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“Subjective Beliefs, Monetary Policy, and Stock Price Volatility”

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Subjective Beliefs, Monetary Policy, and Stock Price Volatility *

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Abstract

The main purpose of this study is to understand how the stance of monetary policy affects stock price volatility in a New Keynesian model with investors who have subjective beliefs about stock price growth. I assume that investors construct subjective beliefs about expected capital gains from stock prices by Bayesian learning from observed growth rates of stock prices. I design the model so that the effects of the existence of subjective households are minimal, i.e., it affects only stock prices. I find that higher monetary policy persistence increases stock price volatilities under the interest rate shock because the subjective beliefs imply myopic pricing in which near-term pricing kernels (or real interest rates) and near-term dividends matter. This result contrasts with stock pricing under the rational expectation, in which future discounted dividends matter.

JEL classification: D83, D84, E44, E52, G12, G14

Keywords: stock price, asset pricing, subjective belief, sticky prices, New Keynesian

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1 Introduction

Empirical studies show that unexpected monetary policy shocks affect stock prices and the effects of such shocks on stock prices change over time. Bernanke and Kuttner (2005) document that unexpected monetary policy actions affect stock prices. Laopodis (2013) finds that the nature of dynamic relationship between the monetary policy of the U.S. and its stock market was different in each of operating regimes under three chairmen of the federal reserve board (pre-Volker, Volker, Greenspan). Chen (2007) argues that its effects are larger in bear markets than bull markets. Paul (2019) empirically shows that stock market reactions to monetary policy are time varying. However, there is no clear consensus about why the reactions of stock prices to monetary policy shocks change over time. In this study, I examine a possibility that the stance of monetary policy affects stock responses to monetary policy shocks, using a model that generates realistic volatilities of stock prices. Understanding the relationship between the stance of monetary policy and stock price behaviors contributes to a discussion on whether monetary policy inertia (gradualism) helps reduce financial market volatilities in the stock market.[1]

To investigate this issue, I examine a possibility that the stance of monetary policy could affect stock price responses to monetary policy shocks, using a model assuming subjective beliefs about stock price growth that generate realistic volatilities of stock prices. Major theoretical research on volatilities of stock price (including habit formation or long-run risks) usually assumes rational expectation.[2] However, rational expectation models have difficulty in generating positive correlation between capital gain expectation and stock price return, which is empirically observed.[3] Another approach to explaining stock price volatility is to assume subjective belief. Adam et al. (2017) show that the subjective belief model can replicate stock price volatilities, where households learn about expected capital gains from past price growths, given exogenous consumption and dividend processes. In addition, this approach is successful in replicating positive correlation between capital gain expectation and

[1] Srour (2001) lists an argument that large surprises in short-term interest rates can cause volatility in financial markets as one reason of smoothing interest rates. Rudebusch (2006) examines a discussion about a rationale for policy gradualism, which is a desire to reduce the volatility in asset prices. González-Páramo (2006) argues that a gradual monetary policy could reduce the likelihood of financial market disruptions. In actual policy making, FOMC Secretariat (1994) records that chairman Alan Greenspan led discussions about whether a 25 basis point policy tightening was preferable to a 50 basis point tightening because some members considered that the larger move had a very high probability of cracking financial markets. To this question, Bernanke (2004) gives no decisive conclusion on whether gradualism provides stability of financial market or asset prices.

[2] This area of research is often called the volatility puzzle.

[3] See Adam et al. (2017).

stock price return.

In this study, I develop a New Keynesian model with households that have subjective beliefs about the capital gains from stock prices. I use a general equilibrium for this argument because under nominal rigidity monetary policy affects real interest rates and pricing kernels, while partial equilibrium models usually assume that consumption path and consequently the pricing kernel are exogenously given. Using this model, I can examine the relations between key structural parameters including those in monetary policy rules and volatility of stock price. Firms pay dividends to households. Households consume goods, supply labor, and save their wealth in stocks. Households construct expectations about capital gains from stock prices by Bayesian learning from the observed growth rates of stock prices. Households do not know the beliefs of other households, although they are homogenous and do not know the pricing function that maps fundamentals to stock prices. These belief structures follow Adam et al. (2017). The model includes habit formation in the household utility and investment adjustment costs in capital formation. To focus on the analysis of stock price behaviors, I design the model so that the effects of the existence of subjective households are minimal, i.e., it affects only stock prices while business cycle properties are kept very standard.

My main finding is as follows. In a general equilibrium model with Bayesian learning about capital gains, the near-term real interest rate is a key variable in explaining stock price volatility. Consequently, a combination of parameters and shocks that increase the volatilities of the near-term real interest rate increases stock price volatility. This implies that the strong inertia of monetary policy rule does not necessarily reduce asset price volatility. Its effects depend on what kind of shock the economy is experiencing as follows.

When the interest rate shock occurs, the persistent monetary policy increases stock price volatility. On the other hand, when the productivity shock occurs, the persistent monetary policy reduces the volatility. The intuitive explanation of this result is as follows. Under the positive monetary shock (shock to increase the nominal interest rate), when monetary policy is persistent, increases of the nominal interest rate sustain for long periods and continue to push up "current" nominal interest rates at each point of time. In addition, low inflation expectation due to persistently high nominal interest rate pushes up real interest rates. Therefore, strong monetary policy persistence implies a sustained high volatility of near-term real interest rates and thus stock prices after the monetary policy shock occurs.

On the other hand, when the economy experiences a positive productivity shock, decreases in today's nominal interest rate responding to the productivity shock become gradual and small when monetary policy is persistent. This is because the monetary policy rule with high interest rate smoothing implies that policy rate is not strongly reactive to real-time changes of inflation because of the high weight allocated to lagged interest rate term in the rule. The gradual reduction of nominal interest rate leads to low expected inflation rate and results in small volatility of today's real interest rate and stock prices.

Strength of consumption habit formation also affects stock price volatilities. When the habit formation parameter is large, under a positive productivity shock, stock prices increase largely because of the high volatility of the near-term stochastic discount factor. Under positive interest rate shocks with a high habit formation parameter, its impact is small.

Theoretical studies that explain the volatility puzzle in stock markets based on rational expectation include, for example, long-run risks by Bansal and Yaron (2004) and habit formation by Campbell and Cochrane (1999). To generate volatilities, these models use pricing kernels that differ from simple time separable utility functions. Research about subjective expectations includes, for example, Timmermann (1993) and Collin-Dufresne et al. (2016). Both studies consider investors who form subjective expectations about fundamentals by learning. Choi and Mertens (2013), Barberis et al. (2015), and Hirshleifer et al. (2015) assume that investors form expectations about fundamentals by extrapolation. Another strand of the belief-based approach focuses on subjective belief about asset price or return rather than on fundamentals. For example, Lansing (2010) considers near-rational bubbles, which imply excess volatility in the long-run equilibrium. In his model, an agent constructs a forecast of a composite variable that depends on both prices and dividends by learning. De Long et al. (1990) consider a model with investors whose beliefs are extrapolative.

Adam et al. (2017) can be categorized in this type of research, which focuses on subjective beliefs about asset prices or returns. They assume that investors who know fundamentals have subjective beliefs about capital gains and do not have knowledge about a pricing function of stock price mapping from fundamentals. Investors' capital gains expectations are influenced by the capital gains observed in the past. They emphasize that survey measures of investors' expected return (or capital gain) available for the US economy correlate positively with the price dividend ratio, while rational return expectations correlate negatively with the price dividend ratio.

In the models mentioned above, including Adam et al. (2017), the consumption and dividend streams are exogenously given. To investigate relations between a wider range of deep parameters, including those of monetary policy and stock price volatility, I need to modify it so that it becomes a general equilibrium model. Usually, general equilibrium models with production under rational expectation face difficulties in generating stock price volatility because consumers smooth away the consumption fluctuation. Agents can easily alter their production plans to reduce fluctuations in consumption. To mitigate these problems, Jermann (1998) shows that a model of a production economy with both capital adjustment costs and habit formation preferences can explain the historical equity premium.[4] De Paoli et al. (2010) discuss implications in asset pricing in a New Keynesian model featuring habit formation, capital adjustment costs, and a staggered price setting mechanism. However, these rational expectation models struggle to predict a positive co-movement of capital gain expectation and stock price, as mentioned in Adam et al. (2017). In addition, to generate large swings of stock price, rational expectation models usually need to assume higher relative risk aversion rate than normally calibrated in dynamic stochastic general equilibrium models.[5] My model is able to generate realistic stock price volatility without high relative risk aversion rate.

Departing from rational expectation, another strand of research examines subjective belief about stock price growth in a general equilibrium model context. Adam and Merkel (2018) propose a real business cycle model to explain business cycles and stock price volatility. This model regards capital price as stock price and is a real model. Their belief structure about capital price growth is based on the Bayesian approach found in Adam et al. (2017). Winkler (2019) analyzes stock price movements and business cycles in his New Keynesian model with learning based asset pricing similar to Adam et al. (2016) and Adam et al. (2017). Winkler (2019) assumes that agents have "conditionally model-consistent expectations". Conditionally model-consistent expectations are consistent with all equilibrium conditions of the model, except those that would convey knowledge of the price that clears the asset market, when agents solve for the perceived law of motion.[6] Stock holders are risk-neutral in his model, while households are risk-averse with access to the risk-free bond market. This eliminates direct pricing kernel effects on asset price, rather makes near-term dividend effects important.[7] In contrast to Winkler (2019), who tries to jointly match asset price and business cycle statistics by combining financial friction and learning process of asset price, this study targets the effects of the monetary policy stance on stock prices by having only minimal

[4]With no habit formation, marginal rates of substitutions are not volatile enough, because people do not care much about consumption volatility. With no adjustment costs, they choose consumption streams to avoid the volatility of marginal rates of substitution.

[5]Jermann (1998) and De Paoli et al. (2010) both assume 5 for the relative risk aversion rate.

[6]Caines and Winkler (2018) study housing price in a New Keynesian model with learning about housing price capital gains. As house quantity is directly included in households' utility, housing price has wealth effects on business cycles.

[7]In Winkler (2019), firms are allowed to hold capital under borrowing constraint linked to stock price. In stock pricing, current dividend plays a major role in determining stock price and the interest rate affects stock price through dividend paid to stock holders as firms' costs in the balance sheet.

effects of subjective beliefs on business cycles properties. I design the model so that the existence of subjective households affects only stock prices to focus on the analysis of stock price behaviors while business cycle properties are kept very standard. For this purpose, my model maintains the direct effects of the pricing kernel on asset pricing assuming risk-averse stock holders (households) to examine the effects of changes in the real interest rate by monetary policy in a straightforward manner. In contrast to Winkler (2019), I assume that agents know the stock market clearing conditions, which is considered to be standard.

The rest of this paper is organized as follows. Section 2 presents my model. Section 3 discusses the quantitative results based on the model. Section 4 investigates relations between deep parameters, including those of monetary policy and stock price volatilities. Section 5 concludes the analysis and discusses the future extension of the analysis.

2 Model

My model is built on a standard New Keynesian model. I assume Rotemberg type price adjustment costs. Firms pay dividends to households, which are output minus investment cash flow and price adjustment costs. Households consume goods, supply labor, and save their wealth in stocks. Households have subjective beliefs about stock price growth. These belief structures follow Adam et al. (2017), a model with exogenous consumption and dividend streams, and Adam and Merkel (2018), a real business cycle model.[8]

2.1 Households

The households' maximization problem basically follows the "internal rationality" discussed by Adam and Marcet (2011) and Adam et al. (2017). Internal rationality requires that agents make fully optimal decisions given a well-defined system of subjective probability beliefs about payoff-relevant external variables that are beyond their control. That is, internal rationality means a standard utility maximization given subjective beliefs about variables that are beyond their control.[9] In this study, following Adam et al. (2017), the households with subjective beliefs do not know a stock price function derived from fundamental variables and the households choose internally optimal plans of stock holdings and consumption under the probability measure "P" of subjective belief about the capital gains from stock

[8]The belief structure in Winkler (2019) is also similar. However, Winkler (2019) assumes that agents have "conditionally model-consistent expectations," which is mentioned in Section 1.

[9]In contrast, external rationality postulates that agents' subjective probability belief equals the objective probability density of external variables as they emerge in equilibrium.

prices. They form this by learning from observed past growth rates of stock prices. Whatever the agents' stock price growth expectations are, the stock price level and consumption plans satisfy the Euler equation with subjective expectations of stock price growth under the probability measure P.

The key assumptions are as follows. The households' homogeneity is not common knowledge. This information structure enables subjective expectations about capital gains of stock prices to deviate from objective expectations. The utility function is concave. This allows the stochastic discount factor to change depending on economic states. Under the probability measure P of subjective belief, the households choose internally optimal plans of stock holdings and consumption. These internally optimal plans need to satisfy market clearing conditions in the equilibrium. They know fundamental structures of dividends, wages, and behaviors of firms. By changing beliefs about capital gain from objective ones to Bayesian learning, equilibrium stock prices become different from a discounted sum of future dividends.

2.1.1 Utility specification and budget constraint

The infinitely lived representative household makes decisions on consumption, savings in stocks, and labor supply. I assume that the households' expectations about wages and dividends are rational. Households' expectation about growth rates of stock prices is subjective, following Adam et al. (2017). The household's utility in each period is presented by the following function with consumption habit formation,

$$U(C_t, L_t) = \frac{1}{1-\gamma}(C_t - \phi C_{t-1})^{1-\gamma} - \chi \frac{L_t^{1+\varphi}}{1+\varphi}, \tag{1}$$

where C_t is consumption at time t, L_t is labor at time t, γ is the rate of relative risk aversion, ϕ is the parameter of habit formation, χ is the weight assigned to labor, and φ is the inverse of Frisch elasticity.

The budget constraint of the household is given by the following equation. The households have accesses to the financial market via stocks.

$$S_t P_t^s + P_t C_t = S_{t-1}(P_t^s + D_t) + W_t L_t, \tag{2}$$

where S_t is stock holdings, P_t^s is nominal stock price per stock at time t, P_t is goods price at time t, D_t is nominal dividend per stock at time t, W_t is nominal wage at time t, and L_t is labor supply at time t.

In real term, the budget constraint becomes

$$S_t p_t^s + C_t = S_{t-1}(p_t^s + d_t) + w_t L_t, \tag{3}$$

where p_t^s is real stock price at time t, d_t is real dividend at time t, and w_t is real wage at time t. The households are under a constraint of minimum and maximum stock holding positions. This allows having maximums in their optimization problem under subjective beliefs.

$$\underline{S} \leq S_t \leq \overline{S}. \tag{4}$$

In equilibrium, this constraint is not binding over the entire time path. I assume that stock supply is a unity.

$$S_t = 1 \ \forall t. \tag{5}$$

The household maximization problem is given by

$$\begin{aligned} &\max \ E_0^P \sum_{t=0}^{\infty} \delta^t \ \exp(Z_t)\, U(C_t, L_t) \\ &\textit{subject to } (3) \textit{ and } (4), \end{aligned} \tag{6}$$

where E_0^P denotes the subjective expectation operator at time 0. This setting basically follows Adam et al. (2017). $\delta \in (0,1)$ is the time preference rate. Z_t denotes the preference shock. The first-order conditions with respect to C_t and S_t using stock supply assumption (5) are given by

$$\lambda_t = \exp(Z_t)(C_t - \phi C_{t-1})^{-\gamma} - \delta\phi E_t^P[\exp(Z_{t+1})(C_{t+1} - \phi C_t)^{-\gamma}] \tag{7}$$

and

$$1 = \delta E_t^P \Big[\frac{\lambda_{t+1}}{\lambda_t}(\frac{p_{t+1}^s}{p_t^s} + \frac{d_{t+1}}{p_t^s})\Big], \tag{8}$$

where λ_t represents a Lagrange multiplier for (3). The expectation operator of this Euler equation is governed by subjective beliefs, as I will explain later.

The preference shock process is formulated with a persistency parameter of ρ_Z as

$$Z_t = \rho_Z Z_{t-1} + \epsilon_t^Z, \tag{9}$$

where ϵ_t^Z represents an i.i.d. stochastic shock regarding the preference shock process.

The first-order condition with respect to labor supply is

$$\chi L_t^{\varphi} = \lambda_t w_t. \tag{10}$$

2.1.2 Subjective growth expectation about stock price

As noted, I assume that the households' expectations about wages and dividends are rational. In contrast, the households' expectations about stock price growth are subjective and use the Kalman filter to form the beliefs. I assume that the household perceives stock prices to evolve according to

$$\frac{p_{t+1}^s}{p_t^s} = \beta_{t+1} + \epsilon_{t+1}, \tag{11}$$

where ϵ_{t+1} is a transitory shock to price growth, $\epsilon_{t+1} \sim N(0, \sigma_\epsilon^2)$. β is unobserved persistent price growth component following Adam et al. (2017). The persistent component of stock prices drifts according to

$$\beta_{t+1} = \beta_t + \nu_{t+1}, \tag{12}$$

where ν_{t+1} is an innovation to price growth, $\nu_{t+1} \sim N(0, \sigma_\nu^2)$. The Kalman filter implies that the price growth component is given by

$$\beta_{t+1} \sim N(m_t, \sigma_\beta^2), \tag{13}$$

where m_t is the conditional expectation of β_{t+1} and σ_β^2 is the steady state Kalman filter uncertainty.

I assume two shocks, ϵ and ν_{t+1}, are independent and that the variances of each variable satisfy

$$\sigma_\nu^2 \ll \sigma_\epsilon^2, \tag{14}$$

so that the Kalman gain for persistent component of stock price growth becomes small.[10] With the optimal constant gain g, the Kalman gain process becomes,

$$m_t = m_{t-1} + g\left(\frac{p_t^s}{p_{t-1}^s} - m_{t-1}\right). \tag{15}$$

[10] A reason that a small value is necessary for the optimal constant Kalman gain g is that when g is not sufficiently small, the Blanchard-Kahn condition is not satisfied in a general equilibrium, as mentioned later. From (16) and (17) shown shortly, g becomes large if I do not assume (14). The model becomes stable when g is smaller than around 0.01.

The optimal constant Kalman gain g is given by

$$g = \frac{\sigma_\beta^2}{\sigma_\beta^2 + \sigma_\epsilon^2}, \tag{16}$$

and a steady state uncertainty σ_β is calculated as

$$\sigma_\beta^2 = \frac{\sigma_\nu^2 + \sqrt{(\sigma_\nu^2)^2 + 4\sigma_\nu^2\sigma_\epsilon^2}}{2}. \tag{17}$$

Internal rationality assumes that the subjective agents make fully optimal decisions given a well-defined system of subjective probability beliefs about stock prices. I assume that subjective households construct and update the expectation of capital gain at time $t+1$, $\frac{p_{t+1}^s}{p_t^s}$, by using m_t in (15) under the subjective probability measure P. Under this assumption with (8), the Euler equation becomes

$$1 = \delta E_t^P \Big[\frac{\lambda_{t+1}}{\lambda_t} (m_t + \frac{d_{t+1}}{p_t^s}) \Big]. \tag{18}$$

Because (15) only includes present and past variables, m_t is not a stochastic variable at time t. Internal rationality implies a fully rational utility maximization given subjective beliefs about stock price growth, m_t. Therefore, I have

$$1 = \delta E_t \Big[\frac{\lambda_{t+1}}{\lambda_t} \Big] m_t + \delta E_t \Big[\frac{\lambda_{t+1}}{\lambda_t} \frac{d_{t+1}}{p_t^s} \Big], \tag{19}$$

where E_t denotes the rational expectation operator given subjective beliefs of stock price growth.

The equation obtained by substituting (15) for m_t in (19) implies simultaneous determination of price beliefs and prices, and could generate multiple solutions of stock price if this model is solved by a non-linear method, because this equation is a quadratic function of the stock price p_t^s. In this study, I use the first-order perturbation method around the steady state to solve this model. Therefore, I can avoid a simultaneity problem by setting a steady state at a point that economically makes sense, although we set the learning process as (15).

However, as often used in learning literature such as in Evans and Honkapohja (2001) and Eusepi and Preston (2011), I can also consider another version of the Kalman gain process by modifying observation timing as

$$m_t = m_{t-1} + g(\frac{p^s_{t-1}}{p^s_{t-2}} - m_{t-1}). \tag{20}$$

To avoid a simultaneity problem, I use a belief system addressed in (20) instead of (15) in this study.

I can recover risk-free rate R^f_t as

$$R^f_t = \Big\{\delta E_t\Big[\frac{\lambda_{t+1}}{\lambda_t}\Big]\Big\}^{-1}. \tag{21}$$

Nominal risk-free rate becomes

$$R^n_t = \Big\{\delta E_t\Big[\frac{\lambda_{t+1}}{\lambda_t}\frac{P_t}{P_{t+1}}\Big]\Big\}^{-1}. \tag{22}$$

When the belief system is objective as opposed to the subjective system, the Euler equation becomes

$$1 = \delta E_t\Big[\frac{\lambda_{t+1}}{\lambda_t}(\frac{p^s_{t+1}}{p^s_t} + \frac{d_{t+1}}{p^s_t})\Big]. \tag{23}$$

2.1.3 Log-linearized stock price equation

Before moving to the firm sector's settings, I intuitively discuss how stock prices are formulated by the assumptions I have shown thus far. To summarize, stock prices react strongly to near-term information under the subjective beliefs defined above, while in a rational expectation case, they are determined by infinite future information about dividends and stochastic discount factors as widely known. To observe this, I first describe a case in which the learning process is given in a real time manner specified in (15). From (8) and (15), I can derive a quadratic equation of the stock price. There are two steady state candidates, one is a point at which stock price increases when dividends increase given that other variables are constant, and the other is a point at which stock price decreases when dividends increase given other variables are constant. By choosing the first steady state, which economically makes sense, a log-linearized stock price becomes,

$$\hat{p^s_t} = \frac{(\delta^{-1}-1)E_t\hat{d}^s_{t+1} + E_t\hat{\lambda}_{t+1} - \hat{\lambda}_t + (1-g)\hat{m}_{t-1} - g\hat{p}^s_{t-1}}{(\delta^{-1}-1-g)}. \tag{24}$$

$\hat{x}$ denotes the log deviation from its steady state value of x. To have a positive stock price value, I need to assume $\delta^{-1} - g - 1 > 0$. It implies that g needs to be sufficiently small. This equation implies that near-term dividends and stochastic discount factor matter to stock price. In addition, the level of Kalman gain g affects stock price response to a change in the stochastic discount factor.

Second, I show a stock price equation in a case that the capital gain expectation is formed with a lag shown by (20). With (8), a stock price equation becomes around the steady state,

$$\hat{p}_t^s = \frac{(\delta^{-1}-1)E_t\hat{d}_{t+1}^s + \delta^{-1}(E_t\hat{\lambda}_{t+1} - \hat{\lambda}_t) + (1-g)\hat{m}_{t-1} + g(\hat{p}_{t-1}^s - \hat{p}_{t-2}^s)}{\delta^{-1}-1}. \tag{25}$$

My model assumes the lagged belief updating (20) and the resulting equation (25). Although (25) implies that near-term stochastic discount factor and dividends are major determinants of stock prices, it also implies that in the lagged belief updating case, stock price reacts less sensitively to changes in near-term dividends and stochastic discount factor than in (24). However, the importance of near-term information for stock pricing does not differ from the first case. $\hat{p_t^s}$ is more directly affected by the historical stock price momentum, $\hat{p_{t-1}^s} - \hat{p_{t-2}^s}$. Regarding the Kalman gain value, g needs to be small in this case as well to avoid explosive paths.

2.2 Firms

Monopolistically competitive intermediate goods firms maximize profits. The intermediate goods firms pay price adjustment costs which are paid by final goods following Rotemberg (1982) when they change their prices. They hire labor from the household, own capital, and conduct capital formations. Capital formations require investment adjustment costs. $j \in [0,1]$ is an intermediate goods firm index. Competitive final goods producers produce final goods by aggregating intermediate goods. The final goods are transformed to consumption goods and investment goods costlessly.

The final goods sector is perfectly competitive and transforms intermediate goods into final goods for consumption and investment by the CES production function,

$$Y_t = \left(\int_0^1 Y_t(j)^{\frac{\eta-1}{\eta}} dj\right)^{\frac{\eta}{\eta-1}}, \tag{26}$$

where Y_t is aggregate output of final goods, $Y_t(j)$ is output of intermediate goods, and $\eta\,(>1)$ is the demand elasticity parameter. The profit maximization of the final goods firm becomes

$$\max_{Y_t(j)} P_t\left(\int_0^1 Y_t(j)^{\frac{\eta-1}{\eta}} dj\right)^{\frac{\eta}{\eta-1}} - \int_0^1 P_t(j)Y_t(j)dj, \tag{27}$$

where $P_t(j)$ is an intermediate good price. This generates a downward-sloping demand for intermediate goods. Intermediate goods demand is set as

$$Y_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\eta} Y_t. \tag{28}$$

The maximization problem of the firm producing intermediate goods j is given by

$$\max E_t \sum_{i=0}^{\infty} M_{t+i}\{d_{t+i}(j)\}, \tag{29}$$

where M_{t+i} is a pricing kernel under the presence of subjective households who own the firm sector. $d_{t+i}(j)$ denotes a real dividend. Real dividend cash flow is governed by,

$$d_t(j) = \frac{P_t(j)}{P_t}Y_t(j) - w_t L_t(j) - I_t(j) - \frac{\zeta^P}{2}(\frac{P_t(j)}{P_{t-1}(j)} - 1)^2 Y_t, \tag{30}$$

where $I_t(j)$ is investment and ζ^P is a price adjustment cost parameter.

Capital formation is defined as

$$K_t(j) = (1-\psi)K_{t-1}(j) + \exp(v_t)I_t(j)\big(1 - \frac{\zeta^I}{2}(\frac{I_t(j)}{I_{t-1}(j)} - 1)^2\big), \tag{31}$$

where $\psi \in (0, 1]$ is the depreciation rate of capital, $\frac{\zeta^I}{2}(\frac{I_t(j)}{I_{t-1}(j)} - 1)^2$ is the investment adjustment cost, and ζ^I is a parameter of investment adjustment costs. v_t represents the investment specific technology shock process at time t. The shock process is formulated with a persistency parameter of ρ_V as

$$v_t = \rho_V v_{t-1} + \epsilon_t^v, \tag{32}$$

where ϵ_t^v represents an i.i.d. stochastic shock regarding the investment specific technology shock process. Production technology of intermediate goods is given by

$$Y_t(j) = \exp(A_t)K_{t-1}(j)^{\xi} L_t(j)^{1-\xi}, \tag{33}$$

where A_t represents the technology level at time t. $\xi \in (0, 1)$ is the capital share. The productivity shock process is formulated with a persistency parameter of ρ_A and an i.i.d. shock to productivity ϵ_t^A as

$$A_t = \rho_A A_{t-1} + \epsilon_t^A. \tag{34}$$

The Lagrangian of this problem can be set as

$$\begin{aligned}\mathcal{L}_t =& E_t \sum_{i=0}^{\infty} M_{t+i} \big\{ \big(\frac{P_{t+i}(j)}{P_{t+i}}\big)^{1-\eta} Y_{t+i} - w_{t+i} L_{t+i}(j) - I_{t+i}(j) - \frac{\zeta^P}{2}(\frac{P_{t+i}(j)}{P_{t+i-1}(j)} - 1)^2 Y_{t+i} \\ &+ \Omega_{t+i}(j)(\exp(A_{t+i}) K_{t+i-1}(j)^{\xi} L_{t+i}(j)^{1-\xi} - \{\frac{P_{t+i}(j)}{P_{t+i}}\}^{-\eta} Y_{t+i}) \\ &+ q_{t+i}(j)((1-\psi) K_{t+i-1}(j) + \exp(v_{t+i}) I_{t+i}(j)\big(1 - \frac{\zeta^I}{2}(\frac{I_{t+i}(j)}{I_{t+i-1}(j)} - 1)^2\big) - K_{t+i}(j))\big\},\end{aligned} \tag{35}$$

where $\Omega_{t+i}(j)$ and $q_{t+i}(j)$ are Lagrange multipliers. I assume that the stochastic discount factor to discount firms' cash flow is that of households. Define Λ as

$$\begin{aligned}\Lambda_t &= \frac{M_{t+1}}{M_t} \\ &= \delta \frac{\lambda_{t+1}}{\lambda_t}.\end{aligned} \tag{36}$$

I assume firms are symmetric. Then, I have the first-order conditions as follows:

$$w_t = \Omega_t (1-\xi) \frac{Y_t}{L_t}. \tag{37}$$

$$q_t = \Lambda_t E_t [\Omega_{t+1} \xi \frac{Y_{t+1}}{K_t} + q_{t+1}(1-\psi)]. \tag{38}$$

$$1 = q_t \exp(v_t)\big\{1 - \zeta^I(\frac{I_t}{I_{t-1}} - 1)\frac{I_t}{I_{t-1}} - \frac{\zeta^I}{2}(\frac{I_t}{I_{t-1}} - 1)^2\big\} + E_t \Lambda_t q_{t+1} \exp(v_{t+1}) \zeta^I (\frac{I_{t+1}}{I_t} - 1)(\frac{I_{t+1}}{I_t})^2. \tag{39}$$

$$\frac{1-\eta}{\eta} - \frac{\zeta^P}{\eta}(\frac{P_t}{P_{t-1}} - 1)\frac{P_t}{P_{t-1}} + E_t \Lambda_t \frac{\zeta^P}{\eta}(\frac{P_{t+1}}{P_t} - 1)\frac{P_{t+1}}{P_t}\frac{Y_{t+1}}{Y_t} + \Omega_t = 0. \tag{40}$$

Aggregate capital evolution is

$$K_t = (1-\psi) K_t + \exp(v_t) I_t (1 - \frac{\zeta^I}{2}(\frac{I_t}{I_{t-1}} - 1)^2). \tag{41}$$

Aggregate real dividend is

$$d_t = Y_t - w_t L_t - I_t - \frac{\zeta^P}{2}(\frac{P_t}{P_{t-1}} - 1)^2 Y_t. \tag{42}$$

Aggregate output is

$$Y_t = \exp(A_t) K_{t-1}^{\xi} L_t^{1-\xi}. \tag{43}$$

Inflation rate is given by

$$\pi_t = \frac{P_t}{P_{t-1}}. \tag{44}$$

2.3 Market clearing and monetary policy

I assume a standard Taylor rule for monetary policy as

$$\frac{R_t^n}{R_{ss}^n} = (\frac{R_{t-1}^n}{R_{ss}^n})^{\theta_M} \{(\frac{P_t}{P_{t-1}})^{\phi_\pi}\}^{(1-\theta_M)} \epsilon_t^M, \tag{45}$$

where $\theta_M \in [0,1)$ is a persistence parameter of the monetary policy, $\phi_\pi (> 1)$ is a reaction parameter of the monetary policy to inflation rate, and ϵ_t^M is an i.i.d. shock to the nominal interest rate. The resource constraint of the entire economy becomes

$$Y_t = C_t + I_t + \frac{\zeta^P}{2}(\frac{P_t}{P_{t-1}} - 1)^2 Y_t. \tag{46}$$

3 Quantitative analysis

In this section, I present the quantitative implications of our model economy. Using the calibrated model, I compute the impulse responses of macroeconomic variables to shocks. In this study, I assume that time frequency is quarterly. The model is solved using the first-order perturbation method. I have confirmed that I can generate a similar volatility of price dividend ratio to Adam et al. (2017), in which dividend and wage are exogenous, using the first-order perturbation method by checking moments of a model with exogenous dividend and wage. This calculation is shown in Appendix A.2.

3.1 Calibration

Table 1 lists the choice of parameter values for my baseline model. Parameter values are calibrated in quarterly rates and assume those of the U.S. economy. I set basic parameters, the rate of relative risk aversion γ, inverse of Frisch elasticity ν, depreciation rate ψ, capital share ξ, and investment adjustment cost parameter ζ^I, following Christiano et al. (2005).

Regarding the habit formation parameter ϕ, I use a lower value than Christiano et al. (2005) who use 0.65. Instead, I refer to Levin et al. (2005) who use 0.29, as a large value of ϕ generates unrealistic high autocorrelation of consumption in this model. The discount rate β is 0.99 which is within a conventional range. The relative utility weight of labor χ is set so that the steady state labor amount becomes 0.3. In Christiano et al. (2005), the Philips curve is not formulated based on Rotemberg type price adjustment costs, which I use in the model. For the demand elasticity parameter η and price adjustment parameter ζ^P, I follow Ireland (2001), who uses Rotemberg type price adjustment costs.

Monetary policy reaction parameter ϕ_π is conventional value, 1.5. Monetary policy persistence parameter θ_M is 0.5, which is lower than 0.8 in Christiano et al. (2005) or 0.81 in Smets and Wouters (2007). I set this to match moments of stock price to the data by reducing the persistence. The autoregressive parameter of productivity ρ_A and investment shock ρ_V follow Smets and Wouters (2007). The autoregressive parameter of preference shock ρ_Z follows Ireland (2001).

The model specific parameter is Kalman gain g. In Adam et al. (2017), its estimated value is 0.02-0.03, depending on assumptions. However, in my general equilibrium model, this range of g results in explosive paths. Adam et al. (2016) estimate this value at 0.007-0.008. I set a value of g at 0.005 so that the model can avoid explosive paths and generate realistic moments of stock price.

Parameters	Value	Description
δ	0.99	Discount rate
γ	1.0	Rate of relative risk aversion
ϕ	0.29	Habit formation parameter
g	$\frac{1}{200}$	Constant Kalman gain
ψ	0.025	Depreciation rate
χ	6.5	Relative utility weight of labor
ν	1	Inverse of Frisch elasticity
ξ	0.36	Capital share
ζ^I	2.5	Investment adjustment cost parameter
ζ^P	77	Price adjustment parameter
η	6	Demand elasticity
ϕ_π	1.5	Monetary policy reaction parameter
θ_M	0.5	Monetary policy persistence parameter
ρ_A	0.95	Autoregressive parameter of productivity shock
ρ_V	0.71	Autoregressive parameter of investment shock
ρ_Z	0.89	Autoregressive parameter of preference shock

Table 1: Baseline parameters

3.2 Steady state values

Table 2 shows the steady state values of our model. The steady state values are in quarterly rates and the subscript ss means a steady state value. These values do not depend on whether the model is of subjective beliefs or objective beliefs, as I set the steady state m at 1. For comparison, this table shows actual data values averaged over 1980-2017.[11] C_{ss}/Y_{ss} and I_{ss}/Y_{ss} are close to the actual value. Actual data for Y in these two fractions is a sum of C and I with a deflator adjustment as explained in Appendix A.1. $K_{ss}/(Y_{ss} * 4)$ is within a plausible range. Actual Y_{ss} in $K_{ss}/(Y_{ss} * 4)$ is real GDP data.

3.3 Model moments

Table 3 compares second moments of our baseline model and the actual data. Moments are calculated on a quarterly basis. Model moments shown here are those of subjective and objective belief cases. The objective belief case uses the belief system shown by (23).

[11]This sample period almost corresponds to periods during which the U.S. central bank targeted the interest rate rather than money growth.

Variables	Steady state value	Actual data
C_{ss}/Y_{ss}	0.79	0.80
I_{ss}/Y_{ss}	0.21	0.20
$K_{ss}/(Y_{ss} * 4)$	2.14	3.16
p^s_{ss}/d_{ss}	99.0	70.4

Table 2: Steady state values

A subscript ss means a steady state value. Actual data are the average of 1980-2017 quarterly data. Y_{ss} for $K_{ss}/(Y_{ss} * 4)$ is real GDP data to be consistent with K_{ss} data. Y_{ss} data for C_{ss}/Y_{ss} and I_{ss}/Y_{ss} is explained in the Appendix. Actual data of p^s_{ss}/d_{ss} is the price earnings ratio.

Theoretical moments of models are based on 0.45% productivity shocks, 0.15% monetary policy shocks, 0.24% preference shocks, and 0.45% investment shocks in standard deviation on a quarterly basis.[12] Actual data moments cover 1980-2017 of the U.S. economy. Details about data source are in Appendix A.1.

The subjective belief model shows realistic standard deviation of stock price without setting a high value of the relative risk aversion rate. This is an advantage of incorporating subjective beliefs. On the other hand, the objective belief model fails to show realistic stock price volatility. The stock price is highly sensitive to real interest rates at time t as implied in (25). The real interest rate and the momentum effect increases the volatilities of stock price in the subjective case.

Adam et al. (2017) claim that survey measures of investors' expected return correlate positively with the price dividend ratio, while rational return expectations correlate negatively with the price dividend ratio. My model is successful in generating a positive correlation between price dividend ratio $\log P^s/d$ and capital gain expectation $\log m$. On the other hand, the objective model fails to show a positive correlation.

The correlation between stock price and dividend is closer to the data in the subjective belief model than the objective model. The data shows a negative correlation between stock price and dividend while the objective model shows higher positive correlations than the subjective case.[13] In the subjective model, effects of dividends are very small because the

[12]In Smets and Wouters (2007), productivity shock is estimated as 0.45%, monetary policy shock is estimated as 0.24% , preference shock is estimated as 0.24%, and investment specific shock is estimated as 0.45% in standard deviation on a quarterly basis. In the model, I use a lower size of monetary policy shock than Smets and Wouters (2007) because the effects of the interest rate shock on stock price are large in the model.

[13]The definition of dividend in the model shown in (42) is not the same as in the statistics of the U.S. Bureau of Economic Analysis because of my model structure. Data is constructed here to be consistent with definition of dividend in the model.

effect of discount factors dominates that of dividends and the correlation between dividends and stock price is low. In (25), the coefficient of $E_t\hat{\lambda}_{t+1} - \hat{\lambda}_t$ is much larger than that of $\hat{d}^s_{t+1}$. In the objective model, because stock price is determined by the sum of future discounted dividends, both dividends and discount rates play major roles in affecting stock prices.

Correlation between stock price and output in the subjective case is lower than that in the objective case and closer to the data. In the subjective case, stock price is affected by dividends relatively lesser than the objective case. Dividends and output have positive correlation both in the two models and the data. In addition, as seen in Section 3.4, the momentum effect on stock prices under the subjective case generates some oscillations that are not observed in the objective case. Therefore, correlation between stock price and output in the subjective case is smaller than in the objective model.

Variables	Model (subjective belief)	Model (objective belief)	Actual data
$SD[\log R^f]$	0.002	0.002	0.024
$SD[\log R^n]$	0.002	0.002	0.036
$SD[\log C]$	0.017	0.017	0.026
$SD[\log P^s]$	0.235	0.019	0.204
$SD[\log d]$	0.025	0.025	0.045
$SD[\log Y]$	0.021	0.021	0.037
$SD[\log W]$	0.019	0.019	0.020
$SD[\log L]$	0.006	0.006	0.028
$SD[\log I]$	0.042	0.042	0.113
$SD[\log K]$	0.021	0.021	0.017
$SD[\log q]$	0.005	0.005	-
$SD[\log \pi]$	0.001	0.001	0.004
$Corr[\log P^s/d, \log m]$	0.78	-0.28	*
$Corr[\log P^s, \log Y]$	0.34	0.91	0.59
$Corr[\log P^s, \log d]$	0.08	0.55	-0.44
$Corr[\log Y, \log d]$	0.35	0.35	0.09
$Autocor[\log Y(-1)]$	0.99	0.99	0.96
$Autocor[\log Y(-2)]$	0.97	0.97	0.89
$Autocor[\log C(-1)]$	0.99	0.99	0.97
$Autocor[\log C(-2)]$	0.96	0.96	0.94
$Autocor[\log P^s(-1)]$	0.79	0.97	0.95
$Autocor[\log P^s(-2)]$	0.55	0.95	0.87
$Autocor[\log d(-1)]$	0.77	0.77	0.96
$Autocor[\log d(-2)]$	0.64	0.64	0.92

Table 3: Second Moments

Actual data are 1980-2017 quarterly U.S. data. * Due to data accessibility to subjective stock price growths, I did not show actual $Corr[\log P^s/d, \log m]$ in this table. Instead, actual correlation between subjective stock price growth expectation and price dividend ratio provided by Adam et al. (2017) based on 1946-2012 data is 0.79.

3.4 Impulse response

This subsection presents impulse responses to productivity and monetary policy shocks. The time frequency is quarterly. Figure 1 indicates impulse responses to a 0.5% positive productivity shock on a quarterly basis. Figure 2 indicates impulse responses to a 0.5% positive monetary policy shock (shock to increase the nominal interest rate) on a quarterly basis. In these figures, I compare the responses of subjective and objective belief cases. Both cases do not show differences except responses of stock price p^s, price dividend ratio p^s/d, and the expectation of stock price growth m. This can be considered an advantage of this model because it can generate a large stock price reaction without sacrificing the responses of fundamental variables other than stock price.

Stock prices in the subjective belief case react to both of productivity and monetary policy shocks much more than in the objective belief case. As I showed in (25), stock price is strongly affected mainly by the near-term marginal rate of substitution or real interest rate, because the coefficient of marginal rate of substitution, $\frac{\delta^{-1}}{\delta^{-1}-1}$, is large. Shocks that lead to large volatility of marginal rate of substitution or real interest rate result in a large movement of stock price. These figures show that in the subjective belief case expected stock price return co-moves with price dividend ratio and stock price, while it does not do so in the objective belief case.

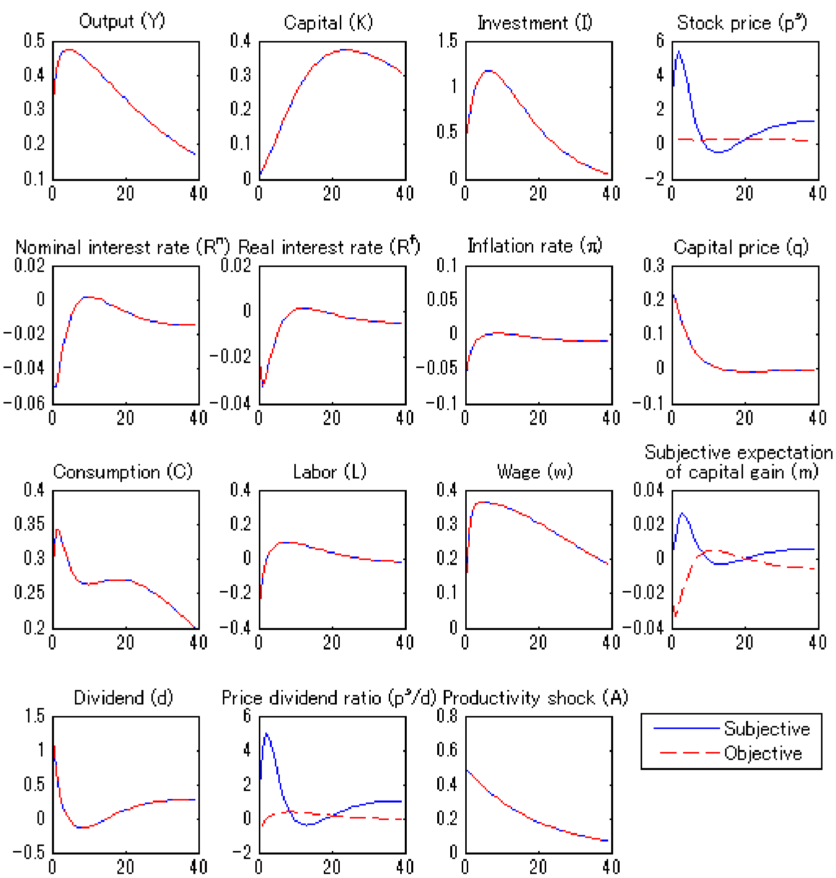

Figure 1: Impulse response of model variables to the productivity shock

Impulse response to a 0.5% positive productivity shock. The solid line indicates the subjective expectation case. The dotted line indicates the objective expectation case. 1 on the vertical axis scale amounts to 1%. Return variables, R^n and R^f are in % point differences from their steady states. Otherwise, variables are shown in percentage deviation from their steady state levels.

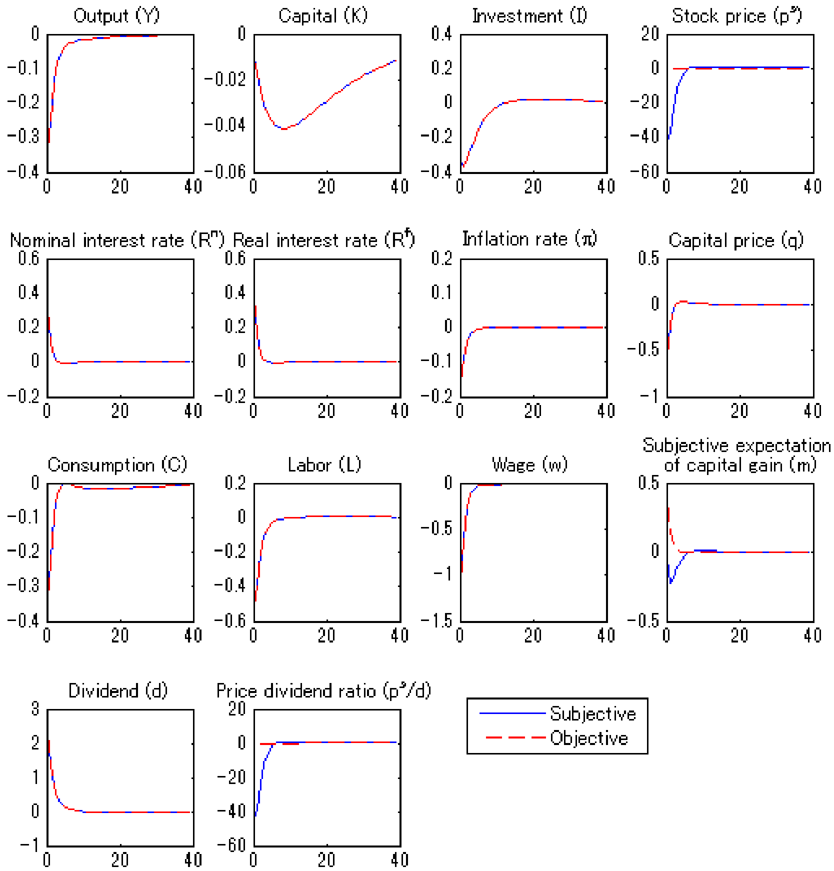

Figure 2: Impulse response of model variables to the monetary policy shock

Impulse response to a 0.5% positive monetary policy shock. The solid line indicates subjective expectation case. The dotted line indicates objective expectation case. 1 on the vertical axis scale amounts to 1%. Return variables, R^n and R^f are in % point differences from their steady states. Otherwise, variables are shown in percentage deviation from their steady state levels.

4 What drives stock price volatilities?

In the previous section, the subjective belief case shows more realistic volatility of stock price compared to actual data than the objective belief case under the baseline parameters. In this section, I show the relevant parameters for magnifying or compressing stock price volatilities. The model is suitable for relating the volatility of stock price with deep parameters, including those related to the stance of monetary policy because by introducing Bayesian learning, the model is able to generate realistic volatilities of stock prices, which are not attained by a standard rational expectation model with conventional values of deep parameters.

4.1 Kalman gain value and stock price

First, I show how a value of Kalman gain g affects a stock price reaction to a productivity shock and a monetary policy shock given other parameters are kept at the baseline values in Figure 3. When Kalman gain g is large, investors react to observed stock price changes more sensitively. The figure includes a range of g from $\frac{1}{100}$ to $\frac{1}{300}$. As g increases, stock price volatility increases under both productivity and monetary policy shock.

For convenience, I show (25) again here.

$$\hat{p}_t^s = \frac{(\delta^{-1} - 1)E_t\hat{d}_{t+1}^s + \delta^{-1}(E_t\hat{\lambda}_{t+1} - \hat{\lambda}_t) + (1-g)\hat{m}_{t-1} + g(\hat{p}_{t-1}^s - \hat{p}_{t-2}^s)}{\delta^{-1} - 1}.$$

When g is large, the last term of the right hand side of this equation generates a larger momentum from a past increase in stock price, while g gives less weight to the persistent component of stock price growth expectation. Thus, higher values of g generate higher volatilities of stock prices.

4.2 Fundamental parameters and stock price

The next experiment is what fundamental parameters including a monetary policy rule in the economy could amplify responses of stock prices to shocks given the baseline Kalman gain value, $g = \frac{1}{200}$.

As I noted in Subsection 2.1.3, an implication of (25) is that first and mostly the stochastic discount factor at t, and second a dividend at $t+1$ affect the stock price level as non-predetermined variables. This is a major contrast to a rational expectation case in which stock price is determined by the infinite sum of dividends discounted by stochastic discount factors.

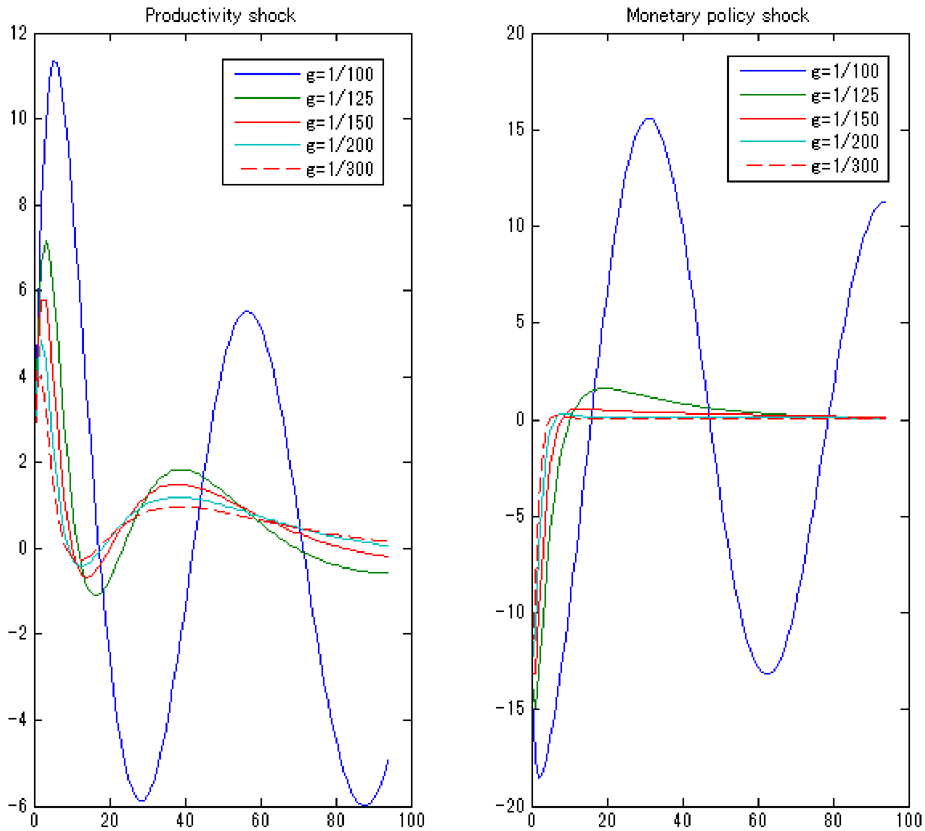

Figure 3: Impulse response of stock prices to productivity and monetary policy shocks under different Kalman gain settings

Impulse response to a 0.45% positive productivity shock and 0.15% positive monetary policy shock under different values of Kalman gain g. 1 on the vertical axis scale amounts to 1% deviation from their steady state levels.

To see this, I show impulse response under different fundamental parameters including monetary policy parameters from the baseline settings. In the following subsections, I examine the monetary policy persistence parameter, $\theta_M = 0.8$ (baseline=0.5), monetary policy reaction parameter, $\phi_\pi = 1.1$ (baseline=1.5), habit formation parameter $\phi = 0.85$ (baseline=0.29), and the rate of relative risk aversion $\gamma = 5$ (baseline=1), respectively. Figure 4 indicates the impulse responses to the positive productivity policy shock and Figure 5 indicates impulse responses to the positive monetary policy shock. Both figures include subjective expectation case, objective expectation case, and these differences.

4.2.1 Monetary policy persistence

Under the positive productivity shock (Figure 4), stock price reaction becomes smaller in the case of the higher monetary policy persistence parameter, $\theta_M = 0.8$ (baseline=0.5). A strong monetary policy persistence implies that real interest rate decreases only gradually responding to decreases in inflation rates associated with the positive productivity shock. As implied by (25), the stochastic discount factor (or real interest rate) at time t has a dominant impact on stock price at time t under the subjective expectation. Therefore, less decrease in the real interest rate from the monetary policy rule given the nominal rigidity generate smaller increases in stock price than in the baseline case. Because investors are less forward-looking in pricing stock, an expected path of real interest rates has less importance than in a rational expectation case. Because the initial stock price reaction is small in the $\theta_M = 0.8$ case, a momentum effect, by which stock price reacts to past growths of itself, also becomes weak in this model as implied in the last term of (25). Therefore, overall impacts on stock price are more muted in the $\theta_M = 0.8$ case than in the base line case.

Responses of stock price to the monetary policy shock (Figure 5) shows a large difference between the $\theta_M = 0.8$ case and the baseline case ($\theta_M = 0.5$). A positive monetary policy shock immediately increases real interest rate at time t as shown in (45) regardless of the monetary policy persistence. The high monetary policy persistence case results in a large drop of stock price under monetary policy shocks. Two reasons lead to this result. The first reason is the path of nominal interest rate. Strong persistence implies that increase in nominal interest rate after unexpected positive nominal interest rate shock sustains for long periods in the case of high persistent parameter. This effect continues to push up "current" nominal interest rates at each point in time. The second reason is inflation expectation. Because of the high persistence of nominal interest rate and forward-looking feature in the New Keynesian model, inflation expectation becomes low. This implies the increase in real interest rate given today's nominal interest rate. Therefore, strong monetary policy persistence implies a sustained high volatility of near-term real interest rates after the occurance of a monetary policy shock. High near-term real interest rates lead to large decreases in stock price. Realized stock price drop generates momentum and downward revisions of capital gain expectation as shown in (25). This analysis implies that monetary policy inertia does not necessarily stabilize stock price behaviors when unexpected monetary policy shock occurs.

4.2.2 Monetary policy reaction

In the case of the low monetary policy reaction parameter, $\phi_\pi = 1.1$ (baseline=1.5), stock price reaction becomes small to the positive productivity shock. Monetary policy is less sensitive to decreases in inflation rates than the baseline case, which reduces the size of decreases in the real interest rate and results in a small reaction of stock price to the productivity shock. Under the positive monetary policy shock, the impact of the change in monetary policy reaction parameter is not large. By the positive monetary shock, the nominal rate hike decreases inflation rate. Responding to the inflation, the monetary policy rule simultaneously adjusts the nominal interest rate. In the case with $\phi_\pi = 1.1$, monetary policy is less reactive than the baseline case. Consequently, the real interest rate in the case with $\phi_\pi = 1.1$ becomes relatively higher than the baseline case at time t. Because stock price reacts to the real interest rate at time t, stock price drops slightly more than the baseline case.

4.2.3 Habit formation parameter and relative risk aversion rate

In addition to policy parameter discussions in the previous subsections, I discuss a habit formation parameter $\phi = 0.85$ case (baseline=0.29). In the productivity shock case (Figure 4), a high habit formation parameter amplifies stock price volatilities. This is because near term real interest rates become volatile, which leads to high volatility of stock prices. Although volatility of near term dividends becomes small, stock prices become more volatile because the impact of the real interest rate on stock prices is much larger than that of dividends.

When a value of habit formation parameter is high, the pricing kernel moves largely given a change in the consumption growth rate. This generates larger volatilities of real interest rate than the baseline case. On the other hand, under a high value of habit formation parameter, the volatility of dividends is smaller than the baseline case. When the sign of productivity shock is positive, the stochastic discount factor increases greatly. As a result, firms are inclined to increase dividends far into the future rather than the near term. Therefore, the increase of $\hat{d}^s_{t+1}$ becomes small by the firms' optimization in a case of strong consumption smoothing. However, the effect of the real interest rate has a much larger magnitude on stock price than the effect of dividends because the term of the real interest rate has much larger weight than that of dividends on the right hand side in (25). Therefore, stock price increases largely in a high habit formation parameter case.

Under the positive monetary policy shock (shock to increase the nominal interest rate), the high habit formation case does not show a notable difference from the baseline case as shown in the "Subjective case" box of Figure 5. A monetary policy shock directly changes the real interest rate given the price rigidity. Therefore, stochastic discount factors which affect stock prices do not show notable differences between the baseline and the high habit formation parameter case, although stock price decreases more in the high habit formation case. The difference mainly comes from the endogenous reactions of the economy, which impacts are limited. Under positive monetary policy shocks with the high habit formation parameter, small decrease in consumption due to high consumption smoothing generates small decrease in inflation, which implies high nominal interest rate via the monetary policy rule. This results in higher real interest rate today and stock price decreases slightly further than the baseline case. Similarly, one can interpret a case of a higher rate of relative risk aversion $\gamma = 5$ (baseline=1) from a point of view of consumption smoothing.

To summarize the results, under the monetary policy shock, the persistence parameter of monetary policy rule has large effects on stock price. Policy inertia does not help to reduce stock price volatility at least under unexpected monetary policy shocks, which is opposite to the usual discussions in the context of "gradualism" of monetary policy. One cannot underestimate the role of the monetary policy stance in affecting the stability of stock prices.

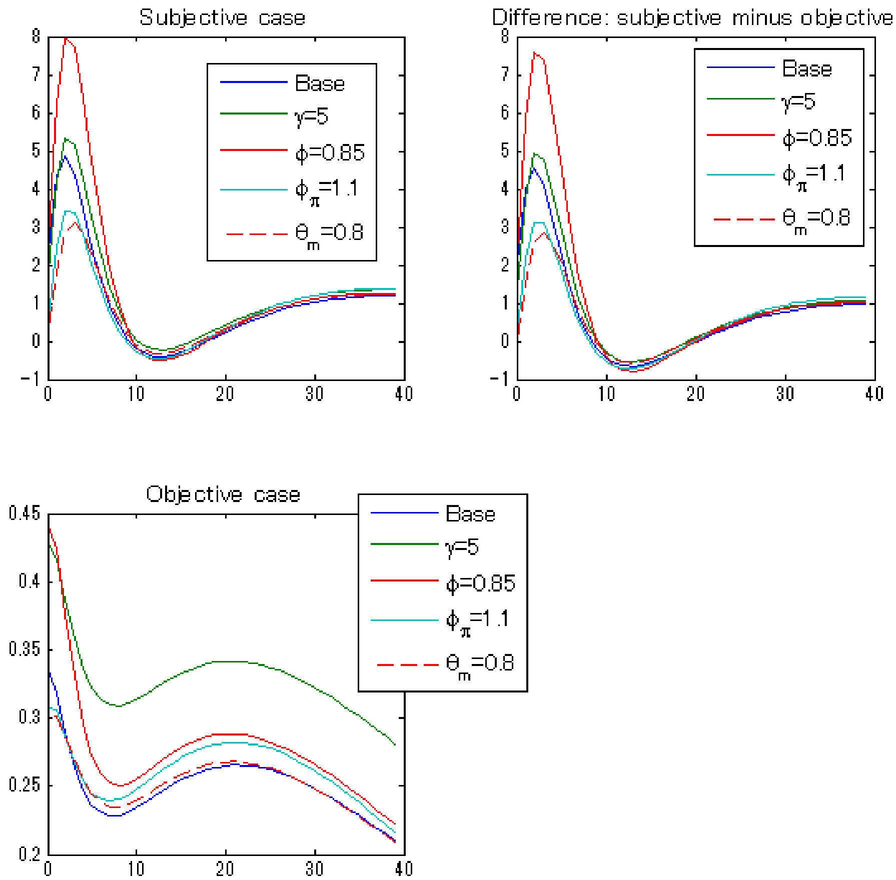

Figure 4: Impulse response of stock prices to productivity shock

Impulse response to a 0.45% positive productivity shock. The solid line indicates subjective expectation case. The dotted line indicates objective expectation case. 1 on the vertical axis scale amounts to 1% deviation from their steady state levels.

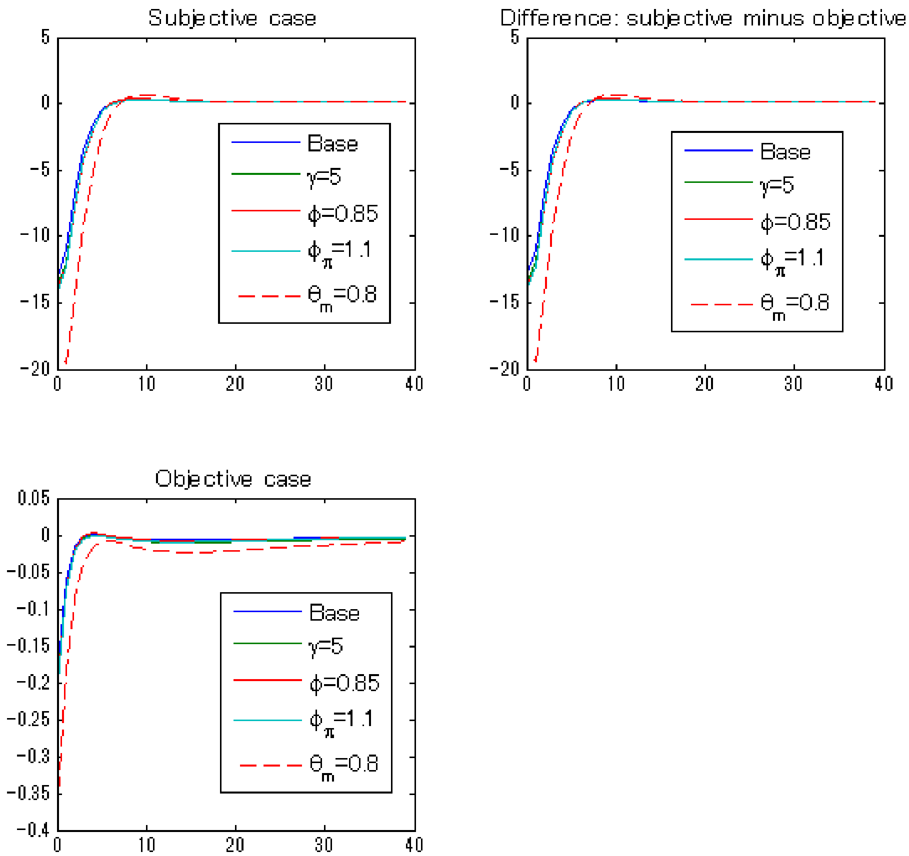

Figure 5: Impulse response of stock prices to monetary policy shock

Impulse response to a 0.15% positive monetary policy shock. The solid line indicates subjective expectation case. The dotted line indicates objective expectation case. 1 on the vertical axis scale amounts to 1% deviation from their steady state levels.

5 Conclusion

How stock price volatilities are related to fundamental parameters, especially those of monetary policy, is the key question in this study. To investigate this issue, I developed a New Keynesian model with subjective belief about capital gains of stock price which can generate realistic volatility of stock price. As my model is a general equilibrium model with nominal rigidity, it allows examining relationships between structural parameters including those of monetary policy and stock price volatilities.

My model provides realistic volatility of stock price without setting a high value of the relative risk aversion rate. In my model with Bayesian learning about capital gains, the near-term real interest rate is an important factor in explaining stock price volatilities. Given this structure, monetary policy stances have large impacts on the volatilities of stock price. When monetary policy persistence or strength of reactions imply a high volatility of near term real interest rates, stock price volatilities become large. This contrasts with rational expectation stock pricing in which the infinite sum of discounted cash flow matters rather than near-term variables.

Finally, the model is solved by the first-order perturbation method for the simplicity of calculation and abstracts higher-order terms. By including these, it would be possible to argue excess return and volatility at the same time. In this study, I assume that households are homogenous for simplicity. Including heterogeneity of beliefs would be also interesting to examine model validity as an expansion of this model. These considerations remain for further research.

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A Appendix

A.1 Data Sources

The actual data formations are conducted by procedures as follows. Actual data used in this paper is of the United States. Real consumption, real investment, and real wage data are from the Federal Reserve Bank of St. Louis FRED economic data base (`https://fred.stlouisfed.org/`). FRED series IDs for these variables are PCECC96, GPDIC1, and LES1252881600Q, respectively. These are seasonally adjusted quarterly data. For real output data, I took a sum of seasonally adjusted nominal consumption and investment, and divided it by the implicit price deflator for gross domestic purchases. Corresponding FRED series ID are PCEC, GPDI, and A712RD3A086NBEA, respectively. Real capital stock data is real net stock (private fixed assets) from the Bureau of Economic Analysis. Total nominal compensation of employee data is from FRED and its FRED series ID is A4102C1Q027SBEA. This is seasonally adjusted. To retrieve the labor amount, I divided total nominal compensation data by real wage and implicit price deflator for gross domestic purchases. For dividend for which development is consistent with the model, I subtracted nominal total compensation and nominal investment (FRED series ID: GDPI) from nominal output, which I define as the sum of nominal consumption (FRED series ID: PCEC) and nominal investment, and divided it by the implicit price deflator for gross domestic purchases. The stock price data is S&P 500 index data. Real stock price data is deflated by the implicit GDP deflator (FRED series ID: GDPDEF). Because the dividend data constructed above covers all U.S. companies which include unlisted companies, I construct dividend data which is a product of the S&P stock index level and its dividend yield for the price dividend ratio in 3.2.

Interest rate data is based on the Federal Reserve Bank of New York' s treasury term premia database (`https://www.newyorkfed.org/research/data_indicators/term_premia.html`). I use one-year fitted zero coupon market yield of U.S. treasury for the nominal interest rate. I deflated it by the actual inflation rates of the implicit gross domestic product deflator. In subsection 3.2, I used gross yields data. Both rates in subsection 3.3 are of the natural log of gross yields.

In subsection 3.3, I took the natural log and de-trend it by the third-order time polynomial regression except interest rate data. I chose third-order because the Akaike information criterion shows the lowest value when I examined fits up to the fourth order. For consistency, I used the same order in time polynomial regressions for other variables too. Real capital data are real net private fixed assets provided on an annual basis. I translated them to quarterly by interpolation for de-trending.

A.2 Model with exogenous consumption and dividend

I solve the model by the first-order perturbation method. I need to confirm whether the solution method can provide similar results to Adam et al. (2017). In Adam et al. (2017), consumption and dividend process are given exogenously by

$$ln\ d_t = ln\ \beta^d + ln\ d_{t-1} + ln\ \epsilon_t^d, \tag{47}$$

where d_t is dividend at time t, β^d is the constant growth rate, and ϵ_t^d is innovation at time t. Instead, in this exercise, I use a zero constant growth rate for simplicity and avoid a random walk by setting a dividend process as

$$d_t = d_{ss}^{(1-\rho_d)} d_{t-1}^{\rho_d} \epsilon_t^d. \tag{48}$$

d_{ss} is a steady state real dividend level and ρ_d is the persistence parameter of the real dividend process.

Adam et al. (2017) set a wage process as

$$(1 + \frac{w_t}{d_t}) = (1 + \frac{w_{ss}}{d_{ss}})^{(1-\rho_w)} (1 + \frac{w_{t-1}}{d_{t-1}})^{\rho_w} \epsilon_t^w, \tag{49}$$

where w_{ss} is the steady state real wage. ρ_w is the persistence parameter of the real wage process. In Adam et al. (2017)'s model, the resource constraint is

$$d_t + w_t l_t = c_t. \tag{50}$$

In addition, labor is normalized to one.

$$l = 1. \tag{51}$$

This implies $1 + \frac{w_t}{d_t} = \frac{c_t}{d_t}$. (19) with (20) constructs the Euler equation.

I solve this model with exogenous dividend and wage by the first-order perturbation method. Model calibration is shown in Table 4. A reason that the value of g is smaller than Adam et al. (2017)'s calibration is that large g results in an explosive path. Even in this linearization case, the moment of price dividend ratio is well replicated as shown in Table 5. The mean of price dividend ratio in the model with exogenous consumption and dividend process is 116.6 and its standard deviation is 75.6. These values are close to Adam et al. (2017) with 120.1 and 95.6, respectively.

Parameters	This model	Model of Adam et al. (2017)
$SD[\epsilon_t^d]$	0.018	0.019
$SD[\epsilon_t^w]$	0.018	0.019
$Corr[\epsilon_t^d, \epsilon_t^w]$	-0.93	-0.98
ρ_d	0.95	-
ρ_w	0.95	0.95
δ	0.992	0.995
g	0.006	0.028
$\frac{w_{ss}}{d_{ss}}$	22	22

Table 4: Calibration comparison of exogenous wage and dividend model

The subscript ss means steady state value.

Parameters	Actual data in Adam et al. (2017)	This model	Model of Adam et al. (2017)
$E[P^s/d]$	139.8	116.6	120.1
$SD[P^s/d]$	65.2	75.6	95.6
$SD[P^s/d]/E[P^s/d]$	0.47	0.65	0.80
$AutoCorr[P^s/d(-1)]$	0.98	0.99	0.98

Table 5: Performance comparison of exogenous wage and dividend model

Actual data in Adam et al. (2017) cover 1946-2012.