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"Can child allowances improve fertility in a gender discrimination economy?"

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Abstract

This paper presents the effects of child allowances on fertility, female labor supply, and economic growth in a gender wage discrimination economy. Child allowances cannot increase fertility in a higher gender discrimination economy. Both theoretical and empirical analyses prove this result. We find that child allowances can increase maternal childcare time. However, the expenditures on market childcare goods and services cannot increase with the decrease of female labor supply and total household income in a higher gender discrimination economy. When both the childcare time and market childcare goods and services are necessary inputs in the parental child care, an increase in child allowances can decrease fertility and per capita output. Moreover, in both the labor market and household, gender equality is critical for encouraging children-bearing. Child allowances can also increase fertility when males actively participate in child care.

Keywords: Child allowances, Gender Wage Discrimination, Female Time Allocation, Fertility, Economic Growth.

JEL: E62, H31.

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1. Introduction

Most of developed countries implement generous child allowances payment, aiming to improve birth rates and female labor supply. However, the consequences of child allowances vary across developed countries. Empirically, Haan and Wrohlich (2011) draw on data from an annual representative sample of over 11,000 households living in Germany and show that higher child care subsidies increase the labor supply of all women as well as the fertility rates of the childless and highly educated women. However, they do not find significant effects on fertility on average. Adda et al. (2017) concentrate on women born in West Germany between 1955 and 1975 and show that the long-run effects of policies that encourage fertility are considerably smaller than short-run effects. Walker (1995) uses Sweden official statistics during 1955–1990, and finds that parental benefits, public childcare availability, and child allowances have reduced the price of fertility since the early 1970s and thus, had a pronatalist effect. However, these effects were small compared to the larger adverse effects of the rise in female wages and return to human capital. Björklund (2006) shows that the expansion of family benefits in Sweden raised the level of fertility and lead to fertility rates fluctuations from the mid-1960s to around 1980. Gauthier (2007) summarizes the literature regarding the impact of cash benefits on fertility in developed countries and shows that small positive policy effects on fertility are found in numerous studies, while insignificant effects are found in others.

Why do the effects of childcare support policies vary in different developed countries and over different periods? This study introduces the gender wage discrimination in the production sector and parental childcare production in the household decision-making into an overlapping generation model and shows that the effects of child allowances on fertility, female labor supply, and per capita output depend on the extent of gender wage discrimination. In a certain gender wage discrimination society, an increase in child allowances can reduce fertility, female labor supply, and per capita output. This model provides theoretical analysis to explain the smaller effects of childcare support policies on fertility and female labor supply in some developed countries. This theory suggests that policies that promote gender equality could have more important impacts on fertility and female labor supply.

A large strand of literature finds a positive correlation between female labor supply and the total fertility rates in developed countries (Ahn and Mira (2002); Rindfuss et al. (2003)). How to explain this positive relationship? Hwang et al. (2018) point out that the substitutability between female childcare time and market childcare can raise both female labor participation rates and total fertility rates. Apps and Rees (2004) suggest that countries with individual taxation and childcare facilities have higher female labor supply and fertility rates. Kemnitz and Thum (2015) and Yakita (2018) indicate that when the price of external child care is lower at high female wage rates, fertility rates can increase as the female wage rates rise. This paper proves that the child allowances policy in a higher gender discrimination economy can lower both fertility and female labor supply.

Even in the most developed countries, women still face severe discrimination in many areas of life, including labor markets (Doepke et al. (2012)). Different from the model of Galor and Weil (1996), which characterizes the gender wage gap from different physical powers between male and female labors, this paper shows that the subjective gender wage discrimination from labor demand side plays a vital role in the effects of child allowances. Recoules (2011) and Cavalcanti and Tavares (2016) also focus on this subjective gender wage discrimination. Recoules (2011) shows a U-shaped relationship between fertility and gender discrimination by using a static general equilibrium model with endogenous fertility. Cavalcanti and Tavares (2016) set up a growth model that endogenizes saving, fertility, and labor market participation and show that while fertility increases with gender wage discrimination, female hours of work in the market decrease with it. This paper is an extension and application of these two models. The effect of gender wage discrimination on fertility depends on the productivity of parental child care and other parameters. When the production of parental child care mainly depends on the maternal time input, fertility is increasing in gender wage discrimination. By contrast, when the parental child care depends mostly on market childcare goods and services, fertility is decreasing in gender wage discrimination.

Our main contributions are listed as follows. (a) We apply a gender-based overlapping generation model with joint parental childcare production and gender discrimination in the labor market. (b) We find that child allowances can not improve fertility in a higher gender discrimination economy. (c) Higher female wage rates cannot decrease fertility, but more severe gender discrimination can. (d) In our empirical analysis, we estimate the cutoff of the discriminatory factor. When the gender discrimination factor is greater than the cutoff, the effects of child allowances on fertility becomes negative. (e) Finally, we show the importance of male childcare time in increasing fertility rates.

The remainder of this paper proceeds as follows. Section 2 introduces the gender-based overlapping generation model and the definition of competitive equilibrium. In Section 3, we analyze the effects of child allowances with gender discrimination. Section 4 shows the empirical evidence on different effects of child allowances on fertility in different gender discrimination society. Section 5 analyzes the effects of child allowances when the male childcare time cannot be substituted for. Section 6 concludes the paper.

2. The Model

The overlapping generations (OLG) model is one of the dominating frameworks of analysis in the study of macroeconomic dynamics and economic growth since Samuelson (1958) and

Diamond (1965). In the OLG model individuals live a finite length of time, long enough to overlap with at least one period of another agent's life. OLG model is also a kind of dynamic general equilibrium models. Compared with comparative statics models, it aims to trace and study the movement of variables across time, and to determine whether these variables tend to move towards equilibrium.

This OLG model in this paper is a natural framework for studing: (a) the life-cycle behavior (consumption, labor supply, and saving for retirement), (b) the implications of resource allocation across generations, such as child allowances on fertility, savings, and per capita income in the long-run, and (c) factors that trigger the fertility transition. In a gender-based OLG model (e.g., Galor and Weil (1996), Zhang et al. (1999), Momota and Futagami (2000), Greenwood et al. (2005), Kimura and Yasui (2010)), our research emphasizes the importance of gender equality in the labor market in increasing fertility and female labor supply in a long term in different developed countries.

Consider an OLG economy where only one homogeneous good is produced, and each agent lives in at most three periods: childhood, adulthood, and retirement (or old age) in a discrete-time framework. They are endowed with one unit of time in childhood and adulthood, and zero units when retired. In adulthood, all individuals work and match up randomly into couples with someone of the opposite sex to form a family, and then these couples become joint decision makers. For simplicity, once married, each will not divorce; couples will retire and die together.

Both females and males must consider two alternatives: market work and raising children. $l_t \in [0,1]$ denotes the female work time, and $h_t \in [0,1]$ the male work time. $\varepsilon_t \in [0,1]$ and $\chi_t \in [0,1]$ are the time spent on maternal and paternal child care, respectively. Thus, the time constraint condition is $l_t + \varepsilon_t = 1$ for females, and $h_t + \chi_t = 1$ for males. The production of child care involves the parental consumption on market childcare goods and services in addition to the time that the wife and husband allocate to parental child care.

In addition to individuals, there exist a continuum of firms and an infinitely lived government. Firms produce the homogeneous good, using male and female labor and capital as inputs. The government only taxes the wage income of individuals in adulthood.

2.1. Firms

We assume a continuum of firms, indexed by $i \in (0,1)$. All firms produce a homogeneous good, which is also a consumer good and physical capital good. Due to a continuum of firms, the production function is

$$Y_t = \int_0^1 Y_t^i \, di = \left(\ell_t L_t^f\right)^{\beta} (h_t L_t^m)^{\beta} K_t^{1-2\beta}, \tag{1}$$

where, the L_t^f and L_t^m are the quantities of female and male labor force in period t. We assume that $L_t^f = L_t^m$ and the population size is $L_t = L_t^f + L_t^m$. The capital input is K_t. The growth rate of labor force is equal to the inter-generational growth rate of the population size, n_t:

$$\frac{L_{t+1}}{L_t} = n_t. \tag{2}$$

We introduce the gender wage discrimination according to the taste model in Becker (1971). This "taste" refers to preference against hiring a group such as women or minorities. The firms with a taste for discrimination against female labors are unwilling to hire female labors. In our model, there is a barrier for women to participate in the labor market. This barrier derives from gender discrimination. The employers or managers in the firms consider both the profits and gender discrimination (preference) on employment. The profit function is

$$\pi_t = Y_t - R_t K_t - w_{f,t}\ell_t L_t^f - w_{m,t} h_t L_t^m.$$

Here, R_t is the rental price of capital in period t, and with full depreciation, $R_t = 1 + r_t$, where r_t is the interest rate. And each effective labor earns $w_{f,t}$ units effective wage for women and $w_{m,t}$ units for men. The firms' or managers' discrimination (disutility) of hiring female labors is $(\delta - 1) w_{f,t}\ell_t L_t^f$, where $\delta > 1$. A higher δ represents a more gender discrimination society. Finally, the firms' or managers' utility is

$$\pi_t - (\delta - 1) w_{f,t}\ell_t L_t^f = Y_t - R_t K_t - \delta w_{f,t}\ell_t L_t^f - w_{m,t} h_t L_t^m. \tag{3}$$

From the first order condition of the managers' utility maximization problem, it is satisfied that

$$R_t = (1 - 2\beta)\left(\ell_t h_t\right)^{\beta} k_t^{-2\beta}, \tag{4}$$

$$w_{f,t} = \frac{1}{\delta}\beta \ell_t^{\beta-1} h_t^{\beta} k_t^{1-2\beta}, \tag{5}$$

$$w_{m,t} = \beta \ell_t^{\beta} h_t^{\beta-1} k_t^{1-2\beta}, \tag{6}$$

where $k_t \equiv K_t / L_t^f$. And $w_{m,t} = \delta \frac{\ell_t}{h_t} w_{f,t}$. We assume $w_{m,t} > w_{f,t}$. The total income of the representative household is $w_t = (1 + \delta \cdot \ell_t) \cdot w_{f,t}$.

2.2. Households

2.2.1. The Production of Parental Child Care

In period t, each couple hass n_t children on average. We assume that the production of child care needs the homogeneous consumption good $c_{R,t}$, the female childcare time ε_t and the male childcare time χ_t. The parental childcare production function is $n_t = (\varepsilon_t + \chi_t)^{\alpha} c_{R,t}^{1-\alpha}$. α is the productivity of parental child care efforts. The childcare cost minimization problem is

$$\min_{\varepsilon_t, \chi_t, c_{R,t}} \quad w_{f,t}\varepsilon_t + w_{m,t}\chi_t + c_{R,t},$$

$$\text{subject to } n_t = (\varepsilon_t + \chi_t)^{\alpha} c_{R,t}^{1-\alpha}.$$

In this problem, because we have $w_{f,t} < w_{m,t}$ in the firms, the above problem is equivalent to

$$\min_{\varepsilon_t, \chi_t, c_{R,t}} \quad w_{f,t}\varepsilon_t + c_{R,t},$$

$$\text{subject to } n_t = \varepsilon_t^{\alpha} c_{R,t}^{1-\alpha},$$

where $h_t = 1$ and $\chi_t = 0$, indicating that the husband allocates all his time to the market work, i.e., the intra-household division of labor and child care. Therefore,

$$\min_{c_{R,t}, \varepsilon_t} \qquad w_{f,t}\, \varepsilon_t + c_{R,t} = A w_{f,t}^{\alpha}\, n_t,$$

where $A \equiv \frac{(1-\alpha)^{\alpha-1}}{\alpha^{\alpha}}$ and $\varepsilon_t = \left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha} w_{f,t}^{\alpha-1}\, n_t$. Also, according to the female time constraint, we have

$$\ell_t = 1 - \left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha} w_{f,t}^{\alpha-1}\, n_t \tag{7}$$

2.2.2. Utility

At the beginning of adulthood in period t, each household produces n_t children on average. The wife's or husband's utility is

$$u^i = \ln c_t^i + \eta \ln n_t + \rho \ln c_{t+1}^i, \;\; i = f, m.$$

c_t^f and c_t^f are wife's and husband's consumption at time t. η is the preference for children, and ρ is the discount factor. And the household welfare function is

$$H = \theta u^f + (1-\theta) u^m,$$

where θ and $1-\theta$ are the gender bargaining power of the wife and the husband, respectively. The gender wage gap δ can affect the intra-household gender bargaining power, and the gender bargaining also impacts the household's decision-making. When the wife's and the husband's preference for children are equal, the gender bargaining power does not affect the demand for children. Because children are a kind of public goods in the household, and this public goods production (parental child care) depends on the parental child care technology, here we have:

$$H = \theta \ln c_t^f + \theta\rho \ln c_{t+1}^f + (1-\theta)\ln c_t^m + (1-\theta)\rho \ln c_{t+1}^m + \eta \ln n_t. \tag{8}$$

In Iyigun and Walsh (2007), there is a similar utility function.

When the wife's and the husband's preference for children are not equal as in Komura (2013a,b), the gender bargaining can affect fertility. However, even in the comparative statics analyses (Iyigun and Walsh (2007); Komura (2013a,b)), the discussion on endogenous gender bargaining and fertility is very complicated. Intuitively, based on Komura (2013b), the wife's bargaining power can increase both female labor supply and fertility. Moreover, the gender wage equality can strengthen the wife's gender bargaining power. Finally, both female labor supply and fertility are increasing in gender wage equality. In this research, we focus on the welfare policy and fertility, and the effects of gender wage discrimination on endogenous gender bargaining and fertility are more complicated in this dynamic general equilibrium model. Therefore we assume the wife and husband have equal preference for children.

The household's budget constraints for the period t and $t+1$ are given by

$$c_t^f + c_t^m + s_t + Aw_{f,t}^{\alpha} n_t = (1-\tau)w_t + gn_t, \tag{9}$$

and

$$c_{t+1}^f + c_{t+1}^m = (1+r_{t+1})s_t, \tag{10}$$

where s_t is the family saving, τ is the income tax rate, w_t is the effective wage income of the couple in period t, g is the child allowances provided by the government, and r_{t+1} is the interest rate. The optimization problem of the household welfare is

$$\max_{n_t, c_t^f, c_{t+1}^f, c_t^m, c_{t+1}^m} \quad H = \eta \ln n_t + \theta \ln c_t^f + \theta\rho \ln c_{t+1}^f + (1-\theta)\ln c_t^m + (1-\theta)\rho \ln c_{t+1}^m,$$

$$\text{subject to} \quad c_t^f + c_t^m + s_t + Aw_{f,t}^{\alpha} n_t = (1-\tau)w_t + gn_t$$

and

$$c_{t+1}^f + c_{t+1}^m = (1+r_{t+1})s_t.$$

The Lagrange equation is thus

$$\mathcal{L} = \eta \ln n_t + \theta \ln c_t^f + \theta\rho \ln c_{t+1}^f + (1-\theta)\ln c_t^m + (1-\theta)\rho \ln c_{t+1}^m +$$

$$\lambda \left[(1-\tau)w_t - c_t^f - c_t^m - \frac{1}{1+r_{t+1}}\left(c_{t+1}^f + c_{t+1}^m\right) - \left(Aw_{f,t}^{\alpha} - g\right) n_t \right].$$

By maximizing the objective function subject to the budget constraints, the consumption in adulthood, the fertility (the quantity of children in each household), and the saving are solved:

$$c_t^f = \frac{\theta}{1+\rho+\eta}(1-\tau)w_t, \quad c_t^m = \frac{1-\theta}{1+\rho+\eta}(1-\tau)w_t, \tag{11}$$

$$n_t = \frac{\eta}{1+\rho+\eta}\frac{(1-\tau)w_t}{Aw_{f,t}^{\alpha} - g}, \tag{12}$$

and

$$s_t = \frac{\rho}{1+\rho+\eta}(1-\tau)w_t. \tag{13}$$

2.3. Government

Since the government taxes the wage income of individuals and spends the tax money on child allowances and other items, we have:

$$n_t\, g = \nu\, \tau\, (1 + \delta\, \ell_t)\, w_t^f. \tag{14}$$

where $\nu \in (0, 1)$ is the expenditures on child allowances as a percentage of government revenues, and is assumed to be constant.

2.4. Savings-Investment Balance

Assuming capital K_t depreciates exponentially at the rate Δ, the law of motion of the capital stock is

$$K_{t+1} = (1 - \Delta)K_t + I_t,$$

where I_t is the investment at (discrete) time t. In the closed economy, the aggregate investment is equal to savings:

$$S_t = I_t = Y_t - C_t - G_t,$$

where Y_t, C_t, and G_t are total output, total consumption, and government expenditures at time t,respectively. Each household saves s_t, and the number of the households is L_t^f. Thus,

$$S_t = s_t L_t^f.$$

Finally, with full depreciation rate ($\Delta = 1$), the law of motion of the capital stock is

$$K_{t+1} = s_t L_t^f. \tag{15}$$

Let $k_t \equiv K_t / L_t^f$, and

$$k_{t+1} = s_t / n_t. \tag{16}$$

2.5. Equilibrium Conditions

A competitive equilibrium for this model is a sequence of consumption and saving choices $\{c_t^f, c_{t+1}^f, c_t^m, c_{t+1}^m, s_t\}_{t=0}^{\infty}$, capital stock and labor inputs $\{K_t, \ell_t L_t^f, h_t L_t^m\}_{t=0}^{\infty}$, the couple's time allocation choices $\{\varepsilon_t, \chi_t, \ell_t, h_t\}_{t=0}^{\infty}$, factor prices $\{w_{f,t}, w_{m,t}, r_{t+1}\}_{t=0}^{\infty}$, quantity of children $\{n_t\}_{t=0}^{\infty}$, available varieties, a constant tax rate, and a constant government spending share such that,

a) Individuals maximize utility subject to their inter-temporal budget constraints;

b) Firms maximize profits, choosing labor and capital, with given input prices;

c) The government budget is balanced; and

d) All markets clear.

3. Child allowances

This section analyzes the effects of the child allowances on the female labor supply, fertility, and economic growth with gender wage discrimination in the production sector.

With child allowances, let $\mu \equiv \frac{\eta}{1+\eta+\rho}(1-\tau)+\nu\tau$ and $\gamma \equiv \frac{\eta}{1+\eta+\rho}(1-\tau)$. Since $n_t = \mu(1+\delta\ell_t)\frac{1}{A}w_{f,t}^{1-\alpha}$ and $\varepsilon_t = \left(\frac{1-\alpha}{\alpha}\right)^{\alpha-1} w_{f,t}^{\alpha-1}n_t$, the female time allocation is constant:

$$\ell \equiv \ell_t = \frac{1-\alpha\mu}{1+\alpha\delta\mu} \text{ and } \varepsilon \equiv \varepsilon_t = \frac{\alpha\mu(1+\delta)}{1+\alpha\delta\mu}. \tag{17}$$

Here, we have $\partial l/\partial\alpha = \frac{-\mu(1+\delta)}{(1+\alpha\delta\mu)^2} < 0$, $\partial l/\partial\delta = \frac{-\alpha\mu(1-\alpha\mu)}{(1+\alpha\delta\mu)^2} < 0$, and

$$\frac{\partial\ell}{\partial\nu} = \frac{-\alpha(1+\alpha\delta\mu)-(1-\alpha\mu)\alpha\delta}{(1+\alpha\delta\mu)^2}\tau = \frac{-\alpha(1+\delta)}{(1+\alpha\delta\mu)^2}\tau < 0. \tag{18}$$

It shows that parental childcare productivity, gender wage discrimination, and child allowances can lower the female labor supply. From $k_{t+1} = s_t/n_t$, the capital-labor ratio in the stable steady state is

$$k = k_{t+1} = k_t = \left[\frac{\gamma}{\mu}A\left(\frac{\beta}{\delta}l^{\beta-1}\right)^{\alpha}\right]^{\frac{1}{1-\alpha+2\alpha\beta}}. \tag{19}$$

We also have:

$$\frac{\partial \ln k}{\partial\nu} = \frac{\tau}{1-\alpha+2\alpha\beta}\left[\frac{\alpha^2(1-\beta)(1+\delta)}{(1-\alpha\mu)(1+\alpha\delta\mu)} - \frac{1}{\mu}\right] \gtreqless 0. \tag{20}$$

The effect of child allowances on the capital-labor ratio is ambiguous, which depends on the gender discrimination factor, the parental childcare productivity and other parameters. And according to Eqs. (17) and (20),

$$\varepsilon \gtreqless \frac{1+\alpha\mu\delta}{1+\alpha\mu\delta+\alpha(1-\beta)}(\equiv \varepsilon_k) \quad \Leftrightarrow \quad \frac{\partial \ln k}{\partial\nu} \gtreqless 0. \tag{21}$$

Let $w_f = w_{f,t}$, and the steady-state female wage rate is

$$w_f = \frac{1}{\delta}\beta\ell^{\beta-1}k^{1-2\beta} = \left(A\frac{\gamma}{\mu}\right)^{\frac{1-2\beta}{1-\alpha+2\alpha\beta}}\left(\frac{\beta}{\delta}\,\ell^{\beta-1}\right)^{\frac{1}{1-\alpha+2\alpha\beta}}. \tag{22}$$

Therefore, the effect of the child allowances on female wage rates in the steady state is

$$\frac{\partial \ln w_f}{\partial\nu} = \frac{\tau}{1-\alpha+2\alpha\beta}\left[\frac{\alpha(1-\beta)(1+\delta)}{(1-\alpha\mu)(1+\alpha\delta\mu)} - \frac{1-2\beta}{\mu}\right] \gtreqless 0, \tag{23}$$

which is also ambiguous. And

$$\varepsilon \gtreqless \frac{1+\alpha\mu\delta}{1+\alpha\mu\delta+\frac{1-\beta}{1-2\beta}}(\equiv \varepsilon_w) \quad \Leftrightarrow \quad \frac{\partial \ln w_f}{\partial\nu} \gtreqless 0. \tag{24}$$

Each household's saving in the steady state is $s = s_t = \gamma(1+\delta l)w_f$. And

$$\varepsilon \gtreqless \frac{1+\alpha\mu\delta+\alpha\delta\mu\frac{1-\alpha+2\alpha\beta}{1-2\beta}}{1+\alpha\mu\delta+\frac{1-\beta}{1-2\beta}+\alpha\delta\mu\frac{1-\alpha+2\alpha\beta}{1-2\beta}}(\equiv \varepsilon_s) \quad \Leftrightarrow \quad \frac{\partial \ln s}{\partial\nu} \gtreqless 0. \tag{25}$$

The steady-state per capita output is $y = l^{\beta}k^{1-\beta}$. When $\alpha(1-\beta) - \frac{\beta(1-\alpha+2\alpha\beta)}{1-2\beta} < 0$, $\partial \ln y/\partial\nu < 0$. And when $\alpha(1-\beta) - \frac{\beta(1-\alpha+2\alpha\beta)}{1-2\beta} > 0$,

$$\varepsilon \gtreqless \frac{1+\alpha\mu\delta}{1+\alpha\mu\delta+\alpha(1-\beta)-\frac{\beta(1-\alpha+2\alpha\beta)}{1-2\beta}} (\equiv \varepsilon_y) \quad \Leftrightarrow \quad \frac{\partial \ln y}{\partial \nu} \gtreqless 0. \tag{26}$$

Finally, the steady-state fertility is

$$n = n_t = \mu(1+\delta\ell)\frac{1}{A}w_f^{1-\alpha}. \tag{27}$$

Proposition 1 *If* $(1-2\beta)(1-\alpha)(1+\alpha\delta\mu) < 1-\alpha+2\alpha\beta$, $\frac{\partial \ln n}{\partial \nu} > 0$ *is satisfied. And If* $(1-2\beta)(1-\alpha)(1+\alpha\delta\mu) > 1-\alpha+2\alpha\beta$, *when* $\frac{1-\alpha+2\alpha\beta}{(1-\alpha)(1+\alpha\mu\delta)} + \frac{\alpha\mu(1+\delta)(1-\beta)}{(1-\alpha\mu)(1+\alpha\delta\mu)} \gtreqless 1-2\beta$, *we have* $\frac{\partial \ln n}{\partial \nu} \gtreqless 0$.

Here $\frac{1-\alpha+2\alpha\beta}{(1-\alpha)(1+\alpha\mu\delta)} + \frac{\alpha\mu(1+\delta)(1-\beta)}{(1-\alpha\mu)(1+\alpha\delta\mu)} \gtreqless 1-2\beta \Leftrightarrow \varepsilon \gtreqless \frac{1+\alpha\mu\delta-\frac{1-\alpha+2\alpha\beta}{(1-\alpha)(1-2\beta)}}{1+\alpha\mu\delta+\frac{1-\beta}{1-2\beta}-\frac{1-\alpha+2\alpha\beta}{(1-\alpha)(1-2\beta)}} (\equiv \varepsilon_n)$. This proposition also shows that there exists a $\tilde{\delta}$, such that if $\delta < \tilde{\delta}$, the child allowances increase fertility. If $\delta > \tilde{\delta}$, the effect of child allowances is ambiguous, which depends on other parameters in the model. And according to Proposition 1, $\tilde{\delta} = \frac{1-\alpha+2\alpha\beta}{\alpha\mu(1-2\beta)(1-\alpha)} - \frac{1}{\alpha\mu}$.

As characterized in this model, a typical woman allocates her time between childcare and market work. With the number of children appearing in the utility function, the optimal response to the reduction in female labor supply would be to increase the fertility rate. However, in terms of the child care, not only the wife's childcare time but also the market childcare goods and services should be taken into account. When the market childcare goods and services are a necessary input, the fertility is increasing in both the female childcare time and the market childcare goods and services. Both the female childcare time and the market childcare goods and services are necessary conditions for increasing fertility. In response to an increase in child allowances, the female childcare time increases more in a high gender wage discrimination society than in a low one. However, in a high gender discrimination society, the expenditures on the market childcare goods and services can not increase as the female childcare time does.

In some cases, the expenditures on the market childcare goods and services decrease in a high gender discrimination society. Due to the decrease in the female labor time and female wage rates, the total income of the household decreases. Finally, with the increase in child allowances, since the expenditures on market childcare goods and services and the wife's childcare time cannot achieve growth at the same time, the former will even fall and thus fertility will decrease.

To summarize these analytical results, we have $\varepsilon_n < \varepsilon_w < \varepsilon_s < \varepsilon_k < \varepsilon_y$. Table 1 shows the effects of child allowances on fertility, female wage rates, savings, capital-labor ratio, and per capita output in different intervals of female time allocation. Here, "−" denotes an adverse impact, and "+" a positive effect.

Table 1: The effects of the child allowances in different female time allocations.

ε	$[0, \varepsilon_n]$	$[\varepsilon_n, \varepsilon_w]$	$[\varepsilon_w, \varepsilon_s]$	$[\varepsilon_s, \varepsilon_k]$	$[\varepsilon_k, \varepsilon_y]$	$[\varepsilon_y, 1]$
$\partial \ln n / \partial \nu$	−	+	+	+	+	+
$\partial \ln w_f / \partial \nu$	−	−	+	+	+	+
$\partial \ln s / \partial \nu$	−	−	−	+	+	+
$\partial \ln k / \partial \nu$	−	−	−	−	+	+
$\partial \ln y / \partial \nu$	−	−	−	−	−	+

Here, the female labor supply is decreasing in child allowances. With the decrease in female labor supply, the fall in female labor earning also reduces fertility. The child allowances, on the other hand, decrease the cost of child-rearing and increases fertility. When the equilibrium maternal child care time $\varepsilon < \varepsilon_n$, the adverse effect of decreasing female labor earning on fertility will be larger than the positive effect of the decreasing costs of child-rearing on fertility, and fertility is thus decreasing in the child allowances. Under this circumstance, due to the fall in female labor earnings, the saving is also reduced by child allowances, which decreases the capital-labor ratio and further the per capita output.

Corollary 1 *There exists a $\bar{\delta}$, such that, when $\delta > \bar{\delta}$, the fertility with no child allowances is higher than the fertility with any child allowances policy, $\nu \in (0, 1)$. In a certain gender wage discrimination society, the child allowances can decrease fertility.*

Similar results also hold for other variables, as shown in Corollary 1. These results suggest that compared with child allowances, reducing gender wage discrimination is more important for increasing fertility, female labor supply, and economic growth. With very high gender wage discrimination, the child allowances policy may not achieve the desired policy goals. Therefore, in this model, the effects of gender wage discrimination should also be analyzed, which are shown in Proposition 2.

Proposition 2 *The gender discrimination factor δ has adverse effects on the female labor supply, wage rates, capital-labor ratio, and per capita economic growth. The impact of the gender discrimination factor on fertility depends on the value of household childcare productivity α and other parameters. When the production of parental child care mainly depends on the maternal time input, the rise in gender wage discrimination increases fertility; By contrast, when both maternal time inputs and market childcare goods and services play essential roles, the rise in gender wage discrimination reduces fertility.*

Intuitively, gender wage discrimination not only decreases the female labor supply and the female effective wage rates but also lowers the male effective wage rates, and further the families' total wage income, which then decreases the saving rates, the capital-labor ratio and finally the per capita economic growth. In the appendix, we prove these results analytically.

However, with the decrease of the wage income as the childcare opportunity cost, the "price" of each child, $A\left(w_f\right)^{\alpha}$, is also lowered, indicating ambiguous effects on fertility. It depends on how the parental child care productivity affects the impacts of wage discrimination on the expenditures for each child. When $\alpha \to 1$, the marginal effect of the gender wage discrimination factor on the costs of each child is higher. An increase in gender discrimination leads to a larger decrease in the opportunity costs of rearing children. Therefore, fertility will be higher with an increase in gender discrimination.

Table 2: The parameters in the numerical simulation

Parameter	Value	Interpretation
β	0.15	The output elasticity of labor in Cobb-Douglas production function.
η	0.5	The preference for children.
ρ	0.01	The discount factor.
τ	0.1	The income tax rate.
ν	0.03	The spending on child allowances as a percentage of government revenues.
$\ell_t + \varepsilon_t$	10000	Total time at time t.

Each parameter is assigned the specific value in Table 2. Based on Eqs. (17), (22) and (27), we present the relation between gender discrimination factor and fertility with 3 different parental childcare productivity parameters, $\alpha = 0.6$, 0.8, 0.9. Comparing Figure 1, 2, and 3 reveals different effects of gender discrimination on fertility. In Figure 1 where $\alpha = 0.6$, gender wage discrimination reduces fertility; In Figure 3, with an increase in the gender discrimination factor, n becomes larger; By contrast, in Figure 2, there exists a non-linear relationship between the gender discrimination factor and fertility. Figure 3 echoes the result that gender wage gap increases fertility in Galor and Weil (1996). However, if we consider a Cobb-Douglas parental child care production with complementary maternal child care time and market childcare goods and services as inputs, the effects of gender discrimination on the opportunity costs for each child and fertility are determined by the parental childcare productivity.

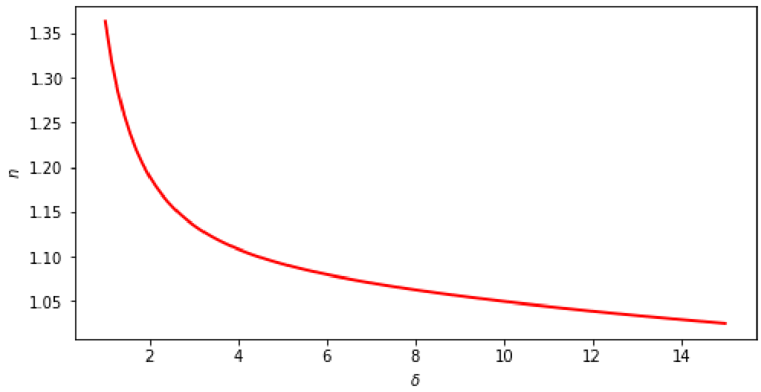

Figure 1: The gender discrimination factor and fertility, $\alpha = 0.6$.

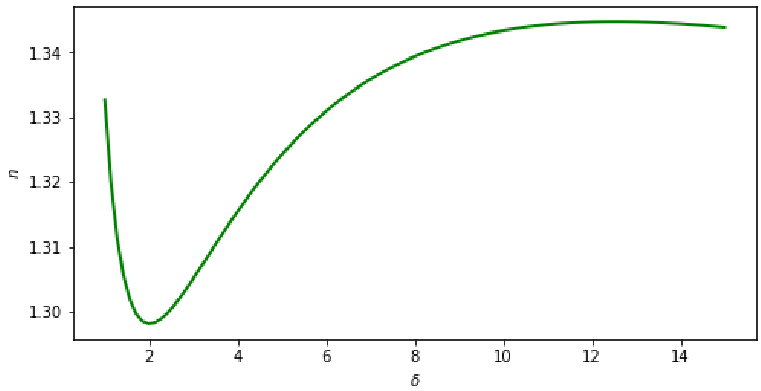

Figure 2: The gender discrimination factor and fertility, $\alpha = 0.8$.

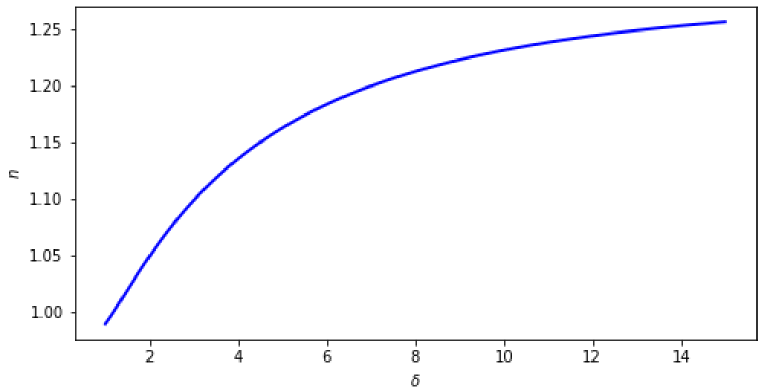

Figure 3: The gender discrimination factor and fertility, $\alpha = 0.9$.

4. Empirical Evidence

To provide some evidence for Corollary 1 and particularly to give a rough estimate of $\bar{\delta}$, we use the data on fertility, family benefits, gender wage gap and other socioeconomic variables of 36 countries (2000–2015) in the OECD Statistics Database, to estimate the following equation:

$$\begin{aligned} ln\ Fert_{c,t} &= \alpha + \gamma ln\ Fert_{c,t-1} + \beta_1 Fam\,Benef_{c,t-1} + \beta_2 Fam\,Benef_{c,t-1} \times Gap_{c,t-1} \\ &+ \beta_3 Fam\,Benefits_{c,t-1} \times Gap^2_{c,t-1} + \boldsymbol{\lambda' W_{c,t-1}} + \theta_c + \mu_{rt} + \varepsilon_{c,t} \end{aligned} \tag{28}$$

$ln\ Fert$, the dependent variable, is the logarithm of total fertility rates. The key variable of interest is $Fam\,Benef$, defined as public spending on family benefits, including financial support that is exclusively for families and children, measured in percentage of GDP. Gap, corresponding to the discriminatory factor in the model, is measured as the difference between median earnings of men and women relative to median earnings of men (for full-time employees).[1] Since our model predicts that the effect of child allowances on fertility depends on the gender wage gap and particularly, in a gender wage discrimination economy, an increase in child allowances can decrease fertility, we interact $Fam\ Benef$ with a quadratic and linear term of Gap to model the nonlinear relationship between family benefits and fertility conditional on gender gap. Therefore, based on the above equation, the impact of child allowances on fertility, $\frac{\partial ln\ Fert}{\partial Fam\,Benef}$, equals $\beta_1 + \beta_2 Gap + \beta_3 Gap^2$. That is to say, if $\beta_3 < 0$, the above expression could be negative when the gender gap is large enough.

$\boldsymbol{W}$ is a large array of control variables, which can potentially impact fertility. See Table 3 for the variables included in $\boldsymbol{W}$ and Table A.1 for their detailed definition. We lag all the variables on the right-hand side of the equation by one year in that there exists a time lag between these socioeconomic variables and fertility. Since fertility is highly autocorrelated, we also include the one-year lagged fertility in the regression. θ_c is the country fixed effects that control for all time-invariant differences between countries and μ_{rt} is a set of continent-year fixed effects, which control for any common shocks experienced across a continent, e.g., probably due to similar cultures or policies. In column (1) of Table 3, we estimate a fixed effects model and in column (2) we further apply the the system GMM approach developed by Blundell and Bond (1998) to deal with the dynamic panel bias in column (1).[2]

In both columns (1) and (2), $\hat{\beta}_3$ is negative and highly significant, indicating a inverse-U relationship between the impact of family benefits on fertility and gender gap. Based on the

[1] Data on the gender wage gap are missing for some county-year observations. To minimize the loss of sample size, we use linear interpolation to impute the missing values.

[2] All the RHS variables except the lagged dependent variable and continent-year fixed effects are treated as endogenous variables. We use variables lagged by two or more periods as instruments for all the endogenous variables in the GMM estimates. The lagged dependent variable is treated as predetermined as usual.

estimate of column (2), we plot the average marginal effect of family benefits on fertility across different values of gender gap and their 95% confidence intervals. As shown in Figure 4, when the gap is larger than 25.5%, the effect of family benefits on fertility becomes significantly negative at the 5% level. However, the effect is indistinguishable from 0 when the gap is less than the cutoff. This provides empirical evidence for Corollary 1 and based on our estimate, the cutoff is around 26%.

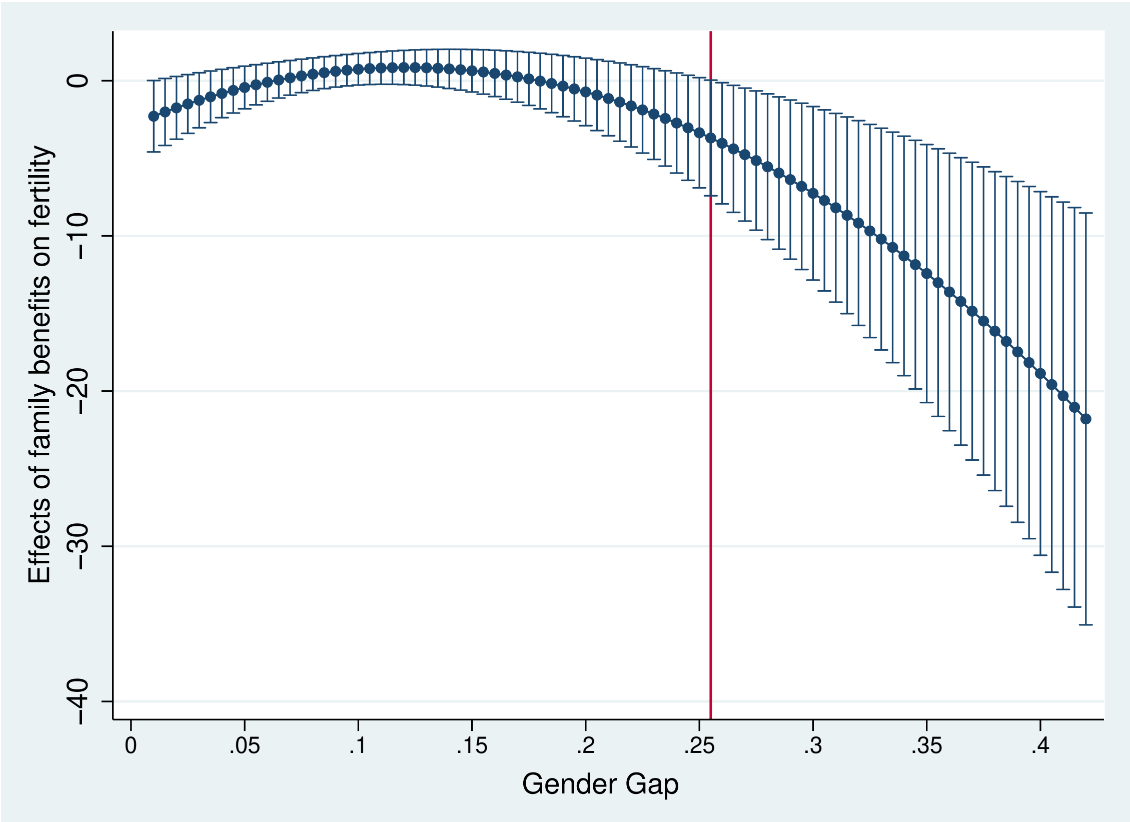

Figure 4: Average Marginal Effects of Family Benefits on Fertility with 95% CIs

Table 3: Family benefits and Fertility.

	(1) FE	(2)GMM
ln Fertility $_{t-1}$	0.801***	0.959***
	(0.038)	(0.043)
Family benefits $_{t-1}$	-3.782	-2.879**
	(2.303)	(1.332)
Family benefits $_{t-1}$ × Gender gap $_{t-1}$	39.050	61.533***
	(26.176)	(18.886)
Family benefits $_{t-1}$ × Gender gap $^2_{t-1}$	-161.275**	-253.738***
	(76.223)	(68.534)
Gender gap $_{t-1}$	-0.870	-1.214**
	(0.792)	(0.506)
Gender gap $^2_{t-1}$	3.397	5.101***
	(2.296)	(1.603)
Female labor participation $_{t-1}$	0.012	0.126**
	(0.051)	(0.052)
Total dependence ratio $_{t-1}$	-0.333***	-0.089
	(0.103)	(0.071)
ln GDP per capita $_{t-1}$	-0.009	0.019
	(0.023)	(0.012)
GDP per capita growth $_{t-1}$	-0.062	-0.103*
	(0.045)	(0.059)
ln Population $_{t-1}$	-0.236***	-0.002
	(0.075)	(0.003)
Household saving rate $_{t-1}$	-0.109	0.023
	(0.132)	(0.060)
Household debt $_{t-1}$	-0.025**	-0.014***
	(0.011)	(0.005)
Country FE	✓	✓
Continent-Year FE	✓	✓
Observations	350	387
Adjusted R^2	0.980	
AR(2) (p-value)		0.584

Note: This table presents the nonlinear relationship between family benefits and fertility conditional on gender gap estimated using the OECD Statistics Database. Column (1) estimates a fixed effect model and column (2) further applies the system GMM approach, in which all the RHS variables except the lagged dependent variable and continent-year fixed effects are treated as endogenous variables. We use variables lagged by two or more periods as instruments for all the endogenous variables in the GMM estimates. The lagged dependent variable is treated as predetermined as usual. ***, ** and * denote significance at 1%, 5% and 10%, respectively.

5. Male time allocation

There is a lot of empirical literature focusing on "daddy-month", paternity leave and the role of fathers in childcare in developed countries (Ekberg et al. (2013); Cools et al. (2015); Yamaguchi (2019); Patnaik (2019)). Since the literature shows the importance and positive role of fathers' efforts as a necessary input in the parental childcare production, we assume that the female and male childcare time are complementary and the quantity of market childcare goods and services is a constant. Thus, the parental childcare production function is $n_t = \varepsilon_t^{\alpha} \chi_t^{1-\alpha}$. The corresponding childcare cost minimization problem is

$$\min_{\varepsilon_t, \chi_t} \quad w_{f,t}\varepsilon_t + w_{m,t}\chi_t,$$

$$\text{subject to } n_t = \varepsilon_t^{\alpha} \chi_t^{1-\alpha}.$$

And we have

$$\min_{\varepsilon_t, \chi_t} \quad w_{f,t}\varepsilon_t + w_{m,t}\chi_t = A w_{f,t}^{\alpha} w_{m,t}^{1-\alpha} n_t,$$

where

$$A \equiv \frac{(1-\alpha)^{\alpha-1}}{\alpha^{\alpha}},$$

$$\varepsilon_t = 1 - \ell_t = \left(\frac{\alpha}{1-\alpha} \frac{w_{m,t}}{w_{f,t}} \right)^{1-\alpha} n_t, \tag{29}$$

and

$$\chi_t = 1 - h_t = \left(\frac{\alpha}{1-\alpha} \frac{w_{m,t}}{w_{f,t}} \right)^{-\alpha} n_t. \tag{30}$$

The firms' production, household decision-making, and government policies do not change.

Because

$$n_t = \frac{\eta}{1+\rho+\eta} \frac{(1-\tau)(w_{m,t}+w_{f,t})}{A w_{f,t}^{\alpha} w_{m,t}^{1-\alpha} - g}$$

and

$$w_{m,t} = \delta \frac{\ell_t}{h_t} w_{f,t}, \tag{31}$$

the fertility is

$$n_t = \frac{1}{A}\mu \left[\left(\delta \frac{\ell_t}{h_t} \right)^{\alpha-1} + \left(\delta \frac{\ell_t}{h_t} \right)^{\alpha} \right]. \tag{32}$$

Combining Eqs. (29)–(30),

$$\ell = \ell_t = \frac{1-\mu}{1-(1-\alpha)\mu + \alpha\mu\delta} = \frac{1-\mu}{1-\mu+\alpha\mu(1+\delta)}$$

and

$$h = h_t = \frac{1-(1-\alpha)\mu}{1+\frac{1}{\delta\ell}(1-\alpha)\mu} = \frac{1-(1-\alpha)\mu}{1+\frac{(1-\alpha)\mu}{\delta(1-\mu)}\left[1-\mu+\alpha\mu(1+\delta)\right]}.$$

Compared with the intra-household gender division of market work and childcare in Subsection 2.2, when the husband participates in the parental childcare production, the child allowances decrease both the wife's and the husband's labor supply, and increase both of their childcare time. Because

$$\frac{\partial \ell}{\partial \mu} = \frac{-\alpha(1+\delta)}{[1-\mu+\alpha\mu(1+\delta)]^2} < 0,$$

and

$$\frac{\partial h}{\partial \mu} = -(1-\alpha)\left[1+\frac{1-\alpha}{\delta}\frac{\mu}{\ell}\right]^{-1} - [1-(1-\alpha)\mu]\left[1+\frac{1-\alpha}{\delta}\frac{\mu}{\ell}\right]^{-2}\frac{1-\alpha}{\delta}\frac{\partial(\mu/\ell)}{\partial \mu},$$

$$\frac{\mu}{\ell} = \frac{1-\mu+\alpha\mu(1+\delta)}{\frac{1-\mu}{\mu}} = \mu + \alpha(1+\delta)\frac{\mu^2}{1-\mu},$$

$$\frac{\partial(\mu/\ell)}{\partial \mu} = 1+\alpha(1+\delta)\frac{2\mu(1-\mu)+\mu^2}{(1-\mu)^2} = 1+\alpha(1+\delta)\frac{2\mu-\mu^2}{(1-\mu)^2} > 0,$$

$$\Rightarrow \frac{\partial h}{\partial \mu} < 0.$$

Finally, fertility is increasing in the child allowances when the husband has to participate in the childcare and can not be replaced. For increasing the fertility rates, gender equality is important in both labor market work and parental child care. Please refer to the empirical studies on "daddy-month" and paternity leave for empirical evidence (see e.g., (Ekberg et al. (2013); Cools et al. (2015); Yamaguchi (2019); Patnaik (2019))).

6. Conclusion

We examined how child allowance policies affect fertility, female labor supply, and economic growth by applying an OLG model with parental child care production and gender wage discrimination in a closed economy. This paper presents nonlinear relationships between the amount of child allowance and other endogenous variables. We reveal the importance of gender wage discrimination in achieving the two policy goals: raising fertility and female labor supply. The prerequisite for the positive role of child allowances is a gender wage equality economy. Finally the empirical evidence supports the analytical results.

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Appendix

The Proof of Proposition 1

About equation (21), Because

$$\frac{\partial \ln k}{\partial \nu} = \frac{\tau}{1-\alpha+2\alpha\beta}\left[\frac{\alpha^2(1-\beta)(1+\delta)}{(1-\alpha\mu)(1+\alpha\delta\mu)} - \frac{1}{\mu}\right],$$

we know

$$\frac{\alpha^2(1-\beta)(1+\delta)}{(1-\alpha\mu)(1+\alpha\delta\mu)} - \frac{1}{\mu} \gtreqless 0 \quad \Leftrightarrow \quad \frac{\partial \ln k}{\partial \nu} \gtreqless 0.$$

Since

$$\frac{\alpha^2(1-\beta)(1+\delta)}{(1-\alpha\mu)(1+\alpha\delta\mu)} - \frac{1}{\mu} \gtreqless 0 \quad \Leftrightarrow \quad \frac{\alpha(1-\beta)}{(1-\alpha\mu)}\cdot\frac{\alpha\mu(1+\delta)}{1+\alpha\delta\mu} \gtreqless 1 \quad \Leftrightarrow \quad \frac{\alpha(1-\beta)}{1+\alpha\delta\mu}\cdot\frac{\alpha\mu(1+\delta)}{1+\alpha\delta\mu} \gtreqless \frac{(1-\alpha\mu)}{1+\alpha\delta\mu}$$

$$\Leftrightarrow \quad \frac{\alpha(1-\beta)}{1+\alpha\delta\mu}\cdot\varepsilon \gtreqless l \quad \Leftrightarrow \quad \frac{\alpha(1-\beta)}{1+\alpha\delta\mu}\cdot\varepsilon \gtreqless 1-\varepsilon$$

$$\Rightarrow \quad \varepsilon \gtreqless \frac{1+\alpha\mu\delta}{1+\alpha\mu\delta+\alpha(1-\beta)}(\equiv \varepsilon_k) \quad \Leftrightarrow \quad \frac{\partial \ln k}{\partial \nu} \gtreqless 0.$$

In the same manner, from Eq. (23), we have Eq. (24).

Regarding Eq. (26), because

$$\ln n = \ln\mu + \ln(1+\delta l) - \ln A + (1-\alpha)\ln w_t,$$

and

$$\frac{\partial \ln n}{\partial \nu} = \tau\frac{\partial \ln n}{\partial \mu},$$

we have

$$\frac{\partial \ln n}{\partial \mu} = \frac{1}{\mu} + \frac{\delta}{1+\delta l}\cdot\frac{\partial l}{\partial \mu} + (1-\alpha)\frac{\partial \ln w_t}{\partial \mu}.$$

Since $l = \frac{1-\alpha\mu}{1+\alpha\mu\delta}$,

$$\frac{\partial \ell}{\partial \mu} = \frac{-\alpha(1+\delta)}{(1+\alpha\mu\delta)^2},$$

and

$$\frac{\delta}{1+\delta l} = \frac{\delta(1+\alpha\mu\delta)}{1+\delta}.$$

From Eq. (23)

$$\frac{\partial \ln w_f}{\partial \mu} = \frac{1}{1-\alpha+2\alpha\beta}\left[\frac{\alpha(1-\beta)(1+\delta)}{(1-\alpha\mu)(1+\alpha\delta\mu)} - \frac{1-2\beta}{\mu}\right],$$

$\Rightarrow$

$$\frac{\partial \ln n}{\partial \mu} = \frac{1}{\mu} + \frac{-\alpha\delta}{1+\alpha\mu\delta} + \frac{1-\alpha}{1-\alpha+2\alpha\beta}\left[\frac{\alpha(1-\beta)(1+\delta)}{(1-\alpha\mu)(1+\alpha\delta\mu)} - \frac{1-2\beta}{\mu}\right]$$

$$= \frac{1}{\mu(1+\alpha\mu\delta)} + \frac{1-\alpha}{1-\alpha+2\alpha\beta}\left[\frac{\alpha\mu(1+\delta)(1-\beta)}{\mu(1-\alpha\mu)(1+\alpha\delta\mu)} - \frac{1-2\beta}{\mu}\right] \gtreqless 0$$

$$\Leftrightarrow \quad \frac{1}{1+\alpha\mu\delta} + \frac{1-\alpha}{1-\alpha+2\alpha\beta}\left[\frac{\varepsilon(1-\beta)}{l(1+\alpha\delta\mu)} - (1-2\beta)\right] \gtreqless 0$$

$$\Leftrightarrow \quad 1 + \frac{1-\alpha}{1-\alpha+2\alpha\beta}\left[\frac{\varepsilon(1-\beta)}{l} - (1-2\beta)(1+\alpha\delta\mu)\right] \gtreqless 0$$

$$\varepsilon \gtreqless \frac{1+\alpha\mu\delta - \frac{1-\alpha+2\alpha\beta}{(1-\alpha)(1-2\beta)}}{1+\alpha\mu\delta + \frac{1-\beta}{1-2\beta} - \frac{1-\alpha+2\alpha\beta}{(1-\alpha)(1-2\beta)}} \quad \Leftrightarrow \quad \frac{\partial \ln n}{\partial \nu} \gtreqless 0,$$

where $(1-2\beta)(1-\alpha)(1+\alpha\delta\mu) > (1-\alpha+2\alpha\beta)$.

When $(1-2\beta)(1-\alpha)(1+\alpha\delta\mu) < (1-\alpha+2\alpha\beta)$, $\frac{\partial \ln n}{\partial \nu} > 0$. Eqs. (27) and (28) can also be proved.

Also, it can be easily proved that $\varepsilon_k < \varepsilon_y$, $\varepsilon_n < \varepsilon_s$, $\varepsilon_w < \varepsilon_k <$, $\varepsilon_s < \varepsilon_w$. Finally, we have $\varepsilon_n < \varepsilon_w < \varepsilon_s < \varepsilon_k < \varepsilon_y$.

The Proof of Corollary 1

From Proposition 1,

$$\frac{\partial^2 \ln n}{\partial \delta\, \partial \mu} = -\frac{1}{\mu}\frac{\alpha\mu}{1+\alpha\mu\delta} + \frac{1-\alpha}{1-\alpha+2\alpha\beta} \cdot \frac{\alpha\mu(1-\beta)}{\mu(1-\alpha\mu)}\left[\frac{1}{1+\alpha\delta\mu} - \frac{(1+\delta)\alpha\mu}{(1+\alpha\mu\delta)^2}\right]$$

$$= -\frac{1}{\mu}\frac{\alpha\mu}{1+\alpha\mu\delta} + \frac{1-\alpha}{1-\alpha+2\alpha\beta}\alpha(1-\beta)\frac{1}{(1+\alpha\delta\mu)^2} < 0,$$

because

$$\frac{\frac{1}{\mu}\frac{\alpha\mu}{1+\alpha\delta\mu}}{\frac{1-\alpha}{1-\alpha+2\alpha\beta}\alpha(1-\beta)\frac{1}{(1+\alpha\delta\mu)^2}} = (1+\alpha\delta\mu)\frac{1-\alpha+2\alpha\beta}{1-\alpha-\beta+\alpha\beta} > 1.$$

When $\nu = 0$ and $\mu_0 = \frac{\eta}{1+\eta+\rho}(1-\tau)$, the fertility is

$$n_0 = \mu_0(1+\delta l_0)\frac{1}{A}w_{f,0}^{1-\alpha},$$

where

$$l_0 = \frac{1-\alpha\mu_0}{1+\alpha\delta\mu_0}$$

and

$$w_{f,0} = \left(A\frac{\gamma}{\mu_0}\right)^{\frac{1-2\beta}{1-\alpha+2\alpha\beta}} \left(\frac{\beta}{\delta}\, l_0^{\beta-1}\right)^{\frac{1}{1-\alpha+2\alpha\beta}}.$$

Therefore,

$$\zeta \equiv \frac{n_0}{n} = \frac{\mu_0(1+\delta l_0)\frac{1}{A}w_{f,0}^{1-\alpha}}{\mu(1+\delta l)\frac{1}{A}w_f^{1-\alpha}}$$

$$= \frac{\mu_0}{\mu} \cdot \frac{(1+\delta)/(1+\alpha\delta\mu_0)}{(1+\delta)/(1+\alpha\delta\mu)} \cdot \left(\frac{\mu}{\mu_0}\right)^{\frac{(1-\alpha)(1-2\beta)}{1-\alpha+2\alpha\beta}} \left(\frac{1-\alpha\mu}{1-\alpha\mu_0}\right)^{\frac{(1-\alpha)(1-\beta)}{1-\alpha+2\alpha\beta}} \left(\frac{1+\alpha\delta\mu_0}{1+\alpha\delta\mu}\right)^{\frac{(1-\alpha)(1-\beta)}{1-\alpha+2\alpha\beta}}$$

$$\Rightarrow \quad \frac{\partial \ln \zeta}{\partial \delta} = \frac{\beta}{1-\alpha+2\alpha\beta} \left[\frac{\alpha\mu}{1+\alpha\delta\mu} - \frac{\alpha\mu_0}{1+\alpha\delta\mu_0}\right] > 0$$

Thus, ζ is increasing in δ, and there exists a $\bar{\delta}$, when $\delta > \bar{\delta}$, $\zeta > 1$, and $n_0 > n$.

The Proof of Proposition 2

Because

$$\ln k = \frac{1}{1-\alpha+2\alpha\beta} \left[\ln\left(\frac{\gamma}{\mu}\right) + \ln A + \alpha \ln\left(\frac{\beta}{\delta}\right) - \alpha(1-\beta)\ln l\right],$$

we have

$$\frac{\partial \ln k}{\partial \delta} = \frac{1}{1-\alpha+2\alpha\beta} \left[-\alpha\frac{1}{\delta} + \alpha(1-\beta)\frac{\alpha\mu}{1+\alpha\mu\delta}\right] = \frac{\alpha/\delta}{1-\alpha+2\alpha\beta} \left[\frac{(1-\beta)\alpha\mu\delta}{1+\alpha\mu\delta} - 1\right] < 0.$$

And

$$\ln w_f = \frac{1-2\beta}{1-\alpha+2\alpha\beta} \ln\left(A\frac{\gamma}{\mu}\right) + \frac{1}{1-\alpha+2\alpha\beta} \left[\ln\frac{\beta}{\delta} - (1-\beta)\ln l\right],$$

thus

$$\frac{\partial \ln w_f}{\partial \delta} = \frac{1}{1-\alpha+2\alpha\beta} \left[(1-\beta)\frac{\alpha\mu}{1+\alpha\delta\mu} - \frac{1}{\delta}\right] < 0.$$

Because $s = \frac{\rho}{1+\eta+\rho}\left(w_f + \beta l^\beta k\right)$ and $y = (l)^\beta (k)^{1-2\beta}$, we have $\frac{\partial s}{\partial \delta} < 0$ and $\frac{\partial y'}{\partial \delta} < 0$.

Since

$$\ln n = \ln \mu + \ln(1+\delta) - \ln(1+\alpha\mu\delta) - \ln A + (1-\alpha)\ln w_f,$$

the marginal effect of the gender wage discrimination on fertility is shown as:

$$\frac{\partial \ln n}{\partial \delta} = \frac{1}{1+\delta} - \frac{\alpha\mu}{1+\alpha\mu\delta} + \frac{1-\alpha}{1-\alpha+2\alpha\beta} \left[\frac{(1-\beta)\alpha\mu}{1+\alpha\delta\mu} - \frac{1}{\delta}\right]$$

$$= \frac{1+\alpha\delta\mu - \alpha\mu\delta - \alpha\mu}{(1+\delta)(1+\alpha\delta\mu)} - \frac{1-\alpha}{1-\alpha+2\alpha\beta} \cdot \frac{1+\alpha\beta\mu\delta}{\delta(1+\alpha\delta\mu)}$$

$$\frac{\partial \ln n}{\partial \delta} = \frac{1}{1+\alpha\mu\delta} \left[\frac{1-\alpha\mu}{1+\delta} - \frac{(1-\alpha)(1+\alpha\beta\mu\delta)}{(1-\alpha+2\alpha\beta)\delta}\right] \gtreqless 0$$

$$\Leftrightarrow \frac{1-\alpha\mu}{1-\alpha} \cdot \frac{1}{1+\frac{1}{\delta}} \cdot \frac{1-\alpha+2\alpha\beta}{1+\alpha\beta\mu\delta} \gtreqless 1$$

$$\Leftrightarrow (1-\alpha\mu)(1-\alpha+2\alpha\beta) \gtreqless (1-\alpha)(1+1/\delta)(1+\alpha\beta\mu\delta)$$

$$\Leftrightarrow 2\alpha\beta \gtreqless \alpha\mu(1-\alpha) + \alpha^2\beta\mu + \frac{1}{\delta}(1-\alpha) + \alpha\beta\mu\delta(1-\alpha) + \alpha\beta\mu.$$

When $\alpha \to 1$, the LHS of this equation is greater than RHS, and thus $\partial \ln n/\partial\delta > 0$. By contrast, when $\alpha \to 0$, the RHS of this equation is greater than the LHS, and thus $\partial \ln n/\partial\delta < 0$.

Table A1: The Description of the Variables

Variables	Description
Family benefits	Family benefits spending refer to public spending on family benefits, including financial support that is exclusively for families and children. Spending recorded in other social policy areas, such as health and housing, also assist families, but not exclusively, and it is not included in this indicator. Broadly speaking there are three types of public spending on family benefits: Child-related cash transfers (cash benefits) to families with children, including child allowances, with payment levels that in some countries vary with the age of the child, and sometimes are income-tested; public income support payments during periods of parental leave and income support for sole parents families. Public spending on services for families (benefits in kind) with children, including direct financing and subsidising of providers of childcare and early education facilities, public childcare support through earmarked payments to parents, public spending on assistance for young people and residential facilities, public spending on family services, including centre-based facilities and home help services for families in need. Financial support for families provided through the tax system, including tax exemptions (e.g. income from child benefits that is not included in the tax base); child tax allowances (amounts for children that are deducted from gross income and are not included in taxable income), and child tax credits, amounts that are deducted from the tax liability. This indicator is broken down by cash benefits and benefits in kind and is measured in percentage of GDP.
Fertility	The total fertility rate in a specific year is defined as the total number of children that would be born to each woman if she were to live to the end of her child-bearing years and give birth to children in alignment with the prevailing age-specific fertility rates.
Gender wage gap	The gender wage gap is defined as the difference between median earnings of men and women relative to median earnings of men. Data refer to full-time employees.
GDP per capita	GDP per capita in 35 OECD countries.

Population	Population is defined as all nationals present in, or temporarily absent from a country, and aliens permanently settled in a country.
Total dependency ratio	The total dependency ratio is a measure of the number of dependents aged zero to 14 and over the age of 65, compared with the total population aged 15 to 64.
Household savings	Net household saving is defined as household net disposable income plus the adjustment for the change in pension entitlements less household final consumption expenditure (households also include non-profit institutions serving households). The adjustment item concerns (mandatory) saving of households, by building up funds in employment-related pension schemes. Household saving is the main domestic source of funds to finance capital investments, a major impetus for long-term economic growth. The net household saving rate represents the total amount of net saving as a percentage of net household disposable income. It thus shows how much households are saving out of current income and also how much income they have added to their net wealth. All OECD countries compile their data according to the 2008 System of National Accounts (SNA).
Household debt	Household debt is defined as all liabilities of households (including non-profit institutions serving households) that require payments of interest or principal by households to the creditors at a fixed dates in the future. Debt is calculated as the sum of the following liability categories: loans (primarily mortgage loans and consumer credit) and other accounts payable. The indicator is measured as a percentage of net household disposable income.

Table A2: The Description of the Variables

	Obs.	Min	Max	Median	Mean	Std.
Family benefits	575	0.00105	0.04089	0.01954	0.02031	0.00969
Fertility	575	1.0800	3.0900	1.6200	1.6766	0.3771
Gender wage gap	408	0.00384	0.41654	0.15807	0.16114	0.078114
GDP	575	8017.35	103787.97	30740.96	32315.61	14155.98
Population	575	281200	320742673	10401062	33877094.1	55496722.7
Total dependency ratio	575	0.4910	0.9840	0.6510	0.6571	0.0753
Female Labor participation	575	0.252	0.855	0.660	0.644	0.1051
Household savings	543	-0.13027	0.2767	0.06421	0.06851	0.06472
Household debt	492	0.03015	3.3978	1.0852	1.1667	0.6532