Stable Profit Sharing in Patent Licensing: General Bargaininig Outcomes

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May, 2004

Abstract

In a generalized framework, we study how much profit sharing the licensor of a patented innovation can gain through negotiations with licensees and how many potential licensees he or she should invite to the negotiation, from a viewpoint of stable coalition structures. The core with coalition structure is empty, unless the grand coalition forms with some condition. The bargaining set with coalition structure is a singleton, if the number of licensees optimal for the licensor is larger than that of non-licensees. The bargaining set coincides with the core, if the core is nonempty.

Keywords : patent licensing, bargaining set with coalition structure, stable profit sharing

JEL Classification Numbers : D45, D43 C71

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1 Introduction

Patent licensing problems in oligopolistic markets have been studied only with noncooperative licensing policies; fixed fees or royalty in Kamien and Tauman (1984, 1986), and auction in Katz and Shapiro (1985, 1986). Kamien, Oren and Tauman (1992) compared those three policies for general demand functions: in the Cournot competition, it is never optimal for an external patent holder to license a cost-reducing innovation by means of the royalty. Muto (1993) studied the optimal licensing in the Bertrand duopoly with differentiated commodities: there is a case where it is optimal for an external patent holder to license by means of the royalty.

On the other hand, licensing through negotiations is also realistic. If there are many potential licensees, it may be best for the licensor to adopt simpler noncooperative policies in order to reduce large "transaction costs" due to persistent negotiations. However, such costs will not be so large if the licensor negotiate with a limited number of potential licensees.

Hence, we study a general cooperative perspective in the patent licensing problems, applying cooperative solutions with coalition structures[1]. Our questions are: (1) how much the licensor 0 can gain through negotiations with potential licensees? (2) which coalition size is most favorable to him?

A key problem with us is how to define the worth of each coalition of players in this model. Driessen, Muto and Nakayama (1992) applied to an information trading a classical characterization of the worth of a coalition in oligopolistic markets: in each coalition of the seller and the buyers of the information on a new technology, not all the buyers are necessarily provided with the information for the efficient sharing among them[2]. However, it seems more natural in reality that every buyer in such coalitions should be provided with the information. We define the worth of each coalition in another way.

[1] See Aumann and Drèze (1974), Thrall and Lucas (1963), Aumann and Maschler (1964) for those concepts.

[2] Let $s = |S|$ be the number of sellers in a coalition $\{0\} \cup S$. Player 0 is the seller of the information. The worth of coalition $\{0\} \cup S$ is defined as $v(\{0\} \cup S) = \max\{tW(t) + (s-t)L(t) | 0 \leq t \leq s\}$, where t is the number of the informed, $W(t)$ and $L(t)$ are the equilibrium payoffs of each informed agent and each non-informed agent, respectively.

Watanabe and Tauman (2003) proposed another definition, taking into account a sofisticated nature of events under a subtle mixture of conflict and cooperation: some licensees can form a cartel S to enhance their oligopolistic power, whereas non-licensees can react by also forming some cartels. The licensees in S might not merge into a single entity, but gather as smaller subcartels in S (e.g., the headquarter-subsidiaries relationship) in such an anticipation of reactions taken by non-licensees.

In this paper, we assume that any cooperations among firms is prohibited (by law) for the sake of a fair comparison with noncooperative licensing policies. A group forms only for negotiations with the licensor. However, such a group formation appears also in Watanabe and Tauman under some conditions (footnote 6); even if firms are allowed to cooperate in the market, firms in any groups will decide not to do so by themselves.

Another key point is that we study the bargaining outcomes in a generalized framework of patent licensing problems that have been studied. In the classical literature, function forms, market structures, and characteristics of the innovation are specified, e.g., linear demand and cost functions, Cournot oligopoly, and a non-drastic cost-reducing innovation under the "perfect" patent protection. However, the stable profit sharing of the licensor can be characterized in quite a less specified model.

We show that: (1) the core with coalition structure is empty, unless the grand coalition forms with some condition. it is always empty in the classical linear model followed by the Cournot competition. (2) the bargaining set with coalition structure is a singleton, if the coalition size is larger than a half of all the players. the stable profit sharing is uniquely determined in that case. (3) the bargaining set coincides with the core, if the core is nonempty. even in a linear model, it can be nonempty, if commodities are differentiated.

The outline of this paper is as follows. For better understanding our generalization, Section 2 gives a classical linear model of patent licensing. Section 3 formalizes our general cooperative licensing game. The core and the bargaining set with coalition structure are the solution concepts we study. Section 4 and 5 provide the results. Some discussions and remarks on related literature are stated in the last section.

2 A Linear Model

There are n firms operating in the market, where $2 \leq n < \infty$. Each firm i produces $q_i (\geq 0)$ units of an identical commodity with the same unit cost $c(> 0)$ of production. The market price p of the commodity is determined by $p = \max(a - \sum_{i \in N} q_i, 0)$, where $a \in (c, \infty)$ is a constant. An external licensor has a patent of an innovation which reduces the unit cost of production from c to $c - \epsilon$, where $0 < \epsilon < c$ and $a - c - \epsilon \geq 0$ (non-drastic innovation[3]).

The profit of firm i is $u_i(q) = pq_i - C(q_i)$. $C(q_i) = (c - \epsilon)q_i$ if i is a licensee of the patented innovation, and $C(q_i) = cq_i$ otherwise. Lacking any production facilities, the licensor takes no action in the market but shares some of the profits of licensees in return for licensing his innovation.

If the licensor sells the license to firms by means of fixed fees only, the game is played as follows[4].The licensor first shows the prices of the patented innovation to firms, and each firm next decides whether or not to purchase it at each price shown by the licensor simultaneously and independently of the other firms. Finally they compete *à la* Cournot in the market, knowing that which firms are licensed or not.

The game is analyzed backwardly in the spirit of the subgame perfection. Given that s firms are licensed, let $W(s)$ and $L(s)$ denote the equilibrium profit of each licensee and that of each non-licensee, respectively. Let $\hat{s} := (a - c)/\epsilon$. Then,

$$W(s) = \begin{cases} ((a - c + (n - s + 1)\epsilon)/(n + 1))^2 & \text{if } s \leq \hat{s} \\ ((a - c + \epsilon)/(\hat{s} + 1))^2 & \text{if } s > \hat{s} \end{cases}$$

$$L(s) = \begin{cases} ((a - c - s\epsilon)/(n + 1))^2 & \text{if } s \leq \hat{s} \\ 0 & \text{if } s > \hat{s} \end{cases}$$

The payoff structure is summarized as

$$W(1) > \cdots > W(s) > \cdots > W(n) > L(0) > \cdots > L(s) > \cdots > L(\hat{s} - 1) \\ \geq L(\hat{s}) = \cdots = L(n - 1) = 0.$$

[3]The drastic case, $a - c - \epsilon < 0$, is trivial to analyze, since every non-licensee stops the production in this linear Cournot model.

[4]The game with licensing by means of royalties only or auction is played in a similar manner. See Kamien and Tauman (1984, 1986) and Katz and Shapiro (1985, 1986) for more details.

3 A Licensing Game with Coalition Structures

We generalize the linear model described in the previous section. There are n firms with the same cost functions and an external licensor of a patented innovation. $N = \{1, \ldots, n\}$ is the set of all the firms and the licensor is denoted as player 0. The market can be either the Cournot or the Bertrand oligopoly for homogeneous or differentiated commodities. The innovation can be a quality-improving technology as well as a drastic or non-drastic cost-reducing one. Let $\{0\} \cup S$ ($S \subseteq N$) denote the set of the licensor and all the potential licensees. No firm outside $\{0\} \cup S$ is licensed.

The game has two stages. It starts with negotiations among the licensor and the firms in S. The patented innovation is licensed to the firms in S, while how much each firm pays to the licensor is determined in negotiations. Next, firms compete in the market, knowing that which firms are licensed or not. No cooperation among firms is allowed in the market for a fair comparison of our results with noncooperative ones.

The equilibrium profit of each firm $i \in N$ in the market is determined for general (symmetric) demand and cost functions. Given that s firms hold the license, $W(s)$ and $L(s)$ denote the equilibrium profit of each licensee and that of each non-licensee, respectively. We require only the following:

$$W(s) > L(0) > L(s) \quad {}^{\forall}s.$$

In the Bertrand duopoly with differentiated commodities, it can be that $W(1) < W(2)$, if the commodities are substitutive at a sufficiently small rate[5]. Our model contains this case. The "spillover" of the patented innovation to non-licensees is also included in this model, since the magnitudes of equilibrium profits are not concerned here. Suppose that non-licensees can also utilize the patented innovation with some probability due to the spillover. Then, $W(s)$ and $L(s)$ are interpreted as the expected equilibrium profits when s firms are officially licensed.

We hereafter formalize this situation as a cooperative game with sidepayments. Denote by s^* the number of licensees such that $s^*(W(s^*) - L(0)) \geq s(W(s) - L(0))$ for any s.

[5] See Muto and Watanabe (2004).

Let $s = |S|$. The worth of each set of players is then characterized by

$$v(\{0\}) = v(\emptyset) = 0 \quad v(\{0\} \cup S) = sW(s)$$
$$v(S) = sL(n-s).$$

The licensor 0 can gain nothing without selling the innovation, since he or she has no production facilities. $v(\{0\} \cup S) = sW(s)$ is the total equilibrium profits of licensees in S. $v(S)$ is the total equilibrium profits that the firms in S can guarantee for themselves even in the worst anticipation that all the other $n-s$ firms are licensed when firms in S jointly break off the negotiations[6]. Any firms can equally be a member of S, since every firm is identical before licensed. Hence, we can apply $v(S)$ to a group S of non-licensees.

Given a set $S \subseteq N$ of firms, negotiations are done only within $\{0\} \cup S$, and so the permissible coalition structure is $P^S = (\{0\} \cup S, \{\{i\}\}_{i \in N \setminus S})$. Since no cooperation is allowed in the market, coalition $\{0\} \cup S$ forms only for negotiations.

The set of imputations under a coalition structure P^S is then defined as

$$X^S = \{x = (x_0, x_1, \cdots, x_n) \in \mathbb{R}^{n+1} | x_0 + \textstyle\sum_{i \in S} x_i = sW(s),$$
$$x_0 \geq 0, \ x_i \geq L(n-1) \ ^{\forall} i \in S, \quad x_i = L(s) \ ^{\forall} i \in N \setminus S\}.$$

The core with coalition structure P^S is defined as

$$C^S = \{x \in X^S | \sum_{i \in T} x_i \geq v(T) \ ^{\forall} T \subseteq \{0\} \cup N, \ T \cap (\{0\} \cup S) \neq \emptyset\}.$$

It can be shown that $C^S = \{x \in X^S | \sum_{i \in T} x_i \geq v(T) \ ^{\forall} T \subseteq \{0\} \cup N\}$.

Let $i, j \in \{0\} \cup S$ and $x \in X^S$. We say that i has an objection (y, T) against j in x if $i \in T$, $j \notin T$, $T \subseteq \{0\} \cup N$, $y_k > x_k \ ^{\forall} k \in T$, and $\sum_{k \in T} y_k \leq v(T)$, and that j has a counter objection (z, R) to i's objection (y, T) if $j \in R$, $i \notin R$, $R \subseteq \{0\} \cup N$, $\ z_k \geq x_k \ ^{\forall} k \in R$, $z_k \geq y_k \ ^{\forall} k \in R \cap T$, and $\sum_{k \in R} z_k \leq v(R)$. We say that i has a *valid* objection (y, T) against j in x if (y, T) is not countered.

[6] See Watanabe and Tauman (2003) and Watanabe (2004a) for models with competition and cooperation among firms. Under conditions that $s \leq (n+1)/2$ and $s \leq \hat{s}$ in the linear model, it is easy to show that our v-function is derived as a special case of theirs.

The bargaining set with coalition structure P^S is defined as

$$M^S = \{x \in X^S | \text{no player in } \{0\} \cup S \text{ has a valid objection in } x\}.$$

The bargaining set contains other several cooperative solutions[7]. It is clear that $C^S \subset M^S$ under any coalition structure P^S by the definitions.

Let $i, j \in N$. We say that i and j are *substitutes* in game v if

$$v(S \cup \{i\}) = v(S \cup \{j\}) \ ^\forall S \subset (N \setminus \{i, j\}).$$

Since all the firms in S are substitutes in game v, the following symmetric sets facilitate our analysis:

$$\begin{aligned} &\tilde{X}^S = \{x \in X^S | x_i = x_j \ ^\forall i, j \in S\} \\ &\tilde{C}^S = C^S \cap \tilde{X}^S, \quad \tilde{M}^S = M^S \cap \tilde{X}^S \end{aligned}$$

4 The Core with P^S

Lemma 1 *If $C^S \neq \emptyset$, then there exists an $x \in C^S$ such that $x_i = \bar{x} \ ^\forall i \in S$.*

Proof: Let $y = (y_0, y_1, \ldots, y_n) \in C^S$. Define $x = (x_0, x_1, \ldots, x_n) \in X^S$ by $x_j = y_j$ if $j \notin S$ and $x_i = \bar{x} = (1/s)\sum_{i \in S} y_i = (1/s)y(S)$ if $i \in S$. Fix a coalition $T \subseteq \{0\} \cup N$ such that $T \cap S \neq \emptyset$. Let $l = |T \cap S|$. Then $\min_{U \subset S,\ |U|=l} y(U) \leq (l/s)y(S) = x(T \cap S)$. Hence,

$$\begin{aligned} x(T) &= x(T \setminus S) + x(T \cap S) &&\geq y(T \setminus S) + \min_{U \subset S,\ |U|=l} y(U) \\ &\geq \min_{U \subset S,\ |U|=l} y((T \setminus S) \cup U) &&\geq v((T \setminus S) \cup U) = v(T), \end{aligned}$$

since $y((T \setminus S) \cup U) \geq v((T \setminus S) \cup U)$ and $v((T \setminus S) \cup U) = v(T)$ by the fact that all the firms in S are substitutes. *Q.E.D.*

Proposition 1 $C^S = \emptyset$ *if* $S \neq N$

Proof: We first show that $\tilde{C}^S = \emptyset$ if $S \neq N$. Suppose $\tilde{C}^S \neq \emptyset$ and take $x \in \tilde{C}^S$. Let $x_i = \bar{x} \ ^\forall i \in S$. Then, we have $\bar{x} > L(0)$. Otherwise, we would have $s\bar{x} + (n-s)L(s) < nL(0)$ since $L(s) < L(0)$, which would imply that coalition N could block x.

[7] It is well-known that the kernel and the nucleolus are the subsets of it

Take a coalition $\{0\} \cup T$ with $|T| = |S|$ where $T \subseteq N \setminus S$ if $|S| \leq n/2$ and $T \supseteq N \setminus S$ if $|S| > n/2$. Let $t = |T|$. Then, we have $x_0 + \sum_{i \in T} x_i < sW(s) = tW(t)$, since $x_0 + s\bar{x} = sW(s)$ and $\bar{x} > L(0) > L(s) (= x_i \; {}^{\forall} i \in N \setminus S)$. Hence, $\tilde{C}^S = \emptyset$ if $S \neq N$.

Now we know $\tilde{C}^S = \emptyset$, which implies that $C^S = \emptyset$ because of Lemma 1. *Q.E.D.*

Proposition 2 *$\tilde{C}^N \neq \emptyset$ if and only if $s^* = n$.*

Proof: ($\Rightarrow$) Suppose $s^* < n$. If $x \in \tilde{C}^N \neq \emptyset$, then we have

$$\bar{x} \geq L(0) \tag{1}$$

$$x_0 + s\bar{x} \geq sW(s), \;\; s = 0, 1, \cdots, n-1, \tag{2}$$

where $x_i = \bar{x} \; {}^{\forall} i \in S$ and $x_0 = nW(n) - n\bar{x}$. Letting $s = s^*$ in (2), we obtain $nW(n) - n\bar{x} + s^*\bar{x} \geq s^*W(s^*)$ or $(n - s^*)\bar{x} \leq nW(n) - s^*W(s^*)$. By (1), we get $(n - s^*)L(0) \leq nW(n) - s^*W(s^*)$ or

$$s^*(W(s^*) - L(0)) \leq n(W(n) - L(0)),$$

contradicting the uniqueness of s^*.

($\Leftarrow$) Take x such that

$$x_i = \begin{cases} n(W(n) - L(0)) & \text{if } i = 0 \\ L(0) & \text{if } i \in N. \end{cases}$$

Since $s^* = n$, it is easily shown that $x \in \tilde{C}^N$. *Q.E.D.*

Remark 1: It is easily confirmed that $s^* < n$ in the linear model described in section 2. Watanabe and Tauman (2003) showed that the core of the linear model is empty as the number of licensees tends to infinity. We could show the same result even in the case of a finite number of players, although our v-function is slightly different from theirs.

We could know that $C^S = \emptyset$ unless $s = s^* = n$ by Proposition 1 and 2. Let us next consider the bargainig set with P^S.

5 The Symmetric Bargaining Set with P^S

It suffices to examine objections and counter objections of the licensor 0 and a licensee $i \in S$, because of the licensees' symmetric payoffs $\bar{x}$.

Lemma 2 *Suppose $n/2 \le s < n$. If $x \in \tilde{M}^S$, then $\bar{x} \le L(0)$.*

Proof : Suppose $\bar{x} > L(0)$ and take the licensor 0's objection $(y, \{0\} \cup T)$ against firm $i \in S$ such that $|T| = s$, $T \supseteq N \setminus S$ and

$$y_k = \begin{cases} x_0 + \epsilon & \text{if } k = 0 \\ \bar{x} + \epsilon & \text{if } k \in T \cap S \\ L(0) + \epsilon & \text{if } k \in T \cap (N \setminus S), \end{cases}$$

where $\epsilon = (n - s)(\bar{x} - L(0))/(s + 1) > 0$.
Note that

$$\begin{aligned} y_0 + \textstyle\sum_{k \in T} y_k &= x_0 + (2s - n)\bar{x} + (n - s)L(0) + (s + 1)\epsilon \\ &= x_0 + (2s - n)\bar{x} + (n - s)L(0) + (n - s)(\bar{x} - L(0)) \\ &= x_0 + s\bar{x} = sW(s). \end{aligned}$$

Since $y_k > L(0)\ {}^\forall k \in T$ and $x_k = \bar{x} > L(0)\ {}^\forall k \in N \setminus T$, any firm $i \in S$ has no counter objection against $(y, \{0\} \cup T)$. Contradiction. *Q.E.D.*

Lemma 3 *Suppose $1 \le s \le n/2$. If $x \in \tilde{M}^S$ and if $s(W(s) - L(0)) \le (n - s)(W(n - s) - L(0))$, then $\bar{x} \le L(0)$.*

Proof: Suppose $\bar{x} > L(0)$. Then $x_0 < s(W(s) - L(0))$. Take the licensor 0's objection $(y, \{0\} \cup (N \setminus S))$ against firm $i \in S$ with

$$y_k = \begin{cases} (n - s)(W(n - s) - L(0)) & \text{if } k = 0 \\ L(0) & \text{if } k \in N \setminus S, \end{cases}$$

Since $y_k > L(0)\ {}^\forall k \in N \setminus S$ and $x_k = \bar{x} > L(0)\ {}^\forall k \in S$, any firm $i \in S$ has no counter objection against $(y, \{0\} \cup (N \setminus S))$. Contradiction. *Q.E.D.*

Lemma 4 *Suppose $1 \le s \le n/2$. If $x \in \tilde{M}^S$ and if $0 < s(W(s) - W(t)) \le (n - s)(W(n - s) - L(0))$, then $\bar{x} \le W(t)$.*

Proof: Suppose $\bar{x} > W(t)$. Then $x_0 < s(W(s) - W(t))$. Since $W(t) > L(0)$, the same argument as in the proof of Lemma 2 applies. *Q.E.D.*

Lemma 5 *If $x \in \tilde{M}^S$, then $x_0 \leq s^*(W(s^*) - L(0))$.*

Proof : Suppose $x_0 > s^*(W(s^*) - L(0))$. Then, $\bar{x} = (sW(s) - x_0)/s < (sW(s) - s^*(W(s^*) - L(0)))/s \leq L(0)$ by s^*. Take an objection (y, N) of $i \in S$ against 0 such that $y_k = L(0)\ ^\forall k \in N$. If 0 had a counter objection $(z, \{0\} \cup T)$, then we would have $z_0 \geq x_0 > s^*(W(s^*) - L(0))$ and $z_k \geq y_k = L(0)\ ^\forall k \in T$, and thus we would reach a contradiction $z_0 + \sum_{k \in T} z_k > s^*(W(s^*) - L(0)) + tL(0) \geq tW(t)$. *Q.E.D.*

Proposition 3 *Let $x \in M^S$. Then, we have the following.*
(a) *If $1 \leq s \leq n/2$ and $s(W(s) - W(t)) \leq (n-s)(W(n-s) - L(0))$, then*

$$s(W(s) - W(t)) \leq x_0 \leq s^*(W(s^*) - L(0)),$$

where $W(t)$ is the lowest one satisfying the above condition.
(b) *If $n/2 \leq s < n$ or $s(W(s) - L(0)) \leq (n-s)(W(n-s) - L(0))$, then*

$$s(W(s) - L(0)) \leq x_0 \leq s^*(W(s^*) - L(0)).$$

(c) *If $s^* < s = n$, then $n(W(n) - L(0)) \leq x_0 \leq s^*(W(s^*) - L(0))$.*

Proof : Lemma 2 to Lemma 5 jointly implies (a) and (b). Consider the case (c). Let $\bar{x} = L(0) + z$ where $z > 0$. If $x_0 = n(W(n) - \bar{x})$, then 0 can make an objection $(y, \{0\} \cup S^*)$ where $y_i > x_i$ for any $i \in \{0\} \cup S^*$, since $n(W(n) - \bar{x}) = n(W(n) - L(0)) - nz < s^*(W(s^*) - L(0)) - s^*z = s^*(W(s^*) - \bar{x})$. Any counter objection cannot be made by $i \in N \setminus S^*$, since $\bar{x} > L(0)$. Lemma 5 completes (c). *Q.E.D.*

Let S^* be a set $S \subseteq N$ with $|S| = s^*$. Proposition 3 (b) directly implies the next corollary.

Corollary 1 *If $n/2 \leq s^* < n$, then $\tilde{M}^{S^*} = \{x^*\}$ where*

$$x_i = \begin{cases} s^*(W(s^*) - L(0)) & \text{if } i = 0 \\ L(0) & \text{if } i \in S^* \\ L(s^*) & \text{if } i \in N \setminus S^*. \end{cases}$$

In the linear model described in section 2, it is easily shown that there exists a threshold $\hat{\epsilon}$ of the cost reduction such that $n/2 \le s^*$ if $\epsilon \le \hat{\epsilon}$.

Note: The bargaining set merely suggests the set of payoffs reachable by a series of objections and counter objections in a real negotiation. In that sense, it has no criterion for the value judgement on a payoff distribution. On the other hand, it generally comtains the "nucleolus" that is uniquely determined in any game v by repeatedly applying the principle of minimizing the maximum complaint on a payoff distribution. Corollary 1 implies that the bargaining set coincides with the nucleolus under a coalition structure P^{S^*} with $n/2 \le |S^*| < n$ and then reflects a "fairness" notion that the nucleolus has.

Proposition 4 *Let x^* be such that*

$$x_i^* = \begin{cases} s^*(W(s^*) - L(0)) & \text{if } i = 0 \\ L(0) & \text{if } i \in S^* \\ L(s^*) & \text{if } i \in N \setminus S^*, \end{cases}$$

where $1 \le s^* \le n$. Then $x^* \in \tilde{M}^{S^*}$.

Proof: Take any objection $(y, \{0\} \cup T)$ of 0 against $i \in S^*$ in x^*. Then, we have $\sum_{k \in T} y_k < tL(0)$. Otherwise, we would obtain $tW(t) \ge y_0 + \sum_{y \in T} y_k > s^*(W(s^*) - L(0)) + tL(0)$, contradicting the definition of s^*. Hence, i has a counter objection (z, N) against $(y, \{0\} \cup T)$ with

$$z_i = \begin{cases} L(0) & \text{if } k \in S^* \setminus T \\ y_k + \epsilon & \text{if } k \in T \\ L(0) & \text{if } k \in (N \setminus S^*) \setminus T. \end{cases}$$

where $\epsilon = (tL(0) - \sum_{k \in T} y_k)/t > 0$. In fact, $\sum_{k \in N} z_k = nL(0)$, $z_k \ge x_k \ ^\forall k \in N$ and $z_k > y_k \ ^\forall k \in T$.

Next take any objection (u, R) of $i \in S^*$ against 0 in x^*. Let

$$u'_k = \begin{cases} u_k & \text{if } k \in R \\ x_k & \text{if } k \in N \setminus R. \end{cases}$$

Order all the n firms according to their payoffs in the non-decreasing order, and take the first s^* firms. Then, we have $\sum_{k\in Q} u'_k < s^*L(0)$ where Q is the set of the first s^* firms. Hence 0 has a counter objection against (u, R). *Q.E.D.*

Lemma 6 *If* $x \in \tilde{M}^{S^*}$*, then* $\bar{x} \geq L(0)$*.*

Proof : Suppose $\bar{x} < L(0)$. Then, a licensee $i \in S^*$ has an objection (y, N) against the licensor 0 in x, where $y_k = L(0)\ ^\forall k \in N$. Suppose that 0 had a counter objection $(z, \{0\} \cup R)$ to i's objection (y, N). Then we would have

$$\begin{aligned} &rW(r) \geq z_0 + \textstyle\sum_{k\in R} z_k \\ &z_0 \geq x_0, \text{ and } z_k \geq y_k = L(0)\ ^\forall k \in R \end{aligned}$$

Since $\bar{x} < L(0)$, it must be that $x_0 = s^*W(s^*) - s^*\bar{x} > s^*W(s^*) - s^*L(0)$. We would then obtain

$$rW(r) \geq z_0 + \sum_{k\in R} z_k > s^*W(s^*) - s^*L(0) + rL(0),$$

contradicting the definition of s^*. Thus, i's objection (y, N) could not be countered by 0. Contradiction. *Q.E.D.*

Proposition 5 *If* $s^* = n$*, then* $\tilde{M}^N = \tilde{C}^N$*.* $x \in \tilde{C}^N$ *is characterized as* $x_0 = n(W(n) - \bar{x})$ *and* $L(0) \leq \bar{x} \leq \min_{s\neq n}(nW(n) - sW(s))/(n - s)$*.*

Proof: $(\supseteq)$ It is clear by the definitions of C^N and M^N.

$(\subseteq)$ Suppose that there exists $x \in \tilde{M}^N$ with $x \notin \tilde{C}^N$. Since $x \in \tilde{M}^N$, we must have $\bar{x} \geq L(0)$ by Lemma 6. Since $x \notin \tilde{C}^N$, there must exist $\{0\} \cup T$ such that

$$x_0 + \sum_{i\in T} x_i < tW(t), \text{ where } t < n.$$

Let $(y, \{0\} \cup T)$ be 0's objection against any $i \in N \setminus T$ in x, where $y_k = x_k + \epsilon\ ^\forall k \in \{0\} \cup T$ and $(t + 1)\epsilon = tW(t) - (x_0 + \sum_{i\in T} x_i) > 0$. Since $\bar{x} \geq L(0)$, i has no counter objection, contradicting that $x \in \tilde{M}^S$.

The system of inequalities to characterize $\tilde{C}^N$ yields $L(n - s) \leq \bar{x} \leq (nW(n) - sW(s))/(n - s)$ for any s. By Lemma 6, $\bar{x} \geq L(0)$. *Q.E.D.*

Even with the linear demand and cost functions, it can be that $\tilde{C}^N \neq \emptyset$ if the commodities are differentiated. See Muto and Watanabe (2004).

Remark 1: Proposition 3, 4 and 5 jointly imply that $s^*(W(s^*) - L(0))$ is the "stable profit sharing" always, but that he cannot gain more than that amount and may not obtain it unless $s = s^*$ and $n/2 \leq s^* < n$. Hence, the licensor should invite $s^*(< n)$ firms to the negotiation if $n/2 \leq s^* < n$, which is the most favorable coalition size for him. When $s^* = n$, there are some cases where it is better for him not to invite all the n firms to the negotiation, if (collective) bargaining power of the firms is quite large[8].

6 Concluding Remarks

Negotiations versus Fees

Recall the linear model in section 2. Kamien and Tauman (1986) showed that it is better for the patent holder to license the innovation by means of fixed fees only than by means of royalties only. In the same model, we can find some cases where negotiations are superior to fixed fees.

Let ϵ denote the magnitude of the cost reduction, and let π^*_{PH} denote the profit of the patent holder who licenses using the fixed fees only. In the case of moderate cost reduction such that $2(a-c)/(3n-2) \leq \epsilon \leq 1 + (a-c)/n$, the stable profit sharing $s^*(W(s^*) - L(0))$ is lower than π^*_{PH}, regardless of any s^*. On the other hand, if $\epsilon \leq 2(a-c)/(3n-2)$ or if $1 + (a-c)/n \leq \epsilon$, it can then be that $s^*(W(s^*) - L(0)) > \pi^*_{PH}$.

When $\epsilon \leq 2(a-c)/(3n-2)$ (or $1 + (a-c)/n \leq \epsilon$), the patent holder licenses to n (or $\hat{s}$) firms by means of fixed fees to obtain π^*_{PH}. In the former case, we know that $s^* < n$ by Remark 1, and obviously $s^* \leq \hat{s}$ by the definition of $\hat{s}$ in the latter case. We can say that negotiations would be superior to fixed fees in such cases that the innovation is trivial or very nice.

Muto and Watanabe (2004) showed that it can be optimal for the licensor to sell the innovation by means of negotiations in the Bertrand duopoly with differentiated commodities. The interpretation of such cases is not so easy, since it depends also on the rate of substitution (complementarity) between the commodities.

[8]For example, it is better for the licensor to invite $n-1$ firms to the negotiations if $(n-1)(W(n-1) - L(0)) \geq n(W(n) - \bar{x})$ where $\bar{x} \geq L(0)$.

Let $\mathrm{Sh}_0(v)$ denote the Shapley value of the licensor[9] and let $x \in M^S$. Lemma 5 shows that $x_0 \leq s^*(W(s^*) - L(0))$. Watanabe and Tauman (2003) also showed in the linear model that $\mathrm{Sh}_0(v) > x_0$ as the number of licensees tends to infinity. It happens in our game if

$$\begin{aligned}&(1/n+1)\textstyle\sum_{s=1}^{n-\hat{s}} sL(0)) + (1/n+1)\sum_{s=n-\hat{s}+1}^{n-1} s(L(0) - L(n-s))\\ &> s^*(W(s^*) - L(0)) - (1/n+1)\textstyle\sum_{s=1}^{n} s(W(s) - L(0)).\end{aligned}$$

It is well known that the Shapley value is not necessarily in the core, but its relationship to the bargaining set has not been studied comprehensively. In this paper, $\hat{s}$ did not play any important role. With more specified assumptions on it, we could have proceed further on that topic.

Limitation of Sidepayments

It is more realistic that no sidepayments are allowed except payments of fees to the licensor: in $\{0\} \cup S$, each $i \in S$ pays p_i to the licensor 0, $^\forall S \subseteq N$, and there is no money transfer in S. Assume the uniform pricing scheme: $p_i = p \ ^\forall i \in S$. We can regain almost the same results even in this setup. Hence, the assumption on the sidepayments does not play any important role for our propositions. Below is the addendum to extend our model.

The permissible coalition structure is

$$P^S = (\{0\} \cup S, \{\{i\}\}_{i \in N \setminus S}), \ ^\forall S \subseteq N,$$

and so the characteristic function is given by

$$\begin{aligned}&V(\{0\} \cup S) = \{(x_i)_{i \in \{0\} \cup S} | x_0 \leq sp, \ x_i \leq W(s) - p, \ 0 \leq p \leq W(s)\}\\ &V(\{0\}) = 0, \quad V(S) = \{(x_i)_{i \in S} | x_i \leq L(n-s)\}.\end{aligned}$$

[9] Let $\mathcal{R}(s+1)$ be an ordering of $n+1$ players where the licensor 0 is at the $(s+1)$-st place. $j\mathcal{R}(s+1)0$ means that firm j precedes the licensor 0 in the ordering $\mathcal{R}(s+1)$. Denote by $\mathcal{P}_0^{\mathcal{R}(s+1)} = \{j \in N | j\mathcal{R}(s+1)0\}$ the set of firms that precedes the licensor in $\mathcal{R}(s+1)$. Since every firm is identical before licensed, there are $n!$ such orderings that have the same marginal contribution $v(\mathcal{P}_0^{\mathcal{R}(s+1)} \cup \{0\}) - v(\mathcal{P}_0^{\mathcal{R}(s+1)})$ of the licensor to coalition $S^0 = \mathcal{P}_0^{\mathcal{R}(s+1)} \cup \{0\}$. The Shapley value of the licensor in our licensing game is

$$\begin{aligned}\mathrm{Sh}_0(v) &= (1/(n+1)!)\textstyle\sum_{s=1,\cdots,n+1} n!\{v(\mathcal{P}_0^{\mathcal{R}(s+1)} \cup \{0\}) - v(\mathcal{P}_0^{\mathcal{R}(s+1)})\}\\ &= (1/n+1)\textstyle\sum_{s=1}^{n} s(W(s) - L(n-s)).\end{aligned}$$

The imputations under a coalition structure P^S is defined by

$$X^S = \{x = (x_0, x_1, \cdots, x_n) \in \mathbb{R}^{n+1} | x_0 = sp,\ x_i = W(s) - p,\ {}^\forall i \in S$$
$$0 \leq p \leq W(s) - L(n-1),\ x_i \geq L(n-s),\ {}^\forall i \in N \setminus S\}$$

The core C^S is defined by

$$C^S = \{x \in X^S | \text{ for any } T \subseteq \{0\} \cup N \text{ with } T \cap (\{0\} \cup S) \neq \emptyset,$$
$$\text{there exists no } y \in V(T) \text{ such that } y_k > x_k,\ {}^\forall k \in T\}.$$

Let $i, j \in \{0\} \cup S$ and $x \in X^S$. i has an objection (y, T) against j in x if $i \in T$, $j \notin T$, $T \subseteq \{0\} \cup N$, $y_k > x_k$ ${}^\forall k \in T$, and $y \in V(T)$. j has a counter objection (z, R) to i's objection (y, T) if $j \in R$, $i \notin R$, $R \subseteq \{0\} \cup N$, $z_k \geq x_k$ ${}^\forall k \in R$, $z_k \geq y_k$ ${}^\forall k \in R \cap T$, and $z \in V(R)$. i has a *valid* objection (y, T) against j in x if (y, T) is not countered.

The bargaining set M^S is defined by

$$M^S = \{x \in X^S | \text{no player in } \{0\} \cup S \text{ has a valid objection in } x\}.$$

Final Remarks

We can obtain similar results, applying other solution concepts such as the strong equilibrium and the coalition-proof Nash equilibrium to our model[10]. We will show them precisely in another paper.

Our model is too general to conduct welfare analysis, since our primary purpose was to show the general bargaining outcomes. The welfare analysis under more specified models are left for our future research.

In this paper, the worth of coalition is derived under the assumption that no cooperation is allowed in the market. However, cooperative actions can be seen in the real world. Taking into account a sophisticated nature of coalition formation with cooperation, Watanabe (2004b) argues how to represent strategic-form games in coalitional form without sidepayments.

[10] For reference, see Muto (1990) and Nakayama and Quintas (1991).

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