}$ for $\{i, j\} = \{S, I\}$. This implies that if firm j chooses to be the leader, $x_j = L$, firm i will be at least as well off by attempting to be the leader, $x_i = L$ (in which case price competition takes place so that i's profit will be $\pi_i^H(p)$), as by accepting to be the follower, $x_i = F$ (in which case i will obtain $\Pi_i^{L^j}$). Thus, $(x_S, x_I) = (L, L)$ is a Nash equilibrium.

If i believes that an interior Bertrand-Chamberlin price, $p_i \in \text{int} Q^B$, it holds that

$$\Pi_S^{L^S} > \pi_S^H(p_i) > \Pi_S^{L^I} \text{ and } \Pi_I^{L^I} > \pi_I^H(p_i) > \Pi_I^{L^S}. \tag{12}$$

This shows that $(x_i, x_j) = (L, F)$ gives a higher profit to i than $(x_i, x_j) = (F, F)$ and that $(x_i, x_j) = (L, L)$ gives a higher profit to i than $(x_i, x_j) =$

(F, L). Neither $(x_i, x_j) = (F, F)$ nor $(x_i, x_j) = (F, L)$ can be a Nash equilibrium. This implies that $(x_i, x_j) = (L, L)$ is the only Nash equilibrium unless each firm believes for certain that the price at an end point of Q^B, $\tilde{p}_I^H$ or p_S^{**} will hold under price competition. ∎

Theorem 1 shows the possibility that an endogenous economic factor generates price competition. This possibility may be explained by a sharp conflict of interest in the assignment of a price leader, which prompts both firms to try to be the price leader than the price follower; this is because, by (12),

$$\Pi_S^{L^S} > \Pi_S^{L^I} \text{ and } \Pi_I^{L^I} > \Pi_I^{L^S}. \tag{13}$$

The existence of this conflict of interest has not been known in the existing literature. Instead, it has been known that the two firms' interests can be coordinated in such a way that both firms prefer the cost superior firm's acting as the price leader to the cost inferior firm's doing so. That is, it may hold that

$$\Pi_S^{L^S} > \Pi_S^{L^I} \text{ and } \Pi_I^{L^S} > \Pi_I^{L^I}. \tag{14}$$

Demonstrating this fact, Ono (1978) shows that if the two firms can cooperate in assigning the price leader, they choose the cost superior firm, S, to be the price leader.

Yano and Komatsubara (2006) reexamine Ono's result in a non-cooperative setting and construct a model in which the profits of firms in the Bertrand-Chamberlin equilibria lie at the ends of inequalities (14), i.e., for any $p \in \text{int} Q^B$, it holds that

$$\Pi_S^{L^S} > \Pi_S^{L^I} > \pi_S^H(p) \text{ and } \Pi_I^{L^S} > \Pi_I^{L^I} > \pi_I^H(p). \tag{15}$$

This relationship shows that a superior price leader market may be formed not only cooperatively but also non-cooperatively. (In this case, as Yano and Komatsubara (2006) show, multiple equilibria exist in which either an inferior price leader market or a superior price leader market is a Nash equilibrium.)

4   Determinants of Price Competition

The analysis below identifies the tightness of a market and the cost differential between the two firms as the main determinants for an equilibrium market organization. The main findings are as follows:

(1) The tighter the market (or, the more expensive it is to produce the product relative to the demand), the more likely the conflict of interest is created in the assignment of a price leader (Theorem 2).

(2) The larger the cost differential between the firms, the more severe the conflict of interest becomes so that both firms insist to act as the price leader; in that case, price competition takes place (Theorem 3.1).

(3) The smaller the cost differential, the less severe the conflict of interest becomes, in which case both firms become more willing to take the follower's role. If the conflict is ignorable, either firm is willing to act as the price follower, provided that the other firm desires to act as the leader. In that case, either superior or inferior price leader market is an equilibrium market organization (multiple equilibria, Theorem 3.3). If the conflict is less severe, only the cost inferior firm is willing to act as the follower, in which case the superior price leader market is formed (Theorem 3.2).

4.1 A Linear Model

We capture the tightness of a market and the cost differential between the firms in a simple linear model. Assume that the market demand function is $D(p) = (\bar{p} - p)/b$. The marginal cost functions of the firms are $C'_j(y) = \alpha_j cy$, $j = S, I$, satisfying that $\alpha_I = \alpha \geq 1$ and $\alpha_S = 1$. In this setting, parameter α represents the cost differential between firms I and S. Parameter c controls the slopes of the firms' marginal cost curves, representing the technological levels. Parameter b is the slope of the consumers' marginal willingness to pay curve.

In this setting, the tightness of a market is captured by the ratio of these slopes, $t = c/b$, which represents the relative size of demand to supply. Either if demand is large (or b is small) or if supply is small (or c is large), tightness t is larger.

By definition, p_i^{**}, $\tilde{p}_i^H$, $\hat{p}_i^H$, and $\hat{p}_i^M$ all depend on ε. In order to characterize the equilibrium market organization by means of α and t, it is useful to deal with the limits of these critical values as $\varepsilon \to 0$. By denoting $\mathbb{P}^0 = \mathbb{R}_+$, those limits are determined by (3), (4), and (5) with $\varepsilon = 0$;

$$p_i^* = \frac{3\alpha_i c\bar{p}}{4b + 3\alpha_i c}; \ \tilde{p}_i^{Ho} = \frac{\alpha_i c\bar{p}}{4b + \alpha_i c}; \ \hat{p}_i^{Ho} = \frac{(2b + \alpha_i c)\bar{p}}{4b + \alpha_i c}; \ \hat{p}_i^{Mo} = \frac{(b + \alpha_i c)\bar{p}}{2b + \alpha_i c}. \qquad (16)$$

By (16), relationships (7), (8), and (9) can be transformed as follows:

(i) $\quad \hat{p}_S^{Ho} < \tilde{p}_I^{Ho} \Longleftrightarrow \alpha > 2 + 4/t,$
$\qquad\qquad$ in which case $\hat{p}_S^H < \tilde{p}_I^H$ for any small $\varepsilon > 0$. $\qquad (17)$

(ii) $\quad \tilde{p}_I^{Ho} < p_S^* \Longleftrightarrow \alpha < 3,$
$\qquad\qquad$ in which case $\tilde{p}_I^H < p_S^{**}$ for any small $\varepsilon > 0$. $\qquad (18)$

$$\text{(iii)} \quad p_S^* < \hat{p}_I^{H^o} \iff \alpha > 3/2 - 2/t,$$
$$\text{in which case } p_S^{**} < \hat{p}_I^H \text{ for any small } \varepsilon > 0. \qquad (19)$$

The next corollary shows that a price competition market can be formed with standard demand and marginal cost curves.

Corollary 1 *For any given α and t satisfying*

$$2 + 4/t < \alpha < 3, \qquad (20)$$

there is $\varepsilon' > 0$ such that if $0 < \varepsilon < \varepsilon'$, $(x_S, x_I) = (L, L)$ is a unique Nash equilibrium in the market organization game (so that a Bertrand-Chamberlin price competition market is formed in the second stage).

Proof. Given (20), by (17), (18) and (19), there is $\varepsilon' > 0$ such that if $0 < \varepsilon < \varepsilon'$, (7), (8) and (9). Thus, the corollary follows from Theorem 1. ∎

The next lemma shows that if $\alpha > 3$, a trivial case arises in which the cost inferior firm can never obtain a positive profit in a Bertrand-Chamberlin equilibrium.

Lemma 5 *Let $\alpha > 3$ and*

$$\bar{Q}^B = \{p : p_S^{**} \leq p \leq \min\{\hat{p}_S^M, \tilde{p}_I^H\} - \varepsilon\}. \qquad (21)$$

Then, there is $\varepsilon' > 0$ such that if $0 < \varepsilon < \varepsilon'$, $\bar{Q}^B$ is the set of Bertrand-Chamberlin prices. In any Bertrand-Chamberlin equilibrium, I's profit is zero.

Proof. Given $\alpha > 3$, by (18), there is $\varepsilon' > 0$ such that $0 < \varepsilon < \varepsilon'$ implies $p_S^{**} < \tilde{p}_I^H$. Since $p_S^{**} < \hat{p}_S^M$, the interval in (21) is well defined. Let $p \in \bar{Q}^B$ and $0 < \varepsilon < \varepsilon'$. Since $p_S^{**} < \tilde{p}_I^H$, by $p \in \bar{Q}^B$, ε' can be chosen in such a way that $\pi_I^H(p) < 0$, $\pi_I^M(p) < 0$, and $\pi_S^M(p + \varepsilon) > \pi_S^H(p + \varepsilon)$. Since $p \leq \hat{p}_S^M - \varepsilon$ by $p \in \bar{Q}^B$, this implies that it is optimal for S to set $p_S = p$ and for I to set $p_I = p + \varepsilon$. Thus, p is a Bertrand-Chamberlin price, in which I's profit is zero.

In order to complete the proof, suppose that there is $p \notin \bar{Q}^B$ that is a Bertrand-Chamberlin price. Let $p \leq p_S^{**} - \varepsilon$. Then, $\pi_I^H(p) < 0$ and $\pi_I^M(p) < 0$. Thus, there is no incentive for I to sell products. Thus, S sells at p. Since, however, it is optimal for S to match I's price, p cannot be a Bertrand-Chamberlin price. Thus, $p \geq \min\{\hat{p}_S^M, \tilde{p}_I^H\}$. Since consumers do not pay a

price $p \geq \bar{p}$ (or they buy none), $\min\{\hat{p}_S^M, \tilde{p}_I^H\} \leq p \leq \bar{p} - \varepsilon$. In this price range, it is optimal for S to undercut I's price whereas it is optimal for I either to match S's price or to undercut S's price. Thus, $p \geq \min\{\hat{p}_S^M, \tilde{p}_I^H\}$ cannot be a Bertrand-Chamberlin price. ∎

If $\alpha > 3$, as Lemma 5 shows, the cost inferior firm has no incentive to participate in the price competition market, for it could receive no profit in a Bertrand-Chamberlin equilibrium. In what follows, we exclude this trivial case by focusing on $1 \leq \alpha < 3$.

4.2 Market Tightness

In what follows, we say that a supply is tight if $t > 4$ (and loose if $t < 4$). We demonstrate that market tightness in this sense is a necessary and sufficient condition for the existence of a conflict of interest in the assignment of a price leader. This conflict of interest is a necessary condition for the endogenous formation of a price competition market.

Note the following:

$$\text{(iv)} \qquad \hat{p}_S^{Ho} < p_S^* \iff t > 4,$$
$$\text{in which case } \hat{p}_S^H < p_S^{**} \text{ for any small } \varepsilon > 0. \qquad (22)$$

If $t < 4$, the inequalities between $\hat{p}_S^H$ and p_S^{**} and between $\hat{p}_S^{Ho}$ and p_S^* change their directions.

Denote as q^{L^S} and q^{L^I}, respectively, the Stackelberg prices in the superior price leader market and the inferior price leader market. The next lemma characterizes these prices.

Lemma 6 *Given α and t, there is $\varepsilon' > 0$ such that $0 < \varepsilon < \varepsilon'$ implies the following:*
1. In the case of $t > 4$,

$$(q^{L^S}, q^{L^I}) = \begin{cases} (\tilde{p}_I^H, p_S^{**}) & \text{if} \quad 2 + 4/t < \alpha < 3 \\ (\hat{p}_S^H, p_S^{**}) & \text{if} \quad 3/2 - 2/t < \alpha < 2 + 4/t \\ (\hat{p}_S^H, \hat{p}_I^H) & \text{if} \quad 1 \leq \alpha < 3/2 - 2/t. \end{cases} \qquad (23)$$

2. In the case of $t < 4$,

$$(q^{L^S}, q^{L^I}) = \begin{cases} (\hat{p}_S^H, p_S^{**}) & \text{if} \quad 2/3 + 4/(3t) < \alpha < 3 \\ (p_I^{**}, p_S^{**}) & \text{if} \quad 1 \leq \alpha < 2/3 + 4/(3t). \end{cases} \qquad (24)$$

Proof. First, let $t > 4$. Under (20), as is shown in the proof of Corollary 1, (7), (8), and (9) hold. Thus, the top expression of (23) follows from Lemmas 3 and 4.

Let $\alpha < 2 + 4/t$. Then, by (17), it is possible to choose ε' sufficiently small so that $0 < \varepsilon < \varepsilon'$ implies $\tilde{p}_I^H < \hat{p}_S^H$; Figure 2 illustrates this case. If S sets its price p_S in price range $\tilde{p}_I^H \leq p_S \leq p_I^{**}$, I will match p_S, in which case S's profit will be $\pi_S^H(p) > 0$. If S sets its price outside of that price range, its profit will be either zero or smaller than or equal to $\pi_S^M(\tilde{p}_I^H)$. Since $\tilde{p}_I^H < \hat{p}_S^H < p_I^{**}$, and since $\pi_S^M(\tilde{p}_I^H) < \pi_S^H(\hat{p}_S^H)$, therefore, it is optimal for S to set its price at $p_S = \hat{p}_S^H$. Thus, the middle and third expressions of (23) hold for S.

Let $1 \leq \alpha < 3/2 - 2/t$. Then, by (19), it is possible to choose ε' so that $0 < \varepsilon < \varepsilon'$ implies $\hat{p}_I^H < p_S^{**}$. Figure 3 illustrates this case. If I sets its price p_I in price range $\tilde{p}_I^H \leq p_I \leq p_S^{**}$, S will match p_I, in which case I's profit will be $\pi_I^H(p_I) \geq 0$. If I sets its price outside of this price range, its profit will be either negative or zero. Since $\tilde{p}_I^H < \hat{p}_I^H < p_S^{**}$, therefore, it is optimal for I to set its price at $p_I = \hat{p}_I^H$. Thus, the bottom expression of (23) holds for I. Let $3/2 - 2/t < \alpha < 2 + 4/t$. Then, by (18) and (19), there is $\varepsilon' > 0$ such that $0 < \varepsilon < \varepsilon'$ implies (8) and (9). Thus, Lemma 4 implies the middle expression of (23) for I.

Let $t < 4$. Then, by (22), it is possible to choose ε' so that $0 < \varepsilon < \varepsilon'$ implies $p_S^{**} < \hat{p}_S^H$. If I sets its price p_I in price range $\tilde{p}_I^H \leq p_I \leq p_S^{**}$, S matches p_I, in which case I's profit is $\pi_I^H(p)$. If, instead, I sets its price outside of this price range, its price is negative or zero. Since $p_S^{**} < \hat{p}_S^H < \hat{p}_I^H$, π_I^H is increasing in the price range. Thus, it is optimal for I to set its price at $p_I = p_S^{**}$. Thus, (24) holds for I.

If S sets its price p_S in price range $\tilde{p}_I^H \leq p_S \leq p_I^{**}$, I will match p_S, in which case S's profit will be $\pi_S^H(p)$. If S sets its price outside of this price range, its profit will be either zero or smaller than or equal to $\pi_S^M(\tilde{p}_I^H)$. Thus, it is optimal for S to set its price at $p_S = p_I^{**}$ if p_I^{**} lies on the increasing part of $\pi_S^H(p)$ and $p_S = \hat{p}_S^H$ if p_I^{**} lies on the decreasing part. Note that

$$\text{(v)} \quad p_I^* < \hat{p}_S^{Ho} \iff \alpha < 2/3 + 4/(3t),$$
$$\text{in which case } p_I^{**} < \hat{p}_S^H \text{ for any small } \varepsilon > 0. \quad (25)$$

Thus, the first and second expressions of (24) hold for S. ∎

Recall that $\Pi_i^{L^j}$ is the profit of firm i in the case in which j acts as the price leader. The next theorem characterizes the conflict of interest in the assignment of a price leader.

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Theorem 2 *Given $1 < \alpha < 3$ and $t > 0$, there is $\varepsilon' > 0$ such that $0 < \varepsilon < \varepsilon'$ implies the following:*

$$\Pi_S^{L^S} > \Pi_S^{L^I} \ \ and \ \Pi_I^{L^I} > \Pi_I^{L^S} \qquad if \quad t > 4. \tag{26}$$

$$\Pi_S^{L^S} > \Pi_S^{L^I} \ \ and \ \Pi_I^{L^S} > \Pi_I^{L^I} \qquad if \quad t < 4. \tag{27}$$

Proof. In order to prove (26), take the case of $t > 4$. Let $1 < \alpha < 3/2 - 2/t$. In this case, by (23), there is $\varepsilon' > 0$ such that if $0 < \varepsilon < \varepsilon'$, it holds that $(q^{L^S}, q^{L^I}) = (\hat{p}_S^H, \hat{p}_I^H)$. By definition, $\Pi_i^{L^j} = \pi_i^H(q^{L^j})$ and $\pi_i^H(\hat{p}_i^H) > \pi_i^H(\hat{p}_j^H)$. Thus, (26) holds. Next, let $2 + 4/t < \alpha < 3$. In this case, by (23), there is $\varepsilon' > 0$ such that if $0 < \varepsilon < \varepsilon'$, it holds that $(q^{L^S}, q^{L^I}) = (\tilde{p}_I^H, p_S^{**})$. Moreover, by (17), (18) and (19), $\varepsilon' > 0$ may be chosen in such a way that if $0 < \varepsilon < \varepsilon'$, it holds that $\hat{p}_S^H < \tilde{p}_I^H < p_S^{**} < \hat{p}_I^H$. This implies $\pi_S^H(\tilde{p}_I^H) > \pi_S^H(p_S^{**})$ and $\pi_I^H(p_S^{**}) > \pi_I^H(\tilde{p}_I^H)$; by Lemma 1, π_i^H is decreasing above $\hat{p}_i^{Ho}$ and is increasing below $\hat{p}_i^{Ho}$. Since, by definition, $\Pi_i^{L^j} = \pi_i^H(q^{L^j})$, (26) holds. Finally, let $3/2 - 2/t < \alpha < 2 + 4/t$. In this case, there is $\varepsilon' > 0$ such that if $0 < \varepsilon < \varepsilon'$, $(q^{L^S}, q^{L^I}) = (\hat{p}_S^H, p_S^{**})$ by (23), and $\hat{p}_S^H < p_S^{**} < \hat{p}_I^H$ by (22) and (19). Thus, for the same reason as above, $\varepsilon' > 0$ may be chosen in such a way that $0 < \varepsilon < \varepsilon'$ implies $\pi_S^H(\hat{p}_S^H) > \pi_S^H(p_S^{**})$ and $\pi_I^H(p_S^{**}) > \pi_I^H(\hat{p}_S^H)$. Since, by definition, $\Pi_i^{L^j} = \pi_i^H(q^{L^j})$, (26) holds.

In order to prove (27), take the case of $t < 4$. Let $2/3 + 4/(3t) < \alpha < 3$. By (24), there is $\varepsilon' > 0$ such that $0 < \varepsilon < \varepsilon'$ implies $(q^{L^S}, q^{L^I}) = (\hat{p}_S^H, p_S^{**})$. By (22), $\varepsilon' > 0$ may be chosen in such a way that if $0 < \varepsilon < \varepsilon'$, it holds that $\hat{p}_S^H > p_S^{**}$. Moreover, $\alpha > 1$ implies $\hat{p}_I^H > \hat{p}_S^H$. Hence, it holds that $\hat{p}_I^H > \hat{p}_S^H > p_S^{**}$. This implies $\pi_S^H(\hat{p}_S^H) > \pi_S^H(p_S^{**})$ and $\pi_I^H(\hat{p}_S^H) > \pi_I^H(p_S^{**})$; π_i^H is increasing below $\hat{p}_i^H$. Since, by definition, $\Pi_i^{L^j} = \pi_i^H(q^{L^j})$, (27) holds. Next, let $1 < \alpha < 2/3 + 4/(3t)$. By (24), there is $\varepsilon' > 0$ such that $0 < \varepsilon < \varepsilon'$ implies $(q^{L^S}, q^{L^I}) = (p_I^{**}, p_S^{**})$. By (25), $\varepsilon' > 0$ may be chosen in such a way that if $0 < \varepsilon < \varepsilon'$, $\hat{p}_S^H > p_I^{**}$. Moreover, $\alpha > 1$ implies $\hat{p}_I^H > \hat{p}_S^H$. Hence, it holds that $\hat{p}_I^H > \hat{p}_S^H > p_I^{**} > p_S^{**}$. This implies $\pi_S^H(p_I^{**}) > \pi_S^H(p_S^{**})$ and $\pi_I^H(p_I^{**}) > \pi_I^H(p_S^{**})$; π_i^H is increasing below $\hat{p}_i^H$. Since, by definition, $\Pi_i^{L^j} = \pi_i^H(q^{L^j})$, (27) holds.

The above discussion holds for $\alpha = 2 + 4/t$ and $3/2 - 2/t$ in the case of $t > 4$ and $2/3 + 4/(3t)$ in the case of $t < 4$. Thus, the lemma is proved. ∎

Theorem 2 can be summarized as follows:

Proposition 1 *The tightness of a market ($t > 4$) is a necessary and sufficient condition for the existence of a conflict of interest in the assignment of a price leader for a sufficiently small minimum trading unit $\varepsilon > 0$.*

4.3 Cost Differential

In this subsection, we relate the type of equilibrium market organization to the size of cost differential α. If $\alpha < 3$ is sufficiently large, a price competition market is formed. If $\alpha \geq 1$ is sufficiently small, both superior and inferior price leader markets can be formed in equilibrium. If α lies in a middle range, a superior price leader market is formed.

Fix $t > 4$, which guarantees the existence of a conflict of interest in the assignment of a price leader (Theorem 2). Note that if $\alpha = 1$, S and I have identical cost functions. In that case, curves π_I^H and π_I^M are identical to curves π_S^H and π_S^M, respectively. As α increases, the former curves will shrink to their upper horizontal intercepts.

Assume that if the price competition market is to hold in the second period, each Bertrand-Chamberlin equilibrium is realized with an equal probability and that each firm maximizes the expected profit. Denote as Π_i^B firm i's expected equilibrium profit in the case in which a price competition market is formed. Since $p \in \mathbb{P}^\varepsilon$, by (10),

$$\Pi_i^B = E(\pi_i^H(p)|p \in Q^B) = \frac{1}{(p_S^{**} - \tilde{p}_I^H)/\varepsilon + 1} \sum_{n=0}^{(p_S^{**} - \tilde{p}_I^H)/\varepsilon} \pi_i^H(\tilde{p}_I^H + n\varepsilon). \quad (28)$$

In order completely to characterize the equilibrium market organization by means of parameters α and t, it is necessary to introduce critical values for parameters at which the nature of an equilibrium market organization changes. For that purpose, we need to deal with profit function $\pi_i^M(p; 0)$ rather than $\pi_i^M(p; \varepsilon) = \pi_i^M(p)$.

For this reason, in this section, think of profit curves π_i^M in Figures 1, 2, and 3 as the graphs of $\pi_i^M(p; 0)$. Moreover, think of critical values p_i^{**}, $\tilde{p}_i^k$, and $\hat{p}_i^k$ in Figures 1, 2, and 3 as p_i^*, $\tilde{p}_i^{ko}$, and $\hat{p}_i^{ko}$, respectively. Profit Π_i^B can be approximated by

$$\Pi_i^{Bo} = \frac{1}{p_S^* - \tilde{p}_I^{Ho}} \int_{\tilde{p}_I^{Ho}}^{p_S^*} \pi_i^H(p)dp, \quad (29)$$

which is the average height of curve π_i^H over the price range $Q^{Bo} = \{p : \tilde{p}_I^{Ho} \leq p \leq p_S^*\}$; note that function $\pi_i^H(p)$, as well as its graph π_i^H, is independent of ε.

If $\alpha < 3$ is close to 3, by (12) and Corollary 1, both firms prefer engaging in price competition to accepting the role of a follower, i.e., $\Pi_i^B > \Pi_i^{L^j}$ for α near 3. If, instead, $\alpha \geq 1$ is close to 1, as (23) shows, $q^{L^S} = \hat{p}_S^H \approx \hat{p}_I^H = q^{L^I}$. This implies $\Pi_i^{L^i} \approx \Pi_i^{L^j}$ Since, moreover, $\Pi_i^{L^i} = \pi_i^H(\hat{p}_i^H) \geq \pi_i^H(p_i)$ for any p_i,

and since $\hat{p}_i^H \in \text{int} Q^B$, $\Pi_i^B < \Pi_i^{L^i}$. These facts imply $\Pi_i^B < \Pi_i^{L^j}$ for α near 1. Thus, there is a cut-off α_i such that $\Pi_i^B > \Pi_i^{L^j}$ for $\alpha > \alpha_i$ and that $\Pi_i^B < \Pi_i^{L^j}$ for $\alpha < \alpha_i$.

For $i = I$, the lemma below approximates this α_i by α_I^o at which $\Pi_I^{B^o} = \pi_I^H(\hat{p}_S^{H^o})$, which is almost equal to $\Pi_I^{L^S}$. Figure 2 illustrates this case. (See Appendix A for a proof.)

Lemma 7 *Let $t > 4$ and*

$$\alpha_I^o = \frac{192 + 96t + 60t^2 + 24t^3 + \sqrt{6(16 + 8t + 3t^2)}(4+t)(4+3t)}{t(-16 + 40t + 23t^2)}. \tag{30}$$

Then, $\alpha_I^o < 2 + 4/t$. Moreover, given α and t, there is $\varepsilon' > 0$ such that if $0 < \varepsilon < \varepsilon'$, the following holds:

$$\Pi_I^B > \Pi_I^{L^S} \quad \text{if} \quad \alpha_I^o < \alpha < 3; \tag{31}$$
$$\Pi_I^B < \Pi_I^{L^S} \quad \text{if} \quad 1 \le \alpha < \alpha_I^o. \tag{32}$$

As (23) shows, $q^{L^I} = p_S^{**}$ in the case of $3/2 - 2/t < \alpha < 2 + 4/t$ and $q^{L^I} = \hat{p}_I^H$ in the case of $1 \le \alpha < 3/2 - 2/t$. For this reason, the other cut-off, α_S, is characterized by two values α_{S1}^o and α_{S2}^o such that, for the former case, $\Pi_S^{B^o} = \pi_S^H(p_S^*)$ and that, for the latter case, $\Pi_S^{B^o} = \pi_S^H(\hat{p}_I^{H^o})$. Figure 3 illustrates the latter case. The next lemma shows that the relationship between Π_S^B and $\Pi_S^{L^I}$ can be characterized by α_{S1}^o and α_{S2}^o. (See Appendix A for a proof.)

Lemma 8 *Let $t > 4$, $\alpha_{S1}^o = \frac{6(8 + 2t + t^2)}{(5t - 4)t}$, $a_{S2}^o = \frac{6t(t-4) + \sqrt{6(4+t)(4+3t)^3}}{2t(4+7t)}$, and*

$$\alpha_S^o = \max\{\alpha_{S1}^o, \alpha_{S2}^o\}. \tag{33}$$

Then, $\alpha_S^o < \alpha_I^o$. Moreover, given t and α, there is $\varepsilon' > 0$ such that if $0 < \varepsilon < \varepsilon'$, the following holds:

$$\Pi_S^B > \Pi_S^{L^I} \quad \text{if } \alpha_S^o < \alpha < 3; \tag{34}$$
$$\Pi_S^B < \Pi_S^{L^I} \quad \text{if } 1 \le \alpha < \alpha_S^o. \tag{35}$$

The next theorem characterizes the Nash equilibrium in our market organization game with respect to the cost differential between the firms, α, for the case of a tight market ($t > 4$).

Theorem 3 *Let $t > 4$. For each given α, there is $\varepsilon' > 0$ such that $0 < \varepsilon < \varepsilon'$ implies that the following holds:*

1. The price competition market with $(x_S, x_I) = (L, L)$ is the unique equilibrium market organization if $\alpha_I^o < \alpha < 3$, $\alpha \neq 2 + 4/t$.[4]

2. The superior price leader market, $(x_S, x_I) = (L, F)$, is the unique equilibrium market organization if $\alpha_S^o < \alpha < \alpha_I^o$.

3. The Stackelberg markets both with the superior price leader and with the inferior price leader, $(x_S, x_I) = (L, F)$ and (F, L), are equilibrium market organizations if $1 \leq \alpha < \alpha_S^o$.

Proof. Let $\alpha \neq 2 + 4/t$. It may be proved that $\Pi_i^{L^i} > \Pi_i^B$ for $i = I, S$. Since $\alpha_S^o < \alpha_I^o$ by Lemma 8, therefore, Theorem 2 and Lemmas 7 and 8 imply the following relationships:

$$\Pi_S^{L^S} > \Pi_S^B > \Pi_I^{L^I} \text{ and } \Pi_I^{L^I} > \Pi_I^B > \Pi_I^{L^S} \quad \text{if } \alpha_I^o < \alpha < 3; \quad (36)$$

$$\Pi_S^{L^S} > \Pi_S^B > \Pi_I^{L^I} \text{ and } \Pi_I^{L^I} > \Pi_I^{L^S} > \Pi_I^B \quad \text{if } \alpha_S^o < \alpha < \alpha_I^o; \quad (37)$$

$$\Pi_S^{L^S} > \Pi_S^{L^I} > \Pi_S^B \text{ and } \Pi_I^{L^I} > \Pi_I^{L^S} > \Pi_I^B \quad \text{if } 1 < \alpha < \alpha_S^o. \quad (38)$$

The theorem readily follows from (36), (37), and (38). ∎

The above result may be summarized as follows:

Proposition 2 *In a tight market, the following holds:*

1. Price competition takes place in the case in which the cost differential is sufficiently large, given that the cost inferior firm's incentive for participation is maintained ($\alpha < 3$).

2. If, in contrast, the cost differential is sufficiently small, a leader-follower relationship is formed, whichever firm might serve as the price leader.

3. In between these two cases, the cost superior firm acts as the price leader while the cost inferior firm acts as the price follower.

4.4 Complete Characterization

In this subsection, by using the above results, we will completely characterize the equilibrium market organization with respect to market tightness t and cost differential α (for the case of $1 \leq \alpha < 3$). This reveals that market tightness ($t > 4$) is a necessary condition for price competition. It also

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[4]It is possible to prove that this statement holds for $\alpha = 2 + 4/t$. It requires a very lengthy argument for proving this fact. For this reason, we simply exclude the case of $\alpha = 2 + 4/t$.

demonstrates that the larger cost differential α, the more severe this conflict of interest, and that the equilibrium market organization depends on the severity of the conflict of interest relative to the profit losses from engaging in price competition.

Figure 4 completely characterizes the equilibrium market organization. Line A illustrates $\alpha = 3$; the dotted curve B illustrates $\alpha = 2 + 4/t$; curves α_i^o illustrates the relationships between α_i^o and t, given by (30) and (33).

If $t > 4$ and $1 \leq \alpha < \alpha_S^o$, the conflict of interest in the assignment of a price leader is small. In this case, as Figure 3 shows, $q^{L^S} = \hat{p}_S^H$ and $q_I^{L^I} = \hat{p}_I^H$ are close to each other and lie in the interior of Q^B. This implies that the profit loss from accepting the follower's role, $\Pi_i^{L^i} - \Pi_i^{L^j}$, is small for either firm, $i = S, I$. If $\alpha < \alpha_S^o$, as (38) shows, those losses are wiped out by the profit losses from engaging in price competition, i.e., $\Pi_i^{L^i} - \Pi_i^{L^j} < \Pi_i^{L^i} - \Pi_i^B$. As a result, either firm can act as the price leader. Thus, both the superior price leader market and the inferior price leader market are equilibrium market organizations in the region below curve α_S^o and to the right of line $t = 4$ in Figure 4.

If $t > 4$ and $\alpha_S^o < \alpha < \alpha_I^o$, the conflict of interest in the assignment of a price leader becomes too large for the cost superior firm, S, to accept the follower's role. In Figure 2, $q^{L^S} = \hat{p}_S^H$ and $q_I^{L^I} = p_S^{**}$ become further apart from each other than in Figure 3. This creates a larger conflict of interest captured by $\Pi_S^{L^S} - \Pi_S^{L^I} = \pi_S^H(\hat{p}_S^H) - \pi_S^H(p_S^{**}) > 0$. By (37), for S, the conflict exceeds the profit losses from engaging in price competition, $\Pi_S^{L^S} - \Pi_S^{L^I} > \Pi_S^{L^S} - \Pi_S^B$. It is, however, not large enough for the cost inferior firm, I, to reject the follower's role; in other words, it remains that $\Pi_I^{L^I} - \Pi_I^{L^S} < \Pi_I^{L^I} - \Pi_I^B$. Thus, in the region between curves α_S^o and α_I^o in Figure 4, the superior price leader market is formed.

If $t > 4$ and $\alpha_I^o < \alpha < 3$, as (36) shows, the conflict of interest becomes too large even for I to accept the follower's role. In Figure 1, $q^{L^S} = \tilde{p}_I^H$ and $q^{L^I} = p_S^{**}$ become even more apart and lie at the ends of segment Q^B. In this case, for either firm, $i = S, I$, the conflict of interest outweighs the profit loss from engaging in price competition, $\Pi_i^{L^i} - \Pi_i^{L^j} > \Pi_i^{L^i} - \Pi_i^B$. Thus, in the region between curve α_I^o and line A, the price competition market is formed. This region is larger than that in Corollary 1, which is between curves A and B. This is because (20) gives the region in which each firm's profit in every price competition equilibrium is larger than in the Stackelberg equilibrium in which that firm acts as the follower, or, (12) holds. In the region between curve α_I^o and line A, in contrast, the firms' expected profits over all price competition equilibria are larger.

If $t < 4$ and $1 \leq \alpha < 3$, both the superior price leader market and the inferior price leader market are equilibrium market organizations. Because

this case was examined in detail by Yano and Komatsubara (2006), we refrain from a discussion.

The complete characterization above shows that the tightness of a market is a necessary condition for the formation of a price competition market.

Proposition 3 *The tightness of a market $(t > 4)$ is a necessary condition for the formation of a price competition market.*

5   Consumers and Market Originations

In this section, we briefly investigate consumers' preference over the three types of market organizations. The main result is that, in general, the equilibrium market organization is not what consumers desire to participate in. Since it can be explained in a manner similar to those in the previous section, we will present only the result.

Denote as CS^{L^S} and CS^{L^I}, respectively, the consumer surpluses in superior price leader and inferior price leader equilibria. Moreover, denote as $E\{CS^B\}$ the expected consumer surplus in a price competition market. The next theorem characterizes consumers' preference over the three types of market organizations. (See Appendix B for a proof.)

Theorem 4 *Let $t > 4$ and*

$$\alpha_C^o = \frac{192 - 36t^2 + 6t^3 + 4(16 + 16t + 3t^2)\sqrt{6t(2+t)}}{t(-16 + 40t + 23t^2)}.$$

Then, given α and t, there is $\varepsilon' > 0$ such that if $0 < \varepsilon < \varepsilon'$, the following holds:

$$CS^{L^S} > E\{CS^B\} > CS^{L^I} \ \ if \ \alpha_C^o < \alpha < 3; \tag{39}$$

$$E\{CS^B\} > CS^{L^S} > CS^{L^I} \ \ if \ 1 < \alpha < \alpha_C^o. \tag{40}$$

Given $t > 4$, it holds that

$$\alpha_S^o < \alpha_C^o < \alpha_I^o. \tag{41}$$

Recall that, by Theorem 3, the equilibrium market organization is of price competition if $\alpha_I^o < \alpha < 3$. Theorem 4 and (41) show, however, that consumers prefer the superior price leader market to the equilibrium market

organization. If $1 \leq \alpha < \alpha_S^o$, both superior and inferior price leader markets are in equilibrium. In this case, by Theorem 4 and (41), consumers prefer the price competition market.

There is a slight chance with which the market organization that consumers desire to participate in coincides with what the market chooses. This occurs in the case in which the superior price leader market is in equilibrium. That is, if $\alpha_C^o < \alpha < \alpha_I^o$, both the market organization that consumers desire to have and the equilibrium market organization are of superior price leadership. If, instead, $\alpha_S^o < \alpha < \alpha_C^o$, consumers prefer the price competition market to what the market chooses (the superior price leader market).

These findings may be summarized as follows:

Proposition 4 *In general, the equilibrium market organization is not what consumers desire to participate in. If, in particular, the price competition market is formed endogenously, consumers prefer the superior price leader market.*

5.1 Off-Equilibrium Process from Monopoly

The analysis above shows that consumers do not necessarily desire price competition among sellers. This result sharply contrasts with the common belief that price competition among sellers benefits consumers. This paradoxical result, however, holds only in the comparison between a price competition market and a Stackelberg price leader market.

In comparison with the monopolistic market, consumers are better off by participating in the market in which more than one firms operate non-cooperatively by either engaging in price competition or forming a leader/follower relationship. This may be explained by comparing the monopoly price with the price that would hold in the case in which an equilibrium market organization is formed. In our setting the monopoly price is $\hat{p}_i^M - \varepsilon$ in the case in which firm i monopolizes the market. Since the set of Bertrand-Chamberlin prices, Q^B, is given by (10), and $p_S^{**} < \hat{p}_S^M - \varepsilon$, it holds that

$$Q^B < \hat{p}_S^M - \varepsilon < \hat{p}_I^M - \varepsilon. \tag{42}$$

Both superior and inferior leader prices are also smaller than monopoly prices, i.e.,

$$\{q^{L^S}, q^{L^I}\} < \hat{p}_S^M - \varepsilon < \hat{p}_I^M - \varepsilon. \tag{43}$$

These inequalities imply that the price that would hold in the market in which more than one firm participate is lower than in the monopolistic

market. This suggests that if a firm initially monopolizes a market, and if a new firm enters the market, price-cutting competition is likely to take place. It is reasonable to assume that during the off-equilibrium price-cutting competition, both the incumbent and the entrant would learn their respective competitor's technology. The equilibrium market organization of this study may be interpreted as a non-cooperative outcome of this leaning process. In this sense, the Hamilton-Slutsky specification adopted for the first-stage game may be thought of as a stylized description of such an off-equilibrium learning process and a resulting equilibrium market organization.

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Appendix A: Proofs of Lemmas 7 and 8

Recall that Π_i^{Bo} is defined by (29). By (1) and (16), it can be expressed as follows:

$$\Pi_I^{Bo} = \frac{c(3-\alpha)(12b+(3+2\alpha)c)\bar{p}^2}{3(4b+3c)^2(4b+\alpha c)} \; ; \tag{44}$$

$$\Pi_S^{Bo} = \frac{c(48(1+\alpha)b^2+4(\alpha^2+24\alpha-9)bc+(7\alpha^2+21\alpha-18)c^2)\bar{p}^2}{3(4b+3c)^2(4b+\alpha c)^2} \; . \tag{45}$$

In order to prove Lemma 7, recall that α_I^o is defined by $\Pi_I^{Bo} = \pi_I^H(\hat{p}_S^{Ho})$.

Proof of Lemma 7. By (1) and (16), we have

$$\pi_I^H(\hat{p}_S^{Ho}) = \frac{(4b+(2-\alpha)c)\bar{p}^2}{2(4b+c)^2}. \tag{46}$$

By (44) and (46), α_I^o can be expressed as (30). Moreover, it holds that $\Pi_I^{Bo} = \pi_I^H(\hat{p}_S^{Ho})$ if and only if $\alpha = \alpha_I^o$. Figure 2 illustrates the case of $\alpha = \alpha_I^o$. As α rises, curve π_I^H will shrink to the upper horizontal intercept, $\bar{p}$. Because $\tilde{p}_I^{Ho}$ also converges to $\bar{p}$, it is clear from this figure that Π_I^{Bo} increases. Since, by (46), $\pi_I^H(\hat{p}_S^{Ho})$ decreases with α, it holds that

$$\Pi_I^{Bo} \gtreqless \pi_I^H(\hat{p}_S^{Ho}) \text{ if and only if } \alpha \gtreqless \alpha_I^o.$$

Now, fix an $\alpha \gtreqless \alpha_I^o$, with $\alpha < 2+4/t$. Then, since $\Pi_I^B \to \Pi_I^{Bo}$ and $\Pi_I^{LS} = \pi_I^H(\hat{p}_S^H) \to \pi_I^H(\hat{p}_S^{Ho})$ as $\varepsilon \to 0$, there is $\varepsilon' > 0$ such that $0 < \varepsilon < \varepsilon'$ implies (31) and (32). ■

Next, we will prove Lemma 8.

Proof of Lemma 8. This lemma needs to be proved separately for the cases of $\alpha > 3/2-2/t$ and $\alpha < 3/2-2/t$. First, take the case of $\alpha > 3/2-2/t$. In this case, by (23), $q^{L^I} = p_S^{**}$, which implies $\Pi_S^{L^I} = \pi_S^H(p_S^{**})$. By (1) and (16), it holds that

$$\pi_S^H(p_S^*) = \frac{4c\bar{p}^2}{(4b+3c)^2}. \tag{47}$$

By (45) and (47), it can be shown that $\Pi_S^{Bo} = \pi_S^H(p_S^*)$ if and only if $\alpha = \alpha_{S1}^o$. As α rises, curve π_I^H will shrink to the upper horizontal intercept, $\bar{p}$. Thus, $\tilde{p}_I^{Ho}$ will converge to $\bar{p}$. Because $\pi_S^H(p_S^*)$ does not change, it holds that, given $\alpha > 3/2-2/t$,

$$\Pi_S^{Bo} \gtreqless \pi_S^H(p_S^*) \text{ if and only if } \alpha \gtreqless \alpha_{S1}^o. \tag{48}$$

Next, take the case of $\alpha < 3/2 - 2/t$. In this case, by (23), $q^{L^I} = \hat{p}_I^H$, which implies $\Pi_S^{L^I} = \pi_S^H(\hat{p}_I^H)$. By using (1) and (16), define

$$\pi_S^H(\hat{p}_I^{H^o}) = \frac{(4b + c(2\alpha - 1))\bar{p}^2}{2(4b + \alpha c)^2}. \tag{49}$$

Thus, by (45) and (49), it can be shown that $\Pi_S^{B^o} = \pi_S^H(\hat{p}_I^{H^o})$ if and only if $\alpha = \alpha_{S2}^o$. Figure 3 illustrates this case. For the same reason as above, it holds that, given $\alpha < 3/2 - 2/t$,

$$\Pi_S^{B^o} \gtrless \pi_S^H(\hat{p}_I^{H^o}) \text{ if and only if } \alpha \gtrless \alpha_{S2}^o. \tag{50}$$

Finally, note that $\alpha_{S1}^o > \alpha_{S2}^o$ if $\alpha > 3/2 - 2/t$ and $\alpha_{S1}^o < \alpha_{S2}^o$ if $\alpha < 3/2 - 2/t$. Define $\alpha_S^o = \max\{\alpha_{S1}^o, \alpha_{S2}^o\}$. Then, (48) and (50) hold for $\alpha_{S1}^o = \alpha_S^o$ and $\alpha_{S2}^o = \alpha_S^o$, respectively. Now, fix an $\alpha \gtrless \alpha_S^o$. Note $\Pi_S^B \to \Pi_S^{B^o}$ as $\varepsilon \to 0$. Moreover, as $\varepsilon \to 0$, $\pi_S^H(p_S^{**}) \to \pi_S^H(p_S^*)$ in the case of (48), and $\pi_S^H(\hat{p}_I^H) \to \pi_S^H(\hat{p}_I^{H^o})$ in the case of (50). Therefore, there is $\varepsilon' > 0$ such that $0 < \varepsilon < \varepsilon'$ implies (34) and (35). $\blacksquare$

Appendix B: Proof of Theorem 4

Denote as $w(D)$ the total willingness to pay for D underlying demand function $D = (\bar{p} - p)/b$. Then, the consumer surplus can be defined as

$$CS(p) = w((\bar{p} - p)/b) - p(\bar{p} - p)/b.$$

Then, $CS^{L^i} = CS(q^{L^i})$ and

$$E\{CS^B\} = \frac{1}{(p_S^{**} - \tilde{p}_I^H)/\varepsilon + 1} \sum_{n=0}^{(p_S^{**} - \tilde{p}_I^H)/\varepsilon} CS(\tilde{p}_I^H + n\varepsilon).$$

Moreover, as in Section 4, think of $E\{CS^B\}$ at the limit case of $\varepsilon = 0$, i.e., $E\{CS^{B^o}\} = \int_{\tilde{p}_I^{H^o}}^{p_S^*} CS(p) dp/(p_S^* - \tilde{p}_I^{H^o})$. The expected consumer surplus, $E\{CS^{B^o}\}$, can be translated into an equivalent consumer surplus by

$$E\{CS^{B^o}\} = CS(q^{B^o})$$

with

$$q^{B^o} = \frac{\bar{p}}{3(4+3t)(4+\alpha t)}[48 + 12(3 + \alpha)t + 9\alpha t^2 \tag{51}$$
$$-4\sqrt{3(48 + 12(3 + \alpha)t + (9 + 3\alpha + \alpha^2)t^2)}].$$

25

Take the α_C^o defined in the theorem. By the definition of q^{B^o} and (16), it can be shown that $q^{B^o} = \hat{p}_S^{H^o}$ if and only if $\alpha = \alpha_C^o$. As α rises, curve $\pi_I^{H^o}$ will shrink to the upper horizontal intercept, $\bar{p}$. Because the lower horizontal intercept, $\tilde{p}_I^{H^o}$, also converges to $\bar{p}$, it is clear from (51) that q^{B^o} increases. Since, by (16), $\hat{p}_S^{H^o}$ does not change with α, it holds that

$$ q^{B^o} \gtreqqless \hat{p}_S^{H^o} \text{ if and only if } \alpha \gtreqqless \alpha_C^o. \tag{52} $$

Moreover, it may be proved that

$$ 3/2 - 2/t < \alpha_C^o < 2 + 4/t. \tag{53} $$

Let $\alpha_C^o < \alpha < 3$. Then, by (23) and (53), there is $\varepsilon' > 0$ such that $0 < \varepsilon < \varepsilon'$ implies $q^{L^I} = p_S^{**}$. Since $p_S^{**} \to p_S^*$ as $\varepsilon \to 0$, and since $p_S^* > q^{B^o}$, ε' can be chosen to guarantee $q^{L^I} = p_S^{**} > q^{B^o}$, which implies $E\{CS^B\} > CS^{L^I}$. By (23), $\alpha_C^o < \alpha < 2 + 4/t$ implies $q^{L^S} = \hat{p}_S^H$; $2 + 4/t < \alpha < 3$ implies $q^{L^S} = \tilde{p}_I^H$. In either case, by (52), $CS^{L^S} > E\{CS^B\}$. Thus, (39) is proved.

Let $1 < \alpha < \alpha_C^o$. By (23), there is $\varepsilon' > 0$ such that $0 < \varepsilon < \varepsilon'$ implies $q^{L^S} < q^{L^I}$, which implies that $CS^{L^S} > CS^{L^I}$. Moreover, by (23) and (53), ε' can be chosen to guarantee $q^{L^S} = \hat{p}_S^H$. Since $\hat{p}_S^H \to \hat{p}_S^{H^o}$ as $\varepsilon \to 0$, by (52), ε' can be chosen to guarantee $E\{CS^B\} > CS^{L^S}$. Thus, (40) is proved.
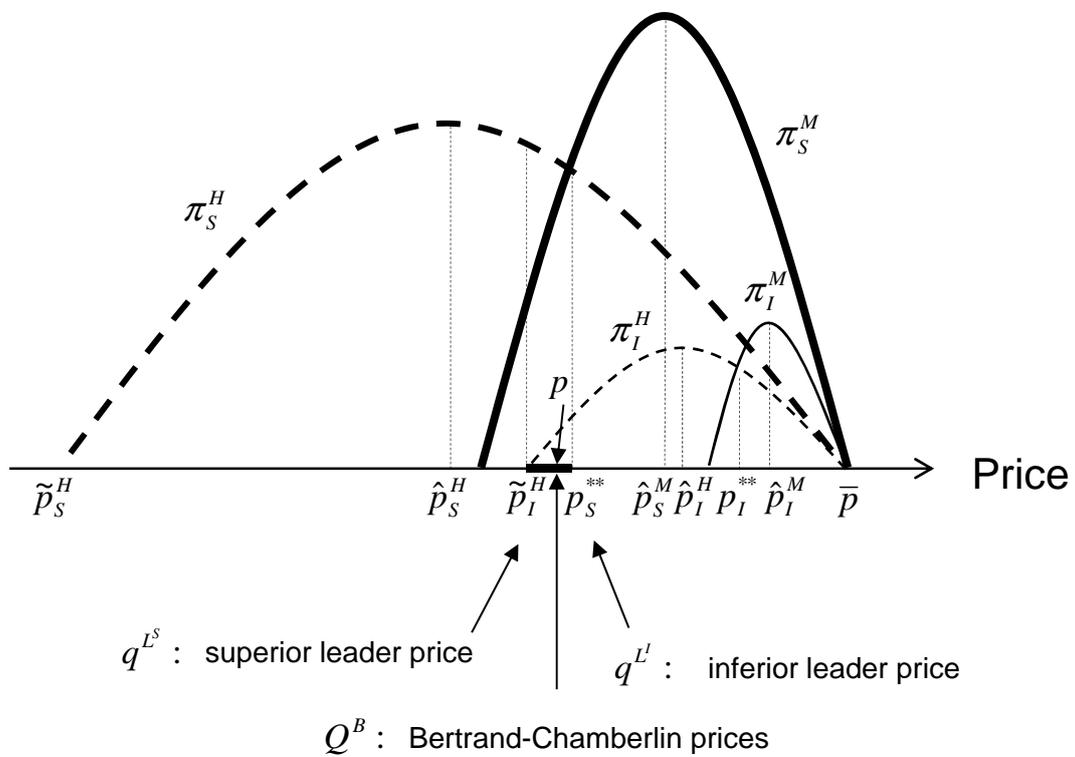
π_S^M

π_S^H

π_I^M

π_I^H

p

Price

$\tilde{p}_S^H$ $\hat{p}_S^H$ $\tilde{p}_I^H$ p_S^{**} $\hat{p}_S^M$ $\hat{p}_I^H$ p_I^{**} $\hat{p}_I^M$ $\bar{p}$

q^{L^S} : superior leader price

q^{L^I} : inferior leader price

Q^B : Bertrand-Chamberlin prices

Figure 1: Price Competition Market

$$2 + 4/t < \alpha < 3$$

π_S^M

π_I^M

π_S^H

π_I^H

Price

$\tilde{p}_S^H$ $\tilde{p}_I^H$ $\hat{p}_S^H$ p_S^{**} $\hat{p}_I^H$ p_I^{**} $\bar{p}$

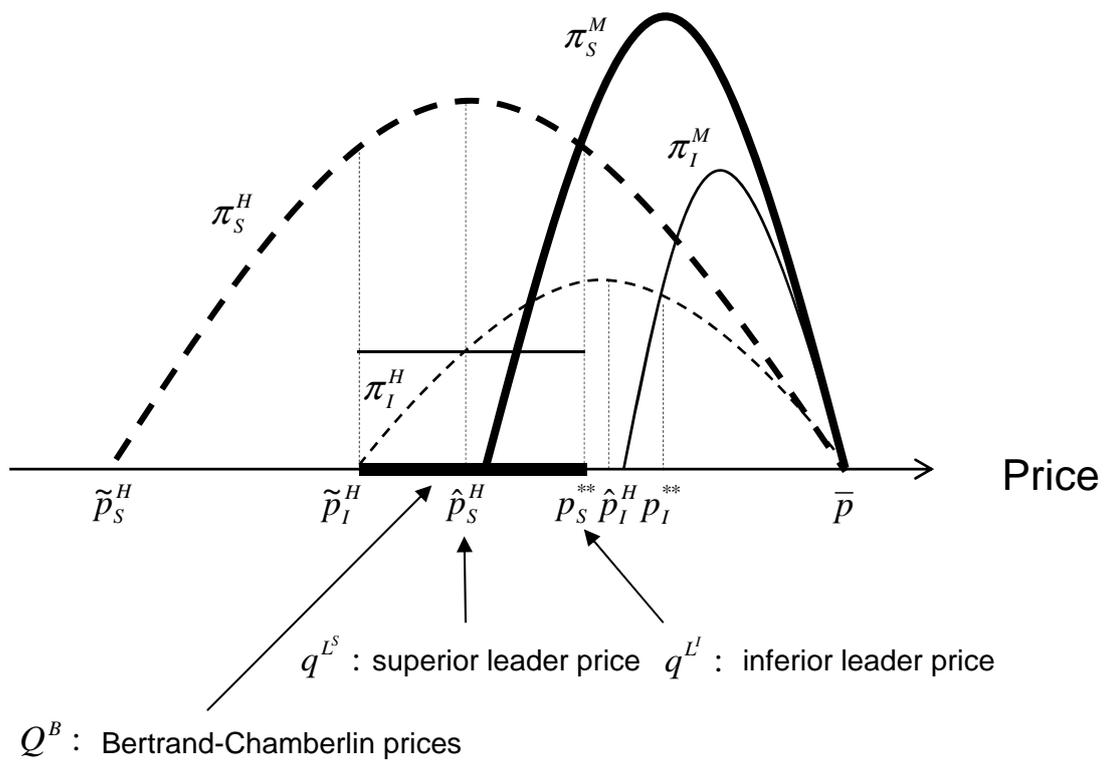q^{L^S} : superior leader price q^{L^I} : inferior leader price

Q^B : Bertrand-Chamberlin prices

Figure 2:
From the Price Competition Market ($\alpha_I^o < \alpha < 3$)
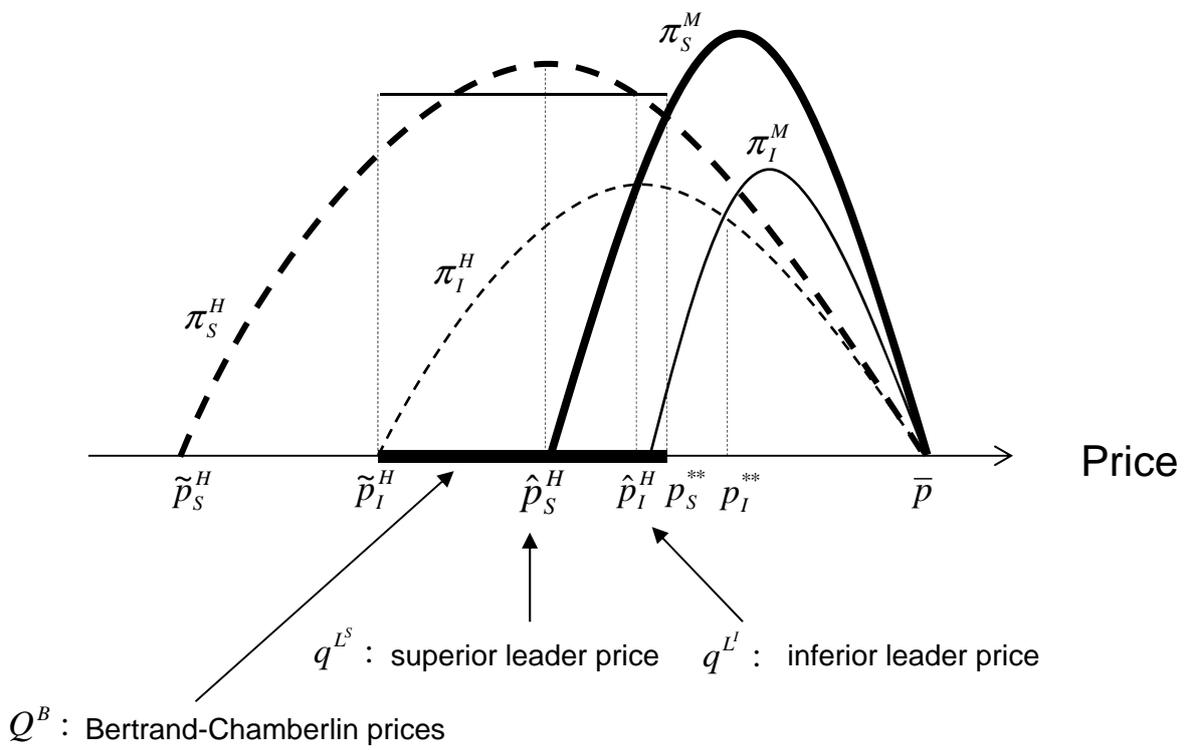to the Superior Leader Market ($\alpha_S^o < \alpha < \alpha_I^o$)

Figure 3:
From the Superior Leader Market ($\alpha_S^o < \alpha < \alpha_I^o$)
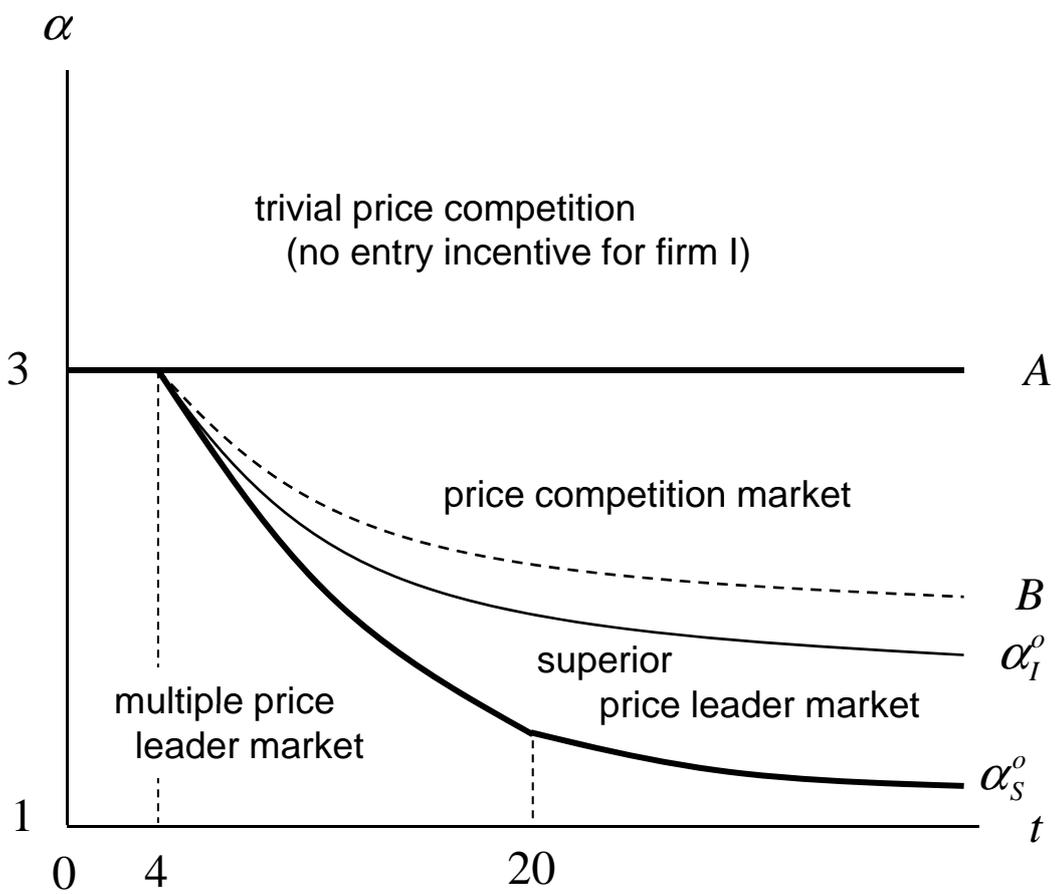to the Multiple Leader Market ($1 \le \alpha < \alpha_S^o$)

Figure 4: Equilibrium Market Organizations:
A Complete Characterization