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“Heterogeneous Beliefs, Monetary Policy, and Stock Price”

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Heterogeneous Beliefs, Monetary Policy, and Stock Price Volatility *

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Abstract

This paper investigates how the stance of monetary policy affects stock price volatilities in an economy where two types of households with subjective and objective beliefs about expected capital gains from stock prices exist. I assume that investors construct subjective beliefs about expected capital gains by Bayesian learning from observed growth rates of stock prices. In a model with only homogenous subjective beliefs, the effect of the interest rate on stock prices tends to be unrealistically strong. In contrast, assuming heterogeneity by including investors with both subjective and objective beliefs improves the fit of theoretical moments to the data and especially helps to explain stock price volatility under interest rate shocks with conventional sizes. This quantitative improvement in stock price reactions to interest rate shocks allows me to conduct realistic analysis about how the stance of monetary policy affects stock price volatilities. Strong inertia of monetary policy rule does not necessarily reduce asset price volatilities. This depends on what kind of shock the economy is experiencing. When the monetary policy is persistent, stock price volatilities magnify under an unexpected monetary policy shock.

JEL classification: D83, D84, E44, E52, G12, G14

Keywords: stock price, asset pricing, heterogeneity, subjective belief, monetary policy, sticky prices, New Keynesian

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1 Introduction

The purpose of this study is to investigate how the stance of monetary policy affects stock price volatilities in an economy in which two types of households with subjective and objective beliefs about capital gains from stock prices exist.

The actual stock price responses to monetary policy shocks are larger than what plain rational expectation models usually produce.[1] In addition, empirical studies show that the effects of unexpected monetary policy shocks on stock prices change over time. Laopodis (2013) finds that the nature of dynamic relationship between monetary policy and the stock market was different in each of operating regimes under three chairmen of the Federal Reserve Board (pre-Volker, Volker, Greenspan). Paul (2019) empirically shows that stock market reactions to monetary policy are time varying. However, existing studies have not reached a clear consensus on why the reactions of stock prices to monetary policy shocks change over time. In this study, I examine whether the stance of monetary policy affects the stock responses to monetary policy shocks using the model that generates realistic stock price volatilities. Understanding the relationship between the stance of monetary policy and stock price behavior contributes to the discussion on whether monetary policy inertia ("gradualism") helps to reduce financial market volatilities in terms of stock market.[2]

To investigate how the stance of monetary policy affects stock price volatilities when there exist two types of households with subjective and objective beliefs about capital gains from stock prices, I develop a New Keynesian model with the two types of households. Households consume goods, supply labor, and save their wealth in stocks and bonds. Households with subjective beliefs construct their expectations about capital gains by Bayesian learning from observed stock price growth rates. The subjective probability belief does not equal the objective probability density as they emerge in equilibrium. The population shares of the subjective and objective households are exogenously given. I use a general equilibrium model because under nominal rigidity a monetary policy affects the real interest rates and pricing

[1] See Challe and Giannitsarou (2014) for a summary of empirical studies about the effect of monetary policy shock on stock prices including Bernanke and Kuttner (2005).

[2] Srour (2001) lists an argument that the large surprises in short-term interest rates can cause volatility in financial markets as one reason for smoothing interest rates. Rudebusch (2006) examines a discussion about a rationale for policy gradualism, which is a desire to reduce the volatility in asset prices. González-Páramo (2006) argues that a gradual monetary policy could reduce the likelihood of financial market disruptions. In actual policy making, FOMC Secretariat (1994) records that there were discussions led by chairman Alan Greenspan on whether a 25 basis point policy tightening was preferable to a 50 basis point tightening because some members considered the larger move to have higher probability of cracking financial markets. To this question, Bernanke (2004) gives no decisive conclusion on whether gradualism of monetary policy provides stability of financial market or asset prices.

kernels, while partial equilibrium models usually assume that the consumption path and consequently pricing kernel are exogenously given. Studies using Bayesian learning about stock price growth, such as a partial equilibrium model of Adam et al. (2017) or general equilibrium models of Adam and Merkel (2018), Oshima (2019), and Winkler (2019) do not consider heterogeneity. I design the model so that the effects of the existence of subjective households are minimal, i.e., it affects only stock prices to focus on the analysis of stock price behaviors while business cycle properties are kept very standard.

The main findings are as follows. First, in the general equilibrium model with heterogeneous beliefs, the overall theoretical second moments match the data well without assuming a high risk aversion rate or high habit formation. The model is successful in generating stock price volatilities close to the data with realistic moments of business cycles properties such as output and inflation. Considering both subjective and objective beliefs is beneficial because it enhances moment matching compared to a model with only subjective beliefs or objective beliefs.

Second, the model can generate empirically plausible stock price drops in response to an interest rate shock. Challe and Giannitsarou (2014) argue that a 100 basis point increase in nominal interest rate is associated with a 2.2%-9% decrease in real stock prices empirically. The model shows a 9.3% decrease in stock prices at the timing of nominal interest rate shock of 100 basis points (annual term) in impulse response analysis. The average response over four quarters at and after the shock is a 4.5% decrease. This cannot be attained in a homogenous subjective belief model because the impacts of interest rate shocks on stock prices are unrealistically large in a homogenous model. For example, Adam and Merkel (2018), Winkler (2019), and Oshima (2019) allow only small interest rate volatilities or shocks to have realistic stock price volatilities.

Third, strong monetary policy inertia does not necessarily decrease stock price volatilities; this depends on what kind of shock the economy is experiencing. When an interest rate shock occurs, a persistent monetary policy magnifies the stock price volatilities. In contrast, when a productivity shock occurs, a persistent monetary policy reduces the stock price volatilities. The intuition of this result is as follows.

In a model with subjective beliefs, the near-term real interest rate is a key variable to explain stock price volatility. Under a positive interest rate shock, because of the high persistence of the nominal interest rate, an increase in interest rate sustains for a long period and continues to push the "current" interest rates up at each time point. Furthermore, low inflation expectation due to persistently high nominal interest rates pushes the real interest rates up. Thus, strong monetary policy persistence implies sustained high volatility of near-term real interest rates and therefore stock prices after a monetary policy shock.

On the other hand, when the economy experiences a positive productivity shock, the decrease in today's nominal interest rate in response to the shock becomes gradual and small when the monetary policy is persistent. This is because the Taylor rule with high interest rate smoothing implies that the policy rate reacts weakly to real-time changes in inflation. A gradual reduction in nominal interest rates leads to low expected inflation rates and results in low volatility of today's real interest rates and hence stock prices.

These results provides policy implications in the context of the "gradualism" of monetary policy. What is important for stock price stability is not simply the size of policy rate changes, but the monetary policy rules and structural shocks that the economy is experiencing.

The rest of the paper is organized as follows. Section 2 reviews the related literature. Section 3 presents my model. Section 4 discusses the quantitative results based on the model. Section 5 investigates the relation between deep parameters and stock price volatilities. Section 6 concludes this study and discusses future extensions.

2 Related literature

Theoretical studies which explain the volatility puzzle based on rational expectation are, for example, long-run risks by Bansal and Yaron (2004) and habit formation by Campbell and Cochrane (1999). Studies which depart from rational expectation are, for example, Timmermann (1993) and Collin-Dufresne et al. (2016). Both consider investors who form expectations about fundamentals by learning. Choi and Mertens (2013), Barberis et al. (2015), and Hirshleifer et al. (2015) assume that investors form their expectations about fundamentals by extrapolation. As another strand of belief-based approach, there is research focusing on subjective beliefs about asset prices or returns such as De Long et al. (1990) and Lansing (2010).

The work of Adam et al. (2017), which my model mainly refers to, can be categorized as this type of research. They consider the homogenous investors with subjective beliefs about stock price growth (capital gains) under exogenous consumption and dividend processes. They assume that investors who know the fundamentals have subjective beliefs about stock price growth and do not have knowledge about a pricing function of stock price mapping from the fundamentals. The investors' expectations of capital gains are influenced by the capital gains observed in the past. In models mentioned above, the consumption and dividend streams are exogenously given. Therefore, it is not suitable for investigating the relations between deep parameters regarding monetary policy and stock price volatilities. My model deals with the fundamental variables endogenously.

Some studies consider stock price in New Keynesian models assuming rational expectation such as De Paoli et al. (2010). To generate large swings of stock prices, rational expectation models tend to assume a higher relative risk aversion rate than what are normally calibrated in dynamic stochastic general equilibrium models.[3] A New Keynesian model by Wei (2009) does not have a high relative risk aversion rate. However, Wei (2009) argues that a highly persistent exogenous monetary policy shock, which is not usual in the literature, is necessary to have enough variations of marginal utility of consumption. My model can generate realistic stock price volatility without a high relative risk aversion rate or highly persistent exogenous monetary policy shock.

Departing from rational expectation, another strand of research studies subjective beliefs about stock price growth in a general equilibrium model context. Adam and Merkel (2018) propose a real business cycle model to explain business cycles and stock price volatility. Investors/households homogenously hold subjective beliefs about capital gains following Adam et al. (2017). By regarding the capital price as stock price, this model generates feedback effects of stock price on output fluctuation. In reality, the capital price itself does not show as large volatility as stock price. The risk-free rate volatility in their model is quite low compared to the actual data while other theoretical moments match data well.

Winkler (2019) analyzes the stock price movements and business cycles in his New Keynesian model with homogenous agents who have subjective beliefs about capital gains. The belief structures are similar to Adam et al. (2016) and Adam et al. (2017).[4] Stock holders are risk-neutral in his model while households are risk-averse with access to the risk-free bond market. This allows to reduce the effects of interest rates on stock prices and making the near-term dividend effects important. In stock pricing, current dividend plays a major role in determining the stock price and the interest rate affects the stock price mainly through the dividend paid to stock holders as firms' costs in the balance sheets. Even under this setting, the stock price reaction to interest rate movements is stronger than what the usual empirical research implies. Oshima (2019) studies the stock price volatility and finds that monetary policy parameters are the keys to stock price volatility. However, he assumes homogenous subjective households and cannot generate stock price reactions to interest rate shocks with

[3] De Paoli et al. (2010) and Challe and Giannitsarou (2014) assume 5 for relative risk aversion rate. The prominent work by Jermann (1998), which is a real model, also assumes 5.

[4] Caines and Winkler (2018) study housing prices in a New Keynesian model with learning about housing price capital gains. Because housing stock quantity is directly included in households' utility, housing price has wealth effects on business cycles.

realistic sizes.

Some studies in the literature consider heterogeneous beliefs in asset pricing. Harrison and Kreps (1978) show that different beliefs about future states and short-selling restrictions lead to higher stock prices because of option values. Scheinkman and Xiong (2003) assume heterogeneity in signals under short-selling restriction. Their model also derives option values in stock price. Shiller et al. (1984) provide a model with rational and noise traders. Brock and Hommes (1998) investigate an asset pricing model with heterogeneous beliefs. Agents select different beliefs or predictors of the future price of a risky asset based on their past performances. However, because these models assume endowment economies, it is difficult to find the relations between stock price and monetary policy.

There is a growing strand of the literature to study macroeconomic questions including monetary policy transmission in New Keynesian models with heterogeneous agents; such models are often referred to as Heterogeneous Agent New Keynesian (HANK) models. These models often assume idiosyncratic shocks to individual income, incomplete market, and financial constraints. Kaplan et al. (2018) study the transmission mechanism of monetary policy focusing on empirically realistic effects of unexpected cuts of interest rates on consumption. Another strand of the literature studies Two Agent New Keynesian (TANK) model, in which two types of agents exist with different accessibilities to the financial market; these studies include Campbell and Mankiw (1990), Galí et al. (2007), Bilbiie (2008), and Debortoli and Galí (2017). My two agent model is heterogeneous in beliefs about stock price growth. This is different from the settings of usual TANK models in which heterogeneity comes from access to financial markets. However, similarly to initial motivations of HANK and TANK, I incorporate two types of agents to provide empirically realistic responses of stock prices to interest rate shocks.

3 Model

The model is built on a standard New Keynesian model. I assume Rotemberg type price adjustment costs. Firms pay dividends to the households, which are output minus investment cash flows and price adjustment costs. Households consume goods, supply labor, and save their wealth in stocks and bonds. Two types of households exist, those with subjective and objective beliefs about stock price growth. The population share of the subjective and objective households is exogenously given. The belief structures of the subjective households follow Adam et al. (2017), a model with exogenous consumption and dividend streams, and

Adam and Merkel (2018), a real business cycle model.[5]

3.1 Households

Households with subjective beliefs form their beliefs about stock price growth based on Kalman filtering with expected capital gain as a state variable and realized capital gain as an observed variable. They do not know other households' beliefs and do not know the pricing function mapping fundamentals to stock prices while they know fundamentals (given stock price).

The subjective household's maximization problem basically follows the "internal rationality" discussed by Adam and Marcet (2011) and Adam et al. (2017). Internal rationality requires that agents make fully optimal decisions given a well-defined system of subjective probability beliefs about payoff-relevant external variables that are beyond their control including stock prices. That is, internal rationality means standard utility maximization given subjective beliefs about variables that are beyond their control.[6] In this study, following Adam et al. (2017), households with subjective beliefs do not know the stock price function derived from fundamental variables, and choose the optimal plans of stock holdings and consumption given the subjective belief about capital gains under the probability measure "P." They form this by learning from observed past growth rates of stock prices. Whatever the agents' expectations about stock price growth are, the stock price level and consumption plans satisfy the Euler equation with subjective expectations of stock price growths.

Key assumptions about the belief structure are as follows. (i) Homogeneity of the households with subjective beliefs is not common knowledge among the subjective households. In addition, they do not know the preferences of households with objective beliefs. This knowledge structure prevents the households from mapping the fundamentals to market-based stock prices based only on their own preferences under any population share of the two types of households and hence enables the subjective expectations of capital gains to deviate from objective expectations.

(ii) I have additional assumptions because of heterogeneity of beliefs in this model, whereas in Adam et al. (2017) and Adam and Merkel (2018) agents are homogenous. Households with

[5]The belief structure in Winkler (2019) is also similar. However, Winkler (2019) assumes that agents have "conditionally model-consistent expectations". Conditionally model-consistent expectations are consistent with all equilibrium conditions of the model, except those that would convey knowledge of the price that clears the asset market, when agents solve for the perceived law of motion. On the other hand, this study follows Adam and Merkel (2018) in which market clearing conditions are known by agents.

[6]On the other hand, external rationality postulates that agents' subjective probability belief equals the objective probability density of external variables as they emerge in equilibrium.

objective beliefs know preferences of the subjective households and their homogeneity, other objective households' preferences and their homogeneity, and the population share of each household in the economy. Because of this belief structure, objective households can apply the law of iterated expectations in stock pricing, given the presence of subjective households.

3.1.1 Households with subjective beliefs

The infinitely lived representative household with subjective beliefs makes decisions on consumption, savings in stock and risk-free bonds, and labor supply. I assume that the household's expectations about wages and dividends are rational. The household's expectations of the growth rate of the stock price is subjective following Adam et al. (2017). The household's utility in each period is presented by the following function with consumption habit formation,

$$U(C_{s,t}, L_{s,t}) = \frac{1}{1-\gamma}(C_{s,t} - \phi C_{s,t-1})^{1-\gamma} - \chi \frac{L_{s,t}^{1+\varphi}}{1+\varphi}, \tag{1}$$

where $C_{s,t}$ is consumption at time t, $L_{s,t}$ is labor at time t, γ is the rate of relative risk aversion, ϕ is the parameter of habit formation, χ is the weight assigned to labor, and φ is the inverse of Frisch elasticity. s is an index for households with subjective beliefs.

The budget constraint of the household is given by the following equation. The households have access to the financial market via stocks and bonds.

$$S_{s,t}p_t^s + C_{s,t} + p_t^s \frac{\zeta^S}{2}(S_{s,t} - S_{ss})^2 + B_{s,t} + \frac{\zeta^B}{2}(B_{s,t} - B_{ss})^2 = S_{s,t-1}(p_t^s + d_t) + w_t L_{s,t} + R_t^f B_{s,t-1}. \tag{2}$$

where $S_{s,t}$ is stock holdings at time t, p_t^s is real stock price at time t, $B_{s,t}$ is real bond holding at time t, R_t^f is real interest rate at time t, d_t is real dividend at time t, and w_t is real wage at time t. S_{ss} is steady state stock holdings. B_{ss} is steady state bond holdings. $p_t^s \frac{\zeta^S}{2}(S_{s,t} - S_{ss})^2$ and $\frac{\zeta^B}{2}(B_{s,t} - B_{ss})^2$ are the stock and bond adjustment costs, respectively.[7] ζ^S is the parameter of stock adjustment cost and ζ^B is the parameter of bond adjustment cost.[8] Subjective households are under the constraint of minimum and maximum stock holding

[7] Both adjustment costs have positive values when the stock and bond holdings deviate from steady state levels. I assume these costs to incorporate demand for stock and bond from the two types of households.

[8] These representations of stock and bond adjustment costs are parsimonious ways to aggregate two types of agents by having a mathematical representation similar to that obtained by the mean-variance optimization under constant absolute risk aversion utility. For example, see Brock and Hommes (1998) and Hanson and Stein (2015).

positions. This allows to have maximums in their optimization problem under subjective beliefs. In equilibrium, this constraint is not binding over the entire time path.

$$\underline{S_s} \leq S_{s,t} \leq \overline{S_s}. \tag{3}$$

The household maximization problem is given by

$$\begin{aligned} &\max \; E_0^P \sum_{t=0}^{\infty} \delta^t \, \exp(Z_t) \, U(C_{s,t}, L_{s,t}) \\ &\textit{subject to } (2) \textit{ and } (3), \end{aligned} \tag{4}$$

where E_0^P denotes the subjective expectation operator at time 0. This setting basically follows Adam et al. (2017). $\delta \in (0,1)$ is the time preference rate. $\exp(Z_t)$ denotes the preference shock. The preference shock process is formulated with persistency parameter ρ_Z as

$$Z_t = \rho_Z Z_{t-1} + \epsilon_t^Z, \tag{5}$$

where ϵ_t^Z represents an i.i.d. stochastic shock regarding the preference shock process.

The first-order conditions with respect to $C_{s,t}$ and $S_{s,t}$ are given by

$$\lambda_{s,t} = \exp(Z_t)(C_{s,t} - \phi C_{s,t-1})^{-\gamma} - \delta\phi E_t^P \Big[\exp(Z_{t+1})(C_{s,t+1} - \phi C_{s,t})^{-\gamma} \Big] \tag{6}$$

and

$$1 + \zeta^S (S_{s,t} - S_{ss}) = \delta E_t^P \Big[\frac{\lambda_{s,t+1}}{\lambda_{s,t}} \big(\frac{p_{t+1}^s}{p_t^s} + \frac{d_{t+1}}{p_t^s} \big) \Big], \tag{7}$$

where $\lambda_{s,t}$ represents the Lagrange multiplier for (2). The expectation operator of this Euler equation is governed by subjective beliefs, as explained later. The bond Euler equation is

$$1 + \zeta^B (B_{s,t} - B_{ss}) = \delta E_t^P \Big[\frac{\lambda_{s,t+1}}{\lambda_{s,t}} \Big] R_t^f. \tag{8}$$

The first-order condition with respect to labor supply is

$$\chi L_{s,t}^{\varphi} = \lambda_{s,t} w_t. \tag{9}$$

3.1.2 Subjective expectation of stock price growth

The households' expectations of stock price growth are subjective and use the Kalman filter to form the belief. I assume that households perceive stock prices to evolve according to

$$\frac{p^s_{t+1}}{p^s_t} = \beta_{t+1} + \epsilon_{t+1}. \tag{10}$$

where ϵ_{t+1} is the transitory shock to price growth, $\epsilon_{t+1} \sim N(0, \sigma^2_\epsilon)$. β is the unobserved persistent price growth component following Adam et al. (2017). The persistent component of stock prices drifts according to

$$\beta_{t+1} = \beta_t + \nu_{t+1}, \tag{11}$$

where ν_{t+1} is the innovation to price growth, $\nu_{t+1} \sim N(0, \sigma^2_\nu)$. The Kalman filter implies that the beliefs are given by

$$\beta_{t+1} \sim N(m_t, \sigma^2_\beta), \tag{12}$$

where m_t is the conditional expectation of β_{t+1} and σ^2_β is the steady state Kalman filter uncertainty.

I assume that ϵ and ν are independent and the variances of each shock satisfy

$$\sigma^2_\nu \ll \sigma^2_\epsilon, \tag{13}$$

so that the Kalman gain for the persistent component of stock price growth becomes small.[9] With the optimal constant gain g, the Kalman gain process becomes

$$m_t = m_{t-1} + g(\frac{p^s_t}{p^s_{t-1}} - m_{t-1}). \tag{14}$$

The optimal constant Kalman gain g is given by

$$g = \frac{\sigma^2_\beta}{\sigma^2_\beta + \sigma^2_\epsilon}, \tag{15}$$

and the steady state uncertainty σ_β is calculated as

[9] The reason why a small value is necessary for the optimal Kalman gain g is that when g is not small enough, the Blanchard-Kahn condition is not satisfied in a general equilibrium, as mentioned later. From (15) and (16) shown shortly, g becomes large if I do not assume (13). The calibrated value for g in this model is $\frac{1}{150}$.

$$\sigma_\beta^2 = \frac{\sigma_\nu^2 + \sqrt{(\sigma_\nu^2)^2 + 4\sigma_\nu^2\sigma_\epsilon^2}}{2}. \tag{16}$$

Internal rationality assumes that subjective agents make fully optimal decisions given a well-defined system of subjective probability beliefs about stock prices. I assume that subjective households construct and update their expectations of capital gains at time $t+1$, $\frac{p^s_{t+1}}{p^s_t}$, by using m_t in (14) under the subjective probability measure P. Under this assumption with (7), the Euler equation becomes

$$1 + \zeta^S(S_{s,t} - S_{ss}) = \delta E_t^P \Big[\frac{\lambda_{s,t+1}}{\lambda_{s,t}} (m_t + \frac{d_{t+1}}{p^s_t}) \Big]. \tag{17}$$

m_t evolves according to (14). Because (14) includes only the present and past variables, m_t is not a stochastic variable at time t. Internal rationality implies fully rational utility maximization given the subjective beliefs about stock price growth, m_t. Therefore, I have

$$1 + \zeta^S(S_{s,t} - S_{ss}) = \delta E_t \Big[\frac{\lambda_{s,t+1}}{\lambda_{s,t}} \Big] m_t + \delta E_t \Big[\frac{\lambda_{s,t+1}}{\lambda_{s,t}} \frac{d_{t+1}}{p^s_t} \Big], \tag{18}$$

where E_t denotes the rational expectation operater given the subjective beliefs of stock price growth.

The equation obtained by substituting (14) for m_t in (18) implies simultaneous determination of the price beliefs and prices, and could generate multiple solutions of stock price because this equation is a quadratic function of the stock price p^s_t. In this study, I use the log-linearization around the steady state to solve this model. Therefore, I can avoid the simultaneity problem by setting a steady state at a point that economically makes sense, even though the learning process is set as (14). However, as often used in the learning literature such as in Eusepi and Preston (2011) and Evans and Honkapohja (2001), I can also consider another version of Kalman gain process by modifying the observation timing as

$$m_t = m_{t-1} + g(\frac{p^s_{t-1}}{p^s_{t-2}} - m_{t-1}). \tag{19}$$

Besides avoiding the simultaneity problem, due to superiority in data fitting, I use the belief system addressed in (19) instead of (14) in this study.

3.1.3 Households with objective belief

The optimization problem of the infinitely lived representative household with objective beliefs is standard. The households' expectation about the growth rate of stock price is objective. The household's utility in each period is presented by the following function with consumption habit formation:

$$U(C_{o,t}, L_{o,t}) = \frac{1}{1-\gamma}(C_{o,t} - \phi C_{o,t-1})^{1-\gamma} - \chi \frac{L_{o,t}^{1+\varphi}}{1+\varphi}, \tag{20}$$

where $C_{o,t}$ is consumption at time t and $L_{o,t}$ is labor at time t. o is the index for households with objective beliefs.

The budget constraint of the household is given by the following equation,

$$S_{o,t}p_t^s + C_{o,t} + p_t^s \frac{\zeta^S}{2}(S_{o,t} - S_{ss})^2 + B_{o,t} + \frac{\zeta^B}{2}(B_{o,t} - B_{ss})^2 = S_{o,t-1}(p_t^s + d_t) + w_t L_{o,t} + r_t B_{o,t-1}, \tag{21}$$

where $S_{o,t}$ is the stock holdings by the households at time t and $B_{o,t}$ is the bond holdings by the households at time t. The household's maximization problem is given by

$$\begin{aligned} &\max\, E_0 \sum_{t=0}^{\infty} \delta^t \, \exp(Z_t)\, U(C_{o,t}, L_{o,t}) \\ &\textit{subject to } (21), \end{aligned} \tag{22}$$

where E_0 denotes the objective expectation operator at time 0. The first-order conditions with respect to $C_{o,t}$ and $S_{o,t}$ are given by

$$\lambda_{o,t} = \exp(Z_t)(C_{o,t} - \phi C_{o,t-1})^{-\gamma} - \delta\phi E_t[\exp(Z_{t+1})(C_{o,t+1} - \phi C_{o,t})^{-\gamma}] \tag{23}$$

and

$$1 + \zeta^S(S_{o,t} - S_{ss}) = \delta E_t\Big[\frac{\lambda_{o,t+1}}{\lambda_{o,t}}(\frac{p_{t+1}^s}{p_t^s} + \frac{d_{t+1}}{p_t^s})\Big], \tag{24}$$

where $\lambda_{o,t}$ represents the Lagrange multiplier for (21). The bond Euler equation is

$$1 + \zeta^B(B_{o,t} - B_{ss}) = \delta E_t\Big[\frac{\lambda_{o,t+1}}{\lambda_{o,t}}\Big] R_t^f. \tag{25}$$

The first-order condition with respect to labor supply is

$$\chi L_{o,t}^{\varphi} = \lambda_{o,t} w_t. \tag{26}$$

3.1.4 Market clearing and stock demand

The stock market clearing with α as the population share of subjective households is

$$\alpha S_{s,t} + (1-\alpha) S_{o,t} = S_t. \tag{27}$$

In addition, I assume that stock supply is unity,

$$S_t = 1 \ \forall t. \tag{28}$$

From (18), (24), (27), and (28), I obtain the stock market clearing condition as

$$1 = \alpha\delta\Big\{E_t\Big[\frac{\lambda_{s,t+1}}{\lambda_{s,t}}\Big]m_t + E_t\Big[\frac{\lambda_{s,t+1}}{\lambda_{s,t}}\frac{d_{t+1}}{p_t^s}\Big]\Big\} + (1-\alpha)\delta E_t\Big[\frac{\lambda_{o,t+1}}{\lambda_{o,t}}\big(\frac{p_{t+1}^s}{p_t^s} + \frac{d_{t+1}}{p_t^s}\big)\Big]. \tag{29}$$

The bond market clearing condition becomes

$$\alpha B_{s,t} + (1-\alpha) B_{o,t} = 0. \tag{30}$$

From (8), (25), and (30), the bond market clearing condition becomes

$$1 = \delta R_t^f E_t\Big[\alpha\frac{\lambda_{s,t+1}}{\lambda_{s,t}}\Big] + \delta R_t^f E_t\Big[(1-\alpha)\frac{\lambda_{o,t+1}}{\lambda_{o,t}}\Big]. \tag{31}$$

The nominal risk-free rate satisfies

$$1 = \delta R_t^n E_t\Big[\alpha\frac{\lambda_{s,t+1}}{\lambda_{s,t}}\frac{P_t}{P_{t+1}}\Big] + \delta R_t^n E_t\Big[(1-\alpha)\frac{\lambda_{o,t+1}}{\lambda_{o,t}}\frac{P_t}{P_{t+1}}\Big]. \tag{32}$$

3.1.5 Log-linearized stock price equation

Before moving to the firm sector's settings, I intuitively discuss how stock prices are formulated in the model. To summarize, stock prices react strongly to near-term information under subjective beliefs, while in a rational expectation case, they are determined by the infinite future information about dividends and stochastic discount factors. To observe this, I show a log-linearized version of the stock price equation. I substitute (19) for m_t in (29) and log-linearize this. Eliminating λ_s and λ_o in this equation using the log-linearized equation of (31) yields the stock price equation as

$$\hat{p}_t^s = -\frac{\delta^{-1}}{\delta^{-1}-\alpha}\hat{R}_t^f + \frac{\delta^{-1}-1}{\delta^{-1}-\alpha}E_t\hat{d}_{t+1}^s + \frac{\alpha(1-g)}{\delta^{-1}-\alpha}\hat{m}_{t-1} + \frac{\alpha g}{\delta^{-1}-\alpha}(\hat{p}_{t-1}^s - \hat{p}_{t-2}^s) + \frac{1-\alpha}{\delta^{-1}-\alpha}E_t\hat{p}_{t+1}^s. \tag{33}$$

$\hat{x}$ denotes the log deviation from the steady state value of x. Again, α is the population share of households with subjective beliefs. This equation implies that the real interest rate at time t affects the stock price significantly when α is close to 1 because $\frac{\delta^{-1}}{\delta^{-1}-\alpha}$ is large. The impact of the interest rate is much larger than that of dividends. In contrast, as α becomes close to 0, this impact becomes weaker because $\frac{\delta^{-1}}{\delta^{-1}-\alpha}$ becomes smaller. In addition, future interest rates and dividends become more important as α becomes closer to 0 because the last term of (33) includes $\hat{p}^s$ at time $t+1$. The level of g determines how sensitively the stock price moves in response to near-term past capital gain.

3.2 Firms

Monopolistically competitive intermediate goods firms maximize their profits. The intermediate goods firms pay price adjustment costs which are paid by final goods following Rotemberg (1982) when they change their prices. They hire labor from the household, own capital, and conduct capital formations. Capital formations require investment adjustment costs. $j \in [0,1]$ is an intermediate goods firm index. Competitive final goods producers produce final goods by aggregating intermediate goods. The final goods are transformed to consumption goods and investment goods costlessly.

The final goods sector is perfectly competitive and transforms intermediate goods into final goods for consumption and investment by the CES production function,

$$Y_t = \left(\int_0^1 Y_t(j)^{\frac{\eta-1}{\eta}} dj\right)^{\frac{\eta}{\eta-1}}, \tag{34}$$

where Y_t is aggregate output of final goods, $Y_t(j)$ is output of intermediate goods, and $\eta\,(>1)$ is the demand elasticity parameter. The profit maximization of the final goods firm becomes

$$\max_{Y_t(j)} P_t\left(\int_0^1 Y_t(j)^{\frac{\eta-1}{\eta}} dj\right)^{\frac{\eta}{\eta-1}} - \int_0^1 P_t(j)Y_t(j)dj, \tag{35}$$

where $P_t(j)$ is an intermediate good price. This generates a downward-sloping demand for intermediate goods. The intermediate goods demand is set as

$$Y_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\eta} Y_t. \tag{36}$$

The maximization problem of the firm producing intermediate goods j is given by

$$\max\ E_t \sum_{i=0}^{\infty} M_{t+i}\{d_{t+i}(j)\}, \tag{37}$$

where M_{t+i} is the pricing kernel under the presence of the two types of households who own the firm sector. $d_{t+i}(j)$ denotes the real dividend of firm j. The real dividend cash flow is governed by

$$d_t(j) = \frac{P_t(j)}{P_t} Y_t(j) - w_t L_t(j) - I_t(j) - \frac{\zeta^P}{2}\left(\frac{P_t(j)}{P_{t-1}(j)} - 1\right)^2 Y_t, \tag{38}$$

where $I_t(j)$ is investment of firm j, and ζ^P is the price adjustment cost parameter.

Capital formation is defined as

$$K_t(j) = (1-\psi)K_{t-1}(j) + \exp(v_t) I_t(j)\left(1 - \frac{\zeta^I}{2}\left(\frac{I_t(j)}{I_{t-1}(j)} - 1\right)^2\right), \tag{39}$$

where $K_t(j)$ is capital of firm j. $\psi \in (0, 1]$ is the depreciation rate of capital, $\frac{\zeta^I}{2}(\frac{I_t(j)}{I_{t-1}(j)} - 1)^2$ is the investment adjustment cost, and ζ^I is a parameter of investment adjustment costs. v_t represents the investment specific technology shock process at time t. The shock process is formulated with the persistency parameter of ρ_V as

$$v_t = \rho_V v_{t-1} + \epsilon_t^v, \tag{40}$$

where ϵ_t^v represents an i.i.d. stochastic shock regarding the investment specific technology shock process. The production technology of intermediate goods is given by

$$Y_t(j) = \exp(A_t) K_{t-1}(j)^{\xi} L_t(j)^{1-\xi}, \tag{41}$$

where A_t represents the technology level at time t. $\xi \in (0, 1)$ is the capital share. The productivity shock process is formulated with the persistency parameter ρ_A and an i.i.d. shock to productivity ϵ_t^A as

$$A_t = \rho_A A_{t-1} + \epsilon_t^A. \tag{42}$$

The Lagrangian of this problem can be set as

$$\begin{aligned}\mathcal{L}_t =& E_t \sum_{i=0}^{\infty} M_{t+i}\Big\{\Big(\frac{P_{t+i}(j)}{P_{t+i}}\Big)^{1-\eta} Y_{t+i} - w_{t+i}L_{t+i}(j) - I_{t+i}(j) - \frac{\zeta^P}{2}\big(\frac{P_{t+i}(j)}{P_{t+i-1}(j)} - 1\big)^2 Y_{t+i} \\ &+ \Omega_{t+i}(j)(\exp(A_{t+i})K_{t+i-1}(j)^{\xi} L_{t+i}(j)^{1-\xi} - \{\frac{P_{t+i}(j)}{P_{t+i}}\}^{-\eta} Y_{t+i}) \\ &+ q_{t+i}(j)((1-\psi)K_{t+i-1}(j) + \exp(v_{t+i})I_{t+i}(j)\big(1 - \frac{\zeta^I}{2}(\frac{I_{t+i}(j)}{I_{t+i-1}(j)} - 1)^2\big) - K_{t+i}(j))\Big\},\end{aligned} \tag{43}$$

where $\Omega_{t+i}(j)$ and $q_{t+i}(j)$ are the Lagrange multipliers. I assume that the stochastic discount factor to discount firms' cash flow is the weighted average of those of the two types of households. Define Λ as

$$\begin{aligned}\Lambda_t &= \frac{M_{t+1}}{M_t} \\ &= \delta\Big\{\alpha\frac{\lambda_{s,t+1}}{\lambda_{s,t}} + (1-\alpha)\frac{\lambda_{o,t+1}}{\lambda_{o,t}}\Big\}.\end{aligned} \tag{44}$$

I assume the firms are symmetric. Then, I have the first-order conditions as below.

$$w_t = \Omega_t(1-\xi)\frac{Y_t}{L_t}. \tag{45}$$

$$q_t = \Lambda_t E_t[\Omega_{t+1}\xi\frac{Y_{t+1}}{K_t} + q_{t+1}(1-\psi)]. \tag{46}$$

$$1 = q_t\exp(v_t)\{1 - \zeta^I(\frac{I_t}{I_{t-1}} - 1)\frac{I_t}{I_{t-1}} - \frac{\zeta^I}{2}(\frac{I_t}{I_{t-1}} - 1)^2\} + E_t\Lambda_t q_{t+1}\exp(v_{t+1})\zeta^I(\frac{I_{t+1}}{I_t} - 1)(\frac{I_{t+1}}{I_t})^2. \tag{47}$$

$$\frac{1-\eta}{\eta} - \frac{\zeta^P}{\eta}(\frac{P_t}{P_{t-1}} - 1)\frac{P_t}{P_{t-1}} + \Lambda_t E_t\frac{\zeta^P}{\eta}(\frac{P_{t+1}}{P_t} - 1)\frac{P_{t+1}}{P_t}\frac{Y_{t+1}}{Y_t} + \Omega_t = 0. \tag{48}$$

Aggregate capital evolution is

$$K_t = (1-\psi)K_t + \exp(v_t)I_t(1 - \frac{\zeta^I}{2}(\frac{I_t}{I_{t-1}} - 1)^2). \tag{49}$$

Aggregate real dividend is

$$d_t = Y_t - w_t L_t - I_t - \frac{\zeta^P}{2}(\frac{P_t}{P_{t-1}} - 1)^2 Y_t. \tag{50}$$

Aggregate output is

$$Y_t = \exp(A_t) K_{t-1}^{\xi} L_t^{1-\xi}. \tag{51}$$

Inflation rate is given by

$$\pi_t = \frac{P_t}{P_{t-1}}. \tag{52}$$

3.3 Market clearing and monetary policy

I assume a standard Taylor rule for monetary policy as

$$\frac{R_t^n}{R_{ss}^n} = (\frac{R_{t-1}^n}{R_{ss}^n})^{\theta_M} \{(\frac{P_t}{P_{t-1}})^{\phi_\pi}\}^{(1-\theta_M)} \epsilon_t^M, \tag{53}$$

where $\theta_M \in [0, 1)$ is the persistence parameter of the monetary policy, $\phi_\pi(> 1)$ is the reaction parameter of monetary policy to inflation rate, and ϵ_t^M is an i.i.d. shock to the nominal interest rate (monetary policy shock). Aggregate labor supply is defined as

$$L_t = \alpha L_{s,t} + (1 - \alpha) L_{o,t}. \tag{54}$$

Aggregate consumption is

$$C_t = \alpha C_{s,t} + (1 - \alpha) C_{o,t}. \tag{55}$$

The resource constraint of the entire economy becomes

$$\begin{aligned} Y_t = {} & C_t + I_t + \frac{\zeta^P}{2}(\frac{P_t}{P_{t-1}} - 1)^2 Y_t \\ & + \alpha\big\{p_t^s \frac{\zeta^S}{2}(S_{s,t} - S_{ss})^2 + \frac{\zeta^B}{2}(B_{s,t} - B_{ss})^2\big\} + (1-\alpha)\big\{p_t^s \frac{\zeta^S}{2}(S_{o,t} - S_{ss})^2 + \frac{\zeta^B}{2}(B_{o,t} - B_{ss})^2\big\}. \end{aligned} \tag{56}$$

4 Quantitative analysis

In this section, I show the quantitative implications of the model economy. Using a calibrated model, I compute the theoretical moments and the impulse responses of macroeconomic variables to shocks. I assume time frequency is quarterly throughout this paper. The model is solved by the first-order perturbation method around the steady state.[10] By applying the first-order perturbation method to the simplified model setting of Adam et al. (2017), where dividends and wages are exogenous, I can generate a volatility of price dividend ratio similar to Adam et al. (2017) as shown in Oshima (2019).

Theoretical moments with the presence of both of two agents show realistic stock price and risk free rate volatilities at the same time. Homogenous models with subjective beliefs have difficulty in having this feature.

4.1 Calibration

Table 1 lists the choice of parameter values for the baseline model. Parameter values are calibrated in quarterly rates and assume those of the U.S. economy. I set the basic parameters, the rate of relative risk aversion γ, habit formation parameter ϕ, inverse of Frisch elasticity ν, depreciation rate ψ, capital share ξ, and investment adjustment cost parameter ζ^I following Christiano et al. (2005). The discount rate β is 0.99, which is within the conventional range. The relative utility weight of labor χ is set so that the steady state labor amount becomes 0.3. In Christiano et al. (2005), the Philips curve is not formulated based on Rotemberg type price adjustment costs, which I use in the model. For the demand elasticity parameter η and price adjustment parameter ζ^P, I follow Ireland (2001), who uses Rotemberg type price adjustment costs.

The monetary policy reaction parameter ϕ_π has a conventional value, 1.5. The monetary policy persistence parameter θ_M is 0.8 following Christiano et al. (2005) or Smets and Wouters (2007). The autoregressive parameters of productivity ρ_A and investment specific shock ρ_V follow Smets and Wouters (2007). The autoregressive parameter of the preference shock ρ_Z follows Ireland (2001).

The model specific parameter is the Kalman gain g. In Adam et al. (2017), its estimated value is 0.02-0.03 depending on the assumptions. However, in my general equilibrium model, this range of g results in explosive paths. As seen in the fourth term of (33), a large value of g leads to an explosive path of stock price when this equation is solved backward. Adam et al. (2016) estimate this value at 0.007-0.008. I set the value of g at 0.007 so that the model can satisfy the Blanchard-Kahn condition and generate realistic moments of stock price. The stock adjustment cost parameter ζ^S and the bond adjustment cost parameter ζ^B are set so that the relative size of the stock adjustment cost becomes larger than the

[10] Winkler (2019) uses the second-order perturbation method to solve his New Keynesian model. I use the first-order perturbation method because excess return of stock is out of my interest.

bond adjustment cost, which seems to be realistic. The calibrated values of the bond and stock adjustment costs in Table 1 do not generate notable differences in allocation variables in impulse responses under any population share of households with subjective beliefs, α. The baseline population share of subjective belief households α is set to fit the theoretical moments of stock price to the data.

Parameters	Value	Description
δ	0.99	Discount rate
γ	1.0	Rate of relative risk aversion
ϕ	0.65	Habit formation parameter
g	$\frac{1}{150}$	Constant Kalman gain
ψ	0.025	Depreciation rate
χ	6.5	Relative utility weight of labor
ν	1	Inverse of Frisch elasticity
ξ	0.36	Capital share
ζ^I	2.5	Investment adjustment cost parameter
ζ^P	77	Price adjustment parameter
η	6	Demand elasticity
ϕ_π	1.5	Monetary policy reaction parameter
α	0.94	Population share of subjective belief households
θ_M	0.8	Monetary policy persistence parameter
ζ^S	10	Stock adjustment cost parameter
ζ^B	0.0005	Bond adjustment cost parameter
ρ_A	0.95	Autoregressive parameter of productivity shock
ρ_V	0.71	Autoregressive parameter of investment specific shock
ρ_Z	0.89	Autoregressive parameter of preference shock

Table 1: Baseline parameters

4.2 Steady state values

Table 2 shows the steady state values of our model. These values are in quarterly rates and the subscript ss indicates steady state value. These values do not depend on whether the model is of subjective or objective beliefs because I set the steady state m at 1. For comparison, this table shows actual data values averaged over 1980-2017.[11] C_{ss}/Y_{ss} and I_{ss}/Y_{ss} are close to the actual value. The actual data for Y in these two fractions is the sum

[11]The sample period almost corresponds to the periods during which the U.S. central bank targeted the interest rate rather than money growth.

of C and I with a deflator adjustment as explained in Appendix A.1. $K_{ss}/(Y_{ss} * 4)$ is within a plausible range. The actual Y_{ss} for $K_{ss}/(Y_{ss} * 4)$ is real GDP data. The price dividend ratio p^s_{ss}/d_{ss} is similar to the data.

Variables	Steady state value	Actual data
C_{ss}/Y_{ss}	0.79	0.80
I_{ss}/Y_{ss}	0.21	0.20
$K_{ss}/(Y_{ss} * 4)$	2.14	3.16
p^s_{ss}/d_{ss}	99.0	70.4

Table 2: Steady state values

The subscript ss means steady state value. Actual data are the average of 1980-2017 quarterly data. Y_{ss} for $K_{ss}/(Y_{ss} * 4)$ is real GDP data to be consistent with K_{ss} data. Y_{ss} data for C_{ss}/Y_{ss} and I_{ss}/Y_{ss} is explained in Appendix. Actual data for p^s_{ss}/d_{ss} is the price earnings ratio.

4.3 Model moments

Table 3 compares the second moments of the baseline model with the actual data. Moments are calculated on a quarterly basis. I show the model moments with four different population shares of households with subjective beliefs (α at 1, 0.94, 0.5, and 0). Theoretical moments of the models are based on 0.7% productivity shocks, 0.3% monetary policy shocks, 0.3% preference shocks, and 0.7% investment specific shocks in standard deviation on a quarterly basis.[12] Actual data moments cover 1980-2017 of the U.S. economy. I take the natural log and de-trend it by third-order time polynomial regression except interest rate data. Details about data source are given in Appendix A.1.

The standard deviation of stock price of the data is 0.204. The cases with $\alpha = 0.94$ show a realistic standard deviation of stock price, 0.187. What to note is that the overall theoretical second moments match the data well without assuming a high risk aversion rate or high habit formation. In contrast, the standard deviation of stock price in $\alpha = 0.5$ case is 0.051 and that in $\alpha = 0$ case is 0.031. These values are too small compared to the data.

Assuming a small share of objective households in addition to subjective households provides quantitative benefits such that the model has realistic stock price and risk free rate volatilities at the same time. In the case of homogenous subjective beliefs ($\alpha = 1$), the

[12] In Smets and Wouters (2007), the productivity shock is estimated as 0.45%, monetary policy shock is estimated as 0.24% , preference shock is estimated as 0.24%, and investment specific shock is estimated as 0.45% in standard deviation on a quarterly basis. The sample period in their study is 1966-2004.

standard deviation of stock price is 0.917, which is much larger than the data, 0.204, with realistic risk-free rate volatility. This is because the reactions of stock prices to interest rate shocks are too large in this homogenous case. In the case of $\alpha = 0.94$, the standard deviation of stock price is close to the data, with the risk-free rate volatilities close to the data as well. When α is less than 1, future dividends and discount rates play the roles as seen in the last term of the right hand side of (33). This reduces the effect of the real interest rate at time t which has strong impacts on stock prices with subjective beliefs.

Several correlations in the model become close to the data when I assume positive α. The correlation between stock price and dividend is negative and close to the data in $\alpha = 1$ and $\alpha = 0.94$ cases. The data show a negative correlation between stock prices and dividends while $\alpha = 0$ and $\alpha = 0.5$ cases show positive correlations.[13] When output increases under negative interest rate shocks, investment increases under the lowered real interest rates in response to these shocks. Since investment is more volatile than output, dividends are squeezed to a certain extent as implied in equation (38). In cases with $\alpha = 1$ and $\alpha = 0.94$, stock prices react strongly to decreases in real interest rates. Hence, stock prices and dividends show a negative correlation in these cases. In the case with $\alpha = 0$ and $\alpha = 0.5$, since stock price is mainly determined by the sum of future discounted dividends in contrast to the cases with $\alpha = 1$ and $\alpha = 0.94$, dividends as well as real interest rates play major roles in affecting stock prices. Therefore, these cases do not show negative correlation between stock price and dividend.

The correlation between stock price and output in $\alpha = 0.94$ case is closer to the data than in other cases. $\alpha = 1$ case indicates a lower correlation than the data, and $\alpha = 0$ or $\alpha = 0.5$ cases shows higher correlation than the data. When α is high, near term real interest rates play an important role in stock pricing and momentum effects, by which the stock price reacts to past growths of itself, strongly affect stock prices as well. As shown in the next subsection, the momentum effects on stock prices in high α cases generate oscillations that are not observed in output responses. These oscillations reduce the correlation between stock price and output. On the other hand, objective beliefs imply that stock prices are affected by future flows of real interest rates rather than near term real interest rates. In a New Keynesian model including this model, future flow of real interest rates is a key determinant of output. Since future real interest rates matter to both stock prices and output, $\alpha = 0$ or $\alpha = 0.5$ case shows higher correlation between stock prices and output than the data and

[13] The definition of dividend in the model shown in (50) is not the same as that in the statistics of U.S. Bureau of Economic Analysis because of my model structure. The data are constructed here to be consistent with the definition of dividend in the model.

$\alpha = 1$ or $\alpha = 0.94$ cases.

Adam et al. (2017) claim that survey measures of investors' expected return correlate positively with the price dividend ratio, whereas rational return expectations correlate negatively with the price dividend ratio.[14] The actual correlation between survey expectations of the stock price growth and price dividend ratio provided by Adam et al. (2017) based on 1946-2012 data is 0.79. When $\alpha = 1$, the correlation between price dividend ratio $\log P^s/d$ and stock price growth expectation $\log m$ in the model is 0.88, which is close to that in Adam et al. (2017). However, I find that the positive correlation between the price dividend ratio and the weighted average of stock price growth expectation, $\alpha m_t + (1-\alpha)E_t[p^s_{t+1}/p^s_t]$, are limited only to the case of $\alpha = 1$ or cases with α very close to 1.

Variables	Model ($\alpha = 1$)	Model ($\alpha = 0.94$)	Model ($\alpha = 0.5$)	Model ($\alpha = 0$)	Actual data
$SD[\log R^f]$	0.020	0.020	0.020	0.020	0.024
$SD[\log C]$	0.026	0.026	0.026	0.026	0.026
$SD[\log P^s]$	0.917	0.187	0.051	0.031	0.204
$SD[\log d]$	0.060	0.060	0.060	0.054	0.045
$SD[\log Y]$	0.032	0.032	0.032	0.032	0.037
$SD[\log I]$	0.068	0.068	0.068	0.068	0.113
$SD[\log \pi]$	0.004	0.004	0.004	0.004	0.004
$Corr[\log P^s, \log Y]$	0.32	0.546	0.886	0.902	0.59
$Corr[\log P^s, \log d]$	-0.62	-0.20	0.12	0.16	-0.44
$Autocor[\log C(-1)]$	0.99	0.99	0.99	0.99	0.97
$Autocor[\log C(-2)]$	0.98	0.98	0.98	0.98	0.94
$Autocor[\log P^s(-1)]$	0.88	0.73	0.92	0.94	0.95
$Autocor[\log P^s(-2)]$	0.68	0.52	0.87	0.89	0.87

Table 3: Second Moments

The actual data are 1980-2017 quarterly U.S. data de-trended by time polynomial regressions except for interest rate data.

[14]Due to data accessibility to subjective stock price growth expectations to cover the same data periods with other variables, I did not show $Corr[\log P^s/d, \log(\alpha m_t + (1-\alpha)E_t[p^s_{t+1}/p^s_t])]$ in Table 3.

4.4 Impulse response

This subsection shows the impulse responses to productivity and monetary policy shocks. I show that heterogeneity in beliefs provides more realistic impulse responses of stock price than homogenous cases. The time frequency is quarterly. Figure 1 indicates the impulse responses to a 1.0% positive productivity shock on a quarterly basis. Figure 2 indicates the impulse responses to a 0.25% positive monetary policy shock (shock to increase the nominal interest rate) on a quarterly basis (1.0% at an annualized rate). In these figures, I compare the responses under different shares of households with subjective beliefs (α=1, 0.94, and 0). The three cases do not show differences except for the responses of stock price p^s and subjective expectation of capital gain m. This can be considered as an advantage of this model because it can generate large stock price reactions without unrealistically increasing the responses of fundamental variables.

As summarized by Challe and Giannitsarou (2014), empirical studies suggest that a 100 basis point increase of the interest rate at an annual rate is associated with a 2.2%-9% decrease in stock prices. The model with $\alpha = 0.94$ shows a 9.3% decrease as initial reaction to a shock of 100 basis points of the nominal interest rate at an annual rate (25 basis points at a quarterly rate) and 4.5% decreases over four quarters on average at and after the shock in its impulse response analysis. This is close to the estimates of empirical studies. In contrast, stock prices in the homogenous subjective belief case ($\alpha = 1$) react to monetary policy shocks too strongly. The model with $\alpha = 1$ shows a 32.3% decrease as initial reaction to a shock of the same size and 33.9% decreases over four quarters on average at and after the shock, which are much larger reactions than the estimates of empirical studies. In the cases of $\alpha = 0$, the reactions are a 0.7% decrease as initial reaction and 0.3% decreases over four quarters on average at and after the shock, which are much smaller than the estimates of empirical studies. Thus, assuming heterogeneity helps to generate realistic stock price responses to interest rate shocks.

Why do levels of α change the stock price responses? The value of α determines to what degree future values of real interest rates and dividends affect stock prices. In (33), as the value of α decreases, the impact of the current real interest rate ($\hat{R}^f_t$) becomes smaller since the share of households with subjective beliefs decreases. On the other hand, as the value of α decreases, the impacts of expectation ($E_t \hat{p}^s_{t+1}$) become more relevant since the share of the households with objective beliefs increases.

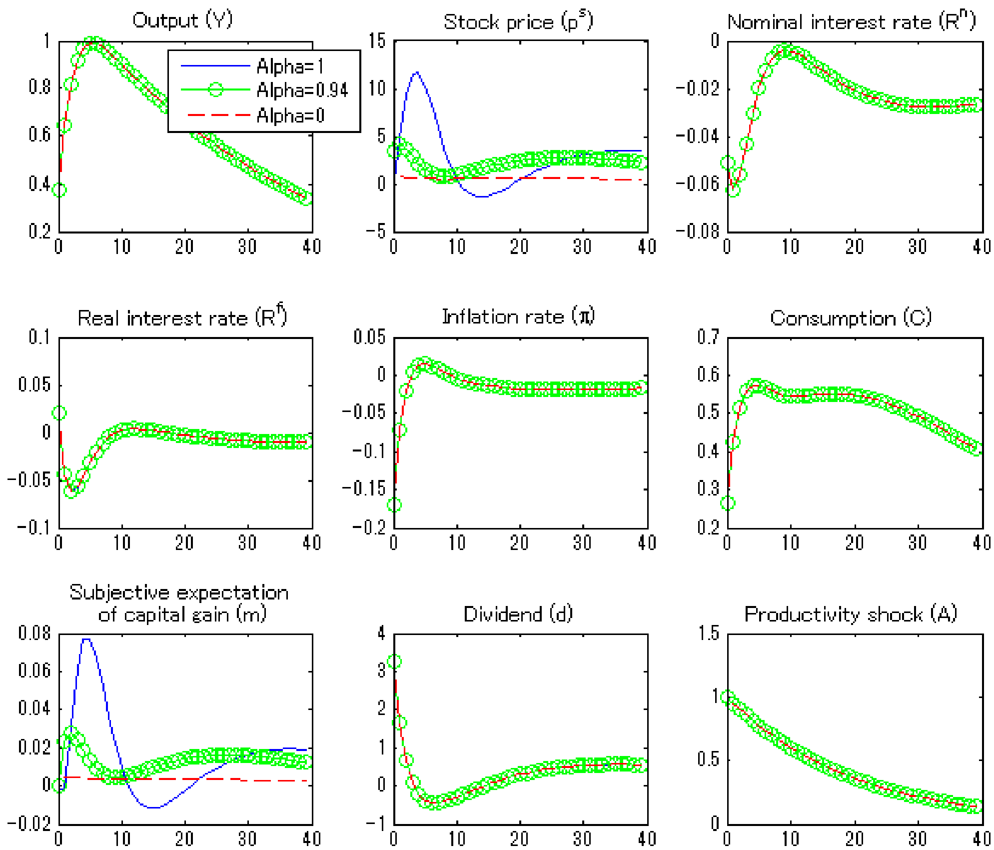

Figure 1: Impulse response of the stock price to the productivity shock

Impulse response to a 1.0% positive productivity shock. The solid line indicates the subjective expectation case. The dotted line indicates the objective expectation case. 1 on the vertical axis scale amounts to 1%. Return variables, R^n and R^f are in percentage point differences from their steady states. Otherwise, variables are shown in percentage deviation from their steady state levels.

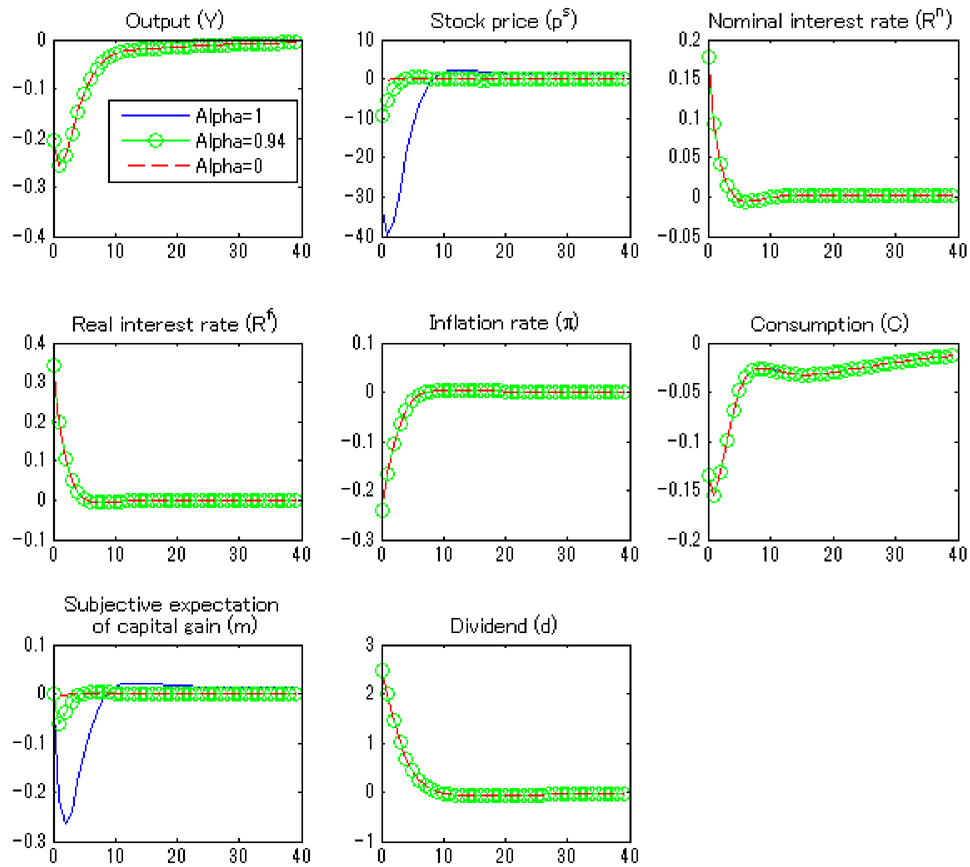

Figure 2: Impulse response of the stock price to the monetary policy shock

Impulse response to a 0.25% positive monetary policy shock (1.0% at an annualized rate). The solid line indicates the subjective expectation case. The dotted line indicates the objective expectation case. 1 on the vertical axis scale amounts to 1%. Return variables, R^n and R^f are in percentage point differences from their steady states. Otherwise, variables are shown in percentage deviation from their steady state levels.

5 Monetary policy and stock price reaction

In this section, to study the effects of monetary policy stances on stock prices, I show how the parameters in the monetary policy rule magnify or compress stock price volatilities under heterogeneous beliefs. By introducing Bayesian learning and heterogeneity, it is possible to relate the realistic volatilities of stock price to parameters that construct monetary policy. This analysis contributes to discussions about "gradualism" because it shows how the stance of monetary policy affects stability of the stock market. The shocks have the same sizes as in section 4.4. A positive productivity shock is 1.0% on a quarterly basis and positive monetary policy shock is 0.25% on a quarterly basis (1.0% at an annualized rate). I set $\alpha = 0.94$ in this section. In Appendix A.2, I investigate the effects on stock price volatilities under the habit formation parameter ϕ with different values. In addition, I show the effects of changes in the Kalman gain g on stock price responses.

5.1 Monetary policy persistence parameter and stock price

How do the monetary policy parameters in (53) amplify stock price volatilities? This question is important because different monetary policy stances would change the stock price volatilities in an economy with heterogeneous beliefs that can generate realistic volatilities.

Figure 3 indicates the impulse responses of stock prices to a positive productivity policy shock under two different persistence parameters of monetary policy, $\theta_M = 0.8$ and $\theta_M = 0$. The other parameters are the same as in the baseline setting in Table 1. In the case with higher monetary policy persistence parameter, $\theta_M = 0.8$, the stock price reaction becomes smaller to the positive productivity shock than $\theta_M = 0$ case. A strong monetary policy persistence implies that the nominal interest rate decreases only gradually in response to decreases in inflation rates associated with a positive productivity shock. Because the decreases in the nominal interest rates are small, inflation rate is low. As implied by (33), the stochastic discount factor (or real interest rate) at time t has a dominant impact on stock prices at time t under subjective expectations. Therefore, less decreases in the real interest rate from the monetary policy rule in $\theta_M = 0.8$ case generate smaller increases in the real stock price than $\theta_M = 0$ case. Because the initial stock price reaction is small, the momentum effect also becomes weak as implied in the last term of (33). Therefore, the overall impacts on stock price are muted in $\theta_M = 0.8$ case.

By contrast, in a response to a positive monetary policy shock (shock to increase the nominal interest rate), stock prices drop further in the case of high monetary policy persistence parameter, $\theta_M = 0.8$, than $\theta_M = 0$ case. There are two reasons to lead to this result. The first reason is the nominal interest rate path. Strong persistence implies that an increase in nominal interest rate after an unexpected positive nominal interest rate shock sustains for long periods. This effect continues to push the "current" nominal interest rates up at each point of time. The second reason is the inflation expectation. Because of the high persistence of the nominal interest rate and forward-looking nature of the New Keynesian model, the inflation expectation becomes low. This implies an increase in the real interest rate given the current nominal interest rate. By these reasons, a strong monetary policy persistence along with a monetary policy shock implies sustained high volatility of the near-term real interest rates and stock prices after a monetary policy shock. A realized stock price drop generates

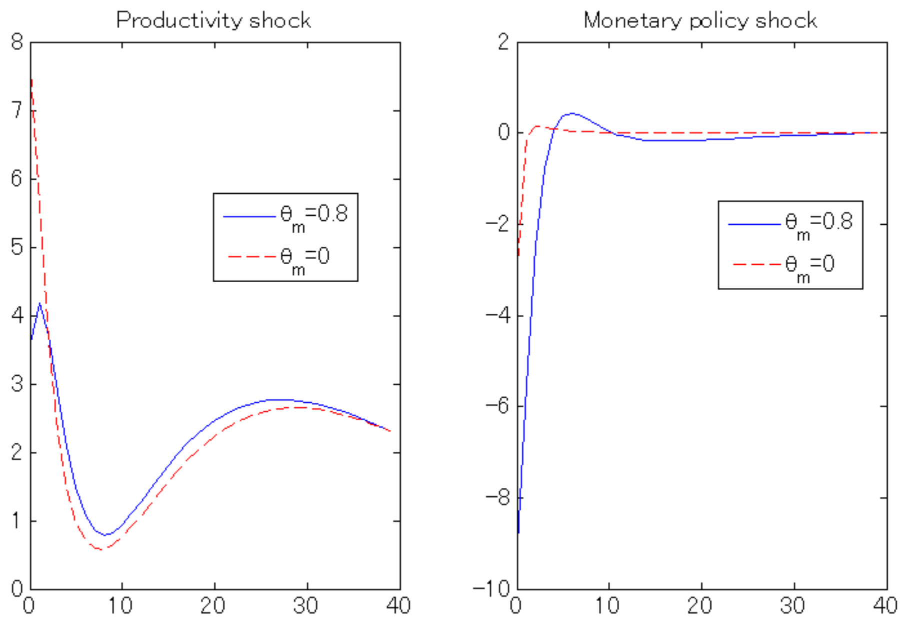

Figure 3: Stock price responses in $\alpha = 0.94$ case with different monetary policy persistence parameters

Impulse response to a 1.0% positive productivity shock and 0.25% (1.0% at an annualized rate) positive monetary policy shock. These sizes are the same as those in section 4.4. The value of α is 0.94. 1 on the vertical axis scale amounts to 1% deviation of stock price from its steady state level.

the momentum as shown in (33) and stock price reaction magnifies.

These analyses provide some policy implications to the "gradualism" of monetary policy management. In Section 1, I mentioned that there were discussions in actual policy making in Federal Open Market Committee in February 1994 with regard to whether a 25 basis point policy tightening was preferable to a 50 basis point tightening, because some members considered the larger move to have a very high probability of "cracking financial markets" (FOMC Secretariat (1994)). The analysis based on the model implies that what is important for stock price stability is not only sizes of changes in policy rates, but also types of structural shocks and the monetary policy rules behind them.

5.2 Monetary policy reaction parameter and stock price

Figure 4 indicates the stock price responses to positive productivity shocks and positive monetary policy shocks under two different reaction parameters of monetary policy to inflation rate, $\phi_\pi = 2$ and $\phi_\pi = 1.1$. Other parameters are the same as in the baseline setting in

Table 1. In the case of the low monetary policy reaction parameter, $\phi_\pi = 1.1$, the stock price reaction to a positive productivity shock becomes small. In $\phi_\pi = 1.1$ case, the monetary policy is less sensitive to decreases in inflation rates than $\phi_\pi = 2$ case and this reduces the size of real interest rate decreases and results in small reactions of stock price to productivity shocks.

Under a positive monetary policy shock, a nominal rate hike decreases inflation rate. In response to the decrease in inflation, the monetary policy rule simultaneously adjust the nominal interest rate. In the case with $\phi_\pi = 1.1$, the monetary policy is less reactive than the case with $\phi_\pi = 2$. Consequently, the real interest rate in the case with $\phi_\pi = 1.1$ becomes relatively higher at time t than the case with $\phi_\pi = 2$. Because stock price reacts strongly to a real interest rate at time t in the model, stock price shows larger drops in $\phi_\pi = 1.1$ case than $\phi_\pi = 2$ case.

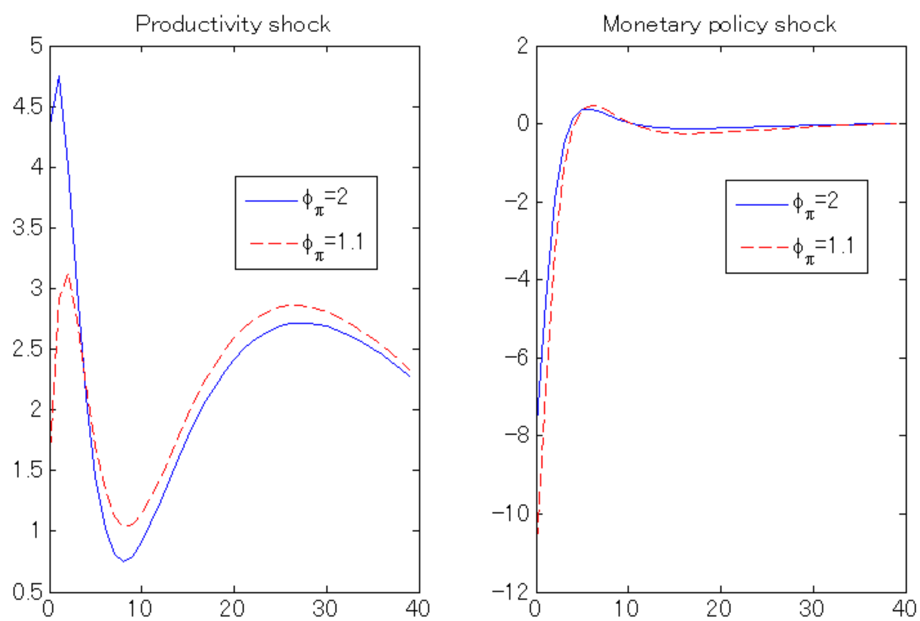

Figure 4: Stock price responses in $\alpha = 0.94$ case with different monetary policy reaction parameters

Impulse responses to a 1.0% positive productivity shock and 0.25% (1.0% at an annualized rate) positive monetary policy shock. These sizes are the same as those in section 4.4. The value of α is 0.94. 1 on the vertical axis scale amounts to a 1% deviation of stock price from its steady state level.

6 Conclusion

How stock price volatilities depend on the stance of monetary policy is a key question in this study. To investigate this issue, I develop a New Keynesian model with two types of households, those with subjective and objective beliefs about capital gains from stock prices as the first step to study the effect of heterogeneity in their beliefs in a general equilibrium model. Households with subjective beliefs construct their expectations of capital gains by Bayesian learning from observed growth rates of stock prices. Because the model is a general equilibrium model with nominal rigidity, it allows to examine the relations between monetary policy parameters and stock price responses.

Assuming two types of households provides several benefits. It enhances moment matching compared to a model with only subjective or objective beliefs without setting the relative risk aversion rate away from the conventional parameter values estimated in dynamic stochastic general equilibrium models. Incorporating two types of agents improves the quantitative performances of the model under monetary policy shocks. The model can generate plausible stock price drops in response to a positive interest rate shock, which cannot be attained in a homogenous subjective beliefs model.

The quantitative improvement of stock price reaction in response to monetary policy shocks allows me to conduct realistic analysis of how the stance of monetary policy affects stock price volatilities. Strong inertia of monetary policy does not necessarily decrease volatilities of stock price. When an interest rate shock occurs, a persistent monetary policy magnifies the volatilities of stock prices. On the other hand, when a productivity shock occurs, a persistent monetary policy reduces them. This result provides an implication to discussions on the "gradualism" of monetary policy management. What is important for stock price stability is not only the sizes of changes in policy rates, but also types of structural shocks and the monetary policy rules behind them.

Finally, the model is solved by the first-order perturbation method for simplicity of calculation and abstracts higher-order terms. By including these, it would be possible to argue excess returns and volatility at the same time. I assume that the population share of each household is fixed exogenously. Endogenous population share changes could further improve moment matching. Bayesian estimations of parameters including the Kalman gain parameter would be also an interesting approach to examine the model validity. These considerations remain for further research.

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A Appendix

A.1 Data Sources

The actual data formations are conducted by procudures as follows. Actual data used in this paper is of the United States. Real consumption, real investment, and real wage data are from the Federal Reserve Bank of St. Louis FRED economic data base (`https://fred.stlouisfed.org/`). FRED series IDs for these variables are PCECC96, GPDIC1, and LES1252881600Q, respectively. These are seasonally adjusted quarterly data. For real output data, I took a sum of seasonally adjusted nominal consumption and investment, and divided it by the implicit price deflator for gross domestic purchases. The corresponding FRED series ID are PCEC, GPDI, and A712RD3A086NBEA, respectively. Real capital stock data is real net stock (private fixed assets) from the Bureau of Economic Analysis. The total nominal compensation of employee data is from FRED and its FRED series ID is A4102C1Q027SBEA. This is seasonally adjusted. To retrieve the labor amount, I divided total nominal compensation data by real wage and implicit price deflator for gross domestic purchases. For dividend for which development is consistent with the model, I subtracted nominal total compensation and nominal investment (FRED series ID: GDPI) from nominal output, which I define as the sum of nominal consumption (FRED series ID: PCEC) and nominal investment, and divided it by the implicit price deflator for gross domestic purchases. The stock price data is S&P 500 index data. Real stock price data is deflated by implicit GDP deflator (FRED series ID: GDPDEF). As the above dividend data covers all U.S. companies, including unlisted companies, I constructed dividend data, which is a product of the S&P stock index level and its dividend yield for the price dividend ratio in 4.2.

Interest rate data is based on the Federal Reserve Bank of New York' s treasury term premia database (`https://www.newyorkfed.org/research/data_indicators/term_premia.html`). I use one-year fitted zero coupon market yield of U.S. treasury for te nominal interest rate. I deflated it by the actual inflation rates of the implicit gross domestic product deflator. In subsection 4.2, I used gross yields data. Both rates in subsection 4.3 are of the natural log of gross yields.

In subsection 4.3, I take the natural log and de-trend by the third-order time polynomial regression, except interest rate data. I chose third-order because the Akaike information criterion shows the lowest value when I examined the fit up to the fourth order. For consistency, I used the same order in time polynomial regressions for other variables too. Real capital data is real net private fixed assets provided on annual basis. I translated them to quarterly basis by interpolation for de-trending.

A.2 Kalman gain, habit formation parameter, and stock price

This appendix subsection shows how parameters other than those of monetary policy affect stock price volatilities.

A.2.1 Kalman gain value

I show how a Kalman gain value g affects the stock price reaction to a productivity and monetary policy shock, given the other parameters are kept at the baseline values in figure 5. The figure includes several values of g, $\frac{1}{50}$, $\frac{1}{150}$, and $\frac{1}{300}$. When the Kalman gain g is large, investors react to observed stock price changes more sensitively. The larger g becomes, the stronger the effect of momentum on stock prices becomes. This feature is the same for productivity and monetary policy shock cases.

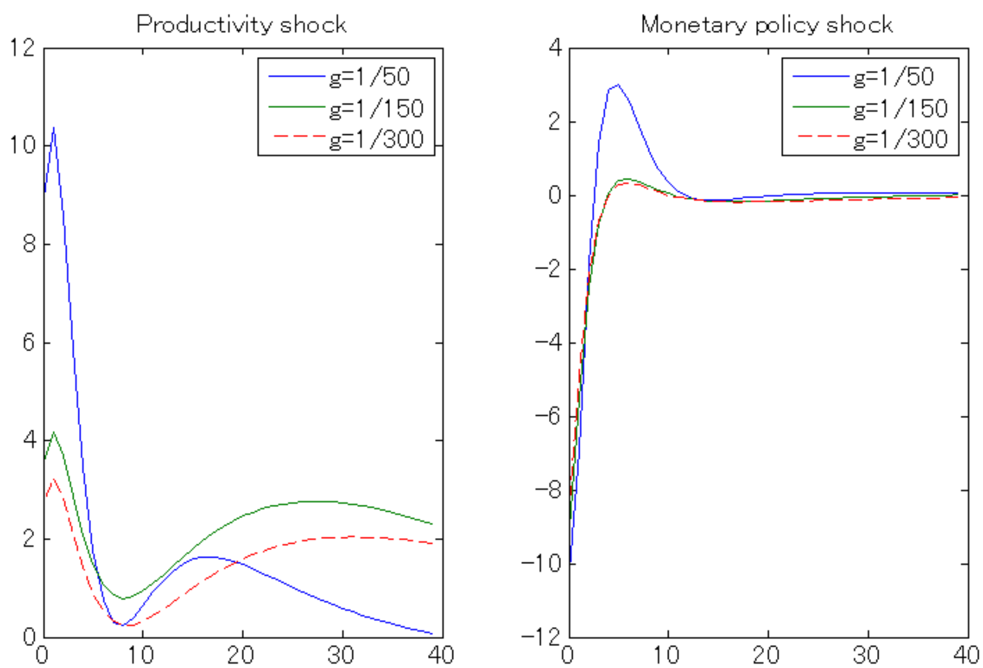

Figure 5: Impulse response of stock price to productivity and monetary policy shocks under different Kalman gain settings

Impulse response to a 1.0% positive productivity shock and 0.25% (1.0% at an annualized rate) positive monetary policy shock under different values of Kalman gain g. 1 on the vertical axis scale amounts to 1% deviation from their steady state levels.

A.2.2 Habit formation parameter

I discuss the effects of the habit formation parameter on stock price volatility. I show the high habit formation case, $\phi = 0.8$, and the low habit formation case, $\phi = 0$, in Figure 6. Under the productivity shock, a high habit formation parameter amplifies stock price volatilities. The intuition behind this is that the strong need for consumption smoothing by high habit formation generates large volatilities of real interest rates. In addition, when α is close to 1, the shape of dividend flows over time matters because $\hat{d}^s$ at time $t+1$ has a strong impact in (33). When consumption smoothing is strong and the stochastic discount factor increases in response to a positive productivity shock, firms are inclined to increase their dividends far into the future. Therefore, $\hat{d}^s_{t+1}$ becomes small by firms' optimization in case of strong consumption smoothing. This implies small stock price reactions from the second term of the right hand side in (33). However, these dividend effects on stock price have a much smaller magnitude than the real interest rate effects, because the second term has less weight than the first term. Therefore, the stock price shows a larger increase in $\phi = 0.8$ case than $\phi = 0$ case.

Under a positive monetary policy shock, the high habit formation parameter case does not differ much from the low habit formation parameter case, as shown in the right-hand side chart in figure 6. A monetary policy shock directly changes the real interest rate given the price rigidity. Therefore, stochastic discount factors that affect stock prices do not show notable differences between two habit formation parameter cases.

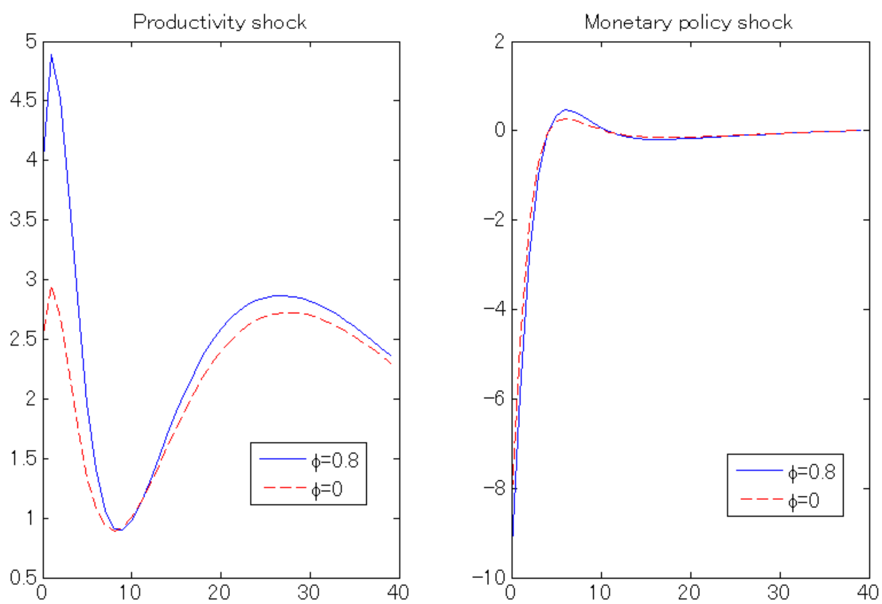

Figure 6: Stock price response with different habit formation parameters

Impulse responses to a 1.0% positive productivity shock and 0.25% (1.0% at an annualized rate) positive monetary policy shock. These sizes are the same as those in section 4.4. The value of α is 0.94. 1 on the vertical axis scale amounts to 1% deviation of stock price from its steady state level.