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"Asset Bubbles, Credit Market Imperfections, and Technology Choice"

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Asset Bubbles, Credit Market Imperfections, and Technology Choice

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Abstract

This paper introduces a bubbly asset into the Matsuyama (2007) model with credit market imperfections and multiple technologies and shows that there can exist multiple bubbly steady states and bubbles may cause underdevelopment traps by preventing the adoption of high productivity technology.

Key words: asset bubbles; credit market imperfections; technology adoption
JEL classification: E44

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1 Introduction

Tirole (1985) shows that, in an overlapping generations model, bubbles can exist if the bubbleless equilibrium of the economy is dynamically inefficient and that the existence of bubbles reduces capital accumulation. Recently, Matsuyama (2007) presents an interesting model with credit market imperfections and multiple technologies and shows that the model can generate a rich variety of growth patterns. This paper introduces a bubbly asset into the Matsuyama (2007) model and examines whether bubbles can exist and how they would affect the choice of technologies in the economy.[1] It is shown that there can exist multiple bubbly steady states and that the existence of bubbles may prevent adoption of high productivity technology.

There are several related studies which also examine the effects of bubbles on economic growth and development in the presence of credit market imperfections. Martin and Ventura (forthcoming) develop a model that shows a positive relationship between bubbles and long-run growth. Farhi and Tirole (forthcoming), Hirano and Yanagawa (2010) and Sakuragawa (2010) construct models in which bubbles can be either a positively or negatively related to growth. In contrast to these studies, by introducing multiple technologies this paper studies the effect of bubbles on the technology choice.

[1] Gokan (2011) studies the dynamic property of monetary equilibria with a single technology. Constructing a similar model with heterogenous agents, Kunieda (2008) examines the dynamic and efficiency properties of the bubbly equilibrium.

2 The Model

The basic structure of the model follows Matsuyama (2007). The economy begins in period 1 and continues toward infinity. A final good is produced by the following production function:

$$Y_t = AK_t^{\alpha} N_t^{1-\alpha}, \tag{1}$$

with $0 < \alpha < 1$, where K_t and N_t represent capital and labor at period t, respectively. Dividing (1) by N_t, we have

$$y_t = Ak_t^{\alpha}, \tag{2}$$

where $y_t = Y_t/N_t$ and $k_t = K_t/N_t$. Capital depreciates fully in one period.

A new generation, a unit measure of homogenous agents, is born in each period and lives two periods. An agent born in period t supplies one unit of labor when young and consumes only when old. In the young period, each agent can become either an entrepreneur or a lender. We denote the ratios of lenders and entrepreneurs in generation t by θ_t and $1-\theta_t$, respectively. There are $\bar{B}$ pieces of a useless asset, which is called a "bubbly asset." When the bubbly asset has a positive value, we say that asset bubbles exist.

If an agent becomes a lender, the agent lends his earnings to entrepreneurs in the competitive loan market at r_{t+1} or holds the bubbly asset. Therefore, the agent solves the following problem:

$$\max_{l_t, b_t} \ c_{t+1}^{l} = r_{t+1} l_t + \frac{p_t}{p_{t+1}} b_t \tag{3}$$

$$\text{s.t.} \quad l_t + b_t = w_t, \tag{4}$$

where l_t, b_t, r_{t+1} and w_t are the quantity of lending, the real value of the bubbly asset, the real interest rate, and the wage rate, respectively. If the bubbly asset is held in equilibrium, the following no-arbitrage condition between the bubbly asset and lending must be satisfied:

$$r_{t+1} = \frac{p_t}{p_{t+1}}. \tag{5}$$

If the agent becomes an entrepreneur, the agent can select one from two types of projects. A type-1 project transforms m_1 units of the final goods into R_1 units of capital while a type-2 project transforms m_2 units of the final goods into R_2 units of capital. When $m_i > w_t$, the entrepreneur must borrow $m_i - w_t$. The entrepreneur's consumption when old is

$$c^e_{t+1} = \rho_{t+1} m_i R_i - r_{t+1}(m_i - w_t),$$

where ρ_{t+1} is the rate of return from capital. The first term in the RHS of the above equation is the revenue from investment and the second represents the repayments to lenders. There are credit constraints in this economy. Each entrepreneur can pledge only up to a constant fraction of the project revenue for the repayment, $\lambda_i m_i R_i \rho_{t+1}$, where $0 \leq \lambda_i \leq 1$ ($i = 1, 2$), and λ_i differs between the two types of projects. The entrepreneur's borrowing must satisfy

$$\lambda_i m_i R_i \rho_{t+1} \geq r_{t+1}(m_i - w_t) \quad \text{for} \quad i = 1, 2. \tag{6}$$

Because entrepreneurs can choose to become lenders, earnings from investment must not be smaller than those from lending, i.e.,

$$c^e_{t+1} \geq c^l_{t+1} \quad \Leftrightarrow \quad \rho_{t+1} R_i \geq r_{t+1}. \tag{7}$$

Thus, the entrepreneur's problem is given by

$$\begin{aligned} &\max \ c^e_{t+1} = \rho_{t+1} m_i R_i - r_{t+1}(m_i - w_t) \\ &\text{s.t.} \quad (6) \ \text{and} \ (7). \end{aligned} \tag{8}$$

3 Market Equilibrium

Under perfect competition, the marginal productivity of each factor is equal to its price:

$$\rho_t \equiv \rho(k_t) = \alpha A k_t^{\alpha-1}, \tag{9}$$

$$w_t \equiv w(k_t) = (1-\alpha) A k_t^{\alpha}. \tag{10}$$

As in Matsuyama (2007), we can show that the equilibrium interest rate is determined by

$$r_{t+1} = \max_{i=1,2} \left\{ \frac{R_i \rho(k_{t+1})}{\max\left\{1, \left(1-\frac{w_t}{m_i}\right)/\lambda_i\right\}} \right\}. \tag{11}$$

This means that the project giving a larger value in the RHS of (11) is adopted by entrepreneurs. Because both arguments in the brace in the RHS of (11) are multiplied by $\rho(k_{t+1})$, by checking the relative size of

$$\frac{R_i}{\max\left\{1, \left(1-\frac{w_t}{m_i}\right)/\lambda_i\right\}} \quad \text{for} \quad i=1,2, \tag{12}$$

we can clarify the equilibrium choice of the project type. Note that, in (12), the credit constraint is binding when

$$\left(1-\frac{w_t}{m_i}\right)/\lambda_i > 1 \quad \Leftrightarrow \quad w_t < (1-\lambda_i)\, m_i.$$

Let us assume the following conditions:

$$(1-\alpha) A R_i^{\alpha} < m_i^{1-\alpha} \;\; \text{for } i=1,2, \tag{A1}$$

$$R_2 > R_1 > \lambda_1 R_1 > \lambda_2 R_2 \text{ and } m_2 \geq m_1. \tag{A2}$$

(A1) is equivalent to $w(k_t) < m_i$ $(i=1,2)$, which means that agents must borrow in order to run the project. (A2) states that there are tradeoffs between the productivity and pledgeability of the projects. Under these assumptions we can depict the loci of (12) as in Figure 1, where we denote the value of w at the intersection of the two loci by

$$w_C = m_2 \left(1 - \frac{R_2\lambda_2}{R_1}\right). \tag{13}$$

Figure 1 shows that type-1 project is adopted if $0 < w_t < w_C$ while type-2 project is employed if $w_C \leq w_t < 1$. Moreover, since (A2) and (13) mean

$$(1-\lambda_1)\, m_1 < w_C \ < (1-\lambda_2)\, m_2,$$

it follows that, if $w_t < (1-\lambda_1)\, m_1$ or $w_C \leq w_t < (1-\lambda_2)\, m_2$, the credit constraint is binding while if $(1-\lambda_1)\, m_1 \leq w_t < w_C$ or $(1-\lambda_2)\, m_2 \leq w_t < 1$, the credit constraint is not binding.

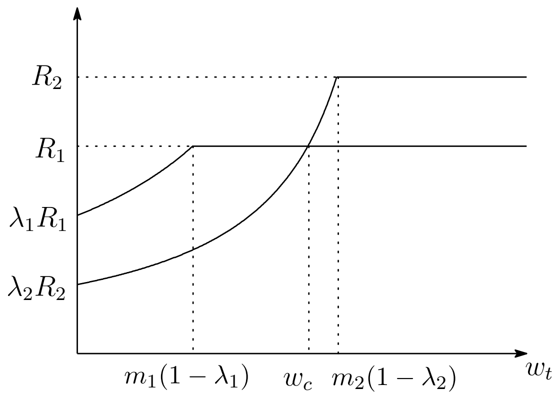


Figure 1: The maximal rate of return under (A2)

Because the wage income of entrepreneurs and the borrowing from lenders are used for investment, the total amount of investment is

$$(1-\theta_t)w_t + \theta_t(w_t - b_t) = w_t - \theta_t b_t.$$

Since one unit of investment yields R_1 units of capital when $0 < w_t < w_C$ and R_2 units of capital when $w_C \leq w_t < 1$, the equilibrium capital accumulation is determined by

$$k_{t+1} = \begin{cases} R_1(w_t - \theta_t b_t) & \text{if } \ 0 < w_t < w_C, \\ R_2(w_t - \theta_t b_t) & \text{if } \ w_C \le w_t < 1. \end{cases} \tag{14}$$

Define

$$\gamma_t \equiv \theta_t b_t / w_t. \tag{15}$$

Then we can rewrite (14) as

$$k_{t+1} = \begin{cases} R_1(1-\gamma_t)w_t & \text{if } \ 0 < w_t < w_C, \\ R_2(1-\gamma_t)w_t & \text{if } \ w_C \le w_t < 1, \end{cases} \tag{16}$$

which describes the equilibrium dynamics of k in terms of γ and w. From (10) and (16), we obtain the equilibrium dynamics of w:

$$w_{t+1} = \begin{cases} (1-\alpha)A[R_1(1-\gamma_t)w_t]^\alpha & \text{if } \ 0 < w_t < w_C, \\ (1-\alpha)A[R_2(1-\gamma_t)w_t]^\alpha & \text{if } \ w_C \le w_t < 1. \end{cases} \tag{17}$$

We next consider asset markets. Since the total supply of the bubbly asset, $\bar{B}$, is fixed and only lenders hold the asset, the equilibrium condition of the bubbly asset is

$$\bar{B} = \theta_t p_t b_t. \tag{18}$$

It is easy to show from (5) and (11) that the equilibrium interest rate satisfies

$$\frac{p_t}{p_{t+1}} = r_{t+1} = \begin{cases} \frac{\lambda_1 R_1 \rho(k_{t+1})}{m_1 - w_t} & \text{if } \ 0 < w_t < m_1\left(1-\lambda_1\right), \\ R_1\rho(k_{t+1}) & \text{if } \ m_1(1-\lambda_1) \le w_t < w_C, \\ \frac{\lambda_2 R_2 \rho(k_{t+1})}{m_2 - w_t} & \text{if } \ w_C \le w_t < m_2\left(1-\lambda_2\right), \\ R_2\rho(k_{t+1}) & \text{if } \ m_2\left(1-\lambda_2\right) \le w_t < 1. \end{cases} \tag{19}$$

Combining (18) and (19) yields

$$r_{t+1}\theta_t b_t = \theta_{t+1} b_{t+1}, \tag{20}$$

which describes the dynamic behavior of bubbles.

Moreover, from (15) and (20), we have

$$\gamma_{t+1} = \frac{\theta_{t+1} b_{t+1}}{w_{t+1}} = \frac{r_{t+1}\theta_t b_t}{w_{t+1}} = \frac{w_t}{w_{t+1}} r_{t+1}\gamma_t. \tag{21}$$

Finally, using (10), (19) and (21), we obtain the dynamics of γ:

$$\gamma_{t+1} = \begin{cases} \frac{\alpha}{1-\alpha}\frac{\gamma_t}{1-\gamma_t}\frac{m_1\lambda_1}{m_1-w_t} & \text{if } \ 0 \le w_t < m_1\left(1-\lambda_1\right), \\ \frac{\alpha}{1-\alpha}\frac{\gamma_t}{1-\gamma_t} & \text{if } \ m_1\left(1-\lambda_1\right) \le w_t < w_C, \\ \frac{\alpha}{1-\alpha}\frac{\gamma_t}{1-\gamma_t}\frac{m_2\lambda_2}{m_2-w_t} & \text{if } \ w_C \le w_t < m_2\left(1-\lambda_2\right), \\ \frac{\alpha}{1-\alpha}\frac{\gamma_t}{1-\gamma_t} & \text{if } \ m_2\left(1-\lambda_2\right) \le w_t \le 1. \end{cases} \tag{22}$$

Sequences $\{\gamma_t, w_t\}_{t=0}^{\infty}$ satisfying (17) and (22) constitute an equilibrium of this economy. The equilibrium ratio of lenders, θ_t, can be derived from the credit market-clearing conditions,

$$\theta_t l_t = (1-\theta_t)(m_i - w_t) \ \text{ for } \ i = 1, 2. \tag{23}$$

The LHS of (23) represents the total supply of lending and the RHS is the total demand for borrowing when type-i ($i = 1, 2$) project is adopted. Combining (4) and (23), we obtain

$$\theta_t = 1 - \frac{(1-\gamma_t)w_t}{m_i} \ \text{ for } \ i = 1, 2.$$

From (17) the $w_{t+1} = w_t$ locus can be depicted as in Figure 2. At $w_t = w_C$ the selected type of projects changes from type-1 to type-2. Because the productivity of type-2 is higher than that of type-2, the $w_{t+1} = w_t$ locus jumps up at $w_t = w_C$. From (22) the $\gamma_{t+1} = \gamma_t$ locus is given by

$$\gamma_t = \begin{cases} 1 - \frac{\alpha}{1-\alpha}\frac{m_1\lambda_1}{m_1-w_t} & \text{if } \ 0 \le w_t < m_1\left(1-\lambda_1\right), \\ \frac{1-2\alpha}{1-\alpha} & \text{if } \ m_1\left(1-\lambda_1\right) \le w_t < w_C, \\ 1 - \frac{\alpha}{1-\alpha}\frac{m_2\lambda_2}{m_2-w_t} & \text{if } \ w_C \le w_t < m_2\left(1-\lambda_2\right), \\ \frac{1-2\alpha}{1-\alpha} & \text{if } \ m_2\left(1-\lambda_2\right) \le w_t \le 1, \end{cases} \tag{24}$$

which is depicted in Figure 3. The locus has an upward jump point at $w_t = w_C$ because the adopted technology switches from type-1 to type-2 at this point.

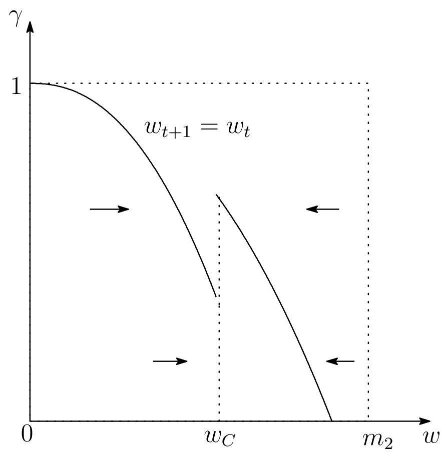


Figure 2: The locus of $w_{t+1} = w_t$

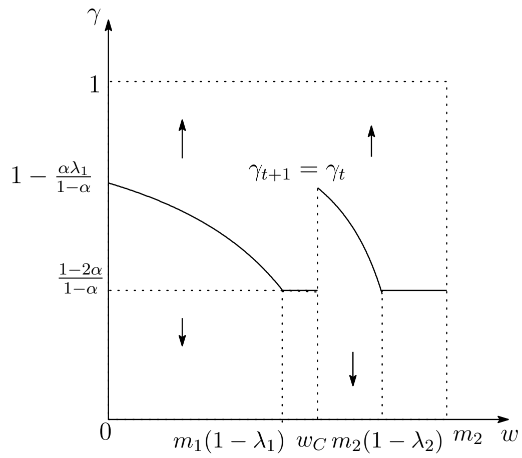


Figure 3: The locus of $\gamma_{t+1} = \gamma_t$

4 Bubbles and Technology Choice

We first derive bubbleless steady states. In this case, $\gamma_t = 0$ for all t and thus the equilibrium dynamics are described by

$$w_{t+1} = \begin{cases} (1-\alpha)A\left(R_1 w_t\right)^{\alpha} & \text{if} \quad 0 < w_t < w_C, \\ (1-\alpha)A\left(R_2 w_t\right)^{\alpha} & \text{if} \quad w_C \leq w_t < 1. \end{cases}$$

Because this is exactly the same as in Matsuyama (2007), this system can have multiple steady states or a unique steady state depending on the parameter values. In the following analysis, however, we restrict our attention to the case in which type-2 technology is always adopted in the bubbleless steady state, i.e., we assume

$$m_2\left(1 - \frac{R_2\lambda_2}{R_1}\right) < \left[(1-\alpha)AR_1^{\alpha}\right]^{\frac{1}{1-\alpha}}.$$

To obtain steady-state equilibria of the model, let us depict the loci of $w_{t+1} = w_t$ and $\gamma_{t+1} = \gamma_t$ simultaneously. Figures 4 show two possible cases in which multiple bubbly steady states exist. The parameters are the same in both cases, $m_1 = m_2 = A = 1$, $\alpha = 0.45$, $R_1 = 2$, $R_2 = 3.1$, and $\lambda_2 = 0.3$, with the exception of λ_1: 0.6 in Figure 4(a) and 0.48 in Figure 4(b). These parameter combinations satisfy (A1) and (A2).

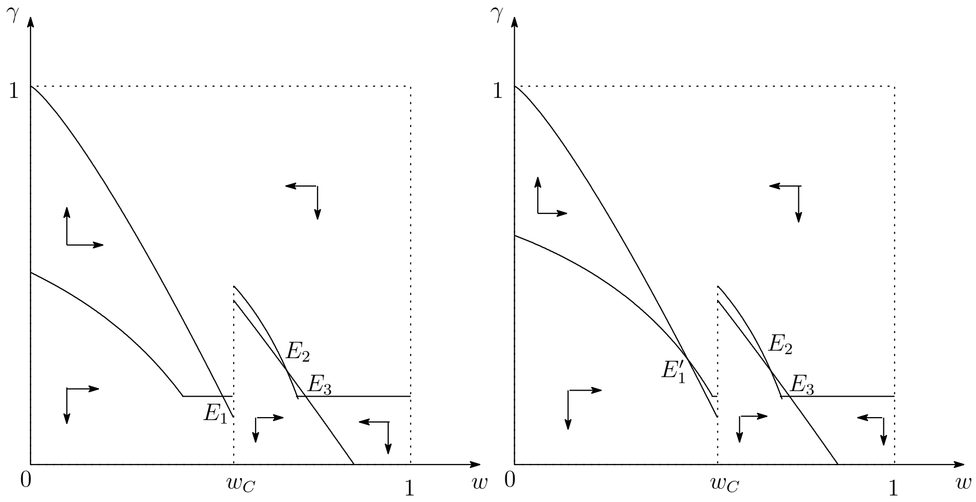
In Figure 4(a), the $w_{t+1} = w_t$ locus always has intersections with the flat parts of the $\gamma_{t+1} = \gamma_t$ locus in both the region in which type-1 technology is adopted and the region in which type-2 technology is employed. In addition, there is a bubbly steady state in which the type-2 project is adopted and the credit constraint is binding.

In steady state E_1, the adopted technology is of type-1 and the wage remains at a lower level. Because we assume that type-2 technology is always adopted in the bubbleless equilibrium, this means that the existence of bubbles prevents the adoption of type-2, which is more productive than type-1, and leads to a lower income equilibrium. We call this situation a *bubbly trap*. On the other hand, in steady state E_2 and E_3, type-2 technology is adopted although bubbles exist in the long run. It should be noted here that in this case the existence of bubbles does not prevent the adoption of high productivity technology but reduces the steady-state capital stock as in Tirole (1985).

Figure 4(b) illustrates the same situation except that the credit constraint is binding in E_1'.

5 Conclusion

Introducing a bubbly asset into the Matsuyama (2007) model, this paper analyzed the effect of bubbles on technology choice, which has not been analyzed by previous research on the relationship between bubbles



(a) High λ_1 (b) Low λ_1

Figure 4: Bubbly traps

and growth. It was shown that there can exist multiple bubbly steady states and that the existence of bubbles may prevent the adoption of high productivity technology and cause a bubbly trap.

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