

Financial Engineering and the Japanese Financial Innovations

Toward Finanssurance

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From a viewpoint of a financial engineering paradigm, we develop an argument on recent movements in finance and insurance in Japan. Our argument is made in view of risk and functional finance for the efficiency of capital. Among others, we foresee a convergence of finance and insurance to “finansurance” in near future, in which households and firms will be able to more efficiently optimize their overall risk-return positions in life-cycle and business. It is also argued that financial technology together with information technology accelerates the trend of the functional finance and will provide products to complete an incomplete system for risk optimization. A theoretical justification is given for a finansurance business. In addition, we demonstrate that a discrete time approach to credit risk analysis in financial engineering makes a bridge between finance and insurance and demonstrate the valuation of the premium of a life insurance and the price of an earthquake bond via the no-arbitrage concept.

In addition, valuation problems in weather derivatives, CBO (collateralized bond obligations), mortgage backed securities, real estate are discussed in association with the Japanese practices.

1 Introduction

To develop an argument on financial innovations in Japan from a financial engineering paradigm, I first define what financial engineering is. **Financial engineering (FE) is in my definition the science of creating thoughts, knowledge and technology on the functional efficiency of finance in view of an efficient use of capital.** Here the concept on the functional efficiency of finance in FE is defined in terms of risk-return optimality in view of capital and will be discussed in detail in Section 2. Such FE need to be an interdisciplinary science associated with economics, accounting, finance, insurance, law, statistics, mathematics, computer science, engineering, etc. It contains such five main areas as (1) Investment, (2) Derivatives, (3) Risk Management, (4) Structured Finance with Legal engineering and (5) Real Options, which will be discussed together with some recent financial innovations in

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Japan in Section 3.

By its definition, financial engineering enables us to make a most efficient utilization of capital and hence it promotes the efficiency of capital allocation and movement in the world via the 3 basic attributes of financial assets and products: risk, returns and time. The fundamental concepts to develop the concept of the functional efficiency of finance in FE are risk and no-arbitrage and the corresponding theories are portfolio theory and no-arbitrage theory that give a scientific basis for FE. Financial technologies implemented by these theories will promote the trend of functional finance together with information technology. Implementation technologies for pursuing the functional finance are naturally means to financial innovations such as the refinement of business concept, the planning of financial systems in government and the development of new commodities and products in financial business. Furthermore they form a basis for the competitive power of financial institutions and a financial system in the international financial system. The technologies require a high level and broad scope of knowledge base in FE and its related fields. In addition, the recent deregulations and the BIS requirement on the capital adequacy in Japan are making the financial industries more exposed to international competitions, through which they will have to pursue the functionally efficiency of finance due to the demands of the efficiency of capital.

Our world economies are getting connected more closely and more complicatedly through the trend of the functional finance with interactions with the development of informational and financial technologies, making the concept of a state or nation more diluted. After all, capital, information and technology, which bring out productivity in finance, will go over sovereignty.

From such a viewpoint of financial engineering associated with the concept of the functional finance thus described, we will discuss on how financial business, in particular, banking business and insurance business, will evolve in future. In fact, in view of capitalism and the FE paradigm of no-free lunch, some types of insurance risks will be better treated with financial risk in the capital market. Besides, the credit risk theory recently developed in FE will make a bridge between credit risk and insurance risk, which have been getting implemented in Japan. This will in turn lower the wall between finance and insurance further. Further the securitization of insurance risk in capital market, as is symbolized in the earthquake bond treated in Section 6, will

promote the stream of a convergence of finance and insurance. Of course, this will be due to the demands for firms' and households' risk optimizations, as has been pointed out by e.g., Shimpi. In our context, this convergence will be understood as a result of the pursuit of the functional efficiency of finance in a system.

In this stream, financial business is now being recognized as risk business (not risky business!) in which risks in broad sense are transformed, transferred, pooled and interchanged for optimalities.

The paper consists of two parts. Part I deals the conceptual development on the evolution of functional finance with some observations and Part II the theoretical formulation of the arguments. In Part I, some recent financial innovations are discussed in view of functionally efficient finance as stated above. In Part II, a theoretical justification is given for a combined business of banking and insurance and an earthquake bond as a finansurance product on earthquake risk is valued by a discrete time approach to the no-arbitrage approach.

In Part II, first the author's result (2000) is introduced. Under an explicit condition, an integrated portfolio of banking and insurance or equivalently a finansurance portfolio is better in asymptotic default probability than any individual portfolio: bank portfolio and insurance portfolio. It can be used to find a strategic position for a finansurance institution and to make a complex product of credit risk and insurance risk.

More specifically, from a viewpoint of the traditional business schemes in banking and insurance, we consider an effectiveness of the combination of banking business and insurance business compared to each separate business. Here we regard banking and insurance businesses as the businesses that respectively make profits by pooling and managing the risks in their loan portfolio and policy portfolio. The effectiveness is evaluated in terms of asymptotic default probability when each portfolio size is large. According to this criterion, the asymptotic default probability of an integrated portfolio is smaller than that of each portfolio under a certain condition.

The concept on which the insurance business relies for pooling and managing risk in its portfolio is a law of large numbers in probability theory. In a large and homogeneous pool of policies or contracts where the probability of the occurrence of an accident is constant, the frequency of accidents relative to the size of policies is regarded as close to the constant. Hence so long as the rate of premium is set more

than the probability of the occurrence, the probability that the collection of premiums received is bigger than the loss paid goes to one as the portfolio size gets large.

In the remaining part, an earthquake bond is valued for a demonstration as an application of a discrete time credit risk analysis and we discuss some related problems.

2 Financial environment and some movements in Japan

2.1 A brief summary on recent financial environment in Japan

As is well known, the Japanese society is basically or at least traditionally a harmonious community governed by the social norms and orders. The social norms are so stubborn that the three co-governing bodies of Japan: politicians, bureaucrats and management people, known as the “iron-triangle”, all believed and still believe to some extent the existence of Japanese own economic system and have been relying on their judgments and regulations, not on the capitalistic mechanism of the dynamic process of capital associated with the dynamic evolution of people’s preferences. What was missing in our society is the concept of risk not only in financial management but also in the management of our country. The Government was basically the body absorbing all the risks of the private sectors, which was possible in the upward trend economy. Naturally it led us to an inefficient and high cost society and hence to the loss of economic competitive power. We also wasted the accumulated capital through the bubbling economy. With severe pains from the long-term depression after the crash of the bubble, which were mostly caused by the wrong policies of our Government, we are now gradually recognizing the importance of economic functions of capital and capital market. With his strong determination Prime Minister Koizumi is now trying to take “what he says all possible means” to make a structural reform of our economy with a view of market economy. But we have already accumulated a huge deficit in the government budget that the future generations have to pay, those firms which declared a few years ago to stick to a Japanese employment system are now restructuring themselves, unemployment rate is rising and our economy is filled with a deflationary sentiment. Hence the politicians, firms and people who favor the traditional system are again demanding the Government to spend more, to take risk more and to take an inflation-target policy. We are now in the mid of a turning point of our history

It was Prime Minister Hashimoto that gave a strong clue to a dramatic change for reformation and deregulation in 1996, known as the Big Bang in Japan. Since his

approach to reformation seemed too abrupt and affirmative and it led to a financial crisis though it was not necessarily his fault, it was not well accepted by people and the iron-triangle. But it made a significant effect on financial industries. The Government had to stop keeping the strong protection for financial industries that it transferred the risks of the industries into people and other firms through taxes and other means. This made the industries more exposed to domestic and international competitions, through which they now have to make business in view of the functional efficiency of finance due to the demand of the capital efficiency.

After the financial crisis in 1997, some big financial firms merged and allied. In the areas of banking and life insurance, in October 1998, the Industrial Bank of Japan and the Dai-ichi Life Insurance made a comprehensive or total alliance for future business. This is a notable movement in the Japanese financial and insurance history. The institutional walls between the financial industries, i.e., banking, securities, insurance, commodities, used to be highly built up for segregation not only by the Government but by each industrial organizations. They had been in fact against any change for the segregation. However, the depression following the crash of the bubble and the BIS requirement on the capital adequacy made a significant change on the situation. Now there are four big financial groups in a line with the corresponding four banking groups: Mizuho, Tokyo-Mitsubishi, Mitsui-Sumitomo, and UFJ. Nomura Securities and Nippon Life Insurance Company, the two giants, are taking each independent line, though they have some allied relations with other financial firms in many ways.

2.4 Brief History on Financial Engineering

Before we discuss on functional approach to finance, let us review some notable movements in financial engineering in the US and in Japan.

In 1972, the CME (Chicago Mercantile Exchange) introduced the foreign exchange futures and options. In 1973, Black and Scholes established the Stock Option Theory (Foundation of Financial Engineering), which was the start of the New Financial Paradigm, i.e., No-Free Lunch Paradigm, filling the vacant spot of the traditional equilibrium theory in finance. In 1980's, financial deregulations were significantly promoted under the Regan's regime and the developments and practices of financially engineered business were made with interactions between the industry and academia. In 1981 Harrison and Pliska reformulated the no-arbitrage theory with a deep insight into

the mathematical structure and developed a Martingale approach to the no-arbitrage, which made a significant impact on the development of FE. In 1990, H. Markowitz, W. Sharpe, and M. Miller shared the economic Nobel Prize, and in 1997 R. Merton and M. Scholes became the Nobel Laureates (Worldwide recognition of Financial Engineering as a science). In 1993 Schachermayer and Delbaen gave a necessary condition for no-arbitrage, which essentially completed the proof of the general theorem on the no-arbitrage.

On the other hand, in Japan the financial institutions and the Government were not willing to admit the importance of the role of financial engineering because the financial institutions and system were important means to keeping the Japanese-style socialistic system. Though interest swaps were made in their international business, they have not introduced new products associated with derivatives till the middle of 90's. In fact, it took 10 years spanning from 1985 through 1995 to make the deregulation of interest rates in saving account. During the period, we had an excess-liquidity problem with regulated interest rates, which is considered a part of the cause of the bubble. In the beginning of 90's there were some information and technology collecting activities through seminars in private sectors. Some famous US professors in finance and financial engineering were invited to deliver some lectures and skills on money management and derivatives in the bubbling economy. In 1993 the Jafee (Japanese Association of Financial Econometrics and Engineering) was founded in collaboration with the financial industry and since then it has been actively promoting the area, which not only made to a greater extent a public recognition on some roles of financial engineering but also made some university professors from the industry. As a business, consulting firms on financial technology appeared. In academia, in 1999 the Tokyo Institute of Technology made a research center and Tokyo University launched a course in FE. In 2000 Kyoto University and Hitotsubashi University launched a course in FE. In addition, Kyoto University started the Research Center for Financial Engineering. In the occasion when the Japanese type REIT(Real Estate Investment Trust) in 2000 got started, the Japan Association of Real Estate Financial Engineering was established to activate the stagnated real estate market and study about relations between real estate market and capital market.

2 Functional Finance

In this section, we discuss on the concept of the functional efficiency of finance on which financial engineering relies.

2.1 Institutional Approach and Functional Approach to Finance

As is well known, there are two approaches to finance or the science of finance: namely,

- (1) Institutional approach
- (2) Functional approach.

In the institutional approach, such institutional system as banking, insurance, securities is the given concept the approach relies upon, hence it cannot have the conceptual and analytical framework to treat the change of the institutional system of these structure, though in reality it is evolving. On the other hand, in the functional approach the analysis is focussed upon the functions of finance and it treats the change of the financial system along its evolution. In this respect, financial engineering takes the functional approach and provides as means not only individual technologies such as pricing methods for various risks or risky products including derivatives but also the conceptual framework for promoting functional finance and developing new concepts for efficiency. The no-arbitrage concept and the associated paradigm, on which the Black-Scholes Option theory is derived, is so insightful and crucial to the essence of financial business. In fact, it will eventually push us to a unification and standardization of the segmented capital markets in the world in terms of the rules of the game, important instruments and systemic risk and to a unification of the segregated industries via functional efficiencies for risk allocation. Another implication of the paradigm is given in Section 5.

The driving force of such functionally efficient finance is the greatly accumulated capital in the world that seeks its efficient use relative to time, cost, risk and return. Naturally capital is a main source of creating future productivity and employment and therefore it has to be used efficiently for future generations. Besides, the concept of efficiency from this viewpoint will lead to a business success in financial industries. In particular, it is important to strategically understand the relations among the functions of finance, the concept of risk and the financial technology for a successful business.

2.2 Functions of Finance and Capital

Next to understand these movements, we discuss on what finance should be to capital.

In the following arguments I share a lot with the views of Professor Robert Merton. Financial products and instruments carry as assets the three basic elements: time, risk, and return. Through financial instruments and products with these elements, the function of finance is to make cashflows between individual households and firms by combining the decisions of consumption and saving with the decisions of production and capital accumulation.

Capital market should promote the efficiency of this function of finance, provide opportunities of risk allocation, risk sharing and risk protection for households and firms and combine internationally diversified decisions efficiently. The efficiency of the capital market is related to the capital cost including the information of the market, trading cost and tax, to the risk involving technology and regulations (rules of the game) and country risk, and to the depth and scope of the market. Financial institutions should exist to promote this function of capital market and should help the capital utilized efficiently. A financial system should exist for the same purpose and not for the controllability of financial policies by government. Regulations based on the political and institutional purpose will delay the development of the financial industries and make the industries lose business competitive power in the international markets. Japan is a typical example for such. Financial technology promotes the efficiency of the functions of capital market and finance together with information technology. It creates more opportunities for risk sharing, decreases trading costs and decrease the agency cost on the asymmetry of information in financial business. Through these opportunities households and firms can attain their far better optimalities in wider choices. Capitalism from a functional perspective should be an open system that is never completed, and hence in its development, it is important to plan it or regulate it as an open dynamic system that should evolve along the need of people and firms.

2.3 Comparisons between the Institutional and the Functional Approaches

On the basis of the above viewpoint, we make some comparisons between the Institutional Approach A and Functional Approach B.

(1) In Approach A, the concept of institutions like banking, securities, insurance etc., is the given concept on which the traditional science of finance relies as its conceptual framework. Prices of financial products tend to be formed based on costs in financial

institutions. While, in B the concept of function is given on which financial engineering relies as its conceptual framework and hence it can treat the change of an institutional system like a convergence of banking and insurance. Prices are formed for risks valued in the markets.

(2) In A, firm-value accounting measures static financial values for value allocation. In risk management, capital is a main source to absorb the risks in a balance sheet. In B, a dynamic aspect of the accounting numbers are more paid attention to with a forward looking analysis and risks are evaluated for an optimal risk-return allocation, which will provide some measures for risk accounting. Risk management is made more based on the concept of risk portfolio and diversification. Hence derivatives are natural means to control risks.

(3) In A, institutional regulations are made from a viewpoint of keeping the institutional system, which typically leads to itemized regulations and a high-cost system. Assets and financial products are legally classified by itemization. While, in B, functional regulations are made from a viewpoint of promoting the functions of finance, leading to comprehensive regulations. Assets and products are classified as composite of risks.

3 Financial Engineering and some recent innovations in Japan

First, from a viewpoint from FE paradigm, we shall briefly observe some actual financial innovations and movements in a big tide of functionally efficient finance and the development of the promoting techniques due to the strong demand of capital efficiency. As stated in Section 1, financial engineering includes **Five main areas**: (1) Investment, (2) Derivatives, (3) Risk Management, (4) Structured Finance with Legal engineering and (5) Real Options.

(1) Investment and asset management

In investment and asset management, market risk is a main object to cope with to generate returns. A main difficulty to cope with the market risk arises from the fact that it includes game-theoretic uncertainty as well as probabilistic uncertainty. Prices change according to the change of behaviors of investors and the behaviors also change when prices change. Hence various approaches and models are used to extract unfound information contained in data. Among others, such sophisticated nonlinear models as threshold autoregressive model, neural network model, chaotic model, etc. are applied

to the construction of portfolios, mutual fund and pension fund products in Japan.

(2) Derivatives

In the area of derivatives, many new products such as credit derivatives, insurance derivatives, credit swap, weather derivatives have been created. These are made possible by the development of risk pricing techniques, by the **demand** of capital for new risk-return enhancement and by the deregulations. However, the Government often mentions the existence of the Gambling Law to intimidate financial institutions. Still derivatives could be intentionally regarded as gambles by this law in Japan. Many recent products are event-triggered derivatives, which may be regarded as an insurance product or a guaranty product. Among others, weather derivatives and earthquake derivatives are notable. For example, a temperature derivative is made as a put on the average value of daily temperatures in a summer period with a fixed exercise temperature, say $24^{\circ}C$. In 2001 Tokyo Electric Power Co. and Tokyo Gas Supply Co., without help of a financial institution, made a risk swap on the average of summer temperatures in Tokyo, where no initial payment was required but either side was obligated to make a certain payment depending on the average. This will mean that non-financial firms are now aware of risks in their business and become capable to evaluate their risks. Recently Tokio Marine & Fire Co. and Swiss Re. also made such a risk swap against typhoon claims in their insurance business. A formal valuation for such a derivative is made possible by the development of the theory on credit risk though there will be a variety of possible models on temperature variations.

As will be discussed in Section 5, the process of such appearance of new products can be viewed as a process of the completion of an incomplete financial system. It is well known that derivatives function as instruments for risk transformation, change of cash flows and payoffs, risk sharing, reduction of cost due to the asymmetry of information and institutional arbitration.

(3) Risk Management

In risk management (3), most Japanese big banks are now in a stage to use an internally developed integrated risk management system that also takes an operational risk into account. Some big banks which developed a credit risk management system sold and are still selling the systems to local banks. Though, the Credit Metrics of J.P. Morgan and the CREDIT RISK+ of the Credit Suisse Financial Products are famous.

Each system seems to describe the loss distribution of a loan portfolio. The default correlations seem to be treated differently in each system.

Financial business is now being recognized as risk business (not risky business!) in which risks in broad sense are transformed, transferred, pooled and interchanged for optimalities.

(4) Securitization and Engineered financing:

Structured finance and securitization is getting popular. They are, among others, CBO (Collateralized Bond Obligations), ABS (Asset-Backed Securities), CB (Convertible Bond) with some options, x-linked bond (or deposit) where x is stock price index, exchange rate, oil-price etc. Also structured insurance such as earthquake bond is also undertaken, which will be discussed in Section 6. The Fuji Securities and the Sakura Securities used the CBO scheme in 1998 and 1999 for some low credit-rated firms to get finance. In the CBO scheme of Fuji Securities, the bonds that the 30 low credit-rated firms issued were used as collaterals to issue the CBO's of the three credit ratings; AAA, mezzanine, no rating where default probabilities of individual firms are transformed into those for the new bonds. The Tokyo Metropolitan Government also used a CLO (Collateralized Loan Obligations) scheme in order for small firms to get finance. This is because due to the bankruptcy of some banks, insurance companies and securities companies in the past 5 years and due to the accumulated bad loans, financial institutions are now very sensitive to credit risks in making their loans and contracts and are subject to the BIS requirements.

(5) Real Options

A real option is an option given to a decision maker with a specific environment and an uncertainty. In particular, a manager in a firm can use some real options to obtain a managerial flexibility and take a strategic position in management. Financial engineering provides some basic concepts to formulate these option problems together with a forward-looking method of analysis and values the options in a similar line with a derivative valuation. For example, a real option valuation method is applied to investment decision problems in development of a real estate or an oil field exploration. The author proposed a dynamic discounted cash flow method for valuing a commercial building with an option of selling it out.

(6) Some other movements

Financial business associated with life-cycle is gradually growing such as mutual

funds, pension, principal-guaranteed products structured with an equity. In October 2001, we now introduced the 401K type pension system in which individuals take market risks. This introduction is due to a crisis of our private pension system because the depression lasted long and it expanded the liability of the firms with the pension system. From March, 2001 on, it is required that the liability of pension funds in firms needs to be listed on the balance sheets in 2001.

4 Financial System as a Competitive Power in International Financial World

From a macroeconomic viewpoint, due to the global nature of capital, which is associated with the reversibility of financial decisions, financial systems in different countries are destined to compete with each other for seeking the functional efficiency of finance. This is because capital is a strategic element for the development of each economy and its logic is very clear and common. Inefficient financial systems fail to generate competitive incomes in financial industries and invite capital inflow for efficiency. In addition, financial decisions between firms and households are not optimal in an inefficient system. Furthermore inefficiencies may be exploited by institutional arbitrage through derivatives. There FE plays a basic role in designing an efficient and innovative system. As has been experienced in the US, there will be three basic elements for competitive and efficient systems.

- (1) Standardized instruments corresponding to different risks
- (2) Deregulated, competitive and standardized markets
- (3) Informative and fair Infrastructure for a financial system

For example, mortgage securities such as GNMA and FNMA in the US are an additional instrument with prepayment risk, which yields return-enhancement and risk-diversification for investors. The instrument is made credit-risk-free by the governmental corporations so that it is of the almost same credit quality as the T-bond. The market is well designed in order that the securities are continuously well supplied and the mortgages collateralizing the securities are well diversified in area and size. Furthermore the information on the prepayment history in each pool of mortgagors is accurately provided for investors in the secondary market. This makes the market liquid and attractive. FE has been supporting the valuation of mortgage securities though the prepayment risk is hard to model. In Japan, some big banks privately issued mortgage

securities on the OTC basis in recent years and hence the default risk is not necessarily well controlled and the diversification of the collateralized mortgages is not well made. In March, 2001, the Government set up an official market based on the mortgage loans made through the Nippon Jutaku Kinyu Kouko (Japan Housing Loan Corporation). But unfortunately information on prepayment behaviors are not provided due to the lack of data and what is worse, the Corporation itself is regarded as an object for Economic Reform under Koizumi' regime though it may happen that it will be made changed as a corporation like GNMA. This makes the market illiquid at present. Besides, the values of the collateralized mortgages might be smaller than the loan values because the land prices went down significantly after the crash of the Bubble. Hence unless the banks takes the credit risk, the investors will have to take a part of it. Assuming the US scheme, theoretical and empirical research on prepayment risk is being done in Japan (e.g., Kariya and Kobayashi (1999)).

The BIS regulations changed the Rules of the Games .

The BIS (Bank of International Settlement) regulation on the capital adequacy has been making significant impacts on the Japanese banking system as well as on the Japanese traditions and social mentality and order in banking business. First of all, the banks are set free in part from the strong bondage to the Ministry of Finance in pursuing their business. Secondly, the current (market) value accounting system has to be adopted, which will take place from 2000 on. This will reveal the current values of the banks and the risk structure. Thirdly, it promotes a trend of quantifying the risks and making objective judgments based on the measurements in banking management, which in turn creates more demands on new financial technologies. It also promotes the disclosure of banks. Fourthly, the local banks are completely separated from the city banks and have to stay as retail banks. Fifthly, the public are made to recognize the default risk of financial institutions and the bankruptcy of some big banks is leading us to forming a deposit payoff system, which will start in 2002. This will lead the public to following the rule of self-responsibility. Finally, the financial institutions in Japan have to seek their own expertized business for controlling risks and stop the business routines of expanding their balance sheets.

5 Finansurance and financial engineering

5.1 Banking and insurance

As is well known, a typical traditional banking business is to get finance via time deposit saving and make loans to firms. In that, a bank manages credit risk in its loan portfolio or equivalently credit risk pool to make a profit from the spread of interests between loan rate and deposit rate. On the other hand, in insurance business, insurance risk is managed in its risk pool of the insured policies to make a profit from the difference between the premiums collected and the loss (damage) actually caused. There the separation or segregation between finance and insurance business is institutionally made on the basis that the function of making finance and the function of providing insurance against risks are regarded as different and separable at least from the side of the suppliers.

However, as the new terms “bancassurance”, “assurebank” and “finassurance” are sometimes used, a trend toward convergence of banking and insurance business is clearly observed in many financial products and business corporations. This trend will correspond to the needs of the firms and consumers that have to inseparably manage financial risk and insurance risk in their value development and life cycles. Merton (1994) and Kariya (2000) discussed the problem in view of the functional approach to finance and insurance and provided clues to understanding the trend as natural .

Gora(1997) provided many actual examples of bancassurance and assurebank in Europe, Canada and USA. According to the author, bancassurance is the case where a bank sells insurance products developed by an insurance company the bank owns, and assurebanking is the reversed case. The author’s basic viewpoint still lies in the institutional approach and he discussed about the merits and demerits of the business schemes of bancassurance and assurebanking.

As Brennan pointed out, a divide between finance and insurance has made insurance segregated from a behavior science as had been in the relation between economics and finance. However, it is clearly observed in a recent business trend that insurance is bridged with finance not only by the demand of people and firms that need a package of financial and insurance products but also by the no-arbitrage asset-pricing theory, developed in financial engineering or more particularly in mathematical finance. The no-arbitrage theory itself may not be a behavior science but includes or is consistent with an equilibrium price theory in economics, because a situation with an arbitrage opportunity is not an equilibrium. In addition, when it prices an asset, it takes into

account its risks involved and plays a role as a risk theory as in pricing a derivative. As a risk theory, it is an individual risk theory, while the insurance theory is a collective risk theory where risks are controlled in a pool by the law of large numbers. Though an insurance contract is not quite a financial derivative, the insurance risks and a portfolio in an insurance firm can be better discussed together with financial risks and a financial portfolio in the framework of financial engineering. As a matter of fact, insurance risk and financial risk cannot always be distinguished clearly and in some areas investment banks and insurance companies provide similar products such as, for example, weather derivatives, event-triggered finance. Now finance and insurance will be simultaneously treated in terms of optimal risk allocation. In other words, finance and insurance will better be combined in view of optimal allocation of capital over risks.

After all, households and firms are required to manage all risks together with returns as portfolios in their time horizons in the globalized world through many channels. Another point we should make on a convergence of finance and insurance is that a structured securitization scheme accelerates the trend of the convergence and capital market is now a market for exchanging all risks including insurance risks and enterprise risks. In other words, it can be regarded as a book for pooling all the risks concerned. An example is a securitization of an earthquake risk, which will be discussed in detail in Section 6.

5.2 Implications of No Free Lunch Theory and Insurance

The no-arbitrage concept is the fundamental concept of valuing relative prices of financial assets and products and it gives a means to producing strategic concepts for business, to developing financial products associated with the concepts and to developing financial systems in promoting the functional efficiency of finance. This is an economic implication of the Black-Scholes mathematical result. The result implies a new recognition about risk and its price. In fact, this implication is confirmed by Harrison –Pliska (1981), where the concept of the no-arbitrage is exploited in terms of relative prices of assets and products via replicability. Since the no-arbitrage concept is a broad concept including the equilibrium concept in economics and the theory based on the concept need not assume formal behaviors of economic agents in pricing the values of assets, it is so understandable and powerful in implementing the result in real world.

The theory states that the prices of financial assets and products are mutually adjusted to exclude arbitrage opportunities, when the market is perfect and complete. Here the perfectness of a market is that there is no frictional cost such as trading cost and tax and any financial position of portfolios of the assets can be taken including arbitrarily large short positions. This is not a realistic assumption. But when a market is very far from the assumption, the concept and the theory enables us to make an arbitration between markets or between institutions in different countries through derivatives or structured schemes. Hence an inefficient system has to adjust itself toward a more efficient one, leading to a standardization. What is more important is that the incompleteness due to the lack of sufficient instruments and products for important risks is eventually completed. The appearance of many new products is viewed as this process of the completion of an incomplete system. Among others, many derivatives appearing in the area of insurance risks can be regarded as such a process. When capital is accumulated and globalized and when the level of life is improved, firms and households are more willing to optimize their risk positions for protection of an achieved life and a business level in a more accurate manner.

And the completion of an incomplete system due to additions of risk products is in fact in a process of making firms and people in the world form comprehensive and optimal risk positions over all risks significant to them. The process of course includes the area of insurance risks and makes the agents better off in business and life-cycle in their dynamic and integrated risk management.

6 Capital market pooling risks

6.1 Credit Risk Analysis Makes a Bridge between Finance and Insurance

The no-arbitrage theory on credit risk analysis enables us to make a bridge between finance and insurance. Risk factors behind an insurance risk such as an automobile accident or a fire accident are in general hard to identify uniquely and thus the market is incomplete in views of risk factors. However, the framework of the continuous time theory does not allow us to use many insurance or statistical models for such risk factors, because the approach requires a semi-Martingale framework, limiting models we can use.

To overcome this situation, I have been proposing a discrete time approach to valuing assets. The theory keeps the no-arbitrage concept with self-financing rule but discards

the replicability of a payoff or price. Then a sufficient condition for the no-arbitrage is that the relative prices are martingale under an equivalent measure, which is the same as in the case of continuous time setting. The theorem is given in Appendix for reference. The loss by doing so is that we lose the concept of hedgeability through replication. But the gains are great. Among others, the class of models we can treat in the discrete time theory is much broader including semi-martingale models. Some models describing insurance or prepayment risks may not be in the semi-martingale and then they can not be treated in the continuous time framework. For example, a model for describing the variations of temperature or the prepayment behaviors in the mortgage loans will be such and hence the discrete time theory is useful for valuing these derivatives and assets. Then the pricing techniques in insurance can be viewed as a special case of the no-arbitrage pricing techniques though the market is incomplete in the sense that long and short positions on the insurance risk are limited.

6.2 Earthquake bond

Until 1997 no insurance company has ever directly provided an insurance against earthquakes. It is because not much data about the occurrence of catastrophic earthquakes has been accumulated and because the reinsurance markets for such catastrophic events are not big enough relative to the sizes of the losses caused by the rare events. Swiss Re Capital Markets and Goldman Sachs came in in 1997, structured the earthquake risk into earthquake bond and sold them in the US capital market. The investors in the capital market can and in fact are willing to take the risk in view of its coupon so that their portfolios are more diversified and more enhanced as the earthquake risk in Japan is independent of any other risks in their portfolio. Thanks to the scheme, the Tokio Marine and Fire Insurance Co. can sell the insurance and those who need it enjoy the secured life. The Mitsui Marine and Fire Insurance Co. also sold earthquake insurance under a slightly different scheme though the Swiss Re again helped it. The Tokyo Disneyland utilized the scheme to protect their asset against earthquake risk. Now we intermediate the risk by derivatives.

What is important is that the catastrophic risk is taken in capital markets, not in the industry of insurance. The capacity of the world capital market is big enough to absorb various risks so long as financial products correspond to the need of investors. The investors, on the other hand, have different capacities for taking risks, which depend on

the risk structure of their portfolios together with risk-return preferences. In the case of the earthquake bond, the collateralized principal is managed safely in the cash market by an SPC (or SPR). Hence in this case the amount of the principal is shifted to the cash market, meaning that there will be neither inflow of new money into the capital market nor outflow from the capital market till a specified earthquake occurs. If the earthquake risk is pooled in the insurance industry, insurance companies have to make reserves of premiums for having the risk, which will be managed safely in the cash market. Hence no essential difference between the two schemes is basically observed in the capital market so long as no earthquake occurs, though the prices may be different. A difference is that the world capital market is of a great capacity to pool, share and transfer risks.

Another notable feature in this example is that nobody knows about the default (occurrence) probability of the earthquakes in details. Usually insurance products are constructed on the law of large numbers for each specific risk that repeats in a period, i.e., on the collective risk theory. But when a transparent scheme is given by a third party with a fair analysis, the cat risk or rare event risk that is not necessarily based on the law of large numbers can be transformed into products in the capital market. This function of the globalized capital market will be paid a greater attention to for risk optimization of firms and households. In addition, some products mixed with different risks in finance and insurance are recently produced in the capital market, leading us to a way to finansurance.

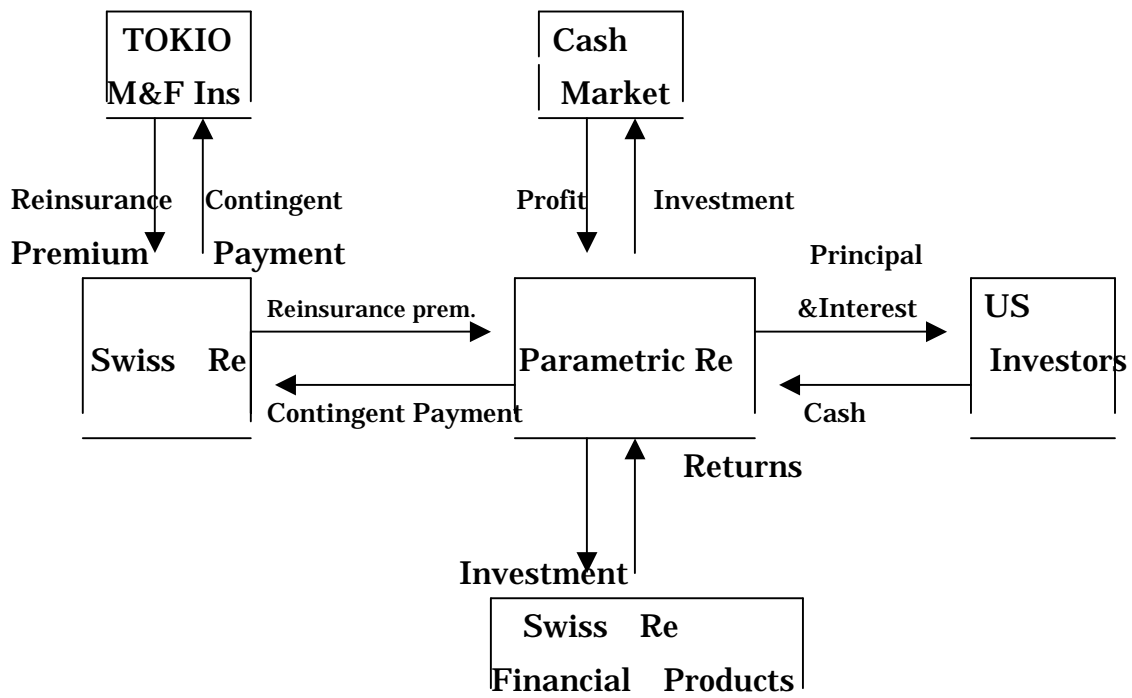
The Structure of the Earthquake Bond (see Shimpi)

Swiss Re Capital Markets and Goldman Sachs placed \$100 million of Catastrophic Bonds in November 1997.

- (1) Sponsor: Tokyo Marine/ Swiss Re
- (2) Issuer: Parametric Re (SPR, Cayman)
- (3) Risk: Japanese Earthquake
- (4) Cover: \$90 million
- (5) Term: 10 years
- (6) Loss Index: JMA (Japan Meteorological Agency) reported EQ magnitude

Bond Structure:

- A \$80 Million Note: LIBOR+430bp, Ba2 Moody's
 Graduated loss starting at 25% for JMA 7.1 and ending at 100% for JMA 7.7
- B \$20 Million Units: LIBOR+206bp, 50% Notes 50% Guaranteed Principal
 Baa3 Moody's



Part II Theoretical Justification for Finassurance

In this part, we first introduce a result in Kariya(2000) to demonstrate a theoretical justification for a finassurance. Then a valuation formula is given for an earthquake bond as an application of credit risk analysis in FE.

7 Combination of banking and insurance accounts

We only treats one-period model for simplicity of arguments. A dynamic extension is left open for complication of modeling. Our result here states that under a condition on default probability of loans, deposit rate, loan rate, premium rate, accident rate, and return rate for premiums, an integrated portfolio of banking and insurance businesses is more effective in asymptotic default probability than any of a bank portfolio and an

insurance portfolio. Consequently the combination of banking and insurance will be theoretically justified. In addition, the condition can be used to find a strategic position for a finansurance institution with loan rate, deposit rate and insurance premium as control variables for default probability.

As has been discussed in Section 1, in the traditional insurance business a pool of insurance risks in a portfolio is managed relative to its reserve of premiums so that the business as it stands can be continued without default provided it is well managed. The theoretical concept on which the scheme of creating the function relies is the law of large numbers in probability theory. To develop our arguments in finansurance, let us briefly state the relevant point and introduce some notation we will use below.

Let the present time be denoted by 0 and consider one period model over $[0,1]$. Supposing a homogeneous insurance population for a specific risk, let n be the size of policies or equivalently the number of the objects (persons) to be insured in an insured portfolio and let \tilde{X}_i denote the claim amount when the i -th insured object gets an accident. Here we assume that each maximum claim for X_i is bounded by policy as $0 < X_i < A$. Then the total loss possibly caused in an insurance portfolio is expressed as

$$(7.1) \quad X = \tilde{X}_1 L_1 + \dots + \tilde{X}_n L_n ,$$

where L_i is the indicator function of the insurance event of the i th insured, defined by

$$(7.2) \quad L_i = \begin{cases} 1 & \text{if the } i\text{-th insured gets an accident ,} \\ 0 & \text{otherwise} \end{cases}$$

We call L_i the accident (event) generation function of the i th object. Here the losses $\tilde{X}_1, \dots, \tilde{X}_n$ are assumed to be independently and identically distributed (*i i d*) and the event generation functions L_1, \dots, L_n are also assumed to be *i i d*.

We here only introduce the following simplified one period insurance problem. Though it is deleted later, we assume that when an accident occurs in the period, a fixed amount A is paid without respect to the size of its damage or loss. This assumption corresponds to the assumption of 100% damageability in insurance. Then in the above setting the problem of controlling the probability that an insurance portfolio (or

company) gets defaulted against a homogeneous insurance population is regarded as the problem of controlling the probability that

$$(7.4) \quad U_2 = AY_2 - n_2x(1+r_2)A$$

is positive. Here x is a premium, r_2 is a rate of interest for the premium reserve, $n_2 \equiv n$, $L_{2i} \equiv L_i$ and

$$(7.5) \quad Y_2 = L_{21} + \dots + L_{2n_2}$$

is the total number of accidents possibly occurring in one period for the portfolio. The first term AY_2 of the right side in (3.1) is the total loss amount the insurance company has to pay at time 1. The second term is the premium revenue n_2xA received at time 0 plus interest received at time 1 under the rate r_2 . Therefore $U_2 > 0$ means a default for the insurance portfolio.

Now to consider a convergence problem of finance and insurance, we first define the default of a loan portfolio in banking for a homogeneous credit risk population where each loan size is assumed to be A . Corresponding to the default problem in insurance, the default of the loan portfolio is defined as the situation that

$$(7.6) \quad \begin{aligned} U_1 &= n_1A(1+r^*) - A(1+r_1)(n_1 - Y_1) \\ &= A(1+r_1)Y_1 - n_1A(r_1 - r^*) \end{aligned}$$

is positive, where r_1 is a loan interest rate, r^* is a saving rate and

$$(7.7) \quad Y_1 = L_{11} + L_{12} + \dots + L_{1n_1}$$

is the total number of defaulted loans in the total size n_1 of the loan portfolio. Here it is assumed that the recovery rate of a defaulted loan is zero and the default generation functions L_{1j} are independent. The first term of the right side of (7.6) in the first equality is the amount the bank has to pay back with interest at time 1 and the second term is the remaining amount when Y_2 firms get bankrupted during the period. The second equality of (7.6) means the difference between the total loss and the revenue from the interest spread $r_1 - r^*$. Hence the default problem is regarded as the problem of controlling the probability that $U_1 > 0$.

In the argument below we view the convergence problem as the problem of finding a condition for an effectiveness of the combined portfolio of the insurance risk portfolio and the credit risk portfolio. To treat this problem, first note that Y_j 's are

independently distributed as binomial;

$$(7.8) \quad Y_j \sim B_i(n_j, p_j) \quad p_j = E[L_{ji}] \quad (j=1,2),$$

where p_j is the default probability of individual firm ($j=1$) or insured object ($j=2$) in one period. Hence as is well known, they are approximated as normal ;

$$(7.9) \quad Z_j \equiv (Y_j - n_j p_j) / (n_j \sigma_{jj})^{1/2} \sim N(0,1) \quad \text{with} \\ \sigma_{jj} = p_j(1 - p_j).$$

Using this fact, we evaluate the default probabilities of individual portfolios and the integrated portfolio approximately.

Default probability of a bank portfolio

To state our result, let

$$(7.10) \quad m_1 = n_1 / (n_1 + n_2), \quad v_{11} = (1+r)^2 \sigma_{11} \quad \text{and} \\ u_1 = (r_1 - r^*) - p_1(1+r_1).$$

Here u_1 is the spread of loan and saving rates when the default probability p_1 of an individual firm is taken into account, which we shall call the default-adjusted spread or simply adjusted bank spread . Here $u_1 > 0$ is equivalent to $(r_1 - r^*) / (1+r_1) > p_1$, which is satisfied when p_1 is small. We assume $u_1 > 0$.

It is easy to see that when u_1 is negative, the default probability is larger than 1/2. This implies that a bank cannot form a loan portfolio unless the discounted spread of loan and saving rates is bigger than the default rate p_1 .

Also let

$$(7.11) \quad m_2 = n_2 / (n_1 + n_2), \quad \text{and} \\ u_2 = (1+r_2)x - p_2.$$

Here u_2 is the difference between the insurance premium $(1+r_2)x$ viewed at time 1 and the default rate p_2 of an insured object, which we shall call the insurance spread. Again when $u_2 \leq 0$, the default probability of an insurance portfolio becomes greater than or equal to 1/2. Hence we assume $u_2 > 0$.

Assume that n_1 and n_2 are large and satisfy

$$(7.12) \quad m_i \rightarrow \delta_i \text{ (constant)} \quad (\min(n_1, n_2) \rightarrow \infty).$$

Theorem7.1 Assume (7.12). Then a necessary and sufficient condition for the

integrated portfolio to be asymptotically more effective than any of the portfolios is

$$(7.13) \quad 1 - \left(\frac{m_2 \sigma_{22}}{m_1 (1+r_1)^2 \sigma_{11} + m_2 \sigma_{22}} \right)^{1/2} < \frac{m_1 u_1}{m_1 u_1 + m_2 u_2} < \left(\frac{m_1 (1+r_1)^2 \sigma_{11}}{m_1 (1+r_1)^2 \sigma_{11} + m_2 \sigma_{22}} \right)^{1/2},$$

where

$$u_1 = (r_1 - r^*) - p_1(1+r_1), \quad u_2 = (1+r_2)x - p_2, \quad \text{and} \quad m_i = n_i / (n_1 + n_2)$$

are respectively the bank spread, the insurance spread and the proportion of the individual portfolio sizes in the integrated portfolio.

The condition is of course a condition on the set

$$(r_1, r_2, p_1, p_2, r^*, x, m_1, m_2).$$

Hence when the portfolio sizes n_1 and n_2 and default rates p_1 in banking and p_2 in insurance are given, this condition gives a restriction on the set $\{r_1, r_2, r^*, x\}$, each of which is the component of the prices {loan rate, return rate for premiums, insurance premium} of the finanssurance business. A strategic choice of a point in this set will give a finanssurance institution a better portfolio position.

Now we delete the assumption of 100% damageability. In the case of non-life (damage and casualty) insurance, when an accident occurs, the full payment of the insurance is not paid, but only the loss caused by the damage is paid with a maximum level A . To treat this general situation, let

$$(7.14) \quad w_i = X_i / A.$$

Also, in the case of loan, when a firm gets bankrupted, some portion of the loan made to the firm is recovered. Hence let the loss rate be w_i , or equivalently let $\tilde{X}_i = w_i A$ in (7.14). Then in either case, w_i is a random variable on $[0, 1]$ and U_1 and U_2 are expressed as

$$(7.15) \quad \begin{aligned} U_1 &= A(1+r_1) \sum_{j=1}^{n_1} w_{1j} L_{1j} - n_1 A(r_1 - r^*), \quad \text{and} \\ U_2 &= A \sum_{j=1}^{n_2} w_{2j} L_{2j} - n_2 x(1+r_2) A. \end{aligned}$$

Under the assumption of the independence of $\{L_{ij}\}$ and $\{w_{ij}\}$

$$(7.16) \quad E[w_{ij}L_{ij}] = \xi_i p_i \equiv \mu_i, \quad \text{with} \quad E[w_{ij}] = \xi_i,$$

$$\text{Var}(w_{ij}L_{ij}) \equiv \sigma_{ii} = E(w_{ij}^2)p_i(1-p_i).$$

Consequently replacing p_1 by μ_1 in (7.7) and (7.8), p_2 by μ_2 in (7.10) and (7.12), σ_{ii} in (7.6) by σ_{ii} in (7.16), and the spreads u_1 and u_2 in (7.8) and (7.11) by

$$(7.17) \quad u_1 = (r_1 - r^*) - p_1 \xi_1 (1 + r_1) \quad \text{and}$$

$$u_2 = (1 + r_2)x - p_2 \xi_2,$$

respectively the argument made above holds as it stands.

Theorem 7.2. In a general case with a common level A for loan and insurance, the condition (7.22) is necessary and sufficient for an integrated portfolio to be more effective, where σ_{ii} and u_i are defined by (7.16) and (7.17)

8 Credit Risk Management in Banking and Securitization

In this section, we briefly treat how the credit risk in a loan portfolio is managed and how the method is applied to securitization.

The loss distribution caused from defaults in loans is basically described as the distribution of

$$(8.1) \quad R_N = A_{1N}(1 - \gamma_{1N})L_{1N} + \dots + A_{MN}(1 - \gamma_{MN})L_{MN}$$

where A_{in} is the loan exposure of the i -th firm at n , γ_{in} the (random) recovery rate of the i -th firm in default at n , which may be replaced by the expected value $\bar{\gamma}_{in}$, and L_{in} the accumulated default generation process with

$$(8.2) \quad L_{in} = \begin{cases} 1 & \text{if the } i\text{-th firm default up to or on } n \\ 0 & \text{otherwise} \end{cases}.$$

When A_{in} and $\bar{\gamma}_{in}$ are given, the distribution of R_N can be in principle derived if the joint distribution of $\vec{L}_N = (L_{1N}, \dots, L_{MN})$ is specified. If the defaults of the firms are independent, the distribution becomes the product of binomial distributions. The problem here is how to specify the correlations of the defaults and how to compute the probabilities of R_N for possible outcomes when the number of loans is large. The correlations are often associated with the ratings of the firms and the industry factors.

On this point, there are a variety of formulations including the CREDIT RISK+. In life insurance, the distribution due to claims can be specified as in (8.1) with $\gamma_{in} = 0$ and A_{in} fixed, though it is often described through Poisson process. This approach can be used in the ALM of life insurance companies, and is used in the risk management by the PBGC (Pension Benefits Guaranty Corporation). Furthermore, it is a basis for CBO or CLO in structured finance.

9 Valuation of an earthquake bond described in Section 6

To value the bond A at n in Section 6, let $n + m_j$ be the time points at which cashflows are generated, and let $C_n(m_j)$ be the cash flow at $n + m_j$, where $j = 1, \dots, M$. Assume that when an earthquake of magnitude greater than or equal to 7.1 occurs in the period $I_j = ((n + m_{j-1})h, (n + m_j)h]$, the corresponding payoff is made at $n + m_j$. Let $\{L_n\}$ be the default (earthquake) generation process, $\{\gamma_n\}$ the recovery process linked with the size of the magnitude and $\{r_n\}$ an interest rate process and assume that these processes are independent. Then the payoff at $n + m_j$ is described as

$$G_n(n + m_j) = C_n(m_j)(1 - L_{n+m_j}) + 100\gamma_{n+m_j}(1 - L_{n+m_{j-1}})L_{n+m_j}$$

where

$$C_n(m_j) = 100(l_{n+m_j} + 0.043)(m_j - m_{j-1})h + 100e_{jM}$$

with $\{l_n\}$ the LIBOR process and $e_{jM} = 1$ if $j = M$ and 0 otherwise. Therefore in the same way as above, the earthquake bond with maturity N is valued as

$$V(n, N) = \sum_{j=1}^M E_n[d(n, n + m_j)G_n(n + m_j)].$$

Here the conditional expectation is

$$100 E_n[l_{n+m_j} + 0.043]d(n, n + m_j)(m_j - m_{j-1})h Q(J > n + m_j) + D(n, n + m_j)\{100e_{jM} Q(J > n + m_j) + \bar{\gamma}_{n+m_j} Q(n + m_{j-1} < J \leq n + m_j)\}$$

where J is the first time to the default. In such a problem, the processes of $\{L_n\}$ and $\{\gamma_n\}$ are hard to specify and they will be out of the semi-martingale framework.

10 Application of the technique to valuing life insurance.

Let $h = 1/12$ year be the time unit for analysis and let $n = 0, 1, \dots, N$ be monthly time points with $n = 0$ the contract month and $n = N$ the maturity. Also let A (yen) denote the amount of insurance for death and $J(y)$ the default time to death where y denotes the age of the insured. Then the default generation process for $J(y)$ is defined by $J(y)$ as

$$(10.1) \quad L_n(y) = \begin{cases} 1 & \text{if } J(y) \leq n \\ 0 & \text{otherwise} \end{cases}$$

just as in credit risk analysis. We assume that when $J(y) = n$, the amount A yen is paid at $n + 1$. Then the payoff of an insurance company at n is described as

$$(10.2) \quad \begin{aligned} U_n(y) &= x(1 - L_n(y)) - AL_{n-1}(y)(1 - L_{n-2}(y)) \\ U_{N+1}(y) &= -AL_N(y)(1 - L_{N-1}(y)) \end{aligned}$$

where x is the monthly premium. To value the premium x , let $\{r_n\}$ be the process of monthly interest rates and let

$$(10.3) \quad d(m, n) = B_m / B_n \quad \text{with } B_m = \exp\left(\sum_{j=0}^{m-1} r_j h\right).$$

Then the no-arbitrage value of the payoff (10.2) at 0 is given by the martingale condition on $\{U_n(y) / B_n\}$;

$$(10.4) \quad u_n(y) = E_0^*[d(0, n)U_n(y)]$$

where $E_0^*(\cdot)$ is the conditional expectation of \cdot with respect to an equivalent martingale measure Q^* . Since the measure is not unique, we may regard the actual processes of $\{L_n(y)\}$ and $\{r_n\}$ as the processes under an equivalent measure. The total payoff of the insurance company is thus valued as

$$(10.5) \quad V(0, N : y) = \sum_{n=1}^{N+1} u_n(y) .$$

Then assuming that $\{L_n(y)\}$ and $\{r_n\}$ are independent and imposing the fair value condition

$$V(0, N : y) = 0,$$

we obtain a fair value premium: $x = C_1 / C_2$ with

$$\begin{aligned} C_1 &= A \sum_{n=1}^{N+1} D(0, n) Q^*(J(y) = n - 1) \\ C_2 &= \sum_{n=1}^{N+1} D(0, n) Q^*(J(y) > n) \end{aligned}$$

where $D(0, n) = E_0^*[d(0, n)]$, the discount factor at n . In the continuous time setting, it is often the case that the default probability is expressed in terms of hazard rates via the Doob-Meyer's theorem. In the discrete time approach, we obtain the following approximation via the Doob's Theorem

$$Q(J(y) > n) \approx E_0 \left[\exp \left(- \sum_{j=0}^{n-1} q_j(y)h \right) \right],$$

where $q_j(y)$ is the discrete time hazard rate at j . For the hazard rate process, we may assume such a model as

$$\Delta q_j(y) = \mu(y)h + \sigma(y)\sqrt{h}\varepsilon_j$$

with $\varepsilon_j \sim \text{iid } N(0,1)$. The argument can be extended to a valuation problem for a person of age y with additional risk factor such as smoking.

Appendix: Discrete time no-arbitrage theory

Here the discrete time version of the no-arbitrage theory we used in sections 5 and 6 is formulated in a general form and we give a sufficient condition for no-arbitrage that the processes of relative prices form Martingales with respect to an equivalent measure. This result is known in a continuous-time setting, which requires a set of regularity conditions to define stochastic integrals and differentials, i.e., it requires a semi-Martingale framework. Our approach does not require the specification of the processes of asset prices and hence it is free from the set of regularity conditions on the stochastic integrals and differentials and Markovness of price processes.

Let X_{in} be the price at n of the i -th asset ($i = 0, 1, \dots, M$), where $\vec{X}_n = (X_{0n}, \dots, X_{Mn})$ follows a stochastic process, and asset 0 is regarded as a standardizing asset with $X_{0n} > 0$. The time horizon is set as $n = 0, 1, \dots, N$, where the unit of time is fixed as h year in advance (e.g., $h = 1/365$). Let Q be the probability measure generating $\{\vec{X}_n : n = 1, \dots, N\}$ given \vec{X}_0 where the distribution of \vec{X}_0 is not specified. We make a portfolio of the M assets at each time n ($n = 0, \dots, N-1$) so that the value of the portfolio at n is expressed as

$$(A.1) \quad V_n(\vec{a}_n) = a_{0n}X_{0n} + a_{1n}X_{1n} + \dots + a_{Mn}X_{Mn},$$

where $\vec{a}_n = (a_{0n}, a_{1n}, \dots, a_{Mn})$ is the portfolio, and depends on the past paths of prices $\{\vec{X}_m : m \leq n\}$. The process $\{\vec{a}_n\}$ formed from \vec{a}_n 's is called a trading strategy. At time $n+1$, the value of the portfolio changes due to the change of prices to

$$(A.2) \quad V_{n+1}(\vec{a}_n) = a_{0n}X_{0n+1} + a_{1n+1}X_{1n+1} + \cdots + a_{Mn+1}X_{Mn+1} \ .$$

Definition 1: A trading strategy $\{\vec{a}_n : n = 1, \dots, N-1\}$ is called a self-financing strategy if at each time n

$$V_n(\vec{a}_{n-1}) = V_n(\vec{a}_n) \quad (n = 1, \dots, N) \ .$$

The concept of the self-financing guarantees the legitimacy of the concept of no-arbitrage.

Definition 2: The M assets are of an arbitrage relation if for some self-financing strategy $\{\vec{a}_n\}$,

$$(A.4) \quad V_0(\vec{a}_0) = 0 \quad \text{and} \quad V_N(\vec{a}_{N-1}) > 0 \quad \text{with probability 1.}$$

When we can obtain neither (A.4) for any trading strategy $\{\vec{a}_n\}$, we say that the M assets are of no arbitrage relation.

Theorem: The M assets are of no-arbitrage relation if the processes of relative prices $\tilde{X}_{in} = X_{in} / X_{0n}$ become Martingales under some measure Q^* which is equivalent to the measure Q generating \tilde{X}_n ($n = 1, \dots, N$) with X_0 arbitrarily given;

$$(A.5) \quad E_{n-1}^*(\tilde{X}_{in}) = \tilde{X}_{in-1} \quad (n = 1, \dots, N) \ .$$

Proof: We only treat the case (A.4) as the other case is similar. First note (A.4) is equivalent to

$$(A.6) \quad \tilde{V}_0(\vec{a}_0) = 0 \quad \text{and} \quad \tilde{V}_N(\vec{a}_{N-1}) > 0 \quad \text{with probability 1,}$$

as $X_{0n} > 0$ where

$$(A.7) \quad \tilde{V}_n(\vec{a}_n) = a_{0n}\tilde{X}_{0n} + a_{1n}\tilde{X}_{1n} + \cdots + a_{Mn}\tilde{X}_{Mn} \ .$$

Here $\tilde{X}_{0n} \equiv 1$. Also note that using the self-financing condition $\tilde{V}_n(\vec{a}_{n-1}) = \tilde{V}_n(\vec{a}_n)$,

$$(A.8) \quad \tilde{V}_N(\vec{a}_{N-1}) = \tilde{V}_0(\vec{a}_0) + \sum_{n=1}^N [\tilde{V}_n(\vec{a}_{n-1}) - \tilde{V}_{n-1}(\vec{a}_{n-1})] \ .$$

Hence if $\{\tilde{X}_{in}\}$ is a Martingale for each i under some Q^* ,

$$(A.9) \quad E_0^*[\tilde{V}_N(\vec{a}_{N-1})] = \tilde{V}_0(\vec{a}_0)$$

since the conditional expectation under Q^* of the second term is

$$\begin{aligned} & \sum_{n=1}^N \sum_{i=0}^M E_0^* [a_{in-1} (\tilde{X}_{in} - \tilde{X}_{in-1})] \\ &= \sum_{n=1}^N \sum_{i=0}^M E_0^* [a_{in-1} [E_{n-1}^* (\tilde{X}_{in}) - \tilde{X}_{in-1}]] = 0. \end{aligned}$$

Here (A.9) means that $\tilde{V}_N(\bar{a}_{N-1}) > 0$ with probability 1 under Q^* implies $\tilde{V}_0(\bar{a}_0) > 0$ with probability 1 under Q^* . But since Q and Q^* are equivalent, (A.4) does not hold, and hence no-arbitrage relation among the M assets obtains. If the prices themselves of the M assets are all Martingales, they are of no arbitrage relation. In fact, the same proof holds for price processes themselves. But such a price as bank deposit $X_{0n} = \exp(rnh)$ where r is a fixed interest rate is not a Martingale under any equivalent measure, though relative prices may become a Martingale under some measure as we will see next.

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