A Dynamic Discounted Cash Flow Method for Valuation of an Office Building1[1]

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This paper proposes a framework and an associated dynamic model for valuing a rental real estate such as an office building as a discounted cash flow (DCF) value of future rents. The future rents are uncertain and described as a stochastic process. Hence the DCF value is also a random variable having a DCF distribution, which we call DDCF (dynamic DCF) distribution. In other words, the value of a rental real estate is in fact the distribution of all possible DDCF values, from which the mean of the DDCF distribution is obtained as a value of the building with a risk measure. Our valuation model in this viewpoint is different from the traditional static DCF valuation model in that we directly treat the dynamic risk or uncertainty factors that stochastically change the patterns and sizes of rental cash flows. Hence the problem of determining on cap rate or risk premium, which is a big issue in the valuation of a real estate by the traditional DCF method, is decomposed into the problem of determining a "fair" trading value of the real estate and the problem of valuing or measuring the risk as an uncertainty of the fair value. This decomposition naturally fits the framework of the traditional portfolio analysis and enables us to include real estates in an investment or financial analysis. Various simulations demonstrate a descriptive capacity and effectiveness of our valuation model. In addition, a valuation model using

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a real option, which is given as an exit strategy for selling out real estate, is formulated as an extended DDCF valuation model.

1 Introduction

This paper proposes a basic analytical framework and a valuation model for a rental real estate when rents are dynamically changing in the market and tenants are moving in and out. In this formulation we take it for granted that market rents follow a stochastic process over an investment time horizon and tenants terminate or renew contracts for occupancy randomly according to various factors including rent. Thus a value of a real estate as a DDCF (dynamic discounted cash flow) value of future net rents is stochastically realized depending on how future market rents are dynamically realized and how tenants move in and out in a stochastic manner. In other words, in our framework the value of a real estate is first viewed as the DDCF distribution itself of cash flows stochastically generated as future rents, where the factor that discounts future rents is a term structure of spot rates, which is common to the case of discounting cash flows from other financial assets and products in financial analysis. Then the mean of the DDCF distribution is understood as a DDCF trading value of a real estate because it is the value that equates the downside expected shortfall for a buy side with the upside expected shortfall for a sell side. This will be discussed in Section 2. Thirdly, a risk factor showing an uncertainty of the DDCF mean value is obtained as a measure of deviation from the mean in the DDCF distribution. By this argument we propose that the expected shortfall will be the first candidate as such a measure. In addition, downside(lower) standard deviation, standard deviation, lower 5% quantile etc. can be also used as a risk factor. Once a risk measure is selected, a risk premium added to spot rate may be defined, if necessary, by the ratio of a risk factor and the DDCF mean, for example. Our valuation model will be in particular suitable for analyzing values of rental real estate in investment or securitization or REIT in terms of return and risk.

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To make an analytical framework, our argument begins with a model of the following model of rental contract :

The rental contract term is 2 years with a fixed rent and the tenant has an option to leave without any penalty before a maturity of the contract by a 6 month advance notification.

This contract style is typical in Japan. The case with no option of early leave is treated as a special case with zero probability for early leave, in which case the contract is a 2-year lease contract.

On this contract scheme our DDCF valuation framework, which is formulated in Section 2, is symbolically described by the structure

$$(X, Q, P : R)$$

Here X represents a rent-generation model (process) describing stochastic variations of market rent, Q a notice probability model for a random notice time at which a tenant gives a 6-month advance notice before leave, P tenant-finding probability model describing a random time till the discovery of a new tenant after the management gets a notice of leave. This describes a vacancy risk. The three models form an integrated model for deriving the DDCF (value) distribution of an office building. In addition, R represents a price process of the real estate itself, which will be used for deriving the DDCF distribution with an option of selling it out. In Section 5 a model for valuing a real estate including a value of this option is formulated.

In Section 3 a specification of each component is made in detail. First what we call a log-DD (discrete time diffusion) process is specified for rent-generation model X, where the drift of the diffusion follows an exponentially smoothing model with the volatility of the log-DD model constant. Thus the process is non-Markovian and the drift changes gradually over time in such a manner that a rise in rent tends to move the drift up. Secondly Q, a simple probability model $\{p(j)\}$ is assumed over time $j = 1,\ldots,18,19$ in month where $p(j)$ is the probability of giving a notice of leave at j for $j < 19$ and $p(19)$ the probability of continuation of renewal. Thirdly for tenant–discovering time process, a negative binomial model is proposed. In Section on 4 this specification, we verify the descriptive power or effectiveness of the model for valuing an office building as a DDCF distribution through Monte Carlo simulations. Naturally each DDCF distribution provides the DDCF mean value as a value of the real estate and a measure of DDCF risk as its standard deviation or lower standard deviation or expected shortfall. The use of the DDCF mean as a fair trading value of a real estate is justified by the fact that the mean makes the downside risk for a buyer and the upside risk for a seller equal. In this view point we in fact define a risk premium by the ratio of the expected shortfall and the mean.

It is observed by various Monte Carlo simulations that our DDCF method is effective and useful for actual valuation of an office building. One of the results shows that the volatility of a rent process plays a most important role in determining the value and risk relation as well as the expected length of tenant discovering time. The risk premium we define as the ratio of the expected shortfall and the DDCF mean value is shown to be close to the volatility of the rent process.

In the literature, it seems to the authors that there has been no such article to directly propose a dynamic valuation method of discounted cash flows in a way as we did in this paper. However, there have been many papers to treat as a stochastic process a value process of a development project in real options in which cash flows from the project supposedly contain rents. McDonald and Siegel (1986) is a seminal paper treating it in view of irreversibility of a project in real options. Dixit and Pyndyck (1994) formulated the problem more systematically to treat the flexibility of decision-making in real options. Sirmans and Sezer (1999) applied it to a problem of real estate development. Williams (1991,1993, 1995, 1997), Grenadier(1996), Child, Ott and Triantis (1996) and Geltner, Riddiough and Stoyanovic (1996) also treated some real option problems in real estate development. A common feature in these papers is to treat a value process of cash flow as a gross process in a continuous time setting and express it by a Brownian motion or a diffusion process to use some results in option theories such as risk neutral pricing. Our method is different from these in that the valuation problem of a rental real estate is directly treated in a discrete time setting, the variations of dynamic cash flows are decomposed into vacancy risk, pre-vacation risk and rent variation risk and the rent process is non-Markovian. In other words, no risk neutrality argument is used to value a real estate and a risk factor is considered for the DDCF distribution. It is noted that in the no-arbitrage pricing theory in a continuous setting a stochastic process needs to be a diffusion that is Markovian.

The content of this paper is as follows.

Section 2 Formulation of our framework

Section 3 Modeling risk

Section 4 Derivation of DDCF distribution via Monte Carlo Simulation

Section 5 Valuation of a whole building and Exit Strategy

2 Formulation of our Framework

In order to formulate an analytical framework for valuation of a rental real estate such as an office building or a residential house, as a time unit for analysis we take monthly unit ($h = 1/12$ year) in terms of year and let the monthly time points be $n = 0, 1, 2, \cdots, N$. Hence time (point) n means nh years from time 0.

(1) Rent Process

For a specific rental space in an office building, which can be a whole space of the building, let Z_n represent the attributes of the space and let its market rent rate at n be denoted by

$$\tilde{X}(n) = \tilde{X}(n, Z) \qquad \$/m^2 , \tag{2.1}$$

where the size of the space is $a\, m^2$ (square meters). Naturally the market rate $\tilde{X}(n)$ changes stochastically and hence $\{\tilde{X}(n)\}$ forms a stochastic process. For a tenant who starts a new contract at 0, the rent at 0 is given as $\tilde{X}(0)$ for 24 months. However, for a tenant who has already rented the space for d months the rent is fixed for the remaining $24 - d$ months by contract, but the rent after termination of the contract is set equal to the market rate $\tilde{X}(24 - d + 1)$. If a tenant renews his contract to continue the rental, a new rate at the renewal may be discounted or adjusted in view of cost of tenant and property management. This will be formulated later. If the present tenant moves out earlier than the date specified by the contract, a market rent at that time applies to a new tenant.

(2) Notification Time and Vacation Time

A tenant who is renting the space at 0 is defined as the first tenant, and the second tenant, the third tenant, and so on, are defined to be respectively those who rent it in the second contract, in the third contract and so on. If the space is vacant at 0, then the tenant who occupies the space next is defined to be the second tenant. Further, if the i-th tenant renews his contract at maturity, he is called the $(i+1)th$ tenant in the next contract period though the same tenant continues to rent the space. Using this definition, let

K_i = occupancy period in months of the i-th tenant from the month of contract to the month of vacation by a 6 month advance notification,

J_i = vacancy period in months after the vacation of the i-th tenant.

Then the time point from time 0 for which the i-th tenant makes occupancy is expressed as

(2.2) $$K_1 + J_1 + \cdots + K_{i-1} + J_{i-1} = \sum_{j=1}^{i-1}(K_i + J_i) \equiv T_{i-1} \quad .$$

Note that vacancy at 0 means $K_1 = 0$. In (2.2) both K_i and J_i are random variables and specifications of the distributions of these variables will be discussed later. In general, vacancy period J_i will depend on the past occupancy and vacancy history. By our assumption, a tenant can leave before or at the end of the contract period by a 6 month advance notification. Hence let M_i denote the number of months after the contract time when a notification is given. Hence $M_i = 1, \cdots, 18$. Here for example $M_i = 10$ and $M_i = 18$ means that the i-th tenant leaves respectively at the end of the 16-th month and at the end of the 24-th month counted from the start of contract. We denote by $M_i = 19$ the event of the renewal of the contract by the i-th tenant. In this case, $J_i = 0$ follows because the i-th tenant continues his occupancy and becomes the $(i+1)$-th tenant. By its definition there is the following relation between occupancy period K_i and notification period M_i.

(2.3) $$K_i = \min(M_i + 6, 24) \qquad (M_i = 1, \cdots, 19).$$

Notification period implies the number of months from the start of a contract till the i-th tenant makes a decision for vacation or renewal. This decision making is made on a change of rental rate, and his attributes, change of regional environment, etc. In other words, the uncertainty on M involves the circumstantial attributes of each tenant, the uncertainty of rental rate, regional changes, etc.

（3） Tenant-Discovering Period and Vacancy Period

When a management receives a notification of leave, it starts to discover a new tenant. Let

$\tilde{J}_i$ = period in months from the time when a management receives a notification of leave till it discovers a tenant.

Using this notation, the vacancy period is described in months as

(2.4) $$J_i = \max(\tilde{J}_i - 6, 0).$$

If $J_i = 0$, either the i-th tenant renews the contract or the $(i+1)$-th tenant moves in right after the i-th tenant leaves.

By the above formulation, the uncertainty or risk factors in evaluation of a rental space or real estate are described as

$$(\{X(n)\},\{M_i:\ i=1,2,\cdots\},\{J_i:\ i=1,2,\cdots\})\ .$$

In the sequel, we will formulate how cash flows from rates are related to these uncertainties.

(4) **Net Present Value (NPV) of Cash Flows from the i-th Tenant.**

By our contractual assumption, the fixed rent for the i-th tenant depends on the market rent $\tilde{X}(T_{i-1}+1)$ at $T_{i-1}+1$ in (2.1). His actual rent $X(T_{i-1}+1)$ is nothing but an adjusted price of the market rent and it is assumed to be expressed by

(2.5) $$X_i(T_{i-1}+1)=\beta_i[\tilde{X}(T_{i-1}+1)-X_{i-1}(T_{i-2}+1)]+X_{i-1}(T_{i-2}+1),$$

where $\beta_i \le 1$. In other words, if the $(i-1)$-th tenant renews his contract and becomes the i-th tenant, the new rent is the old rent plus $100\beta_i$% of the gap between the market rent and the old rent. For example, in the case of renewal of the contract i.e., $M_{i-1}=19$, even if the market rent appreciates greatly or depreciates greatly, the contract may state that an increase or decrease size of rent be moderated for both tenant and management below or above. This is because this not only smoothes the economic fluctuations of cash flows in the both sides but also the management side saves cost for discovering a new tenant and avoids vacancy risk.

An alternative formulation for an actual rent on contract is a collared rent, which gives an incentive of renewal to a tenant. This is expressed as

$$\begin{aligned} X_i(T_{i-1}+1) &= X_{i-1}(T_{i-2}+1)+\max\{\tilde{X}(T_{i-1}+1)-X_{i-1}(T_{i-2}+1),0\} \\ &\quad -\max\{\tilde{X}(T_{i-1}+1)-\beta_1 X_{i-1}(T_{i-2}+1),0\}-\max\{\tilde{X}_{i-1}(T_{i-2}+1)-\tilde{X}(T_{i-1}+1),0\} \\ &\quad +\max\{\beta_2 X_{i-1}(T_{i-2}+1)-\tilde{X}(T_{i-1}+1),0\}, \end{aligned}$$

where $\beta_2 < 1 < \beta_1$. The rent pattern of this specification is drawn in Figure 2.1. For example, in case of $\beta_1 = 1.05$, in the case of renewal, new rent may increase at most by 5% but not more than that.

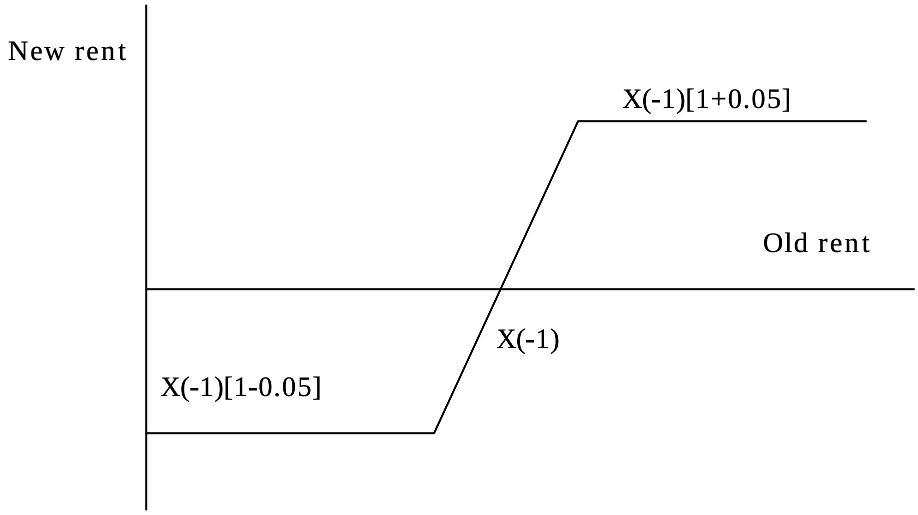


Figure 2.1　Moderation of Rent in Renewal of Contract

No matter how actual rent rate may be defined in contract, the net present value (NPV) at 0 of cash flows from the $i\text{-}th$ tenant is expressed as

(2.6) $$C_{oi}(T_{i-1}, K_i) = [X_i(T_{i-1}+1) - b(T_{i-1}+1)]\{D_0(T_{i-1}+1) + \cdots + D_0(T_{i-1}+K_i)\}\, a\,.$$

Here the inside of [ ] is the monthly net cash flows and the inside of { } is the sum of the discount rates for the occupancy period of the $i\text{-}th$ tenant. Also $C_{oi}(T_{i-1,}\ K_i)$ denotes the NPV of the cash flows generated by the $i\text{-}th$ tenant for the occupancy period from $T_{i-1}+1$ to $T_{i-1}+K_i$, and $b(T_{i-1}+1)$ is a maintenance cost at $T_{i-1}+1$ including tax assumed to be constant for the occupancy period of each tenant. Discount factor $D_0(T_{i-1}+k)$ is the NPV (at 0) of one dollar at $T_{i-1}+k$ and acts as the discount factor for future cash flows.

（5）Choice of Discount Factor

In our viewpoint, a value of a rental real estate is regarded as stochastic and is described as the distribution of DDCF values of future cash flows. Hence not only the mean value of the distribution, which may be regarded as a DDCF value of a real estate, is obtained but also the capitalization risk can be specified as a measure such as downside standard deviation (square root of the lower 2nd moment), expectated shortfall, and so on. In other words, a valuation of the risk component in determining a value of the real estate from the distribution is separated from a derivation of the DDCF distribution. In other words, the DDCF distribution should carry the information on risk and hence in its derivation risk premium should not be included. Hence the problem of selecting the so-called cap rate in the traditional DCF method is separated from the evaluation of the DDCF mean value. This point will be discussed in (8).

Consequently, the distribution that gives both a trading value and a risk measurement should be obtained as the distribution of "fair " DDCF values. Hence the discount rate should be based on spot rates, which are typically obtained as a term structure of interest through government bond prices. Thus letting $r_0(T_{i-1}+k)$ be the $(T_{i-1}+k)h$ year spot rate expressed as annually compounded rate, we specify the discount factor as

(2.7) $$D_0(T_{i-1}+k) = \left(1+r_0(T_{i-1}+k)\right)^{-(T_i+k)h} .$$

Later a cap rate is defined in terms of the mean and lower standard deviation though in our viewpoint this may not be necessary.

In our simulation a flat term structure of interest rates is assumed so that

(2.8) $$r_0(T_{i-1}+k) = r .$$

(6) Specification of Management Cost

For simplicity, one may assume that the management cost in (2.6) is constant ;

$$b(T_{i-1}+1) = b .$$

An alternative specification is to assume that the cost is proportional to rent $X(T_{i-1}+1)$ at $T_{i-1}+1$ as,

(2.9) $$b(T_{i-1}+1) = bX(T_{i-1}+1) .$$

This formulation takes an inflationary effect of the management cost into account. Of course, more directly it may be specified as

$$b(T_{i-1}+1) = b[1+Infl(T_{i-1}+1)],$$

where $Infl(n)$ denotes inflation rate at n. In this case, it is necessary to specify a process for generating inflation rates. In addition, a significant cost is incurred for the period of vacancy after the vacation of the i-th tenant, which is expressed as

(2.10) $$vc_{0i} \equiv c(T_{i-1}+K_i+1)[D_0(T_{i-1}+K_i+1)+\cdots+D_0(T_{i-1}+K_i+J_i)]\, a ,$$

where vc denotes vacancy cost. Again for a specification of this cost, a possibility is proportional to rent level at $T_{i-1}+K_i+1$;

(2.11) $$c(T_{i-1}+K_i+1) = cX(T_{i-1}+K_i+1).$$

We use the specifications (2.9) and (2.11) in our simulations.

(7) DDCF Value of Cash Flow

Under the above set-up, a DDCF value at 0 of total cash flows from a specific space is described as

(2.12) $$C_0^{\infty} = \sum_{i=1}^{\infty} [C_{0i}(T_{i-1}, K_i) - vc_{0i}].$$

Note that this is a random variable depending on future rents and vacancy. We call it a DDCF random variable. Hence on our viewpoint the value of a rental space is the distribution of this DDCF random variable, from which the mean value and a risk measurement are obtained.

However, a building cannot survive forever. Hence letting the duration period of the building be N, we derive the distribution of the DDCF random variable

(2.13) $$C_0^{I^*} = \sum_{i=1}^{I^*} [C_{0i}(T_{i-1}, K_i) - vc_{0i}] \ ,$$

where I^* is the I^*-th tenant up to whom the total period of occupancy and vacancy periods goes over or is equal to N:

(2.14) $$I^* = \min\{ i : T_i \geq N \}.$$

Here since T_i is stochastic, so is I^*. Since the analytical derivation of the distribution of $C_0^{I^*}$ is difficult, we use the MC method. In fact, the DDCF value $F_{0i}(T_{i-1})$ at 0 of all the cash flows from the i-th tenant through the I^*-th tenant is obtained by using the following forward recursive relation (as in dynamic programing)

(2.15a) $$F_{0i}(T_{i-1}) = \sum_{m=1}^{19} \Gamma_i(m)[C_{0i}(T_{i-1}, k(m)) + F_{0i+1}(T_{i-1} + k(m) + J_i)],$$
$$k(m) = \min(m+6, 24),$$

where

(2.15b)

$$\Gamma_i(m) = \begin{cases} 1 & \textit{if the } i\text{-}th \textit{ tenant occupies the space in the } m\text{-}th \textit{ month from the contract} \\ 0 & \textit{otherwise} \end{cases}$$

This states that the DDCF random variable $F_{0i}(T_{i-1})$ at 0 of the total cash flow from the i-th tenant through the I^*-th tenant is equal to the DDCF value $C_{0i}(T_{i-1}, k(m))$ at 0 from the i-th tenant who stays for $k(m)$ months plus the DDCF random variable $F_{0i+1}(T_{i-1} + k(m) + J_i)$ at 0 of the cash flow from the $(i+1)$-th tenant through the I^*-th tenant. Note that $k(19)$ means renewal, implying $J_i = 0$. Clearly the DDCF random variable $F_{0i}(T_{i-1})$ depends on the uncertainties discussed above.

Since we want the distribution of the DDCF random variable $F_{0i}(0)$ at 0 of all the cash flows from the first tenant trough the I^*-th tenant, let us slightly elaborate this case. Suppose that the first tenant has $24-d$ months as remaining rental period. Then if $d \le 18$,

(2.16) $$F_{0i}(0) = \sum_{m=d+1}^{19} \Gamma_1(m)[C_{01}(0, k(m)-d) + F_{02}(k(m)-d+J_1)]$$

If $d \ge 19$, whether or not a notification of lease is given, it is expressed as

$$F_{0i}(0) = [C_{01}(0,l) + F_{02}(l+J_1)] \qquad (l = 1, \cdots, 5).$$

In the case of renewal of contract, $J_1 = 0$. Incidentally substituting (2.15) with $i = 2$ into (2.16) yields

(2.17) $$\begin{aligned} F_{01}(0) = & \sum_{m=d+1}^{19} \Gamma_1(m) C_{01}(0, k(m)-d) \\ & + \sum_{m=d+1}^{19} \Gamma_1(m) \sum_{n=1}^{19} \Gamma_2 [C_{02}(T_1, k(n)) + F_{02}(k(m)-d+k(n)+J_2)] \end{aligned}$$

This shows the complex structure of the DDCF random variable at 0.

If a whole path of the rental rate process is given together with a realization of the tenant-discovering process for each tenant and a realization of the occupancy period of each tenant, which are realized stochastically, the recursive scheme (2.15) enable us to compute a DDCF value for each whole path from backward. This method is less convenient than the MC method which computes $F_{0i}(0)$ directly for each path. Generating thousands of paths via the MC method yields the DDCF distribution of $C_0^{I^*} = F_{0i}(0)$.

（8） DDCF Mean Value

Once the DDCF distribution is derived, the mean m_{DDCF} of the distribution is regarded a theoretical value of a real estate in our viewpoint. This is because m_{DDCF} is the value equating the expected shortfall for buyer

(2.18a) $$put(m) = E[\max(m - C_0, 0)]$$

with the expected shortfall for seller

(2.18b) $$call(m) = E[\max(C_0 - m, 0)].$$

In fact, m_{DDCF} satisfies the equation $put(m) = call(m)$, and $put(m)$ is regarded as a risk measure for a buyer of the real estate while $call(m)$ is regarded as a risk measure for a seller. In addition, $put(m)$ is interpreted as a premium of put option with exercise price m for hedging a downside risk for a long (buy) position, while $call(m)$ is interpreted as a premium of $call$ with exercise price m for hedging an upside risk for a short (sell) position.

Therefore m_{DDCF} gives an equal risk position for both a seller and a buyer in terms of the measures a) and b). Hence, denoting the risk by $es \equiv put(m_{DDCF})$, a risk premium is defined by

(2.19) $$rp = es(m_{DDCF})/m_{DDCF} .$$

We will evaluate this risk premium in simulation. Note that the rate of standard deviation and mean is called a coefficient of variation. The risk premium in (2.19) is a similar version, meaning on average risk in dollars to obtain one dollar in the mean.

3． Modeling Risk

As has been stated, a DDCF value at 0 is exposed to rental rate variation, pre-vacation and vacancy risks. In this section specific stochastic models are proposed for describing these risks.

（1） Rental rate process

We propose as market rental rate process a log-discrete time diffusion (log DD) process given by

(3.1) $$\tilde{X}(n) = \tilde{X}(n-1)\exp(\mu_{n-1}h + \sigma_{n-1}\sqrt{h}\varepsilon_n) \quad ,$$

$$\mu_{n-1} \text{ and } \sigma_{n-1} \text{ depend on past } \tilde{X}(0), \cdots, \tilde{X}(n-1),$$

$$\varepsilon_n \sim iid\ N(0,1).$$

where $iid\ N(0,1)$ denotes “independently and identically distributed” as standard normal distribution. Model (3.1) means that the rate of change in rent from $n-1$ to n is expressed as the sum of a drift and a new innovation

$$\log[\tilde{X}(n)/\tilde{X}(n-1)] = \mu_{n-1}h + \sigma_{n-1}\sqrt{h}\,\varepsilon_n .$$

Here drift μ_{n-1} is assumed to follow

$$\mu_{n-1} = \phi \log[\tilde{X}(n-1)/\tilde{X}(n-2)] + (1-\phi)\mu_{n-2},$$

(3.2)
$$\mu_0 = \frac{1}{m}\sum_{j=-m+1}^{0} \log[\tilde{X}(j)/\tilde{X}(j-1)],$$

and volatility σ_{n-1} , which can depend on past, is assumed to be

(3.3) $$\sigma_{n-1} = \sigma .$$

Hence by (3.2) a big positive innovation ε_n at n moves the drift μ_n upward at $n+1$ by rate ϕ, yielding a tendency of a further appreciation for rent at $n+1$. The drift μ_{n-1} may depend on state variables such as business condition, regional factor, etc. through the smoothing parameter ϕ , though we here assume (3.2). This is a non-Markovian model and will be also used as a price model for a real estate in Section 5. In Kariya, Ushiyama an Pliska (2002) this model was used in valuation of mortgage-backed securities.

（2）Distribution of Notification Time M_i

Since the occupancy period $K_i = \min(M_i + 6, 24)$ of the $i-th$ tenant is a function of notification time M_i, which should be made 6 months in advance before vacation or leave, we specify the distribution of M_i taking m by

(3.5) $$P(M_i = m) = E[\Gamma_i(m)] = p_i(m) \equiv p(m, T_{i-1}) \qquad (m = 1, \cdots, 19),$$

with

$$p_i(m) = q(T_{i-1})^{19-m} \qquad (m = 1, \cdots, 18) \quad (i \geq 2),$$

(3.6)
$$p_i(19) = 1 - \sum_{m=1}^{18} p_i(m),$$

where $\Gamma_i(m)$ is given by (2.15b) and $M_i = 19$ is the event of renewal, implying $J_i = 0$. Note that if the remaining contract period is $24-d$ at 0 with $d \leq 18$, the probability is set as

(3.7) $$p_1(m-d) = q(0)^{19-m} \qquad (m = d+1, \cdots, 18) .$$

As a specification of q-function in (3.6), one may use such a logistic function as

(3.8) $$q(T_{i-1}) = \exp(b_0 \sum_{j=1}^{i-1} J_i + \sum_{i=1}^{e} c_i z_i) \Big/ [1 + \exp(b_0 \sum_{j=1}^{i-1} J_i + \sum_{i=1}^{e} c_i z_i)],$$

where the term $b_0 \sum_{J=1}^{i-1} J_j$ is included as a factor showing a general attribute of the space that a larger past vacancy period will imply a tendency of early notification. As z_i's, one may use some specific hedonic or regional factors of the building. However, at 0 we need to specify and estimate the model to value a rental space or derive the DDCF distribution. This may not be possible because of the lack of data. In our simulation, it is assumed that the $q-function$ is contact.

(3) **Process of Vacancy Period** $\{J_i\}$

Clearly vacancy period is $J_i = \max(\tilde{J}_i - 6, 0)$ with $\tilde{J}_i$ a tenant discovering period. Note that when $M_i = 19$, by definition set

$$\tilde{J}_i(19) \equiv 0.$$

As the distribution of $\tilde{J}_i$, it is assumed that $\tilde{J}_i$ follows negative binomial distribution

$$P(\tilde{J}_i = j) = (1-p_i)^{\alpha}\binom{j+\alpha-1}{j}p_i^j, \quad (j = 0, 1, 2, \cdots). \tag{3.9}$$

This is a unimodal distribution with mean and variance

$$\eta_i \equiv E[\tilde{J}_1] = \frac{\alpha\, p_i}{1-p_i}, \quad \omega_i^2 \equiv Var(\tilde{J}_i) = \frac{\alpha\, p_i}{(1-p_i)^2} \tag{3.10}$$

respectively. Clearly (η_i, ω_i^2) is in one-to-one correspondence with (α, p_i). As p_i, similarly to (3.8) one may assume

$$p_i = \frac{\exp[b_0 \sum_{j=1}^{i-1} J_j + b_1 i + \sum c_i z_i]}{1 + \exp[b_0 \sum_{j=1}^{i-1} J_j + b_1 i + \sum c_i z_i]}. \tag{3.11}$$

However, the existence of data that allows us to estimate this model may be questioned. In our simulation, it is also assumed that it is constant : $p_i = p$. It is remarked that an alternative formulation for tenant discovering time $\tilde{J}_i$ is a Poisson waiting time.

When the above specifications in (1), (2) and (3) are given, the distribution of the DDCF random variable is derived via the MC method. The MC derivation procedure will be described when p_i, q_i functions are given.

1) If $d \leq 18$, generate a random number of M_1, based on (3.6) at 0. If $d \geq 19$, this step is not necessary.
2) If $d \leqq 18$, using the random number m_1, compute $C_{01}(0, k(m) - d)$.

If $d \geq 19$, compute $C_{01}(0, l)$. Here the term structure of discount rates is assumed to be given.

3) If $m_1 = 19$ or if $d \geq 19$ and the contract is renewed, set $J_1 \equiv 0$. If none of them hold, generate a random number of $\tilde{J}_1$ based on (3.9). Letting it be $\tilde{j}_1$, the value $t_1 = k_1 + j_1$ follows with $j_1 = \max(\tilde{j}_1 - 6, 0)$.

4) Using the stochastic process (3.1), generate a path of $\tilde{X}(n)$ and determine the values $\tilde{X}(n)'s \ for \ n = t_1, t_2, \cdots$.

5) Again generate $M_2 = m_2$ and obtain $k_2 = k(m_2)$ and $C_{02}(t_1, k(m_2))$.

6) If $m_2 = 19$, set $J_2 \equiv 0$. Otherwise, generate $\tilde{J}_2 = \tilde{j}_2$ via (3.9) and determine $t_2 = k_2 + j_2$ with $j_2 = \max(\tilde{j}_2 - 6, 0)$.

7) Repeating the above procedure, find the minimum value I for which

$$\sum_{i=1}^{I} t_i \geq N = 240.$$

Then compute the NPV of total cash flows;

$$C_0^I = \sum_{i=1}^{I} [C_{0i}(t_{i-1}, k(m_i)) - vc_{0i}].$$

8) Repeating the procedure from 1) through 7), for example, $S = 10{,}000$ times, one obtain DDCF values; $C_0^1(s) \quad (s = 1, \cdots, S)$.

From these value we obtain the DDCF distribution with mean and variance

$$\overline{C}_0^1 = \frac{1}{S}\sum_{s=1}^{S} C_0^I(s), \quad and \quad v_c^2 = \frac{1}{S}\sum_{s=1}^{S} (C_0^I(s) - \overline{C}_0^I)^2 \quad \text{respectively.}$$

In Section 4, the distributions are derived via this method for various parameter settings.

4 Derivation of DDCF Distribution via Monte Carlo Simulation

To derive DDCF distribution via simulation, assume (3.1) for rent process X_n with constant volatility (3.3), the distribution (3.6) for notification time M_i with constant q-function, and the distribution (3.9) for tenant discovery period $\tilde{J}_i$ with constant mean and variance

$$\eta = E[\tilde{J}_i] = \alpha p/(1-p) \qquad and \qquad \omega^2 = Var(\tilde{J}_i) = \alpha p/(1-p)^2.$$

Then the unknown parameters included in the models are

1) (μ_0, σ, ϕ) for (3.1), 2) q for (3.6),

3) (η,ω) for (3.10).

For the time horizon of our analysis, set $N = 240$ (20 years) and for the initial rent, set $\tilde{X}_0 = one\ thousand\ dollars$. Generating 100,000 paths over the time horizon and computing DDCF values based on each path, the DDCF distribution is derived together with DDCF mean value m_{DDCF}, standard divisions s, lower 5% quintile $z_{0.05}$, the expected shortfall $es \equiv put(m_{DDCF})$, and lower standard deviation defined by

$$s_- = [\frac{1}{N}\sum_{j=1}^{N}\max(m_{DDCF} - z_j, 0)^2]^{1/2}.$$

The measurement unit of these quantities is expressed in our simulation below as 100 thousand dollars. Also the risk premium $rp = es/m_{DDCF}$ is computed. For the management cost, $c = b = 0.1$ in (2.9) and (2.11). Also assuming a flat term structure of interest, we set $r = 0.01$ in (2.8).

Our basic viewpoint of the analysis is that the value of a real estate is the DDCF distribution itself that depends on the model specification of the underlying uncertainty, and hence the DDCF mean value and its risk measures are the functions of the parameters involved in the model. In our case they are viewed as

$$m_{DDCF} = f((\mu_0,\sigma,\phi),\ q,\ (\eta,\omega)), \qquad risk = g((\mu_0,\sigma,\phi),\ q,\ (\eta,\omega)).$$

4．1 Effect of (σ,ϕ,μ_0) on the DDCF distribution

In Table 4-1(a) and Figure 4-1(a), for $\phi = 0.5$, $\mu_0 = 0.0, \eta = 3, \omega^2 = 6$ and $q = 0.25$, or 0.5 held fixed, some effects of changes of σ on the DDCF distribution are investigated. Note that $q = 0.25$ corresponds to the case where one fourth of tenants are vacated on the average though the analysis is pathwise made. In the table the case $q = 0.5$ is also treated. From this table and figure, the following facts are observed:

(1) When σ gets larger, $m_{DDCF}, s, z_{0.05}$, s_- and es get larger. When σ changes twice form 0.1 to 0.2, the downside risk s_- and expected shortfall es become more than twice. But the mean does not become greater than the change of the risk.

(2) When σ becomes greater beyond 0.1, the skewness and kurtosis of the DDCF distribution also become greater, showing an asymmetry of the distribution. Here the kurtosis is measured as deviation from 3.

(3) Figures 4-1(a) and (b) describes the DDCF distributions. The above observations are

confirmed from these graphs and it is found out that the volatility σ of the rent process is important in pricing an office building.

(4)When the case $q = 0.5$ is compared to the case $q = 0.25$ the risk values are almost the same for each σ though m_{DDCF}'s are slightly smaller. This will be because the mean of tenant discovery time is $\eta = 3$ and its standard derivation is $\omega = \sqrt{6}$, and hence a new tenant is very likely to be readily discovered after vacation.

(5) The expected shortfall (es) increases together with σ and the values are almost same for the cases $q = 0.25$ and $q = 0.5$ in both Tables 4-1(a) and 4-1(b). Because of its definition the expected shortfall is smaller than the downside standard deviation for each case.

(6) The risk premium $rp = es / m_{DDCF}$ also increases together with σ. The values are coincidentally similar to those of σ for each case. For example, when $q = 0.25$ and $\sigma = 0.04$ in Table 4-1(a), the risk premium is $rp = 0.041$. One might add this value to interest rate $r = 0.01$ and regard 0.051 as a cap rate .

In Table 4-1(b) and Figure 4-1(b) the case $\eta = 6$ and $\omega^2 = 12$ is treated for comparison. The results do not change much the results in Table 4-1(a), though the DDCF means become slightly smaller for σ large and the s and s_- become a bit larger for each σ.

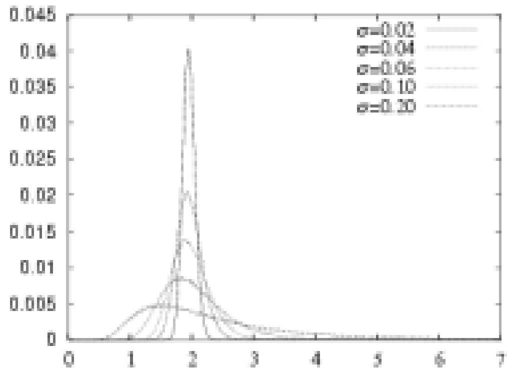


Figure 4-1(a)

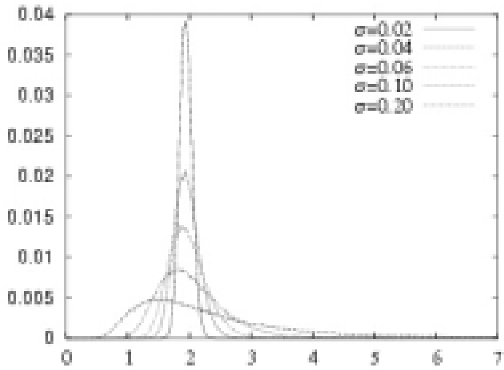


Figure 4-1(b)

q	σ	m_{DDCF}	s	skw	kur	$z_{0.05}$	s_-	es	rp
0.25	0.02	1955.5	100.1	0.18	0.06	1796.1	69.0	39.9	0.020
0.25	0.04	1967.3	201.2	0.39	0.28	1660.1	134.7	79.8	0.041
0.25	0.06	1988.3	306.8	0.60	0.65	1540.5	199.2	120.8	0.061
0.25	0.10	2056.2	544.5	1.07	2.22	1333.7	331.0	209.8	0.102
0.25	0.20	2424.9	1496.2	2.94	19.4	972.9	735.6	512.4	0.211
0.50	0.02	1942.1	102.2	0.17	0.07	1779.1	70.6	40.8	0.021
0.50	0.04	1954.6	202.0	0.39	0.29	1646.1	135.3	80.2	0.041
0.50	0.06	1975.4	308.0	0.60	0.64	1525.7	200.0	121.4	0.061
0.50	0.10	2042.2	547.2	1.08	2.22	1316.0	332.2	210.4	0.103
0.50	0.20	2408.8	1495.3	3.03	22.5	958.6	733.7	513.6	0.213

Table 4-1(a)：Case $\phi = 0.5, \mu_0 = 0.0, \eta = 3, \omega = \sqrt{6}.$

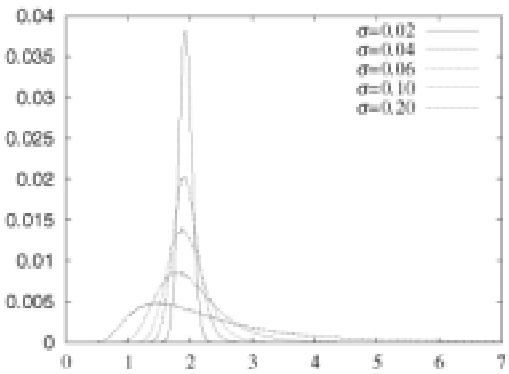


Figure 4-2 (a)

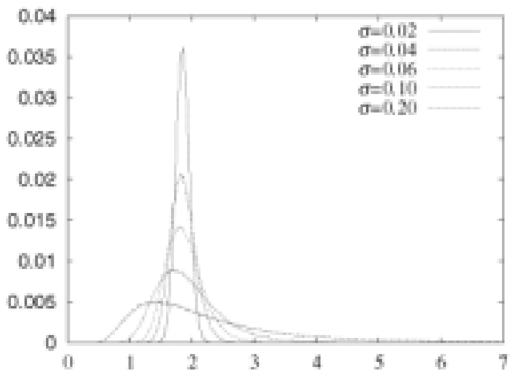


Figure 4-2(b)

q	σ	m_{DDCF}	s	skw	kur	$z_{0.05}$	s_-	es	rp
0.25	0.02	1924.5	105.5	0.11	0.08	1754.4	73.5	42.0	0.022
0.25	0.04	1936.4	201.3	0.38	0.26	1627.8	135.1	79.9	0.041
0.25	0.06	1956.3	304.4	0.60	0.67	1511.0	197.9	120.1	0.061
0.25	0.10	2023.8	537.8	1.07	2.18	1309.8	327.0	207.4	0.102
0.25	0.20	2385.4	1470.0	2.92	18.7	957.3	723.4	505.2	0.212
0.50	0.02	1848.6	112.1	0.09	0.05	1667.2	78.3	44.6	0.024
0.50	0.04	1860.0	199.7	0.37	0.27	1553.2	134.2	79.3	0.043
0.50	0.06	1879.6	298.1	0.61	0.69	1443.6	193.6	117.4	0.062
0.50	0.10	1944.2	521.4	1.07	2.28	1252.6	317.1	201.1	0.103
0.50	0.20	2294.1	1416.4	2.81	16.06	915.0	698.7	488.3	0.213

Table 4-1(b)：Case $\phi = 0.5, \mu_0 = 0.0, \eta = 6, \omega = \sqrt{12}$

4．2 Effect of (μ_0, σ, ϕ) on the DDCF distribution

In Table 4-2(a)(b)(c), for $q = 0.5$, $\eta = 12$, $\omega^2 = 24$ held fixed, some effects of changes of the smoothing parameter ϕ and the initial drift μ_0 on the DDCF distribution are investigated for the cases $\sigma = 0.02$ and 0.2. Note that the larger ϕ is, the more new changes of the rent are absorbed into the current drift and hence the more volatile the drift moves.

(1) Comparisons among the cases (a),(b) and (c) in Table 4-2. When the initial drift is upward (positive), i.e., $\mu_0 = 0.1$ (annual growth rate of rent 10%), the larger ϕ is, the smaller the mean m_{DDCF} and risk measures s, s_- and es are(see Figures 4-2(a) and (b)). When $\mu_0 = -0.1$, the relation is reversed(see Figures 4-2(e) and (f)). When $\mu_0 = 0.0$, the means, skewness and risks in all the cases are not different without respect to the values of ϕ(see Figures 4-2(c)and (d)). In this respect, it is important how properly μ_0 and ϕ are set in valuing the DDCF mean. These facts are also observed from the graphs in Figure 4-2.
On the other hand, the risk premiums are quite stable for changes of ϕ.

(2) For each ϕ and μ_0, the larger the volatility σ of the rent is, the larger the mean and risk of the DDCF distribution are. This is similar to the case with $\mu_0 = 0.0$ in Subsection 4.1. However, the rate of increase in the mean is smaller than the rate of increase in the downside risk measures s_- and *es*, and hence s_- / m_{DDCF} and risk premium *rp* decreases in all the cases.

(3) The dependency of m_{DDCF} on the volatility σ becomes greater for each (ϕ, μ_0) as σ becomes greater. In fact, the change of m_{DDCF} when σ changes from 0.02 to 0.10 is smaller than the change of m_{DDCF} when σ changes from 0.1 to 0.2. On the other hand, the result is reversed for the lower standard deviation s_- and the expected shortfall *es*. For example, in the first column of Table4-1(a), when σ changes from 0.02 to 0.1 the rate of changes of *es* is 182.2/53.2, while when σ changes from 0.1 to 0.2, the rate is 435.5/182.2. Thus the volatility σ is an important parameter for the structure of determining the mean m_{DDCF} and risk.

(4) In the case of μ_0 = 0.1 the expected shortfall decreases gradually for each σ as ϕ increases. But the risk premium remains the same for each σ.In the case of μ_0 = 0.0 *es* does not change and the risk premium remains the same for each σ.In

the case of μ_0 = -0.1 *es* increases gradually for each σ. The risk premium again remains the same for each σ. In other words, the risk premium is robust against changes of the smoothing parameter ϕ.

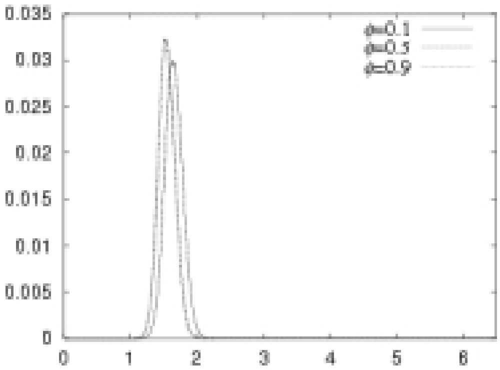


Figure 4-2 (a) σ =0.02, μ_0=0.1

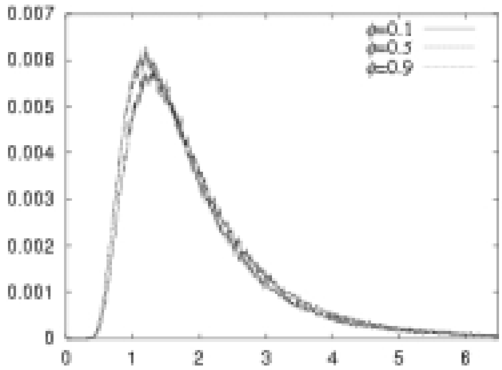


Figure 4-2(b) σ =0.2, μ_0=0.1

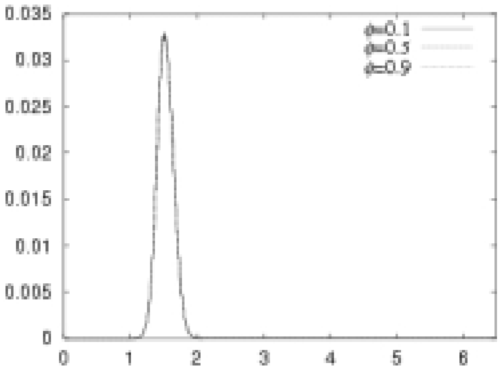


Figure 4-2(c) σ =0.02, μ_0=0.0

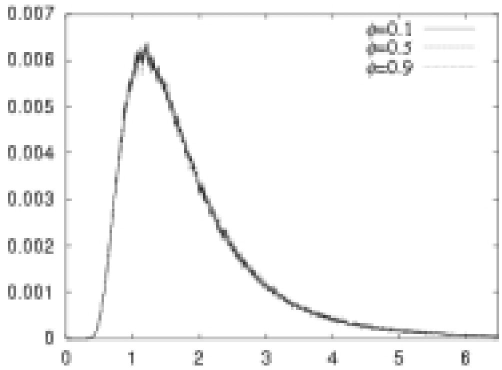


Figure 4-2(d) σ =0.2, μ_0=0.0

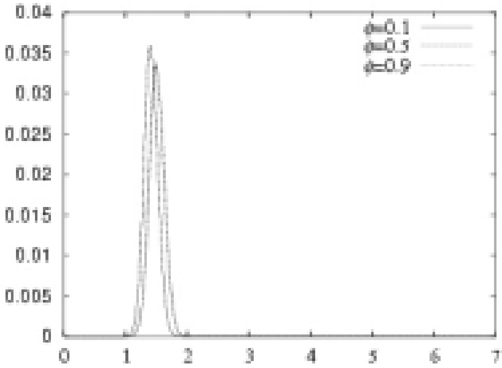


Figure 4-2(e) σ =0.02, μ_0=-0.1

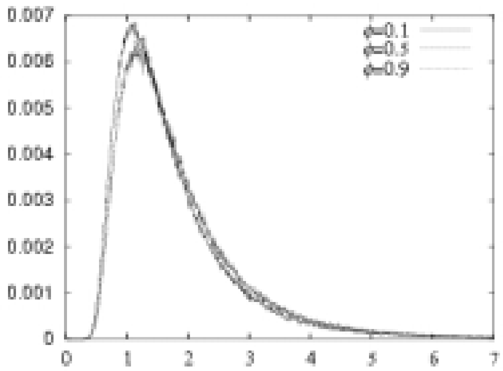


Figure 4-2(f) σ =0.2, μ_0=-0.1

μ_0	σ	m_{DDCF}	s	skw	kur	$z_{0.05}$	s_-	es	rp
0.1	0.02	1654.7	132.9	0.13	0.006	1442.9	92.2	53.2	0.032
0.1	0.10	1738.9	473.8	1.09	2.310	1111.4	288	182.2	0.105
0.1	0.20	2044.6	1264	3.03	21.54	819.1	620	435.5	0.213
0.0	0.02	1529.9	121.6	0.13	0.005	1336.1	84.3	48.7	0.032
0.0	0.10	1606.7	431.1	1.07	2.257	1035.2	262	166.4	0.104
0.0	0.20	1886.0	1154	3.03	21.53	766.5	567	397.8	0.211
-0.1	0.02	1415.7	111.3	0.12	0.016	1238.0	77.2	44.5	0.031
-0.1	0.10	1487.4	395.2	1.07	2.274	962.8	240	152.0	0.102
-0.1	0.20	1742.1	1057	2.92	18.30	717.9	519	363.3	0.209

(a) $\phi = 0.1$

μ_0	σ	m_{DDCF}	s	skw	kur	$z_{0.05}$	s_-	es	rp
0.1	0.02	1554.0	124.1	0.12	0.024	1356.0	86.1	49.6	0.032
0.1	0.10	1635.5	443.3	1.07	2.251	1047.5	269.	170.2	0.104
0.1	0.20	1920.6	1181.6	2.88	17.27	774.6	581	407.2	0.212
0.0	0.02	1529.7	121.9	0.12	0.023	1335.3	84.6	48.7	0.032
0.0	0.10	1607.3	433.6	1.08	2.208	1033.4	263	167.1	0.104
0.0	0.20	1889.7	1160.3	2.88	17.27	764.3	570	399.9	0.212
-0.1	0.02	1506.0	119.7	0.13	0.029	1315.0	83.0	47.8	0.032
-0.1	0.10	1582.9	426.5	1.08	2.261	1017.4	259	164.1	0.104
-0.1	0.20	1861.9	1145.8	2.97	19.87	753.4	561	392.7	0.211

(b) $\phi = 0.5$

μ_0	σ	m_{DDCF}	s	skw	kur	$z_{0.05}$	s_-	es	rp
0.1	0.02	1543.1	123.3	0.12	0.016	1346.4	85.5	49.3	0.032
0.1	0.10	1622.1	439.7	1.09	2.308	1040.7	267	169.2	0.104
0.1	0.20	1905.6	1176.9	3.19	30.68	768.2	576	402.4	0.211
0.0	0.02	1529.6	122.0	0.12	0.016	1334.9	84.6	48.8	0.032
0.0	0.10	1606.6	433.8	1.08	2.369	1031.9	263	167.5	0.104
0.0	0.20	1888.5	1165.1	3.19	30.68	762.6	570	398.4	0.211
-0.1	0.02	1516.9	120.9	0.12	0.016	1323.7	83.9	48.3	0.032
-0.1	0.10	1593.4	430.2	1.09	2.278	1023.7	261	165.8	0.104
-0.1	0.20	1876.0	1161.3	3.10	24.34	758.7	567	394.4	0.210

(c) $\phi = 0.9$

Table 4-2：Case $q = 0.5, \eta = 12, \omega = \sqrt{24}$

4．3 Early Vacation Risk and Vacancy Risk.

In this section, fixing $\sigma = 0.02$ or $0.2, \mu_0 = 0.0$, and $\phi = 0.1$, we consider some effects of changes of the notification parameter q with $(\eta, \omega^2) = (3,4), (12,16)$.

Since in the case of $\eta = 3$, and $\omega = 2$ the mean and standard deviation of the tenant discovery time are respectively 3 months and 2 months, a new tenant is readily found after a 6 month advance notification. While, in the case of $\eta = 12, \omega = 4$, there will be a 6 month vacancy on the average after vacation.

In Table 4-3 and Figure 4-3, the DDCF distribution and some characteristic values are given where the vacancy cost v_c in (2.10) is treated with $c = 0.1$ and $c = 0.5$ in (2.11). Table 4-3 distinguishes the following cases;

(a) $\sigma = 0.02$, $\eta = 3$, $\omega = 2$

(b) $\sigma = 0.02$, $\eta = 12$, $\omega = 4$

(c) $\sigma = 0.2$, $\eta = 3, \omega = 2$

(d) $\sigma = 0.2, \eta = 12, \omega = 4$

(1) In both cases of $\sigma = 0.02$ and $\sigma = 0.2,$ if $q = 0.1$ or $(\eta = 3, \omega = 2)$, tenants are less likely to leave or new tenants are easily found after a notification of leave, the DDCF distributions do not make a big change on the means and risk as in Figures 4-3(a)(c) and Table 4-3 (a)(c).
Naturally the effect of the vacancy cost is small.

(2) But in the case of $q = 0.5$ in (a)-(d) where about half of tenants move out at maturity of contract, the mean m_{DDCF} gets smaller than the case of q=0.1 for each case, and the downside risks $s\_$ and *es* get larger for the cases (a) and (c) and smaller for the cases (b) and (d). In the case of (c) the downside risk $s\_$ does not depend much on q and v_c.

(3) In the case $q = 0.1,\ \eta = 12, \omega = 4$ in (b) and (d), it is difficult to find a new tenant once the space is vacated. Hence in the case of $\sigma = 0.02$ in (b), the mean m_{DDCF} becomes smaller than the same case in (a) and the downside risk becomes larger. This is shown in Figure 4-3 (b). However, in the case of $\sigma = 0.2$ in (d), m_{DDCF} becomes smaller but $s\_$ and *es* also become smaller than each same case of (c). This is because the tenant discovery cost is large when σ is large but vacancy is less likely as $q = 0.1$.

(4) In the case $q = 0.5, \eta = 12, \omega = 4$ in (b) and (d), there are more early notifications and vacancies. In this case, the means m_{DDCF}'s decrease significantly even for $\sigma = 0.02$. But in case of $\sigma = 0.2$, the size of the decrease is smaller. This is because when σ is small and the tenant discovery time is long, it is less likely to recover the cash flow once lost as the rent does not change much. This makes the

downside risk bigger. Also when $q \geq 0.5$, the effect of the tenant discovery cost in case of $\sigma = 0.02$ is larger than that in case of $\sigma = 0.2$.

(5) When (c) and (d) are compared in the case $\sigma = 0.02$ and $q = 0.5$, the downside risk in (d) is smaller than (c).

(6) The risk premium is rather stable in each of (a)-(d). The values are again rather similar to the volatility of the rent process, though they are increasing in (a) and (b) and decreasing in (d).

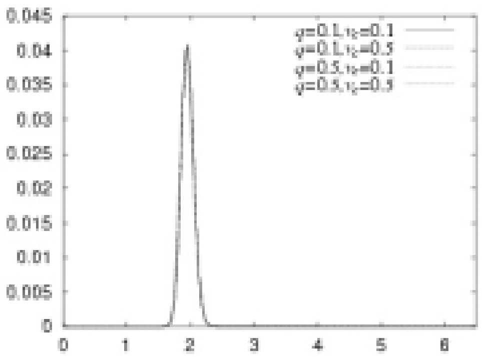


Figure 4-3(a)

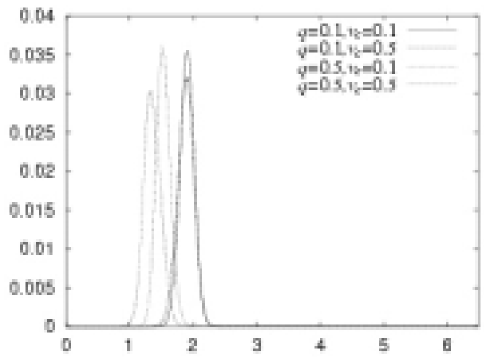


Figure 4-3(b)

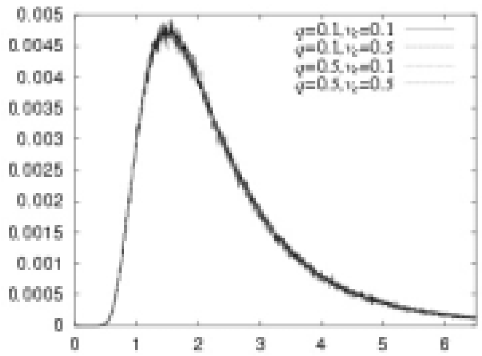


Figure 4-3(c)

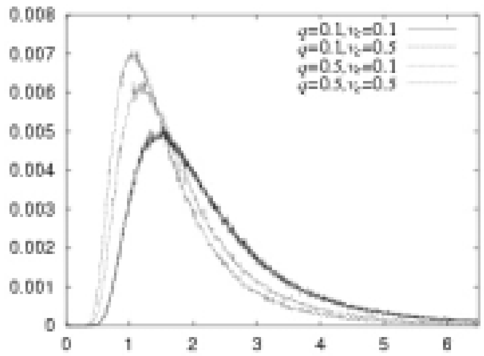


Figure 4-3(d)

q	c	m_{DDCF}	s	skw	kur	$z_{0.05}$	$s\_$	q	c
0.1	0.1	1960.6	98.6	0.20	0.08	1804.0	67.88	39.3	0.0200
0.1	0.5	1960.5	99.0	0.19	0.06	1803.1	68.19	39.5	0.0201
0.5	0.1	1952.4	100.4	0.20	0.08	1793.6	69.08	40.0	0.0204
0.5	0.5	1948.7	100.9	0.19	0.08	1788.0	69.60	40.2	0.0206

(a) $\sigma = 0.02, \eta = 3, \omega = 2$

q	c	m_{DDCF}	s	skw	kur	$z_{0.05}$	s_-	q	c
0.1	0.1	1907.1	113.5	0.002	0.08	1720.0	80.29	45.1	0.0237
0.1	0.5	1885.5	127.6	-0.19	0.16	1667.6	92.57	50.6	0.0268
0.5	0.1	1532.6	110.7	0.19	0.01	1358.3	76.18	44.3	0.0289
0.5	0.5	1361.2	130.4	0.17	-0.03	1158.2	89.72	52.3	0.0385

(b) $\sigma = 0.02, \eta = 12, \omega = 4$

q	c	m_{DDCF}	s	skw	kur	$z_{0.05}$	s_-	q	c
0.1	0.1	2423.1	1479.4	2.87	17.8	980.4	729.94	509.8	0.2104
0.1	0.5	2418.9	1474.9	2.91	19.2	977.6	727.89	507.7	0.2099
0.5	0.1	2417.0	1485.6	2.92	19.9	967.9	732.82	511.6	0.2117
0.5	0.5	2413.3	1490.6	2.99	20.4	967.2	731.77	511.5	0.2119

(c) $\sigma = 0.2, \eta = 3, \omega = 2$

q	c	m_{DDCF}	s	skw	kur	$z_{0.05}$	s_-	q	c
0.1	0.1	2351.9	1433.9	2.92	21.0	953.1	707.55	493.9	0.2100
0.1	0.5	2325.6	1422.7	3.06	26.2	941.7	699.39	488.1	0.2099
0.5	0.1	1887.2	1145.4	2.91	19.5	769.4	565.32	394.6	0.2090
0.5	0.5	1672.2	1020.3	3.10	24.0	685.7	499.26	348.9	0.2086

(d) $\sigma = 0.2, \eta = 12, \omega = 4$

Table 4-3：Dependence of the DDCF distribution on $q\,(\mu_0 = 0.0, \phi = 0.1)$

4．4 Effect of (η, ω) on the DDCF distribution.

So far for the mean η and standard deviation ω of the tenant discovery time given we have investigated various effects of changes of the other parameters on the DDCF distributions. In this subsection, changing (η, ω), we will see how the vacancy risk implied by the tenant discovery time changes the DDCF means and the downside risk.

In Figures 4-4(1) and (2) the cases of $\sigma = 0.02$ and $\sigma = 0.2$ with $q = 0.25$ are treated. The 3-dimentional graphs of relevant statistics of the DDCF distribution are plotted against the axis of (η, ω) in the following cases.

(a) mean m_{DDCF} and 5% lower quantile,

(b) standard deviation s and lower standard deviation s_- ,

(c) skewness, (d)kurtosis .

From Figure 4-4(1) where $\sigma = 0.02$ is treated it is observed;

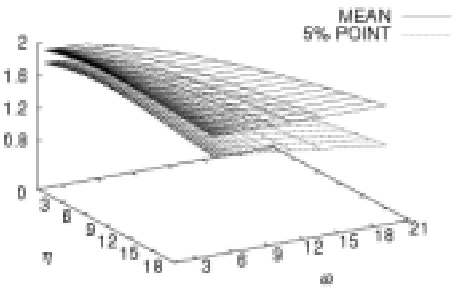
(1) It follows from (a) the graphs of m_{DDCF} and $z_{0.05}$ that as η and ω become larger, m_{DDCF} 's become smaller and the 5% quantiles also become smaller. However, the changes in m_{DDCF} are not very big as a whole.

(2) From (b) the graphs of s and s_- it follows that the risks s and s_- are increasing functions of η and ω. Taking $\eta = 18$ and $\omega = 21$ as an extreme case, s and s_- are about 3 times larger than the case where $\eta = 3$ and $\omega = 3^{1/2}$.

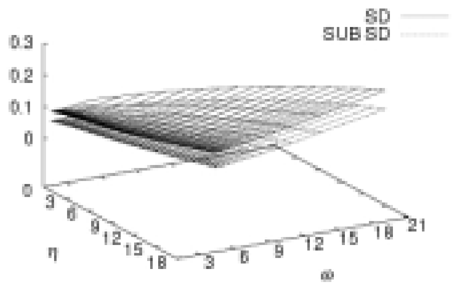
(4) Figure (c) denotes the graph of $skew$. It is observed that as η becomes larger for ω fixed, the skewness decreases from positive values to negative values. This may be interpreted as follows. When the mean η of the tenant discovery time is large, depending on how soon new tenants are found out by chance, DDCF values spread out widely. Since the upper bound is set for the cash flows as $N = 240$, DDCF values tend to spread downward when η is large. This fact is more clearly revealed when ω is large for η fixed.

(5) Figure (d) denotes the case of kurtosis. Letting ω get larger for η fixed, the kurtosis increases rapidly. This tendency is greater when η is smaller. In fact, the distribution of the tenant discovery time is more skewed or spread out to the right when η is small and ω is large, which leads to a fatter tail of the DDCF distribution.

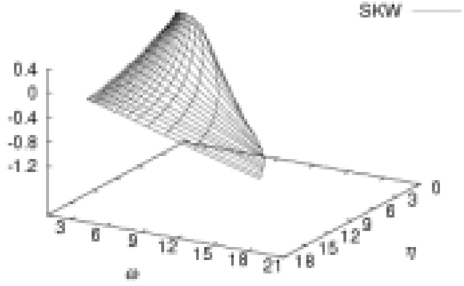
(6) From Figures 4-4(1)(e) and (f) it follows that expected shortfall es and risk premium rp are almost linearly increasing in (η, ω). Both measures are rather stable with respect to changes of (η, ω). For example, even in the case of η=18 and ω= 21, the risk premium is approximately 0.06, while in the case of η=3 and ω= 3 it is 0.02.



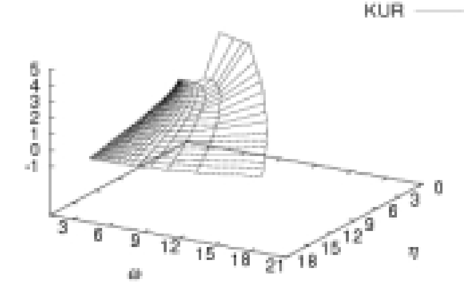
4-4(1)(a) m_{DDCF} and $z_{0.05}$



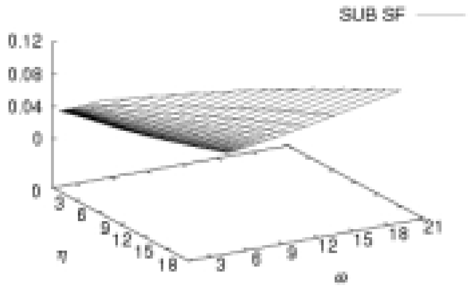
4-4(1)(b) s and s_-



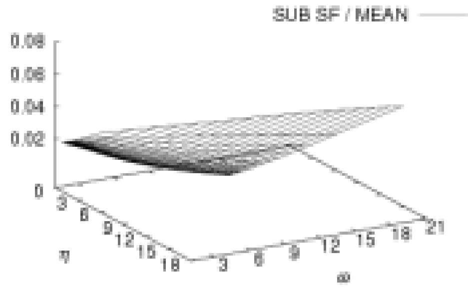
4-4(1)(c) skewness



4-4(1)(d) kurtosis



4-4(1)(e) expected shortfall



4-4(1)(f) risk premium

Figure 4-4(1) Case $\sigma = 0.02$

In Figure 4-4(2) the case $\sigma = 0.2$ is treated. The results are quite different from those in (1).

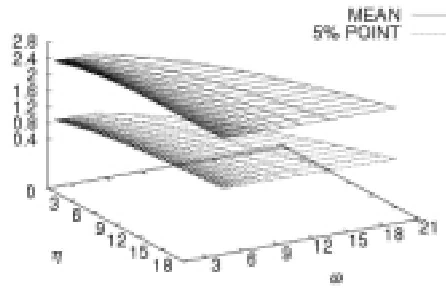
(1) By (a) it is found that the DDCF mean values are larger than the case in (1), though the levels are again rather stable irrespective of (η, ω). Also the 5% quantiles are larger than the case in (1) and increase as (η, ω) increases. This implies that the risks measured by the quantiles are smaller than the case in (1).

(2) From (b) it is observed that the standard deviations and lower standard deviations become larger than the case in (1) as (η, ω) increases.

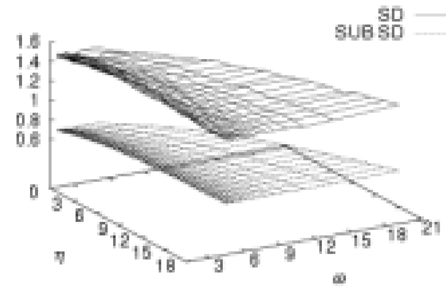
(3) From (c) and (d) it follows that the skewness and kurtosis of the DDCF distribution behave similarly to the case in (1) but more greatly.

(4) The expected shortfall in (e) is decreasing in (η, ω), which contrasts with the case

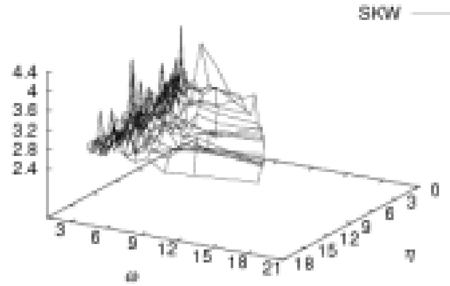
of $\sigma = 0.02$. While, the risk premium in (f) is slowly increasing in (η, ω) and it is stable. In the case of η=18 and ω= 21, it is approximately 0.225, while in the case of η=3 and ω= 3, it is 0.21. This fact together with the fact in the case of $\sigma = 0.02$ will suggest that the volatility of the rent process will be the first approximation to the risk premium.



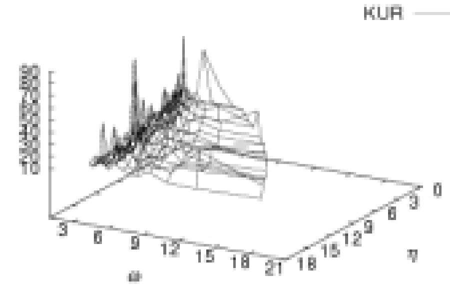
4-4(2)(a) m_{DDCF} and $z_{0.05}$



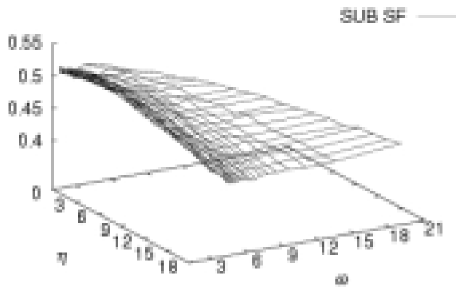
4-4(2)(b) s and s_{-}



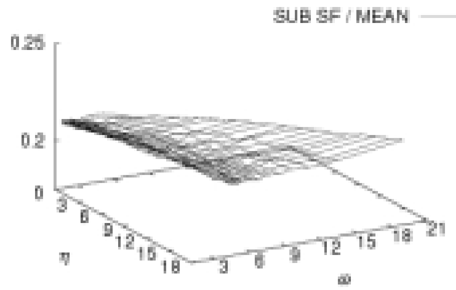
4-4(2)(c) skewness



4-4(2)(d) kurtosis



4-4(2)(e) expected shortfall



4-4(2)(f) risk premium

Table 4-4(2) Case $\sigma = 0.2$

5. Valuation of a whole Building and Exit Strategy

In the Sections 2 through 4, a focus is placed on the valuation of a rental space, which can be regarded as a valuation of a whole building if the argument is applied to a whole building without distinguishing the differences of rental spaces of the building. However, when the differences in the rental spaces are taken into account in the valuation, we need to modify the result as follows. Let

(5.1) $$F_{01}(0:h) \qquad (h=1,\cdots,H)$$

be the DDCF random variable of the h-th space in the building, where $h=1,\cdots,H$. Then the DDCF variable of the whole building itself is the sum of these variables.

(5.2) $$F_{0total}(N) = F_{01}(0:1) + \cdots + F_{01}(0:H)$$

Hence the DDCF mean value of the whole building is obtained as the sum of the DDCF mean value of each space;

(5.3) $$f_0(0,N) = E_0[F_{0total}(N)]$$

However, risk measures such as expected shortfall, standard deviation, etc. depend on the dependent structure of the random variables in (5.1). This dependency may come from the attributive or hedonic structure of the spaces in the building as well as from a portfolio structure of tenants which includes factors associated with industrial business cycles, competition or complementary relation, etc.

Next let us consider a valuation problem of a real estate in which a sell-out option as an exit strategy is included. In this case a capital gain from a sell-out in addition to the income gains as rents is also an object of valuation. Here a capital gain at a future time n is defined as

(5.4) $\Delta_n =$ the price of a real estate at n

− the DDCF mean value at n,

If this value is positive, or equivalently to say, "in the money", it will be better to sell the real estate out at n. Hence it is appropriate to include the value of this sell-out option in valuation. For this purpose, let

(5.5) $$\tilde{V}(n) = \tilde{V}(n-1)\exp[\mu_{vn-1}h + \sigma_{vn-1}\sqrt{h}\varepsilon_{vn}]$$
$$\varepsilon_{vn} \quad iid \quad N(0,1)$$

be a price process of a real estate in question. Here one may use the specifications in (3.2) and (3.3) for drift μ_{vn-1} and volatility σ_{vn-1}. However, in this case it is required to take into consideration the correlation between ε_{vn} in (5.5) and ε_n in (3.1)

$$Correl\ (\varepsilon_{vn}, \varepsilon_n) = \rho_{n-1}.$$

This is because when the price of a real estate is high, rents tend to be high.

In this set-up, a sell-out option over a time horizon [N_1, N] may be defined as

$$\max_{N_1 \le n \le N} \max(\Delta_n, 0) \tag{5.6}$$

If no such a option is included, the DDCF value at 0 is given by $f_0(0, N)$ in (5.3). To value of this option in valuation, let $F_{mtotal}(n)$ denote the DDCF random variable at m (not at 0) of valuing the stochastic rental cash flows over time horizon $[m, n]$. Also replace the discount factors described in (2.6) and (2.7) by $D_m(\cdot)$'s as those discounting future cash flows to values at m.

In this notation, we shall show that when a sell-out option only at n with $N_1 \le n \le N$ is included in valuation, then the DDCF mean value at 0 is given by

$$f_0(0, N) + \max_{N_1 \le n \le N} u_0(n, N) \tag{5.7}$$

where $f_0(0, N)$ is given by (5.3) and

$$u_0(n, N) \equiv E_0\{U(n, N) D_0(n)\} \quad \text{with}$$

$$U(n, N) = \max(V(n) - F_{ntotal}(N), 0)$$

In fact, since the sell-out option at n is expressed as an exchange option with payoff.

$$\begin{aligned} \max(V(n), F_{ntotal}(N)) &= \max(V(n) - F_{ntotal}(N)) + F_{ntotal}(N) \\ &\equiv U(n, N) + F_{ntotal}(N), \end{aligned} \tag{5.8}$$

this option is valued as

$$E_0\{[U(n, N) + F_{ntotal}(N)] D_0(n)\} \equiv u_0(n, N) + f_0(n, N).$$

On the other hand, the DDCF mean value at 0 of cash flows from 0 to $n-1$ is given by

$$f_0(0, n-1) = E_0[F_{0total}(n-1)]$$

Therefore the DDCF mean value at 0 with the sell-out option at n is given by

$$f_0(0, n-1) + f_0(n, N) + u_0(n, N).$$

Here since it follows from a no-arbitrage argument that

$$D_n(m) D_0(n) = D_0(m)$$

we obtain

$$f_0(0, n-1) + f_0(n, N) = f_0(0, N).$$

Consequently, since the sell-out option is in fact given over time $N_1 \le n \le N_2$, the valuation formula (5.7) obtains.

This value is obtained via simulation as has been shown in Section 4, though the volatility and the smoothing parameter and the correlation parameter need to be given in simulation.

6 Conclusion

In this paper a stochastic framework of valuing a rental real estate such as office building together with valuation of risk is formulated and a specific model analyzing the valuation with risks is proposed. The risks considered here are pre-vacation risk, vacancy risk, rent variation risk and property price variation risk. The basic viewpoint is that the value of a real estate is the DDCF distribution itself, and the mean of the distribution is regarded as a fair representative value and the expected shortfall will be a corresponding measure of risk. In the formulation of the framework and modeling, we took a discrete time approached and used a non-Markovian model for rent and property price. In doing so we have discarded the typical framework used in the continuous option approach. A variety of simulations show that the model will be useful for valuing a rental office building.

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