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“Middle-Income Traps and Complexity in Economic Development”

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Middle-Income Traps and Complexity in Economic Development*

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Abstract

In this paper, we develop a simple growth model that exhibits a wide variety of economic development patterns. In particular, our numerical simulations demonstrate that for a given set of parameter values, various types of development patterns such as the middle-income trap, the poverty trap, periodic or chaotic fluctuations, and high-income paths, can coexist, and which pattern is realized depends only on the initial value of capital. For another set of parameter values, we show that due to the pinball effect, an economy starting at a middle-income level can take off to the high-income state or get caught in the poverty trap in a seemingly random way after undergoing transient chaotic motions. Our results can explain observed complicated patterns of economic development in a unified manner.

Keywords: Middle-income traps; Poverty traps; Complex dynamics; Technology choice; CES production function

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1 Introduction

The most standard model of economic growth and development is the Solow model in which, as is well known, any economy exhibits a monotonic development process converging to its steady state. However, this is not necessarily an actual pattern of economic development. Some less developed economies remain at low-income levels for long periods, whereas others experience rapid growth and successfully catch up to the developed countries. Moreover, some economies, which succeed in escaping the low-income level, fail to continue to grow further after certain rapid growth periods. As such, there are various patterns of economic development in the real world. Using data from the Maddison project,¹ we plot the evolutions of gross domestic product (GDP) per capita of Singapore, Cote d'Ivoire, Zimbabwe, and Japan. As the figures show, the GDP per capita of Singapore has continued to grow, whereas Cote d'Ivoire and Zimbabwe exhibit non-monotonic movements in per capita GDP over extended periods.

[Insert Figures 1, 2, 3 and 4]

In the following, to take a closer look at real-world economic development patterns, we classify the per capita income level of each country into three categories: low, middle, and high.

For low-income countries, the traditional focus of analysis has been the “development trap” or “poverty trap” argument, which states that low-income countries cannot escape poverty even in the long run.² Recently, however, because many countries in East Asia and elsewhere have escaped the development trap and started to grow, there has been much interest in the patterns of economic development of countries at the middle-income level.

In East Asia, Latin America, the Middle East, and North Africa, a significant

¹We use “real GDP per capita in 2011\$” from the Maddison Project Database 2020, which is available at <https://www.rug.nl/ggdc/historicaldevelopment/maddison/?lang=en>

²See, for example, Azariadis and Stachurski (2005), Jones (2016), and Matsuyama (2010).

number of countries that escaped from the poverty trap experienced a sharp decline in growth rates after their initial periods of rapid economic growth, thus remaining in the middle-income bracket. In other words, although they successfully reached the middle-income level, these countries were unable to catch up with the high-income developed countries. This phenomenon, referred to as the “middle-income trap” has been analyzed by many researchers.³ However, it should be noted that several countries have succeeded in escaping from the middle-income trap. For example, although Japan remained in the middle-income range for a relatively long period, it did finally escape and experienced very high growth (see Figure 4). Doner and Schneider (2016) point out that from the 1980s to the 1990s, 13 countries escaped from the middle-income trap and moved to the high-income level. Thus, the actual development patterns around the middle-income range vary across countries.

Using GDP per capita for each country relative to that of the United States, Jones (1997) and Kane (2016) investigate the transition processes of countries from low to higher income levels.⁴ Jones (1997) examines transition processes toward the end of the 20th century and finds twin peaks in the world distribution of GDP per capita. This indicates that the countries have been divided into two groups, poor and rich, which suggests the existence of a development trap. However, almost two decades later, Kane (2016) reexamines Jones (1997) and finds that the low-income countries continued to grow as a whole (albeit at a slow pace) and that most of them escaped the development trap. Kane’s finding supports the recent shift in research interest from the development trap to the middle-income trap.

Following Jones (1997) and Kane (2016), we briefly examine how the GDP per capita of each country relative to that of the United States evolved during the period from 2002 to 2017.⁵ In line with the middle-income trap literature, we focus on the

³See, for example, Im and Rosenblatt (2013) and Agenor (2017) for surveys on this literature. Hu et al. (2020) develop a formal general equilibrium model of the middle-income trap. See also Eichengreen et al. (2013).

⁴Both Jones (1997) and Kane (2016) use data from the Penn World Table, using Marks 5.6 and 8.1, respectively.

⁵We use data from the Penn World Table (Mark 9.1). See Feenstra et al. (2015).

transition process between the middle- and high-income brackets. We use a data set of the real GDP per capita of 161 countries, from which countries with populations of 300,000 or fewer are excluded, similar to Kane (2016). Moreover, we limit our attention to the period from 2002 and 2017, as the most recent data available is for 2017 in the Penn World Table (Mark 9.1). Table 1 shows the distribution of relative GDP per capita based on 10 tiers. Tier 1 consists of the richest countries, for which average per capita incomes for 2002 and 2017 are above 90% of that of the United States, whereas tier 10 represents the poorest countries, with average per capita incomes below 10% of per capita income in the United States. Except for tier 1, the cutoff levels between each tier increase by 10%. Tier 1 countries are those with per capita incomes of more than 90% of that of United States. For example, in 2017, Singapore’s income per capita was \$79,872, placing it in tier 1, as the income per capita of the United States was \$56,153.

[Insert Tables 1 and 2]

From these tables, we can observe that there are countries that grew steadily (rising one income rank during the analysis period), those that grew rapidly (rising two or more ranks), those in which growth shrank (falling one rank), and those in which growth shrank significantly (falling two or more ranks).⁶ That is, the development patterns between the middle- and high-income levels are diverse and can be non-monotonic.

Although many recent empirical studies focus on such varying patterns of economic development, there are very few formal models that can explain these observed variations (particularly the development patterns of middle-income countries) in a unified manner. As far as we know, Hu et al. (2020) are the first to construct an infinitely lived representative agent model of economic development with endogenous technology choice on human capital investment. They show that the model

⁶Hu et al. (2020) estimate transition probabilities across the quintiles and obtain a similar result to ours.

can exhibit observed patterns of economic development, such as a poverty trap, a middle-income trap, and a flying geese pattern of economic development.⁷ Their rich results come from their specification of human capital investment. In their model, there are multiple human capital investment technologies that include a kind of threshold externalities; that is, a technology with higher productivity is accompanied by a higher threshold level of human capital. The degree of these externalities plays an essential role in technology choice and enables a unified explanation of the empirically observed development patterns. Our model resembles that of Hu et al. (2020) because both share the endogenous technology choice setting. However, our model is simpler in the sense that it does not include any externalities. Nonetheless, we can produce a wide variety of economic dynamics, including poverty traps, middle-income traps, and complicated dynamics.

For a given set of parameters in our model, numerical simulation results show that the economy can be caught in the middle-income trap or the poverty trap, or that it can converge to the high-income state, depending on the initial values. We consider that this can explain the fact that there exist countries that stay in, drop below, or move up from the middle-income class, as shown in Table 2.

From a theoretical perspective, there are several studies related to ours. The role of an endogenous technology choice by investors has attracted the attention of many researchers in the literature on endogenous business cycles. This line of research includes Iwaisako (2002) and Matsuyama (2007).⁸ In their economy, there are multiple production technologies, and firm owners select one among them, depending on the economic conditions at that time. They graphically show that the selected technology switches endogenously over time as the economic conditions change, and that this switch in the technology choice causes discontinuities and, as a result, rich patterns of economic dynamics.⁹

⁷In the model of Hu et al. (2020), continual technology upgrading in human/knowledge accumulation can occur as the economy develops, a situation that they refer to as flying geese development *à la* Akamatsu (1962).

⁸See also Aghion et al. (1999) for an early attempt in this direction.

⁹Iwaisako (2002) assumes that each investor selects the technology with higher capital returns.

The dynamic properties of these models are analyzed in depth by Kunieda and Shibata (2003), Asano et al. (2012), Matsuyama et al. (2018), Asano and Yokoo (2019), and Umezuki and Yokoo (2019a). By specifying the production technologies as the Cobb–Douglas type, these studies analytically investigate the dynamic properties of these kinds of models. They utilize the fact that when technologies are of the Cobb–Douglas type and preferences are log-linear, the equilibrium dynamics can be transformed into the piecewise linear first-order difference equation, which is known as the Caianiello equation of the neuron model.¹⁰

Although the Cobb–Douglas technology assumption greatly reduces the difficulty in the analysis, it removes important economic implications of endogenous technology choice for economic development and business cycles. Thus, we extend these studies, especially Kunieda and Shibata (2003) and Umezuki and Yokoo (2019a), to the case of the constant elasticity of substitution (CES) production technologies and investigate the properties of the equilibrium dynamics.

2 Basic Settings

We extend the analysis by Kunieda and Shibata (2003) and Umezuki and Yokoo (2019a) on dynamics generated by endogenous technology choice. Although their studies focus on the case of Cobb–Douglas production technologies, we assume a more general class of technology, namely CES technologies, as noted above. The economy begins in period 1 and continues over time toward infinity. Economic

The model of Matsuyama (2007) is more sophisticated. In his model, each agent can become either an entrepreneur or a lender, and there are multiple capital-producing technologies available, with different degrees of productivity and pledgeability (e.g., technology 1 produces more capital than technology 2, but 1 may be less pledgeable than 2). Moreover, because each capital-producing technology has its own minimum input size, the amount of funds required for an entrepreneur to adopt a capital-producing technology differs across technologies. In this environment, the entrepreneur’s choice of technology depends on the market interest rate and the level of the investors’ wealth. Because the equilibrium interest rate and the level of entrepreneurial wealth vary over time as capital accumulates, the choice of technology switches endogenously, and the switches produce discontinuities in the reduced form final goods production function.

¹⁰The business cycle model developed by Ishida and Yokoo (2004) takes a similar form. They show that the model can generate asymmetric periodic cycles of arbitrarily large periods.

agents, except for generation 0, are born at the beginning of each period and live two periods. Generation 0 is present at the inception of the economy and has capital stock k_0 . The population of each generation, L , is constant over time. The goods and factor markets are competitive.

2.1 Household Behavior: Cobb–Douglas Utility

Our formulation of consumer behavior is the same as Iwaisako (2002), Kunieda and Shibata (2003), and Umezuki and Yokoo (2019a). An agent born in period t (called generation t) consumes c_t in period t and d_{t+1} in period $t + 1$. Each agent supplies one unit of labor inelastically in the first period and divides his/her wage income between consumption in that period and saving. In the second period, the agent consumes the proceeds of his/her savings. The maximization problem of generation t is:

$$\begin{aligned} \max_{c_t, d_{t+1}, s_t} \quad & (1 - s) \log c_t + s \log d_{t+1}, \quad s \in (0, 1) \\ \text{s.t.} \quad & s_t + c_t = w_t, \quad d_{t+1} = R_{t+1} s_t, \end{aligned} \tag{1}$$

where w_t denotes the wage rate, and R_{t+1} denotes the rate of return on saving. Then, optimal saving is given by:

$$s_t = s w_t. \tag{2}$$

Throughout this paper, we set the saving rate to be 0.3 ($s = 0.3$).

2.2 Production: CES Technology

Following Kunieda and Shibata (2003) and Umezuki and Yokoo (2019a), we assume that there are m types of technologies for firms (old generations). The technologies are assumed to take the following CES forms:

$$f_i(k) = A_i [(1 - \alpha_i) + \alpha_i k^{-\rho_i}]^{-1/\rho_i} = A_i k [\alpha_i + (1 - \alpha_i) k^{\rho_i}]^{-1/\rho_i},$$

where $\alpha_i \in (0, 1)$ and $A_i > 0$. In this formulation, if $\rho_i = -1$, f_i is linear; if $\rho_i \rightarrow +\infty$, f_i is of the Leontief type; and if $\rho_i \rightarrow 0$, f_i approaches the Cobb–Douglas type.

2.3 Technology Choice

At the beginning of each period, the owners of firms (old generations) select one of the available technologies with the highest return on capital:

$$\max_{i \in M} f'_i(k), \quad (3)$$

where $M = \{1, 2, \dots, m\}$ denotes the set of production technologies.

The marginal productivity schedules of the CES technologies:

$$f'_i(k) = \alpha_i A_i [\alpha_i + (1 - \alpha_i)k^{\rho_i}]^{-(1+\rho_i)/\rho_i}$$

are depicted in Figure 5 for $m = 2$. In Figure 5, there is only one crossing of the f'_i curves. Note that the f'_i curves may cross with each other more than once.

[Insert Figure 5]

2.3.1 Factor Prices

Because of perfect competition, the marginal productivity of each production factor is equalized to its price, that is:

$$R_t = f'_i(k_t), \quad w_t = f_i(k_t) - k_t f'_i(k_t) \equiv w_i(k_t), \quad i \in M. \quad (4)$$

Simple calculation gives:

$$w_i(k) = A_i k \left[[\alpha_i + (1 - \alpha_i)k^{\rho_i}]^{-1/\rho_i} - \alpha_i [\alpha_i + (1 - \alpha_i)k^{\rho_i}]^{-(1+\rho_i)/\rho_i} \right]. \quad (5)$$

Figure 6 depicts a typical wage schedule (multiplied by s) under the CES-type technology with $\rho > 0$. While the wage schedule for the Cobb–Douglas technology is an increasing concave function of k , it can easily be seen that the wage schedule for the CES technology with $\rho > 0$ is increasing but sigmoidal. That is, the graph of $w(k)$ is relatively flat for a small k or large k , but rather steep for a moderate k . As we will see later, the last characteristic of the wage schedule for the CES technology can be a source of complex patterns of economic development.

[Insert Figure 6]

2.4 The Dynamical System

When the selected technology changes, the wage schedule also changes, which causes a discontinuity of the dynamical system at the point of technology switching. For simplicity, we will consider the case where only two technologies are present. As we will see later, two technologies are enough to generate many interesting dynamic phenomena that cannot be reproduced by the one-technology formulation.

Then, the dynamical system that we deal with below can be summarized as follows:

$$k_{t+1} = sw_{\tau}(k_t), \quad (6)$$

$$\tau = \arg \max_{i \in \{1,2\}} f'_i(k_t). \quad (7)$$

Figure 7 presents a typical return map given by the model (6)-(7) when there are two CES technologies corresponding to Figure 5.

[Insert Figure 7]

3 Main Results

From the study of Umezuki and Yokoo (2019a), we know that the model consisting of (6) and (7) can exhibit attracting periodic cycles of arbitrarily large periods when both technologies are of the Cobb–Douglas type. In this study, we demonstrate that when the technologies are of the CES form with $\rho > 0$, which is more general than the Cobb–Douglas form of technologies, the model can reproduce richer and more interesting patterns of economic development.¹¹ In particular, this section shows that a wide variety of economic dynamics, including poverty traps, middle-income traps, and complicated dynamics, can be produced, and that these potential patterns can be realized depending only on the initial values.

¹¹This expression is somewhat inaccurate. Indeed, for a continuum of Cobb–Douglas technologies, Umezuki and Yokoo (2019b) show that a similar model can generate chaotic dynamics.

3.1 Coexistence of the Poverty Trap, Middle-Income Trap, and High-Income State

For a given set of parameter values, numerical simulations demonstrate that various types of dynamic patterns, such as a poverty trap, periodic or chaotic fluctuations, and high-growth paths¹² *simultaneously* emerge, depending on the initial values. In Figure 8, there are four steady states, two of which are stable ($ss_1 = 0$ and ss_4) and two unstable (ss_2 and ss_3). Any trajectory starting from the interval $[ss_1, ss_2)$ monotonically converges to $ss_1 = 0$. This situation is referred to as the *poverty trap*. By contrast, any trajectory starting from a point larger than ss_3 monotonically converges to the highest steady state ss_4 , which we will call the *high-income state*.

[Insert Figure 8]

The behavior of a trajectory starting from the interval (ss_2, ss_3) is more complex. See Figure 9. As the open interval (ss_2, ss_3) contains a discontinuity but no steady state, and as the highest point of the graph of the return map does not exceed ss_3 and the lowest point cannot be less than ss_2 , any trajectory starting from (ss_2, ss_3) cannot escape from that interval and has to keep fluctuating in a periodic or chaotic manner. We call this situation the *middle-income trap*, as the economy is caught in the middle range of income (or capital), neither converging to the poverty trap nor taking off to the high-income state. For the given set of parameter values above, the economy fluctuates in a chaotic manner in the sense of positivity of the Lyapunov exponent¹³ when the economy is caught in the middle-income trap. It is important to recognize that, for the same parameter values noted above, the middle-income trap coexists with the poverty trap and the high-income state, and which situation

¹²We refer to a growth path converging to the high-income state as a *high-growth path*.

¹³For a one-dimensional smooth map $x_{t+1} = f(x_t)$, the Lyapunov exponent λ is calculated according to the formula:

$$\lambda = \lim_{n \rightarrow +\infty} \frac{1}{n} \sum_{t=0}^{n-1} \ln |f'(x_t)|.$$

For the set of parameter values corresponding to Figure 9, it is computed as $\lambda = 0.60$, which indicates that chaos occurs in the middle-income trap.

is attained depends only on the initial value of capital.

[Insert Figure 9]

3.2 Taking Off or Breaking Down: The Pinball Effect

In the previous subsection, we dealt with a situation in which an economy is trapped at the middle-income level. The question then arises: is the economy trapped there forever, without either taking off to the high-income state or reverting to poverty? We would like to give another parametric example in which both takeoff and break-down from the middle-income range can take place, although the situation is a little complicated.

Figures 10 and 11 illustrate our argument. Compared with Figure 9, the parameter values have been changed so that the trapping interval of the middle-income trap “collapses.” In other words, a typical trajectory starting from the middle-income range can escape from that range and may finally get caught in the poverty trap or take off to the high-income state, depending on the initial values. Note that the same parameter values are used for Figures 10 and 11.

[Insert Figures 10 and 11]

Two things are worth noticing. The first concerns the transient behaviors as economies move toward the final states (ss_1 or ss_4). As seen from both Figures 10 and 11, the trajectories fluctuate for a relatively long time in the middle-income range before they settle down to final states. This kind of phenomenon is often referred to as *transient chaos* (see Lai and Tél (2011)). The mechanism for transient chaos could be explained as follows: there is a chaotic invariant Cantor set $\Lambda \subset (ss_2, ss_3)$ such that for each $k_0 \in \Lambda$, the trajectory $\{k_0, k_1, k_2, \dots\}$ stays in Λ .¹⁴ Thus, such an economy would be caught in the middle-income trap. However, Λ is a very “thin,” typically measure-zero set. As a result, almost every trajectory ultimately leaves the middle-income range. However, the closer the initial point k_0

¹⁴See Guckenheimer and Holmes (1983, Section 5) for invariant Cantor sets.

to the chaotic set Λ ($k_0 \notin \Lambda$), the longer the trajectory behaves like a trajectory on Λ before it is finally attracted to a steady state (ss_1 or ss_4), which causes transiently chaotic behaviors.

The second thing is related to the basin of attraction. There are two destinations in our example in Figures 10 and 11: one is good (ss_4) and the other is bad (ss_1). Thus, one would wish to locate the set of initial values for each destination, i.e., the basin of attraction, or to identify the boundaries of these sets. If we consider an economy having only one CES technology with $\rho > 0$ in which three steady states exist, then the middle steady state must be unstable and the other two, left and right, are stable. In such a case, the single point of the middle steady state constitutes the boundary of the basins of attraction for the two stable steady states. In the example we are dealing with, the situation is not as simple. The boundaries of the basins of attraction for ss_1 and ss_4 can be complicated¹⁵ because of the existence of the chaotic invariant set Λ . The invariant set Λ operates like a pinball machine, scrambling the economic trajectories to seemingly random destinations. Indeed, the initial value of the trajectory (reverting to poverty) in Figure 10 is $k_0 = 1.1$, which is larger than the initial value of the trajectory (taking off to the high-income state) in Figure 11, which is $k_0 = 0.95$. Although the system itself is deterministic, there is some sort of randomness in the final states. In such situations, we might say that an economy starting from the middle-income level needs some “luck” to take off to the high-income state.

4 Concluding Remarks

Constructing a simple and standard model, we numerically showed that an economy may revert to a poverty trap, be caught in a middle-income trap, or attain a high-growth path, depending on the initial condition of the economy. These results indicate that our model can explain our empirical observations for the period 2002–

¹⁵In the chaos theory literature, the boundaries we discuss are referred to as *fractal basin boundaries*, although the technical details are beyond the scope of this paper. See Yokoo (2000) for a discussion of fractal basin boundaries in an overlapping generations model.

2017 that some middle-income countries in 2002 remained in the middle-income class in 2017, whereas others reverted to poverty or moved up to the high-income class in 2017. Moreover, we showed that for another set of parameter values, due to the pinball effect, an economy starting at the middle-income level can take off to the high-income state or get caught in the poverty trap in a seemingly random way after experiencing transiently chaotic motions for arbitrarily large periods. The former takeoff case corresponds to the experience of Japan and other countries that escaped from the middle-income trap and became high-growth countries. Thus, our results can explain the observed complicated patterns of economic development, including the takeoff case, in a unified manner. Moreover, the breakdown case, involving reversion to poverty, suggests the possibility that even though a country has remained at the middle-income level for a long period, it could suddenly fail, resulting in the economy shrinking and reverting to a low-income state.

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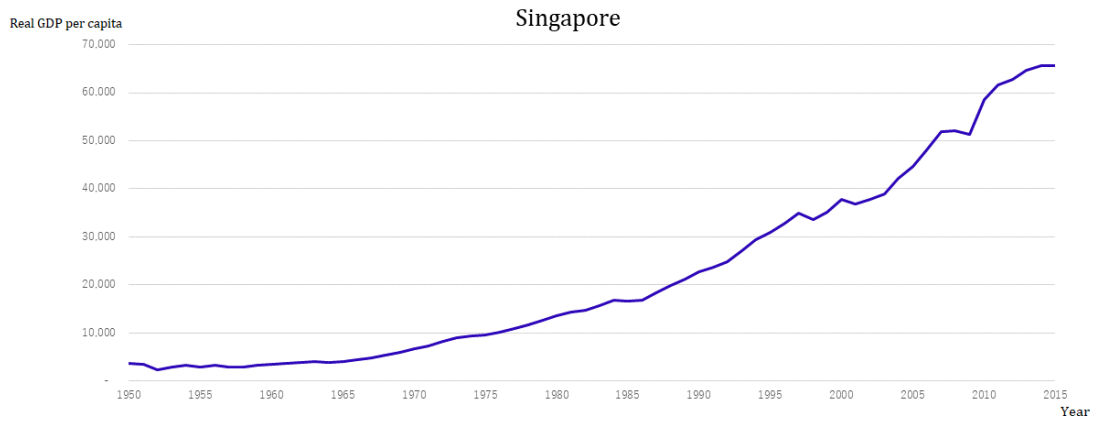


Figure 1: Real GDP per capita in 2011\$ of Singapore

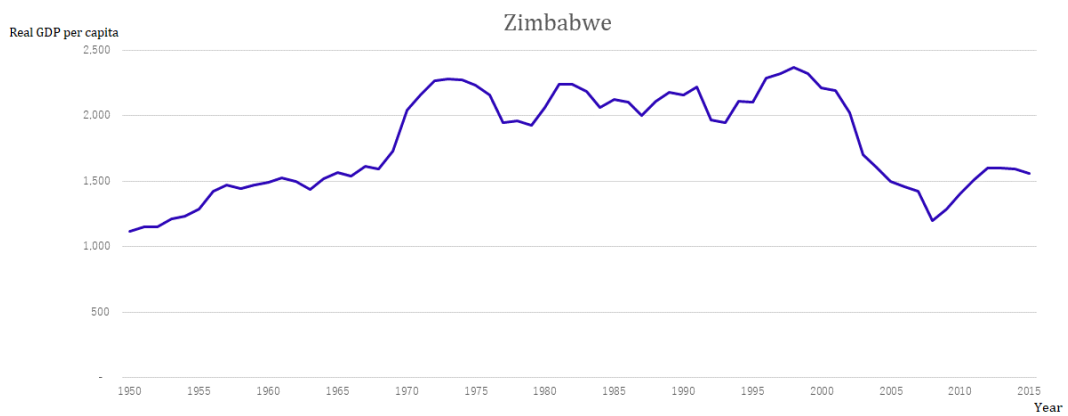


Figure 2: Real GDP per capita in 2011\$ of Zimbabwe

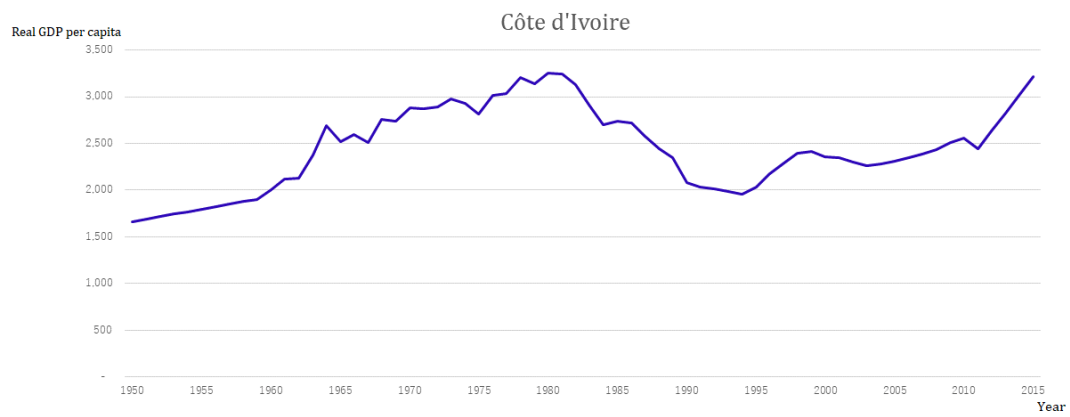


Figure 3: Real GDP per capita in 2011\$ of Côte d'Ivoire CIV

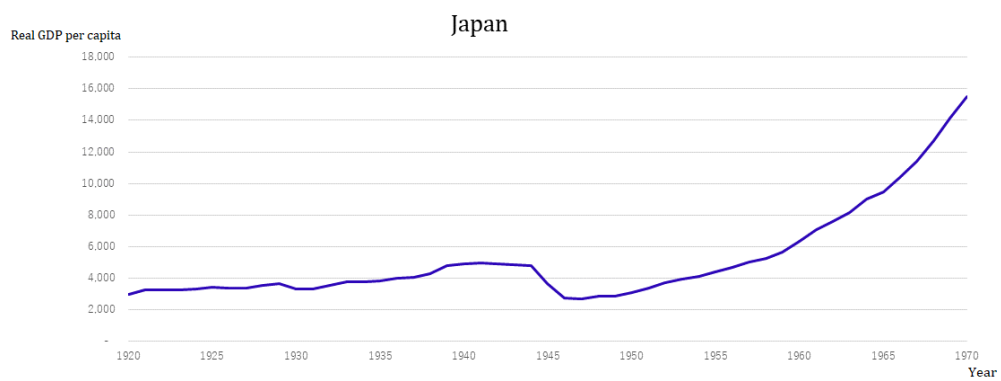


Figure 4: Real GDP per capita in 2011\$ of Japan

| | | 2002 | 2017 |
|---------|--------------------------|------|------|
| Tier 10 | $\hat{y} \leq 0.1$ | 68 | 49 |
| Tier 9 | $0.1 < \hat{y} \leq 0.2$ | 31 | 25 |
| Tier 8 | $0.2 < \hat{y} \leq 0.3$ | 18 | 25 |
| Tier 7 | $0.3 < \hat{y} \leq 0.4$ | 5 | 6 |
| Tier 6 | $0.4 < \hat{y} \leq 0.5$ | 4 | 13 |
| Tier 5 | $0.5 < \hat{y} \leq 0.6$ | 6 | 6 |
| Tier 4 | $0.6 < \hat{y} \leq 0.7$ | 6 | 9 |
| Tier 3 | $0.7 < \hat{y} \leq 0.8$ | 11 | 9 |
| Tier 2 | $0.8 < \hat{y} \leq 0.9$ | 6 | 7 |
| Tier 1 | $0.9 < \hat{y}$ | 6 | 12 |

Table 1: World Income Distributions in 2002 and 2017

| | | 2002 | | | | | |
|------|--------|----------|----------|-----------|----------|----------|-----------|
| | | Tier 5 | Tier 4 | Tier 3 | Tier 2 | Tier 1 | |
| 2017 | Tier 5 | 0 | 0 | 1 | | | 6 |
| | Tier 4 | 5 | 2 | 1 | 0 | | 9 |
| | Tier 3 | 0 | 3 | 2 | 1 | 0 | 9 |
| | Tier 2 | 0 | 0 | 6 | 1 | 0 | 7 |
| | Tier 1 | | 1 | 1 | 4 | 6 | 12 |
| | | 6 | 6 | 11 | 6 | 6 | |

Table 2: Country Transitions among Relative Income Tiers, 2002–2017

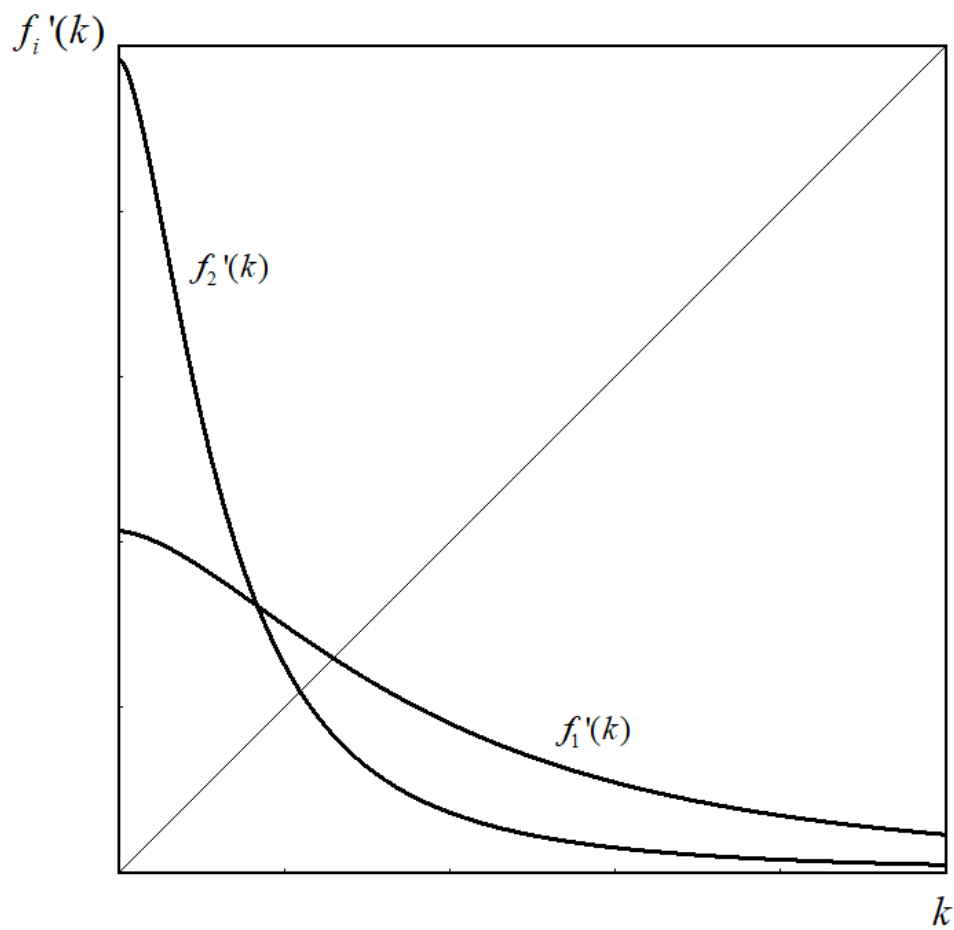


Figure 5: A single crossing of f'_i curves ($i = 1, 2$). $A_1 = 7.5$, $\alpha_1 = 0.8$, $\rho_1 = 1.6$, $A_2 = 9$, $\alpha_2 = 0.2$, $\rho_2 = 1.6$.

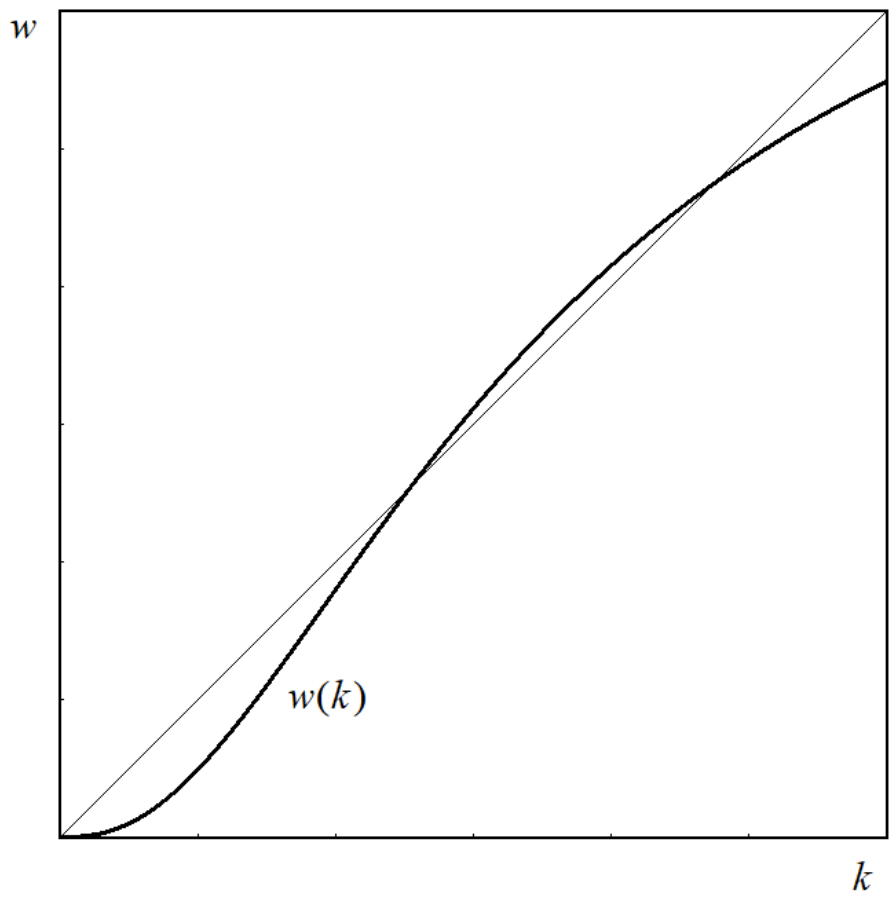


Figure 6: A typical CES wage schedule.

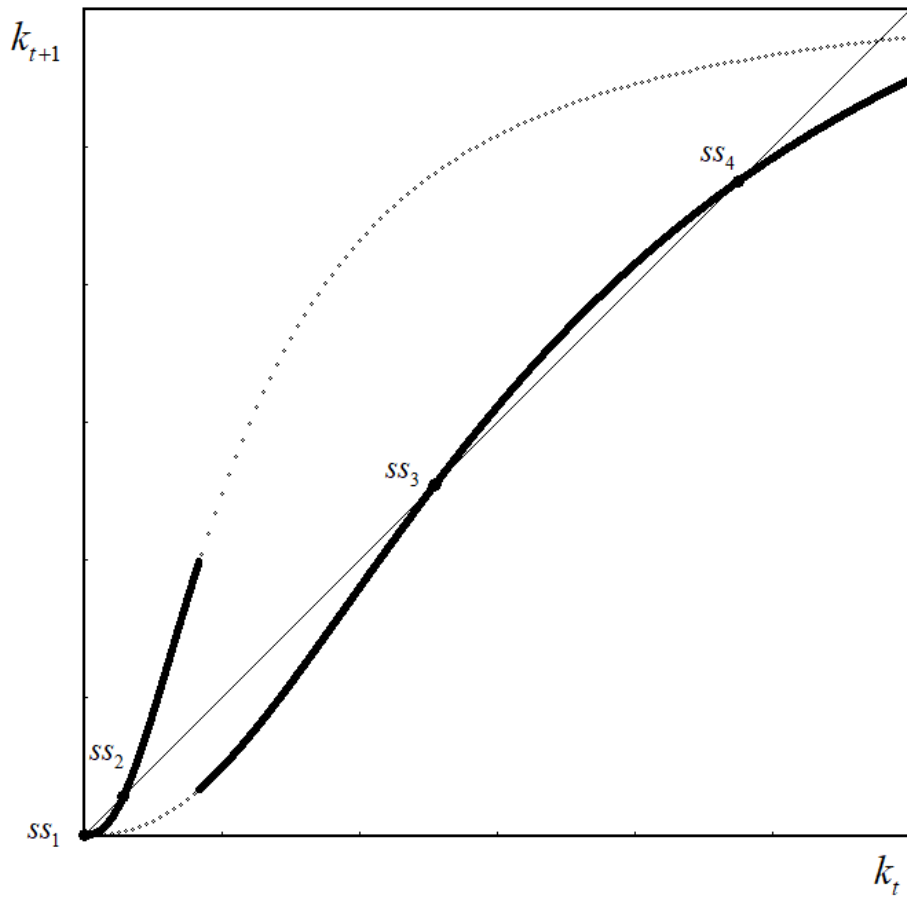


Figure 7: The return map (solid lines). The parameters are the same as in Figure 5.

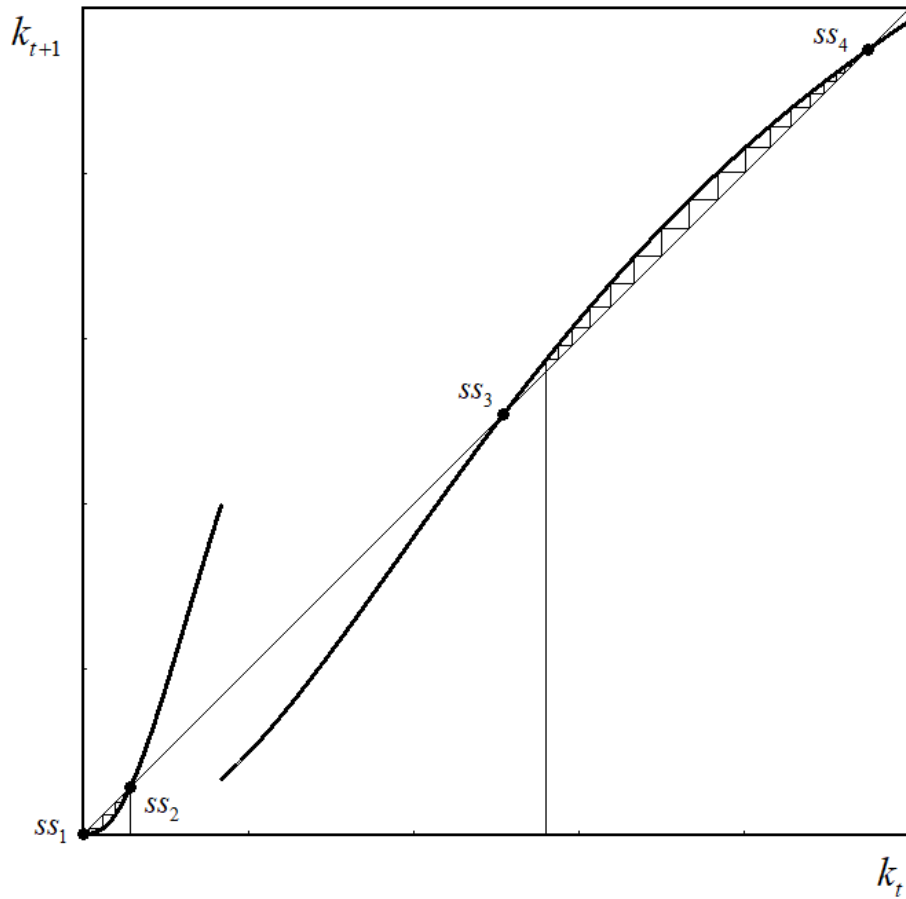


Figure 8: A trajectory getting caught in the poverty trap ($k_0 = 0.14$) and another trajectory taking off to the high-income steady state ($k_0 = 1.4$). The parameters are the same as in Figure 5.

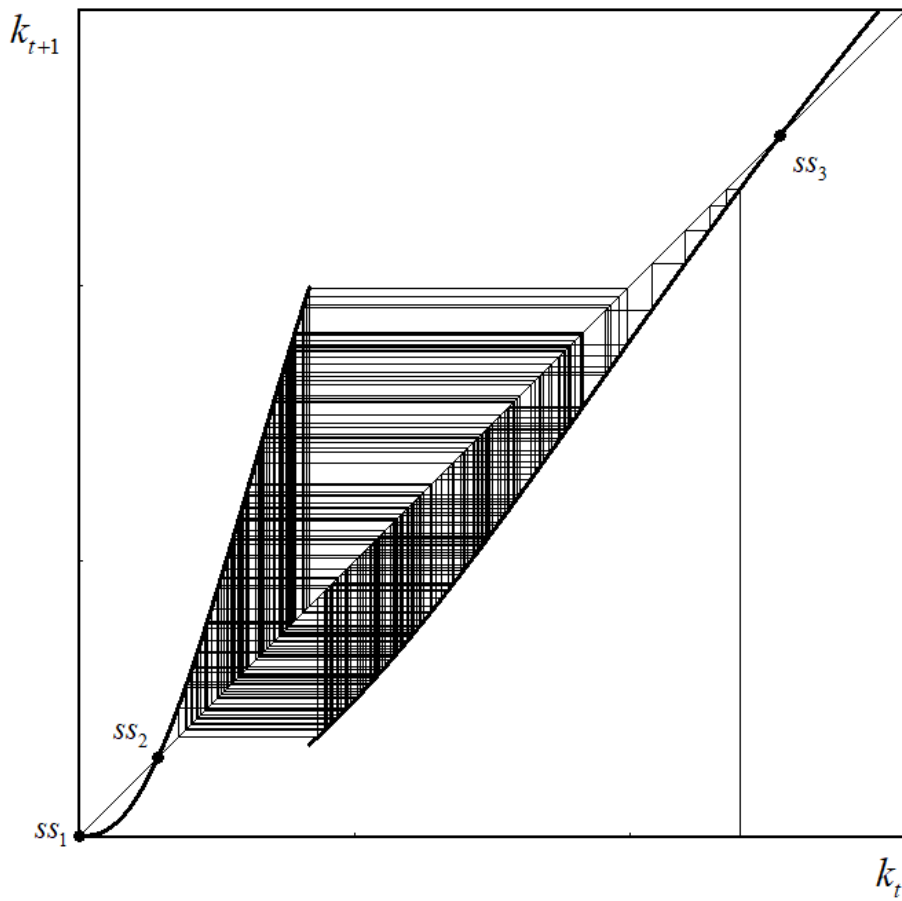


Figure 9: A chaotic trajectory caught in the middle income trap. The parameters are the same as in Figure 5. $k_0 = 1.2$.

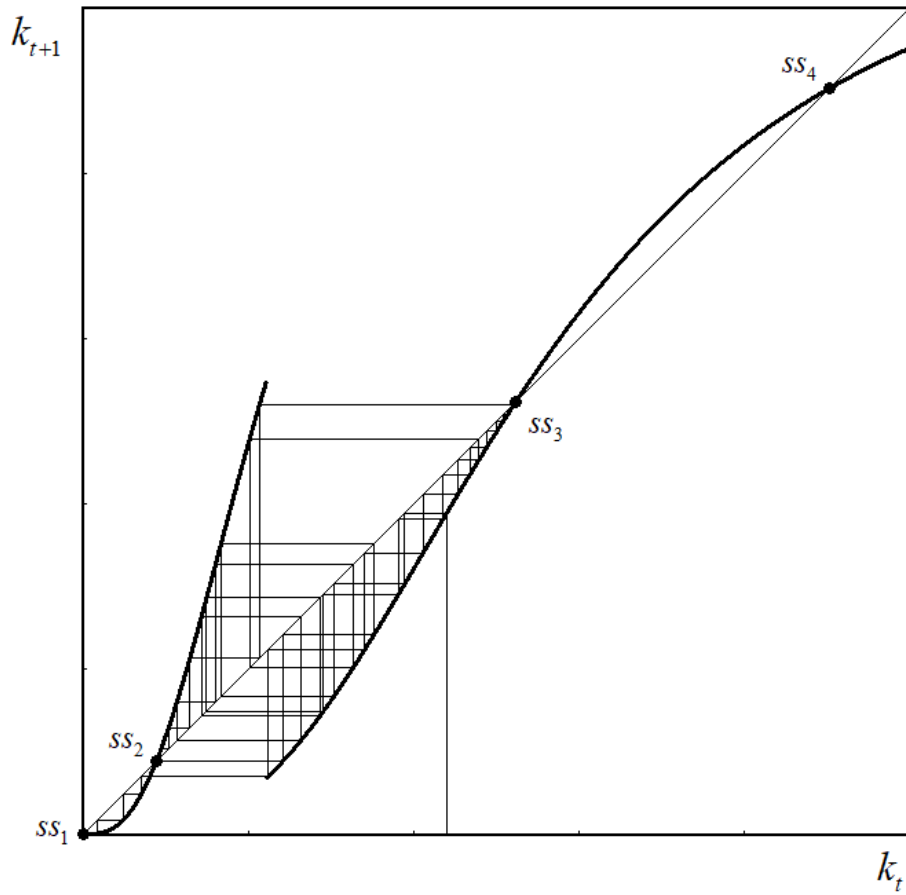


Figure 10: A trajectory getting caught in the poverty trap after exhibiting long-lasting erratic fluctuations. $A_1 = 6.7$, $\alpha_1 = 0.6$, $\rho_1 = 2.5$, $A_2 = 10$, $\alpha_2 = 0.2$, $\rho_2 = 2$, $s = 0.3$, $k_0 = 1.1$.

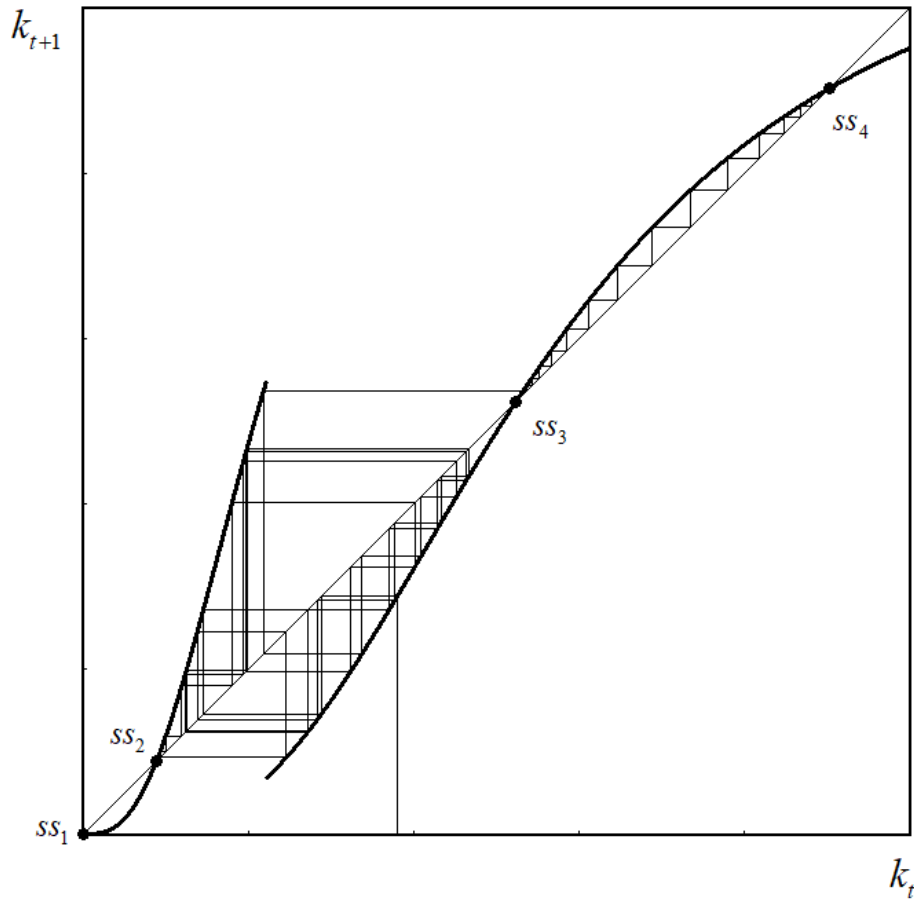


Figure 11: A high-growth path with transiently chaotic fluctuations in the middle-income range. The parameter values are the same as in Fig.10. Only the initial conditions are different between Fig.10 and this figure. For this figure, $k_0 = 0.95$.