Weather Risk Swap Valuation[1]

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In June of 2001, Tokyo Electric Power Company (TEPCO) and Tokyo Gas Supply Company (TGSC) made a zero-cost risk swap contract on the average temperature of August and September of 2001 in Tokyo for their adverse situations. This is an exchange of two options on the average temperature, by which TEPCO and TGSC can respectively hedge against a cold summer and a hot summer. In this paper we develop a theoretical framework to evaluate the fairness or rationality of such a zero-cost weather risk swap, derive some conditions to check the rationality and empirically evaluate the fairness of the above temperature risk swap between the two companies. Since the situation with options or derivatives defined on such a weather index as the average temperature is essentially incomplete in any sense, we can not simply value the options by the no-arbitrage argument and hence it is not sufficient to compare the risk-neutral expected values. In fact, no risk neutral measure exists. In other words, we have to explicitly take a risk factor into account in the evaluation.

First we define the concept of full equivalence and moment equivalence of two options on a weather index and then derive some conditions for full and moment equivalences. Thirdly using the stochastic volatility model in Kariya, Endo and Ushiyama (2003), it is shown that the options in the TEPCO-TGSC risk swap are neither fully equivalent nor moment-equivalent as they stand.

1 Introduction

In corporate risk management typical measures a corporate manager can use to directly hedge against risks inevitably involved in his business are insurances and derivatives. But the cost or equivalently the premium of using these measures for risk management is significantly large when the risk probability and/or insurance money are large. To avoid the cost of this risk hedge, two corporations can form a zero-cost risk swap contract for their "equivalent" risks. Here risk swap is defined to be a contract that satisfies the following conditions.

1) The individual risks to be swapped are expressed as payoffs of options on some risk indices and the "values" of the two options are "equivalent" in some sense.
2) The cost of the exchange of the two options two corporations underwrite each

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other is zero.

In this paper we treat a risk swap on weather risk indices or nonreplicable indices, making it impossible to apply the no-arbitrage (risk neutral) theory for valuation of equivalence of two options. This means that it is necessary to take risk factors involved in the options in the definition of equivalence. Hence we say that two options defined in a risk swap are fully equivalent if the distributions (processes for American options) of the payoffs for exchange are equal (same in distribution for American options), and that they are moment-equivalent up to the k-th order at the time of contract if the moments of the payoffs are equal up to the k-th order.

Of course, if the market or model is complete as in the Black-Scholes world, the two options can be defined to be equivalent if the risk neutral values are the same. But weather derivatives on such indices as average temperature or rainfall are not replicable at all and the underlying processes are typically neither diffusions nor Markovian. Therefore the risk neutrality valuation is not valid in weather derivatives, and it is necessary to consider equivalence of two options under the real probability measures.

In this paper we simply treat a risk swap with two put options being written on a common index, as is represented by the following real example.

Risk swap between TEPCO and TGSC (TT risk swap)

In June of 2001 Tokyo Electric Power Company (TEPCO) and Tokyo Gas Supply Company (TGSC) announced that the two companies made a risk swap on the average temperature of August and September of 2001 in Tokyo, where TEPCO could hedge against a cold summer and TGSC against a hot summer. A gas company loses revenues when summer is too hot.

Their risk swap is an exchange of the following payoffs of European put options

(1.1)
$$\begin{aligned} W_1^{TG} &= Min\{70{,}000 \quad 800 \times 61 Max(X - 26.5\,,\,0\,)\} \\ W_2^{TE} &= Min\{70{,}000 \quad 800 \times 61 Max(25.5 - X\,\,,\,0\,)\} \end{aligned} \qquad \text{(in ten thousand yen)}$$

Here X is the average of 61 daily average temperatures in August and September measured in degrees of Centigrade (C) where the daily average temperature is defined as the average of 24 temperatures the Japan Meteorology Agency records on the hour of each day. In this risk swap one degree of the average temperature above 26.5 degrees or below 25.5 degrees corresponds to 8・61=488 million yen and the maximum limit of the payoffs is 700 million yen. The actual average temperature was 24.5 degrees and TGSC paid 320 million yen to TEPCO. An important question is whether the “values” of the payoffs were equivalent ex ante. It depends on the distribution of X.

The content of this paper is as follows. In Section 2 the concept of full equivalence and moment equivalence is given and in a location family of distributions some conditions derived for the full equivalence of options of the above form. In section 3, when the distribution of X is not symmetric with nonzero skewness and kurtosis, we approximate the distribution by the Gram-Charier approximation and give a formula for the first order moment equivalence of the options to be swapped. In Section 4 when the distribution is not symmetric, a searching procedure for moment equivalence is described. In Section 5 using the stochastic volatility time series model in Kariya, Endo and Ushiyama (2003), we empirically investigate the rationality or the value of the TT risk swap made by TEPCO and TGSC.

2 Equivalence of two payoffs

In this section we first define the concept of full equivalence and moment equivalence of two payoffs to be exchanged in a risk swap and derive some conditions for full equivalence. We only consider the case of European type options for simplicity.

Definition2.1 Two random nonnegative payoffs expressed as two random variables W_1 and W_2 are said to be fully equivalent if the distributions of W_1 and W_2 are completely equal, and moment-equivalent up to the k-th order if the moments of W_1 and W_2 are equal up to the k-th order;

$$E[W_1^{\,j}] = E[W_2^{\,j}] \quad (j = 1,\cdots,k) \quad \text{and}\; P(W_1 > 0) = P(W_2 > 0) \;.$$

Using this definition, we consider the equivalence of the payoffs on a common index defined by

$$\begin{aligned} W_1 &= cMax(X - a,\ 0) \\ W_2 &= dMax(X - b,\ 0) \end{aligned}, \qquad (2.1)$$

where $a > b$, and X may be a weather index such as an average temperature for a certain period or a day count index such as the number of days for a certain period.

Let $F_i(y)$ be the distribution function of W_i. For simplicity our argument is restricted to the case where the distribution of X in (3.1) is continuous in R^1 and let the distribution of X be denoted by $H(x)$ and the pdf $h(x)$. Assume that X has an expectation μ and that X follows a location type distribution. In other words the pdf is expressed as

$$h(x) = g(x - \mu) \qquad (a.e.)$$

for some pdf $g(x)$ with mean 0. In this case $H(x)$ is expressed as

(2.2) $$H(x) = G(x-\mu) = \int_{-\infty}^{x-\mu} g(u)du$$

where $G(x)$ is the cdf of $g(x)$. Needless to say, normal distribution is such a typical one. Note that a location distribution may not be symmetric. The following lemma is rather obvious.

Lemma 2.1 Suppose that X has a location-type cdf in (2.2) with mean μ. Then

（1）for $y \geq 0$, the cdf of W_1 is given by

(2.3a) $$F_1(y) = G(\frac{y + c(a-\mu)}{c}),$$

and for $y > 0$, the pdf is

(2.3b) $$f_1(y) = F_1'(y) = \frac{1}{c} g(\frac{y + c(a-\mu)}{c}).$$

（2）for $y \geq 0$, the cdf of W_2 is

(2.4a) $$F_2(y) = 1 - G(\frac{-y + d(b-\mu)}{d})$$

and for $y > 0$ the pdf is

(2.4b) $$f_2(y) = F_2'(y) = \frac{1}{d} g(\frac{-y + d(b-\mu)}{d})$$

Proof（1）It is clear since for $y \geq 0$

$$F_1(y) = P(W_1 \leq y) = P(Max(X-a,\ 0) \leq \frac{y}{c})$$

$$= P(X \leq a) + P(0 < X - a \leq \frac{y}{c})$$

$$= P(X - \mu \leq a - \mu) + P(a - \mu < X - \mu \leq \frac{y + c(a-\mu)}{c}).$$

Similar for（2）.

Proposition 2.1 Suppose that X follows (2.2) and $G(x)$ is symmetric, i.e.

$$G(x) + G(-x) = 1 \qquad (x \in R^1).$$

Then a necessary and sufficient condition for W_1 and W_2 to be fully equivalent is that $c = d$ and $\mu = (a+b)/2$.

Proof. Since $G(x)$ is symmetric, it follows that

$$F_2(y) = 1 - G(\frac{-y + d(b-\mu)}{d}) = G(\frac{y - d(b-\mu)}{d}).$$

Hence from the monotonically increasing property of G, $F_1(y) = F_2(y)$ $(y \geq 0)$ with (2.3a) implies

$$y(\frac{1}{d} - \frac{1}{c}) = b - \mu + a - \mu \qquad (y \geq 0).$$

Therefore the result follows.

The TT risk swap with payoffs W_1^E and W_2^G is of the fully equivalent structure if the distribution of the average temperature X of August and September is symmetric with mean $\mu = (a+b)/2$. If this is the case, the distribution of X can be any location distribution and the normality of X is not required. In other words, if the distribution is symmetric with mean $\mu = (a+b)/2$, they are equivalent without evaluating the values. However, from an empirical viewpoint it is questioned

1) if the mean $\mu = (a+b)/2$ =26 degrees is empirically valid at the time of the contract as a predictive mean of the average temperature of August and September of 2001, and

2) if the distribution is symmetric.

In addition, it is also questioned if a specification problem on trend is well treated in predicting the distribution of X. These questions are discussed in Section 5 based on a stochastic volatility time series model.

Proposition 2.1 assumes the symmetry of G and derives a necessary and sufficient condition for full equivalence. The converse is an important question to be posed. When X follows a location type distribution, the full equivalence of the payoffs in (2.1) is equivalent to the condition that for $y \geq 0$,

(2.5) $$G(\frac{y + c(a-\mu)}{c}) + G(\frac{-y + d(b-\mu)}{d}) = 1.$$

If this condition holds for all y in R^1, then $c = d$ and $\mu = (a+b)/2$ implies the symmetry of G. However, (2.5) holds only for nonnegative y. In particular, when $y = 0$, it becomes

(2.6) $$G(a-\mu) + G(b-\mu) = 1.$$

This is necessary. In this case the symmetry of G does not follow from the full equivalence. To discuss this case further, let us assume $b \leq \mu \leq a$ without essential loss of generality. The following argument is often made in statistics.

Proposition 2.2 Suppose that $2\mu = a + b$ and μ can take any value in R^1. Then the full equivalence of W_1 and W_2 implies the symmetry of $G(x)$.

Proof Substituting $b=2\mu-a$ into (2.6) yields $G(a-\mu)+G(\mu-a)=1$ and the result is obvious.

However, this is not realistic because a and b are given in advance and because μ is held fixed even though it may be unknown.

Lemma 2.2 When W_1 and W_2 are fully equivalent, a location type pdf g in (2.2) needs to satisfy

(2.7) $$\begin{aligned} & g(x)=eg(-ex+K), \quad x \geq a-\mu \\ & e=c/d, \quad K \equiv K(a,b,\mu:e)=e(a-\mu)+(b-\mu) \end{aligned}.$$

Further $c=d, a+b=2\mu$ implies the tail symmetry of $g(x)$.
Proof. From(2.5), it suffices to equate(2.3a) with(2.4b).

Therefore when the full equivalence of W_1 and W_2 holds, the right tail of $g(x)$ for $x \geq a-\mu$ is simply the linear transformation of the left pdf $g(-x)$ by $-ex+K$. The coefficients e,K of this linear transformation depend on (a,b,c,d,μ). But the (a,b,c,d) needs to be selected for a full equivalence in advance, while for(2.7) to hold, (a,b,c,d)has to depend on μ and the form of $g(\cdot)$. Therefore in general they depend on the skewness and kurtosis of g. It will be clear that there exists an asymmetric pdf $g(x)$ with mean 0 satisfying (2.6) and (2.7). In particular, when $e=1, K=0$, which implies $c=d, a+b=2\mu$, （2. 7）implies the tail symmetry of $g(\cdot)$ for $x \geq a-\mu$.

Corollary When $e=1, K=0$, the tail symmetry of the distribution of $X-\mu$ is necessary for a full equivalence.

When the tail symmetry of $g(\cdot)$ for $x \geq a-\mu$ holds with $c=d, a+b=2\mu$, then (a,b,c,d)can be selected independently of the second and higher moments.

Let us consider the case of asymmetric distribution and derive some necessary condition on the structure of the payoffs. In this case (2.6) needs to holds for a full equivalence. Let

(2.8) $$z=z(y)=\frac{-y+d(b-\mu)}{d}$$

We say that $G(x)$ is asymmetric in tail part when for large y^* there are $z_i=z(y_i)$'s such that at least one of

(2.9a) $$G(z_1)+G(-z_1)<1 \qquad (y_1>0) \text{ and}$$

(2.9b) $$G(z_2)+G(-z_2)>1 \qquad (y_2>0)$$

holds. Further set

$$\frac{y+c(a-\mu)}{c}=-z+D(y) \qquad \text{with}$$

(2.10) $$D(y)=(\frac{1}{c}-\frac{1}{d})y+a+b-\mu\,.$$

Proposition 2.3 Assume a full equivalence.

（1） When(2.9a) holds, $D(y_1)<0$. And $a+b=2\mu$ implies $d<c$.

（2） When(2.9b) holds, $D(y_2)>0$. And $a+b=2\mu$ implies $d>c$.

（3） When (2.9a) and (2.9b) hold, $y_2>y_1>0$ implies $c<d$, while $y_1>y_2>0$ implies $c>d$.

（4） When (2.9a) and (2.9b) hold for $y_2>y_1>0$ and when for some $y_3>y_2$, $G(z_3)+G(-z_3)<1$ holds, then no c,d exist for a full equivalence.

Proof（1） When (2.9a) and $D(y_1)\ge 0$ holds,

$$F_2(y_1)=G(z_1)<1-G(-z_1)\le 1-G(-z_1+D(y_1))=F(y_1)\,.$$

Therefore no full equivalence holds unless $D(y_1)<0$. The second part of (1) follows from (2.10).（2） follows by a similar argument.

（3） follows since by（1） and（2） $-D(y_1)>0$ and $D(y_2)>0$ follow, implying

$$-D(y_1)+D(y_2)=(\frac{1}{c}-\frac{1}{d})(y_2-y_1)>0\,.$$

This implies the result.

(4)is clear since (3)implies $c<d$ and $c>d$.

By this result for a full equivalence to hold depends on the structure of the asymmetry. In any case, when the distribution G is asymmetric in tail part, $a+b=2\mu$ implies $c\neq d$.

From a viewpoint of full equivalence a direct measure for the fairness of a risk swap will be a distance of the distributions of the payoffs. The Kolmogorov-Smirnov distance defined by

(2.11) $d(W_1, W_2)$ = sup abs[F_1(y) – F_2(y)]

is a typical one. If this measure is big, the risk swap is not regarded as a fair swap.

3 Approximation to an asymmetric distribution and moment equivalence.

Judging from data, the distribution of temperature is often found to be asymmetric. The degree of asymmetry depends on the location of the measurement and how it is used. For example, the distributions of daily averaged temperature in Tokyo and Atlanta are judged to be asymmetric where a daily averaged temperature of Atlanta is defined as the mean of maximum and minimum temperature in each day.

As has been observed in Section 2, when the distribution of X is asymmetric, it is difficult to seek the full equivalence of W_1 and W_2 in evaluating the fairness of a rsik swap. Hence in this section, approximating the asymmetric distribution by the Gram-Charier formula, we derive an approximate formula for the first order moment equivalence and a condition on the payoffs.

Now let the mean, variance, skewness and kurtosis of X be denoted by

$$(\mu, \sigma^2, \beta, \xi + 3)$$

and we approximate the first order moment of W_1 and W_2. Let the pdf of X be denoted by $h(x)$. The Gram-Charier approximation formula for $h(x)$ is given by

(3.1) $$h_0(x) = f(x) - \frac{1}{3!}\beta\sigma^3 f^{(3)}(x) + \frac{1}{4!}\xi\sigma^4 f^{(4)}(x).$$

Here $f(x)$ is the pdf of normal distribution $N(\mu, \sigma^2)$ with mean μ and variance σ^2 and $f^{(i')}(x)$ is the i-th order derivative of $f(x)$. It is noted that this approximation guarantees

1) $h_0(x)$ is a pdf; $h_0(x) \geq 0$, $\int_{-\infty}^{\infty} h_0(x)dx = 1$

2) the mean, variance, skewness and kurtosis of $h_0(x)$ are respectively $\mu, \sigma^2, \beta, \xi + 3$. Of course, $\beta = 0, \xi = 0$ implies $N(\mu, \sigma^2)$ and hence the usual practice is included in this approximation.

When X follows (3.1), we first evaluate the mean of $W_1 = c\,Max(X - a, 0)$. For this purpose let

$$\phi(x) = \exp(-\frac{1}{2}x^2)/\sqrt{2\pi},$$

$$\Phi(x) = \int_{-\infty}^{x} \Phi(u)du.$$

and、

$$A = e(a-\mu)/\sigma\ ,\ \ \phi_A = \phi(A)\ ,\ \ \Phi_A = \Phi(A)\ .$$

Lemma 3.1

（1） $\int_A^\infty y\phi(y)dy = \phi_A$,　　（2） $\int_A^\infty y^2\phi(y)dy = A\phi_A + 1 - \Phi_A$,

（3） $\int_A^\infty y^3\phi(y)dy = A^2\phi_A + 2\phi_A$,　　（4） $\int_A^\infty y^4\phi(y)dy = A^3\phi_A + 3[A\phi_A + 1 - \Phi_A]$,

（5） $\int_A^\infty y^5\phi(y)dy = A^4\phi_A + 3[A^2\phi_A + 2\phi_A]$.

Proof　Using integration by parts repeatedly with $y^k\phi(y) = -y^{k-1}\phi^{(1)}(y)$ yealds the result. For example

$$\int_A^\infty y^2\phi\ dy = -y\phi\ |_A^\infty + \int_A^\infty \phi\ dy\ .$$

Proposition 3.1　When X follows(3.1), the means of W_1 and W_2 are respectively

$$\begin{aligned} E[W_1] = c\{&\sigma\phi_A + (\mu-a)(1-\Phi_A) \\ &+\frac{\beta}{3!}[\sigma A^3\phi_A + (a-\mu)(1-A^2)\phi_A] \\ &+\frac{1}{4!}\xi[\sigma(A^4-2A^2-1)\phi_A + (\mu-a)(2(1-\Phi_A) + (A^3-3A)\phi_A]\} \end{aligned} \tag{3.2}$$

and

$$\begin{aligned} E[W_2] = d\{&[\sigma\phi_B + (b-\mu)\Phi_B] \\ &+\frac{\beta}{3!}[\sigma B^3\phi_B + (b-\mu)(1-B^2)\phi_B] \\ &+\frac{1}{4!}\xi[\sigma(B^4-2B^2-1)\phi_B + (\mu-b)(2\Phi_B + (B^3-3B)\phi_B]\} \end{aligned} \tag{3.3}$$

Proof　By simple calculation with Lemma3.1, the first, second and third terms of the right side in(3.1) over (a,∞) are respectively the first, second and third terms of the right side in (3.2). Next to evaluate the mean of W_2, let

$$B = (b-\mu)/\sigma\ ,$$

and $\phi_B = \phi(B), \Phi_B = \Phi(B)$. Then

$$\int_{-\infty}^{b}(b-x)h_0(x)dx = -\int_{-\infty}^{b}(x-b)h_0(x)\,dx\,,$$

and the result in (3.3) follows in the same way as in the case of (3.2).

Corollary The terms with coefficients β and ξ in (3.2) and (3.3) are respectively equal only if $a = b = \mu$.

This follows by direct computation.
By this Corollary if $a = b = \mu$, then the two payoffs are of the first moment equivalence.

4 Moment Equivalence

By the arguments in Sections 2 and 3, it is difficult to get a full equivalence when the distribution of X is asymmetric. Here we describe a searching procedure for finding a payoff structure for moment equivalence, when predictive values of X are obtained by simulation. Suppose that a model for X or the cdf $H(x)$ or pdf $h(x)$ is estimated to generate predictive values $(x_1, x_2, \cdots, x_N)$ by simulation. Let x_i's be ordered as

(4.1)
$$\begin{aligned} &x_{(1)} \le x_{(2)} \le \cdots \le x_{(n)} \le b \\ &\qquad < x_{(n+1)} \le \cdots \le x_{(m)} < a \le x_{(m+1)} \le \cdots \le x_{(N)} \end{aligned}.$$

Here n and m are first selected for finding a and b and they are treated as objects to be specified in our searching procedure. Then using these data, we estimate $E[W_1]$ and $E[W_2]$ by

(4.2)
$$\begin{aligned} \hat{E}[W_1] &= \frac{1}{N}\sum_{j=1}^{N} cMax(x_j - a, 0) = \frac{c}{N}\sum_{j=m+1}^{N}(x_{(j)} - a) \\ &= c[u_1(1) - a(1 - \frac{m}{N})] \\ \hat{E}[W_2] &= \frac{1}{N}\sum_{j=1}^{N} d\,Max(b - x_j, 0) = \frac{d}{N}\sum_{j=1}^{n}(b - x_{(j)}) \\ &= d[\frac{bn}{N} - u_1(2)] \end{aligned}$$

where

(4.3)
$$u_k(1) = \frac{1}{N}\sum_{j=m+1}^{N} x_{(j)}^k, \qquad u_k(2) = \frac{1}{N}\sum_{j=1}^{n} x_{(j)}^k \quad .$$

Further for (n, m) given, the second moments of the payoffs are similarly obtained as

$$\hat{E}[W_1^2] = \frac{c^2}{N}\sum_{j=m+1}^{N}(x_{(j)} - a)^2$$

(4.4)
$$= c^2[u_2(1) - 2au_1(1) + (N-m)a^2 / N]$$

$$\hat{E}[W_2^2] = \frac{d^2}{N}\sum_{j=1}^{n}(b - x_{(j)})^2$$

$$= d^2[u_2(2) - 2bu_1(2)b + nb^2 / N]$$

The third moment of the payoffs are also described as

$$\hat{E}[W_1^3] = \frac{c^3}{N}\sum_{j=m+1}^{N}(x_{(j)} - a)^3$$

(4.5)
$$= c^3[u_3(1) - 3u_2(1)a + 3u_1(1)a^2 - a^3(N-m)/N] .$$

$$\hat{E}[W_2^3] = \frac{d^3}{N}\sum_{j=1}^{n}(x_{(j)} - b)^3$$

$$= d^3[u_3(2) - 3u_2(2)b + 3u_2(1)b^2 - b^3 n / N]$$

Hence equating the first, second and third moments of the payoffs as

(4.6) $$\hat{E}[W_1^k] = \hat{E}[W_2^k] \qquad (k = 1, 2, 3),$$

we obtain a system of simultaneous equations for finding *a, b* and *e*;

(4.7)
$$\begin{aligned} &eu_1(1) - eal_1 = bl_2 - u_1(2) \\ &e^2u_2(1) - 2e^2au_1(1) + e^2al_1 = u_2(2) - 2bu_2(2) + b^2l_2 \\ &e^3u_3(1) - 3u_2(1)e^3a + 3u_1(1)e^3a^2 - e^3a^3l_1 \\ &\quad = u_3(2) - 3u_2(2)b + 3u_2(1)b^2 - b^3l_2 \end{aligned} ,$$

where $l_1 = (N-m)/N$, and $l_1 = (N-m)/N$,

If this system gives a solution of $e = c/d$, a, b , the solution defines the payoffs that are moment-equivalent up to the third order. To find a solution of (4.7), first find b from the equation and substitute it into the second equation. Then getting a from the equation and substituting it into the third equation, we obtain an equation for e, which is a third order polynomial of e. If the system does not give a set of positive solutions of a, b and e, then there are no moment-equivalent payoffs for (*m,n*) given. Changing (*m,n*) repeatedly in a systematic manner, we will eventually find a, b and e so long as the solution exists. A good guess for an initial of (*m,n*) will save computational time.

4 Checking the rationality of the TT Risk Swap

Using one of the stochastic volatility time series model proposed by Kariya, Endo, and Ushiyama(2003), we will examine the fairness or rationality of the TT risk swap in view of our arguments in this paper. In this analysis we ignore the temperature of the 29th , February in each leap year.

The sample period of daily averaged temperatures of Tokyo measured by the Japan Meteorological Agency spans from January 1, 1961 through December 31, 2001. We use the 40 year date of 1996.1.1~2000.12.31 for modeling and derive predictive paths one distributions based on the stochastic volatility time series model. The date of 2001.1~2001.12.31 is reserved for checking. Let $Y_{yr,t}$ denote the average temperature of the day t and year yr . The stochastic volatility time series model in KEU(2003) is given by

(4.1) $$Y_{yr,t} = \mu_{yr,t} + \sigma_{yr,t}\zeta_{yr,t}$$

where $\mu_{tr,t}$ denote the trend given by $\mu_{yr,t} = \alpha_t + \beta_t yr$ and $\sigma_{tr,t}$ is a process of the stochastic volatility and $\{\zeta_{yr,t}\}$ is the corresponding standardized process with mean 0 and variance 1. Here $\{\sigma_{yr,t}\}$ and $\{\zeta_{yr,t}\}$ are assumed to be independent and follow a geometric autoregressive (AR) time series model and an AR model respectively. In view of data, the trend is specified as $T_{yr,t} = a_t + b_t yr$ and the t-test for significance of the slope is used. If b_t is not significant, we use $T_{yr,t} = \overline{Y}_t = \frac{1}{2}\sum_{yr-1}^{N} Y_{tr,t}$.

As a model for $\{\sigma_{yr,t}\}$, the geometric AR model given by

(4.2) $$s_{N,t} = s_{N-1,t}\exp\{u_{N,t}\}$$

$$u_{N,t} = d_1 u_{N,t-1} + d_2 u_{N,t-2} + d_3 u_{N,t-3} + \eta_{N,t},$$

is used, where $s_{N,t}$ is an estimate of $\sigma_{N,t}$ given by

(4.3) $$\mathrm{s}_{N,t} = \sqrt{\frac{1}{N-1}\sum_{yr=1}^{N}(Y_{yr,t} - T_{yr,t})^2}$$

Finally the standardized time series process is viewed as the process of

(4.54) $$V_{yr,t} = \frac{Y_{yr,t} - T_{yr,t}}{s_t}$$

Clearly, the mean and variance of $V_{yr,t}$ is respectively 0 and 1, and $\{s_t\}$ and $\{V_{yr,t}\}$ are uncorrelated. The $\{V_{yr,t}\}$ series is modeled by an AR(4) model with coefficients in the order of lags. Once the models in (4.3) and (4.5) are obtained, we can generate time series paths for the 365 daily temperatures of 2001. Here innovations in the models are generated from the individual empirical distributions based on the residuals in each model, because the residuals do not follow normal distributions. The number of the paths generated by Monte Carlo simulation is 10,000.

Now, to check the rationality of the TT risk swap, we need to derive the distribution of the average temperature of August and September in 2001. This is easily obtained from the corresponding daily temperatures in the 10,000 simulated paths. It is given in Figure 4-1. The mean, standard deviation, skewness and kurtosis of this distribution are respectively 25.46431, 0.557933, －0.04281(1.75) and 3.052411(1.07), where the numbers in the parentheses are the values of the asymptotic normal tests for skewness and kurtosis. The distribution is slightly negatively skewed, where the p-value is 10%. The distribution of each daily(averaged) temperature is more skewed. As an example, the predictive distribution of August 31,2001 is plotted in Figure 4-4. From the argument in Section2 we may say that strictly speaking the payoffs are not fully equivalent.

The distributions of W_1 and W_2 are obtained from the average distribution in Figure 4-1, and they are given in Figure 4-2(a)(b). The basic statistics of the distributions are attached to Figure 4-1(a)(b). Cleary the distributions of W_1 and W_2 are not equal at all nor are their moments. Therefore as they stand, the payoffs are neither fully equivalent nor moment-equivalent. The Kolmogorov–Smirnov distance between the two distribution is

$$d(W_1, W_2) = (9689 - 4757)/10000 = 0.4932$$

which is big. A simple reason for this is that $a + b = 2\mu$ is not satisfied here as

$$(25.5 + 26.5)/2 = 26 \quad vs \quad \bar{x} = 25.46$$

In other words, 26 is not a central value of the distribution. If the payoffs were defined with $a = 25.96$ and $b = 24.96$ so that $(a+b)/2 = 25.46$, the distributions of W_1

and W_2 become these in Figure 4-3(a)(b). In this case Kolmogorv-Smirnov distance is (303-279)/10000= 0.0024, which is much smaller, and hence the two distributions are rather close. The first and second moments of the distributions are correspondingly close though the skewness and kurtosis of the distributions are a bit different. Hence one may say that this modified risk swap is at least almost the second order moment equivalent.

5 Concluding Remark

In this paper, motivated by the TT risk swap on the average temerature, we develop a framework to discuss an equivalence of the payoffs in (2.1) and derive some conditions for full equivalence and moment equivalence. The argument applied to the TT risk swap case. The empirical result revels that the TT risk swap can be regarded as the second order moment-equivalent if the payoffs are modified by a location shift.

Figure 4-1 Distribution of the Average Temperature of August & September, 2001

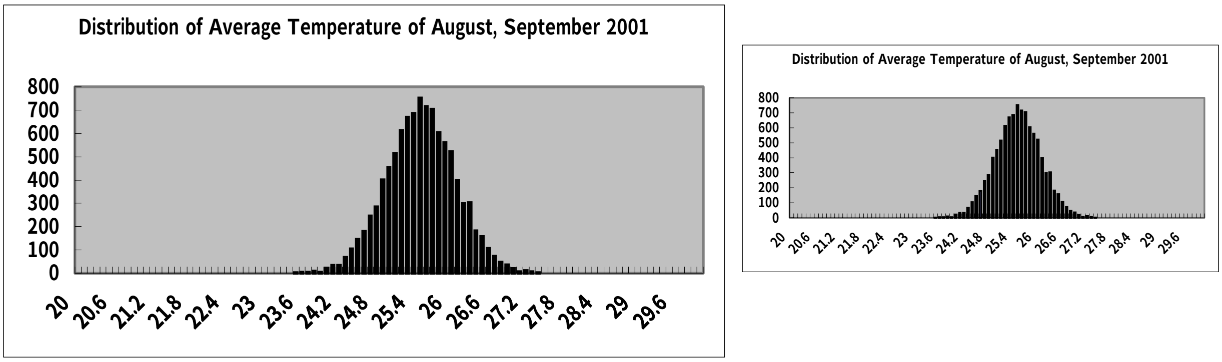
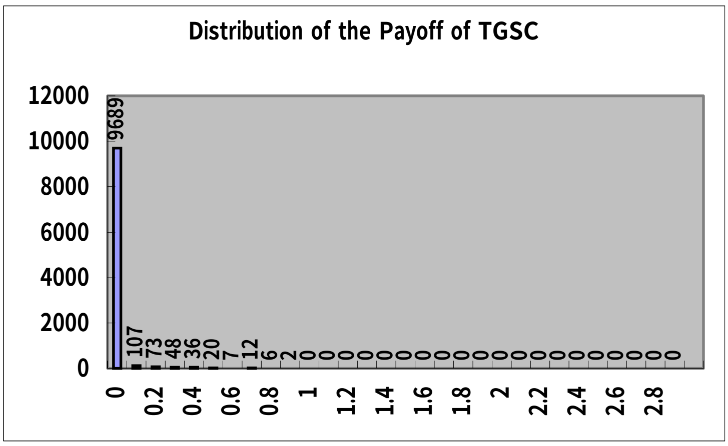
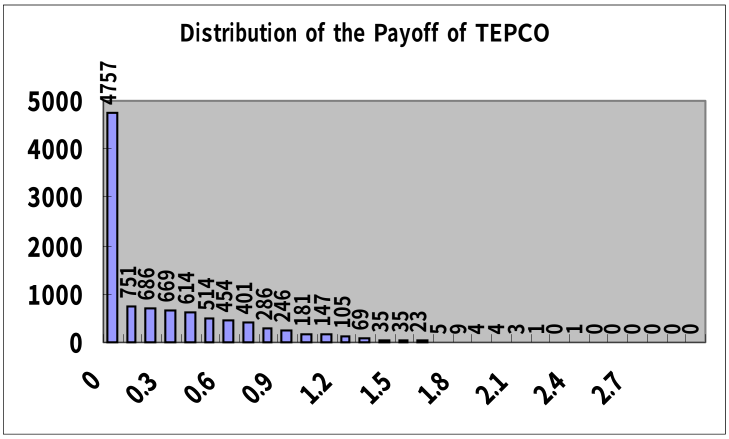


Figure 4-2 Distributions of the Payoffs of TT Risk Swap



Basic statistics of the Payoff of TGSC

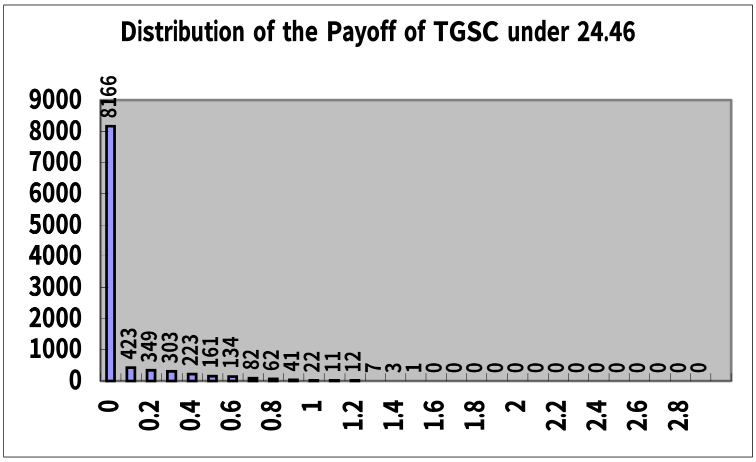
		t-value
mean	0.006599	
standard deviation	0.049285	
skewness	9.853565	402.2499903
kurtosis	112.9988	2306.463858



Basic statistics of the Payoff of TEPCO

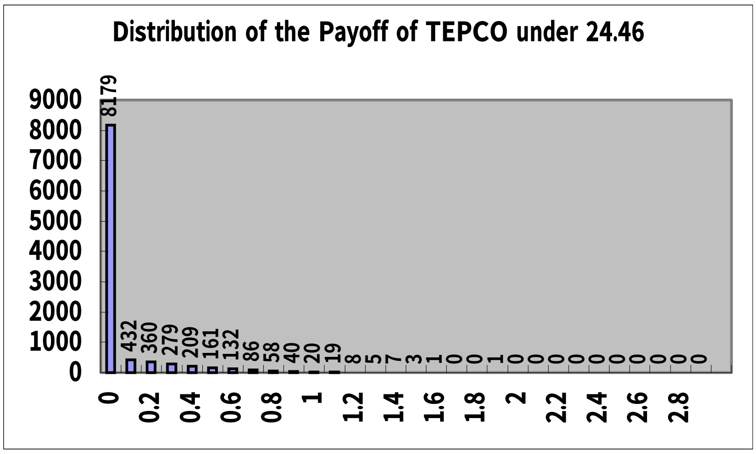
		t-value
mean	0.239925	
standard deviation	0.340832	
skewness	1.606865	65.59670462
kurtosis	2.370243	48.37996537

Figure 4-3 Distributions of the Payoffs of TT Risk Swap with the central value changed to the mean



Basic statistics of the Payoff of TGSC under 24.46

		t-value
mean	0.056453	
standard deviation	0.161217	
skewness	3.642702	148.7052672
kurtosis	15.01009	306.3769057

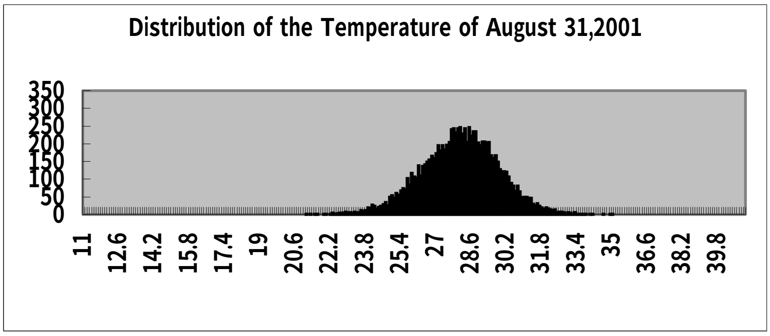


Basic statistics of the Payoff of TEPCO under 24.46

		t-value
mean	0.056502	
standard deviation	0.165135	
skewness	3.859599	157.5595862
kurtosis	17.69829	361.2467281

Distribution of the Payoff of TEPCO under 24.46

Figure 4-4 Distribution of the Temperature of August 31,2001



Basic statistics of the Temperature of August 31,2001

		t-value
mean	28.04676	
standard deviation	1.755324	
skewness	-0.12327	5.032094171
kurtosis	0.223571	4.563388456

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