Firm Size, Hours, and Wages in Search Equilibrium

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April, 2004

Premilinary and Incomplete

Abstract

This paper studies the size distribution of firms in a labor market with search friciton. It constructs a search-matching model in which a firm, if it wishes, engages costly on-the-job search to fill its second vacancy. The model is used to ask what influences a firm's decision regarding whether to expand employment or to stay small. Firms submit take-it-or-leave-it wage offers to workers, and the production function is concave. The model implies wage differential, and that changes in aggregate demand affect hours, wages, and the size distribution of firms.

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1 Introduction

Why do firms want to be larger at some points in time and smaller at different points in time? This paper extends the standard search-matching model of the labor market to explore firm size determination and the size distribution of firms. In particular, this paper focuses on firms' decisions regarding whether to expand employment or to stay small, and asks what influences such decisions by firms.

The search-matching model of Mortensen-Pissarides (1994) type, which is nicely summarized by Pissarides (2000), has received much attention as the framework for studying various labor market issues. The size of a firm in the basic Mortensen-Pissarides (1994) model is irrelevant; each job and worker engages time-consuming search in the labor market, and each matched pair produces. Thus the framework deliberately leaves out the concept of firm and focuses on the match as the production unit. In contrast, Bertola and Caballero (1994), for example, place more emphasis on the firm as the production unit.

There are several models with endogenous firm size. One direction of research, which has been engaged by Burdett and Mortensen (1998), addresses the so-called size-wage differential. Burdett and Mortensen (1998) analyze a search model with wage posting by employers, in which the employed continue to draw new offers and move to a new job if the new wage exceeds the current one. Wage dispersion across firms is generated in equilibrium even though workers are homogenous, and firms are different in size. Firms that offer higher wages are larger in size because higher wages attract more workers and discourage incumbents from quitting. Firms of large size trade off higher wages against the lower search cost due to lower turnover.

The other direction stresses concave production function and optimal firm size. Bertola and Caballero (1994) construct a model of job creation and destruction in which firms of endogenous size face idiosyncratic business condition shocks. Using a structure similar to Bertola and Caballero (1994), Bertola and Garibaldi (2001) find a positive size-wage relationship. Smith (1999) studies a search-matching model in which firms choose the number of employees. The production function is concave and wages are determined by bargaining. Smith (1999) shows that firms over-employ in a model with free entry.

In Bertola and Caballero (1994), Bertola and Garibaldi (2001), and Smith (1999), the number of employees is a continuous variable, and each firm is assumed to choose the size of firm. Their approach is attractive because the size of a firm can be treated as a differentiable variable. However, we argue that in such models it is not clear how each firm decides whether to expand its employment or not. In this paper, we assume that each firm can create and fill only one vacancy at a time; the size of a firm is a discrete variable and each firm faces a discrete choice. In order to employ a second worker, each firm must engage costly search while producing with the worker already employed. Using such a framework we focus on a firm's discrete choice regarding whether to create its second vacancy or to stay small, and explore factors that influence the choice and the size distribution of firms.

The organization of this paper is as follows. Section 2 describes a static frictionless economy, Section 3 presents the model environment. Section 4 characterizes equilibrium with fixed number of firms. Section 5 considers equilibrium with free entry. Section 6 concludes.

2 A Static Model of Firm Size Determination

How is the size of a firm determined? This section presents a simple static, frictionless model of firm size determination. In order to simplify matters as much as possible, we assume that the size of a firm can be either one or two. A novel feature of the model is to focus on the quantity of *task*. Business firms earn profits by getting tasks done. We assume that the quantity of task is given by the aggregate demand, which is taken as exogenous. Let τ denote the amount of task a firm faces. A worker spends $h(\tau)$ hours to complete the task. It is plausible to assume decreasing marginal product of labor, which implies $h'(.) > 0$ and $h''(.) > 0$. A unit of task may be thought of as goods or services each firm provides.

When a firm hires a worker, the firm must pay hourly wage and other fixed cost. The fixed cost is meant to capture various non-wage costs associated with hiring an employee, such as office supplies, desk, and perhaps payroll taxes.

Let w_1 and w_2 denote the hourly wage rate when a firm is of size 1 and 2, respectively. The fixed cost associated with each employee is e. The profits of a firm of size one and two are:

$$p\tau - w_1 h(\tau) - e, \tag{1}$$

$$p\tau - 2w_2 h(\tau/2) - 2e. \tag{2}$$

Specify $h(\tau) = \tau^{\beta}$, $\beta > 1$. It is then easy to show that a firm chooses to be of size 2 if and only if

$$\left(w_1 - \frac{2}{2^{\beta}} w_2\right) \tau^{\beta} > e, \tag{3}$$

where it is easy to verify that $2/2^{\beta} < 1$. According to (3), a firm wishes to be large if (1) the fixed cost is not too large; (2) the quantity of task is large enough; and (3) the wage rate of a large firm is not too large.

3 A Dynamic Model with Search Friction

3.1 Environment

The model is an extension of the standard labor market search model of the Mortensen-Pissarides type. We follow Pissarides (2000) as much as possible, except that we view each job (or, post) as a firm. There are K firms and L workers in the economy. We assume that L is constant and K is either fixed or determined by firms' free entry. The firm size in this paper is characterized by the number of employees in the firm; a firm with n workers is called a firm of size n. The novel feature of this model is that the size of a firm is not restricted to one. Let N be a positive integer. For convenience we assume that a firm can employ up to N workers. The support of the distribution of firms is the set $\mathbf{N} = \{0, 1, ..., N\}$. In order to make our life easy, we focus on the simple case in which $N = 2$. That is, the firm size can be either 0, 1, or 2.

Time is discrete and define $k_n(t)$ as the proportion of firms with n employees at date t. Then,

$$k_0(t) + k_1(t) + k_2(t) = 1. \tag{4}$$

It is easy to check that the total number of employees in this economy, denoted by E_t, is

$$E_t = [k_1(t) + 2k_2(t)]\, K. \tag{5}$$

Thus, it is easy to show that the unemployment rate is $u_t = (L - E_t)/L$.

Let λ denote the probability of job destruction: an employee will be unemployed with probability λ. To simplify matters we preclude firms' firing decisions. Let $\sigma(t)$ denote the probability that a firm of size one decides to seek a second worker at date t, provided that a job destruction does not hit the firm. Then, the law of large number implies that the total number of vacancies in the economy is vK, where the vacancy rate satisfies[1]

$$v_t = k_0(t) + (1 - \lambda)\,\sigma(t)\, k_1(t)\,. \tag{6}$$

[1] It is important to note here that the vacancy rate in this paper is defined differently from the one in Pissarides (2000), in which the vacancy rate measures the number of vacancies *per worker*.

That is, the vacancy rate equals the measure of firms of size zero plus the measure of firms of size one seeking a second worker.

As is standard, the total number of matches in the labor market, denoted by M, is determined by a constant-returns-to-scale matching function:

$$M = m(uL, vK). \tag{7}$$

The probability that a firm meets an employed during a period is $M/vK = m(uL/vK, 1) \equiv q(\theta)$, where $\theta \equiv vK/uL$. Similarly, the probability that a worker meets a firm during a period is $M/uL = (M/vK) \cdot (vK/uL) = \theta q(\theta)$. It is easy to check that $q'(\theta) \leq 0$ and $q \to \infty$ as $\theta \to 0$. Since $\theta q(\theta) = M/uL = m(1, \theta)$, it is easy to check that $\theta q(\theta)$ is increasing in θ. At several points in the analysis below, we shall make use of Cobb-Douglas specification for the matching technology: $m(uL, vK) = A(uL)^{\alpha}(vK)^{1-\alpha}$. It is then easy to verify that $q(\theta) = A\theta^{-\alpha}$ and $\theta q(\theta) = A\theta^{1-\alpha}$. Note that, since $q(\theta)$ and $\theta q(\theta)$ are probabilities, it must be the case that $q(\theta) \in (0, 1)$ and $\theta q(\theta) \in (0, 1)$.

Since λ is the exogenous probability that an employee enters the unemployment pool and $\theta q(\theta)$ is the rate at which an unemployed leaves the unemployment pool, the evolution of the unemployment rate is

$$u_{t+1} = (1 - \theta_t q(\theta_t)) u_t + \lambda (1 - u_t). \tag{8}$$

It is then easy to show that at any steady state,

$$u = \frac{\lambda}{\lambda + \theta q(\theta)}. \tag{9}$$

Use (5) and (6) to rewrite θ_t as

$$\theta_t = \frac{k_0(t) + (1 - \lambda)\sigma(t) k_1(t)}{L/K - k_1(t) - 2k_2(t)}. \tag{10}$$

3.2 Firms

Let δ denote the discount factor. Let V_n denote the value function for a firm of size n of creating a vacancy, and let J_n denote the value of production using n workers. Then,

$$J_1 = p\tau - w_1 h(\tau) - e + \lambda\delta V_0 + (1 - \lambda)\max\{V_1, \delta J_1\}, \tag{11}$$

where

$$V_1 = -c + q(\theta)\,\delta J_2 + (1 - q(\theta))\,\delta J_1. \tag{12}$$

The value for a firm with one worker of production, (11), is interpreted as follows. Production takes place at the beginning of each period, and the firm obtains the net instantaneous payoff of $p\tau - w_1 h(\tau) - e$: The gross sales amounts to $p\tau$; the employee works for $h(\tau)$ hours because the total quantity of task the company has is τ; the firm pays the real hourly wage rate of w_1 to the worker; and there is a fixed cost e, which captures all non-wage costs required to maintain each employment. Job destruction takes place right after the production with probability λ. If a job is destroyed, then the firm must engage costly search to fill its vacancy in the next period. With probability $1 - \lambda$ the job is not destroyed. In that case the firm must decide whether it seeks a second worker or not. The expected returns to search during this period is denoted by V_1. The value is not discounted because the firm engages search *this period*, reflecting that the firm is searching *on the job*.[2] According to (12), the firm must pay c to advertise the post. With probability $q(\theta)$ the firm, if engages costly search, meets an unemployed, in which case the firm increases its size to two. If on the other hand the firm meets no one, then the firm must proceeds to the next period with one worker.

Similarly, the value of a firm of size two is

$$J_2 = p\tau - 2w_2 h(\tau/2) - 2e + \lambda^2 \delta V_0 + 2\lambda(1-\lambda)\,\delta J_1 + (1-\lambda)^2\,\delta J_2. \tag{13}$$

Its interpretation is as follows. The firm obtains the net instantaneous payoff of $p\tau - 2w_2 h(\tau/2) - 2e$: The gross sales amounts to $p\tau$; each employee works for $h(\tau/2)$ hours because the total quantity of task the company has is τ; the firm pays the real hourly wage rate of w_2 to each; and there is a fixed non-wage cost e for *each* employment. Remember that an employee leaves the firm with probability λ after production. Thus, with probability λ^2 a job destruction of size two occurs, in which case the firm is forced to start the company all over again. With probability $2\lambda(1-\lambda)$ a

[2] Note that the formulation here is somewhat similar to models with on-the-job search. The literature such as Pissariades (1994) considers on-the-job search by workers. In the present model, it is the firm that engage on-the-job search in order to seek a second worker.

worker leaves the firm and the firm becomes of size one in the next period. If no job destruction takes place, which happens with probability $(1-\lambda)^2$, then the firm will proceed to the next period with two workers. A firm can employ *at most* two workers, a crucial simplifying assumption.

It is assumed that the vacancy cost, c, is the same whether a firm is creating its first or second vacancy. Then,

$$V_0 = -c + q(\theta)\,\delta J_1 + (1 - q(\theta))\,\delta V_0. \tag{14}$$

According to (14), the firm meets an unemployed and fills its vacancy with probability $q(\theta)$. If the firm meets no one during a period, then it has to continue search in the next period.

3.3 Workers

The expected payoffs for a worker employed in a firm of size 1 and 2 are given by

$$W_1 = w_1 h(\tau)\,\phi(1-h(\tau)) + \lambda\delta U + (1-\lambda)\left[\sigma q(\theta)\,\delta W_2 + (1-\sigma q(\theta))\,\delta W_1\right], \tag{15}$$

$$W_2 = w_2 h(\tau/2)\,\phi(1-h(\tau/2)) + \lambda\delta U + (1-\lambda)\left[\lambda\delta W_1 + (1-\lambda)\,\delta W_2\right], \tag{16}$$

where workers take σ as given. Expression (15) is interpreted as follows. The employee is working for a firm of size one and will receive the real wage w_1 this period. In this section we treat wages as given. We follow Pissarides (2000) to assume that the instantaneous utility during employment is

$$wh\phi(1-h),$$

where $\phi' > 0$, $\phi'' \leq 0$.[3] Separation is exogenous. With probability λ the job is destroyed for some reason, which is not specified in this paper. In that event, the employee must leave the firm and look for a new job opportunity in the next period. The value of being unemployed is U. With probability $1-\lambda$ the job is not destroyed so the employee will work for the firm in the next period. In that event, with probability $\sigma q(\theta)$ the firm seeks a second worker and becomes a firm of size 2, where σ is the probability that the firm decides to seek a second worker and $q(\theta)$ is the probability that the search is successful. With probability $1-\sigma q(\theta)$ the firm either fails to hire a second worker or does not seek it. Whether the firm seeks a second worker or not is determined in equilibrium.

[3] See section 7.3 in Pissarides (2000).

According to (16), the worker receives w_2 at the beginning of this period. With probability λ the job is destroyed and the worker enters the unemployment pool. With probability $1-\lambda$ the worker will stay in the firm. Given that the worker stays, the size of the firm is one in the next period with probability λ and two with probability $1-\lambda$ because the other worker might leave the firm.

Let U denote the value of being a worker without a job. The behavior of a worker without a job is summarized by the following Bellman equation:

$$U = b + \delta\theta q(\theta)\left\{\frac{k_0}{v}W_1 + \frac{(1-\lambda)\sigma k_1}{v}W_2\right\} + \delta(1-\theta q(\theta))U. \tag{17}$$

(17) can be interpreted as follows. A worker without a job receives the unemployment benefit, b, at the beginning of each period. The agent is matched with a firm with probability $\theta q(\theta)$. Since k_0 measures the number of firms of size zero *as a fraction of the total firms*, the probability that the firm turns out to be of size zero, given a match, is k_0/v. Similarly, the probability that the firm turns out to be of size one, given a match, is $\sigma k_1/v$. From (6) it is easy to check that $k_0/v+(1-\lambda)\sigma k_1/v = 1$. If none of the events above occurs during the period, then the unemployed must continue search in the next period. Rewrite (17) as

$$(1-\delta)U = b + \delta\theta q(\theta)\left\{\frac{k_0}{v}(W_1 - U) + \frac{(1-\lambda)\sigma k_1}{v}(W_2 - U)\right\}. \tag{18}$$

The agent works for a firm voluntarily, so it is necessary that $W_1 \geq U$ and $W_2 \geq U$.

3.4 Wage Determination

The majority of papers in the search-matching literature adopts Nash bargaining to determine the wage rates. It is often assumed that a firm and a worker, whenever they form a pair, bargain each period over the share of the joint surplus. Nash bargaining implies that the worker receives a predetermined fraction of the joint surplus. Recent studies question the usefulness of the Nash bargaining approach in labor market search models. Hall (2003) and Shimer (2003), among others, argue that the wage behavior under Nash bargaining is too flexible. They demonstrate that introducing some wage stickiness improves the performance of the Mortensen-Pissarides model.

From the theoretical perspective, the Nash bargaining approach is often criticized for its cooperative nature. It is not clear whether the real-world wage negotiation is as cooperative as the theory assumes. However, it is not plausible to assume fixed wages either. In this paper we have w_1 and w_2, and which one is larger should not be determined arbitrarily. Further, one must make sure that the wages are on the Pareto frontier, as pointed out by Hall (2003).

In what follows we adopt the take-it-or-leave-it pricing protocol, which gives a special case of the Nash solution. In particular, we assume that whenever a match forms the firm offers a take-it-or-leave-it offer to its employee(s), under which each employee is indifferent between accepting and rejecting the offer. Thus, firms offer w_1 and w_2 so that $W_1 = U$ and $W_2 = U$. It is easy to see that under this pricing rule, firms receive all the joint surplus. Yet, the offers are on the Pareto frontier because they are special cases of the Nash solutions.

Substitute $W_1 = U$ and $W_2 = U$ into (15) and (16) to obtain

$$\begin{aligned} (1-\delta) U &= w_1 h(\tau) \phi(1 - h(\tau)), \\ (1-\delta) U &= w_2 h(\tau/2) \phi(1 - h(\tau/2)), \end{aligned}$$

Since $W_1 = U$, $W_2 = U$, and (18) imply $(1-\delta) U = b$, it is easy to establish that under the take-it-or-leave-it pricing protocol,

$$w_1 h(\tau) \phi(1 - h(\tau)) = w_2 h(\tau/2) \phi(1 - h(\tau/2)) = b. \tag{19}$$

With $h(\tau) = \tau^\beta$, $\beta > 1$, and $\phi(x) = x^\gamma$, $\gamma \in (0,1)$, (19) reduces to

$$\begin{aligned} w_2 &= w_1 \Delta(\tau), \\ \Delta(\tau) &\equiv 2^\beta \left(\frac{1-\tau^\beta}{1-(\tau/2)^\beta} \right)^\gamma, \end{aligned} \tag{20}$$

where hours cannot exceed unity. (20) captures the relative wage. It is easy to show that

$$\Delta'(\tau) = -\gamma 2^\beta \left(\frac{1-\tau^\beta}{1-(\tau/2)^\beta} \right)^{\gamma-1} \frac{1-(1/2)^\beta}{\left(1-(\tau/2)^\beta\right)^2} \beta \tau^{\beta-1} < 0.$$

Figure 1 depicts how the wage differential, $(w_2 - w_1)/w_1$, changes as τ changes. The parameters used are $b = 1$, $\beta = 1.2$ and $\gamma = 0.6$. According to the figure, the wage differential is high for small values of τ and is low for large values of τ. Notice that the results are derived without any reference to other equilibrium conditions. Thus, not all values of τ are guaranteed to be part of equilibrium.

3.5 The Distribution of Firms

The evolution of the distribution of the firms is described as follows:

$$
\begin{aligned}
k_2(t+1) &= (1-\lambda)^2 k_2(t) + (1-\lambda)\sigma(t) q(\theta_t) k_1(t), &(21)\\
k_1(t+1) &= (1-\lambda-(1-\lambda)\sigma(t) q(\theta_t)) k_1(t) + q(\theta_t) k_0(t) + 2\lambda(1-\lambda) k_2(t), &(22)\\
k_0(t+1) &= (1-q(\theta_t)) k_0(t) + \lambda k_1(t) + \lambda^2 k_2(t). &(23)
\end{aligned}
$$

For convenience, a firm is said to be in state n if it is of size n $(n = 0, 1, 2)$. According to (21), the number of large firms in the next period equals the number of large firms that encounter no job destruction this period plus the number of small firms that successfully find a second worker. Since $(1-\lambda)^2$ is the probability that no job is destroyed, $(1-\lambda)^2 k_2(t)$ is the number of large firms in which no job destruction occurs at date t. A firm of size one seeks for a second worker with probability $\sigma(t)$. Such a firm successfully employs a second worker with probability $q(\theta_t)$. Since a firm of size one becomes of size zero with probability λ, the total measure flows into state 2 is $(1-\lambda)\sigma(t) q(\theta_t) k_1(t)$. According to (22), with probability λ an employee leaves a size-one firm and the firm becomes of size zero. With probability $(1-\lambda)\sigma(t) q(\theta_t) k_1(t)$, the firm becomes of size two. These are the flow out of state 1. The fraction $2\lambda(1-\lambda)$ of large firms become of size one because of job destruction. The measure of firms flow into state 1 from state 0 is $q(\theta_t) k_0(t)$. (23) asserts that the measure of flow into state zero is $\lambda k_1(t)$ plus $\lambda^2 k_2(t)$ and $q(\theta_t) k_0(t)$ flows out.

4 Equilibrium with a Fixed Number of Firms

4.1 Characterization

This sections characterizes equilibrium of the model assuming that there is no entry or exit of firms. Thus, K is constant and assumed to be sufficiently large. Since there is no entry, one can simplify the analysis by imposing $c = 0$ without loss of generality.

Since wages are exogenous in this section, firms can decide whether they look for a second worker or not without reference to W_1, W_2, and U. From (11) it is evident that a firm of size one expands employment if and only if $V_1 > \delta J_1$. We look for a condition under which firms wish to stay small, in which case

$$J_1 = p\tau - w_1 h(\tau) - e + \lambda\delta V_0 + (1-\lambda)\,\delta J_1. \tag{24}$$

(12) can be rewritten as

$$V_1 - \delta J_1 = q(\theta)\,\delta\,[J_2 - J_1]\,. \tag{25}$$

Thus, $V_1 \leq \delta J_1$ implies $J_2 \leq J_1$. Solve (14) and (24) for J_1 as

$$J_1 = \frac{p\tau - w_1 h(\tau) - e}{1 - (1-\lambda)\,\delta - \frac{\lambda\delta q(\theta)\delta}{1-(1-q(\theta))\delta}}. \tag{26}$$

Solve (13) and (14) for J_2 as

$$J_2 = \frac{p\tau - 2w_2 h(\tau/2) - 2e}{1-(1-\lambda)^2\,\delta} + \frac{\frac{\lambda^2\delta q(\theta)}{1-(1-q(\theta))\delta} + 2\lambda(1-\lambda)}{1-(1-\lambda)^2\,\delta}\delta J_1. \tag{27}$$

Using (26) and (27), we can show, although tedious, that $J_2 \leq J_1$ holds if

$$R(\tau) \leq \frac{1 - \left(1-\lambda^2\right)\delta - \lambda H(\theta)}{1-(1-\lambda)\,\delta - H(\theta)} \equiv \Phi(\theta)\,, \tag{28}$$

where

$$R(\tau) \equiv \frac{p\tau - 2w_2 h(\tau/2) - 2e}{p\tau - w_1 h(\tau) - e}, \tag{29}$$

$$H(\theta) \equiv \frac{\lambda\delta^2 q(\theta)}{1-(1-q(\theta))\,\delta}. \tag{30}$$

It is easy to show that $\Phi(\theta) < 1$ for $\lambda < 1$. It is easy to see that the condition (28) is a general version of the one obtained in section 2; in fact the condition in section 2 is identical to (28) if $\Phi(\theta) = 1$. In a static, frictionless economy, a firm wishes to stay small if and only if $p\tau - 2w_2 h(\tau/2) - 2e \leq p\tau - w_1 h(\tau) - e$, comparing only the instantaneous payoffs. For a firm to seek a second worker, the instantaneous payoff for a large firm must be greater with market frictions than without, and the degree is captured by $\Phi(\theta)$.

Lemma 1 *$\Phi(\theta)$ is decreasing in θ.*

Proof. It is straightforward to compute

$$\Phi(\theta) = \frac{(1-\lambda)(1-\delta) H'(\theta)}{[1-(1-\lambda)\delta - H(\theta)]^2}, \quad H'(\theta) = \frac{(1-\delta)\lambda\delta^2 q'(\theta)}{[1-(1-q(\theta))\delta]^2}.$$

Thus, $q'(\theta) < 0$ implies $\Phi'(\theta) < 0$. ■

Proposition 2 *Let $\widehat{\theta}$ satisfies (28) at equality. An increase in b raises $\widehat{\theta}$ if τ is not too large.*

Proof. (a) Specify the functional forms as $h(\tau) = \tau^{\beta}$, $\beta > 1$, and $\phi(x) = x^{\gamma}$, $\gamma \in (0,1)$. Use (19) to rewrite (29) as

$$R(\tau) = \frac{p\tau - \frac{2b}{\phi(1-h(\tau/2))} - 2e}{p\tau - \frac{b}{\phi(1-h(\tau))} - e}. \tag{31}$$

Differentiate (31) with respect to b to show that $dR/db < 0$ if $\phi(1-h(\tau)) < 2\phi(1-h(\tau/2))$. This condition states that hours at a small firm should not be too large, although concave production function implies $h(\tau) > h(\tau/2)$. Rewrite the condition further to obtain

$$\tau < \left\{ \frac{1-2^{1/\gamma}}{2^{-\beta}-2^{1/\gamma}} \right\}^{1/\beta}.$$

If this condition is violated, then τ is too large in the sense that $\phi(1-h(\tau)) < 2\phi(1-h(\tau/2))$ is violated. ■

Figure 2 shows the determination of $\widehat{\theta}$. By lemma 1, Φ is decreasing in θ and $R(\tau)$ is constant. Thus, firms choose to be small if $\theta \leq \widehat{\theta}$. Proposition 2 asserts that an increase in b shifts the R-locus down, thereby raises $\widehat{\theta}$. In words, an increase in the unemployment benefit raises the likelihood that firms choose to stay small. This is because an increase in b raises w_1 and w_2, thereby reduces the instantaneous payoff for both large and small firms. However, $dR/db < 0$ implies that for large firms an increase in b reduces the instantaneous payoff even more. This makes it more attractive for firms to be small.

4.2 Stationary Equilibrium

As stated earlier, whether a firm seeks a second worker or not is determined by the condition (28). What remains is to compute the steady-state value of θ. Without free entry, (11)-(14) cannot pin down the value of θ; J_1, J_2, V_1, and V_0 can be written in terms of θ. Suppose that all firms prefer to be large. In this case, $\sigma = 1$ holds. Then, manipulate (21)-(23) and $k_0 + k_1 + k_2 = 1$ and solve for k's as

$$k_0 = \frac{\lambda^2 \left[(2-\lambda) + (1-\lambda) q(\theta)\right]}{\lambda^2 (2-\lambda) + \lambda \left(2-\lambda^2\right) q(\theta) + (1-\lambda) q(\theta)^2}, \tag{32}$$

$$k_1 = \frac{\lambda (2-\lambda) q(\theta)}{\lambda^2 (2-\lambda) + \lambda \left(2-\lambda^2\right) q(\theta) + (1-\lambda) q(\theta)^2}, \tag{33}$$

$$k_2 = \frac{(1-\lambda) q(\theta)^2}{\lambda^2 (2-\lambda) + \lambda \left(2-\lambda^2\right) q(\theta) + (1-\lambda) q(\theta)^2}. \tag{34}$$

Substitute (32)-(34) into (10) and impose $\sigma = 1$ to finally obtain

$$\theta = \frac{\lambda}{\frac{L}{K}\frac{D(\theta)}{B(\theta)} - q(\theta)} \equiv \Omega_L(\theta), \tag{35}$$

where $B(\theta) \equiv \lambda(2-\lambda) + 2(1-\lambda) q(\theta)$ and $D(\theta) \equiv \lambda^2 (2-\lambda) + \lambda\left(2-\lambda^2\right) q(\theta) + (1-\lambda) q(\theta)^2$. Since this section assume the number of firms is exogenous, L/K is constant, so the map $\theta = \Omega_L(\theta)$ is fully specified. The fixed point of $\theta = \Omega_L(\theta)$ gives the steady-state value of θ when all firms wish to be large.

Suppose now that all firms prefer to be small. In that case, $\sigma = 0$ holds. Then, (21)-(23) and $k_0 + k_1 + k_2 = 1$ lead to

$$k_0 = \frac{\lambda}{\lambda + q(\theta)}, \tag{36}$$

$$k_1 = \frac{q(\theta)}{\lambda + q(\theta)}. \tag{37}$$

Substitute (36)-(37) into (10) and impose $\sigma = k_2 = 0$ to obtain

$$\theta = \frac{\lambda}{\frac{L}{K}\left[\lambda + q(\theta)\right] - q(\theta)} \equiv \Omega_S(\theta). \tag{38}$$

The fixed point of $\theta = \Omega_S(\theta)$ gives the steady-state value of θ, given that all firms wish to be small.

Proposition 3 *At a stationary equilibrium, $\theta_S < \theta_L$ holds.*

Proof. From (35) and (38), it is easy to show that $\Omega_L(\theta) > \Omega_S(\theta)$ for all θ if and only if $\lambda + q(\theta) > D(\theta)/B(\theta)$. After some algebra, we can show that

$$\lambda + q(\theta) - \frac{D(\theta)}{B(\theta)} = \frac{\lambda(1-\lambda)(2-\lambda) q(\theta)}{\lambda(2-\lambda) + 2(1-\lambda) q(\theta)} > 0.$$

■

Figure 3 show the typical configurations of $\Omega_L(\theta)$ and $\Omega_S(\theta)$. As shown in the figure, $\Omega_L(\theta) > \Omega_S(\theta)$ for all θ so $\theta_S < \theta_L$ holds. If $\widehat{\theta} > \theta_L$, then we should be looking at the map $\Omega_S(\theta)$, thus $\theta = \theta_S$. If $\widehat{\theta} < \theta_S$, then we should be looking at the map $\Omega_L(\theta)$, thus $\theta = \theta_L$. In either cases we can choose one equilibrium. If $\theta_S < \widehat{\theta} < \theta_L$, however, then we will encounter the problem of multiple equilibria; both θ_S and θ_L satisfy the equilibrium conditions. We argue that the source of multiplicity in this economy is that a firm's decision regarding whether it expands employment depends on all other firms' decisions, which are captured by σ. This interdependence creates strategic complementarity among firms. Since multiplicity of equilibria is beyond the scope of this study, in what follows we shall focus on parameter spaces in which unique equilibrium obtains.

In figure 4, we computed how θ, u, v, and the average hours change as τ changes. It is important to note that the equilibrium value of θ is not influenced by changes in τ; the maps Ω_S and Ω_L are determined without any reference to τ, although τ influences which equilibrium is chosen. As shown in the figure, an increase in τ has no effect on equilibrium values of θ, u, and v as long as the economy is in the same equilibrium. For small values of τ, firms are small. At that equilibrium the unemployment rate is high and the vacancy rate is low. For large values of τ all firms wish to be large. At that equilibrium the unemployment rate is low and the vacancy rate is high.

Figure 4d shows how the average hours change as τ changes. As τ *decreases*, the quantity of task decreases and the hours of work required to complete the task decreases. If τ decreases further, then the firms wish to be of size one, raising hours. Thus there is a jump in average hours.

Figures 5 and 6 focus on the equilibrium in which all firms wish to be large, and show how changes in λ influences u, v, and the distribution of firms. Figure 5 implies that as the separation rate increases, both u, and v increase. According to figure 5d, the average hours also rise as λ increases. This reflects that as the separation rate increases the fraction of large firms decreases, as shown in figure 6.

Figures 7 and 8 focus on the equilibrium in which all firms wish to be large, and show how changes in L/K influences u, v, and the distribution of firms. As shown in figures 7b-c, the unemployment rate increases and the vacancy rate decreases as L/K increases. Thus, an increase in population or labor participation relative to the number of firms will tighten the labor market. According to figure 7d, the average hours decreases as L/K increases. This reflects that as L/K increases the fraction of large firms increases, as shown in figure 8.

It is important to note here that b does not appear in equations that characterize stationary equilibrium. Thus, although a change in b influences firms' decisions regarding whether they expand employment, it has no effect on the distribution of firms.

5 Equilibrium with Entry

5.1 Characterization

The previous section has considered equilibrium in which there is a fixed number of firms. This section considers equilibrium with firms' free entry. In such an economy the number of firms is determined endogenously.[4] Free entry requires $V_0 = 0$, from which it is easy to show that (14) reduces to

$$J_1 = \frac{c}{\delta q(\theta)}. \tag{39}$$

Use (11)-(13) to describe

$$V_1 = -c + q(\theta)\,\delta J_2 + (1 - q(\theta))\frac{c}{q(\theta)}, \tag{40}$$

$$J_2 = \frac{p\tau - 2w_2 h(\tau/2) - 2e + 2\lambda(1-\lambda)\frac{c}{q(\theta)}}{1 - (1-\lambda)^2\delta}. \tag{41}$$

Thus, (39) and (40) implies that $V_1 > \delta J_1$ if

$$J_2 > \frac{2c}{\delta q(\theta)}. \tag{42}$$

Substitute (41) into (42) to finally obtain

$$p\tau - 2w_2 h(\tau/2) - 2e + 2\lambda(1-\lambda)\frac{c}{q(\theta)} > \left[1 - (1-\lambda)^2\delta\right]\frac{2c}{\delta q(\theta)}. \tag{43}$$

[4] From a purely mathematical view point, V_0 is given in this section while L/K is constant in the previous section.

In the limiting case where c is arbitrarily small, the condition reduces to $p\tau - 2w_2 h(\tau/2) - 2e > 0$: firms wish to become large as long as there is a positive instantaneous payoff. For $c > 0$, a firm choose to expand employment if

$$q(\theta) > \frac{[1-(1-\lambda)\delta]\,2c}{[p\tau - 2w_2 h(\tau/2) - 2e]\,\delta}. \tag{44}$$

5.2 Stationary Equilibrium

Consider a stationary equilibrium in which all firms choose to be small. Free entry requires $V_0 = 0$, from which it is easy to show that (14) implies $J_1 = c/\delta q(\theta)$. Then, (11) can be rewritten as

$$\frac{c}{\delta q(\theta)} = \frac{p\tau - w_1 h(\tau) - e}{1-(1-\lambda)\delta}, \tag{45}$$

which determines θ.

When is this equilibrium valid? (44) and (45) imply that firms choose to stay small if

$$p\tau - 2w_2 h(\tau/2) - 2e < 2[p\tau - w_1 h(\tau) - e], \tag{46}$$

which states that firms choose the be small if the instantaneous payoff for a large firm does not exceed the twice the payoff for a small firm. Although (44) includes c, λ, and δ, (46) reveals that these parameters do not influence firms' decisions regarding firm size. It is interesting to compare the condition to the one obtained for the static, frictionless model presented in section 2. In the static model, a firm chooses to be small if the instantaneous payoff for a large firm does not exceed the payoff for a small firm.

It is very important to point out that there is no scope for multiplicity. In the preceding section, a firm's decision depends on the size distribution of firms and especially on θ. From (10), θ depends on σ, implying that a firm's decision depends on all other firms' decisions. This causes strategic complementarity. In sharp contrast, the model considered here does not generate multiple equilibria because a firm decides whether it expands employment or not without any reference to the market tightness.

The condition (46) further reduces to

$$w_1 h(\tau) < w_2 h(\tau/2) + \frac{p\tau}{2}. \tag{47}$$

Thus, firms choose to be small if the wage payment per worker at a small firm is sufficiently smaller than the sum of the wage payment per worker at a large firm and a premium. It is important to notice that e disappears in (47). The fixed cost plays an important role in determining the size of a firm in the static model and in the search model without free entry. It has no role here. There is another difference. The price in the product market, p, which is exogenous, plays no role in the static model while it appears in (47). Higher prices induce firms to stay small.

There is one another way to express the condition. Use (19) to rewrite (47) as

$$b < \frac{p\tau}{2} \frac{\phi\left(1-h\left(\tau/2\right)\right)\phi\left(1-h\left(\tau\right)\right)}{\phi\left(1-h\left(\tau/2\right)\right)-\phi\left(1-h\left(\tau\right)\right)} \equiv \Gamma\left(\tau\right). \tag{48}$$

This condition is written in terms of b and τ. It states that for any τ, a firm expands employment if the unemployment benefit is small enough.

Now consider a stationary equilibrium in which all firms choose to be large. From (11)-(13),

$$J_1 = p\tau - w_1 h\left(\tau\right) - e + \lambda\delta V_0 + \left(1-\lambda\right) V_1, \tag{49}$$

$$V_1 = -c + q\left(\theta\right)\delta J_2 + \left(1-q\left(\theta\right)\right)\delta J_1, \tag{50}$$

$$J_2 = p\tau - 2w_2 h\left(\tau/2\right) - 2e + \lambda^2\delta V_0 + 2\lambda\left(1-\lambda\right)\delta J_1 + \left(1-\lambda\right)^2\delta J_2. \tag{51}$$

Substitute $J_1 = c/\delta q\left(\theta\right)$ into (51) to obtain

$$J_2 = \frac{p\tau - 2w_2 h\left(\tau/2\right) - 2e + \frac{2\lambda(1-\lambda)c}{q(\theta)}}{1-\left(1-\lambda\right)^2\delta}. \tag{52}$$

It is then easy to rewrite (50) as

$$V_1 = -c + q\left(\theta\right)\delta\frac{p\tau - 2w_2 h\left(\tau/2\right) - 2e + \frac{2\lambda(1-\lambda)c}{q(\theta)}}{1-\left(1-\lambda\right)^2\delta} + \frac{\left(1-q\left(\theta\right)\right)c}{q\left(\theta\right)}. \tag{53}$$

Substitute (52), (53) and $V_0 = 0$ into (49) to finally obtain

$$\begin{aligned} \frac{c}{\delta q\left(\theta\right)} &= p\tau - w_1 h\left(\tau\right) - e + \left(1-\lambda\right) q\left(\theta\right)\delta\frac{p\tau - 2w_2 h\left(\tau/2\right) - 2e}{1-\left(1-\lambda\right)^2\delta} \\ &\quad - \left(1-\lambda\right)c + \frac{2\lambda\left(1-\lambda\right)^2\delta c}{1-\left(1-\lambda\right)^2\delta} + \frac{\left(1-\lambda\right)\left(1-q\left(\theta\right)\right)c}{q\left(\theta\right)}, \end{aligned} \tag{54}$$

which is a quadratic form in terms of q. The roots are

$$\begin{aligned}
q &= \frac{-a_2 \pm \sqrt{a_2^2 + 4a_1 c\left(1-\left(1-\lambda\right)\delta\right)/\delta}}{2a_1}, \\
a_1 &\equiv \frac{\left(1-\lambda\right)\delta}{1-\left(1-\lambda\right)^2\delta}\left[p\tau - 2w_2 h\left(\tau/2\right) - 2e\right], \\
a_2 &\equiv p\tau - w_1 h\left(\tau\right) - e - 2\left(1-\lambda\right)c\frac{1-\left(1-\lambda\right)\delta}{1-\left(1-\lambda\right)^2\delta}.
\end{aligned}$$

It is easy to show that the smaller root is negative.

Figure 9 shows how θ, u, v, and the average hours change as τ changes. For small values of τ the condition (46) is satisfied, thus all firms choose to be small. For large values of τ, on the other hand, the condition (46) is violated and all firms choose to be large, although not all firms can be large because of the search friction. In contrast to the model without free entry, changes in τ influence θ, u, and v within the same equilibrium.

Figure 10 depicts how changes in τ, which capture changes in the aggregate demand, influence the size distribution of firms. As before, we focus on equilibrium in which all firms choose to be large. As easily predicted, the figure demonstrates that as τ increases the fraction of size-2 firms increases.[5]

Figure 11 shows how changes in b, unemployment benefit, influence θ, u, v, and the average hours. For small values of b all firms choose to stay small and for large values all firms wish to be large, creating a jump in the figures. It is easy to see from the figures that an increase in b raises unemployment and reduces vacancy. That an increase in unemployment benefit raises unemployment is a well-known result; an increase in b raises the value for a worker of waiting. Figure 11c is interesting. As b increases the vacancy rate reduces. However, it also induces firms to expand employment. Vacancy jumps up at the point where all firms choose to expand employment and decreases as b increases further. Figure 12 focuses on the equilibrium in which all firms wish to be large and computes how the size distribution of firms changes as b changes. The fraction of large firms increases as the unemployment benefit increases.

[5] Changes in distribution may look subtle, but this is because we did not allow wide variety in τ.

6 Conclusion

This paper has investigated the size distribution of firms in a labor market search model of Mortensen-Pissarides type. A firm, if it wishes, engages costly on-the-job search to fill its second vacancy. Firms submit take-it-or-leave-it wage offers to workers, and the production function is concave. We have used the model to study how changes in aggregate demand affect hours, wages, and the size distribution of firms.

In this paper the level of aggregate demand plays a central role in determining firm size and we treated the aggregate demand as exogenous. A possible future work is to integrate the product market to determine the aggregate demand.

References

[1] Bertola, Giuseppe, and Ricardo J. Caballero. "Cross-Sectional Efficiency and Labour Hoarding in a Matching Model of Unemployment." *Review of Economic Studies* 61 (1994) 435-456.

[2] Bertola, Giuseppe, and Pietro Garibaldi. "Wages and the Size of Firms in Dynamic Matching Models." *Review of Economic Dynamics* 4 (2001) 335-368.

[3] Camera, Gabriele, and Dean Corbae. "Money and Price Dispersion." *International Economic Review* 40 (1999) 985-1008.

[4] Hall, Robert E. "Wage Determination and Employment Fluctuations." mimeo, 2003.

[5] Mortensen, Dale T. *Wage Dispersion,* MIT Press, Massachusetts, 2003.

[6] Oi, Walter Y., and Todd L. Idson. "Firms Size and Wages." in, O. Ashenfelter and D. Card (editors) *Handbook of Labor Economics*, 1999.

[7] Osborne, Martin J., and Ariel Rubinstein. *Bargaining and Markets.* Academic Press, San Diego, 1990.

[8] Pissarides, Christopher. "Search Unemployment with On-the-Job Search." *Review of Economic Studies* 61 (1994) 457-75.

[9] Pissarides, Christopher. *Equilibrium Unemployment Theory*, 2nd edition, MIT Press, Massachusetts, 2000.

[10] Shi, Shouyong. "Product Market and the Size-Wage Differential." *International Economic Review* 43 (2002) 21-54.

[11] Simer, Robert. "The Cyclical Behavior of Equilibrium Unemployment and Vacancies: Evidence and Theory." mimeo, 2003.

[12] Smith, Eric. "Search, Concave Production, and Optimal Firm Size." *Review of Economic Dynamics* 2 (1999) 456-471.

Figure 1: Size-wage differential

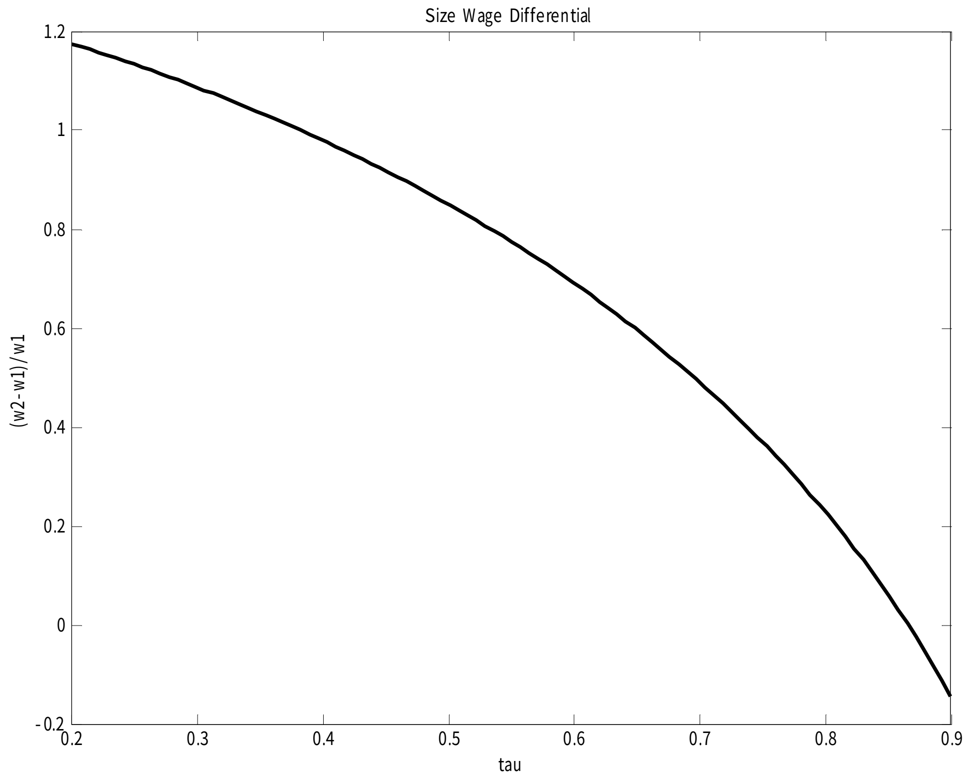


Figure 2: Threshold level of θ

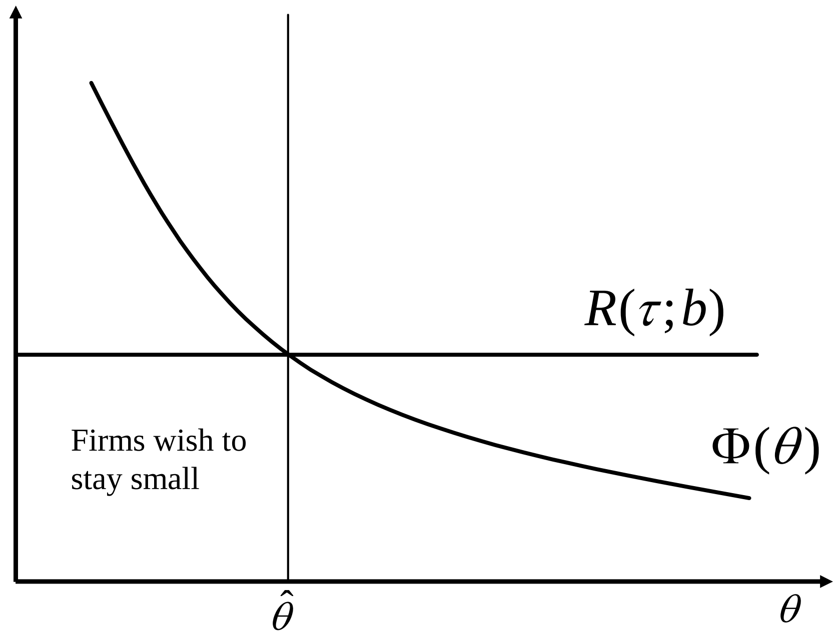


Figure 3: Stationary equilibria

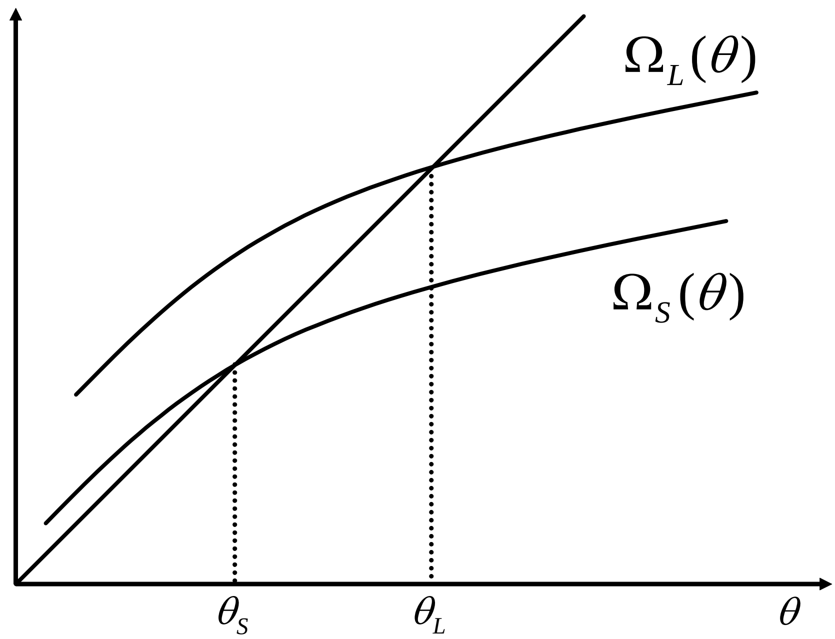
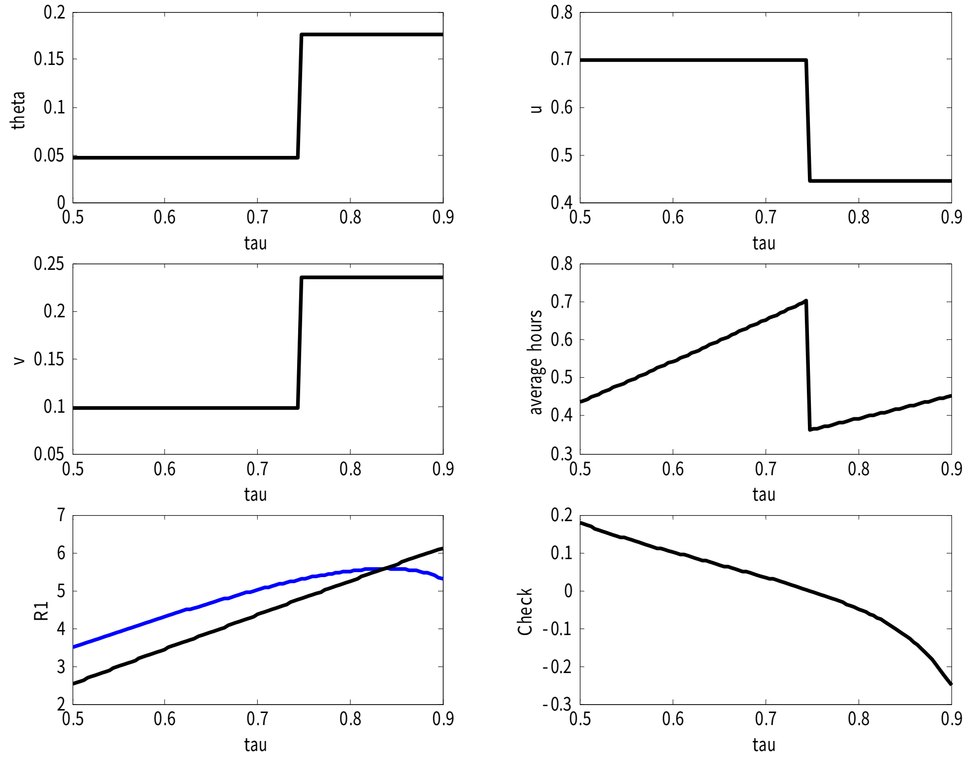


Figure 4: Changes in aggregate demand



(A = 1, α =0.2, β = 1.2, δ = 0.96, λ = 0.2, γ = 0.6, L/K = 3, b = 1, p =10, e = 0.1)

Figure 5: Changes in separation rate

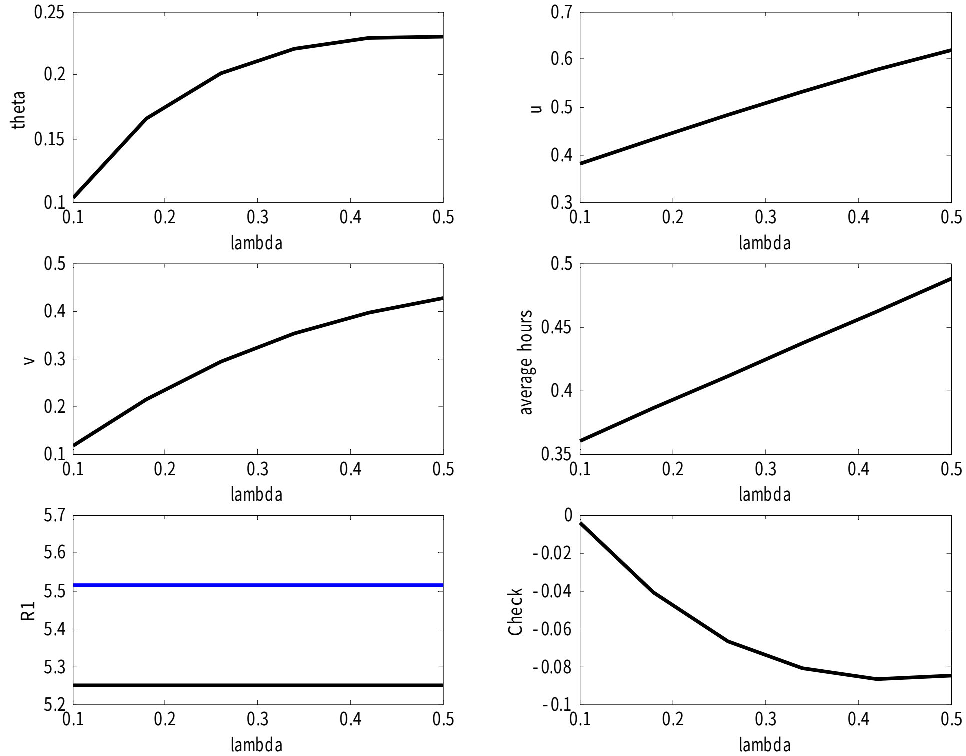


Figure 6: Separation rate and the size distribution of firms

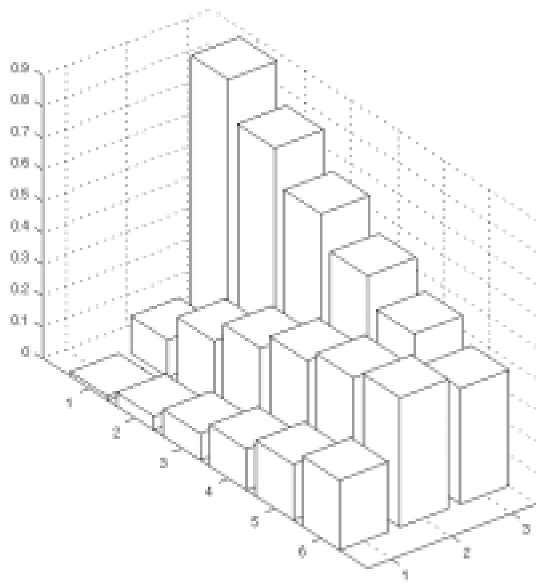


Figure 7: Changes in the relative size of the market

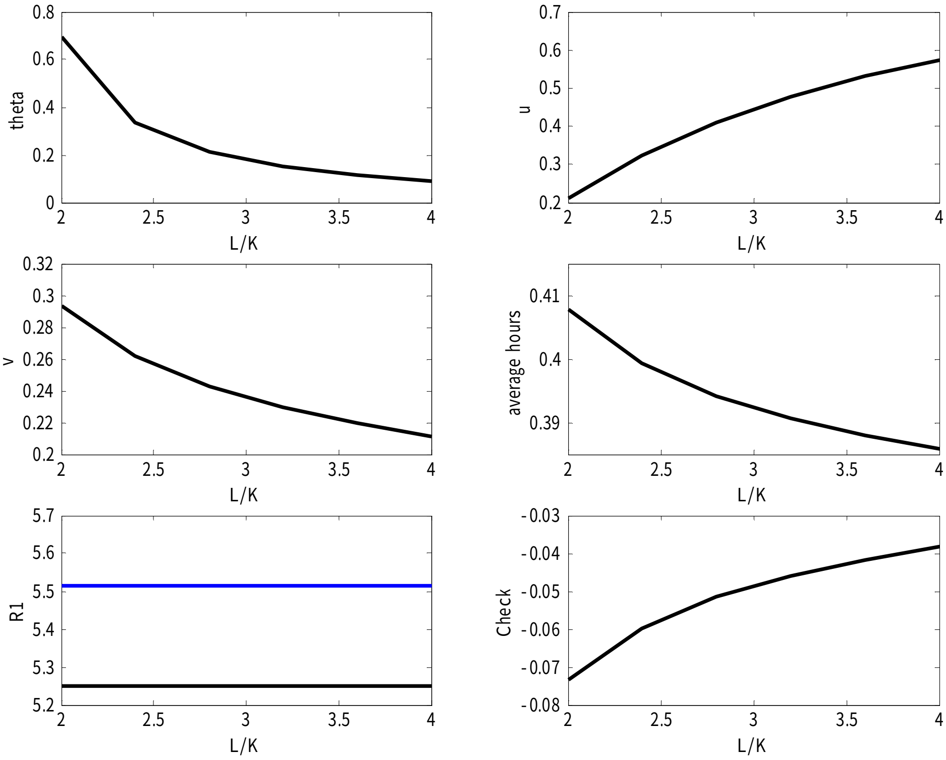


Figure 8: Relative size of the market and the size distribution of firms

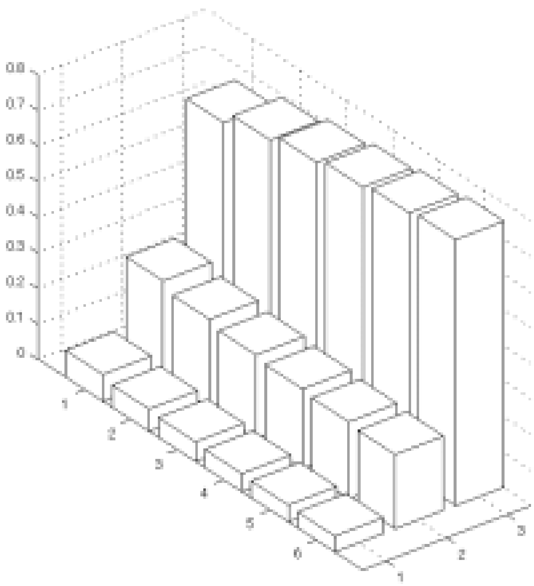
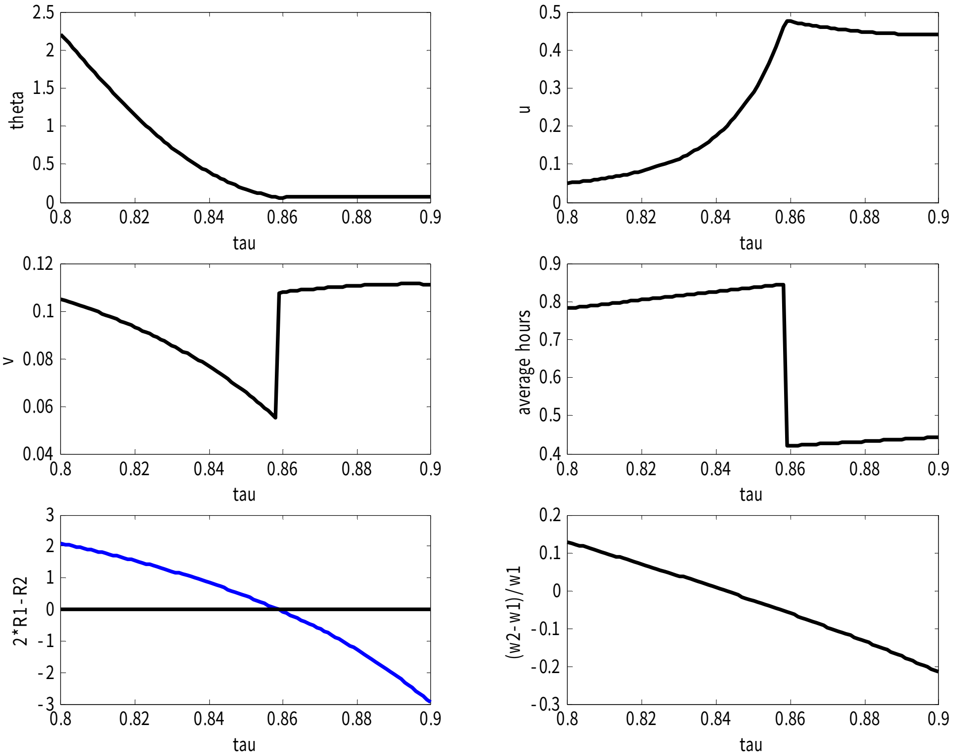


Figure 9: Changes in aggregate demand in a model with free entry



(A = 1, α =0.2, β = 1.1, δ = 0.96, λ = 0.1, γ = 0.6, b = 1, p =10, e = 0.1, c = 10)

Figure 10: Aggregate demand and the size distribution of firms

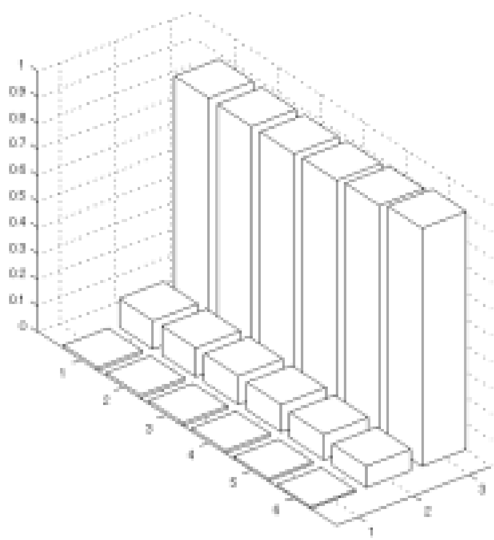


Figure 11: Changes in unemployment benefit

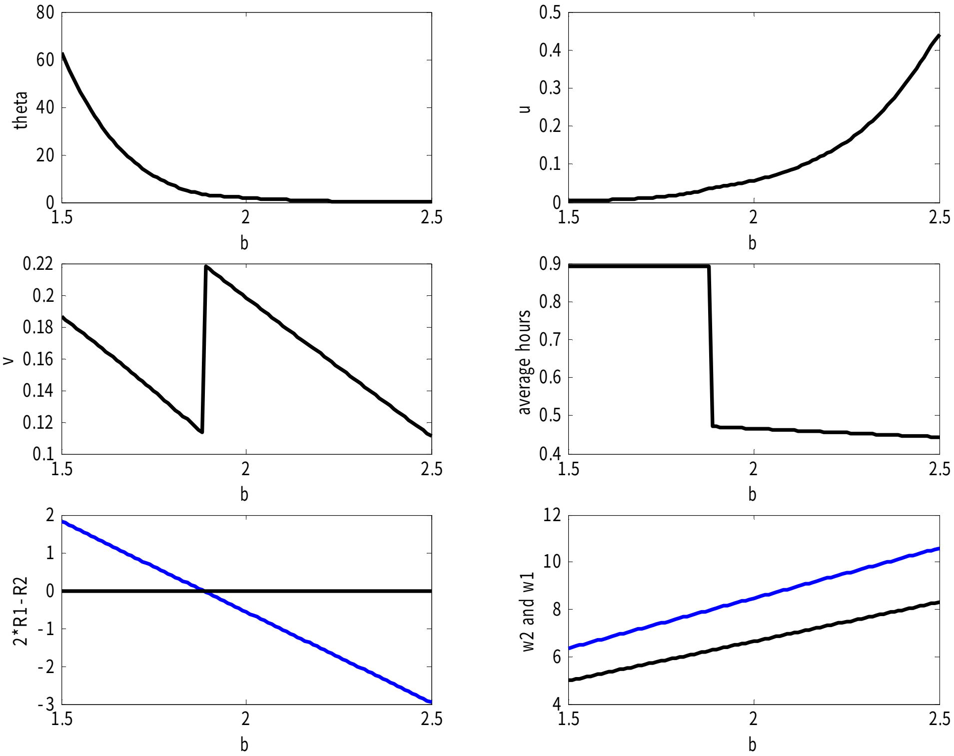


Figure 12: Unemployment benefit and firm distribution