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Spatial Pattern and City Size Distribution

Tomoya Mori

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Tomoya Mori[*],[†]

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Abstract

This paper reviews the extant formal models that explain the spatial pattern together with the size distribution of cities, and discusses the remaining research questions to be answered in this literature. To obtain meaningful results about the spatial patterns of cities, a model needs to depart from the most popular, a two-region and the systems-of-cities frameworks in urban and regional economics in which there is no variation in interregional distance. This is one of the major reasons that only few formal models have been proposed in this literature. To draw implications as much as possible from the extant theories, this review involves extensive discussions on the behavior of the *many-region extension* of the extant models. The mechanisms that link the spatial pattern of cities and the diversity in city sizes are also discussed separately in detail.

Keywords: Cities, Agglomeration, Racetrack geography, Interregional distance, Power laws, Central place theory

JEL Classification: R12, C33

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[*] Institute of Economic Research, Kyoto University, Yoshida-honmachi, Sakyo-ku, Kyoto 606-8501, Japan. E-mail: mori@kier.kyoto-u.ac.jp

[†] Research Institute of Economy, Trade and Industry, 11th floor, Annex, Ministry of Economy, Trade and Industry (METI) 1-3-1, Kasumigaseki Chiyoda-ku, Tokyo, 100-8901 Japan.

1 Introduction

In the past 50 years since the formal analyses of city formation started around the time of Alonso (1964),[1] the spatial pattern of cities has remained as a relatively minor subject in urban economics[2] – despite that economic geographers in the past (e.g., von Thünen, 1826; Christaller, 1933; Lösch, 1940), have commonly suggested the inseparable correspondence between size and spatial distributions of cities (see, e.g., Fujita, 2010).[3]

The mainstream theories in urban economics abstracted from inter-city space by adopting *the systems-of-city model* after the pioneering work by J. Vernon Henderson (Henderson, 1974),[4] or by simply assuming the presence of only two regions in an economy (see a collection of models presented in Baldwin, Forslid, Martin, Ottaviano and Robert-Nicoud, 2003). The mechanism which determines the size of a city has always been a major subject in most of these theories, and for this purpose, the abstraction from interregional space in these approaches substantially simplified the analyses.

As a consequence of this particular evolution of the field, there exist rather limited theoretical as well as empirical literature which relate the spatial pattern and sizes of cities. To my knowledge, there are two major strands of formal models that explicitly deal with the spatial pattern of cities, *new economic geography* (*NEG*) and *social-interactions models*. This paper focuses on the basic structure and implications from these theoretical models in connection to the observed spatial patterns of cities.

In Section 2, I start by making some observations on the relation among sizes, spatial patterns and industrial structure of cities in reality by using data from Japan. In Section 3, generic properties of the canonical models of extant theories are discussed. In particular, while most models were studied in the context of a two-region setup in their original papers, in this paper, by extensively drawing from the work of Akamatsu, Mori, Osawa and Takayama (2018), I summarize their behavior in a many-region setup in which the spatial pattern of cities can be more properly studied. Finally, Section 4 concludes the paper.

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[1] There is a large literature on location theory that preceded urban economics and have important implications on city and agglomeration formation (see, e.g., Thisse et al., 1996, for a survey), although they were not designed to explain city formation per se.

[2] A notable exceptions are Isard (1949, 1956). While no formal models have been proposed by Isard, he foresaw the necessity of increasing returns and imperfect competition in order to explain the formatoin of cities and their spatial pattern. In particular, he envisaged the emergence of new economic geography which played a central role in this literature as will be discussed in Section 3 (see Fujita, 2010).

[3] See an intriguing review by Fujita (2012) on the von Thünen's work and ideas about spatial organization of economy.

[4] See, e.g., Abdel-Rahman and Anas (2004) for a survey. See also Behrens and Robert-Nicoud (2015) for more recent applications of this framework.

2 Facts about size, location and industrial composition of cities

To guide summarizing and classifying the extant theoretical models for the size and spatial patterns of cities, it is useful to have a concrete idea about the basic relationships observed between them in reality. Given that the inter-city space has been largely abstracted in the literature, however, systematic researches on this subject are scarce, and the results published so far provide little decisive evidence (e.g., Dobkins and Ioannides, 2001; Overman and Ioannides, 2001; Ioannides and Overman, 2004). To demonstrate the strong correspondence between theories and facts, here, rather than trying to put together subtle pieces of evidences from the extant empirical literature, I attempt to demonstrate a set of clear-cut facts using data from Japan about the relationship among the spatial pattern, sizes as well as industrial structure of cities, which to a large extent can be explained by the extant theories, *if they were implemented on a many-region geography* as is done in Section 3.

Throughout this section, a *city* is defined to be a contiguous set of (approximately) 1km-by-1km cells with at least 1000 people per km^2 and total population of al least 10,000.[5] The advantage of this simple definition of a city is that the basic regional units (1km-by-1km cells) is consistent in the cross sections of a given country, and across different points in time, unlike more commonly used definitions of metropolitan areas based on administrative regions. Under this definition of a city, the set of all cities in a country account for the population (area) share in the country of 43.6% (2.4%), 44.6% (1.6%), 77.1% (12.4%), 48.7% (2.9%) and 47.0% (3.8%) for the US, Europe, Japan, China and India, respectively.[6]

2.1 Size and spacing of cities

Many large cities in reality are found at locations with certain first nature advantages.[7] But, still, those exogenous features of location are far from fully governing the spatial pattern of cities. In particular, (population) size and spacing of cities are closely related as have long been recognized by economic geographers since von Thünen (1826), Christaller (1933) and Lösch (1940). They (especially, Christaller) suggested *a central place pattern* in the relation between the size and location of cities such that *a larger city is surrounded by smaller cities*, which in turn implies that *a larger cities are more spaced apart*.

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[5] This definition of a city is a variation of that proposed by Rosenfeld et al. (2011).

[6] The estimated population count data at the 1km-by-1km cell level are obtained from Statistics Bureau, Ministry of Internal Affairs and Communication of Japan (2015) for Japan, and from the LandScan by Oak Ridge National Laboratory (2015) for the rest of the countries.

[7] For the role of the natural advantage in the city formation, see, e.g., Davis and Weinstein (2002) for the case of Japan, Bleakley and Lin (2012) and Cronon (1991) for the US, and Michaels and Rauch (2017) for France and the UK.

To see this, let $\mathcal{U}$ be the set of all 450 cities identified in Japan in 2015, s_i be the share of city $i \in \mathcal{U}$ in the national population, and $\|i, j\|$ for $i, j \in \mathcal{U}$ be the road distance between cities i and j.[8] Define the *spacing* of city i by the distance to the closest city of the same or a larger size class:[9]

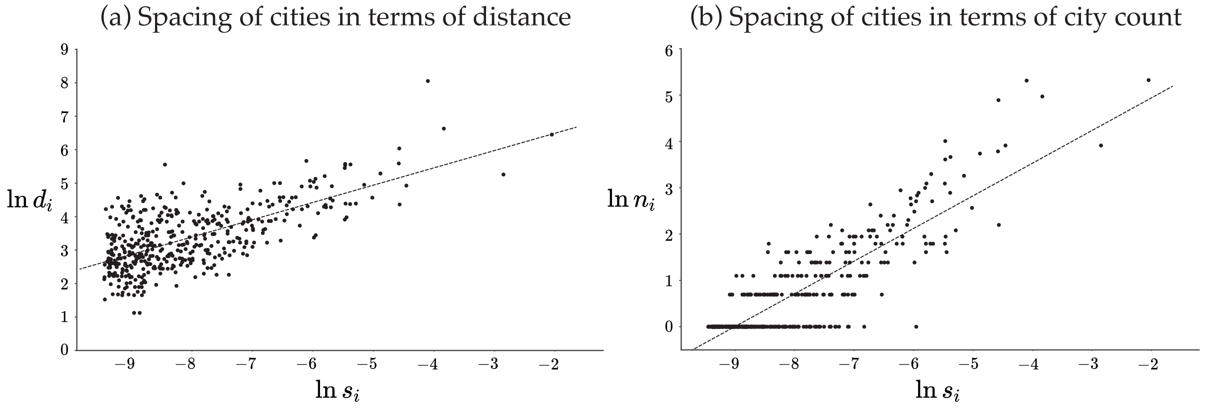
$$d_i = \min_{j \in \{k \in \mathcal{U} \,:\, s_k > 0.75 s_i\}} \|i, j\| \,. \tag{1}$$

Figure 1(a) shows the relationship between d_i and s_i in log scale for each city $i \in \mathcal{U}$. The correlation between them is as high as 0.67.[10] This confirms the spacing-out property of cities mentioned above.

If the number, n_i, of cities within the distance d_i from city $i \in \mathcal{U}$ is counted by

$$n_i \equiv \#\{j \in \mathcal{U} \backslash \{i\} \;:\; \|i, j\| < d_i\} \,, \tag{2}$$

as shown in Figure 1(b), it also has strong correlation, 0.86, with the city size, s_i, in log scale. Thus, indeed it is clear that larger cities are surrounded by smaller cities.[11]


(a) Spacing of cities in terms of distance

(b) Spacing of cities in terms of city count

Figure 1: Spacing of cities

2.2 Size distribution of cities

It is well known that city size distribution within a relatively self-sufficient economic regions – typically in a country – exhibits an approximate power law (e.g., Gabaix and

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[8] The road distance is based on the OpenStreetMap data as of July, 2017. The distance between cities is computed as the distance between the centroids of the most densely populated 1km-by-1km cells in these cities. The computation was done using the Stata interface, osrmtime, of Open Source Routing Machine by Huber and Rust (2016).

[9] The lower threshold share, 0.75, defining the "same size class" in (1) is arbitrary. But, the choice of the threshold value does not alter the qualitative result as long as it is not too far from 1.0.

[10] The dashed line in the figure is the fitted line by Ordinary Least Squares (OLS) regression.

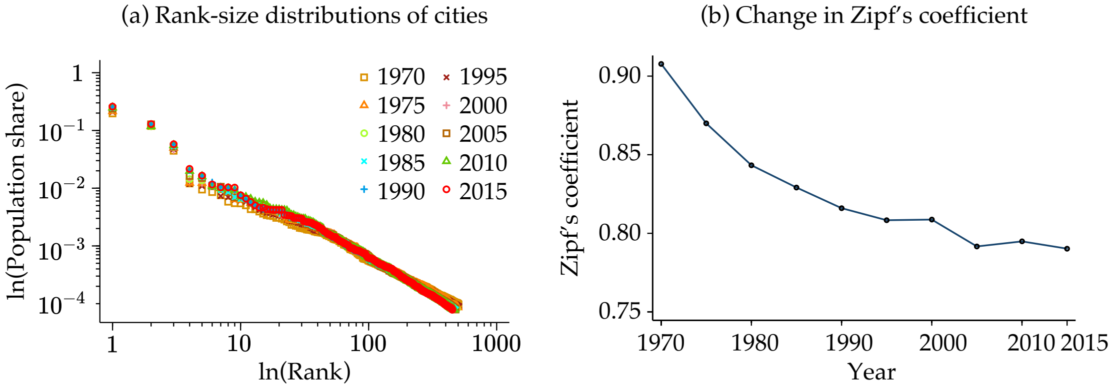
[11] These properties are not specific to Japan. See Mori et al. (2014) for similar results for the US and Germany. Dobkins and Ioannides (2001) found a negative correlation between the size and spacing of cities in the US for the period 1900-1980. But, the specific feature of the US cities needs to be taken into account is their historical development. The formation of cities started in the northeastern region of the US in the 19th century, and then expanded gradually to west and then to south. But, the *effective* distance kept changing in the meantime in response to the advancement in the transport technology. As a consequence, the spacing of the same size class of cities has increased over time. Once, this change is appropriately controlled for, the same positive correlation as in Figure 1 is obtained (refer to, e.g., Mori et al., 2014).

Ioannides, 2004; Batty, 2006; Bettencourt, 2013). To be precise, if a given set of n cities is postulated to satisfy a power law, and if these city sizes are ranked as $s_1 \geq s_2 \geq \cdots \geq s_n$, so that the rank, r_i, of city i is given by $r_i = i$, then for some positive constants c and α,

$$r_i/n \approx P(S > s_i) \approx c s_i^{-\alpha} \Rightarrow \ln s_i \approx b - \frac{1}{\alpha} \ln r_i \tag{3}$$

for $b = \ln(cn)/\alpha$. Japan is an obviously mono-polar economy organized around Tokyo, so that it is a typical case in which the approximate power law for city size holds at the country level.[12]

Figure 2(a) shows the rank-size distributions of cities in every five years between 1970 and 2015, where the city size s_i for city i is expressed in terms of the share in the national population. City sizes are diverse. In 2015, among the 450 cities identified, the largest three cities, Tokyo, Osaka and Nagoya, account for 45% of the national population, where Tokyo alone accounts for 26%.


(a) Rank-size distributions of cities

(b) Change in Zipf's coefficient

Figure 2: Rank-size distribution of cities

There is a strand of literature which informally argue that *Zipf's law* (after Zipf, 1949) holds, i.e., the power law with $\alpha = 1$ holds for city size distribution at the country level (see, e.g. Gabaix and Ioannides, 2004). But, there is no definite evidence for this claim. For the case of Japan, under the present city definition, the estimated Zipf's coefficient, α, is uniformly below 1 for all years, and the value has declined almost steadily (i.e., the distribution has become more skewed towards larger cities) over these 45 years as indicated by Figure 2(b).

Thus, a relatively robust fact appears to be that the city size distribution exhibits an approximate power law at each point in time, although the power coefficient may be country specific (more generally economic region specific), and may be changing over time.

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[12] Not surprisingly, large countries often have multiple largest cities of comparable sizes, and one can see that similar power laws hold in a subregion around each of these cities. Under the same definition of a city as in this paper, for example, the US has two comparable subsystem of cities around New York and Los Angeles, China around Shanghai and Hong Kong, and India may have even three around New Dehli, Kolkata and Mumbai.

2.3 Size and industrial structure of cities

Many evidences (e.g. Glaeser and Maré, 2001; Bettencourt, Lobo, Helbing, Kühnert and West, 2007; Combes, Duranton and Gobillon, 2008; Glaeser and Resseger, 2010; Combes, Duranton, Gobillon, Puga and Roux, 2012; Baum-Snow and Paven, 2013; Davis and Dingel, 2017) have indicated strong correlations between socio-economic quantities and sizes of cities (e.g., wages, education level, gross domestic product, industrial diversity, number of patents produced for positive, amount of crime, and the level of traffic congestion for negative correlations). This section presents one of the clearest representations of such correlations by focusing on industrial location (Mori, Nishikimi and Smith, 2008).

Let $\mathcal{I}$ be the set of all industries that operate in at least one of the cities, and for a given industry $i \in \mathcal{I}$, call a city *a choice city* of this industry if industry i is in operation in the city. These choice cities exhibit a systematic variation in their average population size across industries. To see this, denote by $\mathcal{U}_i\,(\subseteq \mathcal{U})$ the set of all choice cities of industry $i \in \mathcal{I}$, then the average size of choice cities for industry i is given by

$$\bar{s}_i = \frac{1}{\#\mathcal{U}_i} \sum_{i \in \mathcal{U}_i} s_i \,, \tag{4}$$

where $\#\mathcal{U}_i$ means the cardinality of set $\mathcal{U}_i$.

Now, consider three-digit secondary and tertiary industries of the Japanese Standard Industrial Classification (JSIC) that are present both in 2000 and 2015. Of all the 237 such industries, there are 162 and 175 industries that have at least one establishment in cities in 2000 and 2015, respectively.[13] Figure 3 shows the relationship between $\bar{s}_i$ and N_i for $i \in \mathcal{I}$ in log scale, where $N_i \equiv \#\mathcal{U}_i$. The dashed curves indicate the upper and lower bound for the average size of choice cities in 2015, where for each $i \in \mathcal{I}$, the former (latter) is the average size of the largest (smallest) N_i cities.

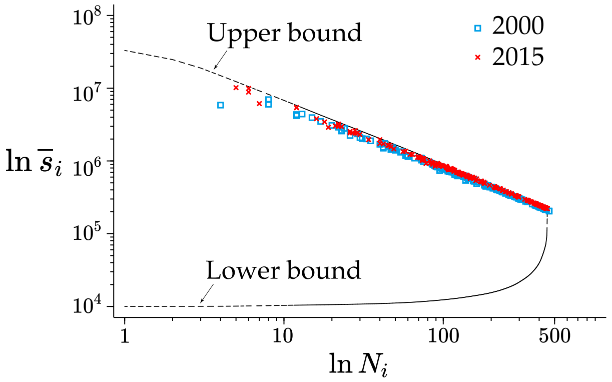

Figure 3: Varieties of economic activities and their choice of cities

There are two key features in these plots. First, the number N_i and average size $\bar{s}_i$ of choice cities exhibit a strong *power law*, which is persistent between 2000 and 2015. Second,

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[13] Data for the locations of establishments were obtained from Statistics Bureau, Ministry of Internal Affairs and Communication of Japan (2001, 2014).

the average sizes of choice cities are almost hitting their upper bound, meaning that the choice cities of an industry $i \in \mathcal{I}$ is roughly the largest N_i cities, which in turn implies that there is roughly a *hierarchical relationship* in the industrial composition between a larger and a smaller cities.[14]

To see this, let $\mathcal{I}_i$ represent the set of industries that are present in city $i \in \mathcal{U}$, and for cities i and $j \in \mathcal{U}$ such that $s_i > s_j$, define the *hierarchy share* for city j with i by

$$H_{ij} = \frac{\#\left(\mathcal{I}_i \cap \mathcal{I}_j\right)}{\#\mathcal{I}_j} \in [0,1]\,, \tag{5}$$

where a larger value of H_{ij} indicates a higher consistency with the hierarchical relationship, and $H_{ij} = 1$ means the perfect hierarchical relationship, i.e., $\mathcal{U}_j \subseteq \mathcal{U}_i$. The average values of the hierarchy shares for all the relevant city pairs,

$$H \equiv \frac{1}{\overline{H}} \sum_{i,j \in \mathcal{U} \text{ s.t. } s_i > s_j} H_{ij} \ \in [0,1] \tag{6}$$

where $\overline{H} \equiv 1/\#\{(i,j) \,:\, i,j \in \mathcal{U},\, s_i > s_j\}$, can be taken as an aggregate measure of spatial coordination among industries. A larger value of H indicates a higher degree of spatial coordination, and the coordination is perfect if $H = 1$. The actual values of H are 0.76 and 0.80 in 2000 and 2015, respectively, which are quite high.[15]

Together with the central place pattern discussed above (see Figure 1), the fact that the spatial coordination of diverse economic activities leads to the diversity in city size has already been suggested informally by Christaller (1933) and Lösch (1940).

A large value of H as in the case of Japan above means that it is not only that industries have different number of agglomerations (i.e., choice cities), but also that their locations tend to coincide, i.e., a more localized industry choose to locate in cities in which a more ubiquitous industries are present. The case of perfect coordination (i.e., $H = 1$) corresponds to the *hierarchy principle* in Christaller (1933).

To close this subsection, it is worth pointing out that while there is a strong tendency of hierarchical relation in the industrial composition between a larger and a smaller cities, it is by no means the rule. Figure 4 shows the distribution of H_{ij} of all the relevant city pairs in 2015. While the mean value is $H = 0.80$, the standard deviation is 0.13, and the range is from 0.18 to 1. Low hierarchy shares are realized for *specialized cities* in which only a limited specific set of industries are concentrated.

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[14] These features are first recognized by Mori et al. (2008); Mori and Smith (2011) for the case of Japan, and Hsu (2012, Appendix A1) and Schiff (2014) for the case of the US. See also Davis and Dingel (2017) for an evidence of the hierarchical industrial structure of the US cities based on an alternative approach.

[15] These values are much higher than the values of H that can be realized under random location of industries after controlling for the industrial diversity (i.e., $\#\mathcal{I}_i$ for $i \in \mathcal{U}$) of cities and locational diversity (i.e., $\#\mathcal{U}_i$ for $i \in \mathcal{I}$) of industries (see, e.g., Mori et al., 2008; Mori and Smith, 2011; Mori, 2017).

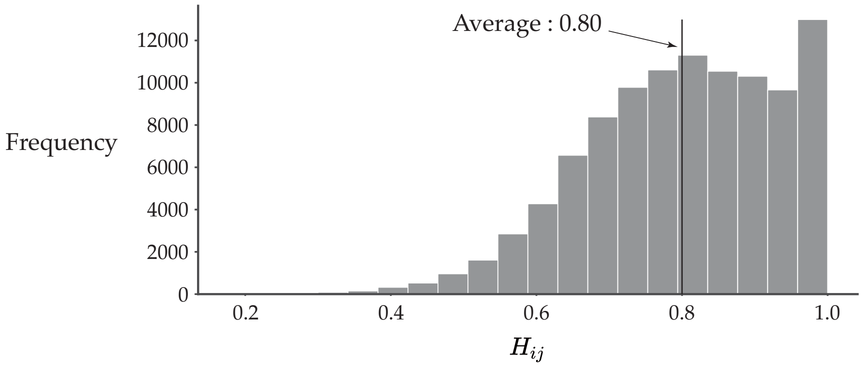

Figure 4: Distribution of hierarchy share in 2015

2.4 Growth of cities

Finally, we look at the characteristics of the growth of individual city sizes in Japan between 1970 and 2015. It is of particular interest to quantify the evolution of city sizes in this period, since it coincides with the period in which the highway and high-speed railway networks were developed almost from scratch to the extent that covers almost the entire nation, where the total highway (high-speed railway) length increased from 879 km (515 km) by more than 16 (10) times to 14,146 km (5,350 km).

The level of interregional transport access has been one of the main key parameters to determine the spatial pattern of cities in the literature. The evolution of the sizes of individual cities should presumably reflect the response to the substantially improved interregional transport access, although the benefit experienced by each city may have varied depending on their relative location. Thus, it is an ideal test case for the models of the size and spatial pattern of cities.

There was substantial movement of population among cities in these 45 years. In particular, there is a clear tendency of *global agglomeration* toward a smaller number of cities, as the number of cities has decreased from 503 to 450.[16]

Figure 5 reveals the thee more key facts about the change in individual city sizes for the 302 cities that have remained throughout the entire period. Panel (a) adds another evidence for global agglomeration: the size of the remained cities in terms of population share (in the country) has grown by 21% on average.[17] Note that it is more meaningful to look at the population share of a city rather than the population size itself to understand the tendency of global agglomeration, because the population shares remove the effects of general population growth and/or urbanization from the population sizes.[18]

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[16] Cities may emerge, disappear, split and merge over time. Cities identified in the consecutive two years are considered to represent the same city if they mutually account for the largest population among all the overlapping cities.

[17] "S.D." in the panels means the standard deviation.

[18] Overman and Ioannides (2001) have shown evidence that there is mild tendency of the decrease in population size of relatively large cities (i.e., metropolitan areas with urban core of at least 50,000 population) of the US for the period 1920-1980. Their result is not directly comparable to the case of Japan here, since their results may be biased for relatively large cities, and the factors driving city sizes during the studied

Despite the tendency of global agglomeration, there is also a clear tendency of *local dispersion* as the areal size of an individual city has almost doubled (Panel, b), while the population density has decreased by 22% on average (Panel, c).

Keep in mind that agglomeration and dispersion taking place exhibit opposite tendency at global and local spatial scales, i.e., *agglomeration took place at the global scale, while dispersion took place at the local scale*.[19]

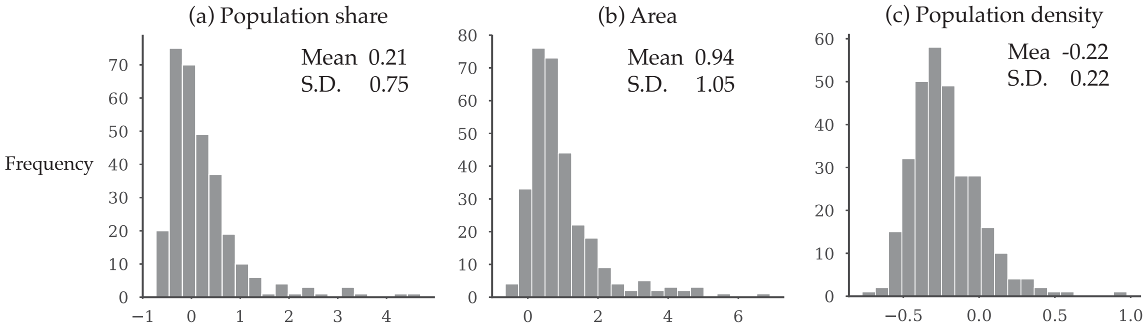

Figure 5: Changes in the sizes of individual cities in Japan between 1970 and 2015

3 Theories

A model capable of explaining the spatial patterns of cities necessarily involves many regions with large variations in interregional distance, such that some cities are close to while others are far from one another. But, the majority of the extant models adopt either two-region or systems-of-city setups in which there is no variation in interregional distance, and thus no meaningful spatial patterns can be expressed by these models.

A recent work by Akamatsu et al. (2018) brought a breakthrough by showing that a wide variety of extant models of endogenous agglomeration can be reformulated in a many-region setup, and can be formally analyzed in a unified framework. More specifically, for a canonical model, i.e., a static model with a continuum of homogeneous mobile agents, each of whom chooses a single location, these reformulated models were shown to boil down to one of the three reduced forms in terms of the spatial pattern of agglomeration and dispersion. A crucial restriction in these models is that there is no variation in the degree of increasing returns. Thus, these models cannot account for the large diversity in city sizes as in reality. Yet, they can explain some essential characteristics of the spatial patterns of cities in reality discussed in Sections 2.1 and 2.4.

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period were not made clear.

[19] The suburbanization in response to the decrease in interregional transport access is one realization of local dispersion, and its evidence for the case of the US metro areas has been reported by Baum-Snow (2007, 2017). For the global agglomeration and dispersion, no clear consensus has been attained at this point in the extant literature (e.g., Duranton and Turner, 2012; Faber, 2014; Baum-Snow, 2017). This is rather evident from the discussion in Section 3 below that the effect of interregional transport access on each individual city size is not monotonic. See Akamatsu et al. (2018, §6) for an extensive discussion on this respect. Ioannides and Overman (2004) examined the change in the distance from each city to its nearest neighbor, and found it was decreasing in the period of 1900 to 1990, which should essentially imply global dispersion. But, there is no discussion on the potential causes of this change in their paper.

Drawing largely from Akamatsu et al. (2018), Section 3.1 reviews the spatial patterns of cities that result from the mechanisms incorporated in the extant models if they were formulated in a many-region setup. To account for the diversity in city sizes and their relation to the spatial pattern of cities, a model typically needs to incorporate variations in the degree of increasing returns. At present, there are only a handful of models that have succeeded in such extensions. These models can account for certain aspects of the observed relationship between spatial pattern of cities and sizes as well as industrial structure of cities as discussed in Sections 2.2 and 2.3. Section 3.2 reviews the theoretical development in this direction.

3.1 Spatial pattern of cities

The most canonical form of the extant theoretical models for city formation (or more generally agglomeration formation) is static and involves a single type of mobile agents subject to the same degree of increasing returns. Many of them were developed in a two-region setup. By formalizing and generalizing the idea proposed by Krugman (1996, Ch.8) based on Turing (1952), however, Akamatsu, Takayama and Ikeda (2012) have shown that in most cases, they can be formally analyzed in a many-region setup by utilizing a symmetric racetrack geography with the help of discrete Fourier transformation.

Using their analytical framework, Akamatsu et al. (2018) have shown that a wide variety of the extant models can be classified in terms of the three distinct reduced forms, despite the difference in their specific mechanisms underlying agglomeration and dispersion. Below, I start by describing the basic setup of this approach.

Basic setup

Consider the location space consisting of a set of K discrete regions, $\mathcal{K} \equiv \{0, 1, \ldots, K-1\}$. There is a continuum of homogeneous mobile agents whose regional distribution is denoted by $\boldsymbol{h} \equiv (h_i)_{i \in \mathcal{K}}$, where h_i is the mass of mobile agents located in region i. Their total mass is a given constant, $H \equiv \sum_{i \in \mathcal{K}} h_i$. All regions in $\mathcal{K}$ are featureless and are placed at an equal interval on a circle. In this racetrack economy, transportation is possible only along the circumference.[20]

Let region index $0, 1, \ldots, K-1$ represent the location on the racetrack in clockwise direction. Transport costs take iceberg form, i.e., if a unit of product is shipped from region i to j, then only the fraction $d_{ij} = d_{ji} \in [0, 1)$ reaches j. The *spatial discounting matrix*, $D = [d_{ij}]$, encapsulates the underlying distance structure of the economy. Typically, iceberg costs are expressed as $d_{ij} = \exp[-\tau \ell_{ij}]$, where ℓ_{ij} is the distance between regions i and j

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[20] The racetrack location space is obviously counterfactual, as it is edge less. Although the presence of the edge tends to make the agglomeration on the edge larger, since there is no competing agglomeration beyond the edge (see, e.g., Fujita and Mori, 1997; Ikeda et al., 2017b), this effect becomes negligible for a large economy, and the agglomeration patterns can be approximated by that in the edge-less economy.

and $\tau \in (0,\infty)$ is the transport technology parameter.

Given the spatial distribution $\boldsymbol{h}$ of agents, the payoff of choosing each region is determined, where the *short-run* payoff function is denoted by $\boldsymbol{v}(\boldsymbol{h}) \equiv (v_i(\boldsymbol{h}))_{i\in\mathcal{K}}$, with $v_i(\boldsymbol{h})$ representing the payoff for an agent located in region $i \in \mathcal{K}$. The relocation of agents is assumed to be much slower than market reactions, so that the short-run equilibrium conditions (such as market clearing and trade balance) determine the payoff (utility or profit) in each region as a function of the regional distribution of agents, $\boldsymbol{h}$.

In the *long-run*, agents are mobile and are free to choose their locations to improve their own payoffs. In (*long-run*) *equilibrium*, it must hold that $v^* = v_i(\boldsymbol{h})$ for all regions i with $h_i > 0$, and $v^* \geq v_i(\boldsymbol{h})$ for any region i with $h_i = 0$, where v^* is the equilibrium payoff level.

A change in endogenous agglomeration pattern is treated as an instance of bifurcation of an equilibrium. To address the stability of equilibria, a standard approach in the literature is to introduce equilibrium refinement based on *local stability* under myopic evolutionary dynamics, where the rate of change in the number of residents h_i in region i is modeled on the basis of the regional distribution of agents, $\boldsymbol{h}$, and that of payoff, $\boldsymbol{v}(\boldsymbol{h})$. Let a deterministic dynamic be denoted by $\dot{\boldsymbol{h}} = \boldsymbol{F}(\boldsymbol{h},\boldsymbol{v}(\boldsymbol{h}))$, where $\dot{\boldsymbol{h}}$ represents the time derivative of $\boldsymbol{h}$, and assume that (i) $\boldsymbol{F}$ satisfies differentiability with respect to both arguments, (ii) agents relocate in the direction of an increased aggregate payoff under $\boldsymbol{F}$, (iii) the total mass of agents is preserved under $\boldsymbol{F}$, and (iv) any spatial equilibrium is a rest point of the dynamic, i.e., if $\boldsymbol{h}^*$ is an equilibrium, it must hold that $\dot{\boldsymbol{h}} = \boldsymbol{F}(\boldsymbol{h}^*,\boldsymbol{v}(\boldsymbol{h}^*)) = \boldsymbol{0}$. The stability of an equilibrium then is defined in terms of asymptotic stability under $\boldsymbol{F}$.

Formation of a city

With a racetrack geography, the uniform distribution of mobile agents is always an equilibrium when the payoff function is symmetric across regions. Call an equilibrium with uniform distribution a *flat-earth equilibrium*, and denote it by $\bar{\boldsymbol{h}} \equiv (h,h,\ldots,h)$ with $h \equiv H/K$.

If the adjustment dynamic is formulated so that the agents migrate in order to maximize their payoff, it follows (Akamatsu et al., 2018, Appendix B) that each eigenvalue of Jacobian matrix $\boldsymbol{J}$ of $\boldsymbol{F}$ and that, $\nabla\boldsymbol{v}$, of $\boldsymbol{v}$ are real, and have a perfect positive correlation at the flat-earth equilibrium. What remains is to identify the direction of the bifurcation at the flat-earth equilibrium, which is equivalent to find the particular eigenvalue (and the corresponding eigenvector) of $\nabla\boldsymbol{v}(\bar{\boldsymbol{h}})$ that first changes its sign from negative to positive.

The sign of the k-th eigenvalue of $\nabla\boldsymbol{v}(\bar{\boldsymbol{h}})$ has been shown to coincide with the sign of the model-specific function of the form:

$$G(f_k) = c_0 + c_1 f_k + c_2 f_k^2, \tag{7}$$

where c_0, c_1 and c_2 are the constants specific to a given model, and f_k is the k-th eigenvalue of the spatial discounting matrix D which is known to be real, and common to all models.

The eigenvector associated with f_k is given by $\boldsymbol{\eta}_k = (\eta_{k,i}) = (\cos[\theta k i])$ for $i \in \mathcal{K}$ with $\theta \equiv 2\pi/K$, and the bifurcation from the flat-earth equilibrium takes place in the direction given by $\boldsymbol{h} = \bar{\boldsymbol{h}} + \epsilon\boldsymbol{\eta}_k$ with $\epsilon > 0$.

The value k here coincides with the number of equidistant regions toward which mobile agents migrate the most. For example, at $k = K/2$, the value $\eta_{K/2,i}$ of each element $i \in \mathcal{K}$ in eigenvetor, $\boldsymbol{\eta}_{K/2}$, is given as depicted for the case of $K = 16$ in Figure 6(a), so that agglomerations start to form at alternate regions, $0, 2, 4, \ldots, K-2\,(=14)$.[21] At $k = 1$, as depicted in Figure 6(b), an unimodal agglomeration will form around region 0.[22]

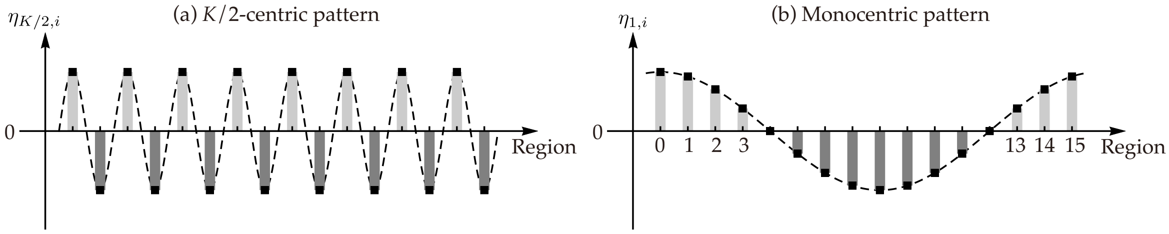

Figure 6: Agglomeration formation at high and low transport costs[23]

There are two key properties of $f_k's$ for the stability of the flat-earth equilibrium:

1. f_k is monotonically increasing in transport cost, τ.
2. $f_1 = \max_{k=1,2,\ldots,K} f_k$ and $f_{K/2} = \min_{k=1,2,\ldots,K} f_k > 0$.[24,25]

Canonical models typically have a positive value of c_1. Since $f_1 > 0$, it means that the second term in (7) represents the agglomeration force, as it works to destabilize the flat-earth equilibrium. In these models, if a stable flat-earth equilibrium exists, then one must have either $c_0 < 0$ or $c_2 < 0$, or both, so that all the eigenvalues of $\nabla \boldsymbol{v}(\bar{\boldsymbol{h}})$ become negative at the flat-earth equilibrium. In particular, the flat-earth equilibrium is stable for a small transport cost if $c_0 < 0$, and for a large transport cost if $c_2 < 0$.

The bifurcation from the flat-earth equilibrium leading to the city formation under $c_0 < 0$ and that under $c_2 < 0$ are, however, qualitatively different in two aspects. The first aspect is the level of transport costs at which the bifurcation takes place. The bifurcation under $c_0 < 0$ takes place when transport costs are sufficiently low, whereas that under $c_2 < 0$ when they are sufficiently high, since f_k is positive and increasing in τ for each $k = 1, 2, \ldots, K-1$.

The second aspect is the form of agglomeration that arises in the two cases. Under high transport costs, the bifurcation takes place in the direction of $\eta_{K/2}$ as depicted in Figure

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[21] The set of regions at which agglomerations take place may be $1, 3, \ldots, K-1$ equally likely.

[22] Of course, the agglomeration can equally take place around any region other than 0.

[23] This figure is the replication of Akamatsu et al. (2018, Figure 3).

[24] f_0 whose corresponding eigenvector is $\eta_0 = (1, 1, \ldots, 1)$ is irrelevant for the stability of equilibria as the total mobile population is preserved.

[25] For simplicity, it is assumed that K is an even integer, although it is not essential.

6(a), i.e., every other region along the racetrack attracts in-migration of mobile agents. The regional distribution of mobile agents that arises in this bifurcation is $\bar{\boldsymbol{h}} + \epsilon \boldsymbol{\eta}_{K/2}$ (for $\epsilon > 0$) as illustrated in Figure 7(a). Under the low transport costs, it takes place in the direction of $\boldsymbol{\eta}_1$ as depicted in Figure 6(b) to form a unimodal agglomeration around a single region. The regional distribution of mobile agents that arises in this case is given by $\bar{\boldsymbol{h}} + \epsilon \boldsymbol{\eta}_1$ as illustrated in Figure 7(b).

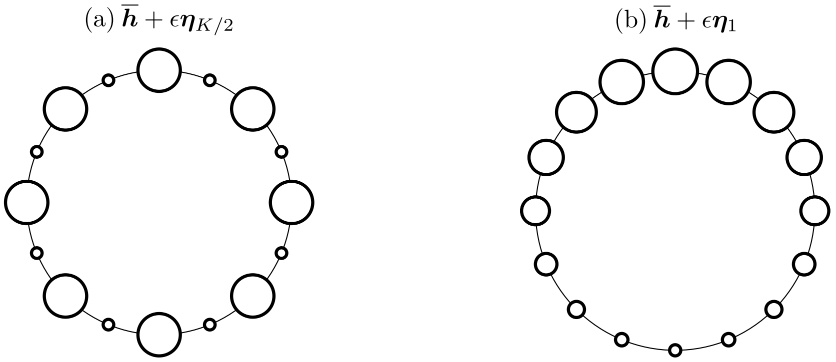

Figure 7: City formation at high and low transport costs[26]

Here, the crucial difference is the source of dispersion force. The one with $c_2 < 0$ which is *dependent* of the distance structure of the economy generates *global* dispersion force, resulting in the *multimodal* agglomeration. On the contrary, the one with $c_0 < 0$ which is *independent* of the distance structure of the economy generates *local* dispersion force, resulting in the *unimodal* agglomeration.

In Akamatsu et al. (2018), the models with only global dispersion force are called Class (i). The models of this class exhibit *period doubling bifurcations* as transport costs decrease, leading to *a smaller number of larger cities with a larger spacing between neighboring cities, until all mobile agents concentrate in one region* (Figure 8a).[27] The models with only local dispersion force are called Class (ii). These models involve only one bifurcation when the flat-earth equilibrium looses stability. *Keeping unimodal regional distribution, the concentration of mobile agents proceeds as transport costs increases, until all mobile agents concentrate in one region* (Figure 8b). The models that incorporate both types of dispersion force are of course the most realistic, and can account for the formation of multiple cities with a positive internal space (Figure 8c). These are called Class (iii).

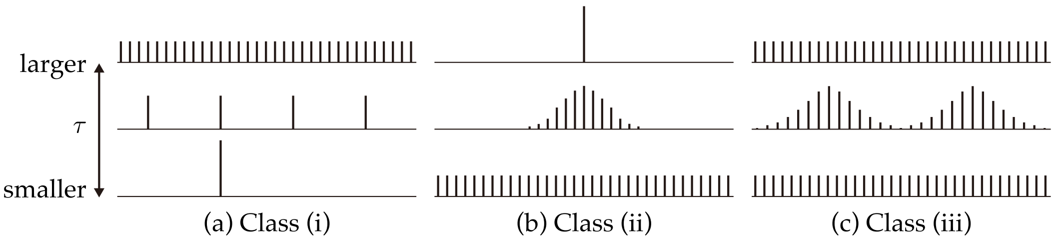

Figure 8: Spatial patterns of cities

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[26] This figure is the replication of Akamatsu et al. (2018, Figure 5).

[27] See Akamatsu et al. (2012) for the formal analyses on the period doubling bifurcations of class (i) models.

Two implications are worth mentioning. First, the heterogeneity among interregional distances is an essential feature of a model to investigate the spatial pattern of cities. In the context of a two region model or a systems-of-city model in which there is no variation in interregional distance, the dispersion of mobile agents in Class (i) and Class (ii) models look exactly the same. But, as indicated by the middle panels of Figure 8(a)(b), these are qualitatively different in spatial scale. The dispersion takes place at the global scale in Class (i) models – in the form of an increase in the number of cities, and at the local scale in Class (ii) models – in the form of a larger spatial extent of a city.

Second, the responses of agglomeration/dispersion to the level of transport costs are *opposite* between global and local spatial scales. More specifically, given the lower interregional transport costs, the agglomeration proceeds at global scale, i.e., the number of cities decrease, the sizes and the spacing of the remaining cities increase, while the dispersion proceeds at local scale, i.e., the population density within a city decrease and the spatial extent of a city increases.[28]

Below, I overview a variety of extant models that fall into one of these three classes, as well as those do not.

New economic geography

NEG (e.g., Fujita, Krugman and Venables, 1999a) commonly utilizes the monopolistic competition together with scale economies in production to explain the endogenous agglomeration. On the one hand, the love for product variety by consumers and the presence of transport costs give an incentive for consumers to locate closer to firms. On the other hand, each indivisible firm subject to scale economies at the plant level has an incentive to locate and supply near consumers.[29]

In this context, the global dispersion force is introduced typically by assuming immobile consumers in each region who generate dispersed demand for the differentiated products (e.g., Krugman, 1991, 1993; Forslid and Ottaviano, 2003; Pflüger, 2004). The assumption of immobility of consumers is nothing but simplification to assure the dispersed demand. It can be obtained endogenously, for example, by introducing land-intensive sectors that also require labor inputs (e.g., Fujita and Krugman, 1995; Puga, 1999), which in turn generates dispersed demand from workers. With transport costs, the proximity to demand matters, and hence, the spatial dispersion of consumers results in the formation of multiple cities, where the firms in each city mainly serves their nearby local market. This distance dependent dispersion force results in the global dispersion force associated with $c_2 < 0$ in (7).

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[28] Of course, the actual evolution of the spatial patterns under the changing level of transport costs is more complicated, as neighboring cities may eventually merge in the case of Class (iii) models. See, Akamatsu et al. (2018, §5.3).

[29] An alternative formulation assumes the product variety in intermediate goods. See, e.g., Fujita et al. (1999a, Ch.14).

The local dispersion force is introduced by assuming consumption of locally scarce land (e.g., Helpman, 1998; Redding and Sturm, 2008; Redding and Rossi-Hansberg, 2017), sometimes together with commuting costs (e.g., Murata and Thisse, 2005).[30] All these costs of concentration are confined within a given region, and are independent of interregional distance. Hence, the dispersion in this case realizes as the spatial sprawl of a given city, rather than the formation of new distinct cities, resulting from the local dispersion force associated with $c_0 < 0$ in (7).

There are models that incorporate both global and local dispersion forces above (Tabuchi, 1998; Pflüger and Südekum, 2008), i.e., of Class (iii) with $c_0 < 0$ and $c_2 < 0$ in (7). While these themselves treat only the two-region case, their many-region extensions can generate a more realistic spatial pattern of cities that involve both global and local dispersion as shown in Figure 8(c) (see Akamatsu et al., 2018, §5.3).[31]

Social interactions model

In the 1970s and 1980s, there were a series of attempts to explain endogenous formation of the central business districts (CBD) *within a city*. The development of the models of this type was initiated by Solow and Vickrey (1971) and Beckmann (1976), then followed by several others (e.g., Borukhov and Hochman, 1977; O'Hara, 1977; Ogawa and Fujita, 1980; Fujita and Ogawa, 1982; Imai, 1982; Tauchen and Witte, 1983; Tabuchi, 1986; Fujita, 1988; Kanemoto, 1990; Fujita, 1990).

In these models, the formation of CBD is explained by introducing positive technological externalities generated from the interaction between each pair of individual agents. While the above mentioned models vary in the specification of positive externalities, Fujita and Smith (1990) have shown that their formulations are essentially equivalent, and reformulated commonly by the so-called *additive interaction function*, $S_i(\boldsymbol{h}) \equiv \sum_{j \in \mathcal{K}} d_{ij} h_j$.

In the simplest specifications, this additive interaction function enters the utility function of consumers directly. Most models assume land consumption by mobile agents, while the production sector is abstracted, i.e., they incorporate only local dispersion force, and hence belong to Class (ii). One exception is Takayama and Akamatsu (2011) who also included global dispersion force by introducing mobile firms and immobile consumers in each region. This model thus contains both local and global dispersion force, i.e., of Class (iii).[32]

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[30] A similar effect can be obtained by assuming local congestion externality that is effective within a given region.

[31] NEG models adopting transport costs that are not iceberg form are not studied in Akamatsu et al. (2018). But, it is known that they can also be classified according to the spatial scale of dispersion. For example, Ottaviano et al. (2002) and Tabuchi et al. (2005), both of which adopt additive transport costs, belong essentially to Class (i) and Class (ii), respectively (see Akamatsu et al., 2018, §3.1).

[32] The social interactions model by Picard and Tabuchi (2013) with non-iceberg transport costs can be shown to belong to Class (iii) (see Akamatsu et al., 2018, §3.1).

Other relevant models

In the NEG literature, a particularly important deviation from the canonical models is to consider different transport cost structures by industry. For example, Fujita and Krugman (1995) included transport costs for (urban) differentiated products as well as land-intensive (rural) homogenous products. In the presence of rural goods that are costly to transport, the delivered price for such goods is lower in regions farther away from cities, which generates a dispersion force. This is similar to the local dispersion force in that even a small deviation from an urban agglomeration will decrease the price of rural goods and increase the payoff of the deviant. However, the advantage of dispersion persists outside the agglomeration, i.e., it depends on the distance structure of the model. This type of dispersion force has been shown to result in the formation of an *industrial belt*, a continuum of agglomeration associated with multiple atoms of agglomeration as demonstrated by the simulations in Mori (1997) and Ikeda, Murota, Akamatsu and Takayama (2017b). The formal characterization of industrial belts, however, remains to be carried out.

Some attempts have been made to develop a systematic framework to study spatial patterns of agglomeration in a two-dimensional geography, utilizing *group-theoretic bifurcation theory* and the *computational bifurcation theory* (e.g., Ikeda et al., 2012a, 2014, 2017a). Their results suggest that the spatial pattern of cities in a two-dimensional geography cannot be fully understood as a straightforward extension of that in the one-dimensional geography, although the basic results of the one-dimensional space persist. More systematic (and formal if possible) characterization in two-dimensional models thus remains to be carried out.

Among the extant social-interactions models, some distinguish location incentives between firms and consumers/workers unlike the canonical models discussed above (e.g., Ogawa and Fujita, 1980; Fujita and Ogawa, 1982; Ota and Fujita, 1993; Lucas and Rossi-Hansberg, 2002). This distinction is especially crucial for explaining the location patterns within a city, while it may be less relevant for the purpose of explaining the spatial pattern of cities. At present, little formal results have been obtained regarding the spatial pattern of cities that arise in these models (see Osawa, 2016, for the recent theoretical development in this direction.)

Other relevant models that were not covered so far include the *spatial oligopoly* models designed to explain the agglomeration of retail stores (e.g., Wolinsky, 1983; Dudey, 1990; Konishi, 2005). In these models, consumers have imperfect information on the types and prices of goods sold by stores before they visit them. The greater the agglomeration of stores, the more likely it is that consumers will find their favorite commodities. The concentration of stores is explained by the market-size effect due to taste uncertainty and/or lower price expectation. The dispersion force is global one, i.e., the exogenous and spatially dispersed demand. Thus, these models are expected to behave similarly to Class (i) models above, although no extensive analyses have been conducted in this direction (see Konishi, 2005, §5, for the discussion on the spacing of retail clusters).[33]

3.2 Diversity in city size

To account for the large diversity in city size observed in reality, the models need to incorporate diversity in increasing returns. While Class (i) models with a global dispersion force discussed above can account for the formation of multiple cities, there is little variation in the sizes of cities to be realized in equilibrium, since each model has only one type of increasing returns.

Initial formal attempts to account for the diversity in increasing returns by introducing multiple increasing returns industries have been made by Beckmann (1958). But, his model lacked microeconomic foundation. Later the models with more explicit mechanisms were developed by Fujita, Krugman and Mori (1999b); Tabuchi and Thisse (2011) in the context of the NEG, and by Hsu (2012) in the context of spatial competition model. In these models, the different degrees of increasing returns among industries result in the different spatial frequencies of agglomeration among industries.

The key to account for the diversity in city size is the *spatial coordination* of agglomerations among industries through inter-industry demand externalities that arise from common consumers among industries. An industry subject to a larger increasing returns agglomerate in a smaller number of cities that are far apart. What is crucial is that these cities are chosen from the ones in which more ubiquitous industries subject to smaller increasing returns are located. Consequently, larger cities are formed at the location in which the coordination of a larger number of industries takes place. This spatial coordination of industries accounts for the positive correlation between the size, spacing and industrial diversity of a city as observed in reality (Sections 2.1 and 2.3).

In particular, Hsu (2012) proposed a unique spatial competition model with product differentiation and scale economies in production, and provided at this point the most far reaching formal explanation for the spatial pattern of cities and the diversity in city size. When the distribution of scale economies in production of each firm (which is expressed in terms of the industry-specific fixed cost for production in his model) is *regularly varying*, then his model replicates the power law for city size distribution (Section 2.2) together with the positive correlation between size and spacing of cities (Section 2.1), the power law for the number and the average size of choice cities of industries (Section 2.3), as well as the hierarchy principle observed in Japan (Section 2.3).

Alternatively, Desmet and Rossi-Hansberg (2009), Desmet and Rossi-Hansberg (2014), Desmet and Rossi-Hansberg (2015), Desmet et al. (2017) incorporated dynamic externalities through endogenous innovation and spillover effects. These models are fundamentally different from all the models discussed so far in that the exogenous heterogeneity

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[33] See Economides and Siow (1988) for a related model that explains the spacing of market areas in which markets are formed due to matching externalities that arise in the exchange of consumption goods.

among regions are essential for city formation, i.e., agglomerations do not form spontaneously. The uneven distribution of mobile agents result as the exogenous regional heterogeneity is magnified by the spillover effects over time. One exception in this strand of literature is Nagy (2017) who incorporated the same dynamic externalities into the NEG framework, so that his model is capable of explaining the spontaneous formation of multiple cities together with the diversity in city sizes. While this model has been applied to replicate the evolution of the US cities in 19th century, its basic properties have not been formally analyzed.

4 Concluding remarks

This paper reviewed the models for the spatial pattern and sizes of cities. A many-region geography with variations in interregional distance is an essential feature of a model, if the spatial pattern of cities were the subject of the study. Naturally, there have been very few formal attempts that explicitly dealt with this high-dimensional problem until recently with notable exceptions by Hsu (2012).

A breakthrough has been brought about by Akamatsu et al. (2012) who proposed to focus on the racetrack economy which involves many regions with heterogeneous interregional distances, while preserving symmetry among the regions. By utilizing the discrete Fourier transformation, they have demonstrated that the spatial patterns of agglomeration that arie in the NEG models in a many region setup can be formally analyzed to a large extent. The same group of researchers have also developed the framework for systematic numerical analysis on a many-region geography based on the *numerical bifurcation theory* and *group-theoretic bifucation theory* (e.g., Ikeda, Akamatsu and Kono, 2012b; Ikeda et al., 2017b). Their numerical approach makes it possible to explore asymmetric geography (e.g., the presence of edges and heterogeneity in regional advantages) as well as two-dimensional location space in a many-region setup.

In this paper, drawing largely from Akamatsu et al. (2018) which applied the analytical tool developed by Akamatsu et al. (2012) to a wide variety of extant agglomeration models, I have reviewed the spatial pattern of cities and its relation to city sizes implied by these models. But, Hsu (2012) continues to be the only tractable model that can account for the large diversity in city size in association with the observed spatial pattern of cities. Thus, much to be expected in the future development in this respect.

Finally, no models so far have been successful in integrating intra- and inter-city space. In the models aiming to explain intra-city spatial patterns, the location behavior of firms and that of workers are typically distinguished, and land consumption and/or land inputs by firms together with commuting are considered (e.g., Fujita and Ogawa, 1982; Ota and Fujita, 1993; Lucas and Rossi-Hansberg, 2002; Picard and Tabuchi, 2013). The models aiming to explain inter-city spatial patterns, on the contrary, typically ignore different location incentives between firms and workers (all models discussed in this paper belong to this group). But, it is not trivial to integrate these two spatial scales in one model.

Some extant NEG models consider commuting and land consumption (e.g., Anas, 2004; Murata and Thisse, 2005). But, such urban structure is by assumption confined within a given region, and does not extend beyond a single region. As is discussed in Section 3.1, in a many-region geography with variations in interregional distance, these models belong to Class (ii), which means that at most unimodal agglomeration forms. Although each region in these models has monocentric urban structure *by assumption*, and hence, it is tempted to be interpreted as a "city", these model can generate essentially at most one "true" city.

To fully account for the spatial pattern of cities, the distinction between inside and outside each city should also be endogenized.

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