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"Communication Enhancement through Information Acquisition by Uninformed Player"

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Communication Enhancement through Information Acquisition by Uninformed Player*

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Abstract

We analyze a situation in which an uninformed decision maker can gather information about states by paying a cost before communicating with an informed sender. We focus on multidimensional information gathering: the decision maker can determine how much time to allocate to gather information about each state. It is shown that communication can be enhanced under multidimensional information gathering compared with no information gathering. We also characterize the optimal investigation, which specifies the state the decision maker gathers information about. Our result demonstrates an advantage of multidimensional information gathering over single-dimensional information gathering.

JEL Classification: C72; C73; D83

Keywords: Cheap talk; Communication; Multidimensional information gathering; Strategic information transmission

1. Introduction

The present paper explores the effect of information acquisition by an uninformed decision maker on the quality of information conveyed in the cheap-talk game. We analyze a variant of Crawford and Sobel [4]: the set of states of nature consists of three states;[1] the sender privately learns the

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[1] In Section 5.1, we briefly discuss a more general case with finitely many states.

realized state; the decision maker does not know the state, but by paying a cost, she can privately obtain noisy signals about the possible states before communicating with the sender.

A motivating example of our model is as follows: Imagine a situation in which a Ph.D. student (decision maker) is deciding her major field of research. There are three possibly promising fields (i.e., states): empirical (industrial organization) IO, theoretical IO, and game theory. Although it is the most favorable for the student to choose the most promising field for her research, she does not know which field it is. She can ask her supervisor (sender) for advice in choosing a field, and the supervisor knows which field is the most promising. However, the most favorable field that the supervisor would like the student to choose does not necessarily coincide with that for the student. Here, suppose that the supervisor's preference is biased toward more theoretical research.[2]

Additionally, before asking the supervisor, the student can privately gather information about which field is the most promising by reading several academic journals. The more she reads papers, the more she becomes knowledgeable about the topics covered by the journals. If the student picks one field and learns about it deeply, she will be well versed in the field. In this case, she can make a good guess about whether this field is promising, whereas she knows little about the others. In contrast, if the student learns every field evenly but shallowly, she possesses information about every state, with a lack of accuracy. That is, the information gathering is multidimensional, which is the key ingredient of our research. Naturally, if the student learns every field deeply by reading several papers in all fields, she may be able to become a generalist who knows everything. However, gathering information is time consuming and costly, and the student cannot become a perfect generalist. Furthermore, the supervisor may convey some pieces of information about which field is promising that the student does not know. Therefore, she is concerned about how much time to spend and which types of journals to read.

We also compare the case of multidimensional investigation with that of the single-dimensional investigation. Single-dimensional investigation is a standard technology, under which more investigation provides an accurate signal about true state with higher probability conditioned on each state. On the other hand, under multidimensional investigation, investigating a particular state provides an accurate signal about that state with a higher probability. We will show the advantage of multidimensional investigation over single-dimensional investigation.

[2]Precisely, if the most promising field is empirical IO, then theoretical IO is the most favorable for the supervisor. If the most promising field is theoretical IO or game theory, then game theory is the most favorable.

There are several economic examples to which our model is applicable. For instance, essentially the same problem is a situation in which a CEO (the decision maker) is interested in the profits of the company, and contrarily, the marketing section (the sender) has a bias toward the showiness of projects. The CEO does not just swallow reports the marketing section submits but can collect information about states by herself or through her direct subordinates.

In the analysis, we confine our attention to a set of parameters under which the decision made by the decision maker never depends on the messages from the sender in any equilibrium when the decision maker cannot acquire information about states at all. Our main result is that information acquisition by the decision maker can enhance communication. Namely, the information conveyed by the sender is more detailed when the decision maker can acquire imperfect information about the states than when she cannot. Information gathering by the decision maker has two effects. The direct one is that the decision regarding project choice improves. This effect is referred to as an *information effect* in Moreno de Barreda [13], which is the research most closely related to ours. The indirect effect is that the direct effect ameliorates communication. As a result, it is endogenously determined how the decision maker gathers information.

The key mechanism underlying the indirect effect mentioned above is as follows. Since the decision maker has imperfect and private information, messages the sender chooses induce a lottery over actions and he is exposed to risk.[3] Moreno de Barreda [13] refers to this effect as a *risk effect*. Roughly speaking, information acquisition by the decision maker enhances communication when misreporting can increase the “threat” that an unintended choice from the sender’s perspective is made.[4] To reduce the possibility that an unfavorable choice is made, the sender sends a more accurate message to the decision maker than in the case where the decision maker has no private information.

In summary, the decision maker’s information acquisition and the sender’s information transmission are mutually dependent. The more the decision maker invests in acquiring information about a particular state, the less the sender of a type has an incentive to tell a lie. Consequently, the sender has more incentive to transmit more accurate information than in the case where the decision maker does not acquire information about any state. In contrast, detailed information

[3] Under the assumption on parameters, in any equilibrium, the decision maker chooses a project independently of the sender’s messages when she cannot gather information, that is, she has no private information.

[4] This idea is also related to that in the model of *noisy communication*, i.e., Krishna and Morgan [11], Blume et al. [3], Goltsman et al. [7], and Ivanov [10]. In these studies, the risk of cheating is generated in communication with error, or noise. In contrast, in our model, this risk is generated by the decision maker’s imperfect information.

transmission weakens the incentive of the decision maker to acquire costly information about a state. Hence, the information conveyed must be coarse to some extent so that the decision maker has an incentive to acquire information on a particular state. We examine this strategic interaction and characterize the decision maker's optimal equilibrium.

Related Literature The seminal work on the cheap-talk game by Crawford and Sobel [4] analyzes unilateral information transmission by a sender. It is assumed that the sender completely knows the true state as in the present paper, but the decision maker does not obtain any information before communicating with the sender. The authors show that in any equilibrium, no full information revelation is possible, and partial information transmission is attainable. It is natural to extend the model to a setting in which the decision maker invests in collecting information about the states by herself in order to make up for imprecise message. For example, a manager of a firm seeks advice from informed experts (such as lawyers, strategic planners, or lower-level managers) but also conducts an investigation by paying a cost (such as marketing research, geological survey, or clinical trial) in order to select a project.

Under our technology of information acquisition, the decision maker obtains noisy signals about the states. Thus, our analysis is closely related to the literature on the cheap-talk game with an imperfectly informed decision maker: Chen [5], Chen [6], Ishida and Shimizu [9], Olszewski [14], and Watson [17].[5] They assume that the decision maker automatically receives private signal about the state, that is, the information structure is exogenously given. In contrast, in the present study, the information structure is endogenously determined.

In the literature on the cheap-talk game, there are only a few papers that analyze information acquisition. Austen-Smith [2], Pei [15], and Venturini [18] analyze the situation in which a sender does not know the true state and can acquire costly information about states before giving advice to the decision maker. In contrast, we investigate the strategic effect of the decision maker's information acquisition on communication.

The studies closely related to ours include Argenziano et al. [1] and Moreno de Barreda [13], which analyze models with the single-dimensional investigation present in our terminology. Argenziano et al. [1] analyze the situation in which either the decision maker or the sender can acquire

[5]Ishida and Shimizu [9] show that welfare is improved when the decision maker has imperfect signals about the states in comparison with the case of having no private information. However, it is not comparable whether the quality of information transmitted by the sender is improved. The other two studies show that the full revelation of information is possible.

costly information about states. They assume that both the decision maker and the sender do not know the true state. In addition, it is assumed that information acquisition is single-dimensional. A player can acquire information about the state by performing "trials," and the authors show how many trials to perform. The cost and precision of the acquired information depend on the number of trials. The authors show that by delegating information acquisition to a biased sender, the decision maker can improve her decision making more than when information is directly acquired by herself.

In contrast to Argenziano et al. [1], we assume that the sender completely knows the true state, but the decision maker does not. Our focus is on whether costly information acquisition by the decision maker enhances communication. Moreover, in our model, the information acquisition is multidimensional, that is, the decision maker can decide which states she gathers information about.

Moreno de Barreda [13] analyzes costly information acquisition by the decision maker although the main part of the analysis is spared in the case of free access to private information. It is also assumed that the sender completely knows the true state as in our model. However, it is assumed that the set of states of nature is a continuum. She stresses that there is a situation in which, in equilibrium, it is worth acquiring costly information, but the communication is worse in comparison with the case in which the decision maker cannot acquire information.

In contrast to Moreno de Barreda [13], we consider a set of parameters under which communication is babbling in any equilibrium when the decision maker cannot acquire information. Thus, the possibility of information acquisition never worsens communication. We emphasize that there is a situation in which the private information of the decision maker about states improves communication with the sender, and more importantly, communication provides the incentive to acquire information about states. We explain the reason multidimensional information acquisition plays an important role (cf. Sections 5 and 7).

The reminder of this paper is organized as follows. In Section 2, we describe our model and define the solution concept. Section 3 derives some properties of the equilibria. In Section 4, we study the benchmark model in which the decision maker cannot acquire information. We show that in any equilibrium, only uninformative communication takes place. Section 5 assumes that the decision maker obtains noisy signals by paying a cost and shows how the decision maker's information acquisition enhances communication. In Section 6, we analyze a situation in which the decision maker can disclose her information obtained by investigation. In Section 7, we compare our multidimensional investigation with single-dimensional investigation. In Section 8, we show that the first-best project choice for the decision maker is attainable in an equilibrium if she can detect a state by paying a cost. Section 9 provides concluding remarks.

2. Model

Here, we describe our model and then define the solution concept, which is a standard perfect Bayesian equilibrium. The basic model is a three-state version of Crawford and Sobel [4].

2.1 Cheap-Talk Game with Information Acquisition by Decision Maker

There are two players, an uninformed decision maker and an informed expert (sender). The decision maker has the authority to choose a project y from a set of projects, $Y \equiv \{0, 1, 2\}$. The payoff for each player depends on the state of nature as well as the chosen project. Let θ be a state of nature, which is distributed over $\Theta \equiv \{0, 1, 2\}$ with a prior probability distribution π. It is assumed that the prior is common knowledge between the decision maker and the sender and satisfies $\pi(\theta) > 0$ for all $\theta \in \Theta$. We impose the following condition throughout the analysis.

Assumption 1. $\pi(1) > \pi(2)$.

Under Assumption 1, there exist only equilibria in which the decision maker chooses optimal projects independently of the messages she receives when she cannot acquire information about states, which will be shown in Section 4. Furthermore, it will be shown that not all equilibria are babbling, and in an equilibrium, partial information transmission arises, but the decision maker chooses a project independently of messages.

The sender can perfectly observe the realized state, and it is his private information. After observing the state, he sends a cheap-talk message m to the decision maker, that is, sending a message is costless. Let M be an arbitrary finite set of messages available for the sender.

The decision maker cannot directly observe the realized state. However, before receiving a message from the sender, the decision maker can acquire noisy signals through her own investigation regarding the possible states.[6] For each state, she can investigate whether the state is realized or not. Let $a_s \geq 0$ be the amount of time the decision maker spends on her investigation of state s.

[6] In Section 8, we consider the case in which the decision maker can acquire complete information about a particular state if she pays a cost.

We call a_s *the level of investigation about state* s. The decision maker decides on the allocation of time, $a = (a_0, a_1, a_2)$, which is called the investigation vector. We assume that time is limited, and $a_0 + a_1 + a_2 \leq 1$. Let us denote the feasible set of the investigation vectors by

$$A \equiv \{\, a \mid a_s \geq 0,\ \forall s \in \Theta,\ \text{and } a_0 + a_1 + a_2 \leq 1 \,\}.$$

The level of investigation of each state provides a noisy signal concerning that state. By investigating state s, the decision maker obtains an imperfect signal $\xi_s \in \Xi_s \equiv \{t, f\}$ regarding state s. It is assumed below that given a positive a_s, signal $\xi_s = t$ is more likely than signal $\xi_s = f$ for $s = \theta$, whereas signal $\xi_{s'} = f$ realizes more likely than signal $\xi_{s'} = t$ for $s' \neq \theta$. After choosing $a = (a_0, a_1, a_2)$, the decision maker privately observes three signals: ξ_0, ξ_1, and ξ_2. Let $\xi = (\xi_0, \xi_1, \xi_2)$ be the signal profile, and $\Xi \equiv \prod_{s=0}^{2} \Xi_s$ be the set of signal profiles. Both a and ξ are assumed to be unobservable to the sender.[7]

Given the realized state θ and the investigation vector a, the probability that the decision maker obtains a signal ξ_s is given by

$$Q(\xi_s = t | a_s, \theta) = \begin{cases} 1/2 + \eta a_s \text{ if } s = \theta, \\ 1/2 - \eta a_s \text{ if } s \neq \theta. \end{cases}$$

Parameter η represents the sensitivity of the signals with respect to the levels of investigation. If Assumption 2 below holds, Q is well defined.

Assumption 2. $0 \leq \eta < 1/2$.

We impose this assumption except for Section 8 in which the signal structure allows the decision maker to detect the true state. Therefore, we redefine Q appropriately in Section 8.

It is assumed that the realization of ξ_s is independent of the levels of investigation of the other states and that the signals are conditionally independent. Namely, signal profile ξ realizes with a probability of $P(\xi | a, \theta) = Q(\xi_0 | a_0, \theta) Q(\xi_1 | a_1, \theta) Q(\xi_2 | a_2, \theta)$.

The decision maker's payoff is given by

$$U^D(y, a, \theta) = -(y - \theta)^2 - c \sum_{s=0}^{2} a_s,$$

[7] Note that the assumption is immaterial that investigation vector a is unobservable. We employ this assumption in order to simplify the description of strategies, and this essentially does not affect our results.

where the last term represents the total cost of information acquisition, and the constant $c > 0$ denotes the marginal cost. If the true state is observable to the decision maker, the optimal project for her would satisfy $y = \theta$ for each $\theta \in \Theta$.

The sender's payoff is given by

$$U^S(y, \theta, b) = -\{y - (\theta + b)\}^2,$$

where the bias $b > 0$ measures how much the sender's interest differs from the decision maker's. We assume that $b \in (1/2, 1)$. Therefore, the most desirable project for the sender is $y = \min\{\theta + 1, 2\}$.

We employ the quadratic function to simplify the representation of the results. The essential assumption is that the best projects for the decision maker and the sender do not coincide for states 0 and 1. The specific functional form is not qualitatively important for our results.

The timing of the game is summarized as follows:

1. Nature randomly draws a state $\theta \in \Theta$ with common prior $\pi(\theta)$, and the sender observes θ privately.

2. The decision maker chooses investigation vector a, and then she observes a signal profile ξ privately. The sender does not observe a and ξ.

3. The sender sends a message $m \in M$ to the decision maker.

4. The decision maker chooses a project $y \in Y$.

We give some intuition about the signal structure we consider. The accuracy of the signal about state s depends on the amount of time the decision maker spends on gathering information about the state. The more she spends time on gathering information about state s, the more likely she is to obtain the signal $\xi_s = t$ when s coincides with the true state, that is, $s = \theta$, and the more likely she obtains the signal $\xi_s = f$ when s does not coincide with the true state, that is, $s \neq \theta$. If the decision maker spends no time on gathering information about state s, then the signal ξ_s is completely uninformative, and it is useless when the decision maker chooses a project. This signal structure captures some property of practical investigations seen in real-life situations such as the implementation of a geological survey or an examination for a certain disease using test drugs.

In previous works (Argenziano et al. [1] and Moreno de Barreda [13]), the investigation activity was assumed to be single-dimensional. That is, it is interpreted that $a_0 = a_1 = a_2$ is satisfied for any $a \in A$. In Section 7, we show that a set of parameters exists such that communication cannot be enhanced when the investigation activity is single-dimensional but it is possible when the investigation activity is multidimensional.

We assume the timing of the game when communication takes place after the investigation by the decision maker. Even if the timing is reversed, the main implication of our analysis does not change. We will discuss this point in Remark 3 in Section 5.1.

2.2 Equilibrium Concept

The solution concept we adopt is a perfect Bayesian equilibrium: both players' strategies are optimal under the given beliefs, and the beliefs are derived from Bayes' rule whenever possible. In order to define some notations used in the next sections, we provide the formal definition of a perfect Bayesian equilibrium. First, we provide the definitions for a strategy for each player and a system of beliefs.

A mixed (behavior) strategy of the sender, $\mu : \Theta \to \Delta M$, specifies the probability distribution of messages depending on $\theta \in \Theta$, where $\mu(m|\theta)$ is the probability that the sender of type θ sends the message m. Denote by $Z(\theta) \equiv \text{supp}(\mu(\cdot|\theta))$ the set of messages that the sender of type θ sends with positive probabilities under μ.

A pure strategy of the decision maker, (a, ρ), is defined as a pair of an investigation vector a and a function ρ. The function $\rho : A \times \Xi \times M \to Y$ specifies a project she takes for each combination of investigation vector, observed profile of signals, and received message. Needless to say, the decision maker can choose any mixed strategy. When necessary, we make explicit the mixed strategies.

A system of beliefs for the decision maker, $\beta : A \times \Xi \times M \to \Delta\Theta$, specifies a probability distribution of states for each combination of investigation vector, observed profile of signals, and received message. On the other hand, we do not mention the beliefs of the sender below. Since the sender observes the true state, the sender's beliefs are obvious. As for the sender's beliefs about the investigation vector, they are uniquely derived in equilibrium.

Here is the definition of a perfect Bayesian equilibrium.

Definition 1. A *perfect Bayesian equilibrium* is a profile of strategies and a system of beliefs

$(\hat{\mu},(\hat{a},\hat{\rho}),\hat{\beta})$ that satisfy the following requirements:

1. The sender maximizes his expected payoff given the decision maker's strategy $(\hat{a},\hat{\rho})$: for each $\theta\in\Theta$, if $\hat{m}\in\hat{Z}(\theta)\equiv\text{supp}(\hat{\mu}(\cdot|\theta))$, then

$$\hat{m}\in\arg\max_{m\in M}\sum_{\xi\in\Xi}P(\xi|\hat{a},\theta)U^S(\hat{\rho}(\hat{a},\xi,m),\theta,b). \tag{1}$$

2. The decision maker maximizes her expected payoff given the sender's strategy $\hat{\mu}$ and the system of beliefs $\hat{\beta}$:

$$\hat{a}\in\arg\max_{a\in A}\sum_{(\theta,\xi,m)\in\Theta\times\Xi\times M}\pi(\theta)P(\xi|a,\theta)\hat{\mu}(m|\theta)U^D(\hat{\rho}(a,\xi,m),a,\theta); \tag{2}$$

$$\hat{\rho}(a,\xi,m)\in\arg\max_{y\in Y}\sum_{\theta\in\Theta}\hat{\beta}(\theta|a,\xi,m)U^D(y,a,\theta)\ \text{ for any } (a,\xi,m)\in\Xi\times M. \tag{3}$$

3. For each $a\in A$, each $\xi\in\Xi$, and each $m\in M$,

$$\hat{\beta}(\theta|a,\xi,m)=\begin{cases}\frac{\pi(\theta)P(\xi|a,\theta)\hat{\mu}(m|\theta)}{\sum_{\theta'\in\Theta}\pi(\theta')P(\xi|a,\theta')\hat{\mu}(m|\theta')}, & \text{if } m\in\bigcup_{\theta\in\Theta}\hat{Z}(\theta),\\ \phi(\theta), & \text{if } m\notin\bigcup_{\theta\in\Theta}\hat{Z}(\theta),\end{cases} \tag{4}$$

where ϕ is an arbitrary probability distribution over Θ.

Requirements 1 and 2 claim that $(\hat{\mu},(\hat{a},\hat{\rho}),\hat{\beta})$ is sequentially rational. Requirement 3 is a consistency condition for the system of beliefs for the decision maker. For any $m\in\bigcup_{\theta\in\Theta}\hat{Z}(\theta)$, the belief at history (a,ξ,m) can be derived from Bayes' rule even if the decision maker deviates in his investigation. For any history such that $m\notin\bigcup_{\theta\in\Theta}\hat{Z}(\theta)$, it is off the equilibrium path, and Bayes' rule is not applied. In this case, we can take an arbitrary belief $\phi\in\Delta\Theta$.

Hereafter, we call the perfect Bayesian equilibrium simply *equilibrium*.

3. Properties of Equilibria

Before moving to the main analysis, we derive some properties of equilibria.

Let $(\hat{\mu},(\hat{a},\hat{\rho}),\hat{\beta})$ be an equilibrium. We say that a message m is a *decisive message* about project y if $\hat{\rho}(\hat{a},\xi,m)=y$ for any $\xi\in\Xi$. Note that a decisive message could be off the equilibrium path. In addition, note that a decisive message m does not necessarily reveal a true state. The definition means that given a decisive message about project y, the decision maker optimally chooses the project y for sure.

First, we show that in any equilibrium, no decisive message about project 2 exists.

Lemma 1. *Suppose that Assumptions 1 and 2 hold. Then, in any equilibrium, no decisive message about project 2 exists.*

Proof. Suppose that an equilibrium exists in which a decisive message m^2 about project 2 exists. Note that it could be off the equilibrium path. Since the sender of type 1 and 2 can ensure the most desirable project by sending m^2, it holds that $\hat{\rho}(\hat{a}, \xi, m) = 2$ for any $\xi \in \Xi$ and any $m \in \hat{Z}(1) \cup \hat{Z}(2)$, that is, any $m \in \hat{Z}(1) \cup \hat{Z}(2)$ is a decisive message about project 2. Furthermore, we have $\hat{Z}(1) \subseteq \hat{Z}(2)$. In contrast, suppose that there exists a message m' such that $m' \in \hat{Z}(1)$ and $m' \notin \hat{Z}(2)$. Then, given message m', the decision maker knows that the true state is not $\theta = 2$, and it is never optimal for the decision maker to choose project $y = 2$. Hence, it is not optimal for the sender of type 1 to choose message m' with a positive probability. This contradicts $m' \in \hat{Z}(1)$.

Since $\hat{Z}(1) \subseteq \hat{Z}(2)$, there exist $m' \in \hat{Z}(1)$ and ξ' such that $\xi' = (\xi_0', t, f)$, and

$$\frac{\hat{\beta}(1|\hat{a}, \xi', m')}{\hat{\beta}(2|\hat{a}, \xi', m')} = \frac{\pi(1)}{\pi(2)} \cdot \frac{\hat{\mu}(m'|1)}{\hat{\mu}(m'|2)} \cdot \frac{Q(t|\hat{a}_1, 1)}{Q(t|\hat{a}_1, 2)} \cdot \frac{Q(f|\hat{a}_2, 1)}{Q(f|\hat{a}_2, 2)} \geq \frac{\pi(1)}{\pi(2)} > 1,$$

where ξ_0' is arbitrary. Note that the above inequality above holds without identifying $\hat{Z}(0)$. Hence, we have

$$-\hat{\beta}(0|\hat{a}, \xi', m') - \hat{\beta}(2|\hat{a}, \xi', m') > -4\hat{\beta}(0|\hat{a}, \xi', m') - \hat{\beta}(1|\hat{a}, \xi', m').$$

This implies that the decision maker strictly prefers project 1 to 2 at history $(\hat{a}, \xi', m')$. This is not consistent with the hypothesis that $\hat{\rho}(\hat{a}, \xi, m') = 2$ for any $\xi \in \Xi$ and any $m' \in \hat{Z}(1) \subseteq \hat{Z}(2)$. □

Secondly, we verify that the first-best for the decision maker is not attainable as an equilibrium. We say that an equilibrium is *DM-optimal* if at each state θ, the decision maker chooses the best project $y = \theta$ with probability one in the equilibrium. Then, Lemma 1 leads to the following proposition.

Proposition 1. *Suppose that Assumptions 1 and 2 hold. Then, no DM-optimal equilibrium exists.*

The proof is straightforward. When η satisfies Assumption 2, any signal profile realizes with a positive probability. Then, in DM-optimal equilibrium, it is necessary that the sender tells a truth at each state, that is, he sends a decisive message about each project. From Lemma 1, no such equilibrium exists. In Section 8, it is shown that a DM-optimal equilibrium can exist when Assumption 2 is not satisfied, that is, $\eta \geq 1/2$.

Thirdly, it is shown that the decision maker never gathers information in equilibrium if there is a decisive message about project 1.

Proposition 2. *Suppose that Assumptions 1 and 2 hold and that there exists a decisive message about project 1 in equilibrium. Then, it holds that* $\hat{\rho}(\hat{a}, \xi, m) = 1$ *for any* $\xi \in \Xi$ *and any* $m \in \hat{Z}(0) \cup \hat{Z}(1) \cup \hat{Z}(2)$*, and* $\hat{a} = (0, 0, 0)$.

In the proof, we confine our attention to the case in which the decision maker chooses only pure strategies in order to make the presentation simple. It is straightforward that the proof goes through, even if the decision maker can choose mixed strategies.

Proof. Suppose that there exists a decisive message m^1 about project 1 in an equilibrium. At this stage, m^1 could be off the equilibrium path. Since $y = 1$ is the best project for the sender of type 0, he can obtain the most favorable expected payoff of $-(1 - b)^2$ by sending message m^1. Thus, it is necessary to hold that

$$\hat{\rho}(\hat{a}, \xi, m) = 1, \quad \forall \xi \in \Xi, \ \forall m \in \hat{Z}(0).$$

That is, any $m \in \hat{Z}(0)$ is a decisive message about project 1.

Next, we show that in any equilibrium, there does not exist a message $m' \in \hat{Z}(1) \cup \hat{Z}(2)$ such that $\hat{\rho}(\hat{a}, \xi', m') \neq 1$ for some signal $\xi' \in \Xi$. Suppose that there exists such a message m'. Then, since sending m' is not optimal for the sender of type 0, the decision maker's belief satisfies $\hat{\beta}(0|\hat{a}, \xi, m') = 0$ for any $\xi \in \Xi$. Thus, it must hold that $\hat{\rho}(\hat{a}, \xi, m') \neq 0$ for any $\xi \in \Xi$. Given $\hat{\rho}$ satisfying the property, types 1 and 2 strictly prefer m' to any decisive message about project 1. Accordingly, we have $\hat{\beta}(0|\hat{a}, \xi, m) = 1$ for any $m \in \hat{Z}(0)$ and any $\xi \in \Xi$. However, given the belief, it is not optimal for the decision maker to choose $y = 1$ when receiving $m \in \hat{Z}(0)$. This contradicts the fact that any $m \in \hat{Z}(0)$ is a decisive message about project 1.

On the equilibrium path, the decision maker chooses project 1 irrespective of signals. Thus, it is optimal for the decision maker not to gather information about any state. □

The communication is not babbling in an equilibrium satisfying the property described in the proposition although the decision maker chooses project 1 for sure, regardless of the message in the equilibrium. After Proposition 4 in the next section, we will explain an example of an equilibrium such that it satisfies the property described in Proposition 2 and the sender sends a partially informative message.

Fourthly, we characterize the babbling equilibria in which the decision maker gathers information. A babbling equilibrium always exists. When c is sufficiently large, the decision maker does not gather information and chooses an optimal project regardless of the signals. Such equilibria are characterized in Proposition 4 in the next section. When c is sufficiently small, a babbling equilibrium can exist in which the decision maker gathers information. Interestingly, no babbling equilibrium in which the decision maker gathers information about state 1 exists, regardless of Assumption 1. This contrasts with the result that the decision maker possibly gathers information in equilibrium when communication is possible.

Proposition 3. *No babbling equilibrium exists in which the decision maker gathers information about state* 1.

Proof. See Appendix A.

Fifthly, from Lemma 1 and Proposition 2, no equilibrium exists in which the decision maker gathers information, and the sender of type 1 sends a decisive message about project 1 or the sender of type 2 sends a decisive message about project 2. On the other hand, an equilibrium might exist in which the sender of type 0 chooses a decisive message about project 0. We will show a necessary and sufficient condition for such an equilibrium to exist in Section 5.

Finally, we note that delegation is never optimal for the decision maker. The decision maker can ensure at least the expected payoff of $-\pi(0) - \pi(2)$. When delegating the decision right to the sender, she obtains the expected payoff of $-\pi(0) - \pi(1)$. By Assumption 1, the delegation is not preferable for the decision maker.

4. Benchmark: No Information Acquisition

In this section, we study the benchmark situation in which the decision maker cannot acquire information about the states. Alternatively, we can interpret the situation as the case in which the marginal cost c of the investigation is large.[8] Suppose that the investigation vector is exogenously fixed as $a^0 = (0, 0, 0)$. Under this assumption, no signal profile conveys information about state

[8] A sufficient condition for which no equilibrium exists such that the decision maker acquires information is $c > 4\eta$.

to the decision maker since $Q(\xi_s = t | a_s^0, \theta) = 1/2$ for any $\theta \in \Theta$ and any $s \in \Theta$. Therefore, this setting essentially corresponds to the situation in which the decision maker obtains no signal. We show that any equilibrium is babbling, that is, any message sent by the sender does not affect the decision maker's decision.

Proposition 4. *Suppose that Assumption 1 holds and that the decision maker cannot acquire information about the states. Then, in any equilibrium, the decision maker chooses a project irrespective of the message sent by the sender on the equilibrium path, and project $y = 2$ is never chosen on and off the equilibrium path.*

Proof. See Appendix B.

We explain the intuition about the proposition by confining our attention to the case in which the sender follows pure strategies.[9] As already stated, it is obvious that no equilibrium exists in which the sender of each type tells a true state. In addition, based on Lemma 1, no equilibrium exists in which the decision maker detects $\theta = 2$. The following strategy does not constitute an equilibrium: types 1 and 2 send a pooling message and type 0 tells a true state. If it is an equilibrium, then, by Assumption 1, the decision maker believes that state 1 is more likely when receiving the pooling message. Then, it is optimal for the decision maker to choose project 1, and it is the best response for the sender of type 0 not to tell a true state and to mimic type 1. Therefore, we have two candidates of equilibria: (i) all types choose a pooling message, and (ii) type 1 tells a true state, and types 0 and 2 choose a pooling message.

There is an equilibrium in which the sender follows strategy (i), which is a babbling equilibrium. Under certain conditions, strategy (ii) can constitute an equilibrium. That is, the proposition does not necessarily mean that, in any equilibrium, no message conveys information about a state. An equilibrium exists in which the sender conveys partial information but the decision maker chooses a particular project irrespective of the message.

To see this, assume $\pi(0) = \pi(2)$ and $M = \{m', m''\}$. Then, by Proposition 4, the decision maker chooses project 1 for sure in any equilibrium.[10] However, an informative equilibrium exists, and the following strategy of the sender constitutes an equilibrium. Type 1 sends m' for sure, and type 0 and 2 send m'' for sure. Given this strategy, the decision maker believes that the true state is $\theta = 1$ if receiving message m', and it is $\theta = 0$ with probability $1/2$ and $\theta = 2$ with probability $1/2$

[9]In Appendix B, each player is allowed to follow mixed strategies.
[10]See case (iii) in Appendix B.

otherwise. Sending message m' reveals that state 1 is realized, and sending message m'' reveals that the true state is not state 1. Given the beliefs, it is optimal for the decision maker to choose $y = 1$ for any message. Therefore, it constitutes an equilibrium.

In the equilibrium, the decision maker knows the true state when receiving m', and a *partially informative communication* is attained. However, the communication is not useful for the decision maker since no message affects the decision of the decision maker. In contrast, the next section shows that the sender can convey useful information for the decision maker if information acquisition is possible.

5. Communication Enhancement by Information Acquisition

In this section, first, we confine our attention to equilibria in which the sender follows pure strategies. We characterize the best equilibrium for the decision maker, in which the sender conveys partial information, that is, he tells the decision maker whether $\theta = 0$. Finally, it is shown that the equilibrium payoff of the decision maker is not improved even if the sender could choose mixed strategies.

Let M be an arbitrary finite set of messages containing two or more messages for the sender. Consider the following pure strategy of the sender: If $\theta = 0$, he sends message m^0 for sure; if $\theta \neq 0$, he sends a message m^* for sure. Denote the strategy of the sender by $\hat{\mu}^0$. As shown in Section 3, no equilibrium exists in which neither type 1 nor 2 reveals the true state. Thus, only $\hat{\mu}^0$ is a candidate of a partially informative equilibrium.

Given $\hat{\mu}^0$, the decision maker believes for sure that the true state is $\theta = 0$ if receiving message m^0, and she believes that the true state is $\theta = 1$ or 2 if receiving message m^*. We assume that if the decision maker receives a message other than m^0 or m^*, she believes for sure that the true state is $\theta = 0$. Given the belief, deviating from $\hat{\mu}^0$ brings the severest punishment to the sender.

We show a necessary and sufficient condition for $\hat{\mu}^0$ constituting an equilibrium, and there can be only two types of equilibria. One is that the decision maker spends all her time on investigating state 1, i.e., $\hat{a}^1 = (0, 1, 0)$. This case is summarized in Proposition 5. The other is that she spends all her time on investigating state 2, i.e., $\hat{a}^2 = (0, 0, 1)$. This case is summarized in Proposition 6.

First, we characterize the optimal choice of projects by the decision maker given $\hat{\mu}^0$ and any investigation vector. Given message m^0, the decision maker's belief satisfies $\hat{\beta}(0|a, \xi, m^0) = 1$ for any investigation vector a and any signal profile ξ. Hence, the optimal project for the decision maker is $\hat{\rho}(a, \xi, m^0) = 0$. Furthermore, the decision maker spends no time on investigating state 0 in equilibrium, i.e., $\hat{a}_0 = 0$.

Given an investigation vector a, signal profile ξ, and message m^*, the belief about state $\theta = 1$ of the decision maker is given by

$$\hat{\beta}(1|a, \xi, m^*) = \frac{\pi(1)Q(\xi_1|a_1, 1)Q(\xi_2|a_2, 1)}{\pi(1)Q(\xi_1|a_1, 1)Q(\xi_2|a_2, 1) + \pi(2)Q(\xi_1|a_1, 2)Q(\xi_2|a_2, 2)},$$

where $\hat{\beta}(1|a, \xi, m^*) + \hat{\beta}(2|a, \xi, m^*) = 1$. Hence, the decision maker obtains the expected payoff of $-\hat{\beta}(2|a, \xi, m^*)$ by choosing project 1 and obtains $-\hat{\beta}(1|a, \xi, m^*)$ by choosing project 2. The equilibrium conditions (3) induce the following lemma.

Lemma 2. *Given $\hat{\mu}^0$ and any (a, ξ, m^*), the optimal project for the decision maker satisfies that*

$$\text{if } \frac{Q(\xi_1|a_1, 1)}{Q(\xi_1|a_1, 2)} \cdot \frac{Q(\xi_2|a_2, 1)}{Q(\xi_2|a_2, 2)} \geq \frac{\pi(2)}{\pi(1)}, \text{ then } \hat{\rho}(a, \xi, m^*) = 1;$$
$$\text{if } \frac{Q(\xi_1|a_1, 1)}{Q(\xi_1|a_1, 2)} \cdot \frac{Q(\xi_2|a_2, 1)}{Q(\xi_2|a_2, 2)} \leq \frac{\pi(2)}{\pi(1)}, \text{ then } \hat{\rho}(a, \xi, m^*) = 2.$$

The proof is straightforward.

For any a_s and $s = 1, 2$, let us denote $e_s \equiv \frac{1+2\eta a_s}{1-2\eta a_s}$. Then, for $s = 1, 2$, the likelihood ratio of signal $\xi_s = t$ is given by

$$\frac{Q(\xi_1 = t|a_1, 1)}{Q(\xi_1 = t|a_1, 2)} = e_1 \text{ and } \frac{Q(\xi_2 = t|a_2, 1)}{Q(\xi_2 = t|a_2, 2)} = \frac{1}{e_2},$$

and the likelihood ratio of signal $\xi_s = f$ is given by

$$\frac{Q(\xi_1 = f|a_1, 1)}{Q(\xi_1 = f|a_1, 2)} = \frac{1}{e_1} \text{ and } \frac{Q(\xi_2 = f|a_2, 1)}{Q(\xi_2 = f|a_2, 2)} = e_2.$$

Since $0 \leq a_s \leq 1$ and $0 \leq \sum_{s=0}^{2} a_s \leq 1$, the feasible set of (e_1, e_2) is

$$L \equiv \left\{ (e_1, e_2) \,\middle|\, e_1, e_2 \in \left[1, \frac{1+2\eta}{1-2\eta}\right] \text{ and } e_2 \leq \frac{1 + \eta(1 + e_1)}{e_1 - \eta(1 + e_1)} \right\}.$$

By Lemma 2, given a, m, and ξ, the optimal project $\hat{\rho}$ for the decision maker is described as follows.

For $(\xi_1, \xi_2) = (t, t)$,

$$\hat{\rho}(a, \xi, m^*) = \begin{cases} 1 & \text{if } e_1/e_2 \geq \pi(2)/\pi(1), \\ 2 & \text{if } e_1/e_2 \leq \pi(2)/\pi(1). \end{cases} \tag{5}$$

For $(\xi_1, \xi_2) = (f, f)$,

$$\hat{\rho}(a, \xi, m^*) = \begin{cases} 1 & \text{if } e_1/e_2 \leq \pi(1)/\pi(2), \\ 2 & \text{if } e_1/e_2 \geq \pi(1)/\pi(2). \end{cases} \tag{6}$$

For $(\xi_1, \xi_2) = (t, f)$,

$$\hat{\rho}(a, \xi, m^*) = 1 \text{ for any } (e_1, e_2) \in L. \tag{7}$$

For $(\xi_1, \xi_2) = (f, t)$,

$$\hat{\rho}(a, \xi, m^*) = \begin{cases} 1 & \text{if } e_1 \cdot e_2 \leq \pi(1)/\pi(2), \\ 2 & \text{if } e_1 \cdot e_2 \geq \pi(1)/\pi(2). \end{cases} \tag{8}$$

Note that for $(\xi_1, \xi_2) = (t, f)$, it holds that $e_1 \cdot e_2 > \pi(2)/\pi(1)$ for any $(e_1, e_2) \in L$. Thus, we have (7).

The conditions (5)–(8) are summarized in Figure 1 and Table 1. Under these conditions, the feasible set L is divided into four regions, and the optimal projects of the decision maker in each region are summarized in Table 1. Abusing the notations, we write $(a_1, a_2) \in A^k$ if (e_1, e_2) belongs to region A^k, where $e_s \equiv (1 + 2\eta a_s)/(1 - 2\eta a_s)$. Each region includes the boundary, and thus the adjacent regions share the boundary. A^0, A^1, and A^3 (similarly, A^0, A^2, and A^3) share one point.

The interpretation of Table 1 is as follows: In region A^0, the decision maker does not spend much time on investigation. Therefore, the sensitivity of signals (ξ_1 and ξ_2) is not too high to affect the project choice. Since the prior probability distribution assigns a higher probability to $\theta = 1$ than to $\theta = 2$, the decision maker always chooses project 1 independent of signals she obtains. In region A^1, the decision maker spends a lot of time investigating state 1. Therefore, the decision maker chooses project 1 if she obtains $\xi_1 = t$, which means that the true state seems to be $\theta = 1$. Otherwise ($\xi_1 = f$), the decision maker chooses project 2. A similar argument holds for region

A^2. In region A^3, the decision maker allocates time to investigate both states 1 and 2. Then, the sensitivity of signals is moderate. Therefore, due to the asymmetry of the prior distribution toward state 1, the decision maker chooses project 2 only when she obtains strong evidence of state 2, i.e., $\xi_1 = f$ and $\xi_2 = t$.

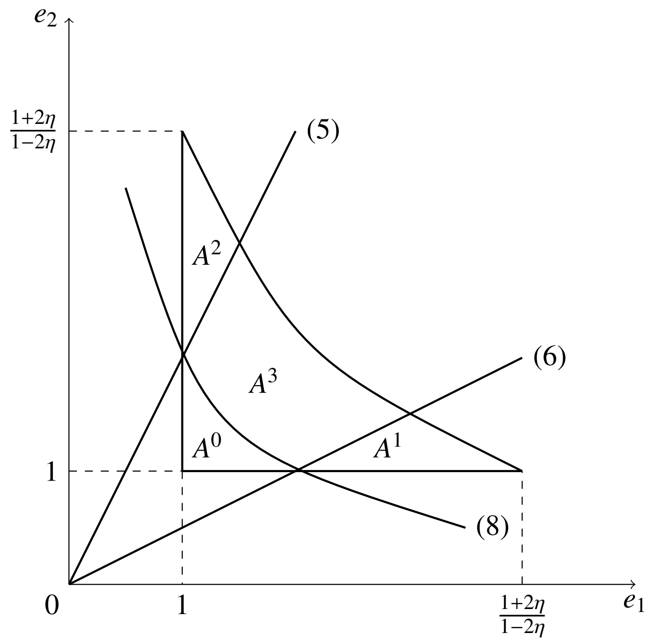

Figure 1: The feasible set of (e_1, e_2)

		(ξ_1, ξ_2)			
		(t,t)	(f,f)	(t,f)	(f,t)
(a_1, a_2)	A^0	$\hat{y} = 1$	$\hat{y} = 1$	$\hat{y} = 1$	$\hat{y} = 1$
	A^1	$\hat{y} = 1$	$\hat{y} = 2$	$\hat{y} = 1$	$\hat{y} = 2$
	A^2	$\hat{y} = 2$	$\hat{y} = 1$	$\hat{y} = 1$	$\hat{y} = 2$
	A^3	$\hat{y} = 1$	$\hat{y} = 1$	$\hat{y} = 1$	$\hat{y} = 2$

Table 1: The optimal projects

Next, we characterize the investigation vector in equilibria. As already seen, $\hat{a}_0 = 0$ must be satisfied in any equilibrium given $\hat{\mu}^0$, and we omit it below. Given (a_1, a_2), the expected payoff for the decision maker is given by

$$\Pi(a_1, a_2) \equiv \sum_{\theta \in \Theta} \sum_{\xi \in \Xi} \sum_{m \in M} \pi(\theta) P(\xi | (0, a_1, a_2), \xi, \theta) \hat{\mu}^0(m|\theta) \left[- \{\hat{\rho}((0, a_1, a_2), \xi, m) - \theta\}^2 \right]$$
$$- c(a_1 + a_2).$$

Note that $\hat{\rho}((0, a_1, a_2), \xi, m^0) = 0$ for any $\xi \in \Xi$ and for any (a_1, a_2).

From Table 1, the expected payoff of the decision maker for $(a_1, a_2) \in A^0$ is

$$\Pi(a_1, a_2) = -\pi(2) - c(a_1 + a_2).$$

Therefore, if an optimal investigation vector is in the region A^0, then $\Pi(a_1, a_2)$ is maximized at $(a_1, a_2) = (0, 0)$, and we write $\hat{\Pi}^0 \equiv \Pi(0, 0) = -\pi(2)$.

For $(a_1, a_2) \in A^1$, the expected payoff is

$$\begin{aligned}
\Pi(a_1, a_2) = &- \pi(1) \{(1/2 - \eta a_1)(1/2 + \eta a_2) + (1/2 - \eta a_1)(1/2 - \eta a_2)\} \\
&- \pi(2) \{(1/2 - \eta a_1)(1/2 + \eta a_2) + (1/2 - \eta a_1)(1/2 - \eta a_2)\} \\
&- c(a_1 + a_2) \\
= &- \{\pi(1) + \pi(2)\}(1/2 - \eta a_1) - c(a_1 + a_2).
\end{aligned}$$

Let us confirm that $(\hat{a}_1, \hat{a}_2) = (1, 0)$ if an equilibrium investigation vector is in the region A^1. Obviously, $\hat{a}_2 = 0$ must be satisfied when $(\hat{a}_1, \hat{a}_2)$ lies in A^1. If $\eta > c/\{\pi(1) + \pi(2)\}$, then $\Pi(a_1, a_2)$ is maximized at $(a_1, a_2) = (1, 0)$ in the region A^1, and we write $\hat{\Pi}^1 \equiv \Pi(1, 0) = -\{\pi(1)+\pi(2)\}(1/2-\eta) - c$. Otherwise, $\Pi(a_1, a_2)$ is less than $-(1/2)\{\pi(1) + \pi(2)\}$ for $(a_1, a_2) \in A^1$. However, $\hat{\Pi}^0 > -(1/2)\{\pi(1) + \pi(2)\}$ holds. Thus, when $\eta \leq c/\{\pi(1) + \pi(2)\}$, no equilibrium investigation vector is in A^1.

For $(a_1, a_2) \in A^2$, the expected payoff is given by

$$\begin{aligned}
\Pi(a_1, a_2) = &- \pi(1) \{(1/2 + \eta a_1)(1/2 - \eta a_2) + (1/2 - \eta a_1)(1/2 - \eta a_2)\} \\
&- \pi(2) \{(1/2 + \eta a_1)(1/2 - \eta a_2) + (1/2 - \eta a_1)(1/2 - \eta a_2)\} \\
&- c(a_1 + a_2) \\
= &- \{\pi(1) + \pi(2)\}(1/2 - \eta a_2) - c(a_1 + a_2).
\end{aligned}$$

This is a symmetric case of $(a_1, a_2) \in A^1$. Thus, only if $\eta > c/\{\pi(1) + \pi(2)\}$, an equilibrium investigation vector is in A^2, and it is $(\hat{a}_1, \hat{a}_2) = (0, 1)$. The expected payoff satisfies $\hat{\Pi}^2 = \hat{\Pi}^1$.

For $(a_1, a_2) \in A^3$, the expected payoff for the decision maker is given by

$$\begin{aligned}\Pi(a_1, a_2) = &-\pi(1)(1/2 - \eta a_1)(1/2 - \eta a_2)\\ &- \pi(2)\{(1/2 - \eta a_1)(1/2 + \eta a_2) + (1/2 + \eta a_1)(1/2 - \eta a_2)\\ &+ (1/2 - \eta a_1)(1/2 - \eta a_2)\} - c(a_1 + a_2)\\ = &-\eta^2\{\pi(1) - \pi(2)\}a_1 a_2 + \eta\{\pi(1) + \pi(2)\}(a_1 + a_2)/2\\ &- \{\pi(1) + 3\pi(2)\}/4 - c(a_1 + a_2).\end{aligned}$$

Suppose that an equilibrium investigation vector $(\overline{a}_1, \overline{a}_2)$ is an interior point of A^3. Then, while satisfying $a_1 + a_2 = \overline{a}_1 + \overline{a}_2$, the decision maker can increase the expected payoff by increasing a_1 or a_2. This contradicts the assumption that $(\overline{a}_1, \overline{a}_2)$ is an equilibrium investigation vector. Therefore, if an equilibrium investigation vector belonging to A^3 exists, then it is necessarily on the boundary of A^3 and A^s for some $s \in \{0, 1, 2\}$. This implies that $0 < \overline{a}_1 < 1$ or $0 < \overline{a}_2 < 1$. The earlier discussion implies that no equilibrium investigation vector belongs to A^3.

Based on the discussion above, we have three candidates of equilibrium investigation vectors: $\hat{a}^0 = (0, 0, 0)$, $\hat{a}^1 = (0, 1, 0)$, and $\hat{a}^2 = (0, 0, 1)$. First, let us ensure that $\hat{a}^0$ does not constitute an equilibrium. Since $\hat{a}^0 \in A^0$, any $m \neq m^0$ is a decisive message about project 1. Then, the sender of type 0 strictly prefers sending m to m^0, and $\hat{\mu}^0$ does not constitute an equilibrium. Therefore, if $\hat{\mu}^0$ constitutes an equilibrium, then the equilibrium investigation vector must be $\hat{a}^1$ or $\hat{a}^2$. It is necessary that $\hat{\Pi}^1 = \hat{\Pi}^2 \geq \hat{\Pi}^0$, and we have

$$\eta \geq \frac{1}{2} - \frac{\pi(2) - c}{\pi(1) + \pi(2)}. \tag{9}$$

Since $\eta < 1/2$, it must be satisfied that

$$\pi(2) > c. \tag{10}$$

Recall that each of $\hat{a}^1$ and $\hat{a}^2$ constitutes an equilibrium only if $\eta > c/\{\pi(1) + \pi(2)\}$. The condition holds if (9) is satisfied.

Next, we consider the optimality of sender's strategy $\hat{\mu}^0$. In what follows, assume that (9) and (10) are satisfied. By the construction of $\hat{\mu}^0$, it is obvious that the sender of type $\theta = 1$ and 2 has no incentive to deviate from the strategy. We have only to check that the sender of type 0 has no incentive to send any message $m \neq m^0$.

First, let us derive the condition under which $\hat{a}^1 = (0, 1, 0)$ constitutes an equilibrium. The sender of type 0 obtains the expected payoff of $-b^2$ by sending m^0 since m^0 is a decisive message about project 0. For m^*, since $\hat{a}^1 \in A^1$, it holds that

$$\hat{\rho}(\hat{a}^1, \xi, m^*) = \begin{cases} 1 \text{ for } \xi_1 = t, \\ 2 \text{ for } \xi_1 = f. \end{cases}$$

Thus, by sending m^*, the sender of type $\theta = 0$ obtains the expected payoff of

$$-(1/2 - \eta)(1 - b)^2 - (1/2 + \eta)(2 - b)^2.$$

Therefore, it is optimal for the sender of type 0 to follow $\hat{\mu}^0$ only if the following condition is satisfied:

$$\eta \geq \frac{6b - 5}{2(3 - 2b)}.$$

From the discussion, we have the following result.

Proposition 5. *A partially informative equilibrium* $(\hat{\mu}^0, (\hat{a}^1, \hat{\rho}), \hat{\beta})$ *exists, if and only if*

$$\pi(2) > c, \text{ and} \tag{11}$$

$$\eta \geq \max\left\{\frac{1}{2} - \frac{\pi(2) - c}{\pi(1) + \pi(2)}, \frac{6b - 5}{2(3 - 2b)}\right\}. \tag{12}$$

The proof of the necessity follows from the discussion above, and the proof of the sufficiency is straightforward.

Second, we derive the condition under which $\hat{a}^2 = (0, 0, 1)$ constitutes an equilibrium. Since $\hat{a}^2 \in A^2$, it holds that

$$\hat{\rho}(\hat{a}^2, \xi, m^*) = \begin{cases} 1 \text{ for } \xi_2 = f, \\ 2 \text{ for } \xi_2 = t. \end{cases}$$

Thus, the sender of type 0 obtains the expected payoff of $-(1/2 + \eta)(1 - b)^2 - (1/2 - \eta)(2 - b)^2$ by sending m^*. Following $\hat{\mu}^0$ is optimal for the sender of type 0 only if the following condition is satisfied:

$$\eta \leq \frac{5-6b}{2(3-2b)}.$$

Now we have the counterpart of Proposition 5.

Proposition 6. *A partially informative equilibrium* $(\hat{\mu}^0, (\hat{a}^2, \hat{\rho}), \hat{\beta})$ *exists, if and only if*

$$\pi(2) > c, \text{and} \tag{13}$$

$$\eta \in \left[\frac{1}{2} - \frac{\pi(2)-c}{\pi(1)+\pi(2)}, \frac{5-6b}{2(3-2b)}\right]. \tag{14}$$

The interval in (14) is non-empty when it holds

$$\frac{\pi(1)+4\pi(2)-3c}{2\{\pi(1)+2\pi(2)-c\}} \geq b.$$

The proof of the necessity follows from the discussion above, and the proof of the sufficiency is straightforward.

Note that the set of parameters satisfying the condition of Proposition 6 is included by the set of parameters satisfying the condition of Proposition 5. Thus, there is an open set of parameters under which multiple partially informative equilibria exist. When η is large, the equilibrium is unique for nondegenerate parameters, which is the one described in Proposition 5. In the equilibrium, the decision maker acquires information only about state $\theta = 1$.

The key mechanism of preventing type 0 sender's exaggeration works through the "threat" that the decision maker will end up choosing project 2 rather than project 1. If the decision maker focuses on investigating state 1, this threat is caused by the decision maker's misinterpretation that $\xi_1 = f$ is strong evidence of $\theta = 2$. The probability with which $\xi_1 = f$ conditional on $\theta = 0$ and $\hat{a}^1$ is $1/2 + \eta$. On the other hand, if the decision maker focuses on investigating state 2, this threat is caused when the decision maker receives $\xi_2 = t$, which she believes strong evidence of $\theta = 2$. The probability with which $\xi_1 = f$ conditional on $\theta = 0$ and $\hat{a}^2$ is $1/2 - \eta$. Clearly, whenever state 0 is ruled out, the threat of project 2 being chosen is much more potent when $\hat{a}^1$ than when $\hat{a}^2$. This is the reason the parameter set in Proposition 5 is a superset of that in Proposition 6.

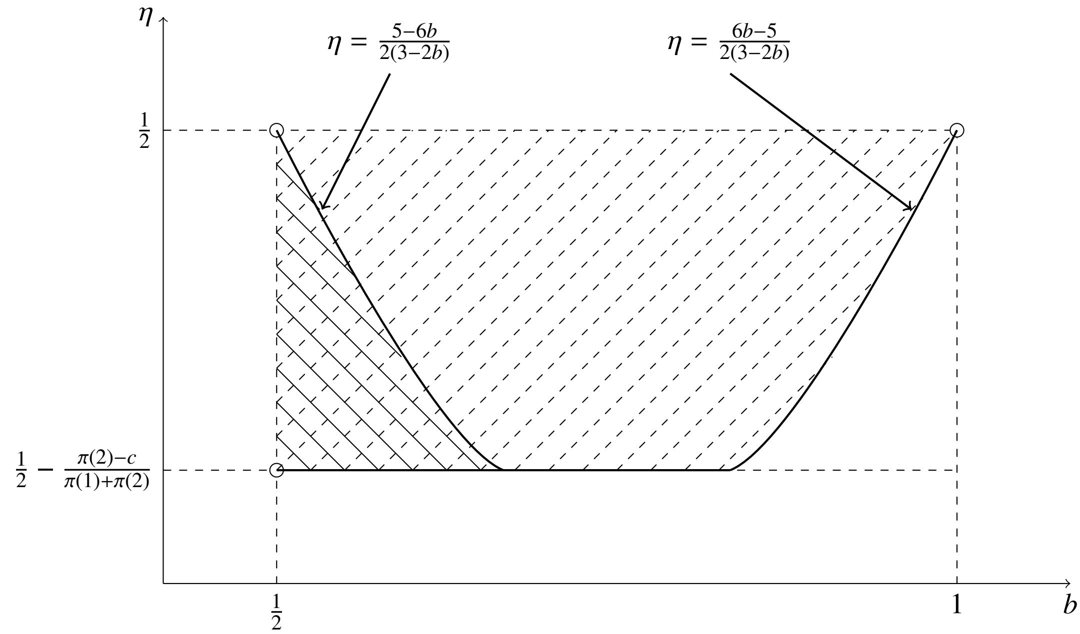

Figure 2: The conditions of (12) and (14)

Let us consider a situation in which $\pi(1) > \max\{\pi(0), \pi(2)\}$. Then, only project $y = 1$ is chosen in equilibrium if the decision maker cannot gather information.[11] In this situation, we may interpret state $\theta = 1$ as the status quo, $y = 1$ as a default project, and $y = 2$ as a challenging project. Let us say that the decision maker is a CEO. One implication of our result is that when the bias parameter is large, the decision maker gathers information about the status quo, and she selects a challenging project by her own will rather than persuaded by her subordinate (the sender). Her subordinate tells the CEO the true state when a realized state is a conservative one ($\theta = 0$).

Finally, we can conclude that the equilibria characterized above are the best for the decision maker, that is, even if the sender follows mixed strategies, no equilibrium exists whose payoff of the decision maker is greater than the equilibrium payoff in the partially informative equilibrium ($\hat{\Pi} \equiv \hat{\Pi}^1 = \hat{\Pi}^2$).

Proposition 7. *Suppose that Assumption 1 holds. Then, no equilibrium exists whose payoff of the decision maker exceeds the partially informative equilibrium payoff of* $\hat{\Pi}$.

The proof is relegated to Appendix C.

[11] See Proposition 4 and case (iii) in the proof.

5.1 Remarks

We give some remarks on the propositions.

Remark 1 (Uniqueness of the partially informative equilibrium)**.** As stated earlier, there exists an open set of parameters that satisfy both conditions of Propositions 5 and 6. That is, multiple partially informative equilibria exist under certain conditions. This comes from the assumption that precision of signals is independent of state. If we allow that the precision depends on state, then partially informative equilibrium is essentially unique for nondegenerate parameters although there are equivalent equilibria in which off-the-equilibrium behaviors or the system of beliefs are different.

Let η_s denote the precision of the signal regarding state s. The analysis completely goes through the same process as mentioned above. Thus, given $\hat{\mu}^0$, the optimal decision about the projects is characterized by Table 1, and the expected payoff of the decision maker given the investigation vector is given by

$$\Pi(a^1) = \Pi(a_1, a_2) = -\{\pi(1) + \pi(2)\}(1/2 - \eta_1 a_1) - ca_1, \text{ if } a^1 \in A^1,$$
$$\Pi(a^2) = \Pi(a_1, a_2) = -\{\pi(1) + \pi(2)\}(1/2 - \eta_2 a_2) - ca_2, \text{ if } a^2 \in A^2.$$

If $\eta_1 > \eta_2$ (resp. $\eta_2 > \eta_1$), then $\hat{a}^1 = (1, 0)$ (resp. $\hat{a}^2 = (0, 1)$) is uniquely optimal. Therefore, for any nondegenerate set of parameters, the partially informative equilibrium is unique.

Remark 2 (Complementarity of communication and information acquisition)**.** Let us emphasize that communication and information acquisition are complementary. To see this, let us consider an extreme case. Suppose that the parameters satisfy

$$\begin{aligned} &\pi(2) > c; \quad b \le 5/6; \\ &\pi(1) = \frac{1}{3} + 2\varepsilon; \quad \pi(0) = \pi(2) = \frac{1}{3} - \varepsilon; \\ &\eta \ge \frac{3(3\varepsilon + 2c)}{2(2 + 3\varepsilon)}. \end{aligned} \tag{15}$$

By Proposition 5, a partially informative equilibrium exists.

Let us take ε and c as arbitrarily small and then set that η satisfying (15) holds with equality. Since ε and c are small, η is also small. Assume that the decision maker cannot communicate with the sender. Then, it is optimal for the decision maker not to acquire information and to choose project $y = 1$, regardless of the signal, and she obtains the expected payoff of $-\pi(0) - \pi(2) = -2/3 - 2\varepsilon$. Since the precision of the signals, η, is small, it is not optimal for the decision maker to choose projects depending on signals even if c is arbitrarily small.

Next, suppose that communication is possible. When she can gather information, partial information transmission is attained since the parameters defined above satisfy the condition in Proposition 5. In the equilibrium, the decision maker obtains the expected payoff of $\hat{\Pi}(\hat{a}^1) = -(2/3 + \varepsilon)(1/2 - \eta) - c \approx -1/3$, which is largely improved in comparison to the case without communication, even though η is small.

Effective communication is supported by the information acquisition of the decision maker. When η is large, the decision maker acquires information without communication. However, when η is small, the incentive for the decision maker to acquire information is supported by the communication with the sender.

Remark 3 (Timing of Communication)**.** The timing of the information acquisition is not material from the viewpoint of the quality of the communication in the three states model. Suppose that the decision maker can acquire information about the states after receiving a message from the sender. It is straightforward that Propositions 5 and 6 hold, except that (11) and (13) are replaced by

$$\frac{\pi(2)}{\pi(1) + \pi(2)} > c. \tag{16}$$

Needless to say, the equilibrium strategies of the decision maker are required to be modified accordingly. In the equilibrium, the decision maker acquires information if receiving message m^* and does not otherwise.

Condition (16) is less restrictive than (11) and (13). These are conditions for the incentive regarding information acquisition, and incentive to acquire information is greater than under the assumption of information acquisition before communication. This is because under information acquisition after communication, there is no possibility that the obtained information is useless. When the decision maker acquires information before communication, the information has no value if the sender sends message m^0.

A characterization of the best equilibrium payoff for the decision maker is done by almost the same argument as the case in which she can acquire information before the communication. The best equilibrium payoff for the decision maker is obtained by a pure strategy equilibrium, and there is no mixed strategy equilibrium whose payoff for the decision maker is greater than the former.

Remark 4 (Generalizations)**.** Generalization to the model with finitely many states and actions complicates the analysis. However, it can be shown that the key mechanism underlying our main findings still works under the variant model mentioned in Remark 3, that is, the decision maker can acquire information about states after receiving a message. Suppose that the state and action space have finitely many elements and the sender is one step upwardly biased: $\Theta = Y = \{1, \dots, N\}$ and $b \in (1/2, 1)$. Since the sender of type θ prefers $y = \theta + 1$, in any equilibrium each element of the equilibrium partition, except for the left-most one, must contain at least two states. It is straightforward to show that there is no equilibrium in which the sender truthfully tells a state $\theta \neq 1$. Hence, the partition whose elements consist of two adjacent states is the finest partition potentially attainable in any equilibrium.[12] By following the argument in the previous section, we can show that if some conditions are satisfied, there is an equilibrium with the prescribed finest partition. In the equilibrium, after receiving a message $m = \{i, i + 1\}$, which means that the true state is i or $i + 1$, the decision maker optimally acquires information about state i when c is small. The sender of type $\theta = i - 1$ prefers $y = i$, but $y = i + 1$ is unfavorable for him if the decision maker chooses $y = i - 1$ with a sufficiently high probability. Therefore, if the precision of the signal is high, the given investigation strategy can prevent the misreporting incentive of the sender $\theta = i - 1$.

Remark 5 (Convex cost functions)**.** We assume that the cost function of information acquisition is linear. The assumption is not material, and it is only for simplification of the analysis. Even if the cost function is assumed to be convex in the total time the decision maker spends, the results do not change qualitatively.

6. Information Disclosure

We consider the situation in which the decision maker can disclose the signals she observes. More precisely, after observing the signals, the decision maker chooses whether to disclose them or not. For each $(a, \xi) \in A \times \Xi$, let $\delta(a, \xi)$ be a decision about disclosure of signals, and it is satisfied that $\delta(a, \xi) \in \{\psi, \xi\}$. When $\delta(a, \xi) = \psi$, the decision maker does not disclose the signal profile. When $\delta(a, \xi) = \xi$, the decision maker discloses the signal profile ξ she observes. We assume that she cannot manipulate the information.

[12]If N is an odd number, the finest equilibrium partition is $\{\{1\}, \{2, 3\}, \dots, \{N - 1, N\}\}$. Otherwise, it is $\{\{1, 2\}, \{3, 4\}, \dots, \{N - 1, N\}\}$.

The sender sends a message depending on both the realized state and the outcome of the decision maker's information disclosure. Abusing notation, let $\mu : \Theta \times (\Xi \cup \{\psi\}) \to M$ be a decision regarding the message, where $\mu(\theta, d)$ denotes a message sent by the sender who observes θ and $d \in \Xi \cup \{\psi\}$. Let $\beta(\theta|a, \xi, d, m)$ be the decision maker's belief given the information the decision maker obtains before choosing a project. The decision about the project is denoted by $\rho(a, \xi, d, m)$.

We have the fact that the decision maker's equilibrium payoff cannot be improved even when the decision maker can disclose her private signals. We implicitly assume that the players can follow mixed strategies.

Proposition 8. *Let $E^* = ((\mu^*, (a^*, \delta^*, \rho^*)), \beta^*)$ be an equilibrium of the game with information disclosure, and let $\hat{E} = (\hat{\mu}^0, (\hat{a}^1, \hat{\rho}), \hat{\beta})$ be an equilibrium in the game without information disclosure, which induces the decision maker to have the best equilibrium payoff. Then, for any E^*, the decision maker's expected payoff under E^* is no more than that under $\hat{E}$.*

We omit the proof of Proposition 8 because it follows the same argument as that of Proposition 7.

The reason disclosing the signals is not beneficial for the decision maker is as follows. When the decision maker discloses the signals, the sender knows the belief of the decision maker about the states. Then, a message the sender chooses update the belief of the decision maker, and given the belief, the decision maker chooses the optimal project. This means that the sender essentially controls the choice of projects, and the decision maker loses an advantage.

7. Single-Dimensional Investigation

In this section, we consider single-dimensional investigation in contrast with multidimensional investigation and show that no informative equilibrium exists when the bias is large, that is, $b > 7/10$. This contrasts with the fact shown in Section 5 that for any $b \in (1/2, 1)$, an open set of parameters exists under which a partial information transmission constitutes an equilibrium.

We say that investigation activity is *single-dimensional* if $a \equiv a_0 = a_1 = a_2$. Here, we assume that the decision maker is restricted to single-dimensional investigation, and $a \in [0, 1]$. Let us refer a to an investigation level. Single-dimensional investigation is a standard technology for acquiring information assumed not only in the literature on the cheap-talk game but also in other literature.

A typical property of the standard technology is that given a state, when the investigation level increases, the probability of a distinct profile of signals becomes the largest. Roughly speaking, the distribution of the profiles of signals becomes sharper by increasing the investigation level. Technologies assumed in Moreno de Barreda [13] satisfy a corresponding property. Moreno de Barreda [13] assumes that investigation is binary, that is, acquiring a signal or not. A signal is a real number, and the set of signals equals the real line. When the decision maker does not acquire a signal, she receives nothing. When she acquires a signal, the signal realizes according to a normal distribution given a state. This also means that information acquisition makes the distribution of signals sharper.[13]

Under multidimensional investigation, the effect of changing the investigation activity is different from that under single-dimensional investigation. Let $\hat{a} = (\hat{a}_0, \hat{a}_1, \hat{a}_2)$ be an investigation vector, and suppose that $\hat{a}$ is an interior point in A, which is close to $(0, 1, 0)$. Given state $\theta = 1$, the probability distribution under $\hat{a}$ has a unique profile of signals whose probability is the largest. When $\hat{a}$ is changed to $(0, 1, 0)$, the probabilities of the four profiles of signals become the largest. The probability distribution of the profiles of signals does not becomes sharper. This property enhances communication under multidimensional investigation when compared to single-dimensional investigation.

Let us confine our attention to pure strategy equilibrium as in Section 5. As discussed in Section 5, in any partially informative equilibrium, the sender of type 0 selects a truthful message, and the sender of type 1 and 2 selects the same message.

From Lemma 2, the optimal decision regarding project, $\hat{\rho}$, corresponding to conditions from (5) to (8), can be derived given investigation level a:

For $(\xi_1, \xi_2) \neq (f, t)$,

$$\hat{\rho}(a, \xi, m) = 1;$$

[13]In Argenziano et al. [1], the investigation level corresponds to the number of trials. By conducting a trial, the decision maker receives a signal, "success" or "failure." Given any non-degenerate state, the distribution of the ratio of success has a single peak. Increasing the number of trials results in a sharper distribution.

For $(\xi_1, \xi_2) = (f, t)$,

$$\hat{\rho}(a, \xi, m) = \begin{cases} 1 & \text{if } e_1 \cdot e_2 \leq \pi(1)/\pi(2); \\ 2 & \text{if } e_1 \cdot e_2 \geq \pi(1)/\pi(2), \end{cases}$$

where

$$e_1 = e_2 = \frac{1 + 2\eta a}{1 - 2\eta a}.$$

The decision maker selects project $y = 2$ only if $e_1 \cdot e_2 \geq \pi(1)/\pi(2)$ is satisfied, that is, $(\xi_1, \xi_2) = (f, t)$. Given this, the expected payoff for the decision maker is given as:

$$-\pi(1)\left(\frac{1}{2} - \eta a\right)^2 - \pi(2)\left\{2\left(\frac{1}{2} - \eta a\right)\left(\frac{1}{2} + \eta a\right) + \left(\frac{1}{2} + \eta a\right)^2\right\} - c \cdot a.$$

The first order condition with respect to a yields

$$a = \frac{1}{2\eta} - \frac{c}{2\eta^2 \{\pi(1) - \pi(2)\}}.$$

Optimal investigation level a is an interior point for an intermediate level of c.

The incentive compatibility condition for the sender of type 0 to select a truthful message is given by

$$-b^2 \geq -\left(\frac{3}{4} + \eta^2 a^2\right)(1 - b)^2 - \left(\frac{1}{4} - \eta^2 a^2\right)(2 - b)^2 \quad \Leftrightarrow \quad \eta^2 a^2 \leq \frac{7 - 10b}{12 - 8b}. \tag{17}$$

When $b > 7/10$, the condition above does not hold, and no informative equilibrium exists. On the other hand, when investigation activity is multidimensional, for each $b \in (1/2, 1)$, there is an open set of parameters under which informative equilibrium exists. Therefore, for meaningful communication, multidimensional investigation is essential when bias b is large.

Let us study comparative statics with respect to η. Under multidimensional investigation, the larger η is, the less restrictive the incentive compatibility condition (12) for the sender of type 0 is. On the other hand, it is obscure under single-dimension investigation because $\eta^2 a^2$ in (17) is not necessarily monotone with respect to η. To highlight the difference in investigation technologies, let us consider extreme parameters under which $a = 1$ is optimal. Then, the larger η is, the more restrictive the incentive compatibility condition (17) for the sender of type 0 is. That is, the communication from the sender (weakly) deteriorates with an increase in the precision of signals. This contrasts with the case of multidimensional investigation.

The comparative statics above for the single-dimensional investigation shares the same logic as in Moreno de Barreda [13]. The higher the precision of the signal is, the smaller the risk effect for the sender is. The first inequality in (17) shows that when η gets larger, the sender of type 0 believes with a higher probability that the decision maker chooses project 1, and the incentive to send a non-truthful message increases. In contrast, when information acquisition is multidimensional, the decision maker gathers information about a particular state (e.g., state 1), that is, she investigates whether the true state is state 1 or not. Then, the higher the precision of the signal is, the sender of type 0 believes with a higher probability that the decision maker will not choose project 1.

Finally, we provide an insight into reality. We can interpret that a person (say, a CEO) with single-dimensional investigation is a generalist type decision maker and one with multidimensional investigation is a specialist type. Our result implies that only a specialist type decision maker can maintain meaningful communication in an organization when the bias of the sender is large, and a generalist type can do this only when the bias is small. The implication seems to be empirically testable.

8. DM-Optimal Equilibrium

Let us consider multidimensional investigation again. So far, we have assumed $0 < \eta < 1/2$. Here we assume $1/2 \leq \eta < 1$. In this case, the decision maker can detect whether only one state is realized or not if she spends a certain amount of time acquiring information about the state. Then, we show a necessary and sufficient condition for a DM-optimal equilibrium to exist. In the case of $\eta \geq 1$, the decision maker can detect which state is realized if she spends enough time. Thus, it is straightforward to derive a necessary and sufficient condition for a DM-optimal equilibrium to exist, and we omit the case here.

Given a state θ and an investigation vector a, the marginal probability with which the decision maker obtains a signal ξ_s is modified as follows:

$$Q(\xi_s = t|a_s, \theta) = \begin{cases} \min\{1/2 + \eta a_s,\ 1\} \text{ if } s = \theta, \\ \max\{1/2 - \eta a_s,\ 0\} \text{ if } s \neq \theta. \end{cases}$$

In this case, the decision maker can know whether the true state is s or not if she spends $a_s \geq 1/(2\eta)$; she can detect only one state and cannot detect two or more states.

Proposition 9. *Suppose that Assumption 1 holds, and assume* $1/2 \leq \eta < 1$. *Then, a DM-optimal equilibrium exists if and only if* $c/(2\eta) \leq \max\{\min\{\pi(0), \pi(1)\}, \pi(2)\}$.

Before jumping into the proof, we explain some features of DM-optimal equilibria. Although multiple DM-optimal equilibria exist, the decision maker gathers information about only state 1 in any equilibrium, that is, she detects whether state 1 is realized or not. In contrast, there are two types of equilibrium strategies for the sender: (i) type 0 sends a truthful message, and type 1 and 2 send a pooling message; (ii) type 2 sends a truthful message, and type 0 and 1 send a pooling message. For each strategy, the decision maker can correctly guess a true state from the message sent by the sender, which is not necessarily truthful. As seen in Section 5, strategy (ii) does not constitute an equilibrium when $0 < \eta < 1/2$. When $1/2 \leq \eta < 1$, a new equilibrium is created.

Proof. We show the "only if" part below. The proof of the "if" part is straightforward, and we omit it.

Let M be an arbitrary finite set of messages containing two or more messages. First, let us show that in any DM-optimal equilibrium, $\hat{a} = (0, \hat{a}_1, 0)$ is satisfied, where $\hat{a}_1 = 1/(2\eta)$. When $\hat{a}_s < 1/(2\eta)$ for $s = 0, 1, 2$, the decision maker always receives imperfect signals. By applying Proposition 1, the equilibrium is not DM-optimal. Thus, $\hat{a}_s \geq 1/(2\eta)$ must be satisfied for some s.

Suppose $\hat{a}_0 \geq 1/(2\eta)$. When the decision maker observes $\xi_0 = f$, the sender must send a truthful message that conveys a true state in order for the equilibrium to be DM-optimal. Let m^θ be a message that the sender of type θ sends on the equilibrium path. For $\theta = 1, 2$, since m^θ is a decisive message about $y = \theta$, the decision maker believes for sure that the true state is θ and chooses project $y = \theta$. Then, it is profitable for the sender of type 1 to send a message m^2. In the case of $\hat{a}_2 \geq 1/(2\eta)$, the same argument follows. Therefore, we have $\hat{a}_1 \geq 1/(2\eta)$.

Based on the result above, we can narrow down the candidates for equilibrium strategies for the sender. Let $\hat{Z}(\theta)$ be the set of messages the sender of type θ chooses with positive probabilities given the equilibrium. We say that the strategy of the sender is *honest* if $\hat{Z}(\theta) \cap \hat{Z}(\theta') = \emptyset$ for each θ and $\theta' \neq \theta$. Given this strategy, the decision maker can ascertain which state is realized by the messages she receives. An honest strategy is not an equilibrium since the decision maker has no incentive to acquire information.

It is necessary that the sender sends a message by which the decision maker can ascertain a true state when the decision maker receives signal $\xi_1 = f$. Only one type reports his type truthfully for sure. Thus, the following strategies are the candidates of equilibrium strategy for the sender:

(i) $\hat{Z}(0) \cap \left(\hat{Z}(1) \cup \hat{Z}(2)\right) = \emptyset$ and $\hat{Z}(1) \cap \hat{Z}(2) \neq \emptyset$.

(ii) $\hat{Z}(2) \cap \left(\hat{Z}(0) \cup \hat{Z}(1)\right) = \emptyset$ and $\hat{Z}(1) \cap \hat{Z}(0) \neq \emptyset$.

We say that a message is *truthful* if, on equilibrium path, a message sent by the sender reveals the true state. For case (i), the sender of type 0 always sends a truthful message, although he possibly follows a mixed strategy. Then, the sender of types 1 and 2 can choose a truthful message with a positive probability, but the probability must be small enough to support the equilibrium.

Next, let us describe the decision about project. Suppose that the decision maker chooses $a_1 \geq \hat{a}_1$. If the decision maker observes signal $\xi_1 = t$, then she chooses project 1 irrespective of signals (ξ_0, ξ_2) and the message. For case (i) (resp. (ii)), if she observes signal $\xi_1 = f$ and receives a truthful message $m \in \hat{Z}(0)$ (resp. $m \in \hat{Z}(2)$), she chooses a corresponding project, irrespective of signals (ξ_0, ξ_2); if she observes signal $\xi_1 = f$ and receives a message $m \in \hat{Z}(2)$ (resp. $m \in \hat{Z}(0)$), she chooses project 2 (resp. project 1) irrespective of signals (ξ_0, ξ_2). If she observes signal $\xi_1 = f$ and receives a message $m \notin \hat{Z}(0) \cup \hat{Z}(1) \cup \hat{Z}(2)$, she chooses project 0, irrespective of signals (ξ_0, ξ_2).

In the case of $a_1 < \hat{a}_1$, the decision maker chooses an optimal project given her belief. The optimal project for the decision maker is described in Table 1 in the proof of Lemma 2. Given the signals and the message, the beliefs of the decision maker are derived from Bayes' rule given the opponent's strategy if possible, and the decision maker believes for sure $\theta = 0$ for case (i) and (ii) if Bayes' rule cannot apply. We do not make the beliefs of the sender explicit since they are trivial.

For case (i), let us show the condition under which the strategy profile and the system of beliefs defined above constitute an equilibrium. Case (ii) completely follows the same argument. Let us check the optimality backward. When receiving a truthful message $m^0 \in \hat{Z}(0)$, the decision maker believes for sure $\theta = 0$, and it is optimal for her to choose project $y = 0$. When receiving a message $m \in \hat{Z}(1) \cup \hat{Z}(2)$, she believes that the true state is either $\theta = 1$ or 2. If $a_1 \geq \hat{a}_1$ and

$\xi_1 = t$, then she believes for sure that the true state is $\theta = 1$, and it is optimal for her to choose project $y = 1$. If $a_1 \geq \hat{a}_1$ and $\xi_1 = f$, then she believes for sure that the true state is $\theta = 2$, and it is optimal for her to choose project $y = 2$. When $a_1 < \hat{a}_1$, the decision about the projects described above is optimal by definition.

Next, we verify the optimality for the sender. Consider the sender of type 0. If he sends a truthful message m^0 or a message $m \notin \hat{Z}(0) \cup \hat{Z}(1) \cup \hat{Z}(2)$, then he believes for sure that the decision maker chooses project $y = 0$, and he obtains the expected payoff of $-b^2$. If he sends a message $m \in \hat{Z}(1) \cup \hat{Z}(2)$, then he believes for sure that the decision maker chooses project $y = 2$, and he obtains the expected payoff of $-(2 - b)^2$. Thus, it is optimal for the sender of type 0 to send m^0 since $b \in (1/2, 1)$.

For type 1, he believes that the decision maker observes signal $\xi_1 = t$ and chooses $y = 1$, irrespective of the message. Thus, it is optimal for the sender of type 1 to send any message. For type 2, if he sends a message the strategy prescribes, then he believes for sure that the decision maker chooses project $y = 2$, and he obtains the expected payoff of $-b^2$. If he sends a message $m \notin \hat{Z}(2)$, he believes for sure that the decision maker chooses project $y = 0$, and he obtains the expected payoff of $-(2 + b)^2$. Thus, it is optimal for the sender of type 2 to follow the strategy.

Finally, let us confirm that $\hat{a}$ is optimal for the decision maker. When she chooses $\hat{a}$, she obtains the expected payoff of $-c\hat{a}_1 = -c/(2\eta)$. If $c/(2\eta) > \pi(2)$, then to choose $\hat{a}$ is not optimal for the decision maker since she obtains the expected payoff of $-\pi(2)$ by not gathering information at all and choosing project 2. Let us show that a DM-optimal equilibrium exists when $c/(2\eta) \leq \pi(2)$. When she deviates and chooses $a \neq \hat{a}$, the expected payoff for the decision maker is at most $-c/(2\eta)$ or $-\pi(2)$ by Lemma 2. The former is obtained when she chooses $a = (0, 0, 1/(2\eta))$, and the latter is obtained when she chooses $a = (0, 0, 0)$. Therefore, the strategy profile and the system of beliefs defined above constitute an equilibrium.

We can derive the condition under which the strategy of case (ii) constitutes an equilibrium. The argument follows the same as that of case (i) except for the decision regarding information acquisition. When $c/(2\eta) > \min\{\pi(0), \pi(1)\}$, to choose $\hat{a}$ is not optimal for the decision maker since she can obtain the expected payoff of $-\min\{\pi(0), \pi(1)\}$ by not gathering information at all and choosing project 0 or 1 for sure. Thus, suppose that $c/(2\eta) \leq \min\{\pi(0), \pi(1)\}$, and let us ensure that $\hat{a}$ is optimal for the decision maker. When she chooses $\hat{a}$, she obtains the expected payoff of $-c\hat{a}_1 = -c/(2\eta)$. On the other hand, when she chooses $a \neq \hat{a}$, the expected payoff for the decision maker is at most $-c/(2\eta)$ or $-\min\{\pi(0), \pi(1)\}$ by Lemma 2. The former is obtained when she chooses $a = (1/(2\eta), 0, 0)$, and the latter is obtained when she chooses $a = (0, 0, 0)$. Therefore, the strategy profile and the system of beliefs defined above constitute an equilibrium. □

Propositions 5 and 9 imply that if the precision of the signals is sufficiently high, the decision maker tends to gather information about the moderate state ($\theta = 1$). Comparing this argument with the example of a firm in Section 7, we can conclude that an important role of CEO is to investigate whether a current state is moderate but not whether it is an extreme state. The investigation enhances the quality of communication, and as a result, the CEO can make a good decision about the project.

9. Concluding Remarks

The present paper analyzed a situation in which the decision maker can acquire costly information about the states and showed that information acquisition enhances communication. This result is mainly due to the fact that communication and information acquisition complement each other. Since information acquisition is multidimensional, the message from the sender provides the decision maker guidance not only on which project should be chosen but also on what information should be acquired. Furthermore, concentrating the information acquisition on a particular state weakens the sender's exaggeration incentive.

Some extensions should be considered. One is a model in which the decision maker can acquire information and communicate with multiple senders. When there are multiple senders, the decision maker may be able to receive more accurate information, but this may weaken the incentive to gather information about states. Another extension is a model in which both the sender and the decision maker can acquire costly information before the decision making and communication, that is, the sender also obtains an imperfect signal about states but he does not know the true states. Which party gathers what information about states will be analyzed. By analyzing such extensions, we can provide a more systematic discussion of the optimal information aggregation under the organizational decision-making process.

Appendix A. Proof of Proposition 3

Before characterizing babbling equilibria, we derive a general property of equilibria.

Lemma 3. *Suppose that $(\mu^*, (a^*, \rho^*), \beta^*)$ is an arbitrary equilibrium such that $a^*_s \in (0, 1)$ for some $s \in \Theta$. We define a' as $a'_s = 1$ and $a'_\theta = 0$ for $\theta \neq s$. Then, (a', ρ^*) is a best response against μ^* given β^*, and given $(\mu^*, (a', \rho^*))$, the expected payoff for the decision maker is the same as in the original equilibrium.*

Proof. We fix an equilibrium $(\mu^*, (a^*, \rho^*), \beta^*)$. Note that the equilibrium may not be a babbling equilibrium, and μ^* and ρ^* are possibly mixed strategies.

Here, suppose that given the equilibrium, it is satisfied that $a^*_s \in (0, 1)$ for all $s \in \Theta$. For the case in which $a^*_s = 0$ for some s, the proof will be straightforward based on the argument below. Let us define $W^*(a)$ as follows:

$$W^*(a) \equiv \sum_{\theta\in\Theta} \sum_{\xi\in\Xi} \sum_{m\in M} \pi(\theta)P(\xi|a), \xi, \theta)\mu^*(m|\theta) \left[- \{\rho^*(a^*, \xi, m) - \theta\}^2\right]$$

$$- c(a_0 + a_1 + a_2).$$

This represents the expected payoff of the decision maker when she deviates from the equilibrium investigation a^* to some investigation vector a and follows $\rho^*(a^*, \cdot, \cdot)$, that is, she chooses projects as if she did not deviate in terms of the investigation. Remember that $P(\xi|a_\theta, a_{-\theta})$ is linear with respect to a_θ when $a_{-\theta}$ is fixed, and thus $W^*(a_\theta, a_{-\theta})$ is also linear with respect to a_θ.

Let us denote by $D^*_\theta(a)$ the partial derivative of $W^*(a)$ with respect to a_θ. Then, in equilibrium, it holds that $D^*_s(a^*) = k \geq 0$ for all s, where k is a constant since a^* constitutes an equilibrium. In contrast, if $D^*_s(a^*) > D^*_{s'}(a^*)$ for some $s' \neq s$, then we have $W^*(\hat{a}) > W^*(a^*)$, where $\hat{a}_s = a^*_s + a^*_{s'}$, $\hat{a}_{s'} = 0$, and $\hat{a}_\theta = a^*_\theta$ for $\theta \neq s, s'$. This contradicts the assumption that a^* constitutes an equilibrium.

Let a' be an investigation vector such that $a'_s = a^*_0 + a^*_1 + a^*_2$. Then, we have $W^*(a') = W^*(a^*)$ since $W^*(a)$ is linear with respect to a_θ for each θ, and it holds that $D^*_\theta(a^*) = k \geq 0$ for each θ. Let us denote $W'(a)$ by replacing $\rho^*(a^*, \cdot, \cdot)$ with $\rho^*(a', \cdot, \cdot)$ in $W^*(a)$. Then, we have $W'(a') = W^*(a')$ since ρ^* is sequentially rational. This implies that (a', ρ^*) is a best response against μ^* given β^*. For the case in which $a'_{s''} = 0$ for some s" and $a'_s \in (0, 1)$ for $s \neq s$", the argument above can be applied.

If $a'_s = 1$, then the proof is completed. Now suppose that $a'_s \in (0, 1)$ and $a'_\theta = 0$ for $\theta \neq s$. $W'(a')$ is linear with respect to a'_s and $D'_s(a') \geq 0$ holds, where $D'_s(a)$ is the partial derivative of $W'(a)$ with respect to a_s. Hence, we have $W'(a") \geq W'(a')$, where a" satisfies $a_s" = 1$. We denote

W"(a) by replacing $\rho^*(a', \cdot, \cdot)$ with $\rho^*(a", \cdot, \cdot)$ in $W'(a)$. Then, we have $W"(a") \geq W'(a")$ since ρ^* is sequentially rational under the given equilibrium. On the other hand, we have $W^*(a^*) \geq W"(a")$ since (a^*, ρ^*) constitutes an equilibrium. Therefore, we have $W"(a") = W^*(a^*)$. This completes the proof. □

Based on the result above, we can focus on babbling equilibria in which the decision maker gathers information about only one state in order to characterize babbling equilibria.

Corollary 1. *Suppose that $(\mu^*, (a^*, \rho^*), \beta^*)$ is a babbling equilibrium such that $a_s^* \in (0, 1)$ for some $s \in \Theta$. Then, there is a corresponding equilibrium $(\mu^*, (a', \rho^*), \beta^*)$ such that $a'_s = 1$.*

Proof. The proof is straightforward from Lemma 3. □

Now, let us show that no babbling equilibrium exists in which the decision maker gathers information about state 1.

Given the investigation vector $a^1 = (0, 1, 0)$, only signal ξ_1 is informative. Thus, the decision maker optimally chooses a project depending on ξ_1. When a^1 is optimal, the decision maker must choose $y = 1$ if $\xi_1 = t$. For $\xi_1 = f$, the optimal project depends on the prior, and thus there are two cases.

Case 1: The optimal project is $y = 0$ when $\xi_1 = f$, and the expected payoff for the decision maker is given by

$$-\pi(0)\left(\frac{1}{2} - \eta\right) - \pi(1)\left(\frac{1}{2} - \eta\right) - \pi(2)\left(\frac{5}{2} + 3\eta\right) - c. \tag{18}$$

Case 2: The optimal project is $y = 2$ when $\xi_1 = f$, and the expected payoff for the decision maker is given by

$$-\pi(0)\left(\frac{5}{2} + 3\eta\right) - \pi(1)\left(\frac{1}{2} - \eta\right) - \pi(2)\left(\frac{1}{2} - \eta\right) - c. \tag{19}$$

When $a^0 = (1, 0, 0)$ is optimal, the decision maker must choose $y = 0$ if $\xi_0 = t$. By Assumption 1, it is optimal for the decision maker to choose $y = 1$ if $\xi_0 = f$. The expected payoff for the decision maker is given by

$$-\pi(0)\left(\frac{1}{2} - \eta\right) - \pi(1)\left(\frac{1}{2} - \eta\right) - \pi(2)\left(\frac{5}{2} - 3\eta\right) - c. \tag{20}$$

When $a^2 = (0, 0, 1)$ is optimal, the decision maker chooses $y = 2$ if $\xi_1 = t$. If $\xi_1 = f$, the optimal project depends on the prior, and thus there are two cases.

Case 1: The optimal project is $y = 0$ when $\xi_2 = f$, and the expected payoff for the decision maker is given by

$$-\pi(0)\,(2 - 4\eta) - \pi(1) - \pi(2)\,(2 - 4\eta) - c. \tag{21}$$

Case 2: The optimal project is $y = 1$ when $\xi_2 = f$, and the expected payoff for the decision maker is given by

$$-\pi(0)\left(\frac{5}{2} - 3\eta\right) - \pi(1)\left(\frac{1}{2} - \eta\right) - \pi(2)\left(\frac{1}{2} - \eta\right) - c. \tag{22}$$

It is easily verified that (18) is strictly smaller than (20), and (19) is strictly smaller than (22). Therefore, it has been shown that no babbling equilibrium exists in which the decision maker gathers information about state 1 with a positive probability. □

Appendix B. Proof of Proposition 4

Let $(\hat{\mu}, \hat{\rho}, \hat{\beta})$ be an arbitrary equilibrium. Here the investigation vector $a^0 = (0, 0, 0)$ and signal profile ξ are omitted. Although the decision maker can choose projects depending on the signals, this does not expand her strategic possibilities. In what follows, $\hat{\rho}(y|m)$ represents the probability that the decision maker chooses project y given a message m, and $\hat{\beta}(\theta|m)$ is the decision maker's belief about state θ given the message m. Let $\hat{Z}$ be the set of the messages chosen with positive probabilities on the equilibrium path, that is, $\hat{Z} \equiv \bigcup_{\theta=0}^{2} \hat{Z}(\theta)$. We prove this proposition in three steps.

Claim 1. *In any equilibrium, there exists no message $m \in M$ such that $\hat{\rho}(0|m) > 0$ and $\hat{\rho}(2|m) > 0$.*

In contrast, suppose that there exists a message m such that $\hat{\rho}(0|m) > 0$ and $\hat{\rho}(2|m) > 0$ in an equilibrium. Then, the decision maker must be indifferent between project 0 and 2, and it must be satisfied that

$$-\hat{\beta}(1|m) - 4\hat{\beta}(2|m) = -4\hat{\beta}(0|m) - \hat{\beta}(1|m).$$

Based on the equation, we have $\hat{\beta}(0|m) = \hat{\beta}(2|m)$. Then, the following inequality holds.

$$-\hat{\beta}(0|m) - \hat{\beta}(2|m) > -\hat{\beta}(1|m) - 4\hat{\beta}(2|m) = -4\hat{\beta}(0|m) - \hat{\beta}(1|m).$$

This implies that project 1 is strictly better for the decision maker than projects 0 and 2. This is a contradiction.

Claim 2. *In any equilibrium, there exists no message $m \in M$ such that $\hat{\rho}(2|m) > 0$.*

First, let us show that project 2 is never chosen with a positive probability on the equilibrium path. Let $\tilde{M}$ be the set of messages such that the sender sends the messages with positive probabilities in the equilibrium, and the decision maker chooses project 2 with a positive probability given any such message:

$$\tilde{M} = \left\{ m \in M \;\middle|\; m \in \hat{Z} \text{ and } \hat{\rho}(2|m) > 0 \right\}.$$

By Claim 1, it holds that

$$\hat{\rho}(2|m) > 0,\ \hat{\rho}(1|m) \geq 0,\ \text{and}\ \hat{\rho}(0|m) = 0, \quad \forall m \in \tilde{M}. \tag{23}$$

Suppose that $\tilde{M} \neq \emptyset$. First, we assume $\hat{Z}(1) \subseteq \tilde{M}$. Then, since $\pi(1) > \pi(2)$, there exists a message $\tilde{m} \in \tilde{M}$ such that $\hat{\beta}(1|\tilde{m})/\hat{\beta}(2|\tilde{m}) \geq \pi(1)/\pi(2)$. Then, we have

$$-\hat{\beta}(0|\tilde{m}) - \hat{\beta}(2|\tilde{m}) > -4\hat{\beta}(0|\tilde{m}) - \hat{\beta}(1|\tilde{m}).$$

This implies that the decision maker strictly prefers project 1 to project 2 given message $\tilde{m}$. This contradicts (23), and $\hat{Z}(1) \subseteq \tilde{M}$ cannot hold in the equilibrium.

Based on the discussion above, there must exist a message m' such that $m' \in \hat{Z}(1)$ and $m' \notin \tilde{M}$. Thus, it must be satisfied that

$$\hat{\rho}(2|m') = 0,\ \hat{\rho}(1|m') \geq 0,\ \text{and}\ \hat{\rho}(0|m') \geq 0, \quad \forall m' \notin \tilde{M}.$$

Since $b \in (1/2, 1)$, $\hat{\rho}(0|m') + \hat{\rho}(1|m') = 1$, and $\hat{\rho}(1|m) + \hat{\rho}(2|m) = 1$ for any $m \in \tilde{M}$, the following inequality holds:

$$
\begin{aligned}
& -\hat{\rho}(1|m)\cdot b^2-\hat{\rho}(2|m)\cdot(1-b)^2>-\hat{\rho}(0|m')\cdot(1+b)^2-\hat{\rho}(1|m')\cdot b^2\\
\Leftrightarrow\quad & -\hat{\rho}(2|m)\cdot(1-2b)>-\hat{\rho}(0|m')\cdot(1+2b).
\end{aligned}
$$

This implies that the sender of type 1 strictly prefers a message $m \in \tilde{M}$ to m'. This contradicts the assumption of $m' \in \hat{Z}(1)$. Therefore, we conclude $\tilde{M} = \emptyset$.

Finally, we show that the decision maker never chooses project 2 with a positive probability even off the equilibrium path. Let m be a message off the equilibrium path, and suppose that $\hat{\rho}(2|m) > 0$. By Claim 1, it must be satisfied that

$$\hat{\rho}(2|m) > 0,\ \hat{\rho}(1|m) \geq 0,\ \text{and}\ \hat{\rho}(0|m) = 0.$$

Then, deviating to send message m is profitable for the sender of type 1 since $\tilde{M} = \emptyset$. Therefore, in any equilibrium, such a message must not exist. This completes the proof of Claim 2, and the last part of Proposition 4 has been shown.

Claim 3. *In any equilibrium, the decision maker chooses a project irrespective of the message sent by the sender on the equilibrium path.*

By Claim 1 and 2, it must hold in any equilibrium that

$$\hat{\rho}(0|m) \geq 0,\ \hat{\rho}(1|m) \geq 0,\ \text{and}\ \hat{\rho}(2|m) = 0, \quad \forall m \in M.$$

Contrary to the assertion of Claim 3, we assume that there exist messages $m', m'' \in \hat{Z}$ such that $\hat{\rho}(0|m') > \hat{\rho}(0|m'')$. Then, it is never optimal for the sender of each type to send message m' and this contradicts the assumption of $m' \in \hat{Z}$. Therefore, the decision maker chooses a project irrespective of message sent by the sender on the equilibrium path.

This completes the proof of Proposition 4.

Additionally, we describe the project choice of the decision maker on the equilibrium path: (i) if $\pi(0) - \pi(1) - 3\pi(2) > 0$, then any equilibrium satisfies $\hat{\rho}(0|m) = 1$ for any $m \in \hat{Z}$; (ii) if $\pi(0) - \pi(1) - 3\pi(2) = 0$, then for any $\hat{\rho}$ such that $\hat{\rho}(0|m) = \hat{\rho}(0|m')$ for any $m, m' \in \hat{Z}$, and $\hat{\rho}(0|m) + \hat{\rho}(1|m) = 1$ for any $m \in \hat{Z}$, a corresponding equilibrium exists; (iii) if $\pi(0) - \pi(1) - 3\pi(2) <$

0, then any equilibrium satisfies $\hat{\rho}(1|m) = 1$ for any $m \in \hat{Z}$.

In what follows, let us consider case (i) and (ii). Case (iii) is obtained analogously to case (i). The following condition is necessary to be optimal for the decision maker to choose project 0:

$$-\hat{\beta}(1|m) - 4\hat{\beta}(2|m) \geq -\hat{\beta}(0|m) - \hat{\beta}(2|m), \quad \forall m \in \hat{Z}.$$

From the inequality, we have

$$\pi(0)\hat{\mu}(m|0) - \pi(1)\hat{\mu}(m|1) - 3\pi(2)\hat{\mu}(m|2) \geq 0, \quad \forall m \in \hat{Z}. \tag{24}$$

Summing up the inequalities with respect to m yields

$$\pi(0) - \pi(1) - 3\pi(2) \geq 0.$$

Let us confirm that if $\pi(0) - \pi(1) - 3\pi(2) > 0$, then $\hat{\rho}(0|m) = 1$ holds for any $m \in \hat{Z}$. In contrast, suppose that there exists a message $m' \in \hat{Z}$ such that $\hat{\rho}(1|m') > 0$. Then, for any $m \in \hat{Z}$, it holds $\hat{\rho}(1|m) > 0$ since $\hat{\rho}(2|m) = 0$ by Claim 2. Thus, choosing project 1 must be optimal for the decision maker, and it must hold that

$$\begin{aligned} & -\hat{\beta}(1|m) - 4\hat{\beta}(2|m) \leq -\hat{\beta}(0|m) - \hat{\beta}(2|m) \\ \Leftrightarrow \quad & \pi(0)\hat{\mu}(m|0) - \pi(1)\hat{\mu}(m|1) - 3\pi(2)\hat{\mu}(m|2) \leq 0, \quad \forall m \in \hat{Z}. \end{aligned} \tag{25}$$

Summing up the last inequalities with respect to m, we have $\pi(0) - \pi(1) - 3\pi(2) \leq 0$. This contradicts the assumption of $\pi(0) - \pi(1) - 3\pi(2) > 0$. Therefore, it is satisfied that $\hat{\rho}(0|m) = 1$ holds for any $m \in \hat{Z}$.

Finally, we make sure that if $\pi(0) - \pi(1) - 3\pi(2) = 0$, then the decision maker is indifferent between project 0 and 1 given any $m \in \hat{Z}$. First, suppose contrarily that there exists a message $m \in \hat{Z}$ for which the decision maker strictly prefers to choose project 1. Then, every type of the sender strictly prefers to send message m, and (25) holds with strict inequality for any $m \in \hat{Z}$. We have $\pi(0) - \pi(1) - 3\pi(2) < 0$. This is a contradiction.

Next, suppose that there exists a message $m \in \hat{Z}$ for which the decision maker strictly prefers to choose project 0. Hence, since $\pi(0) - \pi(1) - 3\pi(2) = 0$, there must exist a message $m' \in \hat{Z}$ for which the decision maker strictly prefers to choose project 1, and then every type of the sender strictly prefers to send message m'. This contradicts $m \in \hat{Z}$.

Based on the discussion, the decision maker must be indifferent between project 0 and 1 for any $m \in \hat{Z}$. By Claim 3, there exists a corresponding equilibrium for any $\hat{\rho}$ such that $\hat{\rho}(0|m) = \hat{\rho}(0|m')$ for any $m, m' \in \hat{Z}$, and $\hat{\rho}(0|m) + \hat{\rho}(1|m) = 1$ for any $m \in \hat{Z}$. □

Appendix C. Proof of Proposition 7

Suppose that $(\mu^*, (a^*, \rho^*), \beta^*)$ is an arbitrary equilibrium in which $a^*_s > 0$ for some $s \in \Theta$, where the sender might randomize messages under μ^*.

By Lemma 3 in Appendix A, given μ^* and β^*, strategy (a', ρ^*) is optimal for the decision maker, where a' is an extreme point of A such that $a' \neq (0, 0, 0)$. The strategy profile $(\mu^*, (a', \rho^*))$ induces the same expected payoff for the decision maker as in the equilibrium. Note that $(\mu^*, (a', \rho^*), \beta^*)$ is not necessarily an equilibrium. Below, we assume $a' = (0, 1, 0)$ and show that the expected payoff of the decision maker given μ^* and (a', ρ^*) does not exceed $\hat{\Pi}$. For $a' = (1, 0, 0)$ and $(0, 0, 1)$, the same argument essentially, and thus, we omit them.

Let us denote a nonempty subset of Θ as σ. A message m^σ represents that given the system of beliefs β^*, the set σ contains all states that the decision maker believes that it is realized with a positive probability when she receives m^σ. By Lemma 1, there is no equilibrium in which $m^{\{2\}}$ exists, and by Proposition 2, the expected payoff of the decision maker does not exceed $\hat{\Pi}$ when $m^{\{1\}}$ exists in equilibrium. Hence, for each m^σ such that $\sigma \in \{\{0\}, \{0, 1\}, \{0, 2\}, \{0, 1, 2\}, \{1, 2\}\}$, we show that given μ^* and (a', ρ^*), the expected loss at each state $\theta \in \sigma$ does not exceed that in the partially informative equilibrium.

In the partially informative equilibrium, the expected loss of the decision maker is zero conditional on state 0, $1/2 - \eta$ at state 1, and $1/2 - \eta$ conditional on state 2 from the viewpoint of *ex ante*.

When the decision maker receives message $m^{\{0\}}$, she believes that the true state is $\theta = 0$. Thus, she optimally chooses $y = 0$ and she obtains no loss conditional on state 0. That is, the loss is the same as in the partially informative equilibrium.

Next, let us consider message $m^{\{1,2\}}$. Note that given the belief, it must not be strictly optimal for the decision maker to choose $y = 1$ or 2 for sure, which follows Lemma 1 and Proposition 2. Thus, suppose that to choose a project for sure is not optimal. Then, the decision regarding projects is completely the same as that in the partially informative equilibrium since ξ_1 is only informative under $a' = (0, 1, 0)$. When to choose $y = 1$ and 2 for sure is indifferent, it is also optimal to follow the same decision regarding projects, that is, to choose a project depending only on ξ_1. Therefore, given $m^{\{1,2\}}$,the expected loss conditional on state $\theta = 1, 2$ is the same as in the partially informative equilibrium.

Finally, let us consider a message m^{σ} such that $\sigma \in \{\{0, 1\}, \{0, 2\}, \{0, 1, 2\}\}$. Again, note that given the belief, it must not be strictly optimal for the decision maker to choose $y = 1$ or 2 for sure, which follows Lemma 1 and Proposition 2. We have only to consider two cases. (1) If she chooses $y = 0$ for sure, then she obtains no loss conditional on state 0 and strictly greater loss conditional on states 1 and 2 than that in the partially informative equilibrium. (2) If the decision maker chooses projects depending on the signals, she chooses projects depending only on signal ξ_1 since $a' = (0, 1, 0)$. Thus, given m^{σ}, the expected loss at state 0 is strictly positive, and the expected loss at state $\theta = 1, 2$ is at most the same as that under the partially informative equilibrium.

Based on the argument above, the expected loss at each state does not exceed that under the partially informative equilibrium from the viewpoint of *ex ante*. □

References

[1] Argenziano, R., Severinov, S., and F. Squintani (2016), "Strategic information acquisition and transmission," *American Economic Journal: Microeconomics*, 8, 119–155.

[2] Austen-Smith, D. (1994), "Strategic transmission of costly information," *Econometrica*, 62, 955–963.

[3] Blume, A., O. Board, and K. Kawamura (2007), "Noisy talk," *Theoretical Economics*, 2, 395–440.

[4] Crawford, V. and J. Sobel (1982), "Strategic information transmission," *Econometrica*, 50, 1431–1451.

[5] Chen, Y. (2009), "Communication with two-sided asymmetric information," mimeo.

[6] Chen, Y. (2012), "Value of public information in sender-receiver games," *Economics Letters*, 114, 343–345.

[7] Goltsman, M., J. Hörner, G. Pavlov, and F. Squintani (2009), "Mediation, arbitration and negotiation," *Journal of Economic Theory*, 144, 1397–1420.

[8] Ishida, J. and T. Shimizu (2015), "Cheap talk with an informed receiver," *Economic Theory Bulletin*, 4, 61–72.

[9] Ishida, J. and T. Shimizu (2019), "Cheap talk when the receiver has uncertain information sources," *Economic Theory*, 68. 303–334..

[10] Ivanov, M. (2010), "Communication via a strategic mediator," *Journal of Economic Theory*, 145, 869–884.

[11] Krishna, V. and J. Morgan (2004), "The art of conversation: eliciting information from experts through multi-stage communication," *Journal of Economic Theory*, 117, 147–179.

[12] Lai, E. (2014), "Sender advice for amateurs," *Journal of Economic Behavior & Organization*, 103, 1–16.

[13] Moreno de Barreda, I. (2013), "Cheap talk with two-sided private information," mimeo.

[14] Olszewski, W. (2004), "Informal communication," *Journal of Economic Theory*, 117, 180–200.

[15] Pei, D. (2015), "Communication with endogenous information acquisition," *Journal of Economic Theory*, 160, 132–149.

[16] Seidmann, D. (1990), "Effective cheap talk with conflicting interests," *Journal of Economic Theory*, 50, 445–458.

[17] Watson, J. (1996), "Information transmission when the informed party is confused," *Games and Economic Behavior*, 12, 143–161.

[18] Venturini, A. (2014), "Cheap talk with transfers," mimeo.