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“Allocating Investments in Conglomerate Mergers:  
A Game Theoretic Approach”

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# Allocating Investments in Conglomerate Mergers: A Game Theoretic Approach\*

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## Abstract

We develop a model of conglomerate mergers. There are two markets that are not related horizontally or vertically. Each market has an oligopoly structure where the firms compete in a Cournot fashion. The firms cannot merge with a firm in the same market, but they are able to with a firm in a different market. Without a merger, we assume that only the firms in one of the markets can invest in technology to reduce the cost of production. After the merger, the new formed conglomerate is able to use the technology in both markets. Using the technology has a cost of opportunity in the merger scenario, hence the conglomerate has to decide how to allocate the technology across both markets. The model predicts that in a monopoly benchmark, the incentives to allocate the technology are to reduce the costs in the markets with better prospects of profits. In an oligopoly structure, the firms merge if they have incentives to transfer the technology from the original market either to invest in the better markets or to avoid technological competition. We fully characterize how the markets' size and the technological compatibility determine the equilibrium market outcomes and the underlying merger decisions. We derive welfare implications of the equilibria.

**Keywords:** Conglomerate Mergers; Corporate Diversification; Game Theory; Resources; Multimarket Competition

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# 1 Introduction

Conglomerate mergers are defined as mergers that are not horizontal nor vertical, i.e., the merging firms' products do not compete in the same market or do not have an input-output relationship (Narver, 1969, p. 2-3). In contrast to the well-known direct effects that the horizontal and vertical mergers have in the market (like increased market power or raising rival costs), the effects of conglomerate mergers are less straightforward. One such effect is to use a strong brand belonging to one of the merging parties to strengthen the market position of another brand or to make contingent sales like bundling or tying (Neven, 2005). Included among these effects is what Narver (1969) denominates "conglomerate market power", defined as the conglomerate's ability to shift resources between its markets. Through the conglomerate market power, the market structure could be affected by the resources that a conglomerate has at their disposal but standalone firms do not, such as research and development (R&D), computer facilities, legal services, and access to capital markets (Goldberg, 1973). Using this pool of resources to, for example, improve the quality of the products, a conglomerate firm might have an advantageous position in the market in comparison to standalone firms.

The concerns over the conglomerate market power are illustrated by the Clorox Case. In 1957, Procter & Gamble (P&G), a producer of a wide variety of personal care and hygiene goods, purchased The Clorox Company (Clorox), which specialized in producing liquid bleach. At the time of the merger, the Federal Trade Commission (FTC) claimed that the products of P&G and Clorox were not substitutes, and therefore this acquisition could be considered as a conglomerate merger. Almost immediately after the acquisition, the FTC challenged the merger arguing that it could lessen the competition in the liquid bleach market, and in the end, the merger was declared unlawful. The main argument for this decision was the cost advantages in advertising that Clorox could obtain from P&G (Peterman, 1968).

The objective of this paper is to analyze theoretically how the ability to transfer technology between industries affects the structure of the market and the firms' decision to form a conglomerate. In our model, we assume two markets that are not related horizontally or vertically. Each market has an oligopoly structure where the firms compete in a Cournot fashion. The firms cannot merge with a firm in the same market, but they are able to with a firm in a different market. In a non-merger situation, only the firms in one

of the markets can invest in a technology that we refer in this paper as the R&D effort of the firm. We assume that investing in R&D reduces the cost of production, though R&D investments are themselves costly. Only through a merger, the conglomerate is able to invest in R&D in both markets. We assume that the technology is not fully compatible in the market without the initial investment opportunity. As the R&D effort has an associated cost, when a firm operates in two markets, there is an opportunity cost to reallocate the R&D investment across markets. Thus, the firm must choose strategically how much to invest in each market in anticipation of its rival's own R&D effort.

We assume that the cost of R&D is quadratic. With this simple form, the model predicts that the firms invest in R&D in only one market. We first develop a monopoly benchmark. In this structure, the firms (weakly) prefer to merge. Given this, we concentrate only on the merger scenario. We demonstrate that the conglomerate in a monopoly structure invests in R&D only in the most profitable market, which directly depends on the sizes of the two markets.

In the duopoly case, we find two patterns of R&D investment behavior. First, if the disparity in sizes of the two markets is very large, similar to the monopoly case, the firms invest only in the greatest market. Second, if the sizes of the two markets are not too dissimilar, the result is an asymmetric outcome where one firm invests only in one of the markets in a way that each market is receiving investments from only one firm. Thus, in contrast to the monopoly benchmark, in duopoly the firms do not necessarily invest in R&D in the greatest market. They can invest in weaker markets in order to avoid technological competition.

Regarding the merger behavior of the firms, we show that in equilibrium the outcome where all firms invest only in the market without the initial opportunity for R&D efforts is consistent only with the scenario with two conglomerates. In regards of the other outcomes in equilibrium, they are consistent with various merger scenarios. For example, the outcome where the firms invest only in the market with the initial opportunity for R&D efforts is consistent with the scenarios without conglomerates and with two conglomerates. However, it does not seem reasonable to form conglomerates if the technology will not be transferred. To refine this result, we assume an infinitesimal cost of merger. This excludes inessential mergers in any equilibrium. With this additional assumption, the outcome where the firms invest only in the market with the initial opportunity for

R&D is now only consistent with the scenarios without conglomerates. Moreover, the asymmetric outcome is only consistent with the scenario with one conglomerate.

Furthermore, we derive the welfare implications of the model. Specifically, we find which equilibrium of the model is the best from the perspective of the producer surplus and the social welfare. We conclude that the asymmetric outcome is the one that maximizes the total producer surplus. This means that if all firms were to maximize their total payoffs, a solution would be to form one conglomerate and then to invest in accordance with the asymmetric outcome.

We find parallels between our total producer surplus' results and some well-known theoretical concepts. First, from the non-merger state, when a pair of firms decide to merge to reach the asymmetric outcome, the total producer surplus increases. However, if the market with the initial opportunity for R&D efforts is large enough in comparison to the other market, the joint-profit of the conglomerate ends being less than the sum of the profit of the non-merged parties. This situation resembles the Merger Paradox. Second, when the firms form two conglomerates to reach the outcome where all firms invest in the market without the initial opportunity for R&D efforts, the total producer surplus results being lower in comparison to the asymmetric outcome. This situation is similar the Tragedy of the Commons, since an excess of mergers is hurtful for the total producer surplus. Third and finally, even though the asymmetric outcome maximizes the total producer surplus, in any equilibrium with that outcome the total payoffs are not allocated fairly between the symmetric firms. This is reminiscent of the battle of sexes coordination game.

With regard to the social welfare, we find that if the market without the initial investment opportunity is too large and the technology compatibility in that market is low enough, then all firms investing in that market is the outcome that maximizes the total social welfare. Under those conditions, that outcome is also the unique equilibrium. If those conditions do not hold, the asymmetric outcome is the one that maximizes the total social welfare. However, the asymmetric outcome is not an equilibrium if the market sizes are too uneven. Since the asymmetric outcome is associated with the scenario where only one pair of firms merge, the policy implication here is that the policy authority should force one merger if none of the firms want to merge and should forbid one merger if all the firms want to merge.

This paper is organized as follows: Section 2 reviews the related literature. Sections 3-6 specify the model structure. Section 3 introduces the three-stage game. Section 4 presents the monopoly benchmark in the third and second stage of the game. Section 5 solves by backward induction the third and second stage of the game in the oligopoly case. Section 6 solves the first stage in the oligopoly case. Section 7 discusses welfare implications. Section 8 concludes.

## 2 Literature review

This paper is related with several strands of literature. First of all, it relates with the merger literature. One of the most famous papers on merger literature is Salant et al. (1983). Their model predicts that horizontal mergers are not usually profitable for the merging parties but can be beneficial for firms excluded from the merger. This theory is known as the Merger Paradox. We find that the asymmetric outcome resembles the Merger Paradox if the market with the initial opportunity for R&D efforts is large enough in comparison to the other market. Specifically, we show that in the asymmetric outcome the merged firms are worse off in comparison to the standalone firms. Thus, the conglomerate merger is more profitable for the non-merging parties than for the conglomerate. Furthermore, we find that the asymmetric outcome is the one that maximizes the total producer surplus. Thus, the conglomerate merger has a positive effect in the total producer surplus even though the merged parties receive the smallest share of the producer surplus.

The majority of theoretical papers about mergers focus on horizontal or vertical mergers, but not on conglomerate mergers. One exception is Granier and Podesta (2010). They propose a theoretical model where an electrical and gas supplier merge endogenously. The merger allows the conglomerate to engage in price discrimination by selling both products in a bundle. Thus, the ability to bundle goods is the means by which a conglomerate merger affects the market structure in Granier and Podesta (2010). In our paper, the market structure is affected through the ability to allocate resources across markets.

Our paper is also related to the literature concerned with the assets in a multimarket firm. Among this kind of research is the capital allocation efficiency literature. These

papers investigate whether the allocation of financial resources across divisions in a multimarket firm matches with the divisions' performance, i.e., whether a high-prospects division receives more than a low-prospects division (Busenbark et al., 2017).

The general idea of the papers supporting the efficiency allocation is that the firm prioritizes the most profitable endeavors over the less profitable ones.<sup>1</sup> This idea of efficiency is consistent with our results in the monopoly structure: the firm invests only in the market with the highest potential profit, i.e., the firm favors the best market and neglects the worst market. Furthermore, it is also consistent with the result in the duopoly structure where the firms allocate their resources only in the best market, which occurs when the sizes of the markets are very dissimilar. However, when the sizes of the markets are rather similar in the duopoly case, while there is a firm that invests in the strongest market, there is another that invests in the weakest one. That result is closer to the papers asserting that the allocation of capital is inefficient. In general, those papers state that an inefficient allocation of capital is caused by agency problems, i.e., managers conflicting with the interest of the overall firm.<sup>2</sup> In our duopoly model, the inefficiency is due to the profitability of the markets for the firms being contingent on the strategy of their rival. Given the strategy of the rival firm, it might be profitable to invest in the worst market because the competition in that market might be weaker in comparison to the best market.

Thus, our model reconciles these opposing views regarding the efficiency of the capital allocation. Depending on the parameters of the model, the firms invest only in the strongest market, or there exists a firm that invests in the weakest market in order to

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<sup>1</sup>The theoretical model of Stein (1997) predicts that the headquarters have incentives to reallocate resources from weaker projects to stronger projects because the headquarters gain a portion of the profits. In Maksimovic and Phillips (2002), the theoretical results show that a firm will specialize in the industry in which it is much more productive; if the firm's productivity in each market is similar, the firm diversifies. They also provide empirical evidence to support their theoretical claims. In the theoretical paper of Brusco and Panunzi (2005), the firm reallocates resources efficiently. However, this reduces the incentives of the least profitable-division manager to exert effort, hurting the overall profit of the firm.

<sup>2</sup>In theoretical research, Rajan et al. (2000) predict that as the diversity increases, the transfers from better-opportunities divisions to worse-opportunities divisions increase. The reason is that allocating resources to the weak division improves the contribution of this division to the joint profit, increasing the strong division's incentives to invest efficiently. In Stein and Scharfstein (2000), the division managers of weak divisions engage in rent-seeking behavior, which is costly for the firm. To mitigate this behavior, the CEO can allocate capital inefficiently to the weak divisions. In Wulf (2009), the core division manager sends distorted information to the headquarters to influence the division of capital in favor of the core division but against the small division. In empirical contributions, Rajan et al. (2000) provide evidence supporting their theoretical hypothesis. In Arrfelt et al. (2013), a backward-looking logic leads to over-investment (under-investment) in low (high) expectations divisions.

avoid competition.

The capital allocation has also been associated with entry deterrence. In the theoretical research of Matsusaka and Nanda (2002), divesting can be an optimal strategy so that the conglomerate commits to a higher level of investment, deterring entry for potential new competitors. In Cestone and Fumagalli (2005), the theoretical model predicts that a business group facing a threat of new entrants reallocates resources to the threatened market or exits that market, depending on the level of internal resources.<sup>3</sup> There exist equilibria in our duopoly model relevant to this idea. One of the firms could invest so heavily in R&D in one market to the point where it is not profitable for the rival to produce a positive output in that market.

Another kind of literature about the assets in a multimarket firm is the resource-based view of corporate diversification literature. Both the resource-based view and capital allocation literature are mainly interested in the firm's assets. However, the focus of the capital allocation research is financial resources, while the resource-based view has a broader concept of resource (Busenbark et al., 2017). In this sense, because our model assumes that the firms transfer the ability to invest in R&D, our paper is closer to the idea of asset in the resource-based view literature.

The resource-based view research proposes that the firms' level of diversification and performance depends to a significant degree on the resources and capabilities that the firm possesses (Wan et al., 2011). One of the main hypothesis of the resource-based view is that the diversification of the firm can be explained by its assets, as Wernerfelt (1984, p.175) stated: "mergers and acquisitions provide an opportunity to trade otherwise non-marketable resources and to buy or sell resources in bundle".

In our model it is assumed that the merging firms share some similarities in their production process. There are various papers in the resource-based view that focus on mergers of firms from related industries. That relatedness can take the form of technological capabilities. Jovanovic and Gilbert (1993) theoretically predict that the firms diversify in related-technology industries searching for profits from cross-products spillovers, and also provide empirical evidence to back their hypothesis. In other empirical contributions, Chatterjee and Wernerfelt (1991) suggest that firms which focus on research or advertising are prone to diversify in related industries, while firms with financial resources

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<sup>3</sup>This result is backed empirically by Boutin et al. (2013).



diversify in unrelated industries. Silverman (1999) shows that firms diversify in markets where their current technological resources can be exploited.

Another focus in this literature is the relationship between diversification and firm performance (Wan et al., 2011). Among the empirical literature, Harrison et al. (1993) conclude that diversifying in industries consistent with the firm R&D emphasis leads to a better corporate financial performance. Similarly, Miller (2006) suggests that technologically diversified firms outperform standalone firms.<sup>4</sup> In empirical literature related with technological outcome, Miller et al. (2007) show that sharing knowledge across divisions in a conglomerate has a positive impact in the technological development of the firm.<sup>5</sup>

In comparison to the capital allocation literature, the research in the resource-based view about how the resources are assigned across industries is scarce. Among them, Matsusaka (2001) proposes a theoretical model where firms try different industries searching for a good match for their capabilities. The model predicts that a firm with a very bad match will exit the original industry and will find a new activity, while with a very good match the firm will specialize. In intermediate cases the firm will diversify, entering new markets without leaving the old ones. In the theoretical paper of Levinthal and Wu (2010), profit-maximizing firms take diversification decisions based on the opportunity cost of sharing a finite resource across industries.

One section of our model is similar to the work of Levinthal and Wu (2010). In their model, the authors assume two multimarket firms competing in a Cournot fashion in two markets. These firms have the ability to relocate a fixed amount of resource across markets. The sequence of events is as follows: first, the firms decide how to allocate the resource, and second, they produce the output. Allocating the resource to one market reduces the marginal cost of that market, and because the resource is finite, there is an opportunity cost in transferring the resource from one market to the other one.

In our model, we assume a different way to model the transfer of resources across markets. Based on Zhao (2015), we assume that the firms can reduce their cost of production by investing in R&D. There is an additional cost to invest in R&D effort, this cost is assumed to be quadratic, reflecting the decreasing returns in R&D investments and capturing Levinthal and Wu's idea of cost of opportunity. With this change, unlike

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<sup>4</sup>Conversely, St. John and Harrison (1999) found empirical evidence that related-diversified firms do not outperform standalone firms or unrelated-diversified firms.

<sup>5</sup>Oppositely to this, Seru (2014) presents empirical evidence that conglomerate firms innovate less in comparison to similar standalone firms.

Levinthal and Wu, our model leads to an analytical solution. This allows us to further analyze the outcomes of the market (quantities, profits, R&D effort levels); moreover, now it is possible to extend the model by including merging as an endogenous choice.

In Levinthal and Wu, the general outline of the equilibria found in the Cournot model is as follows: in the resource allocation stage, one of the firms always uses its resources in only one market (a corner solution), while the other firm uses its resource in one market or in both. In our Cournot model, in the R&D allocation stage, the firms always invest in only one market (a corner solution). In both Levinthal and Wu and this paper, equilibria with asymmetric strategies are interpreted as the firms maximizing their profits through the reduction of the level of competition on the markets.

Levinthal and Wu do not focus on the output stage, so they do not explicitly state results on that matter. However, it is implied that a firm cannot produce in a market without allocating a positive amount of resources to that market. Therefore, in equilibrium, the firm that uses its resources in only one market is also only producing in that market. Moreover, in equilibrium, only one market is in Cournot competition. In contrast, in our model it is possible to produce in one market even if the firm does not allocate R&D in that market, hence there are equilibria where the two markets are in Cournot competition. Therefore, while in Levinthal and Wu competition can be understood simply as two firms facing each other in one market, in our model there are two types of competition: in R&D and in output.

If both pairs of firms merge in our model, the two conglomerates will have multimarket contact. This refers to the situation when rival firms meet each other in several markets (Yu and Cannella, 2013). The multimarket contact and the hypothesis of the markets sharing a similar production process in our model is relevant to the work of Gimeno and Woo (1999). Their empirical results show that firms operating in markets with resource-sharing opportunities are likely to find the same competitors in various markets. There is an hypothesis that multimarket contact weakens rivalry because the firms fear that the rivals might retaliate in other markets (Edwards, 1955; Jayachandran et al., 1999). This lessen rivalry entails an increment in the total profits of the participant firms. However, in our model, when both pairs of firms merge, and thus the multimarket contact occurs, the total profits are not maximized. Rather, the maximization of the total profits is associated with the case where only one pair of firms merge. Thus, our results differ from

this hypothesis from the multimarket contact literature.

### 3 The model

There are two markets that are not related horizontally or vertically, denoted by  $k \in \{A, B\}$ . In each market there are two firms selling a homogeneous good, denoted by  $i \in \{1, 2\}$ . In market  $k$ , firm  $i$  faces the following inverse demand function:

$$p_{ki}(q_{ki}, q_{kj}) = D_k - q_{ki} - q_{kj}$$

where  $D_k$  is a positive constant,  $q_{ki}$  is the output of firm  $i$ , and  $j \in \{1, 2\}$  for  $j \neq i$ .

We assume that mergers between firms in the same market are forbidden by law. However, it is possible for a firm in  $A$  to merge with a firm in  $B$ . Namely, conglomerate mergers are allowed. To avoid coordination problems, we assume that firm  $A1$  can potentially merge only with firm  $B1$ , and the same for firms  $A2$  and  $B2$ . We construct the model so that the firms in the same market are symmetric, so another combination of firms would not change the results.

Initially, all the firms face the same constant marginal cost  $c$ , which is normalized to zero for simplicity. Without a conglomerate merger, only firms in market  $B$  can invest in R&D to reduce their marginal cost. We assume that firm  $Bi$ 's effective marginal cost is  $C_{Bi}(x_{Bi}) = -x_{Bi}$ , where  $x_{Bi}$  is the R&D effort. It is assumed that the cost of investment in R&D is quadratic and is given by  $(\frac{1}{2}x_{Bi}^2)$ , thus reflecting the decreasing returns in R&D investments.

Although the markets are not related horizontally or vertically, we assume that the markets  $A$  and  $B$  share some similarities in their production processes. This is the case, for example, in consumer electronics markets or pharmaceutical markets. This assumption can also be interpreted as two markets producing a similar good (so the production process is similar) but operating in different geographic markets. Because of this, it is possible for the firms in  $A$  to use to some extent the R&D initially available only in market  $B$ .

We assume that the R&D effort can only be shared through a merger between a firm in  $A$  and a firm in  $B$ . Here, the divisions in both markets  $A$  and  $B$  can use the R&D effort to reduce their marginal cost. However, since the technology is not perfectly compatible, the

cost reduction in market  $A$  is  $(\beta x_{Ai})$  with  $\beta \in (0, 1)$ . We assume that in the conglomerate case, when two firms can use the R&D effort, the whole cost of investment in R&D is  $(\frac{1}{2}(x_{Ai} + x_{Bi})^2)$ . Thus, in the conglomerate case the quadratic cost function<sup>6</sup> reflects the decreasing returns in R&D investments and embodies the cost of opportunity of investing in one market or the other.

A three-stage game is considered. In the first stage, the two pairs of firms simultaneously and independently decide whether to merge. In the second stage, the firms that are capable of investing in R&D set simultaneously and independently their R&D effort. Here we assume that the R&D effort is perfectly observable. In the third stage, in each market the firms engage in Cournot competition by simultaneously and independently setting their output.

## 4 Monopoly benchmark

In our model, the effect of a merger in the monopoly structure is just to expand the set of strategies of the firms. Therefore, creating a conglomerate is (weakly) preferred to not merging. Because of that, we only concentrate on the conglomerate case. Moreover, since the first stage of the game is irrelevant in this case, we only analyze the third and second stages.

The joint-profit maximization problem in the third stage is:

$$\max_{q_A, q_B \geq 0} (D_A - q_A + \beta x_A) q_A + (D_B - q_B + x_B) q_B - \frac{1}{2}(x_A + x_B)^2 \quad (1)$$

The optimal output as a function of the R&D effort is:

$$q_A(x_A) = \frac{D_A + \beta x_A}{2}, \quad q_B(x_B) = \frac{D_B + x_B}{2} \quad (2)$$

In the second stage, by substituting (2) into (1), the conglomerate's problem is:

$$\max_{x_A, x_B \geq 0} \left( \frac{D_A + \beta x_A}{2} \right)^2 + \left( \frac{D_B + x_B}{2} \right)^2 - \frac{1}{2}(x_A + x_B)^2 \quad (3)$$

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<sup>6</sup>With this functional form the conglomerate does not achieve a cost reduction in the R&D investments in comparison to its standalone counterpart. Rather, it seems that in the conglomerate case the R&D is more costly considering that  $(x_{Ai} + x_{Bi})^2 \geq x_{Ai}^2 + x_{Bi}^2$ . This is concordant with the concept of conglomerate discount (see for example Berger and Ofek (1995)). This term refers to the situation where the value of the conglomerate is less than the sum of the values of its individual parts.

When the value of either  $x_A$  or  $x_B$  is high enough, the objective function in (3) is decreasing in that variable for any non-negative value of the other variable. Hence, the objective function must be bounded above in the non-negative region. Given that the problem is constrained by  $x_A \geq 0$  and  $x_B \geq 0$ , a solution is guaranteed to exist. Now, the associated Hessian matrix is given by:

$$\begin{bmatrix} \frac{1}{2}\beta^2 - 1 & -1 \\ -1 & -\frac{1}{2} \end{bmatrix}$$

The determinant is  $D = \frac{1}{4}(-\beta^2 - 2) < 0$ , so the objective function is not concave. Then, this problem does not yield an interior solution. Nevertheless, because a solution exists, the optimal R&D effort must be a corner solution. There are two candidates for the optimal solution: the firm invests only in market  $B$  ( $x_A = 0$ ), or the firm invests only in market  $A$  ( $x_B = 0$ ). We denote those cases with the subscripts  $MB$  and  $MA$ , respectively.

First, in the  $MB$  case, the objective function in (3) is concave with respect to  $x_B$  when  $x_A = 0$ . Thus, the First Order Condition (FOC) gives the solution by:

$$\frac{(D_B + x_B)}{2} - x_B = 0$$

Then, the R&D effort, the output, and the profit  $\pi$  are given by:

$$x_A^{MB} = 0, \quad x_B^{MB} = D_B, \quad q_A^{MB} = \frac{D_A}{2}, \quad q_B^{MB} = D_B, \quad \pi_{AB}^{MB} = \frac{D_A^2}{4} + \frac{D_B^2}{2}$$

It is straightforward to see that the R&D effort increases the output and the profit in market  $B$  in comparison to a case without R&D.

Second, in the  $MA$  case, the FOC is:

$$\frac{\beta(D_A + \beta x_A)}{2} - x_A = 0$$

Then, the R&D effort, the output, and the profit are given by:

$$x_A^{MA} = \frac{\beta D_A}{2 - \beta^2}, \quad x_B^{MA} = 0, \quad q_A^{MA} = \frac{D_A}{2 - \beta^2}, \quad q_B^{MA} = \frac{D_B}{2}, \quad \pi_{AB}^{MA} = \frac{D_A^2}{2(2 - \beta^2)} + \frac{D_B^2}{4}$$

Similar to the *MB* case, in the *MA* case the R&D effort increases the output and the profit in market *A* in comparison to a case without R&D. However, the R&D has weaker effects on market *A* than on *B* due to the differences in technology compatibility. It is easy to see that  $\frac{\partial x_A^{MA}}{\partial \beta} > 0$ ,  $\frac{\partial q_A^{MA}}{\partial \beta} > 0$  and  $\frac{\partial \pi_{AB}^{MA}}{\partial \beta} > 0$  for any  $\beta \in (0, 1)$ , so a lower compatibility reduces the R&D effort and also reduces the positive effects of the R&D in the output and profit.

The solution depends on which profit is greater. It holds that  $\pi_{AB}^{MA} \geq \pi_{AB}^{MB}$  if and only if  $\frac{D_A}{D_B} \geq \frac{\sqrt{2-\beta^2}}{\beta}$ . We refer to the quotient between the intercepts of the demand functions of the markets *A* and *B* ( $\frac{D_A}{D_B}$ ) as the *market ratio*. So, if the market ratio is high enough, the firm chooses to invest in market *A*, otherwise, it invests only in market *B*. We state this result formally in the following proposition.

**Proposition 1.** *If  $\frac{D_A}{D_B} \geq \frac{\sqrt{2-\beta^2}}{\beta}$ , MA is the solution to the monopoly benchmark. If  $\frac{D_A}{D_B} \leq \frac{\sqrt{2-\beta^2}}{\beta}$ , MB is the solution to the monopoly benchmark.*

Notice that if  $\beta = 1$ , the condition for *MA* to be the solution would simply be  $D_A \geq D_B$ . Hence, if there is full technology compatibility in both markets, the firm would decide in which market to invest simply based on its relative size. For the firm to invest in market *A* when  $\beta \in (0, 1)$ , to compensate the lack of full compatibility in market *A*, the size of market *A* has to be much bigger than market *B*.

## 5 Second and third stages: R&D effort and Cournot competition

We solve the game using backwards induction. In the first stage, the firms decide whether to merge. With two firms in each market, at most two conglomerates can be formed. Depending on that, there are three possible subgames starting from the second stage: 1) Zero-merger subgame, where none of the firms merge, and thus four standalone firms participate in the subgame. 2) Two-merger subgame, where all firms merge and two conglomerates are created. 3) One-merger subgame, where only one conglomerate is formed and two standalone firms remain. In this section we derive the equilibria in each one of the aforementioned cases for the second and third stages of the game.

## 5.1 Zero-merger subgame

For the firms in market  $A$ , the equilibrium in the third stage is the usual Cournot solution. Thus, firm  $Ai$ 's output and profit are given by:

$$q_{Ai}^{ZM} = \frac{D_A}{3}, \quad \pi_{Ai}^{ZM} = \frac{D_A^2}{9}$$

where the superscript  $ZM$  denotes the zero-merger subgame. On the other hand, for the firms in  $B$ , the equilibrium is the Cournot solution where each firm's marginal cost is  $-x_{Bi}$ . Thus, the equilibrium output for firm  $Bi$ , which is a function of the R&D effort, is:

$$q_{Bi}(x_{Bi}, x_{Bj}) = \frac{D_B + 2x_{Bi} - x_{Bj}}{3}$$

It is straightforward to see that the equilibrium output is increasing in the firm's own R&D effort but decreasing in the rival's R&D effort.

In the second stage, each firm  $Bi$ 's maximization problem given  $x_{Bj}$  is:

$$\max_{x_{Bi} \geq 0} \frac{(D_B + 2x_{Bi} - x_{Bj})^2}{9} - \frac{(x_{Bi})^2}{2}$$

From this, the best response function for the R&D effort is:

$$x_{Bi}(x_{Bj}) = 4D_B - 4x_{Bj}$$

In the zero-merger subgame, we concentrate only on symmetric strategies. In the equilibrium of the second stage, firm  $Bi$ 's R&D effort, output and profit are:

$$x_{Bi}^{ZM} = \frac{4D_B}{5}, \quad q_{Bi}^{ZM} = \frac{3D_B}{5}, \quad \pi_{Bi}^{ZM} = \frac{D_B^2}{25}$$

In comparison to the monopoly case where the firm invests only in market  $B$ , it follows that  $x_{Bi}^{ZM} < x_B^{MB}$ . This means that R&D decisions are strategic substitutes. Intuitively, the competition reduces the individual investments in R&D. However, it also follows that  $2x_{Bi}^{ZM} > x_B^{MB}$ , so the total R&D effort in the market is greater in oligopoly than in monopoly. As in the monopoly case, in oligopoly the output is greater in comparison to a case without R&D investments. However, contrary to the monopoly case, the profit

is lower with R&D than without it.<sup>7</sup> In the oligopoly case with R&D effort, the firms compete with two variables: the quantities and the R&D. The competition in R&D leads to a greater investment in the market in comparison to the monopoly case. This investment increases the total output in the market to the point where there is a loss of profit in comparison to a case without R&D, even though the marginal cost is reduced by the R&D.

## 5.2 Two-merger subgame

In this case, each one of the firms in market  $A$  merges with one of the firms in  $B$ . In the third stage, the merged firm  $ABi$  (for  $i \in \{1, 2\}$ ,  $j \in \{1, 2\}$  and  $j \neq i$ ) chooses non-negative quantities  $q_{Ai}$  and  $q_{Bi}$  given that  $x_{Ai}$  and  $x_{Bi}$  have already been selected in the second stage. The payoff function in the third stage of firm  $ABi$  is:

$$(D_A - q_{Ai} - q_{Aj} + \beta x_{Ai}) q_{Ai} + (D_B - q_{Bi} - q_{Bj} + x_{Bi}) q_{Bi} - \frac{1}{2} (x_{Ai} + x_{Bi})^2 \quad (4)$$

The equilibrium output as a function of the R&D effort is:

$$q_{Ai}(x_{Ai}, x_{Aj}) = \begin{cases} \frac{D_A + 2\beta x_{Ai} - \beta x_{Aj}}{3} & \text{if } \frac{D_A}{\beta} + 2x_{Ai} \geq x_{Aj} \\ & \text{and } \frac{D_A}{\beta} + 2x_{Aj} \geq x_{Ai} \\ 0 & \text{if } \frac{D_A}{\beta} + 2x_{Ai} < x_{Aj} \\ \frac{D_A + \beta x_{Ai}}{2} & \text{if } \frac{D_A}{\beta} + 2x_{Aj} < x_{Ai} \end{cases} \quad (5)$$

$$q_{Bi}(x_{Bi}, x_{Bj}) = \begin{cases} \frac{D_B + 2x_{Bi} - x_{Bj}}{3} & \text{if } D_B + 2x_{Bi} \geq x_{Bj} \\ & \text{and } D_B + 2x_{Bj} \geq x_{Bi} \\ 0 & \text{if } D_B + 2x_{Bi} < x_{Bj} \\ \frac{D_B + x_{Bi}}{2} & \text{if } D_B + 2x_{Bj} < x_{Bi} \end{cases} \quad (6)$$

In the second stage, firm  $ABi$  chooses non-negative  $x_{Ai}$  and  $x_{Bi}$ . By substituting (5) and (6) into (4), the payoff function in the second stage is given by:

$$\pi_{Ai}(x_{Ai}, x_{Aj}) + \pi_{Bi}(x_{Bi}, x_{Bj}) - \frac{1}{2} (x_{Ai} + x_{Bi})^2 \quad (7)$$

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<sup>7</sup>Without R&D in market  $B$ , the equilibrium would be the same as market  $A$ , so the profit in that case would be  $\frac{D_B^2}{9} > \pi_{Bi}^{ZM}$ .



where

$$\pi_{Ai}(x_{Ai}, x_{Aj}) = \begin{cases} \frac{(D_A + 2\beta x_{Ai} - \beta x_{Aj})^2}{9} & \text{if } \frac{D_A}{\beta} + 2x_{Ai} \geq x_{Aj} \\ & \text{and } \frac{D_A}{\beta} + 2x_{Aj} \geq x_{Ai} \\ 0 & \text{if } \frac{D_A}{\beta} + 2x_{Ai} < x_{Aj} \\ \frac{(D_A + \beta x_{Ai})^2}{4} & \text{if } \frac{D_A}{\beta} + 2x_{Aj} < x_{Ai} \end{cases}$$

$$\pi_{Bi}(x_{Bi}, x_{Bj}) = \begin{cases} \frac{(D_B + 2x_{Bi} - x_{Bj})^2}{9} & \text{if } D_B + 2x_{Bi} \geq x_{Bj} \\ & \text{and } D_B + 2x_{Bj} \geq x_{Bi} \\ 0 & \text{if } D_B + 2x_{Bi} < x_{Bj} \\ \frac{(D_B + x_{Bi})^2}{4} & \text{if } D_B + 2x_{Bj} < x_{Bi} \end{cases}$$

At  $x_{Ai} > \frac{D_A}{\beta} + 2x_{Aj}$  the derivative with respect to  $x_{Ai}$  of the payoff function (7) is:

$$\frac{\beta(D_A + \beta x_{Ai})}{2} - x_{Ai} - x_{Bi}$$

which is always negative. On the other hand, at  $x_{Bi} > D_B + 2x_{Bj}$  the derivative with respect to  $x_{Bi}$  of the payoff function (7) is:

$$\frac{D_B + x_{Bi}}{2} - x_{Ai} - x_{Bi}$$

which is always negative. Then, it is suboptimal for firm  $ABi$  to play any  $x_{Ai} > \frac{D_A}{\beta} + 2x_{Aj}$  or  $x_{Bi} > D_B + 2x_{Bj}$ . By symmetry, it is also suboptimal for firm  $ABj$  to play any  $x_{Aj} > \frac{D_A}{\beta} + 2x_{Ai}$  or  $x_{Bj} > D_B + 2x_{Bi}$ . Thus, according to (7), all the equilibria of the two-merger subgame can be found by considering the following payoff function for firm  $ABi$ :

$$\frac{(D_A + 2\beta x_{Ai} - \beta x_{Aj})^2}{9} + \frac{(D_B + 2x_{Bi} - x_{Bj})^2}{9} - \frac{1}{2}(x_{Ai} + x_{Bi})^2 \quad (8)$$

on the region  $\left[0, \frac{D_A}{\beta} + 2x_{Aj}\right] \times [0, D_B + 2x_{Bj}]$ . Now, the associated Hessian matrix of (8) is:

$$\begin{bmatrix} \frac{8}{9}\beta^2 - 1 & -1 \\ -1 & -\frac{1}{9} \end{bmatrix}$$

The determinant of the matrix is  $D = \frac{8}{9}(-\frac{1}{9}\beta^2 - 1) < 0$ , which implies that the

function is not concave on  $\left[0, \frac{D_A}{\beta} + 2x_{Aj}\right] \times [0, D_B + 2x_{Bj}]$ . So, no interior solution exists. However, since (8) is a continuous function on a compact rectangle, a maximum is guaranteed to exist. Therefore, the equilibrium must be a corner solution.

We can easily discard some candidates for corner solutions as equilibria. First consider the case where firm  $ABi$  invests nothing in both markets ( $x_{Ai} = x_{Bi} = 0$ ) and firm  $ABj$  invests nothing in at least one of the markets ( $x_{kj} = 0$ ). Since the derivative of (8) with respect to  $x_{ki}$  is positive at  $x_{Ai} = x_{Bi} = x_{kj} = 0$ , we can discard this case as an equilibrium.

Another possible candidate is two firms setting their R&D effort in the same market equal to one of the upper bounds of the feasible region of (8). The first case would be  $x_{Ai} = x_{Aj} = \frac{D_A}{\beta} + 2x_{Ai}$ , which leads to  $x_{Ai}^* = -\frac{D_A}{\beta}$ . So that solution would be outside the feasible region. The second case would be  $x_{Bi} = x_{Bj} = D_B + 2x_{Bi}$ , which leads to  $x_{Bi}^* = -D_B$ . Again, that solution would be outside the feasible region. Therefore, we discard this case as well.

In the following analysis, we classify the equilibria into two groups: symmetric and asymmetric.

### 5.2.1 Symmetric equilibria

To find a symmetric strategy in equilibrium, let  $x_{Ai}^* = x_{Aj}^*$  and  $x_{Bi}^* = x_{Bj}^*$ . We concentrate on the following two cases:

**Case 1:**  $x_{Ai}^* = 0, \quad x_{Bi}^* > 0$

We denote this case with the subscript  $TMB$ . Here, the function in (8) when  $x_{Ai} = 0$  is concave with respect to  $x_{Bi}$ . Thus, the FOC gives the solution by:

$$\frac{4(D_B + 2x_{Bi} - x_{Bj})}{9} - x_{Bi} = 0$$

From it, the candidate for an equilibrium is:

$$x_{Bi}^* = \frac{4D_B}{5}$$

With this strategy, the R&D effort, the output and the profit are given by:

$$x_{Ai}^{TMB} = 0, \quad x_{Bi}^{TMB} = \frac{4D_B}{5}, \quad q_{Ai}^{TMB} = \frac{D_A}{3}, \quad q_{Bi}^{TMB} = \frac{3D_B}{5}, \quad \pi_{ABi}^{TMB} = \frac{D_A^2}{9} + \frac{D_B^2}{25}$$

Now we examine whether  $TMB$  is an equilibrium. Suppose that the rival plays  $x_{Aj} = 0$  and  $x_{Bj} = x_{Bi}^{TMB}$ , so the objective function of firm  $ABi$  is given by:

$$\frac{(D_A + 2\beta x_{Ai})^2}{9} + \frac{(\frac{D_B}{5} + 2x_{Bi})^2}{9} - \frac{1}{2}(x_{Ai} + x_{Bi})^2 \quad (9)$$

on the region  $\left[0, \frac{D_A}{\beta}\right] \times \left[0, \frac{13D_B}{5}\right]$ .

Suppose that firm  $ABi$  plays the upper bound of  $x_{Bi}$ , that is,  $x_{Bi} = \frac{13D_B}{5}$ . The derivative of (9) with respect to  $x_{Bi}$  evaluated at  $x_{Bi} = \frac{13D_B}{5}$  is  $-\frac{D_B}{5} - x_{Ai}$ , which is always negative. Therefore, any  $(x_{Ai}, \frac{13D_B}{5})$  is not a solution to (9).

The remainder solution candidates to (9) are  $(x_{Ai}, 0)$  and  $(\frac{D_A}{\beta}, x_{Bi})$ , where in the first candidate  $x_{Ai}$  is an interior solution given by the FOC. We start with the first candidate. The function in (9) when  $x_{Bi} = 0$  is concave with respect to  $x_{Ai}$ . Thus, the FOC gives the solution by:

$$\frac{4\beta(D_A + 2\beta x_{Ai})}{9} - x_{Ai} = 0 \quad (10)$$

From this it follows that:

$$x_{Ai} = \frac{4\beta D_A}{9 - 8\beta^2} \quad (11)$$

For (11) to be an interior solution  $\frac{4\beta D_A}{9 - 8\beta^2} < \frac{D_A}{\beta}$  must be satisfied. This expression holds if and only if  $\beta < \sqrt{3}/2$ . Therefore, a necessary condition for (11) to be a solution is  $\beta < \sqrt{3}/2$ . The profit of  $ABi$  in this case, denoted by  $\pi_{ABi}^{D1}$ , is:

$$\pi_{ABi}^{D1} = \frac{D_A^2}{9 - 8\beta^2} + \frac{D_B^2}{225}$$

When  $\frac{D_A}{D_B} > \frac{\sqrt{9 - 8\beta^2}}{5\beta}$ , it follows that  $\pi_{ABi}^{TMB} < \pi_{ABi}^{D1}$ , therefore, firm  $ABi$  deviates unilaterally in this case.

For the second candidate, there are two cases to consider. First, assume the strategy where  $x_{Ai} = \frac{D_A}{\beta}$  and  $x_{Bi}$  is an interior solution given by the FOC. The FOC from (9)

with respect to  $x_{Bi}$  at  $x_{Ai} = \frac{D_A}{\beta}$  is:

$$\frac{4\left(\frac{D_B}{5} + 2x_{Bi}\right)}{9} - x_{Bi} - \frac{D_A}{\beta} = 0$$

From this it is obtained that:

$$x_{Bi} = \frac{4D_B}{5} - \frac{9D_A}{\beta}$$

To satisfy the assumption of  $x_{Bi} > 0$ ,  $\frac{D_A}{D_B} < \frac{4\beta}{45}$  must hold. The profit in this case, denoted by  $\pi_{ABi}^{D2}$ , is:

$$\pi_{ABi}^{D2} = D_A^2 + \frac{D_B^2}{25} - \frac{4D_AD_B}{5\beta} + \frac{4D_A^2}{\beta^2}$$

Given  $\frac{D_A}{D_B} < \frac{4\beta}{45}$ ,  $\pi_{ABi}^{TMB} > \pi_{ABi}^{D2}$  always holds. Thus, firm  $ABi$  does not deviate in this case.

Second, assume that firm  $ABi$  plays  $x_{Ai} = \frac{D_A}{\beta}$  and  $x_{Bi} = 0$ . The profit in this case, denoted by  $\pi_{ABi}^{D3}$ , is:

$$\pi_{ABi}^{D3} = \frac{(2\beta^2 - 1)D_A^2}{2\beta^2} + \frac{D_B^2}{225}$$

When  $\frac{D_A}{D_B} > \frac{4\beta}{5\sqrt{16\beta^2 - 9}}$  and  $\beta > 3/4$ , it follows that  $\pi_{ABi}^{TMB} < \pi_{ABi}^{D3}$ , then firm  $ABi$  deviates unilaterally in this case.

Notice that when  $\beta \in (3/4, \sqrt{3}/2)$ , depending on the market ratio, firm  $ABi$  can deviate to  $\pi_{ABi}^{D1}$  or  $\pi_{ABi}^{D3}$ . Since  $\frac{\sqrt{9-8\beta^2}}{5\beta} < \frac{4\beta}{5\sqrt{16\beta^2-9}}$  for any  $\beta \in (3/4, \sqrt{3}/2)$ , to sustain  $TMB$  as an equilibrium in  $\beta \in (3/4, \sqrt{3}/2)$ , the market ratio must only satisfy  $\frac{D_A}{D_B} \leq \frac{\sqrt{9-8\beta^2}}{5\beta}$ .

In conclusion, when  $\beta < \sqrt{3}/2$  and  $\frac{D_A}{D_B} \leq \frac{\sqrt{9-8\beta^2}}{5\beta}$ , or  $\beta \geq \sqrt{3}/2$  and  $\frac{D_A}{D_B} \leq \frac{4\beta}{5\sqrt{16\beta^2-9}}$ ,  $TMB$  can be sustained as an equilibrium.

The outcome in this case is equivalent to the one derived in the zero-merger subgame, with the only difference being that the joint-profit in the two-merger subgame is split between the standalone firms in the zero-merger subgame. Although the outcome is almost the same, in the two-merger subgame the equilibrium exists only for a range of parameters, a condition not present in the zero-merger solution. Since the firms operate in both markets, the firms specialize in  $B$  only when the market ratio ( $\frac{D_A}{D_B}$ ) is low enough, i.e., if market  $B$  has greater potential profits relative to market  $A$ .

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<sup>8</sup>If  $\frac{D_A}{D_B} \in \left( \frac{\sqrt{9-8\beta^2}}{5\beta}, \frac{4\beta}{5\sqrt{16\beta^2-9}} \right]$ , firm  $ABi$  will deviate to  $\pi_{ABi}^{D1}$ . When  $\frac{D_A}{D_B} > \frac{4\beta}{5\sqrt{16\beta^2-9}}$ , firm  $ABi$  can deviate to  $\pi_{ABi}^{D1}$  or  $\pi_{ABi}^{D3}$ .

The conditions of existence differ, depending on whether the technological compatibility is low ( $\beta < \sqrt{3}/2$ ) or high ( $\beta \geq \sqrt{3}/2$ ). In the former case, the most profitable deviation given  $x_{Bi} = 0$  is to choose  $x_{Ai} < \frac{D_A}{\beta}$ . That is, firm  $ABi$  does not increase the investment to the extent it becomes a monopolist in market  $A$ . In the latter case, the high compatibility makes  $x_{Ai} = \frac{D_A}{\beta}$  the most profitable deviation, establishing firm  $ABi$  as a monopolist in market  $A$ .

**Case 2:**  $x_{Ai}^* > 0$ ,  $x_{Bi}^* = 0$

We denote this case with the subscript  $TMA$ . Here, the function in (8) when  $x_{Bi} = 0$  is concave with respect to  $x_{Ai}$ . Thus, the FOC gives the solution by:

$$\frac{4\beta(D_A + 2\beta x_{Ai} - \beta x_{Aj})}{9} - x_{Ai} = 0$$

From it, the candidate for an equilibrium is:

$$x_{Ai}^* = \frac{4\beta D_A}{9 - 4\beta^2}$$

With this strategy, the R&D effort, the output and the profit are given by:

$$x_{Ai}^{TMA} = \frac{4\beta D_A}{9 - 4\beta^2}, \quad x_{Bi}^{TMA} = 0, \quad q_{Ai}^{TMA} = \frac{3D_A}{9 - 4\beta^2}, \quad q_{Bi}^{TMA} = \frac{D_B}{3},$$

$$\pi_{ABi}^{TMA} = \frac{(9 - 8\beta^2)D_A^2}{(9 - 4\beta^2)^2} + \frac{D_B^2}{9}$$

Similar to the previous case, when both firms specialize in market  $A$ , the total investment in R&D in the market is greater than in the monopoly case ( $2x_{Ai}^{TMA} > x_A^{MA}$ ) and the output is greater than a scenario without R&D. Both the increased quantities and the R&D strengthen the competition in market  $A$ . Due to the increased competition, the profits from market  $A$  are lower in comparison to a case without R&D. As in the monopoly case, an increase in technological compatibility increases both the R&D effort and the output in  $A$  ( $\frac{\partial x_{Ai}^{TMA}}{\partial \beta} > 0$  and  $\frac{\partial q_{Ai}^{TMA}}{\partial \beta} > 0$  for any  $\beta \in (0, 1)$ ). Thus, an increase in the technological compatibility reduces the profits ( $\frac{\partial \pi_{ABi}^{TMA}}{\partial \beta} < 0$  for any  $\beta \in (0, 1)$ ), because a higher compatibility signifies a higher competition in market  $A$ .

Now we verify if  $TMA$  is an equilibrium. Suppose that the rival plays  $x_{Aj} = x_{Ai}^{TMA}$  and  $x_{Bj} = 0$ , so the objective function of firm  $ABi$  is given by:

$$\frac{1}{9} \left( \frac{(9-8\beta^2)D_A}{9-4\beta^2} + 2\beta x_{Ai} \right)^2 + \frac{(D_B + 2x_{Bi})^2}{9} - \frac{1}{2} (x_{Ai} + x_{Bi})^2 \quad (12)$$

on the region  $\left[0, \frac{(9+4\beta^2)D_A}{(9-4\beta^2)\beta}\right] \times [0, D_B]$ .

Suppose that firm  $ABi$  plays the upper bound of  $x_{Ai}$ , that is,  $x_{Ai} = \frac{(9+4\beta^2)D_A}{(9-4\beta^2)\beta}$ . The derivative of (12) with respect to  $x_{Ai}$  evaluated at  $x_{Ai} = \frac{(9+4\beta^2)D_A}{(9-4\beta^2)\beta}$  is  $\frac{(8\beta^2-9)D_A}{(9-4\beta^2)\beta} - x_{Bi}$ , which is always negative. Therefore, any  $\left(\frac{(9+4\beta^2)D_A}{(9-4\beta^2)\beta}, x_{Bi}\right)$  is not a solution to (12).

The remainder solution candidates to (12) are  $(0, x_{Bi})$  and  $(x_{Ai}, D_B)$ , where in the first candidate  $x_{Bi}$  is an interior solution given by the FOC. We start with the first candidate. The function in (12) when  $x_{Ai} = 0$  is concave with respect to  $x_{Bi}$ . Thus, the FOC gives the solution by:

$$\frac{4(D_B + 2x_{Bi})}{9} - x_{Bi} = 0 \quad (13)$$

From this it follows that  $x_{Bi} = 4D_B$ , which is outside the feasible region, therefore, this strategy is not a solution of (12).

For the second solution candidate, there are two cases to consider. First, assume the strategy where  $x_{Bi} = D_B$  and  $x_{Ai}$  is an interior solution given by the FOC. The FOC from (12) with respect to  $x_{Ai}$  at  $x_{Bi} = D_B$  is:

$$\frac{4\beta}{9} \left( \frac{(9-8\beta^2)D_A}{9-4\beta^2} + 2\beta x_{Ai} \right) - x_{Ai} - D_B = 0$$

From this it is obtained that:

$$x_{Ai} = \frac{4\beta D_A}{9-4\beta^2} - \frac{9D_B}{9-8\beta^2}$$

To satisfy the assumption of  $x_{Ai} > 0$ ,  $\frac{D_A}{D_B} > \frac{9(9-4\beta^2)}{4\beta(9-8\beta^2)}$  must hold. The profit in this case, denoted by  $\pi_{ABi}^{D4}$ , is:

$$\pi_{ABi}^{D4} = \frac{(9-8\beta^2)D_A^2}{(9-4\beta^2)^2} + \frac{(9-4\beta^2)D_B^2}{9-8\beta^2} - \frac{4\beta D_A D_B}{9-4\beta^2}$$

Given  $\frac{D_A}{D_B} > \frac{9(9-4\beta^2)}{4\beta(9-8\beta^2)}$ ,  $\pi_{ABi}^{TMA} > \pi_{ABi}^{D4}$  always holds. Thus, firm  $ABi$  does not deviate in this case.

Second, assume that firm  $ABi$  plays  $x_{Bi} = D_B$  and  $x_{Ai} = 0$ . The profit in this case,

denoted by  $\pi_{ABi}^{D5}$ , is:

$$\pi_{ABi}^{D5} = \frac{(9 - 8\beta^2)^2 D_A^2}{9(9 - 4\beta^2)^2} + \frac{D_B^2}{2}$$

When  $\frac{D_A}{D_B} < \frac{(9-4\beta^2)\sqrt{7}}{4\beta\sqrt{9-8\beta^2}}$ , it follows that  $\pi_{ABi}^{TMA} < \pi_{ABi}^{D5}$ , thus firm  $ABi$  deviates unilaterally in this case. Therefore, when  $\frac{D_A}{D_B} \geq \frac{(9-4\beta^2)\sqrt{7}}{4\beta\sqrt{9-8\beta^2}}$ ,  $TMA$  can be sustained as an equilibrium.

Similar to the previous equilibrium, the interpretation of this existence condition is that the firms specialize in market  $A$  only when the market ratio ( $\frac{D_A}{D_B}$ ) is high enough, i.e., if market  $A$  has greater potential profits relative to market  $B$ . Unlike the deviation cases in the  $TMB$  equilibrium, since market  $B$  has perfect compatibility with the technology, when some firm deviates to market  $B$ , that firm is always able to become a monopolist in market  $B$ .

### 5.2.2 Asymmetric equilibria

We concentrate on the strategy profile  $x_{Aj} = x_{Bi} = 0$ ,  $x_{Ai} > 0$  and  $x_{Bj} > 0$ . First, suppose the equilibrium candidate where  $x_{Bj}$  is given by the FOC. The FOC for firm  $ABj$  from (8) with respect to  $x_{Bj}$  at  $x_{Aj} = x_{Bi} = 0$  is the same as (13). Thus, it follows that  $x_{Bj} = 4D_B$ , which is outside the feasible region. Hence, this candidate is not a solution. Second, suppose that  $x_{Bj}$  is equal to the upper bound, that is,  $x_{Bj} = D_B$ . In that case, the objective function of firm  $ABi$  is given by:

$$\frac{(D_A + 2\beta x_{Ai})^2}{9} - \frac{x_{Bi}^2}{18} - \frac{x_{Ai}^2}{2} - x_{Bi}x_{Ai} \quad (14)$$

on the region  $\left[0, \frac{D_A}{\beta}\right] \times [0, 3D_B]$ .

It is easy to see that (14) is strictly decreasing in  $x_{Bi} \geq 0$ . Thus, (14) must be maximized at  $x_{Bi} = 0$ . Since (14) is concave in  $x_{Ai}$  at  $x_{Bi} = 0$ , the optimal  $x_{Ai}$  must be an interior solution if it is inside the feasible region. Here the FOC is equivalent to (10), from which it is obtained that  $x_{Ai} = \frac{4\beta D_A}{9-8\beta^2}$ . That result is inside the feasible region if and only if  $\frac{4\beta D_A}{9-8\beta^2} < \frac{D_A}{\beta}$ . This inequality holds if and only if  $\beta < \sqrt{3}/2$ . Therefore, (14) is maximized at  $x_{Ai} = \frac{4\beta D_A}{9-8\beta^2}$  when  $\beta < \sqrt{3}/2$  and at  $x_{Ai} = \frac{D_A}{\beta}$  when  $\beta \geq \sqrt{3}/2$ . Thus, two cases are analyzed. We denote these cases with the subscript  $TM\Diamond$ , where  $\Diamond = \nabla$  when  $\beta \in (0, \sqrt{3}/2)$  and  $\Diamond = \Delta$  when  $\beta \in [\sqrt{3}/2, 1)$ .

**Case 1:** Consider the equilibrium candidate where  $\beta < \sqrt{3}/2$ . The R&D effort, the output, and the profit are given by:

$$\begin{aligned} x_{Ai}^{TM\nabla} &= \frac{4\beta D_A}{9-8\beta^2}, & x_{Bi}^{TM\nabla} &= 0, & x_{Aj}^{TM\nabla} &= 0, & x_{Bj}^{TM\nabla} &= D_B, \\ q_{Ai}^{TM\nabla} &= \frac{3D_A}{9-8\beta^2}, & q_{Aj}^{TM\nabla} &= \frac{(3-4\beta^2)D_A}{9-8\beta^2}, & q_{Bi}^{TM\nabla} &= 0, & q_{Bj}^{TM\nabla} &= D_B, \\ \pi_{ABi}^{TM\nabla} &= \frac{D_A^2}{9-8\beta^2}, & \pi_{ABj}^{TM\nabla} &= \frac{(3-4\beta^2)^2 D_A^2}{(9-8\beta^2)^2} + \frac{D_B^2}{2} \end{aligned}$$

In this case, each firm specializes in a different market. Hence, unlike the symmetric solutions, there is not competition in R&D. Similar to the monopoly case, here investing in R&D in one market increases the profit in that market in comparison to the case without R&D. This occurs even in market  $A$ , where there is still competition in quantities ( $\pi_{ABi}^{TM\nabla} > \pi_{Ai}^{ZM}$  for any  $\beta \in (0,1)$ ). Furthermore, the profits are increasing in the technological compatibility ( $\frac{\partial \pi_{ABi}^{TM\nabla}}{\partial \beta} > 0$ ).

Now we examine when  $TM\nabla$  is an equilibrium. It suffices to verify the strategy of firm  $ABj$ . When firm  $ABi$  plays  $x_{Ai} = x_{Ai}^{TM\nabla}$  and  $x_{Bi} = 0$ , the objective function of firm  $ABj$  is given by:

$$\frac{1}{9} \left( \frac{3(3-4\beta^2)D_A}{9-8\beta^2} + 2\beta x_{Aj} \right)^2 + \frac{(D_B + 2x_{Bj})^2}{9} - \frac{1}{2} (x_{Aj} + x_{Bj})^2 \quad (15)$$

on the region  $\left[0, \frac{9D_A}{(9-8\beta^2)\beta}\right] \times [0, D_B]$ .

Suppose that firm  $ABj$  plays the upper bound of  $x_{Aj}$ , that is,  $x_{Aj} = \frac{9D_A}{(9-8\beta^2)\beta}$ . The derivative of (15) with respect to  $x_{Aj}$  evaluated at  $\frac{9D_A}{(9-8\beta^2)\beta}$  is  $\frac{[4\beta^2(9-4\beta^2)-27]D_A}{3(9-8\beta^2)\beta} - x_{Bj}$ , which is always negative. Therefore, any  $\left(\frac{9D_A}{(9-8\beta^2)\beta}, x_{Bj}\right)$  is not a solution to (15). Moreover,  $ABj$  playing  $x_{Aj} = 0$  and  $x_{Bj}$  being given by the FOC is not a solution to (15), since it is obtained that  $x_{Bj} = 4D_B$ , which is outside the feasible region.

The remainder solution candidates to (15) are  $(x_{Aj}, 0)$  and  $(x_{Aj}, D_B)$ , where in both cases  $x_{Aj}$  is an interior solution given by the FOC. For the first candidate, the FOC from (15) with respect to  $x_{Aj}$  at  $x_{Bj} = 0$  is:

$$\frac{4\beta}{9} \left( \frac{3(3-4\beta^2)D_A}{9-8\beta^2} + 2\beta x_{Aj} \right) - x_{Aj} = 0$$



From this it follows that:

$$x_{Aj} = \frac{12\beta(3 - 4\beta^2)D_A}{(9 - 8\beta^2)^2}$$

Firm  $ABj$  gains a profit of:

$$\pi_{ABj}^{D6} = \frac{9(3 - 4\beta^2)^2 D_A^2}{(9 - 8\beta^2)^3} + \frac{D_B^2}{9}$$

When  $\frac{D_A}{D_B} > \frac{\sqrt{7}(9-8\beta^2)^{3/2}}{12\beta(3-4\beta^2)}$  and  $\beta < \sqrt{3}/2$ , it follows that  $\pi_{ABj}^{TM\nabla} < \pi_{ABj}^{D6}$ , therefore, firm  $ABj$  deviates unilaterally in this case.

For the second candidate, the FOC from (15) with respect to  $x_{Aj}$  at  $x_{Bj} = D_B$  is:

$$\frac{4\beta}{9} \left( \frac{3(3 - 4\beta^2)D_A}{9 - 8\beta^2} + 2\beta x_{Aj} \right) - x_{Aj} - D_B = 0$$

From this it follows that:

$$x_{Aj} = \frac{12\beta(3 - 4\beta^2)D_A - 9(9 - 8\beta^2)D_B}{(9 - 8\beta^2)^2}$$

To satisfy the assumption of  $x_{Aj} > 0$ ,  $\frac{D_A}{D_B} > \frac{9(9-8\beta^2)}{12\beta(3-4\beta^2)}$  must hold. Given that  $\frac{9(9-8\beta^2)}{12\beta(3-4\beta^2)} > \frac{\sqrt{7}(9-8\beta^2)^{3/2}}{12\beta(3-4\beta^2)}$ , and because it was established previously that  $TM\nabla$  is not an equilibrium when  $\frac{D_A}{D_B} > \frac{\sqrt{7}(9-8\beta^2)^{3/2}}{12\beta(3-4\beta^2)}$ , then this case does not provide new information on the existence conditions.

Thus, the only condition for  $TM\nabla$  to exist as an equilibrium is  $\frac{D_A}{D_B} \leq \frac{\sqrt{7}(9-8\beta^2)^{3/2}}{12\beta(3-4\beta^2)}$  and  $\beta < \sqrt{3}/2$ . The condition involving the market size ensures that the firm specializing in  $B$  does not deviate when the potential profit in  $A$  is high enough. Such condition is not necessary for the firm specializing in  $A$ , because that firm cannot deviate due to the high investments in R&D in market  $B$ .

**Case 2:** Consider the equilibrium candidate where  $\beta \geq \sqrt{3}/2$ . Here the R&D effort, the output, and the profit are given by:

$$\begin{aligned} x_{Ai}^{TM\Delta} &= \frac{D_A}{\beta}, & x_{Bi}^{TM\Delta} &= 0, & x_{Aj}^{TM\Delta} &= 0, & x_{Bj}^{TM\Delta} &= D_B, \\ q_{Ai}^{TM\Delta} &= D_A, & q_{Aj}^{TM\Delta} &= 0, & q_{Bi}^{TM\Delta} &= 0, & q_{Bj}^{TM\Delta} &= D_B, \end{aligned}$$

$$\pi_{ABi}^{TM\Delta} = \frac{(2\beta^2 - 1)D_A^2}{2\beta^2}, \quad \pi_{ABj}^{TM\Delta} = \frac{D_B^2}{2}$$

In this case firm  $ABi$  becomes a monopolist in market  $A$  while firm  $ABj$  becomes a monopolist in market  $B$ . In market  $B$ , where the technology is fully compatible, firm  $ABj$  is able to seize the totality of market  $B$  by investing an amount of R&D effort equivalent to the one in the monopoly case. However, in the case of market  $A$  where the technology is not fully compatible, firm  $ABi$  has to invest more than it would do in the monopoly case ( $x_{Ai}^{TM\Delta} > x_A^{MA}$ ) in order to seize the totality of market  $A$ . Even when firm  $ABi$  is the only firm in market  $A$ , the over-investment in R&D prevents  $ABi$ 's profit from market  $A$  to reach the level of the monopoly case.

Now we verify if  $TM\Delta$  is an equilibrium. It suffices to verify the strategy of firm  $ABj$ . When firm  $ABi$  plays  $x_{Ai} = x_{Ai}^{TM\Delta}$  and  $x_{Bi} = 0$ , the problem of firm  $ABj$  is given by:

$$\frac{(D_B + 2x_{Bj})^2}{9} - \frac{(9 - 8\beta^2)x_{Aj}^2}{18} - \frac{x_{Bj}^2}{2} - x_{Bj}x_{Aj} \quad (16)$$

on the region  $\left[0, \frac{3D_A}{\beta}\right] \times [0, D_B]$ .

The function in (16) is strictly decreasing in  $x_{Aj} \geq 0$ . Thus, (16) must be maximized at  $x_{Aj} = 0$ . Moreover,  $ABj$  playing  $x_{Aj} = 0$  and  $x_{Bj}$  being given by the FOC is not a solution to (16), since it is obtained that  $x_{Bj} = 4D_B$ , which is outside the feasible region. Hence, the only solution of (16) is the strategy in  $TM\Delta$ . Therefore, the only condition for  $TM\Delta$  to exist as an equilibrium is  $\beta \geq \sqrt{3}/2$ .

When the firms  $ABi$  and  $ABj$  play  $x_{Ai}^{TM\Delta} = \frac{D_A}{\beta}$  and  $x_{Bj}^{TM\Delta} = D_B$ , respectively, each firm supplies the totality of the market in which they are specializing, thus no firm can unilaterally deviate by setting a positive R&D effort in the market in which its rival is specializing. Then, unlike the previous cases, there is not a condition involving the market sizes because the firms cannot deviate to the other market due to the rival's high R&D effort in that market.

The core difference between  $TM\Delta$  and  $TM\nabla$  is that in the former firm  $ABi$  is able to seize the totality of market  $A$  and become a monopolist, while in the latter firm  $ABi$  is unable to do so. When  $\beta < \sqrt{3}/2$  it follows that  $x_{Ai}^{TM\Delta} > x_{Ai}^{TM\nabla}$ . This implies that a low technological compatibility makes the R&D effort more costly. Hence, in this case firm  $ABi$  is incapable of invest in the necessary amount of R&D effort to become a monopolist.

When the technological compatibility is low ( $\beta < \sqrt{3}/2$ ),  $TM\nabla$  is not an equilibrium if the market ratio is high enough. In that case, as firm  $ABi$  does not invest heavily in market  $A$ , the other firm finds profitable to deviate to that market. On the other hand, when the technological compatibility is high ( $\beta \geq \sqrt{3}/2$ ),  $TM\Delta$  is an equilibrium even if the market ratio is high. Because the high investments of firm  $ABi$  in market  $A$ , it is not profitable for the rival firm to deviate to that market regardless of the value of the market ratio.

### 5.3 One-merger subgame

In this case, it is assumed that only one firm in  $A$  merges with one firm in  $B$ . Let  $i = \{M, N\}$ , with  $M$  denoting the merged firms, and  $N$  denoting the non-merged firms. In comparison to the two-merger subgame, the set of strategies is reduced in the one-merger subgame. Specifically, the standalone firms can only operate in their own markets. Thus, the standalone firm in market  $A$  cannot invest in R&D and the standalone firm in market  $B$  can only invest in R&D in its own market. On the other hand, the conglomerate can invest in both markets as in the two-merger subgame. Thus, a symmetric equilibrium in this case can only be one where the firms invest only in market  $B$ . Furthermore, an asymmetric equilibrium entails that the conglomerate invests only in market  $A$ , while the standalone firm in market  $B$  invests only its own market.

The equilibrium candidates in the one-merger subgame are very similar to the ones in the two-merger subgame. However, the equilibrium conditions can differ. In the two-merger subgame, an asymmetric equilibrium might fail if the firm prescribed to invest in market  $B$  finds that it is more profitable to invest in market  $A$ . This kind of unilateral deviation is not possible in the one-merger subgame. Therefore, the details of the solution for this case are omitted and we only show the outcomes in equilibrium and their existence conditions. Once again, we distinguish the symmetric and asymmetric equilibria.

#### 5.3.1 Symmetric equilibria

As stated before, the only possible symmetric equilibrium in the one-merger subgame is when two firms invest in market  $B$  and none of the firms invest in market  $A$ . We denote this equilibrium with the superscript  $OMB$ . Here the R&D effort, the output, and the

profit are given by:

$$x_{Ai}^{OMB} = 0, \quad x_{Bi}^{OMB} = \frac{4D_B}{5}, \quad q_{Ai}^{OMB} = \frac{D_A}{3}, \quad q_{Bi}^{OMB} = \frac{3D_B}{5},$$

$$\pi_{ABM}^{OMB} = \frac{D_A^2}{9} + \frac{D_B^2}{25}, \quad \pi_{AN}^{OMB} = \frac{D_A^2}{9}, \quad \pi_{BN}^{OMB} = \frac{D_B^2}{25}$$

In this case the existence conditions are the same as the  $TMB$  case. Therefore, when  $\beta < \sqrt{3}/2$  and  $\frac{D_A}{D_B} \leq \frac{\sqrt{9-8\beta^2}}{5\beta}$ , or  $\beta \geq \sqrt{3}/2$  and  $\frac{D_A}{D_B} \leq \frac{4\beta}{5\sqrt{16\beta^2-9}}$ ,  $OMB$  can be sustained as an equilibrium.

The outcome and the existence conditions in  $OMB$  are equivalent to the ones in  $TMB$ . This is because in this case there are not incentives to focus on market  $A$ , so it is irrelevant whether there are one or two conglomerates. The only minor difference is that the joint-profit in  $TMB$  is split in  $OMB$  between the standalone firms.

### 5.3.2 Asymmetric equilibria

Equilibria that are analogous to  $TM\Diamond$  are possible. Thus, we consider two cases. We denote these cases with the subscript  $OM\Diamond$

**Case 1:** Consider the equilibrium candidate  $OM\nabla$ , which is analogous to  $TM\nabla$ . Here the R&D effort, the output, and the profit are given by:

$$x_{AM}^{OM\nabla} = \frac{4\beta D_A}{9-8\beta^2}, \quad x_{BM}^{OM\nabla} = 0, \quad x_{BN}^{OM\nabla} = D_B,$$

$$q_{AM}^{OM\nabla} = \frac{3D_A}{9-8\beta^2}, \quad q_{AN}^{OM\nabla} = \frac{(3-4\beta^2)D_A}{9-8\beta^2}, \quad q_{BM}^{OM\nabla} = 0, \quad q_{BN}^{OM\nabla} = D_B,$$

$$\pi_{ABM}^{OM\nabla} = \frac{D_A^2}{9-8\beta^2}, \quad \pi_{AN}^{OM\nabla} = \frac{(3-4\beta^2)^2 D_A^2}{(9-8\beta^2)^2}, \quad \pi_{BN}^{OM\nabla} = \frac{D_B^2}{2}$$

Here the conditions of existence can be obtained following a similar process as in  $TM\nabla$ . First, given that firm  $BN$  plays  $x_{BN} = x_{BN}^{OM\nabla}$ , the problem of firm  $ABM$  is equivalent to (14). From it, the necessary condition  $\beta < \sqrt{3}/2$  is obtained. Second, given that firm  $ABM$  plays the strategy in  $OM\nabla$ , the problem of firm  $BN$  is similar to (15), but without the profit from market  $A$  and without the ability to invest in market  $A$ . With these restrictions, the only solution for firm  $BN$  in this case is the strategy in  $OM\nabla$ . Therefore, the only condition to sustain  $OM\nabla$  as an equilibrium is  $\beta < \sqrt{3}/2$ .

The conditions of existence for  $OM\nabla$  differ from  $TM\nabla$ . In both equilibria the condition  $\beta < \sqrt{3}/2$  is present, but in  $OM\nabla$  there is not a condition involving the sizes of the market. This occurs because firm  $BN$ , as a standalone firm, is unable to deviate to market  $A$ .

**Case 2:** Consider the equilibrium candidate  $OM\Delta$ , which is analogous to  $TM\Delta$ . Here the R&D effort, the output, and the profit are given by:

$$\begin{aligned} x_{AM}^{OM\Delta} &= \frac{D_A}{\beta}, & x_{BM}^{OM\Delta} &= 0, & x_{BN}^{OM\Delta} &= D_B, \\ q_{AM}^{OM\Delta} &= D_A, & q_{AN}^{OM\Delta} &= 0, & q_{BM}^{OM\Delta} &= 0, & q_{BN}^{OM\Delta} &= D_B, \\ \pi_{ABM}^{OM\Delta} &= \frac{(2\beta^2 - 1)D_A^2}{2\beta^2}, & \pi_{AN}^{OM\Delta} &= 0, & \pi_{BN}^{OM\Delta} &= \frac{D_B^2}{2} \end{aligned}$$

The existence condition for  $OM\Delta$  is the same as  $TM\Delta$ . Hence,  $OM\Delta$  can be sustained as an equilibrium when  $\beta \geq \sqrt{3}/2$ .

## 5.4 Conditions for the existence of equilibrium

Here we summarize the conditions for the existence of equilibria in the second stage. For this, we first define the following thresholds:

$$\theta_A = \frac{(9 - 4\beta^2)\sqrt{7}}{4\beta\sqrt{9 - 8\beta^2}}, \quad \hat{\theta}_A = \frac{\sqrt{7}(9 - 8\beta^2)^{3/2}}{12\beta(3 - 4\beta^2)}, \quad \theta_B = \begin{cases} \frac{\sqrt{9 - 8\beta^2}}{5\beta} & \text{if } \beta < \frac{\sqrt{3}}{2} \\ \frac{4\beta}{5\sqrt{16\beta^2 - 9}} & \text{if } \beta \geq \frac{\sqrt{3}}{2} \end{cases}$$

These are the thresholds of the existence conditions related to the market size that appeared in the previous analysis. The threshold  $\hat{\theta}_A$  is paired with the technological compatibility condition  $\beta < \sqrt{3}/2$ . It is easy to see that  $\hat{\theta}_A$  is well defined for that range of  $\beta$ . It holds that  $\theta_B < \theta_A$  for any  $\beta$  and  $\theta_A < \hat{\theta}_A$  for any  $\beta \in (0, \sqrt{3}/2)$ .

Now, we state the results in the following propositions:

**Proposition 2.** *There always exists an equilibrium in the second stage in the zero-merger subgame.*

**Proposition 3.** *(a) When  $\beta \in (0, \sqrt{3}/2)$ , if  $\frac{D_A}{D_B} \leq \hat{\theta}_A$ ,  $TM\nabla$  is an equilibrium in the two-merger subgame. If  $\frac{D_A}{D_B} \geq \theta_A$ ,  $TMA$  is an equilibrium in the two-merger subgame. If  $\frac{D_A}{D_B} \leq \theta_B$ ,  $TMB$  is an equilibrium in the two-merger subgame.*

(b) When  $\beta \in [\sqrt{3}/2, 1)$ ,  $TM\Delta$  is an equilibrium in the two-merger subgame. If  $\frac{D_A}{D_B} \geq \theta_A$ ,  $TMA$  is an equilibrium in the two-merger subgame. If  $\frac{D_A}{D_B} \leq \theta_B$ ,  $TMB$  is an equilibrium in the two-merger subgame.

(c) There always exists an equilibrium in the second stage in the two-merger subgame.

We represent the statements of Proposition 3 in Figures 1 and 2.

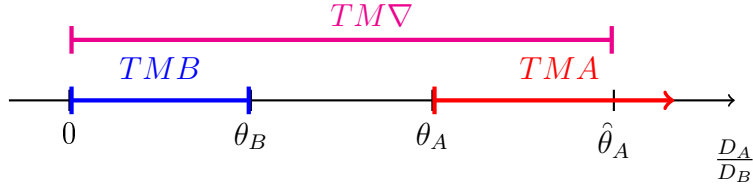


Figure 1: Existence of equilibria in the two-merger subgame. ( $\beta \in (0, \sqrt{3}/2)$ )

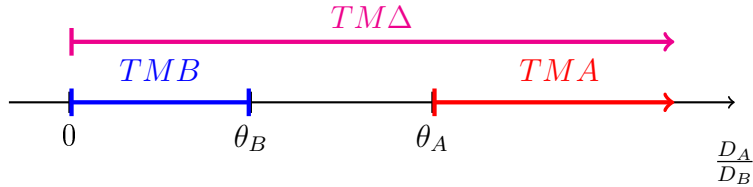


Figure 2: Existence of equilibria in the two-merger subgame. ( $\beta \in [\sqrt{3}/2, 1)$ )

**Proposition 4.** (a) When  $\beta \in (0, \sqrt{3}/2)$ ,  $OM\nabla$  is an equilibrium in the one-merger subgame. If  $\frac{D_A}{D_B} \leq \theta_B$ ,  $OMB$  is an equilibrium in the one-merger subgame.

(b) When  $\beta \in [\sqrt{3}/2, 1)$ ,  $OM\Delta$  is an equilibrium in the one-merger subgame. If  $\frac{D_A}{D_B} \leq \theta_B$ ,  $OMB$  is an equilibrium in the one-merger subgame.

(c) There always exists an equilibrium in the second stage in the one-merger subgame.

We represent the statements of Proposition 4 in Figure 3.

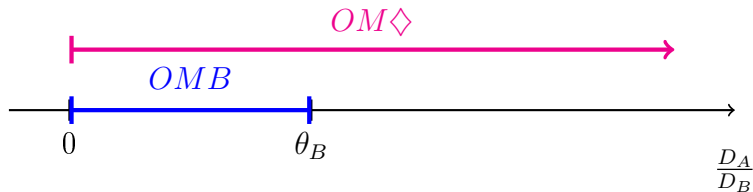


Figure 3: Existence of equilibria in the one-merger subgame.

As in the monopoly case, there are equilibria in the duopoly case where the firms invest only in one of the markets. Comparing the threshold in the monopoly case and the ones in this section, it follows that  $\theta_B < \frac{\sqrt{2-\beta^2}}{\beta} < \theta_A$  for any  $\beta$ . Thus, for the equilibrium

where the firms invest in market  $B$  to exist in the two and one-merger subgames, the market ratio has to be lower in comparison to the monopoly case. Moreover, for the equilibrium where all firms invest in market  $A$  to exist in the two-merger subgame, the market ratio has to be greater in comparison to the monopoly case.

## 6 First stage: The merger game

We solve here the first stage of the game where the firms decide whether to merge. A pair of firms choose to merge when the joint-profit of the merged firm is larger than the sum of the separated firms' profits. Those profits depend on what equilibrium is played in the second stage after the firms' merger decisions. It is assumed that player 1 is a team consisting of  $A1$  and  $B1$ , while player 2 is  $A2$  together with  $B2$ . The set of actions is to merge ( $M$ ) or not ( $DM$ ). The payoffs are the profits derived in the second stage. Since it was established that the existence of equilibria on the second stage depends on the market ratio and the technology compatibility, different versions of the merger game exist depending on how the parameters are configured. Moreover, for the same configuration of parameters, multiple equilibria in the second stage might exist. Hence, multiple versions of the merger game can be constructed for the same range of parameters, depending on which of the multiple equilibria is set as the payoff of the merger game.

Even with the multiplicity of equilibria, because there exist equivalent outcomes across the three merger subgames of the second stage, the payoffs of the merger game can be characterized in a simple manner. Specifically, all the equilibria in the second stage can be categorized into three groups. The main criterion for this categorization is that all the equilibria belonging in one group have identical total payoffs. We refer to this groups as *market outcomes*.

The first market outcome contains the  $OMB$ , the  $TMB$  and the  $ZM$  equilibria. We denominate this market outcome as the B-outcome because in this group of equilibria the firms invest only in market  $B$ . In the merger game, the payoffs in the B-outcome are symmetric. We denote the payoff of any team in this outcome with  $\pi^B$ .

The second market outcome contains only the  $TMA$  equilibrium. We denominate this market outcome as the A-outcome because in the  $TMA$  equilibrium the firms invest only in market  $A$ . In the merger game, the payoffs in the A-outcome are symmetric. We

denote the payoff of any team in this outcome with  $\pi^A$ .

The third market outcome contains the  $OM\nabla$  and the  $TM\nabla$  equilibria when  $\beta < \sqrt{3}/2$  and the  $OM\Delta$  and the  $TM\Delta$  equilibria when  $\beta \geq \sqrt{3}/2$ . We denominate this market outcome as the asymmetric outcome because the payoffs are asymmetric in all the equilibria of these groups. For this group, we use  $\pi^{\diamond A}$  to denote the payoff of the team investing in market  $A$  and  $\pi^{\diamond B}$  for the payoff of the team investing in market  $B$ .

Then, the possible payoffs on the merger game are:

$$\pi^B = \frac{D_A^2}{9} + \frac{D_B^2}{25}, \quad \pi^A = \frac{(9-8\beta^2)D_A^2}{(9-4\beta^2)^2} + \frac{D_B^2}{9},$$

$$\pi^{\diamond A} = \begin{cases} \frac{D_A^2}{9-8\beta^2} & \text{if } \beta < \frac{\sqrt{3}}{2} \\ \frac{(2\beta^2-1)D_A^2}{2\beta^2} & \text{if } \beta \geq \frac{\sqrt{3}}{2} \end{cases}, \quad \pi^{\diamond B} = \begin{cases} \frac{(3-4\beta^2)^2 D_A^2}{(9-8\beta^2)^2} + \frac{D_B^2}{2} & \text{if } \beta < \frac{\sqrt{3}}{2} \\ \frac{D_B^2}{2} & \text{if } \beta \geq \frac{\sqrt{3}}{2} \end{cases}$$

The normal-form of the game is presented in Table 4.

Table 4: The Merger Game

		A2,B2	
		Do not merge	Merge
A1,B1	Do not merge	$\pi^B, \pi^B$	$\pi_1^{DM,M}, \pi_2^{DM,M}$
	Merge	$\pi_1^{M,DM}, \pi_2^{M,DM}$	$\pi_1^{M,M}, \pi_2^{M,M}$

Where –contingent on the parameters–  $(\pi_1^{M,M}, \pi_2^{M,M}) \in \{(\pi^{\diamond B}, \pi^{\diamond A}), (\pi^{\diamond A}, \pi^{\diamond B}), (\pi^A, \pi^A), (\pi^B, \pi^B)\}$ ,  $(\pi_1^{M,DM}, \pi_2^{M,DM}) \in \{(\pi^{\diamond A}, \pi^{\diamond B}), (\pi^B, \pi^B)\}$  and  $(\pi_1^{DM,M}, \pi_2^{DM,M}) \in \{(\pi^{\diamond B}, \pi^{\diamond A}), (\pi^B, \pi^B)\}$ .

Notice that in the profiles  $(DM, M)$  and  $(M, DM)$  there is only one way to allocate the payoffs from the asymmetric outcome: the team that chooses to merge always invests only in market  $A$ . This characteristic is not present in the profile  $(M, M)$ . Thus, there are two possible ways to configure the payoffs from the asymmetric outcome in the profile  $(M, M)$ , depending on how the payoffs are allocated to the players.

To better understand the construction of the merger game, we explain two polar cases. First consider the scenario where  $\beta \in (0, \sqrt{3}/2)$  and  $\frac{D_A}{D_B} > \hat{\theta}_A$ . Here it holds that  $(\pi_1^{M,M}, \pi_2^{M,M}) = (\pi^A, \pi^A)$ ,  $(\pi_1^{M,DM}, \pi_2^{M,DM}) = (\pi^{\diamond A}, \pi^{\diamond B})$  and  $(\pi_1^{DM,M}, \pi_2^{DM,M}) = (\pi^{\diamond B}, \pi^{\diamond A})$ . Thus, the merger game is uniquely defined for that range of parameters. Second, consider  $\frac{D_A}{D_B} \leq \theta_B$  for any  $\beta$ . In this case it holds that  $(\pi_1^{M,M}, \pi_2^{M,M}) \in \{(\pi^{\diamond B}, \pi^{\diamond A}), (\pi^{\diamond A}, \pi^{\diamond B}), (\pi^B, \pi^B)\}$ ,  $(\pi_1^{M,DM}, \pi_2^{M,DM}) \in \{(\pi^{\diamond A}, \pi^{\diamond B}), (\pi^B, \pi^B)\}$  and



$(\pi_1^{DM,M}, \pi_2^{DM,M}) \in \{(\pi^{\diamond B}, \pi^{\diamond A}), (\pi^B, \pi^B)\}$ . Thus, there are 12 cases of the merger game for this set of parameters. We define 9 main scenarios, with a total 28 cases of the merger game. We solve each case in the Appendix. Before stating the results, we define the following threshold:

$$\hat{\theta}_B = \begin{cases} \frac{3\sqrt{9-8\beta^2}}{10\sqrt{2}\beta} & \text{if } \beta < \frac{\sqrt{3}}{2} \\ \frac{3\sqrt{2}\beta}{5\sqrt{16\beta^2-9}} & \text{if } \beta \geq \frac{\sqrt{3}}{2} \end{cases}$$

This threshold results from comparing  $\pi^B$  with  $\pi^{\diamond A}$ . More precisely,  $\pi^B \geq \pi^{\diamond A}$  iff  $\frac{D_A}{D_B} \leq \hat{\theta}_B$  for any  $\beta$ . It holds that  $\theta_A > \hat{\theta}_B > \theta_B$  for any  $\beta$ . We state the results in terms of the market outcomes in the following proposition.

**Proposition 5.** (a) When  $\beta \in (0, \sqrt{3}/2)$ , if  $\frac{D_A}{D_B} \geq \theta_A$  and the A-outcome is set in the profile  $(M, M)$ , the unique equilibrium of the merger game corresponds to the A-outcome. If  $\hat{\theta}_A \geq \frac{D_A}{D_B} > \hat{\theta}_B$  and an asymmetric outcome is set in the profile  $(M, M)$ , any equilibrium of the merger game corresponds to the asymmetric outcome. If  $\frac{D_A}{D_B} = \hat{\theta}_B$ , any equilibrium of the merger game is either the asymmetric outcome or the B-outcome. If  $\frac{D_A}{D_B} < \hat{\theta}_B$ , any equilibrium of the merger game is the B-outcome.

(b) When  $\beta \in [\sqrt{3}/2, 1)$ , if  $\frac{D_A}{D_B} \geq \theta_A$  and the A-outcome is set in the profile  $(M, M)$ , the unique equilibrium of the merger game corresponds to the A-outcome. If  $\frac{D_A}{D_B} > \hat{\theta}_B$  and an asymmetric outcome is set in the profile  $(M, M)$ , any equilibrium of the merger game corresponds to the asymmetric outcome. If  $\frac{D_A}{D_B} = \hat{\theta}_B$ , any equilibrium of the merger game is either the asymmetric outcome or the B-outcome. If  $\frac{D_A}{D_B} < \hat{\theta}_B$ , any equilibrium of the merger game is the B-outcome.

The proof of Proposition 5 is in the Appendix. We represent the statements of Proposition 5 in Figures 5 and 6.

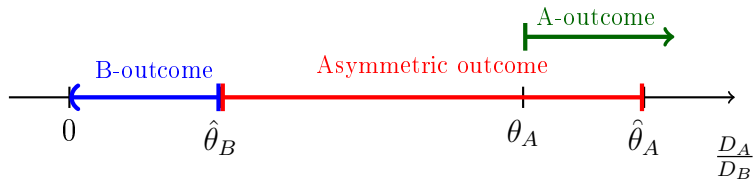


Figure 5: Merger Game's Equilibria. Market Outcomes. ( $\beta \in (0, \sqrt{3}/2)$ )

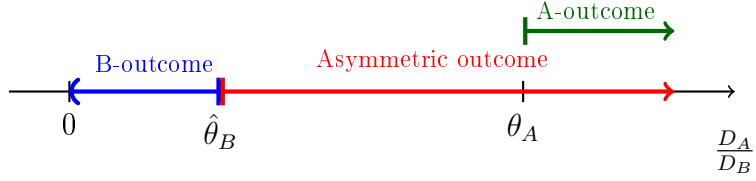


Figure 6: Merger Game's Equilibria. Market Outcomes. ( $\beta \in [\sqrt{3}/2, 1)$ )

Proposition 5 states various results. First, for any  $\beta$ , when the market ratio is small enough, the firms invest only in market  $B$  in any equilibrium. When the market ratio is small enough, the profiles in the merger game contain either the asymmetric or the B-outcome. In this case, it holds that  $\pi^{\diamond A} < \pi^B < \pi^{\diamond B}$  for any  $\beta$ , that is, market  $B$  is much more profitable relative to  $A$ , even taking into account the increased competition in the B-outcome in comparison to the asymmetric outcome. Thus, the asymmetric outcome fails to be an equilibrium because the team that merges and invests in market  $A$  prefers to invest in market  $B$  to secure either  $\pi^B$  or  $\pi^{\diamond B}$ . On the other hand, the B-outcome in the profile  $(DM, DM)$  is guaranteed to be an equilibrium because  $\pi^{\diamond A} < \pi^B$ .

Second, Proposition 5 provides the conditions for any equilibrium to sustain an asymmetric outcome. From part (a), when the technology compatibility is low enough ( $\beta \in (0, \sqrt{3}/2)$ ) and an asymmetric outcome is set in the profile  $(M, M)$ , if neither the relative size of market  $A$  or  $B$  is big enough, at least one conglomerate is formed and it invests only in market  $A$ , and one firm invests only in market  $B$  and becomes a monopolist in that market. Part (b) is similar to part (a), however, in part (b) the market ratio condition is relaxed and is only required to not be too low.

Given that the market ratio is not low enough, here it holds that  $\pi^B < \pi^{\diamond A}$  for any  $\beta$ . Since the firms benefit from the lesser competition in the asymmetric outcome in comparison to the B-outcome, this inequality does not necessarily mean that market  $A$  is better than  $B$ . Thus, the B-outcome fails to be an equilibrium mainly because the firms gain by avoiding the strong competition of the B-outcome by deviating to the asymmetric outcome.

Setting the asymmetric outcome in the profile  $(M, M)$  implies that the A-outcome is not present in the merger game, so the merger game only contains the asymmetric or the B-outcome. This results in the asymmetric outcome being the equilibrium of the merger game. Conversely, when the A-outcome is present in the merger game, it always holds that  $\pi^{\diamond B} < \pi^A$  for any  $\beta$ . Therefore, the asymmetric outcome always fails to be

an equilibrium. Thus, when  $\beta \in (0, \sqrt{3}/2)$  and the market ratio is high enough, the asymmetric outcome is not an equilibrium because the merger game always contains the A-outcome. However, when  $\beta \in [\sqrt{3}/2, 1)$ , no matter how high is the market ratio, it is possible to construct a merger game without a profile containing the A-outcome.

In summary, when the market ratio and the technology compatibility are high enough, if no profile contains the A-outcome, there always exists a profile with the asymmetric outcome such that, even though the team investing in market  $B$  might be better off by investing in market  $A$ , there is not a possible deviation from that team that results in the team investing in market  $A$ . Therefore, the equilibrium of the merger game ends being the asymmetric outcome.

Third and finally, Proposition 5 states that, for any  $\beta$ , when the market ratio is high enough and the A-outcome is set in the profile  $(M, M)$ , all firms invest only in market  $A$  in equilibrium. Due to the high market ratio, it holds that  $\pi^B < \pi^{\diamond A}$  and  $\pi^{\diamond B} < \pi^A$  for any  $\beta$ . Hence, market  $A$  is much more profitable than market  $B$ , even taking into account the increased competition in the A-outcome in comparison to the asymmetric outcome. Thus, the firms deviate from the B-outcome to the asymmetric outcome to invest in market  $A$ . Moreover, given that the A-outcome is present in this case of the merger game, the team investing in market  $B$  in the asymmetric outcome deviate to the A-outcome to invest in market  $A$ .

In Proposition 1 it was stated that the monopoly solution depends on which market is deemed more profitable, that is, which market is the greatest taking into account the disadvantage of investing in market  $A$  as a consequence of the technological compatibility. This also occurs in the oligopoly case when a symmetric outcome is an equilibrium, i.e., when all the teams invest only in the greatest and hence the most profitable market. Comparing the threshold in the monopoly case and the ones in Proposition 5, it follows that  $\hat{\theta}_B < \frac{\sqrt{2-\beta^2}}{\beta} < \theta_A$  for any  $\beta$ . Thus, for the B(A)-outcome to be an equilibrium of the merger game in the duopoly case, the market ratio requires to be lower (greater) in comparison to the monopoly case.

However, as seen in Proposition 5, investing in the best market is not the only behavior in the oligopoly case. When an asymmetric outcome is the equilibrium of the merger game, one of the teams invests in the weakest market, even in the cases where investing in the strongest market is a possibility. Although the strongest market could potentially

be more profitable, one of the teams prefers to invest in the weakest market to avoid R&D competition in the strongest market. In conclusion, in oligopoly the firms maximize their profits by investing in the best market and/or by avoiding R&D competition.

The threshold that the market ratio has to be below to sustain the B-outcome as an equilibrium in the merger game and as an equilibrium in the two and one-merger subgames is different. However, the threshold that the market ratio has to exceed to sustain the A-outcome as an equilibrium in the merger game and as an equilibrium in the two-merger subgame is the same. When  $\frac{D_A}{D_B} \in \left(\theta_B, \hat{\theta}_B\right]$  the B-outcome is a continuation equilibrium only in the zero-merger subgame. Therefore, as the zero-merger subgame is an equilibrium in the merger game, even though the B-outcome is not a continuation equilibrium in the two and one-merger subgame, the B-outcome is an equilibrium in the merger game. Intuitively, if there is at least one conglomerate in the second stage, the conglomerate has incentives to deviate from investing in market  $B$ . Nevertheless, there are not incentives to create at least one conglomerate in the first stage. Thus, the final result is that the B-outcome is an equilibrium of the whole game. A similar situation does not occur for the equilibrium with the A-outcome. This disparity is a consequence of the structure of the game: The A-outcome is possible only in the two-merger subgame, while the B-outcome is possible in any of the merger subgames.

Now we define the following threshold:

$$\bar{\theta} = \begin{cases} \frac{(9-8\beta^2)}{4\sqrt{2}\beta\sqrt{1-\beta^2}} & \text{if } \beta < \frac{\sqrt{3}}{2} \\ \frac{\beta}{\sqrt{2\beta^2-1}} & \text{if } \beta \geq \frac{\sqrt{3}}{2} \end{cases}$$

This threshold results from comparing the payoffs of the asymmetric outcome. More precisely,  $\pi^{\diamond A} \geq \pi^{\diamond B}$  iff  $\frac{D_A}{D_B} \geq \bar{\theta}$  for any  $\beta$ . It also holds that  $\hat{\theta}_B < \bar{\theta} < \theta_A$  for any  $\beta$ . When  $\frac{D_A}{D_B} \in \left(\hat{\theta}_B, \bar{\theta}\right)$ , in any equilibria of the merger game there is only one conglomerate (see the Appendix). In that equilibrium, the conglomerate firm and the standalone firm in market  $B$  are always better-off in comparison to the zero-merger structure. The conglomerate firm profits by obtaining a cost advantage in market  $A$ . On the other hand, the standalone firm in market  $B$  profits due to the elimination of the competition in both quantities and R&D in that market. However, the profit of the conglomerate is lower than the sum of the profits of the standalone firms. This situation resembles the Merger Paradox of Salant et al. (1983). In our model, a pair of firms becomes a conglomerate

and invest only in market  $A$ , which is the weakest market in this specific case. On the other hand, one of the standalone firms becomes the only one that invest in market  $B$ , the strongest market. Thus, the act of merging results in the conglomerate investing in the worst market, while one of the standalone firms remains as the only investor in the best market. The situation of that standalone firm can be described as free-riding: the standalone firm gains with the conglomerate merger without being a member of such merger.

In our merger game's results so far, in equilibrium the A-outcome is connected with the two-merger structure. The interpretation here is that both teams choose to merge to transfer their R&D effort to market  $A$ . For the remaining equilibria of the merger game, the economic interpretation of the relation between the market outcomes and the merger structures is not so clear. This is because the asymmetric and B-outcome are not always consistent with the merger decisions in equilibrium. For example, consider the merger game where  $\frac{D_A}{D_B} \leq \theta_B$  and  $(\pi_1^{M,M}, \pi_2^{M,M}) = (\pi_1^{M,DM}, \pi_2^{M,DM}) = (\pi_1^{DM,M}, \pi_2^{DM,M}) = (\pi^B, \pi^B)$ . Here all the profiles are an equilibrium of the merger game. Moreover, all the profiles contain the B-outcome. Thus, this case suggest that the B-outcome is consistent with any of the merger scenarios. However, it does not seem reasonable to create a conglomerate if none of the firms will not transfer their R&D effort to market  $A$ . This occurs because the lack of a merger cost in our model. More precisely, the lack of a merger cost creates a lot of instances in the merger game where the firms are indifferent between merge or not merge. So, if we were to add this cost to the model, the firms would actually prefer not to merge in those cases of indifference. This would eliminate the equilibria with inessential mergers, allowing us to focus on the equilibria with a clearer economic reasoning. To keep things simple, instead of adding a merger cost to the model, we just make the following assumption.

**Assumption M.** *Given the rival's strategy, if both strategies of a player yield the same payoff, the player will choose DM.*

With Assumption M, we state the equilibria of the merger game in terms of merger decisions in the following proposition.

**Proposition 6.** *(a) When  $\beta \in (0, \sqrt{3}/2)$ , if  $\frac{D_A}{D_B} \geq \theta_A$  and the A-outcome is set in the profile  $(M, M)$ , two firms merge in the unique equilibrium. With Assumption M, if*

$\hat{\theta}_A \geq \frac{D_A}{D_B} > \hat{\theta}_B$  and an asymmetric outcome is set in the profile  $(M, M)$ , one firm merges in any equilibrium. With Assumption M, if  $\frac{D_A}{D_B} \leq \hat{\theta}_B$ , none of the firms merge in any equilibrium.

(b) When  $\beta \in [\sqrt{3}/2, 1)$ , if  $\frac{D_A}{D_B} \geq \theta_A$  and the A-outcome is set in the profile  $(M, M)$ , two firms merge in the unique equilibrium. With Assumption M, if  $\frac{D_A}{D_B} > \hat{\theta}_B$  and an asymmetric outcome is set in the profile  $(M, M)$ , one firm merges in any equilibrium. With Assumption M, if  $\frac{D_A}{D_B} \leq \hat{\theta}_B$ , none of the firms merge in any equilibrium.

The proof of Proposition 6 is in the Appendix. We represent the statements of Proposition 6 in Figures 7 and 8.



Figure 7: Merger Game's Equilibria. Merger Structures. ( $\beta \in (0, \sqrt{3}/2)$ )



Figure 8: Merger Game's Equilibria. Merger Structures. ( $\beta \in [\sqrt{3}/2, 1)$ )

Part (a) of Proposition 6 provides the solution to the merger game when the technology compatibility is low enough. If the market ratio is so high as to support an equilibrium where both firms invest in market A when both teams merge, they choose to merge and play the equilibrium. If the market ratio is low enough, both teams decide to not merge. For intermediate values of the market ratio, one conglomerate firm is formed, and two standalone firms remain. In this case, when  $\hat{\theta}_A \geq \frac{D_A}{D_B} \geq \theta_A$ , two solutions (both players merging and one player merging) are possible depending on the continuation equilibrium of the two-merger subgame.

Part (b) of Proposition 6 contains the solution to the merger game when the technology compatibility is high enough ( $\beta \in [\sqrt{3}/2, 1)$ ). The results in part (b) are similar to part (a), with the exception that in part (b), the equilibrium with a single conglomerate

always exists when the market ratio is high enough. Thus, in part (b) the equilibrium with two conglomerates always coexists with the one with one conglomerate. Again, which of these merger structures is the equilibrium of the merger game depends on the continuation equilibrium of the two-merger subgame.

As stated above, in equilibrium the A-outcome is associated only with the two-merger scenario. Now, with the refinement of Assumption M, in equilibrium the asymmetric outcome is associated only with the one-merger scenario and the B-outcome with the zero-merger scenario. Thus, the intuition is that the firms create conglomerates if the profits strictly increase by the transfer of technology from market  $B$  to market  $A$ .

## 7 Welfare analysis

In this section, first we briefly examine the producer surplus. Afterwards, we compute and analyze the society's overall welfare.

### 7.1 Producer surplus

We define the total producer surplus as  $TPS = PS_A + PS_B$ , where  $PS_k$  is the producer surplus in market  $k$ , which is the sum of the profits of all firms in market  $k$ . Thus, the total producer surplus in the A, B and asymmetric outcome are defined as  $TPS^A = 2\pi^A$ ,  $TPS^B = 2\pi^B$  and  $TPS^\diamond = \pi^{\diamond A} + \pi^{\diamond B}$ , respectively. If we compare the total producer surplus of the asymmetric outcomes with the symmetric ones, it follows that  $TPS^B < TPS^\diamond$  and  $TPS^A < TPS^\diamond$  for any  $\beta$ . From this we are able to state the following result:

**Proposition 7.** *The asymmetric outcome is the one that always maximizes the total producer surplus.*

The statement in Proposition 7 is not surprising because in the asymmetric outcome the firms gain with their R&D investments, while in the symmetric outcomes the R&D competition harms the firms. Thus, there is the question of why the symmetric outcomes, which do not maximize the total producer surplus, can be supported in equilibrium. By the nature of asymmetric outcome, the teams' profits are different, thus there exists a "winner" and a "loser" team. Evidently, the greatest share of the total producer surplus corresponds to the profit of the winner team.

When the market ratio is above  $\bar{\theta}$ , the loser team is the one investing in market  $B$ . If the market ratio surpasses  $\theta_A$  and the continuation equilibrium of the two-merger subgame is the A-outcome, the asymmetric outcome is not supported in equilibrium because the team investing in market  $B$  prefers to merge and deviate to the more profitable A-outcome. On the other hand, if the market ratio is below  $\bar{\theta}$ , the loser team is the one investing in market  $A$ . If the market ratio drops behind  $\hat{\theta}_B$ , the asymmetric outcome is not supported in equilibrium because the team investing in market  $A$  deviates to a more profitable B-outcome or for another asymmetric outcome where it becomes a winner. In both cases, the loser team's deviation from the asymmetric outcome increases its profit but decreases in a greater proportion the profit of the winner team. Thus, the deviation reduces the total producer surplus.

We can draw parallels between our previous discussion on the total producer surplus and some well-known theoretical concepts. First, in light of Proposition 7, the Merger Paradox that we already discussed in the previous section becomes more paradoxical. The loser team, by taking the decision to merge, not only increases its rival's profit, but also the total producer surplus. Second, the A-outcome in equilibrium is reminiscent of the Tragedy of the Commons, where the act of merging is the common resource. When the non-merged team in an asymmetric outcome has incentives to merge and deviate to the A-outcome, the profit of that team increases but decreases both its rival's profit and the total producer surplus. Thus, an excess of mergers is harmful to the common wellness, measured in this case by the total producer surplus. Third and finally, the existence of a loser and a winner in the asymmetric outcome in equilibrium resembles the battle of sexes coordination game. Even though the asymmetric outcome maximizes the total producer surplus, any equilibrium with that outcome results unfair for one of teams, although the teams are symmetric.

## 7.2 Social welfare

We begin this section defining the consumer surplus. We assume that a representative consumer in market  $k$  has a quasi-linear utility function with the form  $U_k(q_0, q_{k1}, q_{k2}) = q_{k0} + v_k(q_{k1}, q_{k2})$ , where  $q_{k0}$  is the quantity of the numeraire good and  $v_k(q_{k1}, q_{k2})$  is given by:

$$v_k(q_{k1}, q_{k2}) = D_k(q_{k1} + q_{k2}) - \frac{1}{2} (q_{k1} + q_{k2})^2$$



Then, the consumer surplus can be computed by:

$$CS_k = v_k(q_{k1}^*, q_{k2}^*) - p_{k1}(q_{k1}^*, q_{k2}^*) * q_{k1}^* - p_{k2}(q_{k1}^*, q_{k2}^*) * q_{k2}^*$$

where  $q_{ki}^*$  is the equilibrium quantity of firm  $i$  in market  $k$ . In equilibrium,  $p_{k1}(q_{k1}^*, q_{k2}^*) = p_{k2}(q_{k1}^*, q_{k2}^*)$  holds. Hence, the last expression can be simplified as follows:

$$CS_k = \frac{1}{2}(q_{k1}^* + q_{k2}^*)^2$$

Now, we define the overall social welfare as  $W = TPS + CS_A + CS_B$ . The social welfare in each market outcome is:

$$W^B = \frac{4D_A^2}{9} + \frac{4D_B^2}{5}, \quad W^A = \frac{4D_A^2}{9 - 4\beta^2} + \frac{4D_B^2}{9},$$

$$W^\diamond = \begin{cases} \frac{4(9-14\beta^2+6\beta^4)D_A^2}{(9-8\beta^2)^2} + D_B^2 & \text{if } \beta < \frac{\sqrt{3}}{2} \\ \frac{(3\beta^2-1)D_A^2}{2\beta^2} + D_B^2 & \text{if } \beta \geq \frac{\sqrt{3}}{2} \end{cases}$$

To compare the social welfare between market outcomes, first we define the following thresholds:

$$\gamma_{A\diamond} = \frac{(9-8\beta^2)\sqrt{5(9-4\beta^2)}}{6\beta\sqrt{2(9-23\beta^2+12\beta^4)}}, \quad \gamma_{AB} = \frac{\sqrt{9-4\beta^2}}{\sqrt{5}\beta}, \quad \bar{\beta} = \frac{\sqrt{23-\sqrt{97}}}{2\sqrt{6}}$$

Here it holds that  $\bar{\beta} < \sqrt{3}/2$ . Now, we can compare the social welfare from each market outcome as follows:  $W^\diamond > W^B$  for any  $\beta$ ,  $W^A \geq W^B$  if and only if  $\frac{D_A}{D_B} \geq \gamma_{AB}$ , and  $W^A \geq W^\diamond$  if and only if  $\frac{D_A}{D_B} \geq \gamma_{A\diamond}$  and  $\beta < \bar{\beta}$ . From this comparison, we state the following proposition:

**Proposition 8.** (a) When  $\frac{D_A}{D_B} \geq \gamma_{A\diamond}$  and  $\beta < \bar{\beta}$ , the best market outcome from the perspective of the social welfare is the A-outcome.

(b) Otherwise, the best market outcome from the perspective of the social welfare is the asymmetric outcome.

Part (a) of Proposition 8 establishes the conditions for the A-outcome to be the best market outcome from the perspective of the social welfare. When the market ratio is high, market A is the one with higher potential profits for the firms and the one that contributes most to the total consumer surplus. Moreover, when the technological compatibility is low

enough, the level of competition in the A-outcome is less intense. Thus, the consumers in market  $A$  benefit from the competition in quantities and R&D effort without greatly hurting the profits of the firms. For this reason, the B-outcome is never the best market outcome from the perspective of the social welfare, because in market  $B$  the technology is fully compatible.

To discuss the merger policies implications of part (a) of Proposition 8, we first relate the welfare results with the equilibrium results. It follows that  $\gamma_{A\Diamond} > \hat{\theta}_A$  for any  $\beta \in (0, \bar{\beta})$ . Therefore, if the best market outcome from the perspective of the social welfare is the A-outcome, then it is also an equilibrium. Moreover, under these conditions, the A-outcome is the unique equilibrium. Thus, the policy authority has to do nothing in this case.

Part (b) of Proposition 8 states that if the conditions of part (a) are not satisfied, then the asymmetric outcome is the best market outcome from the perspective of the social welfare. Here the interests of the firms in terms of the total producer surplus align with the interests of the society. In this case, the best market outcome from the social welfare perspective might not be sustained in equilibrium if the market ratio is too large or too small. Thus, the policy implications depend on the market ratio. If the market ratio is low enough such that in equilibrium there are not mergers, then the policy authority should force one pair of firms to merge. On the other hand, if the market ratio is high enough such that in equilibrium there are two mergers, then the policy authority should prohibit the merger of only one of the pairs of firms.

## 8 Conclusion

This paper studied merger decisions in a conglomerate framework under Cournot competition where firms in one market can share their technology with firms in other markets. One main theoretical prediction of the model is how conglomerates allocate their R&D across their markets. In a monopoly structure, the conglomerate just chooses in which market to invest depending on which is more profitable. In duopoly, the firms also follow a profit-maximization behavior. However, the outcomes are not limited to allocate the R&D into the best markets, they can also allocate R&D effort into markets that are devoid of the rival's R&D effort to avoid competition, even if that market is weaker.

Another main theoretical prediction of the model is the decision of whether to merge. We find that in equilibrium, the A-outcome is consistent only with the scenario with two conglomerates. On the other hand, the asymmetric and B-outcome are consistent with various merger scenarios. Nevertheless, this result is not intuitive because there are equilibria with meaningless mergers. Namely, firms choose to merge in the first stage even though the technology is not transferred in the second stage. To refine this result, we utilize Assumption M, which imposes that the firms prefer to not merge when there is indifference between the merger decisions. With Assumption M, the asymmetric outcome is consistent only with the scenario with one conglomerate, and the B-outcome is only consistent with the scenario without mergers. Thus, the intuition is that the firms choose to merge when the joint-profit of the conglomerate strictly improves due to the transfer of technology from one market to the other.

In our welfare analysis, we find that the asymmetric outcome always maximizes the producer surplus. However, this market outcome is not always an equilibrium. For extreme values of the market ratio, the distribution of profits between the teams is very uneven. Thus, the team that receives the lower profit has incentives to deviate to one of the symmetric outcomes. We find that the total social welfare is maximized by the A-outcome when the market ratio is high enough and the technological compatibility is low enough. Under those conditions, the A-outcome is the unique equilibrium. If those conditions do not hold, the asymmetric outcome is the one that maximizes the total social welfare. However, since the asymmetric outcome is not necessarily an equilibrium in this case, the policy implication is that the policy authority should enforce one merger if the equilibrium is the one without conglomerates, and should forbid one merger if the equilibrium is the one with two conglomerates.

For intermediates values of the market ratio, in equilibrium there is a firm that invests only in the weakest market. In terms of the capital allocation literature, this result could be interpreted as “inefficient”. In our model, the inefficient firm, given its rival’s strategy, finds more profitable to invest in the weakest market because the competition is lessened in that market. However, as stated at the beginning of this paper, the capital allocation literature generally associates this result of inefficiency with agency problems. Following this, one possible extension is to include shareholders and managers with conflicting interests into the model to analyze how the original results change. The idea is that

the shareholders would pursue the maximization of the conglomerate's joint-profit and would decide how to allocate the technology across markets. Nevertheless, they would not know the demand structure in each market where the conglomerate participates, so they would rely on the division managers to obtain that information. On the other hand, the managers would pursue their own interests and only maximize the profit of the division, so the managers could send distorted information to the shareholders, possibly hurting the conglomerate's joint-profit.

It would be interesting to extend the model to research not only merging decisions, but also divestitures. One possibility to study this is by extending the model into a dynamic framework, so that the players can decide in each period whether to merge, to divest, or to do nothing. The dynamic framework would also be useful to study cooperative behavior. Specifically, the players would be able to cooperate in quantities, R&D effort, or in the merger decisions. This might change the situation where all firms merging does not maximize the total producer surplus. Extending the model into a repeated game might need a more complex way to model the R&D, for example, by assuming that the R&D effort accumulates to the next period, but it depreciates at some rate in each period.

Another possible extension is to verify if the main results hold under more general assumptions, for example, general demand functions, asymmetric players, different R&D costs, sequential movements, incomplete information, many finite firms, many finite markets, and so on. These and other extensions are left for future research.

## Appendix

As established in the main text, the existence of the equilibria in the one and two-merger subgames depends on the market size and the value of  $\beta$ , so different scenarios are analyzed. To classify these scenarios, we restate a set of thresholds from the main text and define new ones. These thresholds come from the conditions for the existence of equilibria of the second stage and also from comparing the payoffs of the merger game. First, for the previously defined thresholds, we set  $\theta_1 = \hat{\theta}_A$ ,  $\theta_2 = \theta_A$ ,  $\theta_4 = \bar{\theta}$ ,  $\theta_5 = \hat{\theta}_B$ , and  $\theta_6 = \theta_B$ . Second, we define the following new thresholds:

$$\theta_3 = \begin{cases} \frac{\sqrt{7}(9-8\beta^2)(9-4\beta^2)}{12\beta\sqrt{81-180\beta^2+128\beta^4-32\beta^6}} & \text{if } \beta < \frac{\sqrt{3}}{2} \\ \frac{\sqrt{7}(9-4\beta^2)}{3\sqrt{2(9-8\beta^2)}} & \text{if } \beta \geq \frac{\sqrt{3}}{2} \end{cases}, \quad \theta_7 = \begin{cases} \frac{3\sqrt{23}(9-8\beta^2)}{20\beta\sqrt{9-10\beta^2}} & \text{if } \beta < \frac{\sqrt{3}}{2} \\ \frac{3\sqrt{23}}{5\sqrt{2}} & \text{if } \beta \geq \frac{\sqrt{3}}{2} \end{cases}$$

It follows that  $\theta_1 > \theta_2$  for any  $\beta \in (0, \sqrt{3}/2)$ ,  $\theta_2 > \theta_3 > \theta_4 > \theta_5 > \theta_6$  for any  $\beta$  and  $\theta_7 > \theta_5$  for any  $\beta$ . Regarding the thresholds originated from comparing the profits of the merger game, it follows that  $\pi^B \leq \pi^{\diamond A}$  iff  $\frac{D_A}{D_B} \geq \theta_5$ ,  $\pi^{\diamond B} \leq \pi^A$  iff  $\frac{D_A}{D_B} \geq \theta_3$ ,  $\pi^{\diamond A} \geq \pi^{\diamond B}$  iff  $\frac{D_A}{D_B} \geq \theta_4$  and  $\pi^B \geq \pi^{\diamond B}$  iff  $\frac{D_A}{D_B} \geq \theta_7$ .

**Scenario 1:**  $\frac{D_A}{D_B} > \theta_1$  for any  $\beta \in (0, \sqrt{3}/2)$ . For the two-merger subgame, the equilibrium is  $TMA$ , and for the one-merger subgame is  $OM\nabla$ . Here it holds that  $\pi^B < \pi^{\diamond A}$  and  $\pi^{\diamond B} < \pi^A$ . If player 2 plays  $(DM)$ , the best strategy for player 1 is to play  $(M)$ . If player 2 plays  $(M)$ , the best strategy for player 1 is to play  $(M)$ . Then, the dominant strategy for player 1 is  $(M)$ . Since the payoffs are symmetric, the dominant strategy for player 2 is also  $(M)$ . Thus, the unique equilibrium in the merger game is the profile  $(M, M)$ .

Therefore, when  $\frac{D_A}{D_B} > \theta_1$  and  $\beta \in (0, \sqrt{3}/2)$ , the equilibrium of the merger game corresponds to the A-outcome. Moreover, in that equilibrium two conglomerates are formed.

**Scenario 2:**  $\theta_1 \geq \frac{D_A}{D_B} \geq \theta_2$  for any  $\beta \in (0, \sqrt{3}/2)$ . For the two-merger subgame, both  $TMA$  and  $TM\nabla$  exist as an equilibrium, and for the one-merger subgame,  $OM\nabla$  is the equilibrium. Here it holds that  $\pi^B < \pi^{\diamond A}$ ,  $\pi^{\diamond B} < \pi^A$  and  $\pi^{\diamond B} < \pi^{\diamond A}$ .

**Scenario 2.1:** The equilibrium is  $TMA$ . This scenario is analogous to Scenario 1. Hence, the equilibrium in the merger game is the profile  $(M, M)$ .

**Scenario 2.2:** The equilibrium is  $TM\nabla$  and player 1's payoff in the profile  $(M, M)$  is  $\pi^{\diamond A}$ . If any player plays  $(DM)$ , the best strategy for the other player is to play  $(M)$ . If player 2 plays  $(M)$ , the best strategy for player 1 is to play  $(M)$ . When player 1 plays  $(M)$ , player 2 is indifferent between  $(DM)$  and  $(M)$ . Thus, the equilibria in the

merger game are the profiles  $(M, DM)$  and  $(M, M)$ . With Assumption M, only the profile  $(M, DM)$  is an equilibrium.

**Scenario 2.3:** The equilibrium is  $TM\nabla$  and player 1's payoff in the profile  $(M, M)$  is  $\pi^{\diamond B}$ . By symmetry with the Scenario 2.2, the equilibria in the merger game are the profiles  $(DM, M)$  and  $(M, M)$ . With Assumption M, only the profile  $(DM, M)$  is an equilibrium.

Therefore, when  $\theta_1 \geq \frac{D_A}{D_B} \geq \theta_2$  and  $\beta \in (0, \sqrt{3}/2)$ , if the equilibrium played in the two-merger subgame is  $TMA$ , the equilibrium of the merger game corresponds to the A-outcome. Moreover, in that equilibrium two conglomerates are formed.

If the equilibrium played in the two-merger subgame is  $TM\nabla$ , then any equilibrium of the merger game corresponds to the asymmetric outcome. Adding Assumption M, in any equilibrium of the merger game always occurs the outcome with one conglomerate

**Scenario 3:**  $\frac{D_A}{D_B} \geq \theta_2$  for any  $\beta \in [\sqrt{3}/2, 1)$ . For the two-merger subgame, the equilibria are  $TMA$  and  $TM\Delta$ , and for the one-merger subgame is  $OM\Delta$ . Here it holds that  $\pi^B < \pi^{\diamond A}$ ,  $\pi^{\diamond B} < \pi^A$  and  $\pi^{\diamond B} < \pi^{\diamond A}$ .

**Scenario 3.1:** The equilibrium is  $TMA$ . This scenario is analogous to Scenario 1 and Scenario 2.1. Thus, the equilibrium in the merger game is the profile  $(M, M)$ .

**Scenario 3.2:** The equilibrium is  $TM\Delta$  and player 1's payoff in the profile  $(M, M)$  is  $\pi^{\diamond A}$ . This scenario is analogous to Scenario 2.2. Thus, the profiles  $(M, DM)$  and  $(M, M)$  are the equilibria of the merger game. With Assumption M, only the profile  $(M, DM)$  is an equilibrium.

**Scenario 3.3:** The equilibrium is  $TM\Delta$  and player 1's payoff in the profile  $(M, M)$  is  $\pi^{\diamond B}$ . This scenario is analogous to Scenario 2.2. Thus, the profiles  $(DM, M)$  and  $(M, M)$  are the equilibria of the merger game. With Assumption M, only the profile  $(DM, M)$  is an equilibrium.

Therefore, when  $\frac{D_A}{D_B} \geq \theta_2$  and  $\beta \in [\sqrt{3}/2, 1)$ , if the equilibrium played in the two-merger subgame is  $TMA$ , the equilibrium of the merger game corresponds to the A-outcome. Moreover, in that equilibrium two conglomerates are formed.

If the equilibrium played in the two-merger subgame is  $TM\Delta$ , then any equilibrium of the merger game corresponds to the asymmetric outcome. Adding Assumption M, in any equilibrium of the merger game always occurs the outcome with one conglomerate.

**Scenario 4:**  $\theta_2 > \frac{D_A}{D_B} > \theta_4$  for any  $\beta$ . For the two-merger subgame, the equilibrium is  $TM\Diamond$ , and for the one-merger subgame is  $OM\Diamond$ . Here it holds that  $\pi^B < \pi^{\Diamond A}$  and  $\pi^{\Diamond B} < \pi^{\Diamond A}$ .

**Scenario 4.1:** Player 1's payoff in the profile  $(M, M)$  is  $\pi^{\Diamond A}$ . This scenario is analogous to Scenario 2.2 and Scenario 3.2. Hence, the equilibria in the merger game are the profiles  $(M, DM)$  and  $(M, M)$ . With Assumption M, only the profile  $(M, DM)$  is an equilibrium.

**Scenario 4.2:** Player 1's payoff in the profile  $(M, M)$  is  $\pi^{\Diamond B}$ . This scenario is analogous to Scenario 2.3 and Scenario 3.3. Hence, the equilibria in the merger game are the profiles  $(DM, M)$  and  $(M, M)$ . With Assumption M, only the profile  $(DM, M)$  is an equilibrium.

**Scenario 5:**  $\frac{D_A}{D_B} = \theta_4$  for any  $\beta$ . For the two-merger subgame, the equilibrium is  $TM\Diamond$ , and for the one-merger subgame is  $OM\Diamond$ . Here it holds that  $\pi^{\Diamond B} = \pi^{\Diamond A} > \pi^B$ . Thus, the payoffs in the profile  $(M, M)$  are symmetric. If any player plays  $(DM)$ , the best strategy for the other player is to play  $(M)$ . The profiles  $(M, M)$ ,  $(M, DM)$  and  $(DM, M)$  have the same symmetric payoffs. Thus,  $(M, M)$ ,  $(M, DM)$  and  $(DM, M)$  are the equilibria of the merger game. With Assumption M, only the profiles  $(M, DM)$  and  $(DM, M)$  are equilibria.

**Scenario 6:**  $\theta_4 > \frac{D_A}{D_B} > \theta_5$  for any  $\beta$ . For the two-merger subgame, the equilibrium is  $TM\Diamond$ , and for the one-merger subgame is  $OM\Diamond$ . Here it holds that  $\pi^B < \pi^{\Diamond A}$  and  $\pi^{\Diamond B} > \pi^{\Diamond A}$ .

**Scenario 6.1:** Player 1's payoff in the profile  $(M, M)$  is  $\pi^{\diamond A}$ . If any player plays  $(DM)$ , the best strategy for the other player is to play  $(M)$ . If player 2 plays  $(M)$ , the best strategy for player 1 is to play  $(DM)$ . When player 1 plays  $(M)$ , player 2 is indifferent between  $(DM)$  and  $(M)$ . Therefore, there are two equilibria in the merger game, the profiles  $(M, DM)$  and  $(DM, M)$ .

**Scenario 6.2:** Player 1's payoff in the profile  $(M, M)$  is  $\pi^{\diamond B}$ . By symmetry with the Scenario 6.1, the equilibria in the merger game are the profiles  $(M, DM)$  and  $(DM, M)$ .

Therefore, when  $\theta_2 > \frac{D_A}{D_B} > \theta_5$  for any  $\beta$ , any equilibrium of the merger game corresponds to the asymmetric outcome. Moreover, with Assumption M, in any equilibrium of the merger game always occurs the outcome with one conglomerate.

**Scenario 7:**  $\frac{D_A}{D_B} = \theta_5$  for any  $\beta$ . For the two-merger subgame, the equilibrium is  $TM\Diamond$ , and for the one-merger subgame is  $OM\Diamond$ . Here it holds that  $\pi^{\diamond B} > \pi^{\diamond A} = \pi^B$ .

**Scenario 7.1:** Player 1's payoff in the profile  $(M, M)$  is  $\pi^{\diamond A}$ . If player 2 plays  $(M)$ , the best strategy for player 1 is to play  $(DM)$ . If player 1 plays  $(M)$ , player 2 is indifferent between  $(DM)$  and  $(M)$ . If any player plays  $(DM)$ , the other player is indifferent between  $(DM)$  and  $(M)$ . Thus, the equilibria of the merger game are the profiles  $(DM, DM)$ ,  $(M, DM)$  and  $(DM, M)$ . With assumption M, the only equilibrium is the profile  $(DM, DM)$ .

**Scenario 7.2:** Player 1's payoff in the profile  $(M, M)$  is  $\pi^{\diamond B}$ . By symmetry with the Scenario 7.1, the equilibria of the merger game are the profiles  $(DM, DM)$ ,  $(M, DM)$  and  $(DM, M)$ . With assumption M, only the profile  $(DM, DM)$  is an equilibrium.

Therefore, when  $\frac{D_A}{D_B} = \theta_5$  for any  $\beta$ , one of the equilibria is the profile  $(DM, DM)$ , which corresponds to the B-outcome. Any other equilibrium corresponds to the asymmetric outcome. Moreover, with Assumption M, in any equilibrium of the merger game always occurs the outcome without conglomerates.

**Scenario 8:**  $\theta_5 > \frac{D_A}{D_B} > \theta_6$  for any  $\beta$ . For the two-merger subgame, the equilibrium is  $TM\Diamond$ , and for the one-merger subgame is  $OM\Diamond$ . Here it holds that  $\pi^{\diamond B} > \pi^B > \pi^{\diamond A}$ .



**Scenario 8.1:** Player 1's payoff in the profile  $(M, M)$  is  $\pi^{\diamond A}$ . If player 2 plays  $(M)$ , the best strategy for player 1 is to play  $(DM)$ . If any player plays  $(DM)$ , the best strategy for the other player is to play  $(DM)$ . Therefore, the equilibrium in the merger game is the profile  $(DM, DM)$ .

**Scenario 8.2:** Player 1's payoff in the profile  $(M, M)$  is  $\pi^{\diamond B}$ . By symmetry with the Scenario 8.1, the equilibrium of the merger game is the profile  $(DM, DM)$ .

**Scenario 9:**  $\frac{D_A}{D_B} \leq \theta_6$  for any  $\beta$ . For the two-merger subgame, the equilibria are  $TMB$  and  $TM\diamond$ , and for the one-merger subgame are  $OMB$  and  $OM\diamond$ . Here it holds that  $\pi^{\diamond B} > \pi^B > \pi^{\diamond A}$ .

**Scenario 9.1:**  $OM\diamond$  is the equilibrium in the profiles  $(DM, M)$  and  $(M, DM)$ ,  $TM\diamond$  is the equilibrium in the profile  $(M, M)$  and player 1's payoff in the profile  $(M, M)$  is  $\pi^{\diamond A}$ . This scenario is analogous to Scenario 8.1. Hence, the equilibrium of the merger game is the profile  $(DM, DM)$ .

**Scenario 9.2:**  $OM\diamond$  is the equilibrium in the profiles  $(DM, M)$  and  $(M, DM)$ ,  $TM\diamond$  is the equilibrium in the profile  $(M, M)$  and player 1's payoff in the profile  $(M, M)$  is  $\pi^{\diamond B}$ . This scenario is analogous to Scenario 8.2. Hence, the equilibrium of the merger game is the profile  $(DM, DM)$ .

**Scenario 9.3:**  $OMB$  is the equilibrium in the profiles  $(DM, M)$  and  $(M, DM)$  and  $TMB$  is the equilibrium in the profile  $(M, M)$ . Both players are indifferent between  $(DM)$  and  $(M)$  regardless of the other player's strategy. Thus, all the profiles are equilibria. With assumption M, only the profile  $(DM, DM)$  is an equilibrium.

**Scenario 9.4:**  $OM\diamond$  is the equilibrium in the profiles  $(DM, M)$  and  $(M, DM)$  and  $TMB$  is the equilibrium in the profile  $(M, M)$ . If player 2 plays  $(DM)$ , the best strategy for player 1 is to play  $(DM)$ . If player 2 plays  $(M)$ , the best strategy for player 1 is to play  $(DM)$ . Then, the dominant strategy for player 1 is  $(DM)$ . Since the payoffs are symmetric, the dominant strategy for player 2 is also  $(DM)$ . Thus, the equilibrium in the merger game is the profile  $(DM, DM)$ .

**Scenario 9.5:**  $OMB$  is the equilibrium in the profiles  $(DM, M)$  and  $(M, DM)$ ,  $TM\Diamond$  is the equilibrium in the profile  $(M, M)$  and player 1's payoff in the profile  $(M, M)$  is  $\pi^{\Diamond A}$ . If player 2 plays  $(M)$ , the best strategy for player 1 is to play  $(DM)$ . If player 1 plays  $(M)$ , the best strategy for player 2 is to play  $(M)$ . If any player plays  $(DM)$ , the other player is indifferent between  $(DM)$  and  $(M)$ . Hence, the profiles  $(DM, M)$  and  $(DM, DM)$  are equilibria of the merger game. With Assumption M, only the profile  $(DM, DM)$  is an equilibrium.

**Scenario 9.6:**  $OMB$  is the equilibrium in the profiles  $(DM, M)$  and  $(M, DM)$ ,  $TM\Diamond$  is the equilibrium in the profile  $(M, M)$  and player 1's payoff in the profile  $(M, M)$  is  $\pi^{\Diamond B}$ . By symmetry with the Scenario 9.5, the profiles  $(M, DM)$  and  $(DM, DM)$  are equilibria of the merger game. With Assumption M, only the profile  $(DM, DM)$  is an equilibrium.

**Scenario 9.7:**  $OMB$  is the equilibrium in the profile  $(DM, M)$ ,  $OM\Diamond$  is the equilibrium in the profile  $(M, DM)$  and  $TMB$  is the equilibrium in the profile  $(M, M)$ . If player 1 plays  $(M)$ , the best strategy for player 2 is to play  $(DM)$ . If player 2 plays  $(DM)$ , the best strategy for player 1 is to play  $(DM)$ . Finally, since the profiles  $(DM, DM)$ ,  $(DM, M)$  and  $(M, M)$  have the same symmetric payoffs, then the equilibria of the merger game are the profiles  $(DM, M)$  and  $(DM, DM)$ . With Assumption M, only the profile  $(DM, DM)$  is an equilibrium.

**Scenario 9.8:**  $OM\Diamond$  is the equilibrium in the profile  $(DM, M)$ ,  $OMB$  is the equilibrium in the profile  $(M, DM)$  and  $TMB$  is the equilibrium in the profile  $(M, M)$ . By symmetry with the Scenario 9.7, the profiles  $(M, DM)$  and  $(DM, DM)$  are equilibria of the merger game. With Assumption M, only the profile  $(DM, DM)$  is an equilibrium.

**Scenario 9.9:**  $OMB$  is the equilibrium in the profile  $(DM, M)$ ,  $OM\Diamond$  is the equilibrium in the profile  $(M, DM)$ ,  $TM\Diamond$  is the equilibrium in the profile  $(M, M)$  and player 1's payoff in the profile  $(M, M)$  is  $\pi^{\Diamond A}$ . If player 1 plays  $(DM)$ , player 2 is indifferent between  $(DM)$  and  $(M)$ . The dominant strategy for player 1 is  $(DM)$ . Thus, the equilibria of the merger game are the profiles  $(DM, M)$  and  $(DM, DM)$ . With Assumption M, only the profile  $(DM, DM)$  is an equilibrium.

**Scenario 9.10:**  $OM\Diamond$  is the equilibrium in the profile  $(DM, M)$ ,  $OMB$  is the equilibrium in the profile  $(M, DM)$ ,  $TM\Diamond$  is the equilibrium in the profile  $(M, M)$  and player 1's payoff in the profile  $(M, M)$  is  $\pi^{\Diamond B}$ . By symmetry with the Scenario 9.9, the profiles  $(M, DM)$  and  $(DM, DM)$  are equilibria of the merger game. With Assumption M, only the profile  $(DM, DM)$  is an equilibrium.

**Scenario 9.11:**  $OM\Diamond$  is the equilibrium in the profile  $(DM, M)$ ,  $OMB$  is the equilibrium in the profile  $(M, DM)$ ,  $TM\Diamond$  is the equilibrium in the profile  $(M, M)$  and player 1's payoff in the profile  $(M, M)$  is  $\pi^{\Diamond A}$ . If player 1 plays  $(DM)$ , the best strategy for player 2 is to play  $(DM)$ . If player 1 plays  $(M)$ , the best strategy for player 2 is to play  $(M)$ . If player 2 plays  $(M)$ , the best strategy for player 1 is to play  $(DM)$ . Finally, when player 2 plays  $(DM)$ , player 1 is indifferent between  $(DM)$  and  $(M)$ . Thus, the only equilibrium of the merger game is the profile  $(DM, DM)$ .

**Scenario 9.12:**  $OMB$  is the equilibrium in the profile  $(DM, M)$ ,  $OM\Diamond$  is the equilibrium in the profile  $(M, DM)$ ,  $TM\Diamond$  is the equilibrium in the profile  $(M, M)$  and player 1's payoff in the profile  $(M, M)$  is  $\pi^{\Diamond B}$ . By symmetry with the Scenario 9.11, the only equilibrium of the merger game is the profile  $(DM, DM)$ .

Therefore, when  $\frac{D_A}{D_B} \leq \theta_5$  for any  $\beta$ , any equilibrium of the merger game corresponds to the B-outcome. Moreover, with Assumption M, in any equilibrium of the merger game always occurs the outcome without conglomerates.

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